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NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2019

TECHNICAL MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours

150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

QUESTION 1

$$\begin{aligned}
 1.1 \quad AB &= \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} \quad \text{correct formula} \\
 &= \sqrt{(200 - 100)^2 + (0 - 250)^2} \quad \text{substitution} \\
 &= \sqrt{(100)^2 + (-250)^2} \\
 &= 50\sqrt{29} \text{ units} \quad \text{simplified surd form}
 \end{aligned}$$

$$\begin{aligned}
 1.2 \quad m_{AB} &= \frac{y_B - y_A}{x_B - x_A} \\
 &= \frac{0 - 250}{200 - 100} \quad \text{substitution} \\
 &= -2,5 \quad \text{simplification}
 \end{aligned}$$

$$\begin{aligned}
 1.3 \quad M &= \left(\frac{x_A + x_O}{2} ; \frac{y_A + y_O}{2} \right) \\
 M &= \left(\frac{100 + 0}{2} ; \frac{250 + 0}{2} \right) \\
 M &= (50 ; 125) \quad \text{x-coordinate} \quad \text{y-coordinate}
 \end{aligned}$$

$$\begin{aligned}
 1.4 \quad \tan B &= m \\
 \tan B &= -2,5 \quad \text{substitution} \\
 \therefore \hat{OBA} &\approx 68,2^\circ \quad \text{angle } \hat{OBA} \\
 \therefore \hat{AOB} &= 68,2^\circ \quad \text{angle } \hat{AOB} \\
 \therefore \theta &= 180^\circ - 68,2^\circ - 68,2^\circ = 43,6^\circ \quad \text{simplification}
 \end{aligned}$$

QUESTION 2

$$2.1 \quad 2.1.1 \quad m_{LP} \times m_{LN} = -1$$

$$-1 \times m_{LN} = -1$$

$$\therefore m_{LN} = 1 \quad \text{LN gradient}$$

$$\therefore y = x + c \quad \text{with } M(-4;4)$$

$$\therefore 4 = -4 + c$$

$$\therefore 8 = c$$

$$\therefore y = x + 8 \quad \text{equation}$$

$$\text{OR} \quad y - y_1 = m(x - x_1)$$

$$y - 4 = 1(x + 4)$$

$$y = x + 8 \quad \text{equation}$$

$$2.1.2 \quad \therefore y = x + 8 = -x + 2 \quad \text{follow up from Question 2.1.1}$$

$$\therefore 2x = -6$$

$$\therefore x = -3$$

$$\therefore y = x + 8$$

$$\therefore y = -3 + 8 = 5$$

$$\therefore L(-3;5) \quad \text{coordinates}$$

$$m\left(\frac{x_L + x_N}{2}; \frac{y_L + y_N}{2}\right) = (-4; 4)$$

$$\frac{-3 + x_N}{2} = -4 \quad \frac{5 + y_N}{2} = 4 \quad \text{equations}$$

$$x_N = -5 \quad y_N = 3$$

$$\therefore N(-5;3) \quad \text{coordinates}$$

$$2.1.3 \quad y = -x + 2$$

$$\text{x-intercept: } 0 = -x + 2$$

$$x = 2 \quad \therefore P(2;0) \quad \text{coordinates}$$

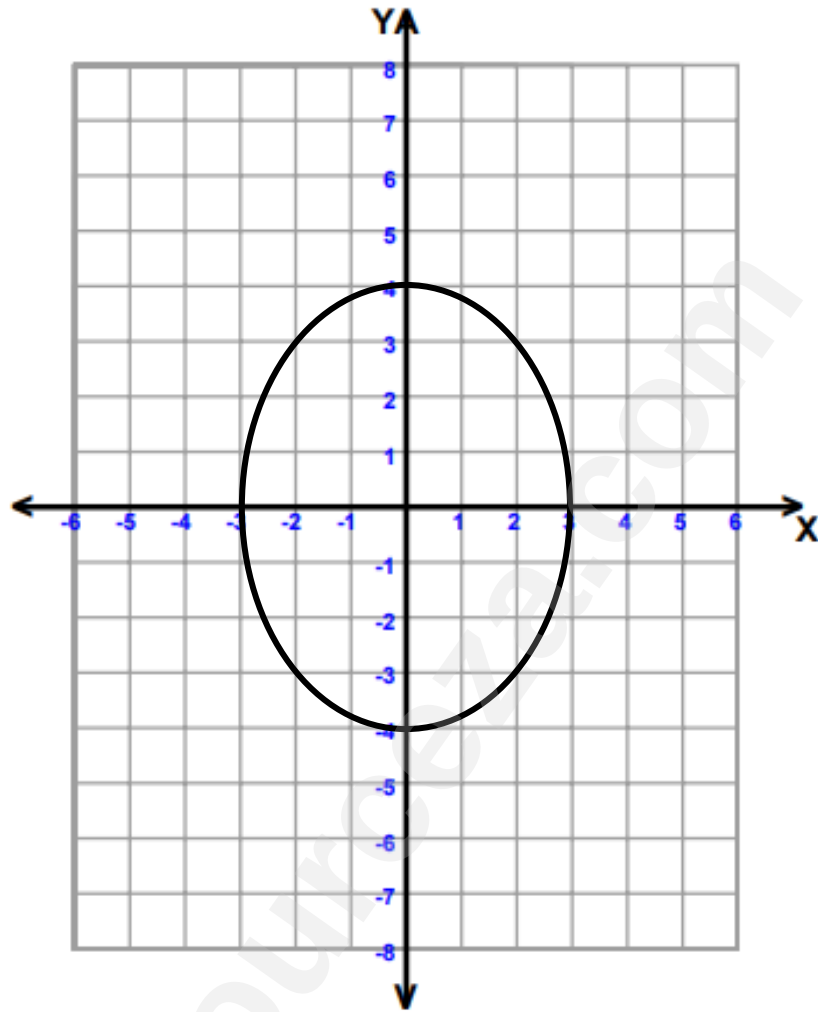
$$2.1.4 \quad x^2 + y^2 = r^2 \quad p(2;0)$$

$$\therefore (2)^2 + (0)^2 = r^2 \quad \text{substitution}$$

$$4 = r^2$$

$$\therefore x^2 + y^2 = 4 \quad \text{equation}$$

2.2



x-intercepts y-intercepts shape

QUESTION 3

3.1 3.1.1 $x^2 + y^2 = r^2$

$$(2\sqrt{3})^2 + (-2)^2 = r^2 \quad \text{substitution}$$

$$16 = r^2$$

$$4 \text{ units} = r \quad \text{simplification}$$

3.1.2 $\tan \theta = \frac{-2}{2\sqrt{3}}$ function

$$(\text{Ref angle}) = 30^\circ \quad \text{method}$$

$$\theta = 360^\circ - 30^\circ = 330^\circ \quad \text{simplification}$$

3.2 $\sec(a - b)$

$$= \sec(2,695 - 1,112) \quad \text{substitution}$$

$$= \sec(1,583)$$

$$= \frac{1}{\cos(1,583)}$$

$$\approx -81,9 \quad \text{simplification}$$

3.3 $\frac{\sin 210^\circ \tan 45^\circ \cos 315^\circ}{\sin 45^\circ \cos 60^\circ}$

$$= \frac{-\sin 30^\circ \tan 45^\circ \cos 45^\circ}{\sin 45^\circ \cos 60^\circ}$$

$$= \frac{\left(\frac{-1}{2}\right) \cdot (1) \cdot \left(\frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right) \cdot \left(\frac{1}{2}\right)}$$

$$= -1$$

OR

$$= \frac{-\sin 30^\circ \times \frac{\sin 45^\circ}{\cos 45^\circ} \times \cos 45^\circ}{\sin 45^\circ \times \sin 30^\circ}$$

$$= -1$$

3.4 LHS: $\tan x \cdot \sin x$

$$= \frac{\sin x}{\cos x} \cdot \sin x$$

$$= \frac{\sin^2 x}{\cos x}$$

RHS: $\sec x - \cos x$

$$= \frac{1}{\cos x} - \cos x$$

$$= \frac{1 - \cos^2 x}{\cos x}$$

$$= \frac{\sin^2 x}{\cos x}$$

$$\therefore LHS = RHS$$

OR LHS: $\tan x \cdot \sin x$

$$= \tan x (\cos x \tan x)$$

$$= \cos x \cdot \tan^2 x$$

$$= \cos x (\sec^2 x - 1)$$

$$= \cos x \left(\frac{1}{\cos^2 x} - 1 \right)$$

$$= \frac{1}{\cos x} - \cos x$$

$$= \sec x - \cos x$$

$$\therefore LHS = RHS$$

$$\begin{aligned}
 \text{OR LHS : } \tan x \cdot \sin x &= \frac{\sin x}{\cos x} \cdot \frac{\sin x}{1} \\
 &= \frac{\sin^2 x}{\cos x} \\
 &= \frac{1 - \cos^2 x}{\cos x} \\
 &= \frac{1}{\cos x} - \frac{\cos^2 x}{\cos x} \\
 &= \sec x - \cos x \\
 \therefore LHS &= RHS
 \end{aligned}$$

$$\begin{aligned}
 \text{OR RHS : } \sec x - \cos x &= \frac{1}{\cos x} - \cos x \\
 &= \frac{1 - \cos^2 x}{\cos x} \\
 &= \frac{\sin^2 x}{\cos x} \\
 &= \sin x \times \frac{\sin x}{\cos x} \\
 &= \sin x \cdot \tan x \\
 \therefore LHS &= RHS
 \end{aligned}$$

3.5 3.5.1 $\operatorname{cosec} 2x = 2,114$ for $2x \in [0^\circ; 180^\circ]$

$$\frac{1}{\sin 2x} = 2,114$$

$$\frac{1}{2,114} = \sin 2x$$

$$0,473... = \sin 2x$$

$$\text{Ref angle} \approx 28,2316^\circ$$

$$2x = 28,23^\circ \quad \text{or} \quad 2x = 180^\circ - 28,23^\circ \quad \text{correct quadrants}$$

$$x = 14,12^\circ \quad \text{or} \quad x = 75,88^\circ \quad \text{both answers}$$

3.5.2
$$\frac{\sin(360^\circ - x) \cdot \cos(180^\circ - x) \cdot \tan(180^\circ + x)}{\cos^2 x \cdot \sin \frac{5}{6}\pi}$$

$$= \frac{(-\sin x)(-\cos x)(\tan x)}{(\cos^2 x)\left(\frac{1}{2}\right)} \checkmark$$

$$= \frac{\sin x \cdot \tan x}{\cos x \cdot \frac{1}{2}}$$

$$= 2 \tan^2 x$$

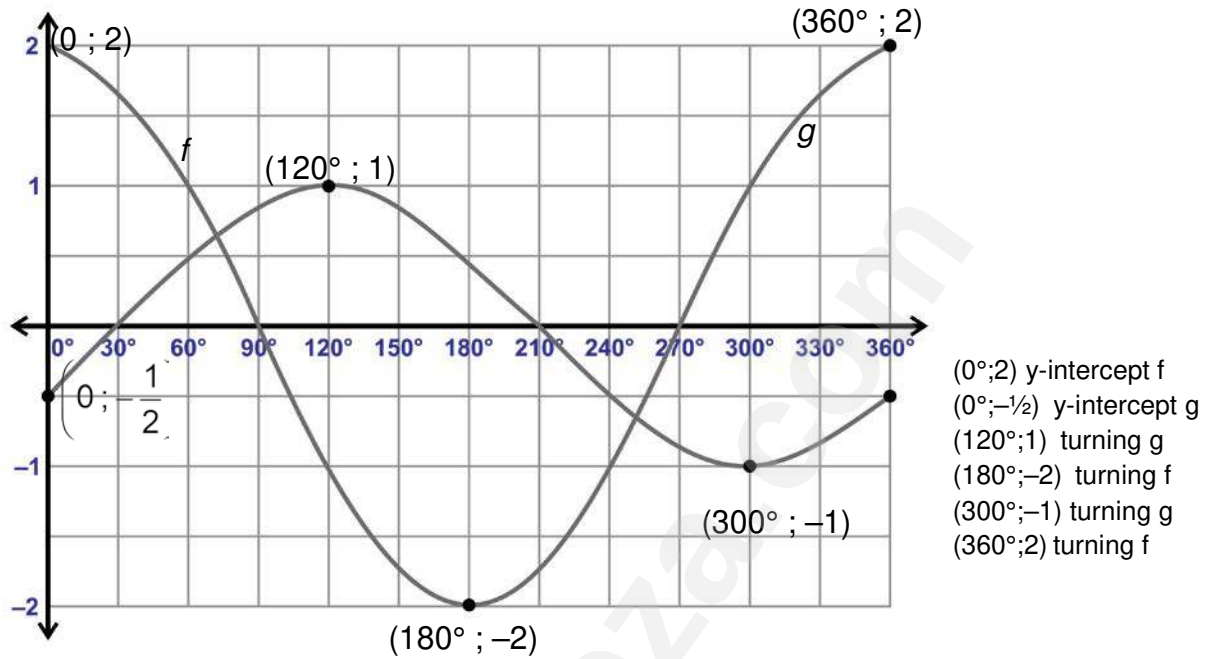
$$\text{OR } \frac{(-\sin x)(-\cos x)(\tan x)}{(\cos^2 x)\left(\sin \frac{5}{6}\pi\right)}$$

$$= \frac{(-\sin x)(-\cos x)\left(\frac{\sin x}{\cos x}\right)}{(\cos^2 x)\left(\frac{1}{2}\right)}$$

$$= 2 \tan^2 x$$

QUESTION 4

4.1



4.2 2

4.3 360°

4.4 $x \in (0^\circ; 70^\circ) \cup (250^\circ; 360^\circ)$ give leeway with $70^\circ (\pm 5^\circ)$ and $250^\circ (\pm 5^\circ)$
 OR $0^\circ < x < 70^\circ \cup 250^\circ < x < 360^\circ$

QUESTION 5

$$5.1 \quad \sin 58^\circ = \frac{AC}{2} \text{ ratio} \quad \text{OR} \quad \frac{AC}{\sin 58^\circ} = \frac{2}{\sin 90^\circ}$$

$$\therefore AC \approx 1,7 \text{ m answer} \quad \therefore AC \approx 1,7 \text{ m}$$

$$5.2 \quad \text{Area of } \triangle ABC = \frac{1}{2}(1,7 \text{ m})(2,3 \text{ m}) \sin 108^\circ \text{ formula substitution}$$

$$\approx 1,9 \text{ m}^2$$

$$5.3 \quad \frac{BC}{\sin 108^\circ} = \frac{2,3 \text{ m}}{\sin 42^\circ} \quad \text{OR} \quad BC^2 = 2,3^2 + 1,7^2 - 2(2,3)(1,7) \cos 108^\circ$$

$$BC \approx 3,3 \text{ m}$$

$$5.4 \quad DB^2 = (1,1)^2 + (2,3)^2$$

$$\therefore DB \approx 2,5 \text{ m}$$

$$BC^2 = DC^2 + DB^2 - 2DC \cdot DB \cdot \cos \hat{D}$$

$$(3,3)^2 = (2)^2 + (2,5)^2 - 2(2)(2,5) \cdot \cos \hat{D}$$

$$\therefore \hat{D} \approx 93,7^\circ$$

QUESTION 6

$$6.1 \quad 6.1.1 \quad \hat{P}_2$$

$$\hat{P}_1 + \hat{P}_2 = 180^\circ \text{ (Angles on a straight line)}$$

$$\hat{P}_1 = 60^\circ \text{ (Angle at centre} = 2 \times \text{angle on circumference of circle)}$$

$$60^\circ + \hat{P}_2 = 180^\circ$$

$$\hat{P}_2 = 120^\circ$$

$$6.1.2 \quad \hat{R}_1 = \hat{T} \text{ (Angles at equal sides)}$$

$$\text{and } \hat{P}_2 = \hat{R}_1 + \hat{T} = 180^\circ \text{ (int. angles of triangle)}$$

$$\therefore 120^\circ + 2\hat{R}_1 = 180^\circ$$

$$2\hat{R}_1 = 60^\circ$$

$$\hat{R}_1 = 30^\circ$$

6.2 6.2.1 $\hat{D}_2 = 50^\circ$ (Tan-chord theorem)

6.2.2 $\hat{B}_1$

$\hat{A}_1 = 50^\circ$ (Tan-chord theorem)

$\hat{A}_1 + 110^\circ + \hat{B}_1 = 180^\circ$ (Interior angles of triangle)

$50^\circ + 110^\circ + \hat{B}_1 = 180^\circ$

$\hat{B}_1 = 20^\circ$

6.2.3 $\hat{D}_1$

$\hat{B}_1 = \hat{C}_3 = 20^\circ$ (Angles in same segment)

and $\hat{C}_2 = 20^\circ$ (Given)

$\therefore \hat{D}_1 = 20^\circ$ (Angles in same segment)

6.3 6.3.1 $a = 49^\circ$ (Tan-chord theorem)

6.3.2 $\hat{P}\hat{T}\hat{R} = 78^\circ$ (Tan-chord theorem)

$32^\circ + \hat{T}_1 = 78^\circ$

$\hat{T}_1 = 46^\circ$

$\therefore b = 46^\circ$ (Tan-chord theorem)

OR $\hat{Q}\hat{P}\hat{R} = 32^\circ$ (angles in same segment)

$\therefore C = 78^\circ - 32^\circ$

$= 46^\circ$

6.3.3 $c + 78^\circ = 180^\circ$ (Angles on same line)

$\therefore c = 102^\circ$

QUESTION 7

7.1 7.1.1 QM

$$KQ : QM = 3 : 1 \quad (\text{Given})$$

$$\therefore KM : QM = 4 : 1 \quad (\text{Proportionality theorem})$$

$$\therefore \frac{KM}{QM} = \frac{4}{1}$$

$$\therefore \frac{20 \text{ units}}{QM} = \frac{4}{1}$$

$$\therefore 20 = 4QM$$

$$\therefore 5 \text{ units} = QM$$

$$\begin{aligned} \text{OR } QM &= \frac{1}{4} \times 20 \text{ units} \\ &= 5 \text{ units} \end{aligned}$$

7.1.2 KP

$$\frac{KQ}{QM} = \frac{KP}{PL}$$

(Proportionality theorem $PQ \parallel LM$)

$$\frac{3}{1} = \frac{KP}{4 \text{ units}}$$

$$KP = 12 \text{ units}$$

7.1.3 KB

$$\frac{KM}{QM} = \frac{KP}{BP}$$

(Proportionality theorem $BQ \parallel PM$)

$$\frac{20 \text{ units}}{5 \text{ units}} = \frac{12 \text{ units}}{BP}$$

$$20BP = 60$$

$$BP = 3 \text{ units}$$

$$KB + BP = KP$$

$$KB + 3 = 12$$

$$KB = 9 \text{ units}$$

7.2 7.2.1 In $\triangle KPM$ and $\triangle KBQ$

$$\hat{M}KP = \hat{Q}KB \quad (\text{Common angle})$$

$$\hat{K}MP = \hat{K}QB \quad (\text{Corresponding angles } BQ \parallel PM)$$

$$\hat{K}PM = \hat{K}BQ \quad (\text{Corresponding angles } BQ \parallel PM)$$

$$\therefore \triangle KPM \parallel \triangle KBQ \quad (\text{Angle, angle, angle})$$

$$7.2.2 \quad \frac{KQ}{KM} = \frac{BQ}{PM}$$

($\triangle KPM \parallel \triangle KBQ$)

$$\frac{3}{4} = \frac{BQ}{10 \text{ units}}$$

$$4BQ = 30$$

$$BQ = 7,5 \text{ units}$$

QUESTION 8

$$8.1 \quad v = \pi D n$$

$$8,75 = \pi(50)n$$

$$0,0557 = n \quad \text{OR} \quad \frac{7}{40\pi}$$

$$w = 2\pi n$$

$$= 2\pi(0,0557\dots) \quad \text{OR} \quad 2\pi\left(\frac{7}{40}\pi\right)$$

$$\approx 0,35\text{rad/sec} \quad \frac{7}{20}\text{rad/sec}$$

$$8.2 \quad 8.2.1 \quad s = r\theta$$

$$s = 28 \text{ cm} \times \left(240^\circ \times \frac{\pi}{180^\circ}\right)$$

$$= 117,3 \text{ cm}$$

$$8.2.2 \quad \hat{APC} = 90^\circ \text{ (radius} \perp \text{ tangent)}$$

$$AB = BC = 28 \text{ cm (Given)}$$

$$\therefore AC = AB + BC$$

$$= 28 \text{ cm} + 28 \text{ cm}$$

$$= 56 \text{ cm}$$

$$AC^2 = AP^2 + PC^2 \text{ (Pyth)}$$

$$(56 \text{ cm})^2 = (28 \text{ cm})^2 + PC^2$$

$$\therefore \mathbf{PC = 48,5 \text{ cm}}$$

$$\therefore \mathbf{QC = 48,5 \text{ cm}}$$

$$\text{OR} \quad \frac{PC}{28} = \tan 60^\circ$$

$$PC = 48,5 \text{ cm}$$

$$QC = 48,5 \text{ cm} \checkmark$$

$$\frac{RC}{12} = \tan 60^\circ$$

$$RC = 20,8 \text{ cm}$$

$$\therefore TC = 20,8 \text{ cm}$$

$$\text{Total} = 117,3 + 50,3 + 2(48,5) + 2(20,8)$$

$$\approx 306,2 \text{ cm}$$

$$\hat{ERC} = 90^\circ \text{ (radius} \perp \text{ tangent)}$$

$$ED = DC = 12 \text{ cm (Given)}$$

$$\therefore EC = ED + DC$$

$$= 12 \text{ cm} + 12 \text{ cm}$$

$$= 24 \text{ cm}$$

$$\text{(Pyth)}$$

$$EC^2 = ER^2 + RC^2$$

$$(24 \text{ cm})^2 = (12 \text{ cm})^2 + RC^2$$

$$\therefore \mathbf{RC = 20,8 \text{ cm}}$$

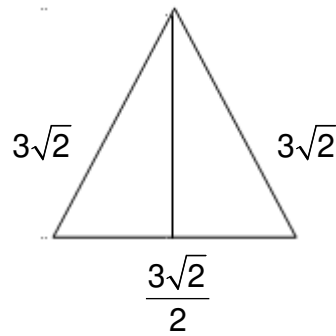
$$\therefore \mathbf{TC = 20,8 \text{ cm}}$$

$$\text{Total belt length} = 117,286 \text{ cm} + 50,3 \text{ cm} + 2(48,5 \text{ cm}) + 2(20,78 \text{ cm})$$

$$= 306,11 \text{ cm}$$

QUESTION 9

9.1



9.1.1 Perpendicular height of $\triangle ABE$: $(3\sqrt{2})^2 = (sh)^2 + \left(\frac{3\sqrt{2}}{2}\right)^2$

$$18 = (sh)^2 + \frac{18}{4}$$

$$sh = \frac{3\sqrt{6}}{2} \text{ units or } 3,7 \text{ units}$$

OR $\frac{h}{3\sqrt{2}} = \sin 60^\circ$

$$h = \frac{3\sqrt{6}}{2}$$

9.1.2 Area $\triangle ABE = \frac{1}{2} \times \text{base} \times \perp \text{ height}$

$$= \frac{1}{2} \times 3\sqrt{2} \times \frac{3\sqrt{6}}{2}$$

$$= \frac{9\sqrt{3}}{2} \text{ or } 7,794$$

Total area of octahedron

$$= 8 \times \frac{9\sqrt{3}}{2}$$

$$= 36\sqrt{3} \text{ or } 62,4 \text{ units}^3$$

$$\begin{aligned} 9.2 \quad \text{Volume cylinder} &= \pi \times r^2 \times h \\ &= \pi \times (30 \text{ mm})^2 \times 70 \text{ mm} \\ &= 197\,920\,3372 \text{ mm}^3 \text{ or } 63\,000 \pi \end{aligned}$$

$$\text{Volume dome} = \frac{2}{3} \pi r^3$$

$$\begin{aligned} &= \frac{2}{3} \pi (30 \text{ mm})^3 \\ &= 56\,548\,668 \text{ mm}^3 \text{ or } 18\,000 \pi \end{aligned}$$

$$\begin{aligned} \text{Volume centre removed} &= \pi \times r^2 \times h \\ &= \pi \times (15 \text{ mm})^2 \times 70 \text{ mm} \\ &= 49\,480\,084 \text{ mm}^3 \text{ or } 15\,750 \pi \end{aligned}$$

$$\begin{aligned} \text{Total volume} &= 197\,920\,3372 \text{ mm}^3 + 56\,548\,668 \text{ mm}^3 - 49\,480\,084 \text{ mm}^3 \\ &= 204\,988\,92 \text{ mm}^3 \text{ or } 29\,250 \pi \end{aligned}$$

$$9.3 \quad a = 6,5 \text{ m} \div 5 = 1,3 \text{ m}$$

$$\text{Area} = a(m_1 + m_2 + m_3 + m_4 + m_5)$$

$$= 1,3 \left(\frac{0+0,8}{2} + \frac{0,8+1,3}{2} + \frac{1,3+1,1}{2} + \frac{1,1+0,5}{2} + \frac{0,5+0}{2} \right)$$

$$= 1,3(0,4 + 1,05 + 1,2 + 0,8 + 0,25)$$

$$= 4,81 \text{ m}^2$$

TOTAL: 150 marks