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INTERNATIONAL SECONDARY CERTIFICATE EXAMINATION
NOVEMBER 2022

MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours

150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A**QUESTION 1**

$$\begin{aligned}
 \text{(a)} \quad RQ^2 &= (-5 - 10)^2 + (9 - 0)^2 \\
 &= 225 + 81 \\
 &= 306 \\
 RQ &= \sqrt{306} \quad \text{or} \quad 17,5 \quad \text{or} \quad 3\sqrt{34}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad M_{QR} &= \frac{9 - 0}{-5 - 10} \\
 &= \frac{9}{-15} \\
 &= \frac{-3}{5} \\
 y - 0 &= \frac{-3}{5}(x - 10) \quad \text{or} \quad y - 9 = \frac{-3}{5}(x + 5) \\
 y &= \frac{-3}{5}x + 6
 \end{aligned}$$

$$\text{(c)} \quad S(0,6)$$

$$\begin{aligned}
 \text{(d)} \quad P(-8,0) \quad S(0,6) \quad M(-12,k) \\
 M_{PS} &= \frac{6 - 0}{0 + 8} \\
 &= \frac{6}{8} \\
 &= \frac{3}{4} \\
 M_{SM} &= \frac{k - 6}{-12 - 0} \\
 \frac{3}{4} &= \frac{k - 6}{-12} \\
 -36 &= 4k - 24 \\
 -12 &= 4k \\
 k &= -3
 \end{aligned}$$

$$\begin{aligned}
 \text{(e)} \quad &= \frac{\cancel{\frac{1}{2}} \times \cancel{18} \times 9}{\cancel{\frac{1}{2}} \times 10 \times 6} \\
 &= \frac{27}{10} \\
 &= 2,7
 \end{aligned}$$

QUESTION 2

(a) (1) $x = 60$
 $y = 74$

(2) Skewed left

(b) (1) $n(s) = 40$ $n(g) = x$ $n(y) = 20$ $n(b) = 20 - x$

$$\frac{20 - x}{40} = \frac{1}{8} \quad \text{or} \quad 40 \times \frac{1}{8} = 5$$

$$160 - 8x = 40 \quad \therefore 5 \text{ blue}$$

$$120 = 8x$$

$$x = 15 \quad 40 - (20 + 5) = x$$

(2) $P(B) = \frac{15}{60}$
 $= \frac{1}{4}$

(3) $n(s) = 100$ $n(y) = 20 + 30$

$$p(y) = \frac{50}{100}$$

$$= \frac{1}{2}$$

(c) (1) $x = 13$ or they could write $x \geq 13$

$$x = 20$$

(2) $10 + 12 + 7 + 20 + 3 + 2$
 $= 54$

Thus, we need between the 26th and 27th piece of data
 So median is 2

(3) $10 + 12 + 7 + x + 3 + 2 = 50$
 $34 + x = 50$
 $x = 16$
 $n = 50$

QUESTION 3(a) Constr : join OD and OB

$$\hat{O}_1 = 2\hat{A} \quad (\angle \text{ at centre} = 2\angle \text{ at circle})$$

$$\hat{O}_2 = 2\hat{C} \quad (\angle \text{ at centre} = 2\angle \text{ at circle})$$

$$\hat{O}_1 + \hat{O}_2 = 360^\circ \quad (\angle \text{'s around a point})$$

$$\therefore 2\hat{A} + 2\hat{C} = 360^\circ$$

$$\hat{A} + \hat{C} = 180^\circ$$

$$(b) \quad (1) \quad \hat{BCD} = 180^\circ - (28^\circ + 36^\circ) \quad \text{opp } \angle \text{ of cyclic quad} \\ = 116^\circ$$

$$(2) \quad \hat{B}_1 + \hat{B}_2 = 90^\circ$$

rad $\perp$ tan

$$\hat{O}_1 = 72^\circ$$

 $\angle$ at centre = 2 $\angle$ at circle

$$\hat{E} = 180^\circ - (90^\circ + 72^\circ)$$

 $\angle$'s in a Δ

$$\hat{E} = 18^\circ$$

$$(3) \quad \hat{DBC} = 28^\circ$$

($\angle$'s in same segment)

$$(4) \quad C_3 = 180^\circ - (18^\circ + 36^\circ)$$

($\angle$'s in a Δ)

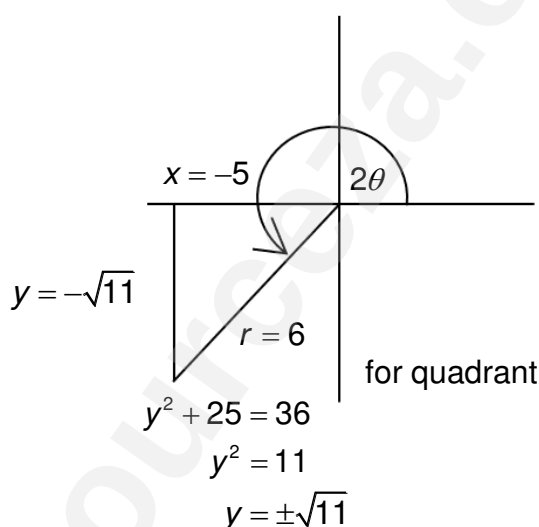
$$C_3 = 126^\circ$$

QUESTION 4

$$\begin{aligned}
 \text{(a)} \quad &= \frac{(-\sin x)(\tan x)^2(\cos x)}{(-\tan x)(\sin x)(\sin x)} \\
 &= \frac{\sin x}{\cos x} \times \frac{\cos x}{1} \quad \text{for changing tan} \\
 &= 1
 \end{aligned}$$

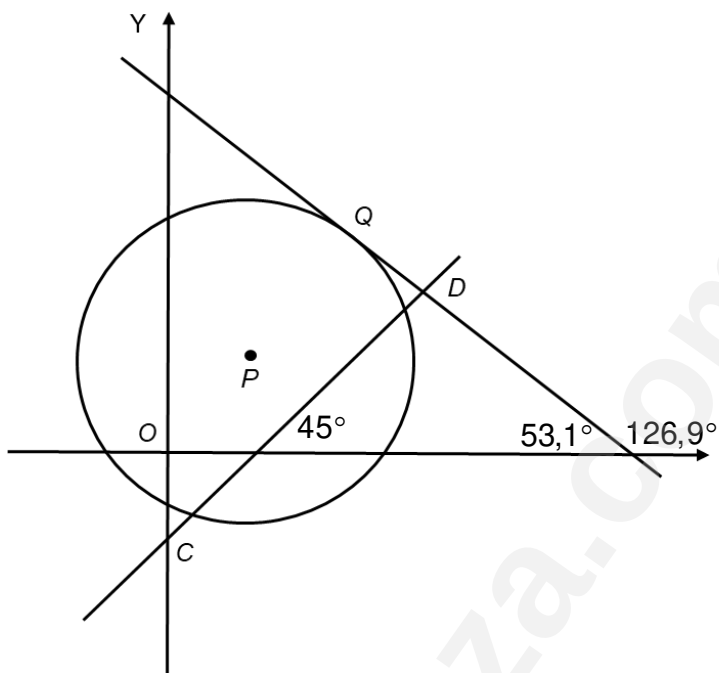
$$\begin{aligned}
 \text{(b)} \quad \text{LHS} &= \cos^2 \theta \left(1 - \frac{\sin^2 \theta}{\cos^2 \theta}\right) \quad \text{distribution} \\
 &= \cos^2 \theta - \sin^2 \theta \\
 &= \cos 2\theta \\
 &= \text{RHS}
 \end{aligned}$$

$$\text{(c)} \quad (1) \quad \sin 2\theta = \frac{-\sqrt{11}}{6}$$



$$\begin{aligned}
 (2) \quad \cos 2\theta &= 1 - 2\sin^2 \theta & \text{or} & \quad \cos 2\theta = 2\cos^2 \theta - 1 \\
 -\frac{5}{6} &= 1 - 2\sin^2 \theta & & \quad -\frac{5}{6} = 2\cos^2 \theta - 1 \\
 2\sin^2 \theta &= 1 + \frac{5}{6} & & \quad 2\cos^2 \theta = 1 - \frac{5}{6} \\
 2\sin^2 \theta &= \frac{11}{6} & & \quad 2\cos^2 \theta = \frac{1}{6} \\
 \sin^2 \theta &= \frac{11}{12} & & \quad 2(1 - \sin^2 \theta) = \frac{1}{6} \\
 & & & \quad 2 - 2\sin^2 \theta = \frac{1}{6} \\
 & & & \quad 2 - \frac{1}{6} = 2\sin^2 \theta \\
 & & & \quad \frac{11}{6} = 2\sin^2 \theta \\
 & & & \quad \sin^2 \theta = \frac{11}{12}
 \end{aligned}$$

QUESTION 5



(a) $x^2 - 4x + 4 + y^2 - 6y + 9 = 12 + 4 + 9$
 $(x - 2)^2 + (y - 3)^2 = 25$
 centre $(2 ; 3)$ $r = 5$

(b) LHS $= (7 - 2)^2 + (4 - 3)^2$
 $= 25 + 1$
 $= 26$

LHS > RHS $\therefore$ outside circle to fit appropriate calculation

(c) $t^2 + t^2 = 4t + 6t + 12$
 $2t^2 - 10t - 12 = 0$
 $t^2 - 5t - 6 = 0$
 $(t + 1)(t - 6) = 0$

$t = -1$ or $t = 6$
 N/A

$$M_{CD} = 1$$

$$\tan \theta = 1$$

$$\theta_1 = 45^\circ$$

$$\hat{QDC} = 53,1^\circ + 45^\circ$$

$$= 98,1^\circ \text{ ext } \angle \text{ of triangle}$$

OR

Draw in parallel line to x-axis at D

$$M_{CD} = 1$$

$$\tan \theta = 1$$

$$\theta_1 = 45^\circ$$

$$\hat{QDC} = 53,1^\circ + 45^\circ$$

$$= 98,1^\circ$$

(2,3)	(6,6)
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$$M_{rad} = \frac{6-3}{6-2} = \frac{3}{4}$$

$$M_{tan} = -\frac{4}{3}$$

$$\tan \theta_2 = -\frac{4}{3}$$

$$\theta_2 = 126,9^\circ$$

$$M_{rad} = \frac{6-3}{6-2} = \frac{3}{4}$$

$$M_{tan} = -\frac{4}{3}$$

$$\tan \theta = \frac{4}{3}$$

$$\theta = 53,1^\circ$$

QUESTION 6

$$(a) \quad \frac{\text{area of base Box B}}{\text{area of base Box A}} = \frac{3\,375}{1\,500} = \frac{9}{4}$$

$$k^2 = \frac{9}{4}$$

$$\therefore k = \frac{3}{2}$$

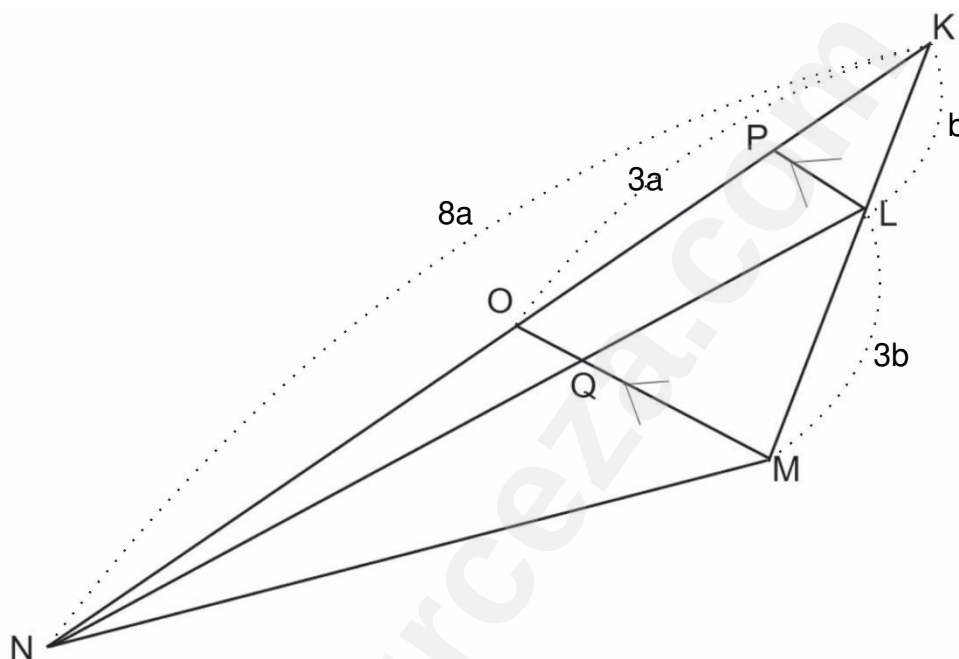
$$(b) \quad A \times \frac{3}{2} = 99$$

$$A = \frac{2}{3} \times 99$$
$$= 66 \text{ cm}$$

SECTION B

QUESTION 7

- (a) 'The line drawn parallel to one side of a triangle divides **the other two sides proportionally**'
- (b)



(1) In $\triangle KOM$ $\frac{PO}{OK} = \frac{LM}{KM}$ (line $\parallel$ to one side of $\triangle$)

$$\frac{PO}{3a} = \frac{3b}{4b}$$

$$\therefore PO = \frac{9}{4}a$$

In $\triangle NPL$ $\frac{LQ}{QN} = \frac{PO}{ON}$ (line $\parallel$ to one side of $\triangle$)

$$\therefore \frac{LQ}{QN} = \frac{\frac{9}{4}}{5}$$

$$= \frac{9}{20}$$

$$(2) \quad \frac{\frac{1}{2} \cdot 3a \cdot MO \cdot \sin \hat{KOM}}{\frac{1}{2} \cdot 5a \cdot MO \cdot \sin \hat{MON}}$$

Since $\sin \hat{KOM} = \sin \hat{MON}$

$$= \frac{3}{5}$$

OR

Drop a perpendicular height from M to NK. Call it h

$$\begin{aligned} & \frac{\frac{1}{2} \cdot h \cdot KO}{\frac{1}{2} \cdot h \cdot NO} \\ &= \frac{3}{5} \end{aligned}$$

QUESTION 8

(a) $\cos x \cos 60^\circ - \sin x \sin 60^\circ + 2 \sin x = 0 \quad = \frac{3}{5}$

$$\frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x + 2 \sin x = 0$$

$$\cos x = \sqrt{3} \sin x - 4 \sin x$$

$$\cos x = \sin x(\sqrt{3} - 4)$$

$$\tan x = \frac{1}{\sqrt{3} - 4}$$

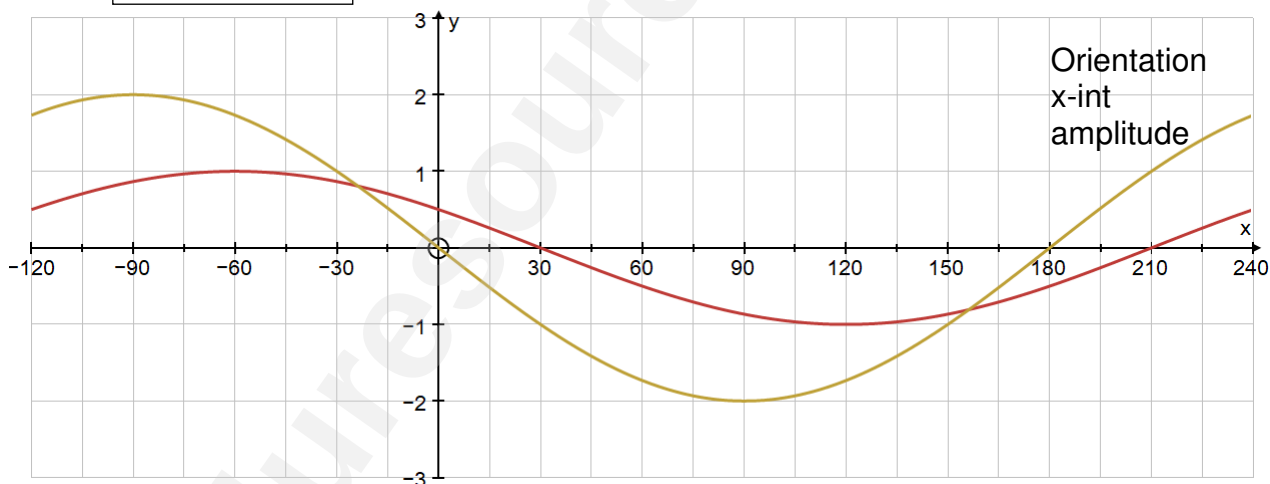
(b) $\tan x = \frac{1}{\sqrt{3} - 4}$

$$x = -23,8^\circ + k180^\circ$$

$$x = -23,8^\circ \quad \text{or} \quad x = 156,2^\circ$$

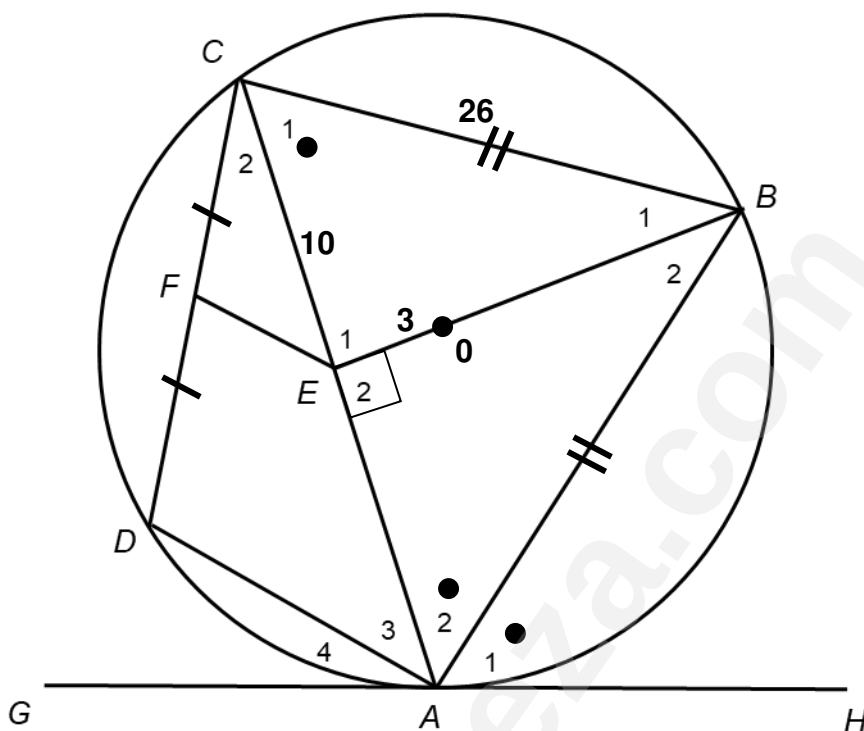
(c)

$$f(x) = -2 \sin x$$



(d) $x \in [-23,8^\circ, 156,2^\circ]$

QUESTION 9



- (a) $\hat{A}_1 = \hat{C}_1$ tan chd Th
 $\hat{C}_1 = \hat{A}_2$ $\angle$'s opp = sides
 $\therefore \hat{A}_1 = \hat{A}_2$
 $\therefore AB$ bisects $\hat{CAH}$
- (b) $CE = EA$ (line from centre $\perp$ to chd)
 $\therefore \frac{CF}{FD} = \frac{CE}{EA}$ (prop sides) or (Midpoint Theorem)
 $\therefore FE \parallel DA$
- (c) $CE = EA = 10$ (proven above)
 let $r = OB$
 $\therefore 26^2 = 10^2 + BE^2$ (implied use of Pyth Th)
 $576 = BE^2$
 $BE = -24$ or $BE = 24$
 N/A $r = 21$
 $\therefore$ radius is 21 units

OR

$$CE = EA = 10 \quad (\text{proven above})$$

Let $OB = r$ Thus $EB = r + 3$

In triangle CEB

$$10^2 + (r + 3)^2 = 26^2$$

$$100 + (r + 3)^2 = 576$$

$$(r + 3)^2 = 476$$

$$r + 3 = 21,8$$

$$r = 18,8$$

OR

$$CE = EA = 10 \quad (\text{proven above})$$

OC = r

In triangle CEO

$$r^2 = 3^2 + 10^2$$

$$r^2 = 9 + 100$$

$$r^2 = 109$$

$$r = 10,4$$

QUESTION 10

$$43\,700\text{ cm}^3$$

$$\begin{aligned} V_{\text{sphere}} &= \frac{1}{2} \cdot \frac{4}{3} \pi (23)^3 \\ &= 25\,482,51\text{ cm}^3 \end{aligned}$$

$$\begin{aligned} V_{\text{cylinder}} &= 43\,700 - 25\,482 \\ &= 18\,217,5 \end{aligned}$$

$$\pi r^2 h = 18\,217,5$$

$$\begin{aligned} h &= \frac{18\,217,5}{\pi (23)^2} \\ h &= 10,96\text{ cm} \end{aligned}$$

OR

$$V = \frac{2}{3} \pi r^3 = \pi r^2 h$$

$$43\,700 = \frac{2}{3} \pi (23)^3 = \pi (23)^2 h$$

$$h = \frac{43\,700 - \frac{2}{3} \pi (23)^3}{\pi (23)^2}$$

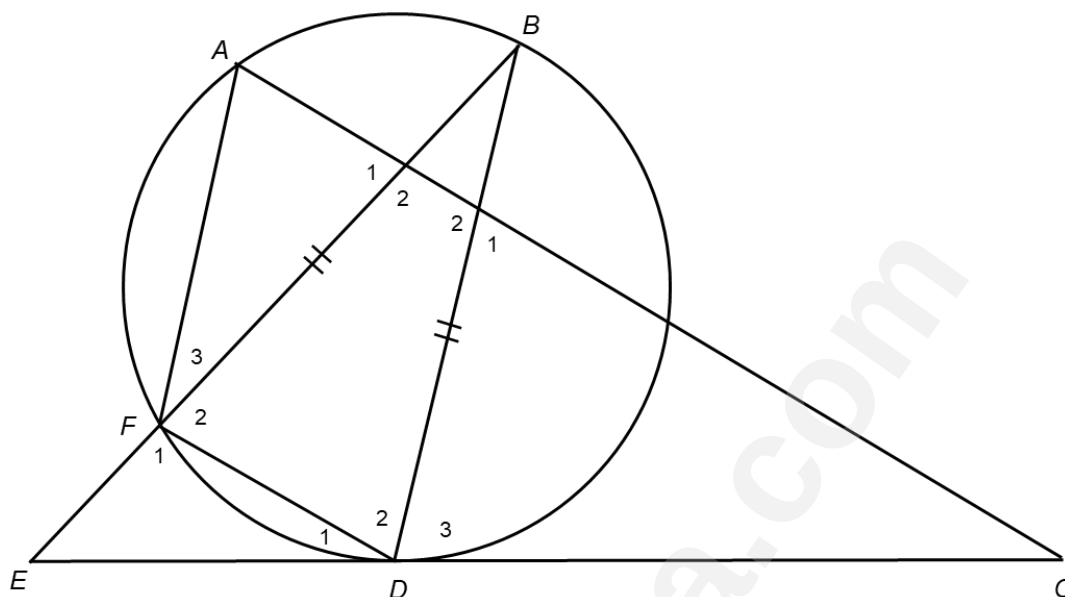
$$h = 10,96$$

QUESTION 11

- (a) (1) $p(A \cap B) = 0$
 $\therefore p(A) + p(B) = p(A \text{ or } B)$
 $0,4 + x = 0,9$
 $x = 0,5$
- (2) $p(A) \cdot p(B) = p(A \cap B)$
 $p(A \text{ or } B) = p(A) + p(B) - p(A \text{ and } B)$
 $0,9 = 0,4 + x - 0,4x$ (for 0,4x) (for substitution)
- $$0,5 = 0,6x$$
- $$x = \frac{5}{6}$$
- (b) (1) $= 13 \times 12 \times 11 \times 10 \times 9$ OR ${}^{13}P_5$
 $= 154\,440$
- (2) $n(\text{no pupils}) = 7 \times 6 \times 5 \times 4 \times 3$ or ${}^7P_5$
 $= 2\,520$
- $n(\text{at least one pupil}) = 154\,440 - 2\,520$
 $= 151\,920$
- (3) $n(\text{all teachers}) = 10 \times 9 \times 3 \times 2 \times 1$
 $= 540$
- (c) (1) $\bar{x} = 70,5$
 $\sigma = 14,8$ If all correct.
- (2) $70,5 - 14,8 < x < 70,5 + 14,8$
 $55,7 < x < 85,3$
- 8 pupils lie within one standard deviation
- (3) $A = 16,4$ $B = 0,8$
 $y = 16,4 + 0,8x$
 $r = 0,932$
- (4) $y = 16,4 + 0,8(55)$
 $= 60,4$

Since the correlation is a very strong positive one this is a realistic expectation.

QUESTION 12



(a) In $\triangle EFD$ and $\triangle EDB$

(1) $\hat{E} = \hat{E}$ shared

$$(2) \quad \hat{D}_1 = \hat{B} \quad \text{tan chd}$$

$$(3) \quad \hat{F}_1 = \hat{D}_1 + D_2 \quad 3^{\text{rd}} \angle \text{ of } \Delta$$

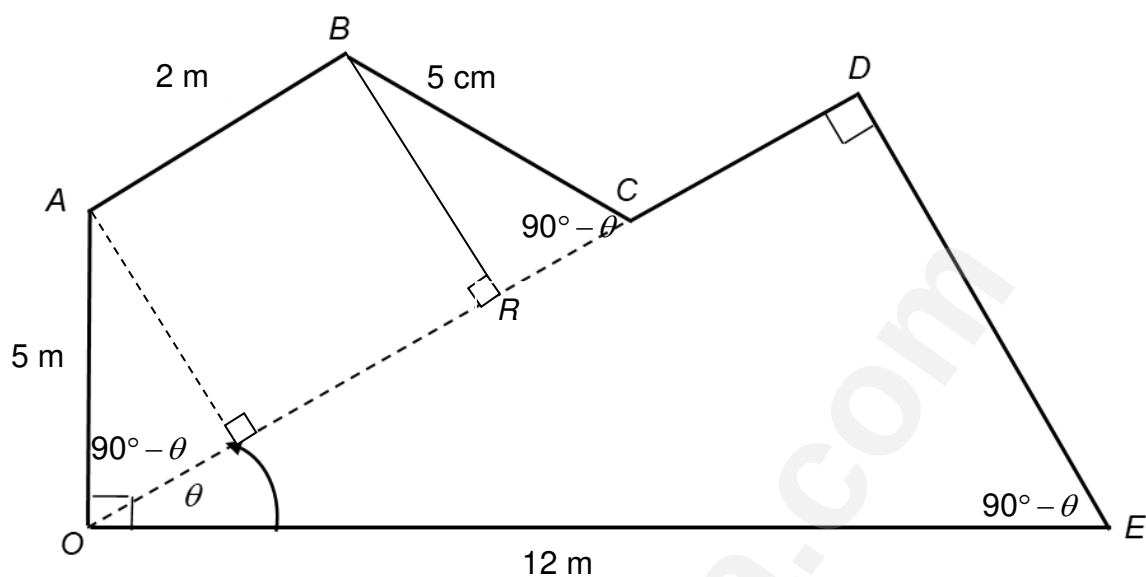
$$\therefore \triangle EDF \cong \triangle EBD \quad (\text{AAA})$$

(b) from (a) $\frac{ED}{EB} = \frac{EF}{ED} = \frac{EF}{BD}$

$$\begin{aligned} ED^2 &= EF \times EB \\ &= EF(EF + FB) \\ &= EF^2 + EF \times FB \end{aligned}$$

but $FB = BD$ given
 $\therefore ED^2 - EF^2 = EF \times BD$

QUESTION 13



(a) $\frac{RC}{5} = \cos(90^\circ - \theta)$

$RC = 5 \sin \theta$

$\therefore OC = 2(5 \sin \theta) + 2 = 10 \sin \theta + 2$

(b) $\frac{DE}{12} = \sin \theta$

$DE = 12 \sin \theta$

$\frac{DO}{12} = \cos \theta$

$DO = 12 \cos \theta$

$S = 5 + 2 + 5 + 12 + 12 \sin \theta + (12 \cos \theta - (10 \sin \theta + 2))$

$S = 24 + 12 \sin \theta + 12 \cos \theta - 10 \sin \theta - 2$

$S = 22 + 2 \sin \theta + 12 \cos \theta$

Total: 150 marks