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NATIONAL SENIOR CERTIFICATE EXAMINATION  
NOVEMBER 2023

**MATHEMATICS: PAPER II**

**MARKING GUIDELINES**

Time: 3 hours

150 marks

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These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

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**NOTE:**

- If a candidate answers a question more than once, only mark the FIRST attempt.
- Consistent Accuracy applies in all aspects of the marking memorandum.

**SECTION A****QUESTION 1**

|        |                                                                                                                                            |                                               |
|--------|--------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------|
| (a)(1) | $95 - a = 80$<br>$a = 15$                                                                                                                  | $a = 15$                                      |
| (a)(2) | $b - 40 = 30$<br>$b = 70$                                                                                                                  | $b = 70$                                      |
| (b)    | $Q_1 - 1,5 \times \text{IQR}$<br>$= 40 - 1,5 \times 30$<br>$= -5$<br>Since the minimum value is greater than -5, it is not an outlier.     | -5<br>not an outlier                          |
| (c)(1) | $y = a + bx$<br>$y = 19,259 + 0,552x$<br>$y = 19,259 + 0,552(180)$<br>$y = 118,603$<br>The learner would obtain 100%.                      | 19,259<br>0,552<br>Learner would obtain 100%. |
| (c)(2) | No,<br><br>180 min is outside of the data set, therefore extrapolation. Implies that anyone who studies for over 180 min will obtain 100%. | No<br><br>Explanation                         |

**QUESTION 2**

|        |                                                                                                                                                                                                                                                                   |                                                                                |
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| (a)(1) | $m_{BC} = \frac{7-4}{6-0}$ $\therefore m_{BC} = \frac{1}{2}$                                                                                                                                                                                                      | $m_{BC} = \frac{7-4}{6-0}$ $m_{BC} = \frac{1}{2}$                              |
| (a)(2) | $m_{CD} = -2$ <p>Eq. of CD:</p> $y = -2x + c \dots \text{sub. (6;7)}$ $c = 19$ $\therefore y = -2x + 19$ <p>Alternate:</p> $m_{CD} = -2$ $y - 7 = -2(x - 6)$ $y = -2x + 19$                                                                                       | $m_{CD} = -2$ <p>sub (6;7)</p> $y = -2x + 19$                                  |
| (b)    | <p>Equation AD:</p> $y = \frac{1}{2}x + c \dots \text{sub (4;1)}$ $c = -1$ $\therefore y = \frac{1}{2}x - 1$ <p>Point of intersection of CD and AD:</p> $-2x + 19 = \frac{1}{2}x - 1$ $-5x = -40$ $x = 8$ $\therefore y = -2(8) + 19$ $y = 3$ $\therefore D(8;3)$ | $y = \frac{1}{2}x + c$ $c = -1$ $-2x + 19 = \frac{1}{2}x - 1$ $x = 8$ $D(8;3)$ |

|        |                                                                                                                                                                                                                                                                                                                |                                                                                         |
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|        | <p>Alternate:<br/>Equation AD: sub (4;1)</p> $y - 1 = \frac{1}{2}(x - 4)$ $y = \frac{x}{2} - 1$ <p>Point of intersection of CD and AD:</p> $-2x + 19 = \frac{x}{2} - 1$ $-5x = -40$ $x = 8$ $\therefore y = -2(8) + 19$ $y = 3$ $\therefore D(8;3)$                                                            |                                                                                         |
| (c)(1) | $E\left(\frac{6+4}{2}; \frac{7+1}{2}\right)$ $\therefore E(5;4)$ <p>Midpoint AE <math>\left(\frac{9}{2}; \frac{5}{2}\right)</math></p> $F\left(\frac{x+8}{2}; \frac{y+3}{2}\right)$ $\therefore F(1;2)$                                                                                                        | $E(5;4)$ <p>Midpoint AE <math>\left(\frac{9}{2}; \frac{5}{2}\right)</math></p> $F(1;2)$ |
| (c)(2) | <p>From Q2b: <math>m_{AD} = \frac{1}{2}</math></p> $\tan \theta_1 = \frac{1}{2}$ $\theta_1 = 26,6^\circ$ <p>Gradient AC:</p> $m_{AC} = \frac{7-1}{6-4}$ $\therefore m_{AC} = 3$ $\tan \theta_2 = 3$ $\therefore \theta_2 = 71,6^\circ$ $\hat{EAD} = 71,6^\circ - 26,6^\circ$ $\therefore \hat{EAD} = 45^\circ$ | $\theta_1 = 26,6^\circ$ $m_{AC} = 3$ $\theta_2 = 71,6^\circ$ $\hat{EAD} = 45^\circ$     |

|        |                                                                                                                                                                                                                                                                                                                                                                                                                                                    |                                                                                                                                                                                                                                  |
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| (d)(2) | $\text{Area EAD} = \frac{1}{2} \text{EA} \times \text{AD} \times \sin \hat{\text{EAD}}$ $\text{Dist EA} = \sqrt{10}$ $\text{Dist AD} = 2\sqrt{5}$ $\text{Area EAD} = \frac{1}{2} \times \sqrt{10} \times 2\sqrt{5} \times \sin 45^\circ$ $\therefore \text{Area EAD} = 5 \text{ units}^2$ $\therefore \text{Area EFA} = 5 \text{ units}^2 \quad \dots \text{diag of } //^m \text{ bisect area}$ $\therefore \text{Area ADEF} = 10 \text{ units}^2$ | $\text{Dist EA} = \sqrt{10}$ $\text{Dist AD} = 2\sqrt{5}$ $\text{Area EAD}$ $= \frac{1}{2} \times \sqrt{10} \times 2\sqrt{5} \times \sin 45^\circ$ $\text{Area EAD} = 5 \text{ units}^2$ $\text{Area ADEF} = 10 \text{ units}^2$ |
|--------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

**QUESTION 3**

|     |                                                                                                                        |                                                                                                                                                              |
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| (a) | $\bar{x} = \frac{(10 \times 1) + (30 \times 7) + (50 \times 10) + (70 \times 5) + (90 \times 4)}{27}$ $\bar{x} = 53$   | midpt x frequency<br>divide by 27<br><br>$\bar{x} = 53$                                                                                                      |
| (b) | $\frac{5}{27} \times 100$ $= 18,5\%$                                                                                   | $= 18,5\%$                                                                                                                                                   |
| (c) | <p data-bbox="316 622 970 1261"> </p> <p data-bbox="316 1294 363 1328">Alt:</p> <p data-bbox="316 1384 970 1977"> </p> | <p data-bbox="1034 645 1106 678">(0;0)</p> <p data-bbox="1034 723 1297 757">Plotting Endpoints</p> <p data-bbox="1034 790 1361 824">Cumulative Frequency</p> |

|            |                                                                              |                                    |
|------------|------------------------------------------------------------------------------|------------------------------------|
| (d)(1)(i)  |                                                                              | see graph                          |
| (d)(1)(ii) |                                                                              | see graph                          |
| (d)(2)     | Mean > Median<br>Positively skewed<br><br>OR<br>Mean = Median<br>Symmetrical | mean > median<br>positively skewed |

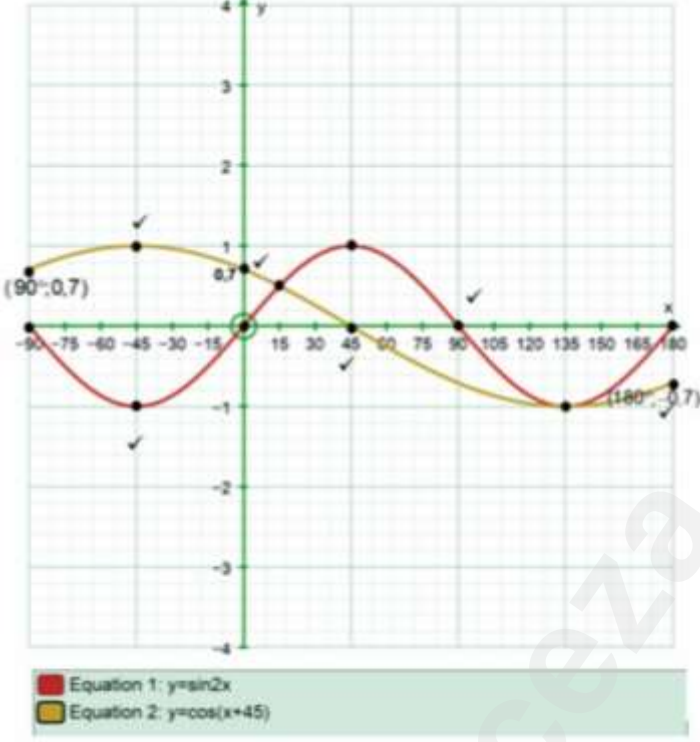
**QUESTION 4**

|     |                                                                                                                                                                                                                                                 |                                                                                           |
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| (a) | $\hat{C}_1 = 25^\circ$ ... angles opp = sides/radii<br><br>$\hat{COA} = 180^\circ - 50^\circ$ ... int. angles of triangle<br>$= 130^\circ$                                                                                                      | $\hat{C}_1 = 25^\circ$<br>$= 130^\circ$                                                   |
| (b) | $\hat{C}_2 = 25^\circ$ ... alt. angles OA//CB<br>$\hat{C}_1 + \hat{C}_2 = \hat{B}$ ... angles opp = sides/radii<br>$\therefore \hat{B} = 50^\circ$<br>$\hat{O}_1 = 180^\circ - 100^\circ$ ... int. angles of triangle<br>$\hat{O}_1 = 80^\circ$ | $\hat{C}_2 = 25^\circ$<br>$\hat{C}_1 + \hat{C}_2 = \hat{B}$<br><br>$\hat{O}_1 = 80^\circ$ |

## QUESTION 5

|     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|-----|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| (a) | <p>Construct:</p> <p>Join DC and BE</p> $\frac{\text{Area } \triangle ADE}{\text{Area } \triangle BDE} = \frac{\frac{1}{2} \times AD \times EF}{\frac{1}{2} \times DB \times EF}$ $\frac{\text{Area } \triangle ADE}{\text{Area } \triangle BDE} = \frac{AD}{BD} \quad \dots (1)$ $\frac{\text{Area } \triangle ADE}{\text{Area } \triangle DEC} = \frac{\frac{1}{2} \times AE \times DG}{\frac{1}{2} \times EC \times DG}$ $\frac{\text{Area } \triangle ADE}{\text{Area } \triangle DEC} = \frac{AE}{CE} \quad \dots (2)$ <p>But: <math>\text{Area } \triangle BDE = \text{Area } \triangle DEC</math> ... same base,<br/>between // lines</p> $\therefore \frac{AD}{BD} = \frac{AE}{CE}$ | <p>construction</p> $\frac{\text{Area } \triangle ADE}{\text{Area } \triangle BDE} = \frac{\frac{1}{2} \times AD \times EF}{\frac{1}{2} \times DB \times EF}$ $\frac{\text{Area } \triangle ADE}{\text{Area } \triangle DEC} = \frac{\frac{1}{2} \times AE \times DG}{\frac{1}{2} \times EC \times DG}$ <p><math>\text{Area } \triangle BDE = \text{Area } \triangle DEC</math></p> <p>reason</p> $\therefore \frac{AD}{BD} = \frac{AE}{CE}$ |
| (b) | <p>In <math>\triangle HIJ</math>:</p> $(HI)^2 = (29)^2 - (21)^2 \quad \dots \text{pythag}$ <p><math>HI = 20</math> units</p> $\frac{IL}{IJ} = \frac{IK}{IH} \quad \dots \text{line } // \text{ to one side of } \triangle$ $\frac{12}{21} = \frac{IK}{20}$ $\therefore IK = 11\frac{3}{7}$ <p>or <math>IK = 11,4</math> units</p>                                                                                                                                                                                                                                                                                                                                                           | $(HI)^2 = (29)^2 - (21)^2$ <p><math>HI = 20</math> units</p> $\frac{IL}{IJ} = \frac{IK}{IH}$ <p>reason</p> $IK = 11\frac{3}{7}$                                                                                                                                                                                                                                                                                                              |

## QUESTION 6

|        |                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                                                                                      |
|--------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| (a)    |                                                                                                                                                                                                                                                    | <p>Graph of <math>f</math><br/>amplitude<br/>frequency<br/>turning pts, endpts and<br/>intercepts and shape correct</p> <p>Graph of <math>g</math><br/>turning pts<br/>frequency<br/>endpts and intercepts and<br/>shape correct</p> |
| (b)    | $\sin 2x = \cos(x + 45^\circ)$ $\cos(90^\circ - 2x) = \cos(x + 45^\circ)$ $90^\circ - 2x = x + 45^\circ + 360^\circ k ; k \in \mathbb{Z}$ $x = 15^\circ + 120^\circ k$ $\therefore x = 15^\circ$ <p>OR</p> $90^\circ - 2x = -x - 45^\circ + 360^\circ k$ $x = 135^\circ + 360^\circ k ; k \in \mathbb{Z}$ $\therefore x = 135^\circ$ | $\cos(90^\circ - 2x)$ $\cos(90^\circ - 2x) = \cos(x + 45^\circ)$ $90^\circ - 2x = x + 45^\circ + 360^\circ k$ $\therefore x = 15^\circ$<br>$\therefore x = 135^\circ$                                                                |
| (c)(1) | $x \in [15^\circ; 180^\circ]$ <p>Alt:<br/><math>15^\circ \leq x \leq 180^\circ</math></p>                                                                                                                                                                                                                                            | $x \in [15^\circ; 180^\circ]$                                                                                                                                                                                                        |
| (c)(2) | $x \in (-90^\circ; 0^\circ) \cup (45^\circ; 90^\circ)$ <p>Alt:<br/><math>-90^\circ &lt; x &lt; 0</math> or <math>45^\circ &lt; x &lt; 90^\circ</math></p>                                                                                                                                                                            | $x \in (-90^\circ; 0^\circ) \cup (45^\circ; 90^\circ)$                                                                                                                                                                               |

## SECTION B

## QUESTION 7

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                               |
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| <p>In <math>\triangle ABC</math>:</p> <p><math>\hat{B} = 90^\circ</math> ... tangent perp to radius<br/> <math>AO = 7</math> ... radius</p> <p><math>\therefore \cos 38^\circ = \frac{14}{AC}</math></p> <p><math>AC = \frac{14}{\cos 38^\circ}</math><br/> <math>AC = 17,766...</math></p> <p>Construct: DB</p> <p><math>\hat{ADB} = 90^\circ</math> ... angle in semi-circle<br/> <math>\cos 38^\circ = \frac{AD}{14}</math><br/> <math>\therefore AD = 11,032...</math></p> <p><math>\therefore CD = 17,766... - 11,032...</math><br/> <math>\therefore CD \approx 6,7</math> units</p> <p>Alt:</p> <p><math>\hat{B} = 90^\circ</math> ... tangent perp to radius<br/> <math>AO = 7</math> ... radius</p> <p>In <math>\triangle ABC</math>:</p> <p><math>\tan 38^\circ = \frac{CB}{14}</math><br/> <math>\therefore CB = 10,9</math> units</p> <p>Construct: DB</p> <p><math>\hat{ADB} = 90^\circ</math> ... angle in semi-circle</p> <p>In <math>\triangle DCB</math>:</p> <p><math>\hat{C} = 52^\circ</math><br/> <math>\therefore \cos 52^\circ = \frac{CD}{CB}</math><br/> <math>\therefore CD = 6,7</math> units</p> | <p><math>\hat{B} = 90^\circ</math> and reason</p> <p><math>\cos 38^\circ = \frac{14}{AC}</math></p> <p><math>AC = 17,766...</math></p> <p><math>\hat{ADB} = 90^\circ</math><br/> reason<br/> <math>\cos 38^\circ = \frac{AD}{14}</math></p> <p><math>\therefore AD = 11,032...</math></p> <p><math>\therefore CD \approx 6,7</math> units</p> |
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## QUESTION 8

|     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |                                                                                                                                                                |
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| (a) | $a > b > 0$ and $\sin \theta < 0$<br>$\therefore$ Quadrant 4<br>$x = a^2 - b^2$<br>$r = a^2 + b^2$<br>$\therefore y^2 = (a^2 + b^2)^2 - (a^2 - b^2)^2 \dots \text{pythag}$<br>$y^2 = a^4 + 2a^2b^2 + b^4 - a^4 + 2a^2b^2 - b^4$<br>$\therefore y^2 = 4a^2b^2$<br>$y = \pm \sqrt{4a^2b^2}$<br>$\therefore y = -2ab \dots \text{quad 4}$<br>$\therefore \tan \theta = -\frac{2ab}{a^2 - b^2}$                                                                                                                      | $\therefore$ Quadrant 4<br>$\therefore y^2 = (a^2 + b^2)^2 - (a^2 - b^2)^2$<br>$\therefore y^2 = 4a^2b^2$<br>$\therefore \tan \theta = -\frac{2ab}{a^2 - b^2}$ |
| (b) | $\tan \theta = -\frac{2ab}{a^2 - b^2}$<br>$\tan \theta = -\frac{2 \times (3) \times (2)}{(3)^2 - (2)^2}$<br>Ref angle: $-67,4^\circ$<br>$\theta = -67,4^\circ + 360^\circ(k) \dots k \in \mathbb{Z}$<br>$\theta \in \{292,6^\circ; 652,6^\circ\}$<br><b>Alternate:</b><br>$\cos \theta = \frac{(3)^2 - (2)^2}{(3)^2 + (2)^2}$<br>$\therefore \cos \theta = \frac{5}{13}$<br>Ref angle: $67,4^\circ$<br>$\theta = 292,6^\circ + 360^\circ(k) \dots k \in \mathbb{Z}$<br>$\theta \in \{292,6^\circ; 652,6^\circ\}$ | $\tan \theta = -\frac{2 \times (3) \times (2)}{(3)^2 - (2)^2}$<br>Ref angle<br>$\theta \in \{292,6^\circ; 652,6^\circ\}$                                       |

## QUESTION 9

|        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                            |
|--------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| (a)    | $\frac{\frac{1}{2}\cos(90^\circ + \theta) - \sin\theta \cdot \sin(\theta - 90^\circ)}{\cos^2(180^\circ - \theta) - 2\cos(-\theta) + \cos^2(\theta + 90^\circ)}$ $= \frac{-\frac{1}{2}\sin(\theta) - (\sin\theta) \cdot (-\cos(\theta))}{\cos^2(\theta) - 2\cos(\theta) + \sin^2(\theta)}$ $= \frac{-\frac{1}{2}\sin\theta + \sin\theta \cdot \cos\theta}{1 - 2\cos\theta}$ $= \frac{-\frac{1}{2}\sin\theta(1 - 2\cos\theta)}{1 - 2\cos\theta}$ $= -\frac{1}{2}\sin\theta$                                                                                                                                                                                                                                                                                         | $-\frac{1}{2}\sin(\theta)$ $(-\cos(\theta))$ $\cos^2(\theta)$ $-2\cos(\theta)$ $\sin^2\theta + \cos^2\theta = 1$ $-\frac{1}{2}\sin\theta(1 - 2\cos\theta)$ $= -\frac{1}{2}\sin\theta$                                                                                      |
| (b)(1) | $\text{LHS} = \sin\theta \times \frac{\sin\theta}{\cos\theta} \div \left[ \frac{\sin 2\theta}{\cos 2\theta} \times \left( 1 - \frac{\sin^2\theta}{\cos^2\theta} \right) \right]$ $= \frac{\sin^2\theta}{\cos\theta} \div \left[ \frac{\sin 2\theta}{\cos 2\theta} \times \left( \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta} \right) \right]$ $= \frac{\sin^2\theta}{\cos\theta} \div \left[ \frac{\sin 2\theta}{\cos 2\theta} \times \left( \frac{\cos 2\theta}{\cos^2\theta} \right) \right]$ $= \frac{\sin^2\theta}{\cos\theta} \div \left[ \frac{2\sin\theta \cdot \cos\theta}{\cos^2\theta} \right]$ $= \frac{\sin^2\theta}{\cos\theta} \times \left[ \frac{\cos^2\theta}{2\sin\theta \cdot \cos\theta} \right]$ $= \frac{\sin\theta}{2}$ $= \text{RHS}$ | $\frac{\sin\theta}{\cos\theta}$ $\left( \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta} \right)$ $\cos 2\theta$ $2\sin\theta \cdot \cos\theta$ $= \frac{\sin^2\theta}{\cos\theta} \times \left[ \frac{\cos^2\theta}{2\sin\theta \cdot \cos\theta} \right]$ $= \text{RHS}$ |
| (b)(2) | <p><b>Working: Not valid for:</b></p> $\sin 2\theta = 0$ $\therefore \theta = 90^\circ k \quad \dots k \in \mathbb{Z}$ $\cos 2\theta = 0$ $\therefore \theta = 45^\circ + 90^\circ k \quad \dots k \in \mathbb{Z}$ $\cos\theta = 0 \quad \therefore \theta = \pm 90^\circ + 360^\circ k \quad \dots k \in \mathbb{Z}$ $\tan^2\theta = 1$ $\therefore \theta = \pm 45^\circ + 180^\circ k \quad \dots k \in \mathbb{Z}$ <p><b>Therefore, not valid for <math>\theta = 45^\circ k \quad \dots k \in \mathbb{Z}</math></b></p>                                                                                                                                                                                                                                       | $\checkmark \tan 2\theta = 0$ $\checkmark \text{General solution}$ $\checkmark 1 - \tan^2\theta = 0$ $\checkmark \text{General solution}$ $\checkmark \theta = 45^\circ k \quad \dots k \in \mathbb{Z}$                                                                    |

|  |                                                                                                                                                                                                                                                                                                                                        |  |
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|  | <p>Alt:</p> <p><math>\tan 2\theta = 0</math></p> <p><math>\therefore 2\theta = 180^\circ k</math></p> <p><math>\therefore \theta = 90^\circ k</math> and <math>\tan 2\theta</math> is undefined for:</p> <p><math>\theta = 45^\circ + 90^\circ k</math></p> <p>Thus: <math>\theta = 45^\circ k \quad \dots k \in \mathbb{Z}</math></p> |  |
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**QUESTION 10**

|     |                                                                                                                                                                                                                                    |                                                                                                                                          |
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| (a) | <p><math>\hat{A}_1 = 90^\circ \quad \dots</math> tangent perp to radius</p> <p><math>\hat{D}_3 = 90^\circ \quad \dots</math> tangent perp to radius</p> <p><math>\therefore</math> AODE is a cyclic quad... Opp. angles suppl.</p> | <p><math>\hat{A}_1 = 90^\circ</math></p> <p>tangent perp to radius</p> <p><math>\hat{D}_3 = 90^\circ</math></p> <p>Opp angles suppl.</p> |
| (b) | <p><math>\hat{O}_2 = 2(68^\circ) \quad \dots</math> angle at centre</p> <p><math>\hat{O}_2 = 136^\circ</math></p> <p><math>\therefore \hat{E} = 44^\circ \quad \dots</math> opp. angles of cyclic quad</p>                         | <p><math>\hat{O}_2 = 136^\circ</math></p> <p>reason</p> <p><math>\therefore \hat{E} = 44^\circ</math></p> <p>reason</p>                  |

## QUESTION 11

|     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                                                   |
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| (a) | $x^2 - 12x + y^2 - 4y = -p$ $(x-6)^2 + (y-2)^2 = -p + 36 + 4$ $(x-6)^2 + (y-2)^2 = -p + 40$ $C(6;2) \therefore \text{radius} = 2$ $-p + 40 = 2^2$ $\therefore p = 36$                                                                                                                                                                                                                                                                                                                                 | $(x-6)^2$ $(y-2)^2$ $\text{radius} = 2$ $-p + 40 = 2^2$ $\therefore p = 36$                                                                                                                                       |
| (b) | <p>Draw CB</p> $m_{CB} = \frac{0-2}{12-6}$ $m_{CB} = -\frac{1}{3}$ <p>Draw CD: radius perp. to tangent OB at D</p> $\tan \hat{CBD} = \frac{1}{3}$ $\therefore \hat{CBD} = 18,4349\dots$ $\triangle ECB \equiv \triangle DCB \dots \text{RHS}$ $\therefore \hat{DBA} = 2 \times 18,4349\dots$ $\hat{DBA} = 36,8698\dots$ $m_{AB} = \tan(180^\circ - 36,8689\dots)$ $m_{AB} \approx -0,75 = -\frac{3}{4}$ $y = -\frac{3}{4}x + c \dots \text{sub. } (12; 0)$ $c = 9$ $\therefore y = -\frac{3}{4}x + 9$ | $m_{CB} = -\frac{1}{3}$ $\therefore \hat{CBD} = 18,4349\dots$ $\triangle ECB \equiv \triangle DCB \dots \text{RHS}$ $\hat{DBA} = 36,8698\dots$ $m_{AB} = -\frac{3}{4}$ $c = 9$ $\therefore y = -\frac{3}{4}x + 9$ |

|     |                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                                                                |
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| (c) | <p>Second Circle:</p> $x^2 + (y - 9)^2 = r^2 \quad \dots \text{sub. } (2;3)$ $r^2 = 40$ $\text{Alt: } r = \sqrt{(2 - 0)^2 + (3 - 9)^2} = \sqrt{40}$ <p>Distance between centres</p> $= \sqrt{(6 - 0)^2 + (2 - 9)^2}$ $= \sqrt{85}$ $\approx 9,2$ <p>Sum of radii = <math>\sqrt{40} + 2</math></p> $\approx 8,3$ <p>They do not intersect since the distance between centres is greater than the sum of the radii.</p> | $r^2 = 40$ $\approx 9,2$ $\approx 8,3$ <p>They do not intersect.</p> <p>The distance between centres is greater than the sum of the radii.</p> |
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## QUESTION 12

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|-----|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| (a) | <p>In <math>\triangle ABC</math> and <math>\triangle OFC</math>:</p> <p><math>\hat{C}_1 = \hat{C}_2</math> ... given</p> <p><math>\hat{B} = 90^\circ</math> ... angle in semi-circle</p> <p><math>\therefore \hat{B} = \hat{F}_1</math></p> <p><math>\hat{A}_2 = \hat{O}_2</math> ... third angle</p> <p><math>\therefore \triangle ABC \parallel \triangle OFC</math> ... equiangular</p>                                                                                                                                                                                                                                                                                                                                                                                                                                             | <p><math>\hat{C}_1 = \hat{C}_2</math></p> <p><math>\hat{B} = 90^\circ</math> ...</p> <p>angle in semi-circle</p> <p><math>\therefore \triangle ABC \parallel \triangle OFC</math> ...</p> <p>equiangular</p>                                                                                                                                                                                   |
| (b) | <p><math>\frac{BC}{FC} = \frac{AC}{OC}</math> ... <math>\parallel</math> triangles; sides in prop</p> <p>Let: <math>OC = x</math></p> <p><math>\therefore AC = 2x</math> ... radii</p> <p><math>\therefore BC : FC = 2 : 1</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | <p><math>\frac{BC}{FC} = \frac{AC}{OC}</math></p> <p><math>\therefore BC : FC = 2 : 1</math></p>                                                                                                                                                                                                                                                                                               |
| (c) | <p><math>LHS = \left(\frac{AC}{2}\right)^2 - \left(\frac{AB}{2}\right)^2</math></p> <p><math>LHS = \frac{(AC)^2 - (AB)^2}{4}</math></p> <p><math>= \frac{(BC)^2}{4}</math> pythag</p> <p><math>\frac{(BC)^2}{4} = (CF)^2</math> from 12(b)</p> <p>RTP: <math>(FC)^2 = DF \times FA</math></p> <p>In <math>\triangle CFA</math> and <math>\triangle DFE</math></p> <p><math>\hat{E} = \hat{A}</math> ... angle in same segment</p> <p><math>\hat{C}_1 = \hat{D}</math> ... angle in same segment</p> <p><math>\therefore \triangle CFA \parallel \triangle DFE</math> ... equiangular</p> <p><math>\therefore \frac{CF}{DF} = \frac{FA}{FE}</math></p> <p><math>CF = FE</math> ... line from centre perp to chord</p> <p><math>\therefore \frac{CF}{DF} = \frac{FA}{CF}</math></p> <p><math>\therefore (FC)^2 = DF \times FA</math></p> | <p><math>\frac{(BC)^2}{4}</math></p> <p><math>\frac{(BC)^2}{4} = (CF)^2</math></p> <p><math>\hat{E} = \hat{A}</math> and <math>\hat{C}_1 = \hat{D}</math></p> <p>reason</p> <p><math>\therefore \triangle CFA \parallel \triangle DFE</math></p> <p><math>\therefore \frac{CF}{DF} = \frac{FA}{FE}</math></p> <p><math>CF = FE</math></p> <p><math>\therefore (CF)^2 = DF \times FA</math></p> |

Alternate:

In  $\triangle OFC$ :  $(OC)^2 = (CF)^2 + (OF)^2$  ... pythag

$$(CF)^2 = (OC)^2 - (OF)^2$$

Since  $AC = 2 \cdot OC$

$$(CF)^2 = \left(\frac{AC}{2}\right)^2 - \left(\frac{AB}{2}\right)^2$$

RTP:  $(CF)^2 = DF \times FA$

In  $\triangle CFA$  and  $\triangle DFE$

$\hat{E} = \hat{A}$  ... angle in same segment

$\hat{C}_1 = \hat{D}$  ... angle in same segment

$\therefore \triangle CFA \parallel \triangle DFE$  ... equiangular

$$\therefore \frac{CF}{DF} = \frac{FA}{FE}$$

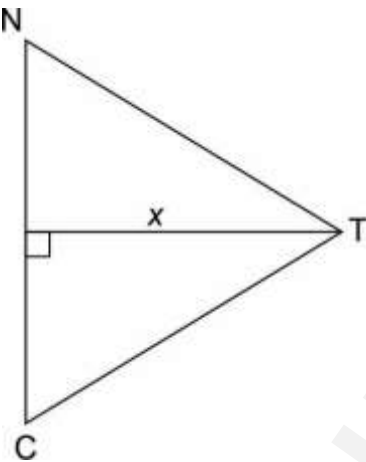
$CF = FE$  ... line from centre perp to chord

$$\therefore \frac{CF}{DF} = \frac{FA}{CF}$$

$$\therefore (CF)^2 = DF \times FA$$

$$\text{Then: } \therefore \left(\frac{AC}{2}\right)^2 - \left(\frac{AB}{2}\right)^2 = DF \times FA$$

## QUESTION 13

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                                                                                                                                                                                                                  |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\hat{N}TC = 46^\circ \dots \text{int } \angle \text{ of } \Delta$<br><br>For CT:<br>$\frac{10}{\sin 46^\circ} = \frac{CT}{\sin 69^\circ}$<br>$CT = 12,978295\dots$<br><br>In $\Delta ATC$ :<br>$\tan 43,5^\circ = \frac{\text{Height of tree}}{12,978\dots}$<br>$\therefore \text{Height} = 12,3 \text{ m}$<br><br><br>$\frac{x}{12,978295} = \sin 65^\circ$<br>$\therefore x = 11,7623\dots$<br>$\therefore \text{The tree will hit the house.}$ | $\hat{N}TC = 46^\circ \dots \text{int } \angle \text{ of } \Delta$<br><br>$\frac{10}{\sin 46^\circ} = \frac{CT}{\sin 69^\circ}$<br>$CT = 12,978295\dots$<br>$\tan 43,5^\circ = \frac{\text{Height of tree}}{12,978\dots}$<br>$\therefore \text{Height} = 12,3 \text{ m}$<br>$\frac{x}{12,978295} = \sin 65^\circ$<br>$\therefore x = 11,7623\dots$<br>conclusion |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

Total: 150 marks