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NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2019

MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours

150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A**QUESTION 1**

(a) $A(0; 3)$

$$\begin{aligned}
 \text{(b)} \quad -2x + 3 &= 3x - 7 \\
 -5x &= -10 \\
 x &= 2 \\
 y &= -1
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad d &= \sqrt{(3 - (-1))^2 + (0 - 2)^2} \\
 d &= \sqrt{20}
 \end{aligned}$$

$$\text{(d)} \quad (x - 2)^2 + (y + 1)^2 = 20$$

$$\text{(e)} \quad (4; -5)$$

$$\begin{aligned}
 \text{(f)} \quad 0 &= -2x + 3 \\
 x &= \frac{3}{2} \\
 0 &= 3\left(\frac{3}{2}\right) + c \\
 y &= 3x - \frac{9}{2}
 \end{aligned}$$

QUESTION 2

$$\text{(a)} \quad (1) \quad \text{Method of finding third side}$$

$$\sqrt{1 - m^2}$$

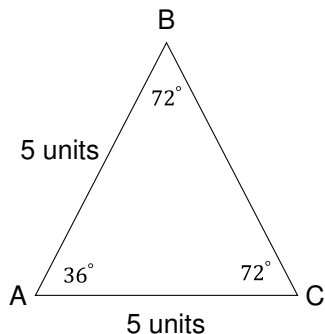
$$\begin{aligned}
 (2) \quad 2\cos^2 25^\circ - 1 & \quad \text{or} \quad \cos^2 25^\circ - \sin^2 25^\circ \\
 = 2m^2 - 1 & \quad \text{or} \quad m^2 - (\sqrt{1 - m^2})^2
 \end{aligned}$$

$$(3) \quad \cos 30^\circ \cos 25^\circ - \sin 30^\circ \sin 25^\circ$$

$$\frac{\sqrt{3}m}{2} - \frac{\sqrt{1 - m^2}}{2}$$

$$\begin{aligned}
 \text{(b)} \quad & \frac{\sin(21^\circ + w)}{\sin(w + 21^\circ)} - \frac{\sin^2 \beta}{\cos^2 \beta} \cdot \cos^2 \beta \\
 &= 1 - \sin^2 \beta \\
 &= \cos^2 \beta
 \end{aligned}$$

(c)



$$\begin{aligned}
 \text{Area } \triangle ABC &= \frac{1}{2} \times 5 \times 5 \times \sin 36^\circ \\
 \text{Area } \triangle ABC &= 7,3 \text{ units}^2
 \end{aligned}$$

QUESTION 3

(a) $p = 2$

(b) 180°

$$\begin{aligned}
 \text{(c)} \quad & 3 \sin 2x = 2 \sin x \\
 & 6 \sin x \cos x = 2 \sin x \\
 & 2 \sin x (3 \cos x - 1) = 0
 \end{aligned}$$

$$\cos x = \frac{1}{3}$$

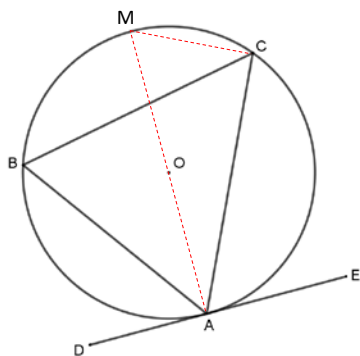
$$x = 70,5^\circ$$

$$B(289,5^\circ; -1,9)$$

(d) $k > 3$

QUESTION 4

(a)



Construction AM and MC

$$\hat{EAM} = 90^\circ \quad (\text{Line from centre perpendicular to tangent})$$

$$\hat{MCA} = 90^\circ \quad (\text{Angle in semi-circle})$$

$$\hat{CMA} + \hat{MAC} = 90^\circ \quad (\text{Angles in a triangle})$$

Therefore

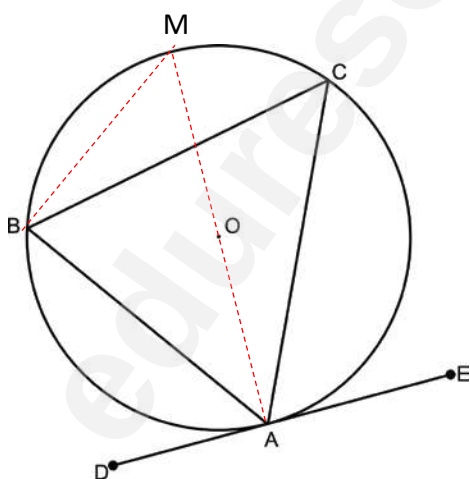
$$\hat{AMC} = \hat{EAC}$$

but

$$\hat{AMC} = \hat{ABC} \quad (\text{Angles in same segment})$$

Hence

$$\hat{EAC} = \hat{ABC}$$

Alternate solution (there are a number of others)

Construction AM and MB

$$\hat{MBA} = 90^\circ \quad (\text{Angle in semi-circle})$$

$$\hat{MAE} = 90^\circ \quad (\text{Line from centre perpendicular to tangent})$$

$$\hat{MBC} = \hat{MAC} \quad (\text{Angles in same segment})$$

$$\hat{CBA} = \hat{CAE}$$

- (b) $\hat{F}_1 = 55^\circ$ (Tan chord theorem)
 $\hat{F}_2 = 35^\circ$ (Angles in semi-circle)
 $\hat{E}_1 = 35^\circ$ (Angles in same segment)
- (c) $\hat{C}_1 = 110^\circ$ (CF = CG radii; Isos triangle)
 $\hat{D}_1 = 80^\circ$ (Angle @ centre)
 $\hat{D}_3 = 80^\circ$ (Vert opp)
 $\hat{D}_3 + \hat{B} = 180^\circ$

Therefore DHBJ is a cyclic quad (Converse: Opp. angles of a cyclic quad are supplementary)

Alternate solution

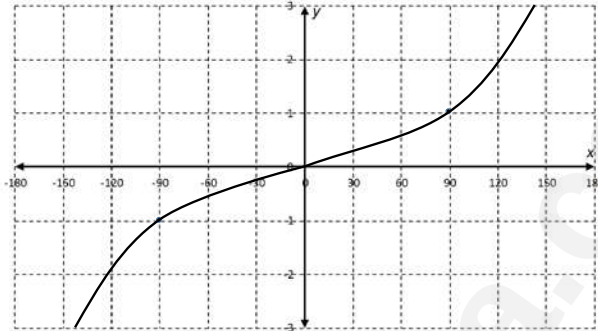
- $\hat{C}_1 = 110^\circ$ (CF = CG radii; Isos triangle)
 $\hat{D}_1 = 80^\circ$ (Angle @ centre)
 $\hat{D}_2 = 100^\circ$ (Angles on a straight line)
 $\hat{D}_2 = \hat{B}$

Therefore DHBJ is a cyclic quad (Converse: Exterior angle of a cyclic quad is equal to the interior opposite angle)

QUESTION 5

(a) (1) $x = 180^\circ + k.360$

(2) Shape of graph

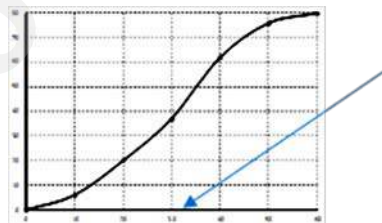
Points on $(-90^\circ; -1)$ and $(90^\circ; 1)$ Understanding of Asymptotes at -180° and 180° 

$$\begin{aligned}
 (b) \quad &= \frac{\cos \theta \sin^2 \theta + \cos^3 \theta}{\cos \theta} - 2\sin^2 \theta && \cos 2\theta \\
 & && = 1 - 2\sin^2 \theta \\
 &= \frac{\cos \theta (\sin^2 \theta + \cos^2 \theta)}{\cos \theta} - 2\sin^2 \theta \\
 &= 1 - 2\sin^2 \theta \quad (5)
 \end{aligned}$$

QUESTION 6

(a) 16

(b) (1) Look at the diagram



(2) $IQR = 38 - 20 = 18$

(c) (1) $A = -2,6$ and $B = 2$

(2) Extrapolation or prediction is way outside HV used



(2) The values of those to the right of median are more spread out thus the mean is pulled to the right of the median.

(e) Yes; it needs a service as one standard deviation from the mean shows that a large percentage of the logs are between 9,2 and 10,8 metres. This shows that the machine is inaccurate.

SECTION B**QUESTION 7**

- (a) $-0,8$
- (b) (1) Get closer to -1 or decrease
 (2) Decrease or get steeper
 (3) A person who plays professional sport and studies via the internet
- (c) (1) More of the data is closer to the mean
 OR 4 low and 3 high hence further apart
 (2) Mean would decrease and standard deviation will decrease
 (3) Decrease the price on any day from Monday to Saturday, increase in sales and data will be more dispersed **OR** any intervention that would increase the sales on a day between Monday and Saturday.

QUESTION 8

(a) (1) $\frac{CN}{NE} = \frac{DG}{GE} = \frac{2}{3}$

$$DG = GK$$

Therefore

$$\frac{EK}{KG} = \frac{1}{2}$$

(2) $\frac{DH}{HF} = \frac{4}{1}$

$$DE = 5m \text{ and } DF = 5k$$

$$\frac{\frac{1}{2} \times 2m \times 4k \times \sin \hat{D}}{\frac{1}{2} \times 5m \times 5k \times \sin \hat{D}}$$

$$= \frac{8}{25}$$

(b) (1) $\hat{E}_2 = x$ (Angle at centre)
 $\hat{N}_3 = x$ (Alternate angles // lines)
 $\hat{G}_1 = x$ (Tan chord theorem)

ALT: $\hat{E}_2 = \hat{N}_3 = \hat{G}_1$ (With the reasons above)

Therefore
 $GN = NE$ (Converse: isos triangle)

(2) $\hat{GNE} = 180^\circ - 2x$ (Angles in a triangle)
 $\hat{H}_2 = 2x$ (Opp. angles of cyclic quad)
 $\hat{E}_1 = \hat{N}_1 = 90^\circ - x$ (Angles in isos triangle)
 $\triangle GON \parallel \triangle GHE$ (A.A.A)

(3) Working with HE
 $\frac{HE}{ON} = \frac{GE}{GN}$ (Similar triangles)
 $\frac{HE}{EP} = \frac{HB}{BC}$ (Prop theorem)

QUESTION 9

(a) $FH = \sqrt{50}$
 $HG = \sqrt{65}$
 $10^2 = 50 + 65 - 2 \times \sqrt{50} \times \sqrt{65} \cos \hat{H}$
 $\hat{H} = 82,4^\circ$

ALT: Cosine rule for any two sides and angle

$$\text{Area } \triangle FGH = \frac{1}{2} \times \sqrt{50} \times \sqrt{65} \times \sin 82,4^\circ$$

$$\text{Area } \triangle FGH = 28,3 \text{ m}^2$$

(b) (1) Coordinates of C
 $x^2 - 6x + 8 = 0$
 $(x - 4)(x - 2) = 0$
 $C(4; 0)$

$$(2) \quad x^2 - 6x + 9 + y^2 - 4y + 4 = -8 + 13$$

$$(x-3)^2 + (y-2)^2 = 5$$

Centre of circle (3; 2)

$$\text{Gradient of line } AB \times \frac{1}{2} = -1$$

$$\text{Gradient of line } AB = -2$$

$$3 = -2(5) + c$$

$$c = 13$$

$$y = -2x + 13$$

Coordinates of B

$$0 = -2x + 13$$

$$x = 6,5$$

$$B(6,5;0)$$

Therefore CB = 2,5 units

QUESTION 10

(a) $y = \frac{2}{3}x + \frac{8}{3}$

$$\tan \hat{FEA} = \frac{2}{3}$$

$$\hat{FEA} = 33,7^\circ$$

$$\tan \hat{EFA} = -1$$

$$\hat{EFA} = 45^\circ$$

Therefore

$$\hat{EAF} = 101,3^\circ$$

(b) $\frac{AF}{\sin 33,7^\circ} = \frac{\sqrt{52}}{\sin 45^\circ}$

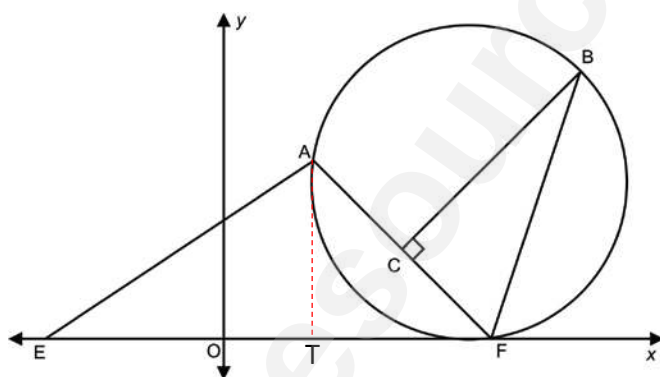
$$AF = 5,66$$

$$CF = 2,83$$

(Line from centre perpendicular to chord)

$$CB^2 = \sqrt{40^2} - 2,83^2$$

$$CB = 5,7$$

Alternate solution

$$\frac{AT}{\sqrt{52}} = \sin 33,7^\circ$$

$$AT = 4$$

$$TF = 4 \text{ units}$$

(Isos Triangle)

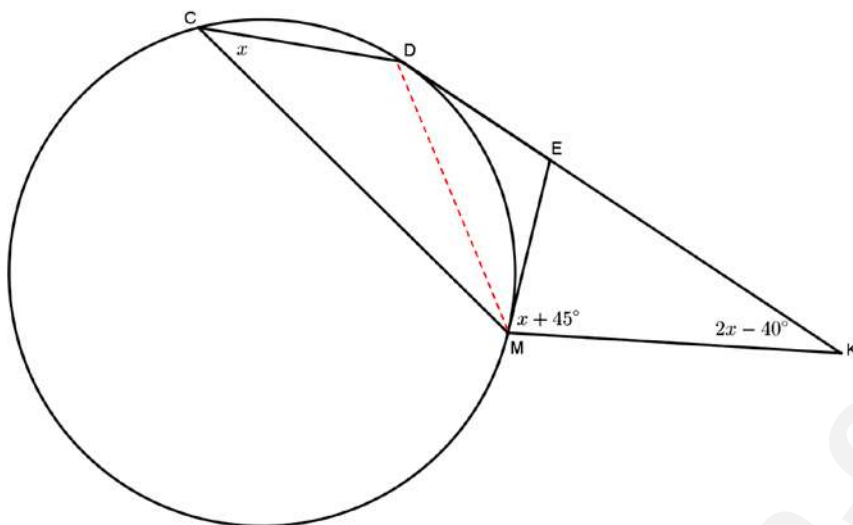
$$AF = \sqrt{32}$$

$$CF = 2,83$$

(Line from centre perpendicular to chord)

$$CB^2 = \sqrt{40^2} - 2,83^2$$

$$CB = 5,7$$

QUESTION 11

Construction MD

$$\hat{MDE} = x \quad (\text{Tan chord theorem})$$

$$\hat{DME} = x \quad (\text{Tan chord theorem}) \text{ OR (isos triangle; tangents from common point)}$$

$$\hat{DEM} = 180^\circ - 2x$$

$$180^\circ - 2x = x + 45^\circ + 2x - 40^\circ \quad (\text{Ext angle of triangle})$$

$$-5x = -175^\circ$$

$$x = 35^\circ$$

QUESTION 12

(a) $5x + 12y = 60$

$$A(0; 5) \text{ and } B(12; 0)$$

$$AB^2 = 5^2 + 12^2$$

$$AB = 13$$

(b) $\triangle ADC : \triangle BCD = 8 : 18$

$$AC = 13 \times \frac{8}{26}$$

$$AC = 4 \text{ units}$$

$$(5k)^2 + (12k)^2 = 4^2$$

$$169k^2 = 16$$

$$k^2 = \frac{16}{169}$$

$$k = \frac{4}{13}$$

Coordinates of C

$$C\left(\frac{48}{13}; \frac{45}{13}\right)$$

Alternate solution

$$\triangle ADC : \triangle BCD = 8 : 18$$

$$AC = 13 \times \frac{8}{26}$$

$$AC = 4 \text{ units}$$

If C(a; b) then:

$$\frac{a}{12} = \frac{4}{13} \quad (\text{line parallel to side of } \triangle)$$

Coordinates of C

$$C\left(\frac{48}{13}; \frac{45}{13}\right)$$

Alternate solution

$$\triangle ADC : \triangle BCD = 8 : 18$$

$$AC = 13 \times \frac{8}{26}$$

$$AC = 4 \text{ units}$$

$$\hat{CBO} = 22,62^\circ$$

$$\sin 22,62^\circ = \frac{y}{9}$$

$$y = 3,5 \text{ units}$$

$$\cos 22,62^\circ = \frac{m}{9}$$

$$m = 8,3$$

$$x = 12 - 8,3 = 3,7$$

$$C(3,7;3,5)$$

Total: 150 marks