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NASIONALE SENIOR CERTIFIKAAT-EKSAMEN
NOVEMBER 2019

TEGNIESE WISKUNDE: VRAESTEL I

NASIENRIGLYNE

Tyd: 3 uur

150 punte

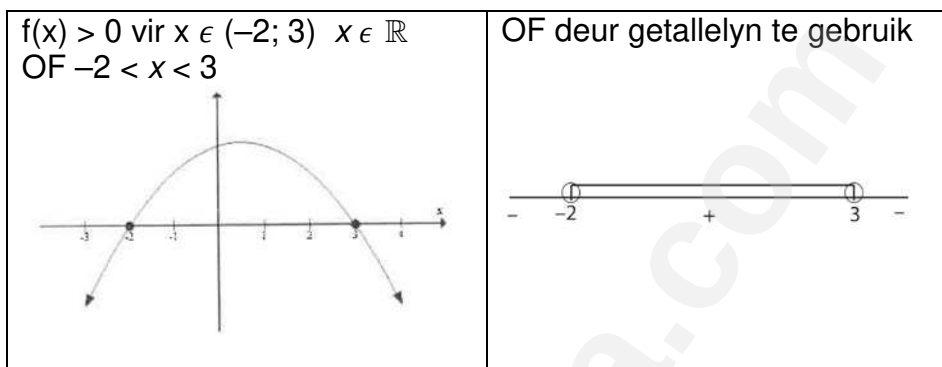
Hierdie nasienriglyne is opgestel vir gebruik deur eksaminators en hulp-eksaminators van wie verwag word om almal 'n standaardiseringsvergadering by te woon om te verseker dat die riglyne konsekwent vertolk en toegepas word by die nasien van kandidate se skrifte.

Die IEB sal geen bespreking of korrespondensie oor enige nasienriglyne voer nie. Ons erken dat daar verskillende standpunte oor sommige aangeleenthede van beklemtoning of detail in die riglyne kan wees. Ons erken ook dat daar sonder die voordeel van die bywoning van 'n standaardiseringsvergadering verskillende vertolkings van die toepassing van die nasienriglyne kan wees.

VRAAG 1

1.1 1.1.1 $6 - x^2 + x = 0$
 $x^2 - x - 6 = 0$
 $(x - 3)(x + 2) = 0$
 $x = 3$ of $x = -2$

1.1.2



1.2 $3^y = 3^{4x}$ of $y = x^2 - 6x + 9 \dots\dots\dots (2)$

$\therefore y = 4x \dots\dots\dots (1)$
 Vervang (1) $\rightarrow$ (2):

$4x = x^2 - 6x + 9$
 $0 = x^2 - 10x + 9$
 $0 = (x - 9)(x - 1)$
 $x = 9$ of $x = 1$
 $y = 36$ of $y = 4$

Vervang in (1)

1.3 1.3.1 $-(x - 2)^2 + 3 = 5$
 $-x^2 + 4x - 4 - 2 = 0$
 $x^2 - 4x + 6 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(6)}}{2(1)}$$

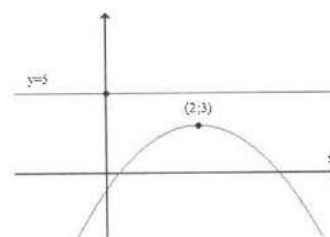
$$x = \frac{4 \pm \sqrt{-8}}{2} = \frac{4 \pm 2\sqrt{-2}}{2}$$

$$x = 2 \pm \sqrt{-2} \text{ OF } x = 2 \pm \sqrt{2}i$$

1.3.2 $-x^2 + 4x - 1 = 5 + k$ OF

$0 = x^2 - 4x + 6 + k$
 $\Delta = (-4)^2 - 4(1)(6 + k)$
 $= 16 - 24 - 4k$
 $= -4k - 8$

Dus 2 reële, verskillende wortels $-4k - 8 > 0$
 $-4k > 8$
 $k < -2$



Deur inspeksie van grafiek $g(x) + k$ ontmoet f twee keer indien $5 + k < 3$
 $k < -2$

$$\begin{aligned}
 1.4 \quad & \frac{1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0}{1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0} \\
 &= \frac{16 + 8 + 2 + 1}{32 + 16 + 2 + 1} \\
 &= \frac{27}{51} \\
 &= \frac{9}{17}
 \end{aligned}$$

$$\begin{aligned}
 1.5 \quad & \varepsilon = \frac{\Delta L}{L} \\
 0,77 &= \frac{182 - L}{L} \\
 0,77L + L &= 182 \\
 L &= \frac{182}{1,77} \\
 &= 102,824858 \dots\dots \\
 &\approx 1,02825 \times 10^2
 \end{aligned}$$

VRAAG 2

$$\begin{array}{ll}
 2.1 & \left((x+2)^{\frac{3}{4}} \right)^{\frac{4}{3}} = (27)^{\frac{4}{3}} \quad \begin{array}{l} x+2 > 0 \\ x > -2 \end{array} \quad \text{OF} \quad (x+2)^{\frac{3}{4}} = 27 \\
 & (x+2)^{\frac{1}{4}} = 3 \\
 & x+2 = 3^4 \\
 & x+2 = 81 \\
 & x = 79 \text{ Geldig}
 \end{array}$$

$$2.2 \quad 2.2.1 \quad (2\sqrt{3} - \sqrt{3} - 2\sqrt{2})(2\sqrt{3} - \sqrt{3} + 2\sqrt{2})$$

$$\begin{aligned}
 &= (\sqrt{3} - 2\sqrt{2})(\sqrt{3} + 2\sqrt{2}) \\
 &= 3 - 8 \\
 &= -5
 \end{aligned}$$

$$2.2.2 \quad \frac{3 \cdot 2^{2x+1} - 2^{2x-2} + 4^x}{4 \cdot 2^{2x-3}}$$

$$= \frac{3 \cdot 2^{2x} \cdot 2^1 - 2^{2x} \cdot 2^{-2} + 2^{2x}}{4 \cdot 2^{2x} \cdot 2^{-3}}$$

$$= \frac{\cancel{2^{2x}} (3 \cdot 2^1 - 2^{-2} + 1)}{4 \cdot \cancel{2^{2x}} \cdot 2^{-3}}$$

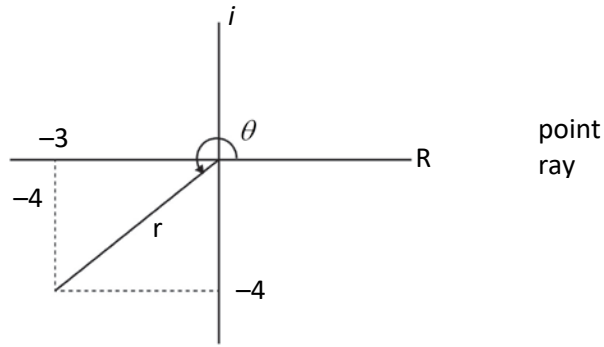
$$= \frac{6 - \frac{1}{4} + 1}{4 \cdot \frac{1}{8}}$$

$$= \frac{27}{4} \times \frac{2}{1}$$

$$= \frac{27}{2}$$

$$\begin{aligned}
 2.3 \quad & 2(5-2i) - i(6i-1) \\
 &= 10 - 4i - 6i^2 + i \\
 &= 10 - 4i + 6 + i \\
 &= 16 - 3i
 \end{aligned}$$

2.4 2.4.1

2.4.2 $r = |p|$

$$r^2 = (-3)^2 + (-4)^2$$

$$= 9 + 16 = 25$$

$$r = 5$$

$$\tan \theta = \frac{-4}{-3} = \frac{4}{3}$$

$$\theta = 180^\circ + 53,13^\circ$$

$$= 233,13^\circ$$

$$P = (5; 233,13^\circ) \text{ OF } (5 \cos 233,13^\circ ; 5 \sin 233,13^\circ)$$

$$\text{OF } p = +5 \cos 233,13^\circ + 5 \sin 233,13^\circ$$

$$\text{OF } p = 5 \text{ cis } 233,13^\circ$$

$$\text{OF } p = 5 \underline{233,13^\circ}$$

VRAAG 3

3.1 3.1.1 By B: $x + 2 = 0$
 $x = -2$
 B is $(-2; 0)$

3.1.2 Wortels is -12 en -2

Vergelyking is $y = a(x + 12)(x + 2)$

Vervang $(-13; -11)$: $-11 = a(-1)(-11)$

$$-11 = 11a$$

$$-1 = a$$

$$y = -1(x + 12)(x + 2)$$

$$y = -x^2 - 14x - 24$$

3.1.3 $x_E = -7$ (deur simmetrie)

OF $f'(x) = 0$
 $-2x + 14 = 0$
 $x = 7$

$$y_E = -(-7)^2 - 14(-7) - 24$$

$$= -49 + 98 - 24$$

$$= 25$$

$x_F = x_E = -7$: Vervang in g : $y_F = -7 + 2 = -5$

$$EF = y_E - y_F$$

$$= 25 - (-5) = 30$$

3.1.4 $x_E = x_G = -7$

G is $(-7; 3)$

By K, $y = 3$: $3 = x + 2$

$$1 = x$$

K is $(1; 3)$

$GF = 3 - (-5) = 8$ $GK = 1 - (-7) = 8$

Oppervlakte $\Delta GFK = \frac{1}{2}GF \cdot GK$

$$= \frac{1}{2} \times 8 \times 8$$

$$= 32$$

3.2 3.2.1 Laat $y = 0$

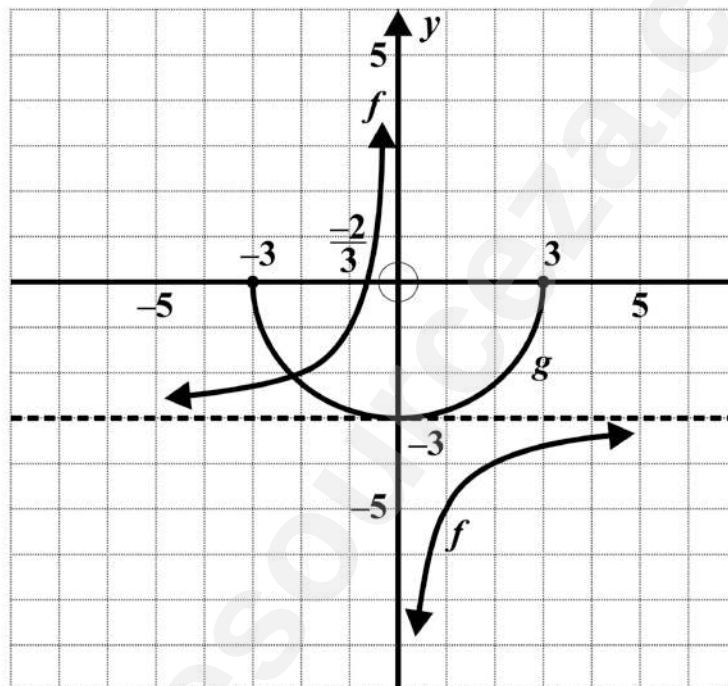
$$0 = \frac{-2}{x} - 3$$

$$3 = \frac{-2}{x}$$

$$3x = -2$$

$$x = -\frac{2}{3} \text{ OR } \left(-\frac{2}{3}; 0\right)$$

3.2.2



Vir $g: r = 3$

3.2.3 $x \in \left[-\frac{2}{3}; 0\right)$ OF $-\frac{2}{3} \leq x < 0$

3.3 Asimptoot $y = -3$

d.w.s. $q = -3$

$$y = a \cdot b^x - 3$$

Vervang $(0; -2)$: $-2 = a \cdot b^0 - 3$

$$1 = a$$

Vervang $(1; -1)$: $-1 = b^1 - 3$

$$2 = b$$

d.w.s. $y = 2^x - 3$

VRAAG 4

$$4.1 \quad 4.1.1 \quad 1 + i_{eff} = \left(1 + \frac{0,072}{12}\right)^{12}$$

$$i_{eff} = 0,074424 \dots\dots$$

d.w.s. effektiewe koers $\approx 7,44\%$

$$4.1.2 \quad 150\,000 = 120\,000 \left(1 + \frac{0,072}{12}\right)^n$$

$$\frac{15}{12} = \left(1 + \frac{0,072}{12}\right)^n$$

$$\log_{\left(1 + \frac{0,072}{12}\right)} \left(\frac{5}{4}\right) = n$$

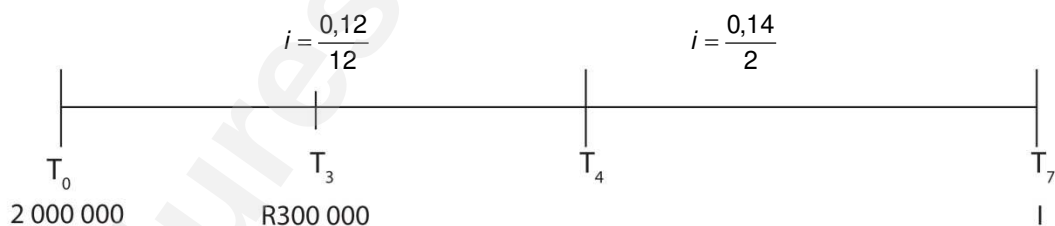
$n \approx 37,3\dots$ maande d.w.s. 38 maande

$$4.2 \quad I = 2 \times 10^6 \left(1 + \frac{0,12}{12}\right)^{48} \left(1 + \frac{0,14}{2}\right)^6 - 300\,000 \left(1 + \frac{0,12}{12}\right)^{12} \left(1 + \frac{0,14}{2}\right)^6$$

$$\approx R4\,331\,715,06$$

$$\text{OF} \quad \left[2 \times 10^6 \left(1 + \frac{0,12}{12}\right)^{36} - 300\,000 \right] \left(1 + \frac{0,12}{12}\right)^{12} \left(1 + \frac{0,14}{2}\right)^6$$

$$\approx R4\,331\,715,06$$



4.3 4.3.1 Verminderende saldo

$$4.3.2 \quad 110\,940 = 150\,000 (1 - i)^2$$

$$\sqrt{\frac{110\,940}{150\,000}} = 1 - i$$

$$i \approx 0,14$$

Koers is 14%

$$4.3.3 \quad A = 150\,000 (1 - 0,14)^7$$

$$\approx R52\,189,17$$

Die boekwaarde van motor na 7 jaar.

VRAAG 5

$$\begin{aligned}
 5.1 \quad f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{-3(x+h) + 1 - (-3x + 1)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{-3x - 3h + 1 + 3x - 1}{h} \\
 &= \lim_{h \rightarrow 0} \frac{3h}{h} \\
 &= 3
 \end{aligned}$$

$$\begin{aligned}
 5.2 \quad 5.2.1 \quad f(x) &= \frac{2\sqrt{x}}{\sqrt{x}} - \frac{5}{\sqrt{x}} \\
 &= 2 - 5x^{-\frac{1}{2}} \\
 f'(x) &= \frac{5}{2}x^{-\frac{3}{2}}
 \end{aligned}$$

$$\begin{aligned}
 5.2.2 \quad y &= 2x(1-x)^2 \\
 &= 2x - 4x^2 + 2x^3 \\
 \therefore \frac{dy}{dx} &= 2 - 8x + 6x^2
 \end{aligned}$$

$$\begin{aligned}
 5.3 \quad f(x) &= 3x^2 + 13x \\
 f'(x) &= 6x + 13 \\
 m_{\tan} &= \tan 45^\circ = 1 \\
 \text{d.w.s. } f'(x) &= 1 \\
 6x + 13 &= 1 \\
 6x &= -12 \\
 x &= -2
 \end{aligned}$$

5.4 5.4.1 Vergelyking is $y = (x+2)(x-1)(x-6)$

$$= (x+2)(x^2 - 7x + 6)$$

$$= x^3 - 7x^2 + 6x + 2x^2 - 14x + 12$$

$$= x^3 - 5x^2 - 8x + 12$$

$$b = -5; c = -8; d = 12$$

5.4.2 $f'(x) = 3x^2 - 10x - 8$

By D en E, $3x^2 - 10x - 8 = 0$

$$(3x+2)(x-4) = 0$$

$$x_D = -\frac{2}{3} \quad x_E = 4$$

$$y_D = \frac{400}{27} \quad y_E = -36$$

(Gebruik sakrekenaar)

$$D \text{ is } \left(-\frac{2}{3}; \frac{400}{27}\right) \quad E \text{ is } (4; -36)$$

VRAAG 6

6.1 Teken $A = 1$ Tyd is tussen t_B en t_C By B en C , $1 = -t^3 + 2t^2$

$$t^3 - 2t^2 + 1 = 0$$

Laat $f(t) = t^3 - 2t^2 + 1$

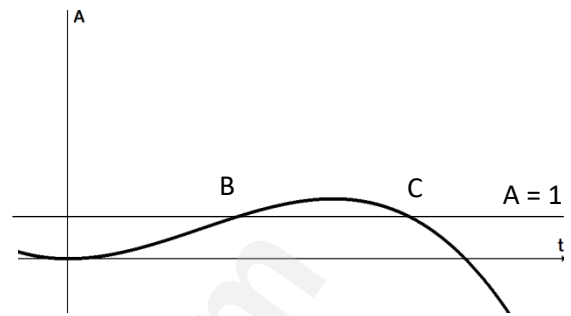
$$f(1) = 1 - 2 + 1 = 0$$

 $\therefore t - 1$ is 'n faktor

$$(t - 1)(t^2 - t - 1) = 0$$

$$t = 1 \text{ of } t = \frac{1 \pm \sqrt{1 + 4}}{2}$$

$$t = \frac{1 + \sqrt{5}}{2} \approx 1,6 \quad (t > 0)$$

 $\therefore$ tyd is $1,6 - 1 = 0,6$ uur = 36 minute6.2 Omtrek = $2\pi r + 2L$

$$400 = 2\pi r + 2L$$

$$200 = \pi r + L$$

$$200 - \pi r = L$$

$$S = \pi r^2 + 2rL$$

$$= \pi r^2 + 2r(200 - \pi r)$$

$$S = 400r - \pi r^2$$

$$\frac{ds}{dr} = 400 - 2\pi r$$

By maksimum, $400 - 2\pi r = 0$

$$r = \frac{200}{\pi} \text{ m}$$

6.3 $\int (2x^{-1} + 3x^2 - 1) dx$

$$= 2 \cdot \ln x + \frac{3x^{2+1}}{2+1} - x + c$$

$$= 2 \ln x + x^3 - x + c$$

6.4 By A en B, $2x^2 - 8x + 6 = 0$

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$x = 3 \text{ of } x = 1$$

$$\therefore \text{Oppervlakte} = \int_A^B f(x) dx$$

$$= \int_1^3 (2x^2 - 8x + 6) dx$$

$$= \left[\frac{2x^3}{3} - \frac{8x^2}{2} + 6x \right]_1^3$$

$$= \left[\frac{2(27)}{3} - 4(9) + 6(3) \right] - \left[\frac{2}{3} - 4 + 6 \right]$$

$$= \left| -2\frac{2}{3} \right|$$

$$\text{Oppervlakte} = 2\frac{2}{3}$$

Totaal: 150 punte