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INTERNASIONALE SEKONDÊRE SERTIFIKAAT-EKSAMEN
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VERDERE STUDIES WISKUNDE (UITGEBREID): VRAESTEL II

NASIENRIGLYNE

Tyd: 1 uur

100 punte

Hierdie nasienriglyne is opgestel vir gebruik deur eksaminators en hulpeksaminators van wie verwag word om almal 'n standaardiseringsvergadering by te woon om te verseker dat die riglyne konsekwent vertolk en toegepas word by die nasien van kandidate se skrifte.

Die IEB sal geen bespreking of korrespondensie oor enige nasienriglyne voer nie. Ons erken dat daar verskillende standpunte oor sommige aangeleenthede van beklemtoning of detail in die riglyne kan wees. Ons erken ook dat daar sonder die voordeel van die bywoning van 'n standaardiseringsvergadering verskillende vertolkings van die toepassing van die nasienriglyne kan wees.

MODULE 2 STATISTIEK**VRAAG 1**

1.1 (a) 0,5

(b) $X \sim N(12,5; \sigma^2)$

(1) $P(X > 14) = 0,06$

$$1,55 = \frac{14 - 12,5}{\sigma}$$

$$\sigma = 0,97$$

of

$$1,5548 = \frac{14 - 12,5}{\sigma}$$

$$(\text{of } 0,96)$$

(2) $P(X > 12) = P\left(Z > \frac{12 - 12,5}{0,97}\right)$

$$= P(Z > -0,52)$$

$$= 0,5 + 0,1985$$

$$= 0,6985$$

of

$$= 0,6969$$

$$\begin{aligned}
 (3) \quad P(X < 12,5 | X > 12) &= \frac{P(12 < X < 12,5)}{P(X > 12)} \\
 &= \frac{0,6985 - 0,5}{0,6985} \\
 &= 0,2842
 \end{aligned}$$

1.2 (a) 'n 94%-vertrouensinterval vir μ is

$$331,25 \pm 1,88 \left(\frac{3,3}{5} \right)$$

$$(330,01; 332,49)$$

(b) 330 ml is nie in die interval nie, gevolglik kan die brouer tot die gevolgtrekking kom dat die bierbottels te vol gemaak word. OF Die interval is meer as 330 ml, gevolglik kan die brouer vertroue hê dat hy aan die wet voldoen en verseker dat die klante minstens 330 ml kry.

(c) Die wydte van die vertrouensinterval sal afneem.

VRAAG 2

2.1 (a) $(0,5)(0,8) = 0,4$

(b) $X \sim B(5; 0,4)$

$$P(X \leq 1) = \binom{5}{0}(0,4)^0(0,6)^5 + \binom{5}{1}(0,4)^1(0,6)^4 \\ = 0,337$$

2.2 (a) $E[X] = 200(0,6) = 120$

$$\sigma_x = \sqrt{200(0,6)(0,4)} = 4\sqrt{3}$$

(b) $X \sim B(200; 0,6)$ ongeveer $X \sim N(120; \sqrt{48}^2)$

$$P(111 < X < 130) \rightarrow P(111,5 < X < 129,5) \\ = P\left(\frac{111,5 - 120}{\sqrt{48}} < Z < \frac{129,5 - 120}{\sqrt{48}}\right) \\ = P(-1,23 < Z < 1,37) \\ = 0,3907 + 0,4147 \\ = 0,8054$$

(c) Dit is 'n geldige benadering, aangesien beide $np = 200(0,6) > 5$ en $nq = 200(0,4) > 5$.

VRAAG 3

$$3.1 \quad (a) \quad 3k + \frac{1}{3} = 1$$

$$3k = \frac{2}{3}$$

$$k = \frac{2}{9}$$

$$(b) \quad E[X] = 1\left(\frac{1}{4}\right) + 2\left(\frac{2}{9}\right) + 3\left(\frac{4}{9}\right) = 2,0278$$

$$(c) \quad P(X=2 \cap X=3) \times 2 + P(X=3 \cap X=3)$$

$$= \left(\frac{2}{9}\right)\left(\frac{4}{9}\right) \times 2 + \left(\frac{4}{9}\right)^2$$

$$= \frac{32}{81}$$

$$3.2 \quad (0,1;4) \quad (0,5;0)$$

$$m = \frac{4-0}{0,1-0,5} = -10$$

$$y = -10(x-0,5)$$

$$= -10x + 5$$

$$\therefore \int_m^{0,5} -10x + 5 \, dx = 0,5$$

$$-5x^2 + 5x \Big|_m^{0,5} = 0,5$$

$$\frac{5}{4} - (-5m^2 + 5m) = \frac{1}{2}$$

$$5m^2 - 5m + 0,75 = 0$$

$$m \neq 0,82 \quad \text{of} \quad m = 0,1838$$

VRAAG 4

$$4.1 \quad -2,05 < \frac{q-50}{\frac{10}{6}} < 2,05$$

$$-3,42 < q - 50 < 3,42$$

$$46,6 < q < 53,4$$

$$4.2 \quad -2,05 < \frac{48-m}{\frac{10}{6}} < 2,05 \quad \text{of} \quad -3,42 < 48 - m < 3,42$$

$$44,6 < m < 51,4$$

VRAAG 5

$$5.1 \quad \frac{\binom{3}{2}\binom{4}{1} + \binom{3}{3}\binom{4}{0} + \binom{2}{2}\binom{5}{1} \times 2}{\binom{7}{3}} = \frac{23}{35}$$

$$5.2 \quad (a) \quad \frac{5 \times 2 \times 8!}{10!} = \frac{1}{9}$$

$$(b) \quad \begin{matrix} \textcircled{12} & \textcircled{23} & \textcircled{34} & \textcircled{45} \\ \textcircled{67} & \textcircled{78} & \textcircled{89} & \textcircled{910} \end{matrix}$$

Saam is $4 \times 2 \times 2 \times 8!$

Dus is afsonderlik $10! - 16(8!) = 2\,983\,680$

Totaal vir Module 2: 100 punte

MODULE 3 FINANSIES EN MODELLERING**VRAAG 1**

$$(a) \quad 6500000 \left(1 + \frac{0,125}{12}\right)^8 = \frac{x \left[1 - \left(1 + \frac{0,125}{12}\right)^{-292}\right]}{\frac{0,125}{12}}$$

$$x = R77\,311,31$$

$$(b) \quad \text{Uitstaande saldo} = \frac{77\,311,31 \left[1 - \left(1 + \frac{0,125}{12}\right)^{-252}\right]}{\frac{0,125}{12}}$$

$$= R6\,876\,896,48$$

$$(c) \quad 3\,876\,896,49 = \frac{x \left[1 - \left(1 + \frac{0,125}{12}\right)^{-192}\right]}{\frac{0,125}{12}}$$

$$= R46\,781,34$$

$$(d) \quad \Delta \text{Uitstaande saldo} = 7\,061\,831,755 - 6\,876\,896,48$$

$$= 184\,935,28$$

$$\text{Rente} = 40 \times 77\,311,31 - 184\,935,28$$

$$= 2\,907\,517,12$$

VRAAG 2

$$(a) \quad 0,15 = \left(1 + \frac{r}{12}\right)^{12} - 1$$

$$r = 0,1406$$

$$F = \frac{15000 \left[\left(1 + \frac{0,1406}{12}\right)^{240} - 1 \right]}{\frac{0,1406}{12}} \times \left(1 + \frac{0,1406}{12}\right) + \frac{15000 \left[(1 + 0,15)^{20} - 1 \right]}{0,15} \times (1 + 0,15)$$

$$= R21\,673\,253,41$$

$$(b) \quad (1) \quad 21673253 = \frac{700000 \left[1 - \left(1 + \frac{0,12}{4}\right)^{-n} \right]}{\frac{0,12}{4}}$$

$$n = 89,41 \dots$$

d.w.s. 90 betalings

(2) Uitstaande saldo na 89:

$$= 21673253 \left(1 + \frac{0,12}{4}\right)^{89} - \frac{700000 \left[\left(1 + \frac{0,12}{4}\right)^{89} - 1 \right]}{\frac{0,12}{4}}$$

$$= R284\,808,92$$

$$FB = 284808,92 \left(1 + \frac{0,12}{4}\right)$$

$$= R293\,414,99$$

VRAAG 3

(a) Toekomstige annuïteit

$$(b) \quad 40\,200,00 = a(20\,000) + b$$

$$60\,602,00 = a(40\,200) + b$$

$$\therefore 20\,200a = 20\,402$$

$$\therefore a = 1,01 \text{ en } b = 20\,000$$

$$F_4 = 81\,208,01 \times 1,01 + 20\,000$$

$$= R102\,020,09$$

$$(c) \quad 0,01 \times 12 = 0,12$$

$$\therefore r_{eff} = \left(1 + \frac{0,12}{12}\right)^{12} - 1$$

$$= 12,68\%$$

VRAAG 4

$$(a) \quad r = b - d$$

$$8,5 = 0,4 \times x \times 50 - 1$$

$$x = 0,475$$

$$(b) \quad (1) \quad P_{n+1} = P_n$$

$$\therefore P_4 = P_4(9,5) - x$$

$$\therefore x = 8,5 \times 3258025$$

$$\therefore x = 27693213$$

$$(2) \quad \frac{3258025}{9,5^4} = 400$$

VRAAG 5

(a) $\text{Roofdier} = 20; \text{Prooi} = 100$

(b) $22 - 13 = 9$

(c) $\frac{1}{0,05} = 20 \text{ siklusse}$

(d)
$$R_E = \frac{c}{fb}$$
$$= \frac{0,05}{0,0008}$$
$$\approx 63$$

(e)
$$F_E = \frac{a}{b} \left(1 - \frac{c}{fbK} \right)$$
$$\therefore 12 = \frac{0,15}{b} \left(1 - \frac{0,05}{0,0008 \times 30} \right)$$
$$\therefore b = 0,009895$$
$$\therefore bF_0R_0 \approx 20$$

OF

$$F_E = \frac{a}{b} \left(1 - \frac{R_E}{K} \right)$$
$$\therefore 12 = \frac{0,15}{b} \left(1 - \frac{63}{300} \right)$$
$$\therefore b = 0,009875$$

VRAAG 6

(a)

$$K = 180$$

$$50 = \frac{180}{1+P}$$

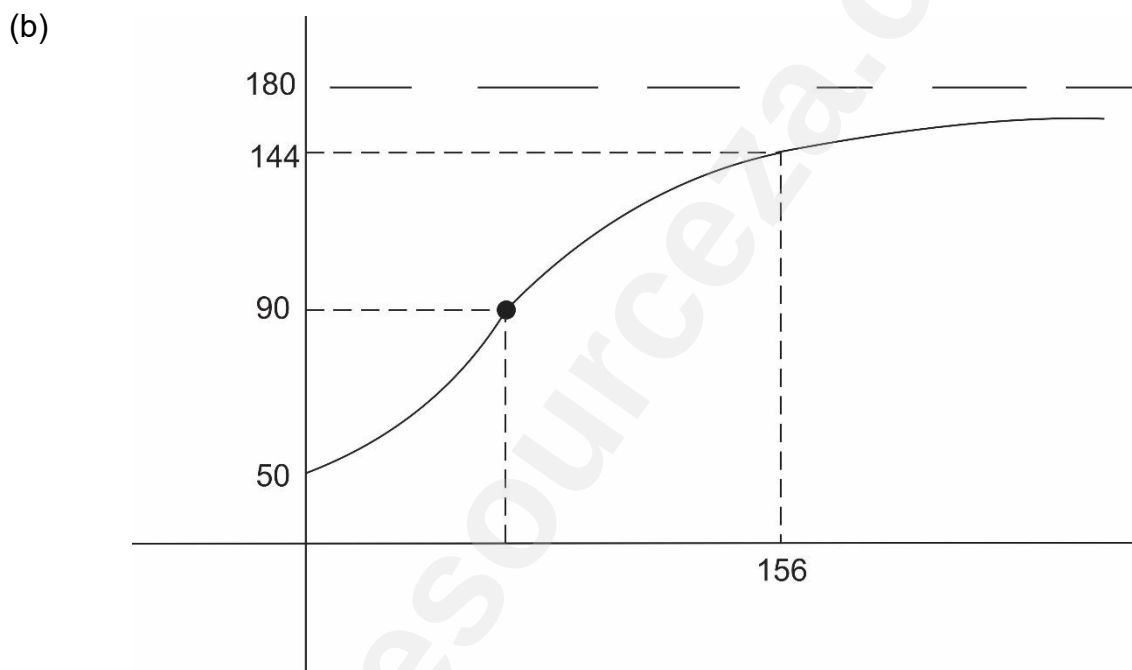
$$\therefore 1+P = \frac{180}{50}$$

$$\therefore P = 2,6$$

$$\therefore 144 = \frac{180}{1+2,6e^{-156Q}}$$

$$\therefore e^{-156Q} = 0,09615\dots$$

$$Q = 0,015$$



(c)

$$90 = \frac{180}{1+2,6e^{-0,015t}}$$

$$\therefore t \approx 64 \text{ maande}$$

Totaal vir Module 3: 100 punte

MODULE 4 MATRIKSE EN GRAFIEKTEORIE**VRAAG 1**

1.1 (a) C vierkantige matriks

(b) $B = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

(c) C nodusmatriks

1.2 (a) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 2 & 6 & 4 \\ 2 & 1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} -1 & 2 & 6 & 4 \\ -2 & -1 & -2 & -3 \end{pmatrix}$
matriks
volgorde

(b) tekens
matriks
volgorde

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 2 & 6 & 4 \\ 2 & 1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} -2 & -1 & -2 & -3 \\ 1 & -2 & -6 & -4 \end{pmatrix}$$

(c) $\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 2 & 6 & 4 \\ 2 & 1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} -3 & 6 & 18 & 12 \\ 2 & 1 & 2 & 3 \end{pmatrix}$
volgorde
matriks

1.3 volgorde matriks

$$\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} 0 & 3 & 7 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0,574 & 2,457 & 6,308 \\ 0,819 & -1,721 & -3,196 \end{pmatrix}$$

1. $-\sin \theta = 0,574$ enige 2 vergelykings

2. $3 \cos \theta = 2,457$

3. $7 \cos \theta - \sin \theta = 6,308$

4. $\cos \theta = 0,819$

5. $3 \sin \theta = -1,721$

6. $7 \sin \theta + \cos \theta = -3,196$

Los op vir θ Identifiseer $\theta = 35; -35$ Kies $\theta = -35$ of $\theta = 325$

1.4 $A = \begin{pmatrix} 3 & k & 8 \\ -2 & 2 & 8 \\ 1 & 1 & 1 \end{pmatrix}$

teken

$$\det A = 1(8k - 16) - 1(24 + 16) + 1(6 + 2k) = 0$$

$$0 = -50 + 10k \quad \therefore k = 5$$

VRAAG 2

$$2.1 \quad \text{Toegevoegde } A = \begin{bmatrix} -1 & -2 & -1 \\ -38 & -4 & 22 \\ -31 & -2 & 17 \end{bmatrix}$$

$$2.2 \quad \det A = \begin{vmatrix} 2 & -3 & 4 \\ 3 & 4 & -5 \\ 4 & -5 & 6 \end{vmatrix}$$

$$\begin{aligned} \det A &= 2(24 - 25) + 3(18 + 20) + 4(-15 - 16) \\ &= 2(-1) + 3(38) + 4(-31) \\ &= -12 \end{aligned}$$

$$2.3 \quad A^{-1} = -\frac{1}{12} \begin{pmatrix} -1 & -2 & -1 \\ -38 & -4 & 22 \\ -31 & -2 & 17 \end{pmatrix}$$

$$2.4 \quad A^{-1}A = -\frac{1}{12} \begin{pmatrix} -1 & -2 & -1 \\ -38 & -4 & 22 \\ -31 & -2 & 17 \end{pmatrix} \begin{pmatrix} 2 & -3 & 4 \\ 3 & 4 & -5 \\ 4 & -5 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{1}{12} \begin{pmatrix} -1 & -2 & -1 \\ -38 & -4 & 22 \\ -31 & -2 & 17 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{aligned} \therefore x &= 1 \\ y &= 0 \\ z &= 0 \end{aligned}$$

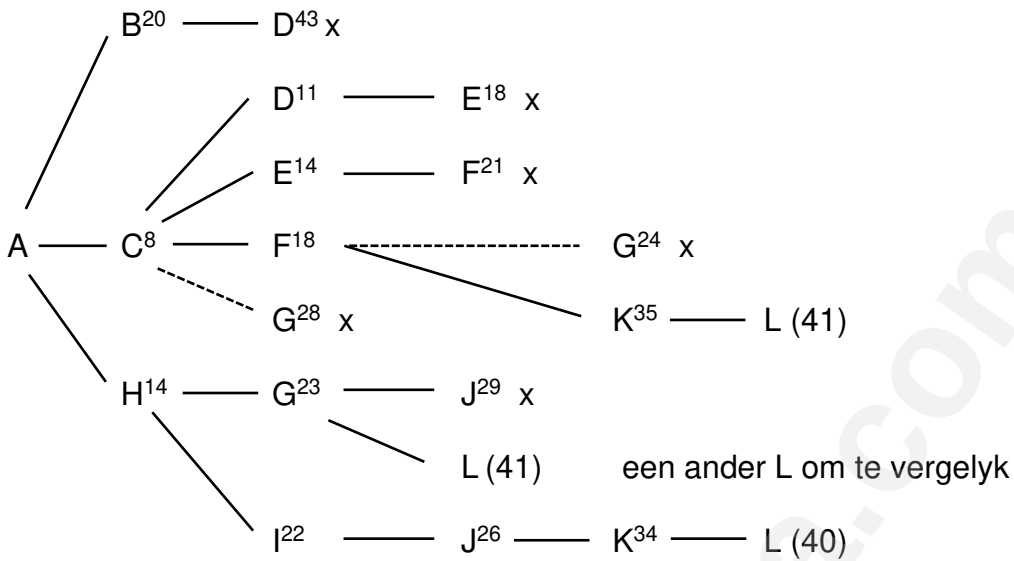
VRAAG 3

3.1 B Alle grafieke moet nodusse hê.

3.2 A hul komplemente isomorfies is

3.3 C dieselfde nodusse as P het en net al die skakels wat nie in P vervat is nie

VRAAG 4



A – H – I – J – K – L (40)

VRAAG 5

12 30 40 53 72 85 99 120 137 154
5.1 A – E – I – H – G – C – D – B – A – F – A

5.2 [Dit moet 'n lys van skakels wees, nie nodusse nie. AH, HI, ens.]

HI	10	
AH	12	
AE	12	
GH	13	
CD	13	
BD	14	
IF	17	(of AF) slegs een van IF en AF
CG	19	

Gewig: 110

5.3 Herhaal nodusse met onewe graad: I; E; H; G; C; D

CD	13
EI	18
GH	13

Gewig van alle skakels: 215

Tel terug by, totaal: 259

VRAAG 6

6.1 $B^T B = I$
 $(B^T B) B^{-1} = I(B^{-1})$ vir B^{-1} vir vermenigvuldiging van regs
 $B^T (B B^{-1}) = B^{-1}$
 $B^T I = B^{-1}$
 $\therefore B^T = B^{-1}$

6.2 $A^T = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$
 $A^T A = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$
 $A^T A = \begin{pmatrix} \cos^2 \theta + \sin^2 \theta & -\cos \theta \sin \theta + \sin \theta \cos \theta \\ -\sin \theta \cos \theta + \cos \theta \sin \theta & \cos^2 \theta + \sin^2 \theta \end{pmatrix}$
 $A^T A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

Totaal vir Module 4: 100 punte