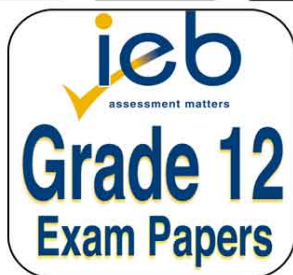




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NATIONAL SENIOR CERTIFICATE EXAMINATION  
NOVEMBER 2020

**MATHEMATICS: PAPER I**

**MARKING GUIDELINES**

Time: 3 hours

150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

**NOTE:**

- If a candidate answers a question more than once, only mark the FIRST attempt.
- Consistent Accuracy applies in all aspects of the marking memorandum.

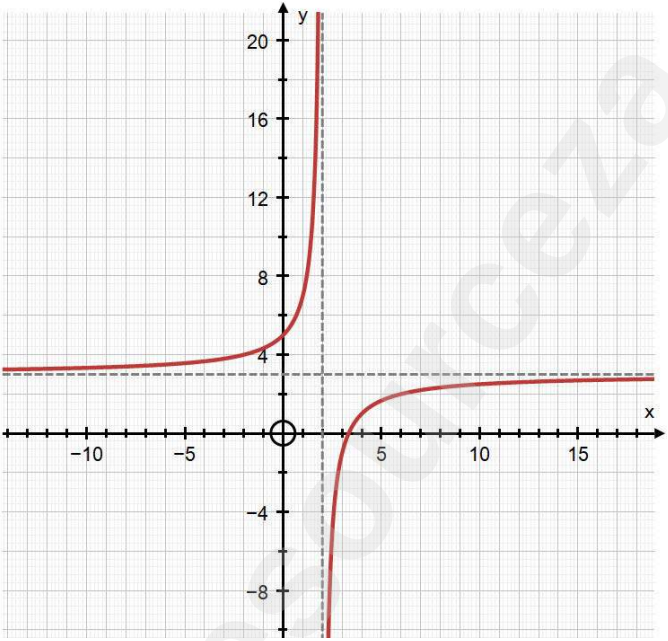
**SECTION A****QUESTION 1**

|        |                                                                                                                                                                                                        |                                                                                           |
|--------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------|
| (a)(1) | $px^2 + 2x - 3 = 0$<br>$x = \frac{(-2) \pm \sqrt{(2)^2 - 4(p)(-3)}}{2(p)}$<br>$x = \frac{-2 \pm \sqrt{4 + 12p}}{2p}$<br>$x = \frac{-2 \pm 2\sqrt{1 + 3p}}{2p}$<br>$x = \frac{-1 \pm \sqrt{1 + 3p}}{p}$ | use of quadratic formula<br>$x = \frac{-2 \pm \sqrt{4 + 12p}}{2p}$<br>simplified solution |
| (a)(2) | Non-real roots for: $1 + 3p < 0$<br>$p < -\frac{1}{3}$                                                                                                                                                 | $\Delta < 0$<br>$p < -\frac{1}{3}$<br>No marks for $\Delta > 0$                           |
| (b)    | $\sqrt{x-2} + 4 = x$<br>$x^2 - 9x + 18 = 0$<br>$(x-2) = (x-4)^2$<br>$x-2 = x^2 - 8x + 16$<br>$x = 6$ or $x = 3$<br>n/v for $x = 3$                                                                     | Isolate surd<br>$x^2 - 4x + 4$<br>$x^2 - 8x + 16$<br>factors<br>answer with selection     |
| (c)    | $(x+3)(x-1) \geq 0$<br>Crit. values: $-3$ ; $1$<br><br>$x \leq -3$ or $x \geq 1$                                    | Number line/graph<br>$x \leq -3$ or $x \geq 1$                                            |

## QUESTION 2

|     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                                                                                                                                                                                                                   |
|-----|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| (a) | $x^{\frac{2}{3}} = 4$ $\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = \left(2^2\right)^{\frac{3}{2}}$ $x = \pm 8$ <p><b>Alternate:</b></p> $\sqrt[3]{x^2} = 4$ $\left(\sqrt[3]{x^2}\right)^3 = (4)^3$ $x^2 = 64$ $x = 8 \text{ or } x = -8$                                                                                                                                                                                                                                                                                                                                | $\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = \left(2^2\right)^{\frac{3}{2}}$ $x = 8$ $x = -8$ $\left(\sqrt[3]{x^2}\right)^3 = (4)^3$ $x = 8$ $x = -8$                                                                                            |
| (b) | $x^2 + 1 = x - y$ <p>Sub: <math>y = 2 - 3x</math></p> $x^2 + 1 = 4x - 2$ $x^2 - 4x + 3 = 0$ $x = 1 \text{ or } x = 3$ <p>When <math>x = 1</math> ; <math>y = -1</math><br/> When <math>x = 3</math> ; <math>y = -7</math></p> <p><b>Alternate:</b></p> $y = 2 - 3x \quad \dots \text{eq 1}$ $3^{x^2+1} = 3^{x-y} \quad \dots \text{sub. eq 1}$ $3^{x^2+1} = 3^{x-(2-3x)}$ $3^{x^2+1} = 3^{4x-2}$ $x^2 + 1 = 4x - 2$ $x^2 - 4x + 3 = 0$ $x = 1 \text{ or } x = 3$ <p>When <math>x = 1</math> ; <math>y = -1</math><br/> When <math>x = 3</math> ; <math>y = -7</math></p> | $x^2 + 1 = x - y$ <p>Sub: <math>y = 2 - 3x</math></p> $x^2 + 1 = 4x - 2$ $x = 1$ $y = -1$ $x = 3$ $y = -7$ $3^{x^2+1} = \frac{3^x}{3^y} \quad \dots \text{sub. eq 1}$ $3^{x^2+1} = 3^{4x-2}$ $x^2 + 1 = 4x - 2$ $x = 1$ $y = -1$ $x = 3$ $y = -7$ |
| (c) | $A = P(1+i)^n$ $25\,000 = 20\,000\left(1 + \frac{4}{100}\right)^n$ $\frac{5}{4} = (1,04)^n$ $n = \log_{1,04}\left(\frac{5}{4}\right)$ $n \approx 5,7 \text{ years}$ <p>After 6 years</p>                                                                                                                                                                                                                                                                                                                                                                                 | $25\,000 = 20\,000\left(1 + \frac{4}{100}\right)^n$ $n = \log_{1,04}\left(\frac{5}{4}\right)$ $n \approx 5,7 \text{ years}$ <p>6 years</p>                                                                                                        |

QUESTION 3

|        |                                                                                                                               |                                                                                      |
|--------|-------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------|
| (a)    | $f(0) = 3 - \frac{4}{0-2}$ $f(0) = 5$                                                                                         | $f(0) = 5$                                                                           |
| (b)    | $3 - \frac{4}{x-2} = 0$ $3(x-2) - 4 = 0 \quad \text{restr. } x \neq 2$ $3x - 6 - 4 = 0$ $x = \frac{10}{3}$ $x = 3\frac{1}{3}$ | $3(x-2) - 4 = 0$ $x = 3\frac{1}{3}$                                                  |
| (c)    |                                             | <p>Shape</p> <p>Vertical Asymptote</p> <p>Horizontal Asymptote</p> <p>Intercepts</p> |
| (d)(1) | $f(x+p) = 3 - \frac{4}{x+p-2}$ $f(x+p) = -\frac{4}{[x+(p-2)]} + 3$                                                            | $f(x+p) = 3 - \frac{4}{x+p-2}$                                                       |
| (d)(2) | Graph of $f$ will shift $p$ units to the right                                                                                | Explanation                                                                          |

|        |                                                                                                                                                                                                                                                                                                                                                                    |                                                                                                                                                                                                                                      |
|--------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| (e)(1) | <p>For <math>f^{-1}(x)</math>: <math>x = 3 - \frac{4}{y-2}</math></p> $x = 3 - \frac{4}{y-2}$ $\frac{4}{y-2} = 3 - x$ $4 = (3 - x)(y - 2)$ $4 = 3y - 6 - xy + 2x$ $3y - xy = 4 + 6 - 2x$ $y(3 - x) = 10 - 2x$ $y = \frac{10 - 2x}{3 - x}$ $\therefore f^{-1}(x) = \frac{10 - 2x}{3 - x}$ <p><b>Alternate final answer:</b></p> $f^{-1}(x) = \frac{2x - 10}{x - 3}$ | $x = 3 - \frac{4}{y-2}$ $4 = (3 - x)(y - 2)$ $\therefore f^{-1}(x) = \frac{10 - 2x}{3 - x}$ <p><b>Alternate final answer:</b></p> $f^{-1}(x) = \frac{2x - 10}{x - 3}$ <p><b>Alternate final answer:</b></p> $y = -\frac{4}{x-3} + 2$ |
| (e)(2) | Domain of $f^{-1}(x)$ : $x \in R$ ; $x \neq 3$                                                                                                                                                                                                                                                                                                                     | $x \in R$ ; $x \neq 3$                                                                                                                                                                                                               |

QUESTION 4

|                        |                                                                                                                                                                                                                                                                                                                      |                                                                                                                            |    |                        |    |          |  |   |   |    |                  |  |  |   |   |                        |                                                                                                                     |          |                  |                        |
|------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------|----|------------------------|----|----------|--|---|---|----|------------------|--|--|---|---|------------------------|---------------------------------------------------------------------------------------------------------------------|----------|------------------|------------------------|
| (4)(a)                 | $ar^2 = 7$<br>$ar^5 = -2\,401$<br>$\therefore \frac{ar^5}{ar^2} = -\frac{2\,401}{7}$<br>$\therefore r^3 = -343$<br>$\therefore r = -7$<br><br>$T_n = a(-7)^{n-1}$<br>$T_3 = a(-7)^{3-1} = 7$<br>$a = \frac{7}{49}$<br>$\therefore a = \frac{1}{7}$                                                                   | $ar^2 = 7$<br>$ar^5 = -2\,401$<br>$\therefore \frac{ar^5}{ar^2} = -\frac{2\,401}{7}$<br><br>$r = -7$<br>$a = \frac{1}{7}$  |    |                        |    |          |  |   |   |    |                  |  |  |   |   |                        |                                                                                                                     |          |                  |                        |
| (4)(b)(1)              | <table><tr><td>3</td><td>7</td><td>15</td><td>27</td><td>Sequence</td></tr><tr><td></td><td>4</td><td>8</td><td>12</td><td>First Difference</td></tr><tr><td></td><td></td><td>4</td><td>4</td><td>Constant second differ</td></tr></table>                                                                          | 3                                                                                                                          | 7  | 15                     | 27 | Sequence |  | 4 | 8 | 12 | First Difference |  |  | 4 | 4 | Constant second differ | <table><tr><td>Sequence</td></tr><tr><td>First Difference</td></tr><tr><td>Constant second differ</td></tr></table> | Sequence | First Difference | Constant second differ |
| 3                      | 7                                                                                                                                                                                                                                                                                                                    | 15                                                                                                                         | 27 | Sequence               |    |          |  |   |   |    |                  |  |  |   |   |                        |                                                                                                                     |          |                  |                        |
|                        | 4                                                                                                                                                                                                                                                                                                                    | 8                                                                                                                          | 12 | First Difference       |    |          |  |   |   |    |                  |  |  |   |   |                        |                                                                                                                     |          |                  |                        |
|                        |                                                                                                                                                                                                                                                                                                                      | 4                                                                                                                          | 4  | Constant second differ |    |          |  |   |   |    |                  |  |  |   |   |                        |                                                                                                                     |          |                  |                        |
| Sequence               |                                                                                                                                                                                                                                                                                                                      |                                                                                                                            |    |                        |    |          |  |   |   |    |                  |  |  |   |   |                        |                                                                                                                     |          |                  |                        |
| First Difference       |                                                                                                                                                                                                                                                                                                                      |                                                                                                                            |    |                        |    |          |  |   |   |    |                  |  |  |   |   |                        |                                                                                                                     |          |                  |                        |
| Constant second differ |                                                                                                                                                                                                                                                                                                                      |                                                                                                                            |    |                        |    |          |  |   |   |    |                  |  |  |   |   |                        |                                                                                                                     |          |                  |                        |
| (b)(2)                 | $2a = 4 \quad \therefore a = 2$<br><br>$3a + b = 4 \quad \therefore b = -2$<br><br>$a + b + c = 3 \quad \therefore c = 3$<br><br>$T_n = 2n^2 - 2n + 3$<br><br><b>Alternate:</b><br>$T_n = 7(n-1) - 3(n-2) + \frac{(n-1)(n-2)}{2} \times (4)$<br>$T_n = 7n - 7 - 3n + 6 + (n^2 - 3n + 2)(2)$<br>$T_n = 2n^2 - 2n + 3$ | <p>Method</p><br>$a = 2$<br><br>$b = -2$<br><br><br>$c = 3$<br><br><p>Method</p><br>$a = 2$<br><br>$b = -2$<br><br>$c = 3$ |    |                        |    |          |  |   |   |    |                  |  |  |   |   |                        |                                                                                                                     |          |                  |                        |

## QUESTION 5

|     |                                                                                                                                                                                                                                                                                                             |                                                                          |
|-----|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------|
| (a) | $g(x) = \log_t x$ sub. $(2; -1)$<br>$-1 = \log_t 2$<br>$t^{-1} = 2$<br>$t = \frac{1}{2}$                                                                                                                                                                                                                    | $-1 = \log_t 2$<br>$t = \frac{1}{2}$                                     |
| (b) | X-int. of normal/standard log graph is always:<br>$(1; 0)$ since $\log_t 1 = 0$<br>$\therefore C(1; 0)$<br><br><b>Alternate:</b><br>For Co-ord. of C: X-int, let $y = 0$<br>$y = \log_{\frac{1}{2}} x$<br>$0 = \log_{\frac{1}{2}} x$<br>$x = \left(\frac{1}{2}\right)^0$<br>$x = 1$<br>$\therefore C(1; 0)$ | $\therefore C(1; 0)$<br><br>$\therefore C(1; 0)$                         |
| (c) | $f(x) = 2p^x + q$<br>$q = -1$ since asymptote passes through $A(2; -1)$<br>$f(x) = 2p^x - 1$ ... sub. $(1; 0)$<br>$0 = 2p^1 - 1$<br>$\therefore p = \frac{1}{2}$                                                                                                                                            | $q = -1$<br>$0 = 2p^1 - 1$<br>$p = \frac{1}{2}$                          |
| (d) | D is the y-int of $f$ : let $x = 0$<br>$f(x) = 2 \times \left(\frac{1}{2}\right)^x - 1$ ... sub. $x = 0$<br>$y = 2 \times \left(\frac{1}{2}\right)^0 - 1$<br>$y = 1$<br>$\therefore D(0; 1)$                                                                                                                | $y = 2 \times \left(\frac{1}{2}\right)^0 - 1$<br>$\therefore D(0; 1)$    |
| (e) | $f(x) = 2\left(\frac{1}{2}\right)^x - 1$ ... sub. $B(2; y)$<br>$f(x) = 2\left(\frac{1}{2}\right)^2 - 1$<br>$f(x) = y = -\frac{1}{2}$<br>Length of AB = $\frac{1}{2}$                                                                                                                                        | $f(x) = 2\left(\frac{1}{2}\right)^2 - 1$<br>Length of AB = $\frac{1}{2}$ |
| (f) | Range of $f$ : $y > -1$                                                                                                                                                                                                                                                                                     | $y > -1$                                                                 |

## QUESTION 6

|     |                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                        |
|-----|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------|
| (a) | $f(x) = 1 - 2x + x^2$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{1 - 2(x+h) + (x+h)^2 - (1 - 2x + x^2)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{1 - 2x - 2h + x^2 + 2xh + h^2 - 1 + 2x - x^2}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-2h + 2xh + h^2}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{h(-2 + 2x + h)}{h}$ $f'(x) = \lim_{h \rightarrow 0} (-2 + 2x + h)$ $2x - 2$ | $1 - 2(x+h) + (x+h)^2$ $-(1 - 2x + x^2)$ <p>Squaring and distributing</p> <p>Factorisation</p> <p>Notation</p> <p>Sub. to get: <math>2x - 2</math></p> |
| (b) | $y = x^{10} + 10x$ $\frac{dy}{dx} = 10x^9 + 10$                                                                                                                                                                                                                                                                                                                                                                                      | $10x^9$ $10$                                                                                                                                           |
| (c) | $y = \frac{5}{x^3} + \frac{x^{\frac{1}{2}}}{x^3}$ $y = 5x^{-3} + x^{-\frac{5}{2}}$ $\frac{dy}{dx} = -15x^{-4} - \frac{5}{2}x^{-\frac{7}{2}}$                                                                                                                                                                                                                                                                                         | $y = 5x^{-3} + x^{-\frac{5}{2}}$ $\frac{dy}{dx} = -15x^{-4} - \frac{5}{2}x^{-\frac{7}{2}}$ <p>Penalise 1 for notation</p>                              |

## SECTION B

## QUESTION 7

|     |                           |                       |
|-----|---------------------------|-----------------------|
| (a) | For: $x < -4$ and $x > 1$ | $x < -4$<br>$x > 1$   |
| (b) |                           | Shape<br>X-Intercepts |
| (c) | $k > p$ or $k < q$        | $k > p$<br>$k < q$    |
| (d) | $x > -1\frac{1}{2}$       | $x > -1\frac{1}{2}$   |

## QUESTION 8

|        |                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                    |
|--------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| (a)(1) | $8^6$                                                                                                                                                                                                                                                                                                                                                                                                             | $8^6$                                                                                                                                                                                                                              |
| (a)(2) | $= 20\,160$                                                                                                                                                                                                                                                                                                                                                                                                       | $8 \times 7 \times 6 \times 5 \times 4 \times 3$<br>$20\,160$                                                                                                                                                                      |
| (b)(1) |                                                                                                                                                                                                                                                                                                                                                                                                                   | $\frac{3}{15}$ ; $\frac{5}{15}$ and $\frac{7}{15}$<br><br>$\frac{\square}{14}$<br>Branches with correct values                                                                                                                     |
| (b)(2) | $\left(\frac{5}{15} \times \frac{7}{14}\right) + \left(\frac{7}{15} \times \frac{5}{14}\right)$<br>$= \frac{1}{3}$                                                                                                                                                                                                                                                                                                | $\left(\frac{5}{15} \times \frac{7}{14}\right)$<br><br>$\left(\frac{7}{15} \times \frac{5}{14}\right)$<br>$= \frac{1}{3}$                                                                                                          |
| (c)    | $P(A \cap B) = P(A) \times P(B)$<br>$\therefore P(A \cap B) = 0,08 \times 0,02$<br>$= 0,0016$<br>but $P(A \cup B) = P(A) + P(B) - P(A \cap B)$<br>$\therefore P(A \cup B) = 0,08 + 0,02 - 0,0016$<br>$= 0,0984$<br><br><b>Alternate:</b><br><br>$P(\text{at least one win})$<br>$= P(\text{one or more wins})$<br>$= 1 - P(\text{no wins})$<br>$= 1 - P(L) \times P(L)$<br>$= 1 - 0,98 \times 0,92$<br>$= 0,0984$ | $\therefore P(A \cap B) = 0,0016$<br>$P(A \cup B) = P(A) + P(B) -$<br>$P(A \cap B)$<br>$\therefore P(A \cup B) = 0,08 + 0,02 - \dots$<br>$= 0,0984$<br><br>$= 1 - P(\text{no wins})$<br><br>$0,98$<br><br>$0,92$<br><br>$= 0,0984$ |

## QUESTION 9

|     |                                                                                                                                                                                                                                                                                                                                              |                                                                                                                                                                                                               |
|-----|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| (a) | $a = 725$<br>$b = 190$                                                                                                                                                                                                                                                                                                                       | $a = 725$<br>$b = 190$                                                                                                                                                                                        |
| (b) | $h = k(x - a)^2 + b$<br>$h = k(x - 725)^2 + 190$ sub. (0;315)<br>$315 = k(0 - 725)^2 + 190$<br>$k = \frac{1}{4\,205}$<br>$h = \frac{1}{4\,205}(x - 725)^2 + 190$ ... sub. (x;210)<br>$210 = \frac{1}{4\,205}(x - 725)^2 + 190$<br>$x = 1\,015$ or $x = 435$<br>Therefore the horizontal distance of hygrometer from the left tower is 435 m. | $h = k(x - 725)^2 + 190$<br>$k = \frac{1}{4\,205}$<br>$210 = \frac{1}{4\,205}(x - 725)^2 + 190$<br>$x = 1\,015$ or $x = 435$<br>Therefore the horizontal distance of hygrometer from the left tower is 435 m. |

## QUESTION 10

|     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                                                                                                                                                                                                                                                                                                                                                                 |
|-----|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| (a) | $F = 8\,755 \left[ \frac{\left(1 + \frac{6,7}{400}\right)^{(5 \times 4)} - 1}{\frac{6,7}{400}} \right]$ $F = 205\,973,485$ <p>Total cost of shares = <math>8\,755 \times 4 \times 5</math><br/>Total cost of shares = 175 100</p> <p>Total Profit = 30 873,485</p> $\% \text{ Profit} = \frac{30\,873,485}{175\,100} \times 100$ $= 17,6319\%$ $\approx 17,6\%$ <p><b>Alternate:</b></p> $F = 8\,755 \left[ \frac{\left(1 + \frac{6,7}{400}\right)^{(5 \times 4)} - 1}{\frac{6,7}{400}} \right]$ $F = 205\,973,485$ <p>Total cost of shares = <math>8\,755 \times 4 \times 5</math><br/>Total cost of shares = 175 100</p> $\therefore \% \text{ Profit} = \frac{205\,973,485}{175\,100}$ $= 1,176319$ $\therefore 17,6\%$ | <p>Inside square bracket</p> <p>Correct <b>X</b> in correct formula</p> $F = 205\,973,485$ <p>175 100</p> <p>30 873,485</p> <p><math>\approx 17,6\%</math></p> <p>Inside square bracket</p> <p>Correct <b>X</b> in correct formula</p> $F = 205\,973,485$ <p>175 100</p> $\% \text{ Profit} = \frac{205\,973,485}{175\,100}$ <p><math>\approx 17,6\%</math></p> |
|-----|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

|            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
|------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>(b)</p> | $300\,000 = x \left[ \frac{1 - \left(1 + \frac{9,5}{1200}\right)^{-(15 \times 12)}}{\frac{9,5}{1200}} \right]$ $x = 3\,132,674049$ <p>Balance of loan = <math>A - F</math></p> $A = 300\,000 \left(1 + \frac{9,5}{1200}\right)^{12 \times 5}$ $A = 481\,502,8408$ $F = 3\,132,674049 \left[ \frac{\left(1 + \frac{9,5}{1200}\right)^{(12 \times 5)} - 1}{\frac{9,5}{1200}} \right]$ $F = 239\,405,9954$ <p>Balance of loan</p> $= (481\,502,8408) - (239\,405,9954)$ $= 242\,096,8454$ $\approx 242\,096,85$ <p><b>Alternate:</b></p> $P = 3\,132,674049 \left[ \frac{1 - \left(1 + \frac{9,5}{1200}\right)^{-(10 \times 12)}}{\frac{9,5}{1200}} \right]$ $P = 242\,096,8454$ $\approx 242\,096,85$ <p>No, there would be a shortfall of R36 123,36</p> | <p>300 000</p> <p>Inside the square bracket</p> $x = 3\,132,674049$ $A = 481\,502,8408$ $F = 239\,405,9954$ $= (481\,502,8408) - (239\,405,9954)$ <p>Comparison between 10a and 10b with conclusion</p> <p>.....</p> <p>300 000</p> <p>Inside the square bracket</p> $x = 3\,132,674049$ $P = 3\,132,674049 \left[ \frac{1 - \left(1 + \frac{9,5}{1200}\right)^{-(10 \times 12)}}{\frac{9,5}{1200}} \right]$ <p>No. of years: -120</p> $\approx 242\,096,85$ <p>Comparison between 10a and 10b with conclusion</p> |
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## QUESTION 11

|        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                                                                                                                                                             |
|--------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| (a)    | $\sum_{i=1}^{\infty} \frac{k}{2^i} + \sum_{i=1}^{10} 2^{2i} > 1\,000\,000$ <p>Working with: <math>\sum_{i=1}^{\infty} \frac{k}{2^i}</math></p> $T_1 = \frac{k}{2} \quad ; \quad T_2 = \frac{k}{4} \quad ; \quad T_3 = \frac{k}{8}$ <p>Common ratio: <math>\frac{k}{4} \div \frac{k}{2}</math></p> $r = \frac{1}{2}$ $S_{\infty} = \frac{a}{1-r} \quad \text{for } -1 < r < 1$ $S_{\infty} = \frac{\frac{k}{2}}{1 - \frac{1}{2}}$ $S_{\infty} = k$ <p>Working with: <math>\sum_{i=1}^{10} 2^{2i}</math></p> $T_1 = 2^2 \quad ; \quad T_2 = 2^4 \quad ; \quad T_3 = 2^6$ <p>Common ratio: <math>r = 4</math></p> $S_n = \frac{a(r^n - 1)}{r - 1} \quad ; \quad r \neq 1$ $S_{10} = \frac{4(4^{10} - 1)}{4 - 1}$ $S_{10} = 1\,398\,100$ <p><math>\therefore \sum_{i=1}^{\infty} \frac{k}{2^i} + \sum_{i=1}^{10} 2^{2i} &gt; 1\,000\,000</math> can be rewritten as</p> $k + 1\,398\,100 > 1\,000\,000$ $k > -398\,100$ $\therefore k = -398\,099 \quad (k \in \mathbb{Z})$ | $r = \frac{1}{2}$ <p>Correct substitution into correct formula to get</p> $S_{\infty} = k$<br>$r = 4$ <p>Correct substitution into correct formula to get</p> $S_{10} = 1\,398\,100$<br>$k + 1\,398\,100 > 1\,000\,000$ $\therefore k = -398\,099 \quad (k \in \mathbb{Z})$ |
| (b)(1) | $5 + \frac{15}{2} + 10 + \dots + \frac{505}{2}$ <p>Common difference of <math>\frac{5}{2}</math>; series is arithmetic</p> $T_n = a + (n-1)d$ $\frac{505}{2} = 5 + (n-1)\left(\frac{5}{2}\right)$ $250 = \frac{5}{2}n$ $n = 100$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | $d = \frac{5}{2}$ <p>Correct substitution in the correct formula</p> $n = 100$                                                                                                                                                                                              |

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| (b)(2) | <p>Middle 30 terms would be: <math>T_{36}</math> to <math>T_{65}</math></p> $T_{36} = 5 + (35)\left(\frac{5}{2}\right)$ $T_{36} = \frac{185}{2}$ <p>Let <math>a = \frac{185}{2}</math>; <math>d = \frac{5}{2}</math></p> $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{30} = \frac{30}{2}\left[2\left(\frac{185}{2}\right) + (29)\left(\frac{5}{2}\right)\right]$ $S_{30} = 3\,862,5$ <p><b>Alternate:</b></p> <p>Middle 30 terms would be: <math>T_{36}</math> to <math>T_{65}</math></p> $T_{36} = 5 + (35)\left(\frac{5}{2}\right)$ $T_{36} = \frac{185}{2}$ $T_{65} = 5 + (64)\left(\frac{5}{2}\right)$ $T_{65} = 165$ $S_n = \frac{n}{2}(a + l)$ $S_{30} = \frac{30}{2}\left(\frac{185}{2} + 165\right)$ $S_{30} = 3\,862,5$ | $T_{36}$ $T_{36} = \frac{185}{2}$ <p>Correct substitution into correct formula</p> $S_{30} = 3\,862,5$<br>$T_{36}$ $T_{36} = \frac{185}{2}$ <p>Correct substitution into correct formula</p> $S_{30} = 3\,862,5$ |
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## QUESTION 12

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| 12 | <p>Let: <math>g(1) = h(1)</math><br/> <math>(1)^3 - a(1)^2 + 6 = 2(1)^2 + b(1) + 3</math><br/> <math>1 - a + 6 = 2 + b + 3</math><br/> <math>a = 2 - b \quad \dots \text{eq1}</math></p> <p><math>g'(x) = 3x^2 - 2ax</math><br/> <math>h'(x) = 4x + b</math></p> <p><math>g'(1) = h'(1)</math><br/> <math>3(1)^2 - 2a(1) = 4(1) + b</math><br/> <math>3 - 2a = 4 + b \quad \dots \text{sub eq1: } a = 2 - b</math><br/> <math>3 - 2(2 - b) = 4 + b</math><br/> <math>b = 5</math><br/> <math>a = -3</math></p> <p><math>h(x) = 2x^2 + 5x + 3</math><br/> <math>h(1) = 10</math></p> <p>Point of contact is: <math>(1;10)</math></p> | <p><math>g(1) = h(1)</math><br/> <math>a = 2 - b \quad \dots \text{eq1}</math></p> <p><math>g'(x) = 3x^2 - 2ax</math><br/> <math>h'(x) = 4x + b</math></p> <p><math>g'(1) = h'(1)</math><br/> <math>2a + b = -1</math><br/> <math>b = 5</math></p> <p><math>(1;10)</math></p> |
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## QUESTION 13

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| 13 | $8x + 4x + 4h = P$<br>$P = 12x + 4h$<br>$P - 12x = 4h$<br>$\therefore h = \frac{1}{4}P - 3x$<br>$V = l \times b \times h$<br>$V = (2x)(x)(h) \dots \text{sub.: } h = \frac{1}{4}P - 3x$<br>$V = (2x)(x)\left(\frac{1}{4}P - 3x\right)$<br>$V = \frac{1}{2}x^2P - 6x^3$<br>$V' = Px - 18x^2$<br>$0 = x(P - 18x)$<br>$x = 0 \text{ or } x = \frac{P}{18}$<br>Hence, length of the box is $2x = \frac{P}{9}$<br>$\therefore$ length of box is $\frac{1}{9}P$ cm when the volume is a maximum. | $8x + 4x + 4h = P$<br>$h = \frac{1}{4}P - 3x$<br>$V = (2x)(x)\left(\frac{1}{4}P - 3x\right)$<br>$V = \frac{1}{2}x^2P - 6x^3$<br>$V' = Px - 18x^2$<br>$0 = x(P - 18x)$<br>$P = 18x$<br>Length of the box is $2x$ and $P = 18x$ |
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Total: 150 marks