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NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2021

MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours

150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

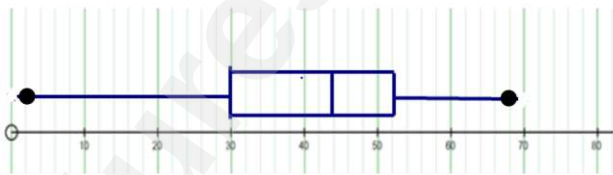
NOTE:

- If a candidate answers a question more than once, only mark the FIRST attempt.
- Consistent Accuracy applies in all aspects of the marking memorandum.

SECTION A**QUESTION 1**

(a)	$A = -160,645$ $B = 21,505$ $y = -160,645 + 21,505x$	$A = -160,645$ $B = 21,505$ rounding
(b)	$y = -160,645 + 21,505(90)$ $y = 1774,81$ Alternate with calculator: R 1774,79	R1 774,81 Alt: R 1774,79
(c)	Extrapolation has its risks, i.e. when working outside the boundaries of the given data.	Extrapolation
(d)	$r = 0,912$	$r = 0,912$
(e)	Very strong positive correlation	Very strong positive correlation

QUESTION 2

(a)	 Correct box and whisker plot accordingly	Shape: Box & Whisker Min: 2 Max: 68 Q1: 30 Q2: 44 Q3: 52 Max. 2 if box & whisker has errors
(b)	Skewed left / negatively skewed	negatively skewed
(c)	Since $\text{Range A} > \text{Range B}$ and $\text{IQR}_A > \text{IQR}_B$, the heights of the plants grown in Environment A were more spread out.	as described

QUESTION 3

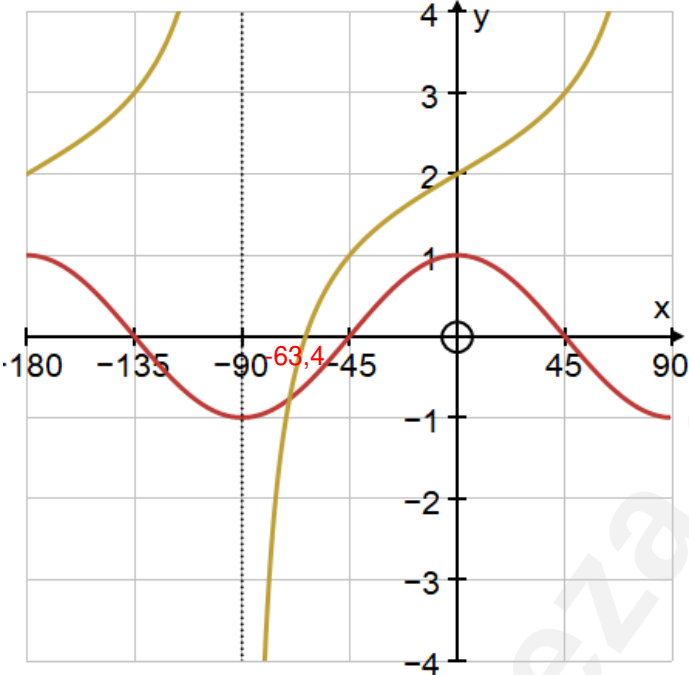
(a)	$\text{Length AB} = \sqrt{(x^2 - x_1)^2 + (y_2 - y_1)^2}$ $\text{Length AB} = \sqrt{(11 - 6)^2 + (12 - 16)^2}$ $\text{Length AB} = \sqrt{25 + 16}$ $\text{Length AB} = \sqrt{41}$	$= \sqrt{(11 - 6)^2 + (12 - 16)^2}$ Sub in dist. formula $= \sqrt{41}$
(b)	$m_{AB} = \frac{16 - 12}{6 - 11}$ $m_{AB} = -\frac{4}{5}$ $m_{DE} = \frac{-11 + 3}{6 + 4} = -\frac{8}{10}$ $m_{DE} = -\frac{4}{5}$ Gradients are equal $\therefore AB \parallel DE$	Gradients $m_{AB} = -\frac{4}{5}$ $m_{DE} = -\frac{4}{5}$
(c)	Eq. of line DB: $y = mx + c$ sub. ($m_{DB} = 1$) $y = x + c$ sub. $(-4; -3)$ or $(11; 12)$ $-3 = -4 + c$ $c = 1$ $\therefore y = x + 1$ For point of int. sub. $x = 6$ $\therefore y = 7$ $\therefore k = 7$	sub. ($m_{DB} = 1$) $c = 1$ $x = 6$ $\therefore y = 7$
(d)	$m_{AB} = -\frac{4}{5}$ $\tan \theta = m$ $\theta \approx 38,7^\circ$ $AE \perp x\text{-axis} \therefore \alpha = 90^\circ$ $\hat{BAC} = 180^\circ - (90^\circ + 38,7^\circ) \text{ (int. } \angle \text{ of } \Delta)$ $\hat{BAC} = 51,3^\circ$	$\theta \approx 38,7^\circ$ $AE \perp x\text{-axis} \therefore \alpha = 90^\circ$ $\hat{BAC} = 51,3^\circ$

(e)	$\frac{\text{Area } \triangle ABC}{\text{Area } \triangle EDC} = \frac{\frac{1}{2}(AB)(BC)\sin \hat{B}}{\frac{1}{2}(CD)(DE)\sin \hat{D}}$ $\triangle ABC \sim \triangle EDC \quad (\text{equiangular})$ $\therefore \frac{AB}{ED} = \frac{BC}{DC} = \frac{AC}{EC}$ Since $\hat{D} = \hat{B}$ (alt $\angle$s; // lines) and $\frac{AB}{DE} = \frac{BC}{DC}$ (/// Δs, sides in Prop) $\therefore \frac{\text{Area } \triangle ABC}{\text{Area } \triangle EDC} = \frac{(AB)^2}{(DE)^2}$ $\therefore \frac{\text{Area } \triangle ABC}{\text{Area } \triangle EDC} = \frac{(\sqrt{41})^2}{(2\sqrt{41})^2}$ $\therefore \frac{\text{Area } \triangle ABC}{\text{Area } \triangle EDC} = \frac{1}{4}$ Alternate 1: $\frac{\text{Area } \triangle ABC}{\text{Area } \triangle EDC} = \frac{\frac{1}{2}(AC)(AB)\sin \hat{A}}{\frac{1}{2}(CE)(ED)\sin \hat{E}}$ $\frac{\text{Area } \triangle ABC}{\text{Area } \triangle EDC} = \frac{9 \times \sqrt{41}}{18 \times 2\sqrt{41}} = \frac{1}{4}$ Alternate 2: $\frac{\text{Area } \triangle ABC}{\text{Area } \triangle EDC} = \frac{\frac{1}{2}(AC)(h_B)}{\frac{1}{2}(CE)(h_D)}$ $\frac{\text{Area } \triangle ABC}{\text{Area } \triangle EDC} = \frac{9 \times 5}{18 \times 10} = \frac{1}{4} \quad \dots h_B = 12 - 7$	$= \frac{\frac{1}{2}(AB)(BC)\sin \hat{B}}{\frac{1}{2}(CD)(DE)\sin \hat{D}}$ $\hat{D} = \hat{B} \quad (\text{alt } \angle\text{s; // lines})$ $\frac{AB}{DE} = \frac{BC}{DC} \quad (\text{/// } \Delta\text{s, sides in Prop})$ $\frac{\text{Area } \triangle ABC}{\text{Area } \triangle EDC} = \frac{(AB)^2}{(DE)^2}$ $\frac{\text{Area } \triangle ABC}{\text{Area } \triangle EDC} = \frac{1}{4}$ $\frac{\frac{1}{2}(AC)(AB)\sin \hat{A}}{\frac{1}{2}(CE)(ED)\sin \hat{E}}$ Cancelling $\frac{9 \times \sqrt{41}}{18 \times 2\sqrt{41}} = \frac{1}{4}$ $\frac{\frac{1}{2}(AC)(h_B)}{\frac{1}{2}(CE)(h_D)}$ Perp heights 5 and 10 Values 9 and 18 $\frac{9 \times 5}{18 \times 10} = \frac{1}{4}$
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QUESTION 4

(a)(1)	$\text{Length AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\text{Length AB} = \sqrt{(2 - 1)^2 + (8 + 1)^2}$ $\text{Length AB} = \sqrt{1 + 81}$ $\text{Length AB} = \sqrt{82}$ Alternate: $AB = \sqrt{82}$	$= \sqrt{(2 - 1)^2 + (8 + 1)^2}$ Sub in dist. formula $= \sqrt{82}$ $ AB = \sqrt{82}$
(a)(2)	AB is a diameter: For centre: $\text{MidPt AB} \left(\frac{2+1}{2}; \frac{8-1}{2} \right)$ $\text{MidPt AB} \left(\frac{3}{2}; \frac{7}{2} \right)$ $r = \frac{\sqrt{82}}{2}$ $\left(x - \frac{3}{2} \right)^2 + \left(y - \frac{7}{2} \right)^2 = \frac{41}{2}$	$\text{MidPt AB} \left(\frac{3}{2}; \frac{7}{2} \right)$ $r = \frac{\sqrt{82}}{2}$ $\left(x - \frac{3}{2} \right)^2 + \left(y - \frac{7}{2} \right)^2 = \frac{41}{2}$
(a)(3)	$m_{\text{diam}} = \frac{8+1}{2-1} \therefore m_{\text{diam}} = 9$ $\therefore m_{\text{tan}} = -\frac{1}{9}$ $y = -\frac{1}{9}x + c \text{ sub. } (2;8)$ $c = 8\frac{2}{9}$ $y = -\frac{1}{9}x + 8\frac{2}{9}$ $9y = -x + 74$	$m_{\text{diam}} = \frac{8+1}{2-1} \therefore m_{\text{diam}} = 9$ $\therefore m_{\text{tan}} = -\frac{1}{9}$ $c = 8\frac{2}{9}$ $9y = -x + 74$
(b)	Construct AO $\therefore \text{AO} = 10$ units Radius $\text{AO} \perp \text{AM}$ Tan $\perp$ Rad $(\text{AM})^2 = (13)^2 - (10)^2$ Pythag $\text{AM} = \sqrt{69}$	AO = 10 units $\text{AO} \perp \text{AM} \therefore \text{Tan} \perp \text{Rad}$ $(\text{AM})^2 = (13)^2 - (10)^2$ $\text{AM} = \sqrt{69}$

QUESTION 5

(a)	 Equation 2: $y = \tan x + 2$ Equation 3: $x = -90$	$f(x)$: shape & (endpoints) turning points x-intercepts $g(x)$: shape & (endpoints) asymptote x-intercept $(63, 4^\circ)$ Do not penalise twice for incorrect endpoints
(b)	$\cos 2x \leq \tan x + 2$ $x \in [-180^\circ; -90^\circ) \cup [-70, 1^\circ; 90^\circ)$ Allow range of $[-65^\circ; -75^\circ]$ for point of intersection – reading from graph	$[-180^\circ; -90^\circ)$ $[-70, 1^\circ; 90^\circ)$

QUESTION 6

(a)	Construction: B through centre O Proof: $\hat{O}_1 = \hat{A} + \hat{B}_1$ (ext. $\angle$ of Δ) $\hat{A} = \hat{B}_1$ (Isos Δ / Radii) Similarly, in the other triangle: $\hat{O}_1 = 2 \times \hat{B}_1$ $\hat{O}_2 = 2 \times \hat{B}_2$ $\therefore \hat{AOC} = 2 \times \hat{ABC}$	B through centre O $\hat{O}_1 = \hat{A} + \hat{B}_1$ (ext. $\angle$ of Δ) $\hat{A} = \hat{B}_1$ (Isos Δ / Radii) $\hat{O}_1 = 2 \times \hat{B}_1$ $\hat{O}_2 = 2 \times \hat{B}_2$ $\therefore \hat{AOC} = 2 \times \hat{ABC}$
(b)(1)	$\hat{C}_1 = \hat{A}_1$ (Tan. from pt / isosceles Δ) $2\hat{A}_1 + \hat{T} = 180^\circ$ $\hat{A}_1 = 59^\circ$ (Int. $\angle$s of Δ)	$\hat{A}_1 = 59^\circ$ (Tan. from pt / isosceles Δ) (Int. $\angle$s of Δ)
(b)(2)	$\hat{A}_1 + \hat{A}_2 = 90^\circ$ (radius $\perp$ tangent) $\hat{A}_2 = 90^\circ - 59^\circ$ $\hat{A}_2 = 31^\circ$ $\hat{A}_2 = \hat{C}_2$ (Isos. Δ; CO=AO radii) $\therefore \hat{O}_1 = 118^\circ$ (int. $\angle$s of Δ) ALTERNATE: $\hat{A}_1 = \hat{B}$ (Tan-Chord Th) $\hat{A}_1 = 59^\circ$ (From (b)(1)) $\therefore \hat{O}_1 = 118^\circ$ ($\angle$ at centre = 2 X $\angle$ at cir)	$\hat{A}_1 + \hat{A}_2 = 90^\circ$ (radius $\perp$ tangent) $\hat{A}_2 = 90^\circ - 59^\circ$ $\hat{A}_2 = 31^\circ$ $\hat{A}_2 = \hat{C}_2$ $\therefore \hat{O}_1 = 118^\circ$ (int. $\angle$s of Δ) $\hat{A}_1 = \hat{B}$ (Tan-Chord Th) $\hat{A}_1 = 59^\circ$ (From (b)(1)) $\therefore \hat{O}_1 = 118^\circ$ ($\angle$ at centre = 2 X $\angle$ at cir)

QUESTION 7

(a)	$DO = 3$ units $AD:DO = 4:3$ $\frac{AD}{DO} = \frac{AE}{EC} = \frac{AF}{FB}$ (Prop Th – $DE \parallel OC$ & $EF \parallel CB$) $\therefore AF:FB = 4:3$	$AD:DO = 4:3$ $\frac{AD}{DO} = \frac{AE}{EC} = \frac{AF}{FB}$ with reason $\therefore AF:FB = 4:3$
(b)	$\triangle AHF \parallel \triangle AGB$ (Equiangular) $\therefore \frac{AH}{AG} = \frac{HF}{GB} = \frac{AF}{AB}$ (similar triangles, sides in prop) $AB = 7x$ $\therefore \frac{HF}{GB} = \frac{4x}{7x}$ $\therefore GB:HF = 7:4$	$\triangle AHF \parallel \triangle AGB$ with reason $\therefore \frac{AH}{AG} = \frac{HF}{GB} = \frac{AF}{AB}$ with reason $\therefore GB:HF = 7:4$
(c)	$AE:EC = 4:3$ (Prop Th) $EG = GC = 1\frac{1}{2}k$ $\therefore AE:EG = 4:\frac{3}{2}$ or $8:3$	$AE:EC = 4:3$ (Prop Th) $EG = GC = 1\frac{1}{2}k$ $\therefore AE:EG = 4:\frac{3}{2}$ or $8:3$

SECTION B

QUESTION 8

(a)	$\sin 3x = -\frac{3}{4}$ $3x = -48,6^\circ + k360^\circ ; k \in \mathbb{Z}$ $x = -16,2^\circ + k120^\circ ; k \in \mathbb{Z}$ or $3x = 180 - (-48,6^\circ) + k360^\circ ; k \in \mathbb{Z}$ $x = 76,2^\circ + k120^\circ ; k \in \mathbb{Z}$ $x = \{-16,2^\circ; -43,8^\circ\}$	$3x = -48,6^\circ + k360^\circ; k \in \mathbb{Z}$ $x = -16,2^\circ + k120^\circ; k \in \mathbb{Z}$ $x = 76,2^\circ + k120^\circ ; k \in \mathbb{Z}$ Quadrants $x = \{-46,2^\circ; -73,8^\circ\}$
(b)	$\tan x = \sin 2x$ $\frac{\sin x}{\cos x} = 2 \sin x \cos x$ $\sin x = 2 \sin x \cos^2 x$ $2 \sin x \cos^2 x - \sin x = 0$ $(2 \cos^2 x - 1) = 0$ $\cos 2x = 0$ $2x = \pm 90^\circ + k360^\circ ; k \in \mathbb{Z}$ $\therefore x = \pm 45^\circ + k180^\circ ; k \in \mathbb{Z}$ Alternate: $\tan x = \sin 2x$ $\frac{\sin x}{\cos x} = 2 \sin x \cos x$ $\sin x = 2 \sin x \cos^2 x$ $2 \sin x \cos^2 x - \sin x = 0$ $(2 \cos^2 x - 1) = 0$ $\cos^2 x = \frac{1}{2}$ $\cos x = \pm \sqrt{\frac{1}{2}}$ $x = \pm 45^\circ + k360^\circ ; k \in \mathbb{Z} \text{ or }$ $x = \pm 135^\circ + k360^\circ ; k \in \mathbb{Z}$	$\frac{\sin x}{\cos x}$ $2 \sin x \cos x$ $\sin x = 2 \sin x \cos^2 x$ $(2 \cos^2 x - 1) = 0$ $\cos 2x = 0$ $\therefore x = \pm 45^\circ + k180^\circ ; k \in \mathbb{Z}$ $\frac{\sin x}{\cos x}$ $2 \sin x \cos x$ $\sin x = 2 \sin x \cos^2 x$ $(2 \cos^2 x - 1) = 0$ $\cos x = \pm \sqrt{\frac{1}{2}}$ $x = \pm 45^\circ + k360^\circ ; k \in \mathbb{Z}$ or $x = \pm 135^\circ + k360^\circ ; k \in \mathbb{Z}$

QUESTION 9

(a)	$\sin(\hat{C} - \hat{D}) = \sin \hat{C} \cdot \cos \hat{D} - \cos \hat{C} \cdot \sin \hat{D}$ $= \left(\frac{12}{13}\right)\left(\frac{3}{5}\right) - \left(\frac{5}{13}\right)\left(-\frac{4}{5}\right)$ $= \frac{56}{65}$	$= \sin \hat{C} \cdot \cos \hat{D} - \cos \hat{C} \cdot \sin \hat{D}$ $\left(\frac{12}{13}\right)$ $\left(\frac{3}{5}\right)$ $\left(\frac{5}{13}\right)$ $\left(-\frac{4}{5}\right)$ $= \frac{56}{65}$
(b)	$\cos(90^\circ + 60^\circ) \cdot \cos 28^\circ + \cos 60^\circ \cdot \cos 62^\circ$ $- \sin 60^\circ \sin 62^\circ + \cos 60^\circ \cos 62^\circ$ $\cos(60^\circ + 62^\circ)$ $\cos 122^\circ$ $= \cos(180^\circ - 58^\circ)$ $= -\cos 58^\circ$ $= -k$ Alternate: $- \sin 60^\circ \cos 28^\circ + \cos 60^\circ \sin 28^\circ$ $= -\sin(60^\circ - 28^\circ)$ $= -\sin 32^\circ$ $= -\cos 58^\circ$ $= -k$ Alternate: $- \cos 30^\circ \cos 28^\circ + \sin 30^\circ \sin 28^\circ$ $= -\cos(30^\circ + 28^\circ)$ $= -\cos 58^\circ$ $= -k$	$\cos(90^\circ + 60^\circ)$ $\cos(60^\circ + 62^\circ)$ $= \cos(180^\circ - 58^\circ)$ $= -\cos 58^\circ$ $= -k$

QUESTION 10

(a)	In $\triangle AEC$ $\frac{EC}{\sin 60^\circ} = \frac{80}{\sin 45^\circ}$ $EC = \frac{80 \sin 60^\circ}{\sin 45^\circ}$ $EC \approx 98 \text{ m}$ In $\triangle EDC$: $\hat{CED} = 135^\circ$ (adj $\angle$s on str line) $(CD)^2 = (53)^2 + (97,98)^2 - 2(53)(97,98) \times \cos 135^\circ$ $CD = 140,54499\dots \text{ m}$ $CD \approx 140,5 \text{ m}$	$\frac{EC}{\sin 60^\circ} = \frac{80}{\sin 45^\circ}$ $EC = \frac{80 \sin 60^\circ}{\sin 45^\circ}$ $EC \approx 98 \text{ m}$ $\hat{CED} = 135^\circ$ $(CD)^2 = (53)^2 + (97,98)^2 - 2(53)(97,98) \times \cos 135^\circ$ $CD = 140,5 \text{ m}$
(b)	In $\triangle ACB$: $\tan 37^\circ = \frac{BC}{AC}$ $BC = 80 \tan 37^\circ$ $BC = 60,284 \text{ m}$ Let M be the midpoint of BC: In $\triangle DMC$: $MC = \frac{1}{2}BC$ $\therefore MC = 30,142 \text{ m}$ $\tan \hat{CDM} = \frac{MC}{CD}$ $\tan \hat{CDM} = \frac{30,142}{140,55}$ $\hat{CDM} \approx 12,1^\circ$ The angle of elevation of M from D is $12,1^\circ$.	In $\triangle ACB$: $\tan 37^\circ = \frac{BC}{AC}$ $BC = 60,284 \text{ m}$ $\tan \hat{CDM} = \frac{MC}{CD}$ $MC = 30,142 \text{ m}$ $\hat{CDM} \approx 12,1^\circ$

QUESTION 11

(a)	Circle with centre P: $x^2 - 6x + y^2 - 12y = -41$ $(x - 3)^2 + (y - 6)^2 = 4$ Centre: P(3;6) Radius: 2 units	$(x - 3)^2 + (y - 6)^2 = 4$ P(3;6) Radius: 2 units
(b)	Centre: Q(9;3) Distance PQ = $\sqrt{(9 - 3)^2 + (3 - 6)^2}$ Distance PQ = $\sqrt{45}$ Distance PQ = $3\sqrt{5}$ $\therefore 3\sqrt{5} - (2 + 2)$ $= 2,7$	$= \sqrt{(9 - 3)^2 + (3 - 6)^2}$ $= 3\sqrt{5}$ $\therefore 3\sqrt{5} - (2 + 2)$
(c)	Volume of block = $lbh - 2 \times (\pi r^2 h)$ $= (20 \times 14 \times 10) - 2(\pi(4)(20))$ $= 2800 - 160\pi$ $\approx 2297,3 \text{ units}^3$	$= lbh - 2 \times (\pi r^2 h)$ $= 2800 - 160\pi$ $\approx 2297,3 \text{ units}^3$

QUESTION 12

(a)	$\bar{x} = \frac{5a+5b}{10}$ $\bar{x} = \frac{a+b}{2}$	$\bar{x} = \frac{a+b}{2} \quad (1)$
(b)	$\sigma^2 = \frac{5\left[a - \frac{a+b}{2}\right]^2 + 5\left[b - \frac{(a+b)}{2}\right]^2}{10}$ $\sigma^2 = \frac{\left(\frac{a-b}{2}\right)^2 + \left(\frac{b-a}{2}\right)^2}{2}$ $\sigma^2 = \frac{\frac{(a-b)^2}{4} + \frac{(a-b)^2}{4}}{2}$ $\sigma^2 = \frac{(a-b)^2}{4}$ $\sigma = \frac{(a-b)}{2}$	$\sigma^2 = \frac{5\left[a - \frac{a+b}{2}\right]^2 + 5\left[b - \frac{(a+b)}{2}\right]^2}{10}$ $\sigma^2 = \frac{\left(\frac{a-b}{2}\right)^2 + \left(\frac{b-a}{2}\right)^2}{2}$ $\sigma = \frac{(a-b)}{2}$

QUESTION 13

(a)	Let: $\hat{O}_1 = 2x$ $\therefore \hat{D} = x$ ($\angle$ at centre = $2x$) $\therefore \hat{A}_1 = x$ (alt $\angle$s; AC//BC) $\therefore \hat{C} = x$ ($\angle$ in same segment) In $\triangle CAE$: $\hat{E}_1 = 180^\circ - 2x$ (int $\angle$s of $\triangle$) $\therefore \hat{E}_2 = 2x$ (adj $\angle$s on str. lines) Since $\therefore \hat{E}_2 = 2x = \hat{O}_1$ And these are subtended by AB, Then AEOB is cyclic ($\angle$s in same segment =)	$\hat{D} = x$ ($\angle$ at centre = $2x$) $\therefore \hat{A}_1 = x$ (alt $\angle$s; AC//BC) $\hat{C} = x$ ($\angle$ in same seg) $\hat{E}_2 = 2x$ (adj $\angle$s on str. line) And these are subtended by AB, then AEOB is cyclic ($\angle$s in same segment =)
(b)	Let: $\hat{D}_1 = x$ $\therefore \hat{B}_2 = x$... equal chords subtend = $\angle$s Let: $\hat{E}_1 = y$ $\therefore \hat{B}_1 = y$... ext. $\angle$ of cyclic quad = int opp $\therefore \hat{A}_1 = y - x$... ext. $\angle$s of $\triangle$ = sum int opp $\therefore \hat{B}_1 - \hat{B}_2 = \hat{A}_1$	$\therefore \hat{B}_2 = x$ equal chords subtend = $\angle$s $\therefore \hat{B}_1 = y$ ext. $\angle$ of cyclic quad = int opp $\therefore \hat{A}_1 = y - x$ ext. $\angle$s of $\triangle$ = sum int opp $\therefore \hat{B}_1 - \hat{B}_2 = \hat{A}_1$
(c)	Draw a perp. From P to SQ Call perp. PU $\therefore UQ = 5 - 3$ $UQ = 2$ cm $\therefore PQ = 3 + 5$ $PQ = 8$ cm $(PU)^2 = (PQ)^2 - (UQ)^2$ pythag $(PU)^2 = (8)^2 - (2)^2$ pythag $PU = \sqrt{60}$ $PU = 7,7$ cm PU = RS (rectangle) $\therefore RS = 7,7$ cm	Draw a perp. From P to SQ $UQ = 2$ cm $PQ = 8$ cm $(PU)^2 = (8)^2 - (2)^2$ pythag $PU = \sqrt{60}$ PU = RS (rectangle)

QUESTION 14

(a)	In $\triangle BOC$: $\hat{C} = 90^\circ - \theta \quad (\text{Isos } \triangle; \text{Radii; Int } \angle\text{s of } \triangle)$ In $\triangle OCF$: $\therefore \hat{C} = \theta$ $\frac{CF}{8} = \cos \theta$ $CF = 8 \cos \theta$ $OF = 8 \sin \theta$ $\therefore P = 2 \times CF + 4 \times OF$ $\therefore P = 16 \cos \theta + 32 \sin \theta$	$\therefore \hat{C}_2 = \theta$ $CF = 8 \cos \theta$ $OF = 8 \sin \theta$ $\therefore P = 2 \times CF + 4 \times OF$
(b)	$P = 16 \cos \theta + 32 \sin \theta \quad \text{and}$ $P = 16\sqrt{5} \sin(\theta + \alpha)$ $P = 16\sqrt{5} \sin \theta \cdot \cos \alpha + 16\sqrt{5} \cos \theta \cdot \sin \alpha$ $\therefore 16\sqrt{5} \sin \alpha = 16$ $\text{and } 16\sqrt{5} \cos \alpha = 32$ $\alpha \approx 26,6^\circ$	$16\sqrt{5} \sin \theta \cdot \cos \alpha + 16\sqrt{5} \cos \theta \cdot \sin \alpha$ $\therefore 16\sqrt{5} \sin \alpha = 16$ $16\sqrt{5} \cos \alpha = 32$ $\alpha \approx 26,6^\circ$

Total: 150 marks