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NATIONAL SENIOR CERTIFICATE EXAMINATION
SUPPLEMENTARY EXAMINATION 2015

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours

150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1

a) 1) $10x = 3x^2 - 8$
 $3x^2 - 10x - 8 = 0$
 $(3x+2)(x-4) = 0$ 3)
 $x = -\frac{2}{3}$ or $x = 4$

2) $x + \sqrt{x-2} = 4$
 $\sqrt{x-2} = 4 - x$
 $x - 2 = 16 - 8x + x^2$
 $x^2 - 9x + 18 = 0$
 $(x-3)(x-6) = 0$
 $x = 3$ or $x = 6$
Check $x = 3$
LHS = $3 + \sqrt{3-2}$ 5)
 $= 4 = \text{RHS}$
Check $x = 6$
LHS = $6 + \sqrt{6-2}$
 $= 8 \neq \text{RHS}$

3) $x(2x-1) \geq 15$
 $2x^2 - x - 15 \geq 0$
 $(2x+5)(x-3) \geq 0$ 5)

$$\begin{array}{ccccccc} & & \frac{5}{2} & & 3 & & \\ & & \downarrow & & \downarrow & & \\ + & 0 & - & 0 & + & & \end{array}$$

 $x \leq -\frac{5}{2}$ or $x \geq 3$

4) $\log_x 5 = 3$ 2)
 $x^3 = 5$
 $x = \sqrt[3]{5}$
 $\approx 1,7$

b) 1) $P = \frac{4^{x+3} + 4^x}{8^{x+2} + 8^x}$
 $= \frac{2^{2x+6} + 2^{2x}}{2^{3x+6} + 2^{3x}}$
 $= \frac{2^{2x}(2^6 + 1)}{2^{3x}(2^6 + 1)}$
 $= 2^{-x}$ or $\left(\frac{1}{2}\right)^x$

ALTERNATIVELY:

$$P = \frac{4^x(4^3 + 1)}{8^x(8^2 + 1)}$$

$$= \frac{4^x(65)}{8^x(65)}$$

$$= \left(\frac{1}{2}\right)^x$$

3)

2) $P = 8$

$$2^{-x} = 2^3$$

$$x = -3$$

2)

- c) 1) Irrational
2) Rational
3) Non-real

1)

1)

1)

d) $f(x) = cx^3 - 6x + 2c$

$$f(1) = c - 6 + 2c = 0$$

$$3c = 6$$

$$c = 2$$

3)

[26]

QUESTION 2

a) 12345,6 123456,7

1)

b) $\sum_{k=2}^5 \frac{3^{k-1}}{k}$

$$= \frac{3}{2} + \frac{3^2}{3} + \frac{3^3}{4} + \frac{3^4}{5}$$

$$= \frac{549}{20} \quad (27,45)$$

3)

c) 1) $T_3 + T_4 = 167$

$$a + 2d + a + 3d = 167$$

$$2a + 5d = 167 \dots\dots ①$$

$$T_{21} = -4$$

$$a + 20d = -4$$

$$a = -20d - 4 \dots\dots ②$$

$$① : 2(-20d - 4) + 5d = 167$$

$$-40d - 8 + 5d = 167$$

$$-35d = 175$$

$$d = -5$$

4)

$$2) \quad a = -20(-5) - 4 \\ = 96$$

$$S_{21} = \frac{21}{2}[2a + 20d] \\ = \frac{21}{2}[2 \times 96 + 20(-5)] \\ = 966$$

4)

$$d) \quad 72 ; 48 ; 32$$

$$\text{G.S. } a = 72 \quad r = \frac{2}{3}$$

$$S_{\infty} = \frac{72}{1 - \frac{2}{3}} \\ = 216 \text{ cm}$$

3)
[15]**QUESTION 3**

$$a) \quad f(x) = 3x^2 + 2$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(3(x+h)^2 + 2) - (3x^2 + 2)}{h} \\ = \lim_{h \rightarrow 0} \frac{3(x^2 + 2xh + h^2) + 2 - 3x^2 - 2}{h} \\ = \lim_{h \rightarrow 0} \frac{6xh + 3h^2}{h} \\ = \lim_{h \rightarrow 0} (6x + 3h) \\ = 6x$$

5)

$$b) \quad y = \frac{5x - 2}{10x^2}$$

$$= \frac{x^{-1}}{2} - \frac{x^{-2}}{5}$$

$$\frac{dy}{dx} = \frac{-x^{-2}}{2} + \frac{2x^{-3}}{5} \\ = -\frac{1}{2x^2} + \frac{2}{5x^3}$$

5)

c)
$$f(x) = \sqrt{x}(x+2)$$

$$= x^{\frac{3}{2}} + 2x^{\frac{1}{2}}$$

$$f'(x) = \frac{3x^{\frac{1}{2}}}{2} + 2 \times \frac{x^{-\frac{1}{2}}}{2}$$

$$f'\left(\frac{1}{4}\right) = \frac{3}{2}\left(\frac{1}{4}\right)^{\frac{1}{2}} + \left(\frac{1}{4}\right)^{-\frac{1}{2}}$$

$$= \frac{3}{2} \times \frac{1}{2} + 2$$

$$= \frac{11}{4} \quad (2,75)$$

5)

d)
$$f(x) = \frac{2}{3}x^3 - x^2 - 3x$$

$$f(3) = \frac{2}{3} \times 3^3 - 3^2 - 3 \times 3$$

$$= 0$$

$$f'(x) = \frac{2}{3} \times 3x^2 - 2x - 3$$

$$= 2x^2 - 2x - 3$$

$$f'(3) = 2 \times 3^2 - 2 \times 3 - 3$$

$$= 9$$

Eqn. of tangent : $y - 0 = 9(x - 3)$
 $y = 9x - 27$

5)
[20]

QUESTION 4

a) $12\,500(1-i)^5 = 5\,546,32$
 $(1-i)^5 = 0,4437056$
 $1-i = 0,8500001102$
 $i = 0,1499998898$
 $I \approx 15\%$ 3)

b) 1)
$$\frac{500 \left[\left(1 + \frac{0,06}{12} \right)^{180} - 1 \right]}{\frac{0,06}{12}}$$

 $= 145\,409,3562$
 $\approx R145\,409,36$ 3)

2) $150\,000 \left(1 + \frac{0,085}{4} \right)^{20}$
 $= 228\,419,223$
 $\approx R228\,419,22$ 3)

3) $150\,000 \left(1 + \frac{0,085}{4} \right)^{4n} = 10^6$
 $\left(1 + \frac{0,085}{4} \right)^{4n} = \frac{20}{3}$
 $4n = \log_{1,02125} 6,66666$
 $= 90,22147031$
 $n = 22,55536758$
 $= 22,6$ 5)
[14]

75 marks

SECTION B**QUESTION 5**

(a) 1) i) $\frac{40}{500}$ 1)
 $= \frac{2}{25}$ 0,08)

ii) $\frac{200}{500}$ 1)
 $= \frac{2}{5}$ 0,4)

iii) $\frac{160}{200}$ 2)
 $= \frac{4}{5}$ 0,8)

2) P_{Male}
 $= \frac{230}{500}$ 0,46)
 $P_{\text{Male}} \times P_{\text{Living with Parents}}$
 $= \frac{23}{50} \times \frac{2}{5}$
 $= \frac{23}{125}$ 0,184)
 $\neq P_{\text{Male and Living with Parents}} = 0,08$
 $\therefore$ Living with Parents is NOT independent of gender.

ALTERNATIVELY:

P_{Female}
 $= \frac{270}{500}$ (0,54)
 $P_{\text{Female and Living with Parents}}$
 $= \frac{160}{500}$ (0,32)
 $P_{\text{Female}} \times P_{\text{Living with Parents}}$
 $= \frac{27}{50} \times \frac{2}{5}$
 $\frac{27}{125}$ (0,216) 3)
 $\neq P_{\text{Female and Living with Parents}} = 0,32$

- (b) 1) $7!$
 $= 5\,040$ 1)
- 2) $4 \times 3 \times 5!$
 $= 1\,440$ 3)
- 3) $4! \times 3! \times 4$
 $= 576$ 3)
[14]

QUESTION 6

- a) 1)
$$\begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 \\ 1 & 0 & 5 & 16 & 33 \\ 1^{\text{st}} \text{ diff.} & 1 & 5 & 11 & 17 \\ 2^{\text{nd}} \text{ diff.} & 6 & 6 & 6 \\ \text{Any valid method:} \\ \text{a} & \text{b} & \text{c} \end{array}$$

$$\begin{aligned} T_n &= 0 + (n-1) \times 5 + \frac{(n-1)(n-2)}{2} \times 6 \\ &= 5n - 5 + 3(n^2 - 3n + 2) \\ &= 5n - 5 + 3n^2 - 9n + 6 \\ &= 3n^2 - 4n + 1 \\ \therefore a &= 3, b = -4 \text{ and } c = 1 \end{aligned}$$

ALTERNATIVELY :

$$2a = d = 6$$

$$\therefore a = 3$$

$$T_0 = 1 = c$$

$$T_1 = 3 + b + 1 = 0$$

$$\therefore b = -4$$

5)

- 2) $3n^2 - 4n + 1 = 5896$
 $3n^2 - 4n - 5895 = 0$

$$n = \frac{4 \pm \sqrt{16 + 12 \times 5895}}{6}$$

$$= 45 \text{ or } -\frac{131}{3}$$

 $\therefore 45^{\text{th}} \text{ term}$

3)

b) $f(x) = \frac{2}{x^2} + 1$

$$f(x^{-1}) = \frac{2}{(x^{-1})^2} + 1$$

$$= 2x^2 + 1$$

$$f(-1) = \frac{2}{(-1)^2} + 1$$

$$= 3$$

$$\therefore f(x^{-1}) - x^2 f(-1)$$

$$= 2x^2 + 1 - x^2 \times 3$$

$$= -x^2 + 1$$

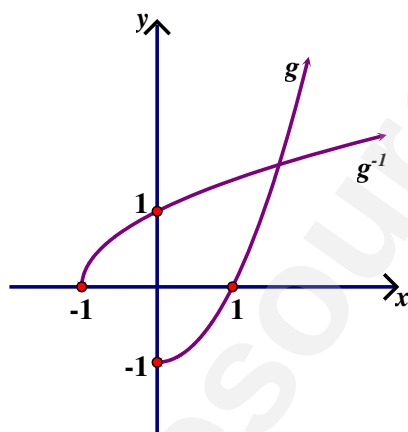
4)

c) $g(x) = x^2 - 1$ with $x \geq 0$

1) $y \geq 0$

1)

2)



4)

d) 1) $m_{AB} = \frac{3+1}{1-3}$

$$= -2$$

2)

2) $\frac{y}{x} - 3 = -2(x - 1)$

$$\frac{y}{x} = -2x + 2 + 3$$

$$y = -2x^2 + 5x$$

4)

[23]

QUESTION 7

a) 1) $f(0) = 30$ 1)
 $\therefore 30 \text{ m}$

2) $-\frac{x^2}{1\,000} + \frac{x}{5} + 30 = 0$
 $x^2 - 200x - 30\,000 = 0$
 $(x - 300)(x + 100) = 0$
 $x = 300 \text{ or } x = -100$
 $\therefore 300 \text{ m}$ 3)

3) $x_{TP} = -\frac{1}{5} \div -\frac{2}{1\,000}$
 $= 100$
 $f(100) = \frac{-100^2}{1000} + \frac{100}{5} + 30$
 $= 40$
 i.e. Max height is 40 m. 4)

4) $x \in [0 ; 300]$ 1)

b) 1) $h(x) = \frac{k}{x-2} + 1$
 $3 ; 4) : 4 = \frac{k}{3-2} + 1$
 $k = 3$
 $h(x) = \frac{3}{x-2} + 1$ 4)

2) $y = f(x) = 3 \cdot 2^{x-p} + q$
 $q = 1$
 $3 ; 4) : 4 = 3 \cdot 2^{3-p} + 1$
 $3 = 3 \cdot 2^{3-p}$
 $2^{3-p} = 1$
 $3 - p = 0$
 $p = 3$ 4)

3) $2 < x \leq 3$ 2)
[19]

QUESTION 8

(a) $2x + 2y = 64$

$$y = 32 - x$$

$$x + h = 20$$

$$h = 20 - x$$

$$\text{Area ABEDC} = \frac{1}{2}xh + xy$$

$$= \frac{x}{2}(20 - x) + x(32 - x)$$

$$= 10x - \frac{x^2}{2} + 32x - x^2$$

$$= -\frac{3x^2}{2} + 42x$$

6)

b) $\frac{d\text{Area}}{dx} = -3x + 42 = 0$

$$-3x = -42$$

$$x = 14$$

ALTERNATIVELY:

$$x = \frac{-42}{2(-\frac{3}{2})}$$

$$= 14$$

$$h = 6$$

$$y = 18$$

4)
[10]

QUESTION 9

(a) $y = kx^3 + lx^2$

At 1 ; 5) : $5 = k + l \dots\dots ①$

$$\frac{dy}{dx} = 3kx^2 + 2lx$$

At 1 ; 5) : $m = 3k + 2l = 12 \dots\dots ②$

$$3(5 - l) + 2l = 12$$

$$15 - 3l + 2l = 12$$

$$l = 3$$

$$k = 2$$

5)

b) $y = ax^3 + bx^2 + cx + d$

$$\frac{dy}{dx} = 3ax^2 + 2bx + c$$

$$\frac{d^2y}{dx^2} = 6ax + 2b$$

At a point of inflection, $\frac{d^2y}{dx^2} = 0$

$$6ax = -2b$$

$$x = \frac{-2b}{6a}$$

$$= \frac{-b}{3a}$$

4)

[9]**75 marks****Total: 150 marks**