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INTERNATIONAL SECONDARY CERTIFICATE EXAMINATION
MAY 2024

FURTHER STUDIES MATHEMATICS (STANDARD): PAPER I

MARKING GUIDELINES

Time: 2 hours

200 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

QUESTION 1

Prove that $1 \times 2 + 2 \times 3 + 3 \times 4 + \dots + n \times (n + 1) = \frac{n(n + 1)(n + 2)}{3}$ for $n \in \mathbb{N}$.

For $n = 1$

$$L = 1 \times 2 = 2$$

$$R = \frac{(1)(1 + 1)(1 + 2)}{3} = 2$$

$n=1$

Substituting

Conclusion

Therefore, the equation holds true for $n = 1$.

Assume for $n = k$:

$$1 \times 2 + 2 \times 3 + 3 \times 4 + \dots + k \times (k + 1) = \frac{k(k + 1)(k + 2)}{3}$$

for $k \in \mathbb{N}$.

$n=k$

Assumption
statement

$n=k+1$

For $n = k + 1$

$$1 \times 2 + 2 \times 3 + 3 \times 4 + \dots + k(k + 1) + (k + 1) \times (k + 1 + 1)$$

$$= \frac{k(k + 1)(k + 2)}{3} + (k + 1) \times (k + 2)$$

$$= \frac{k(k + 1)(k + 2) + 3(k + 1)(k + 2)}{3}$$

$$= \frac{(k + 2)(k + 1)(k + 3)}{3}$$

$$\text{RHS} = \frac{(k + 1)(k + 1 + 1)(k + 1 + 2)}{3}$$

$$= \frac{(k + 1)(k + 2)(k + 3)}{3}$$

Replacing k
terms

LCD

Factoring the
numerator

Simplifying the
right

Therefore, the equation holds true for $n = k + 1$.

This means that the equation holds for $n \in \mathbb{N}$, hence

$$1 \times 2 + 2 \times 3 + 3 \times 4 + \dots + n \times (n + 1) = \frac{n(n + 1)(n + 2)}{3}$$

Conclusion for

$n=k+1$

Conclusion of

PMI

QUESTION 2(a) Solve for $x \in \mathbb{C}$:

(1) $(x^2 + 1)(e^x - 1) = 0$

$$x^2 = -1$$

$$x = \pm i$$

two options

A Answer

OR

$$e^x = 1$$

$$x = 0$$

A Answer

(2) $|x + 2| - 2x = 1$

$$|x + 2| = 2x + 1$$

$$x + 2 \geq 0 : x + 2 = 2x + 1$$

$$x \geq -2 : 1 = x$$

Making the abs
value subject
domains (or
inferred)

First option

CA first x-value

second half of sum

CA solving x

$$x + 2 < 0 : -(x + 2) = 2x + 1$$

$$x < -2 : -x - 2 = 2x + 1$$

$$-3 = 3x$$

$$-1 \neq x$$

(b) Given: $m - 8m^{\frac{1}{2}} = 9$

(1) Solve the equation for $m \in \mathbb{R}$.

$$m - 8m^{\frac{1}{2}} - 9 = 0$$

$$\left(m^{\frac{1}{2}} + 1\right)\left(m^{\frac{1}{2}} - 9\right) = 0$$

$$m^{\frac{1}{2}} \neq -1 \text{ OR } m^{\frac{1}{2}} = 9$$

$$m = 81$$

Alternatively:

$$-8\sqrt{m} = 9 - m$$

$$64m = (9 - m)^2$$

$$64m = 81 - 18m + m^2$$

$$0 = 81 - 82m + m^2$$

$$0 = (81 - m)(1 - m)$$

$$m = 81 \text{ OR } m \neq -1$$

using $m^{\frac{1}{2}}$

two factors
zero-product rule
CA Answer

Isolating square
root
Squaring both
sides
factorisation
zero-product rule
CA Answer

(2) Hence, or otherwise, solve the resultant equation:

$$64^x - 8^{x+1} = 9$$

$$64^x - 8 \cdot 8^x = 9$$

$$64^x - 8 \cdot (64^x)^{\frac{1}{2}} = 9$$

$$\therefore 64^x = 81$$

$$x = \log_{64}(81)$$

$$x \approx 1,1$$

Using exponential
laws
relating to
previous sum
Using logs to
solve
CA Answer

Students can use
solve function for
the final 2 marks

(c) Given: $z = 1 + ai$ and $\frac{1}{z} = \frac{1}{10} + bi$.

Calculate the values of a and b , if $a > 0$.

$$\begin{aligned}\frac{1}{z} &= \frac{1}{1 + ai} \\ &= \left(\frac{1}{1 + ai} \right) \times \frac{1 - ai}{1 - ai} \\ &= \frac{1 - ai}{1 - a^2 i^2} \\ &= \frac{1}{1 + a^2} - \frac{a}{1 + a^2} i \\ \therefore \frac{1}{10} + bi &= \frac{1}{1 + a^2} - \frac{a}{1 + a^2} i \\ \therefore \frac{1}{10} &= \frac{1}{1 + a^2} \\ 10 &= 1 + a^2 \\ 9 &= a^2 \\ a &= 3 \\ b &= -\frac{3}{1 + 3^2} \\ \therefore b &= -\frac{3}{10}\end{aligned}$$

reciprocal

multiplying by

complex conj.

simplification

equating real parts

CA Answer

equating complex
parts

CA substituting a in

CA Answer

Alternatively:

$$\begin{aligned}\frac{1}{z} &= \frac{1}{10} + bi \\ \frac{1}{z} &= \frac{1 + 10bi}{10} \\ 10 &= z(1 + 10bi) \\ \therefore 10 &= (1 + ai)(1 + 10bi) \\ 10 &= 1 + 10bi + ai + 10abi^2 \\ 10 &= 1 - 10ab + (10b + a)i\end{aligned}$$

$$10 = 1 - 10ab + (10b + a)i$$

$$\therefore 10 = 1 - 10ab \text{ AND } 10b + a = 0$$

$$9 = -10ab$$

$$a = -10b$$

$$\therefore 9 = -10(-10b)b$$

$$\therefore \frac{9}{100} = b^2$$

$$b = \pm \frac{3}{10}$$

$$a = -10 \left(+ \frac{3}{10} \right) \neq -3$$

$$a = -10 \left(- \frac{3}{10} \right) = 3$$

- (d) The function $f(x) = 2x^3 + ax^2 + bx - 10$ has a root of $1 + i$ and a, b are rational. Determine the roots of $f(x)$.

$$x = 1 + i$$

$$x^* = 1 - i$$

$$x^2 - (1 + i + 1 - i)x + (1 + i)(1 - i) = 0$$

$$x^2 - 2x + 2 = 0$$

$$f(x) = 2x^3 + ax^2 + bx - 10$$

$$f(x) = (x^2 - 2x + 2) \times (2x - 5)$$

$$x = \frac{5}{2}$$

other root

sum of roots

product of roots

CA quadratic

equation

$2x$

-5

CA final root

QUESTION 3

Josephine's company makes pots for major supermarkets. Her company employed a market researcher to estimate the production cost as a function. The cost function, applicable up to 600 pots, was:

$$C(x) = 20\,000 + 25x - 0,02x^2 + 250e^{rx}$$

where x represents the number of units produced, and $C(x)$ represents the total cost of the units produced. The cost to make 20 pots is R20 496,58.

- (a) Calculate the value of r .

$$20\,496,58 = 20\,000 + 25(20) - 0,02(20)^2 + 250e^{r \times 20}$$

sub in both sides

$$4,58 = 250 \times e^{20r}$$

rewriting in log-form

$$0,01832 = e^{20r}$$

$$20r = \ln(0,01832)$$

ans

$$r = \frac{\ln(0,01832)}{20}$$

$$r \approx -0,2$$

sub in both sides

Alternative:

ans

$$20\,496,58 = 20\,000 + 25(20) - 0,02(20)^2 + 250e^{r \times 20}$$

Use solve function calculator:

$$r \approx -0,2$$

- (b) The marginal cost is represented by $C'(x)$. Marginal cost at n units represents the cost to make the $(n + 1)$ -th unit.

- (1) Determine an expression for the marginal cost function.

$$C'(x) = 25 - 0,04x + 250 \times (-0,2)e^{-0,2x}$$

CA

$$= 25 - 0,04x - 50e^{-0,2x}$$

$$\frac{d}{dx}(e^{-0,2x}) = -0,2x \times e^{-0,2x}$$

$$\frac{d}{dx}(20\,000 + 25x) = 25$$

$$\frac{d}{dx}(-0,02x^2) = -0,04x$$

- (2) How much will it cost to produce the 300
- th
- unit?

$$C'(299) = 25 - 0,04(299) - 50e^{-0,2(299)}$$

$$= 13,04$$

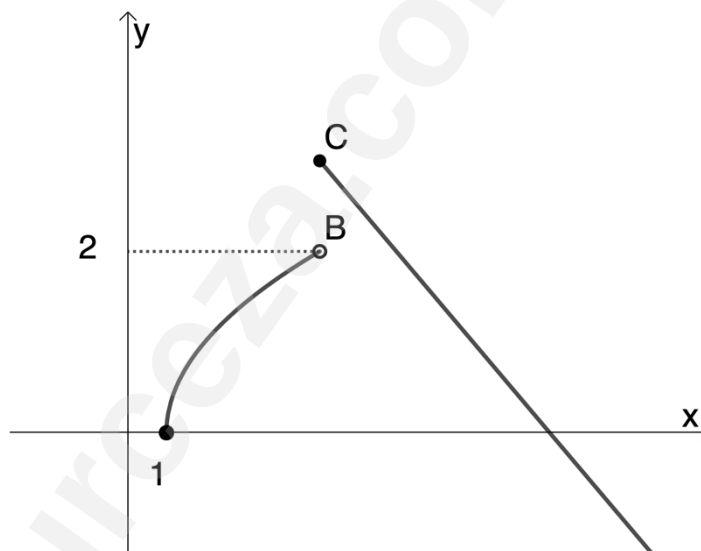
Sub

CA Ans

QUESTION 4

- (a) The graph shows

$$f(x) = \begin{cases} \sqrt{x-1} & x < 5 \\ \frac{-x+11}{2} & x \geq 5 \end{cases}$$



- (1) Discuss the continuity of
- $f(x)$
- at
- $x = 1$
- .

The graph's domain is: $x \geq 1$ this means that the graph is continuous at $x = 1$. It is all about the domain.

looking at
domain
using definition
of continuity
Conclusion

$f(1)$
Limit from right
Conclusion

Alternative:

$$f(1) = 0$$

$$\lim_{x \rightarrow 1^+} f(x) = 0$$

This means that the graph is continuous at $x = 1$

- (2) Transform $f(x)$ to be continuous at $x = 5$ by using the following transformation:

$$f(x) = \begin{cases} \sqrt{x-1} & x < 5 \\ \frac{-x+11}{2} + b(x) & x \geq 5 \end{cases} \text{ where } b(x) \text{ is a polynomial function.}$$

Give two possible functions for $b(x)$ that will ensure continuity at $x = 5$.

$$\lim_{x \rightarrow 5^+} f(x) = \frac{-5 + 11}{2} = 3$$

finding value at
 $x=5$

$$b(x) = -1 \text{ OR}$$

$b(x)$ can be any polynomial function
such that $b(5) = -1$ e.g.

$$b(x) = -1$$

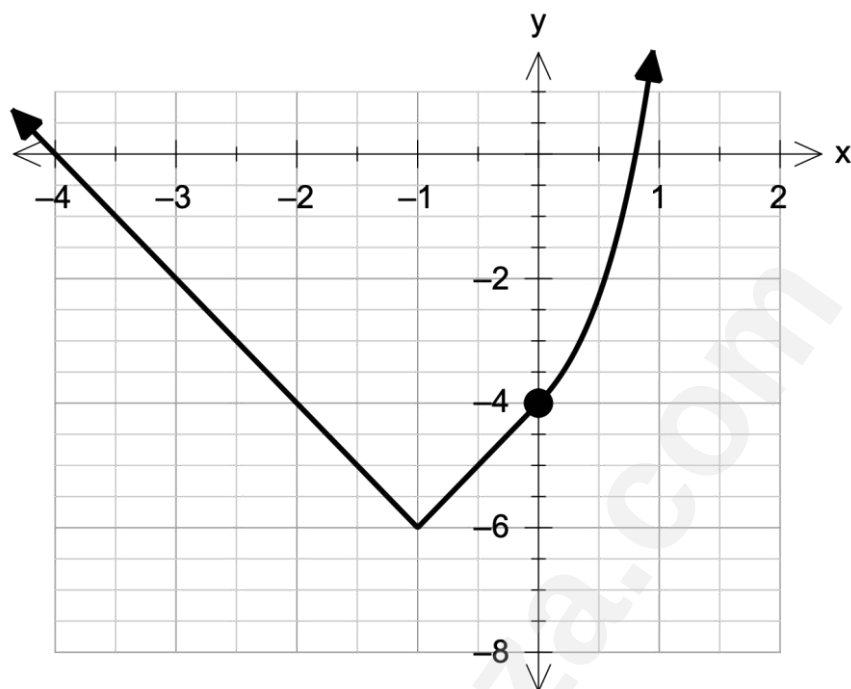
$$b(x) = -\frac{1}{5}x \text{ OR } b(x) = -\frac{1}{25}x^2$$

second $b(x)$
expression

- (b) Given:

$$g(x) = \begin{cases} e^{2x} - 5 & x \geq 0 \\ 2|x + 1| - 6 & x < 0 \end{cases}$$

- (1) Complete the sketch of $g(x)$ on the diagram below. Clearly indicate the intercepts with the axes and the salient point of the graph.



Space for calculations:

x-intercept

y-intercept

Salient point

form of graph

- (2) Using mathematical notation, discuss the differentiability of $g(x)$ at $x = 0$, if $g(x)$ is continuous for $x \in \mathbb{R}$.

$$g'(x) = \begin{cases} 2e^{2x} & x \geq 0 \\ 2 & x < 0 \end{cases}$$

$$\lim_{x \rightarrow 0^+} (g'(x)) = \lim_{x \rightarrow 0^+} (2e^{2x})$$

$$= 2$$

$$\lim_{x \rightarrow 0^-} (g'(x)) = \lim_{x \rightarrow 0^-} (2)$$

$$= 2$$

chain rule

2

e^{2x}

$m = 2$

$$\lim_{x \rightarrow 0^-} (f(x)) = 2$$

$$\lim_{x \rightarrow 0^+} (f(x)) = 2$$

Conclusion

Therefore, the graph is differentiable at $x = 0$.

QUESTION 5

- (a) If $f(x) = \frac{3}{\sqrt{x}}$, determine an expression for $f'(x)$ by using first principles.

$$f(x) = \frac{3}{\sqrt{x}}$$

$$f(x+h) = \frac{3}{\sqrt{x+h}}$$

f(x+h)

$$f'(x) = \lim_{h \rightarrow 0} \left(\frac{3}{\sqrt{x+h}} - \frac{3}{\sqrt{x}} \right) \times \frac{1}{h}$$

formula /
notation

$$= \lim_{h \rightarrow 0} \frac{3\sqrt{x} - 3\sqrt{x+h}}{\sqrt{x} \times \sqrt{x+h}} \times \frac{1}{h}$$

sub into
formula

$$= \lim_{h \rightarrow 0} \frac{3\sqrt{x} - 3\sqrt{x+h}}{\sqrt{x} \times \sqrt{x+h}} \times \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}} \times \frac{1}{h}$$

LCD

$$= \lim_{h \rightarrow 0} \frac{3x - 3(x+h)}{\sqrt{x} \times \sqrt{x+h} \times (\sqrt{x} + \sqrt{x+h})} \times \frac{1}{h}$$

Multiply by root
conjugate

$$= \lim_{h \rightarrow 0} \frac{3x - 3x - 3h}{\sqrt{x} \times \sqrt{x+h} \times (\sqrt{x} + \sqrt{x+h})} \times \frac{1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-3h}{\sqrt{x} \times \sqrt{x+h} \times (\sqrt{x} + \sqrt{x+h})} \times \frac{1}{h}$$

Simplified form
including h

$$= \lim_{h \rightarrow 0} \frac{-3}{\sqrt{x} \times \sqrt{x+h} \times (\sqrt{x} + \sqrt{x+h})}$$

simplification

$$= -\frac{3}{\sqrt{x} \times \sqrt{x} \times (\sqrt{x} + \sqrt{x})}$$

substitute h=0

$$= -\frac{3}{x \times (2\sqrt{x})}$$

Ans

$$= -\frac{3}{2\left(\frac{3}{x^2}\right)}$$

- (b) Determine $\frac{dy}{dx}$ for $4 - 2xy + e^y = 16x^2 + \cot x$.

$$\begin{aligned}\frac{d}{dx}(4 - 2xy + e^y) &= \frac{d}{dx}(16x^2 + \cot x) && \text{deriving both sides} \\ -2y - 2x\left(\frac{dy}{dx}\right) + e^y \times \frac{dy}{dx} &= 32x - \operatorname{cosec}^2 x && 32x \\ -2x\left(\frac{dy}{dx}\right) + e^y \times \frac{dy}{dx} &= 32x - \operatorname{cosec}^2 x + 2y && -\operatorname{cosec}^2 x \\ \frac{dy}{dx}(-2x + e^y) &= 32x - \operatorname{cosec}^2 x + 2y && \text{product rule} \\ \frac{dy}{dx} &= \frac{32x - \operatorname{cosec}^2 x + 2y}{-2x + e^y} && \text{chain rule} \\ &&& \text{CA factoring of } dy/dx \\ &&& \text{CA Answer}\end{aligned}$$

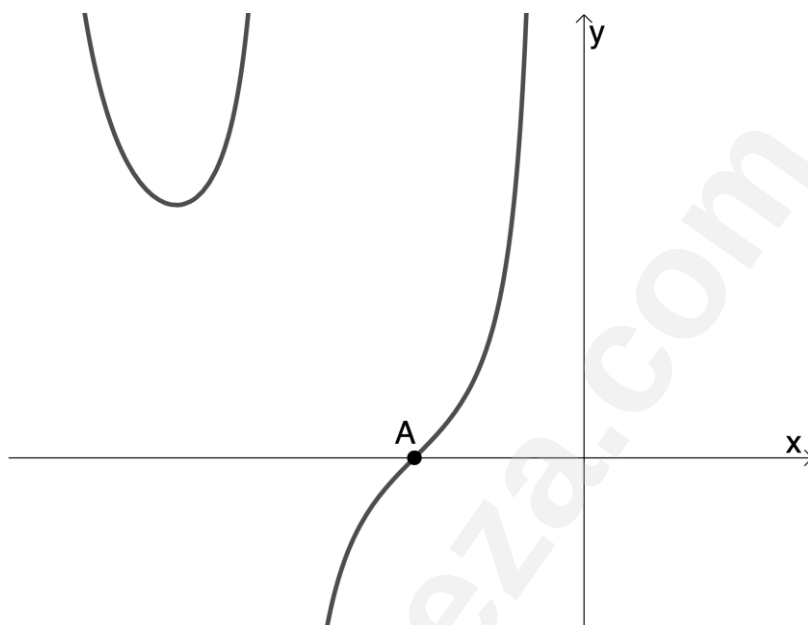
- (c) Determine the $h'(x)$ if $h(x) = \ln(x^2 - 6x + 9)$. Simplify your answer completely.

$$\begin{aligned}h(x) &= \ln(x^2 - 6x + 9) && \text{chain rule} \\ h'(x) &= \frac{1}{x^2 - 6x + 9} \times 2x - 6 && \frac{1}{x^2 - 6x + 9} \\ h'(x) &= \frac{1}{(x-3)^2} \times 2(x-3) && \times (2x-6) \\ h'(x) &= \frac{2}{x-3} && \text{factorisation} \\ &&& \text{CA Simplified form}\end{aligned}$$

Alternative:

$$\begin{aligned}h(x) &= \ln(x^2 - 6x + 9) \\ h(x) &= \ln((x-3)^2) && \text{Factorisation} \\ h(x) &= 2\ln(x-3) && \text{simplified log form} \\ h'(x) &= \frac{2}{x-3} && \text{CA } \frac{2}{x-3}\end{aligned}$$

- (d) The graph below shows a portion of the function $y = \frac{x+1}{\cos(2x+2)}$ with A as the x-intercept.



Determine the equation of the tangent at A.

$$0 = \frac{x+1}{\cos(2x)}$$

$$x = -1$$

$$A(-1, 0)$$

$$\frac{dy}{dx} = \frac{1\cos(2x+2) - (x+1)(-\sin(2x+2) \times 2)}{\cos^2(2x+2)}$$

$$\frac{dy}{dx} = \frac{\cos(2(-1)+2) - ((-1)+1)\sin(2(-1)+2)}{\cos^2(2(-1)+2)}$$

$$m = 1$$

$$y = x + c$$

$$0 = 1(-1) + c$$

$$c = 1$$

$$\therefore y = x + 1$$

0

A's coordinates

quotient rule

$$\frac{dy}{dx}(x+1) = 1$$

Denominator

$$-\sin(2x+2) \times 2$$

substitute in

CA m

sub

A equation

QUESTION 6Given: $\cot(x) - 2\operatorname{cosec}(2x) = -\tan(x)$

- (a) Prove the identity, ignoring all restrictions of
- x
- .

$$\begin{aligned}
 \text{LHS} &= \frac{\cos x}{\sin x} - \frac{2}{\sin(2x)} \\
 &= \frac{\cos x}{\sin x} - \frac{2}{2\sin x \cos x} \\
 &= \frac{\cos^2 x - 1}{\sin x \cos x} \\
 &= \frac{-\sin^2 x}{\sin x \cos x} \\
 &= -\frac{\sin x}{\cos x} \\
 &= -\tan x
 \end{aligned}$$

changing cosec(2x)
changing sin(2x)
changing cotx
LCD
LCD
square identities
simplified
expression
- tanx

- (b) Determine values of
- $\theta \in [-\pi; \pi]$
- for which
- $f(\theta) = \cot(\theta) - 2\operatorname{cosec}(2\theta)$
- will be concave down.

$$f(\theta) = -\tan(\theta)$$

$$f'(\theta) = -\sec^2 \theta$$

$$f''(\theta) = -2\sec(\theta) \times \sec(\theta)\tan(\theta)$$

$$f''(\theta) = -\sec^2 \theta \times 2\tan\theta$$

f'
2secθ
chain rule
f''(θ) < 0

$$-\sec^2 \theta < 0 \text{ for all } \theta$$

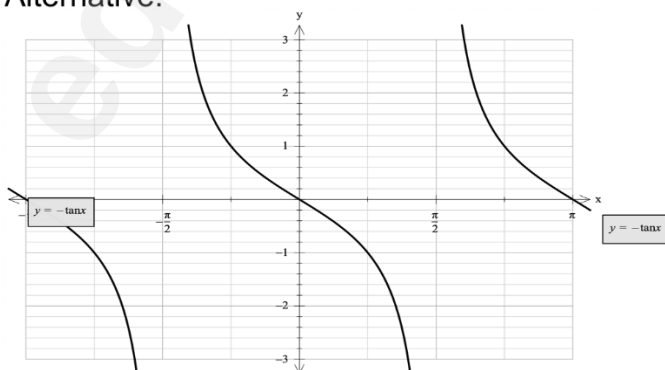
ans

$$f''(x) < 0 \text{ when } \tan x > 0$$

ans

$$0 \leq \theta \leq \frac{\pi}{2} \text{ or } -\pi \leq \theta \leq -\frac{\pi}{2}$$

Alternative:



QUESTION 7

(a) Given:

$$y = \frac{x^3 - 3x^2}{x^2 - 25}$$

Give the equation(s) of the vertical asymptote(s) of the graph.

$$x^2 - 25 = 0$$

$$(x - 5)(x + 5) = 0$$

$$x = 5 \text{ or } -5$$

Denominator = 0

ans

(b) Determine the equation of the horizontal asymptote of

$$h(x) = \frac{3 + 2x - x^2}{x^2 - 4}$$

$$y = \lim_{x \rightarrow \infty} \left(\frac{3 + 2x - x^2}{x^2 - 4} \right)$$

$$= \lim_{x \rightarrow \infty} \left(\frac{\frac{3}{x^2} + \frac{2}{x} - 1}{1 - \frac{4}{x^2}} \right)$$

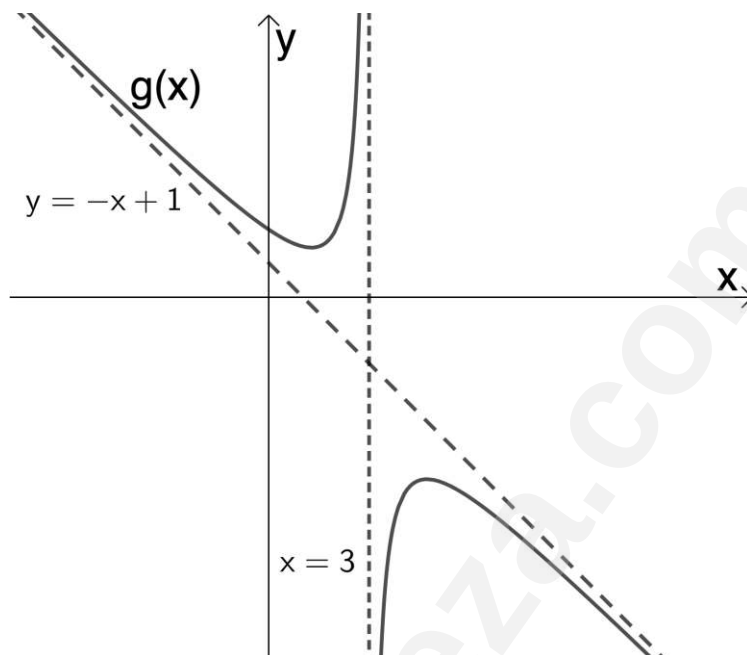
$$= -1$$

lim to infinity

dividing by x^2

ans

- (c) The graph below shows $g(x) = \frac{ax^2 + bx + 2}{x + c}$ and all its asymptotes:



Determine the values of a , b and c .

$$\begin{aligned}\frac{ax^2 + bx + 2}{x + c} &= -x + 1 + \frac{k}{x-3} \\ &= \frac{(-x + 1)(x-3) + k}{x-3} \\ &= \frac{-x^2 + 4x - 3 + k}{x-3}\end{aligned}$$

$$a = -1$$

$$b = 4$$

$$c = -3$$

c-value

Expression

form

$x+1$) in

expression

$x-3$) in

expression

LCD

simplified

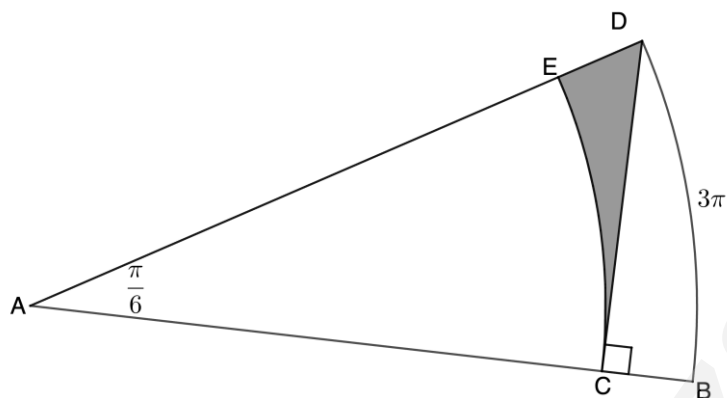
expression

a-value

b-value

QUESTION 8

Two circles with the same midpoint (A) are drawn below. A tangent from point C is drawn to D which on the circumference of the larger circle such that $AB \perp CD$ and the length of the arc BD is 3π cm. The angle formed between the radii is $\frac{\pi}{6}$.



- (a) Determine the length of AD.

$$l = r\theta$$

formula

$$3\pi = r\left(\frac{\pi}{6}\right)$$

sub

$$3\pi \times \frac{6}{\pi} = r$$

$$18 = r$$

ans

- (b) If $AC:AB$ is $\sqrt{3}:2$, calculate the area of CDE.

$$AC = 18 \times \frac{\sqrt{3}}{2} = 9\sqrt{3} = 15.588$$

AC

$$CD^2 = 18^2 - (9\sqrt{3})^2$$

$$CD = \sqrt{81} = 9$$

Pyth

Ans

Area = Area of $\triangle ACD$ – Area of sector ACE

$$Area = \frac{1}{2} \times 9\sqrt{3} \times (9) - \frac{1}{2} (9\sqrt{3})^2 \times \frac{\pi}{6}$$

Area of ACD

Area of ACE

–

$$\approx 6,53\text{cm}^2$$

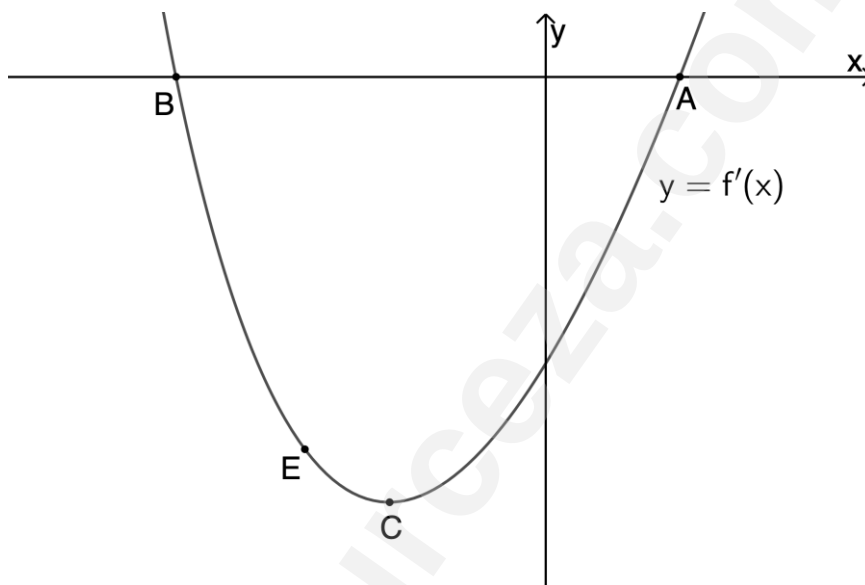
sub

ans

QUESTION 9

Given: $f(x) = (x^2 - 1)\ln(x + 1)$

The graph below represents $y = f'(x)$. A and B are the points where the graph of $f'(x)$ intersects the x -axis, C the turning point of the graph of $f'(x)$; and E $(-0,5; -0,81)$.



- (a) Peter wants to calculate the x -value of B by using the 'SOLVE' function of the calculator, which uses Newton-Raphson's method. He must choose between the 'START'-value of 0 or the x -value at E.

Which value must he choose? Justify your answer

E – The x -value at 0 will converge to A.

E

Explanation

- (b) Determine the x -value of B's coordinates, correct to 4 decimal places, using the Newton-Raphson method. Clearly state your starting point and show the answer of your first iteration.

$$\begin{aligned} f'(x) &= 2x\ln(x + 1) + (x^2 - 1) \times \left(\frac{1}{x + 1} \right) \\ &= 2x\ln(x + 1) + (x + 1)(x - 1) \times \left(\frac{1}{x + 1} \right) \\ &= 2x\ln(x + 1) + (x - 1) \\ f''(x) &= 2\ln(x + 1) + \frac{2x}{x + 1} + 1 \end{aligned}$$

$2x$

product rule

$$\frac{1}{x + 1}$$

simplification

product rule

$$\frac{2x}{x + 1}$$

$$x_0 = -0,5$$

$$x_1 = x_0 - \frac{2x \ln(x_0 + 1) + (x_0 - 1)}{2 \ln(x_0 + 1) + \frac{2x_0}{x_0 + 1} + 1}$$

$$x_1 = -0,8381...$$

...

$$x \approx -0,7024$$

Newton-

Raphson

sub correct

into N-R

x1

ans

QUESTION 10

Evaluate the following:

(a)
$$\int \frac{4x-18}{x^2-9x} dx$$

$$2 \int \frac{2x-9}{x^2-9x} dx$$

$$= 2\ln|x^2-9x| + c$$

Taking out 2
common

correct form

integration

Absolute value

c

Choosing u

deriving

Isolating dx

Alternative:

$$\int \frac{4x-18}{x^2-9x} dx$$

Let $u = x^2-9x$

$$\frac{du}{dx} = 2x-9$$

$$\frac{du}{2x-9} = dx$$

$$\int \frac{4x-18}{u} \left(\frac{du}{2x-9} \right)$$

$$2 \int \frac{1}{u} du$$

$$= 2\ln|u| + c$$

$$= 2\ln|x^2-9x| + c$$

Integration

absolute

values

c

$$\begin{aligned}
 \text{(b)} \quad & \int \sin^2(4x) \, dx \\
 & \int (\sin(4x) \times \sin(4x)) \, dx \\
 & = \frac{1}{2} \int (\cos(4x - 4x) - \cos(4x + 4x)) \, dx \\
 & = \frac{1}{2} \int (\cos(0) - \cos(8x)) \, dx \\
 & = \frac{1}{2} \int (1 - \cos 8x) \, dx \\
 & = \frac{1}{2} \left(\int 1 \, dx - \frac{1}{8} \int 8 \cos(8x) \, dx \right) \\
 & = \frac{1}{2} x - \frac{1}{16} \sin(8x) + c
 \end{aligned}$$

Trig form
correct form for
int
 $\cos(8x)$
 $1/2 x$

Alternative:

$$\begin{aligned}
 \sin^2 4x &= \frac{1}{2} (1 - \cos(8x)) \\
 &= \frac{1}{2} \int (1 - \cos 8x) \, dx \\
 &= \frac{1}{2} \left(\int 1 \, dx - \frac{1}{8} \int 8 \cos(8x) \, dx \right) \\
 &= \frac{1}{2} x - \frac{1}{16} \sin(8x) + c
 \end{aligned}$$

Trig form

correct form for
int
 $\cos(8x)$
 $1/2 x$

(c) $\int \frac{3x-3}{x^2-9} dx$

$$\frac{3x-3}{x^2-9} = \frac{3x-3}{(x-3)(x+3)} = \frac{A}{x-3} + \frac{B}{x+3}$$

$$3x-3 = A(x+3) + B(x-3)$$

$$x = 3 :$$

$$3(3)-3 = A(3+3) + B(3-3)$$

$$6 = 6A$$

$$1 = A$$

$$x = -3 :$$

$$3(-3)-3 = A(-3+3) + B(-3-3)$$

$$-12 = -6B$$

$$2 = B$$

$$\begin{aligned} \int \frac{3x-3}{x^2-9} dx &= \int \left(\frac{1}{x-3} \right) dx + \int \left(\frac{2}{x+3} \right) dx \\ &= \ln|x-3| + 2\ln|x+3| + c \end{aligned}$$

Alternative:

$$\begin{aligned} &\int \frac{3x-3}{x^2-9} dx \\ &= \int \frac{3x}{x^2-9} dx - \int \frac{3}{x^2-9} dx \end{aligned}$$

$$\begin{aligned} \int \frac{3x}{x^2-9} dx &= \frac{3}{2} \int \frac{2x}{x^2-9} dx \\ &= \frac{3}{2} \ln|x^2-9| \end{aligned}$$

Factored form

Partial fractions

form

Multiply by

LCD

$x=3$

A's answer

B's answer

Form

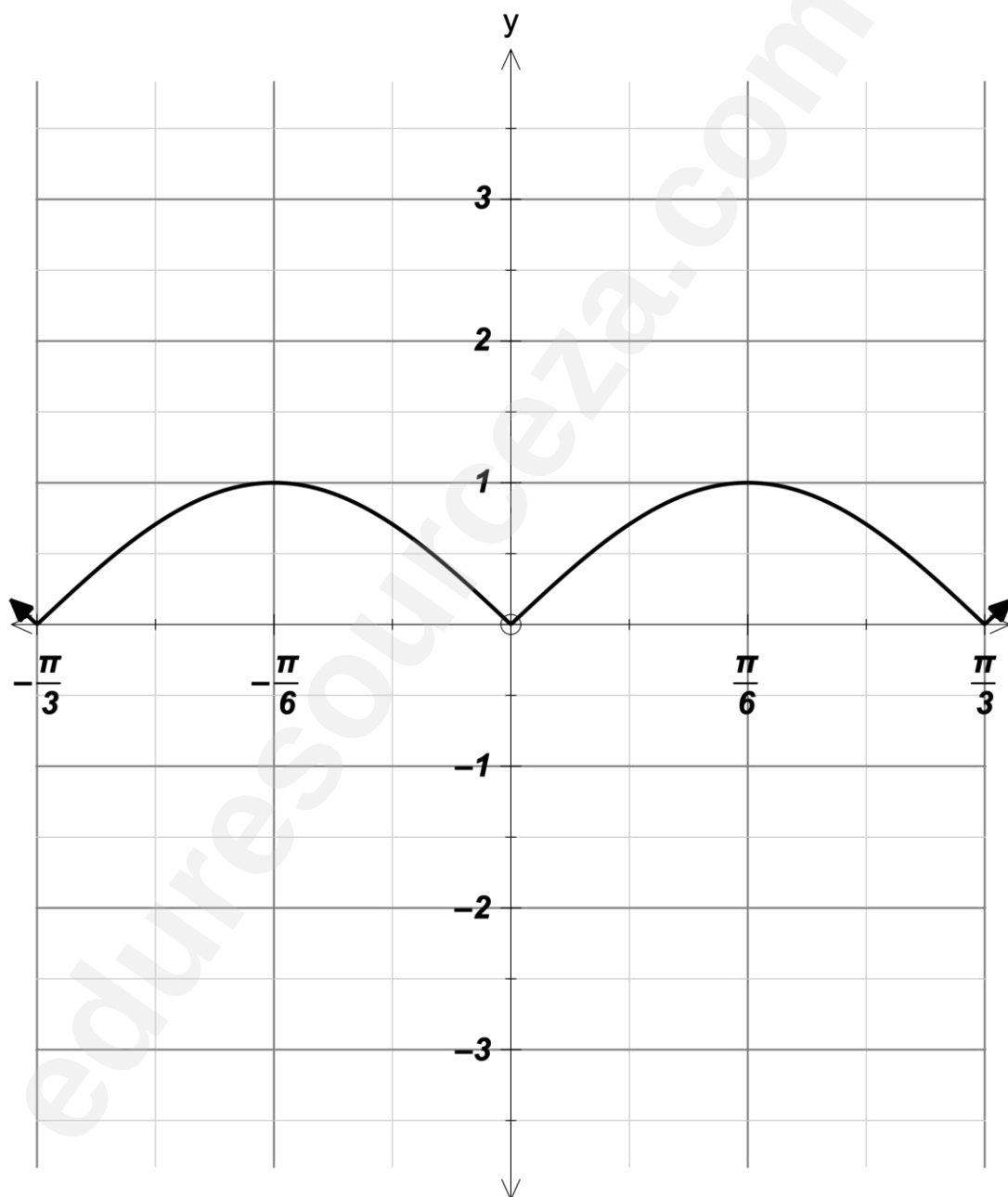
Integrals

$$\begin{aligned}\frac{3}{x^2-9} &= \frac{3}{(x-3)(x+3)} \\ \frac{3}{(x-3)(x+3)} &= \frac{a}{x-3} + \frac{b}{x+3} \\ 3 &= a(x+3) + b(x-3) \\ (x=3) \quad a &= \frac{1}{2} \\ (x=-3) \quad b &= -\frac{1}{2} \\ \frac{3}{(x-3)(x+3)} &= \frac{\frac{1}{2}}{x-3} - \frac{\frac{1}{2}}{x+3} \\ \int \frac{\frac{1}{2}}{x+3} - \frac{\frac{1}{2}}{x-3} dx &= \frac{1}{2} \ln|x+3| - \frac{1}{2} \ln|x-3| \\ &= \int \frac{3x}{x^2-9} dx - \int \frac{3}{x^2-9} dx \\ &= \frac{3}{2} \ln|x^2-9| - \frac{1}{2} \ln|x-3| + \frac{1}{2} \ln|x+3| + c\end{aligned}$$

QUESTION 11

Consider $f(x) = \sin 3x$.

- (a) Make a neat sketch of $y = |f(x)|$ for $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ on the diagram below.



Period

maximum

Absolute values

intercepts

- (b) Leah was asked to evaluate the area between the x -axis and the graph for the x -values between $x = -a$ and $x = a$ where $a \in \left[0, \frac{\pi}{2}\right]$. Her attempt

below is:

$$\int_{-a}^a \sin(3\theta) d\theta = 0.$$

One of the methods she could use to determine this enclosed area is by doing the following:

$$2 \times \int_0^a (\sin(3\theta)) d\theta$$

Give two more options to determine the exact area between the x -axis and the graph between $x = -a$ and $x = a$.

$$\left| \int_{-a}^0 \sin(3\theta) d\theta \right| + \int_0^a \sin(3\theta) d\theta$$

Absolute value
integral
integral

Alternatively:

$$\int_0^{-a} \sin(3\theta) d\theta + \int_0^a \sin(3\theta) d\theta$$

swopping 0 and -a
around

Alternatively:

$$-2 \times \int_{-a}^0 (\sin(3\theta)) d\theta$$

2
integral

- (c) If the exact area is $\frac{2}{3}$ units² determine the value of a .

$$2 \times \int_0^a (\sin(3\theta)) d\theta = \frac{2}{3}$$

using integral
from (c)

$$\int_0^a (\sin(3\theta)) d\theta = \frac{1}{3}$$

$\frac{2}{3}$

$$\frac{1}{3} \int_0^a (3\sin(3\theta)) d\theta = \frac{1}{3}$$

correct form
integrating sin
sub

$$\frac{1}{3} [-\cos(3\theta)]_0^a = \frac{1}{3}$$

$$\frac{1}{3} (-\cos(3a) - (-\cos(0))) = \frac{1}{3}$$

isolating cos

$$\frac{1}{3} (-\cos(3a) + 1) = \frac{1}{3}$$

$\frac{\pi}{2}$

$$\cos(3a) = 0$$

answer

$$3a = \frac{\pi}{2}$$

$$a = \frac{\pi}{6}$$

Total: 200 marks