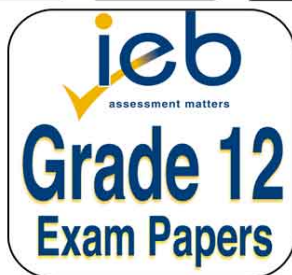




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NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2017

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours

150 marks

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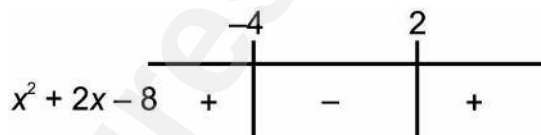
SECTION A

QUESTION 1

(a) (1) $(x-1)^2 = 2(1-x)$
 $(x-1)^2 = -2(x-1)$
 $(x-1)^2 + 2(x-1) = 0$
 $(x-1)(x-1+2) = 0$
 $(x-1)(x+1) = 0$
 $x=1 \quad x=-1$

(2) $5^{-x} \cdot 5^{x-2} = \frac{25^{2x}}{5}$
 $5^{-x} \cdot 5^{x-2} = \frac{5^{4x}}{5^1}$
 $5^{-x+x-2} = 5^{4x-1}$
 $-2 = 4x-1$
 $x = -\frac{1}{4}$

(b) $(x+1)^2 < 9$
 $\therefore x^2 + 2x + 1 < 9$
 $\therefore x^2 + 2x - 8 < 0$
 $\therefore (x+4)(x-2) < 0$
Critical Values: $-4; 2$



Solution: $\{x : -4 < x < 2\}$

Alternative

$(x+1)^2 < 9$
 $\therefore -3 < x+1 < 3$
 $\therefore -4 < x < 2$

(c) $(x-2)(x+4) = 0$
 $x^2 + 2x - 8 = 0$
 $\therefore b=2 \text{ and } c=-8$

Alternative

$(x+1)^2 < 9$
 $x^2 + 2x - 8 < 0$
Sketch: $y = x^2 + 2x - 8$
 $x - \text{int} : x = -4; x = 2$

Solution: $\{x : -4 < x < 2\}$

(d) (1) $x - 2 = \frac{-4}{x-2} - 4$ let $x - 2 = y$
 $y = -\frac{4}{y} - 4$ LCD: y
 $y^2 = -4 - 4y$
 $\therefore y^2 + 4y + 4 = 0$

(2) $(y + 2)^2 = 0$
 $\therefore y = -2$
Roots are real and equal.

Alternative

$$y^2 + 4y + 4 = 0$$
$$\therefore \Delta = 4^2 - 4(1)(4)$$
$$\therefore \Delta = 0$$

$\therefore$ Roots are real and equal.

Alternative

$$x - 2 = \frac{-4}{x-2} - 4$$
$$\therefore (x - 2)^2 = -4 - 4(x - 2)$$
$$\therefore x^2 - 4x + 4 = -4 - 4x + 8$$
$$\therefore x^2 = 0$$

$\therefore$ Roots are real and equal.

QUESTION 2

(a) (1)

(2) $E = \{HTT, THT, TTH\}$

$$\therefore P(2 \text{ tails and } 1 \text{ head}) = \frac{3}{8}$$

(b) (1) $P(A \cap B) = 0$

(2) (i) You cannot pick a R2 and a R5 coin at the same time.

(ii) $P(\text{either a R5 or a R2})$
 $= P(A \text{ or } B)$
 $= P(A) + P(B) \text{ mutually exclusive}$
 $= 0,36 + 0,47$
 $= 0,83$

(c) (1)

(2) $P(\text{exactly one machine is stamping R5 coins})$
 $= 0,3 + 0,2$
 $= 0,5$
 $\therefore 50\%$

QUESTION 3

- (a) $480\,163 \div 0,502 = \text{R}956\,500$
- (b) $\text{R}956\,500 \times 5\% = \text{R}47\,825$
- (c) Cost of machinery including import charges = $\text{R}956\,500 + \text{R}47\,825$
= $\text{R}1\,004\,325$

$$A = P(1+i)^n$$

$$1\,004\,325 = 225\,450 \left(1 + \frac{9,5}{100}\right)^n$$

$$\frac{1\,004\,325}{225\,450} = \left(\frac{219}{200}\right)^n$$

$$\log_{\left(\frac{219}{200}\right)} \left(\frac{1\,004\,325}{225\,450}\right) = n$$

$$n = 16,46171594$$

$$\therefore n \approx 16,46$$

$$\therefore \text{approx. 17 years}$$

- (d) (1) Loan required: $\text{R}1\,004\,325 - \text{R}225\,450 = \text{R}778\,875$

$$P = x \left[\frac{1 - (1+i)^{-n}}{i} \right]$$

$$778\,875 = x \left[\frac{1 - \left(1 + \frac{12}{1200}\right)^{(-4 \times 12)}}{\frac{12}{1200}} \right]$$

$$x = \text{R}20\,510,76607$$

$$\therefore x = \text{R}20\,510,77$$

(2) Outstanding Balance = $A - F$

$$A = 778\,875 \left(1 + \frac{12}{1200} \right)^{24}$$

$$A = 988\,964,5744$$

$$A \approx 988\,964,57$$

$$F = 20\,510,76607 \left[\frac{\left(1 + \frac{12}{1200} \right)^{24} - 1}{\frac{12}{1200}} \right]$$

$$F = 553\,246,4277$$

$$F \approx 553\,246,43$$

$$\text{Outstanding balance} = 988\,964,5744 - 553\,246,4277$$

$$= R435\,718,1467 \approx R435\,718,15$$

NB: If A and F are rounded to the nearest cent, consider

$$\text{Outstanding balance} = 988\,964,57 - 553\,246,43$$

$$= R435\,718,14$$

Alternative

$$\text{Outstanding balance} = 20\,510,76607 \left[\frac{1 - \left(1 + \frac{12}{1200} \right)^{-24}}{\frac{12}{1200}} \right]$$

$$= R435\,718,1466$$

$$\approx R435\,718,15$$

QUESTION 4

(a) (1)

Constant second difference

$$(2) \quad T_n = an^2 + bn + c$$

$$T_1 = a + b + c = 1$$

$$T_2 = 4a + 2b + c = 3$$

$$T_3 = 9a + 3b + c = 6$$

$$\therefore 3a + b = 2 \text{ and } 5a + b = 3$$

Substitute $b = 2 - 3a$ into $5a + b = 3$

$$\therefore 5a + (2 - 3a) = 3$$

$$2a = 1$$

$$a = \frac{1}{2}$$

$$b = \frac{1}{2} \text{ and } c = 0$$

$$\therefore T_n = \frac{1}{2}n^2 + \frac{1}{2}n$$

$$(b) \quad T_3 = 52 \text{ cm}$$

$$T_7 = 78 \text{ cm}$$

$$T_3 = a + 2d = 52$$

$$T_7 = a + 6d = 78$$

$$4d = 26 \quad \therefore d = 6\frac{1}{2}$$

$$\therefore a = 39 \text{ cm}$$

$$T_{43} = 39 + 42\left(6\frac{1}{2}\right)$$

$$T_{43} = 312 \text{ cm}$$

QUESTION 5

(a) (1) $f(x) = x^2 - 6x + 9$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^2 - 6(x+h) + 9 - (x^2 - 6x + 9)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - 6x - 6h + 9 - x^2 + 6x - 9}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{2xh + h^2 - 6h}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{h(2x + h - 6)}{h}$$

$$f'(x) = 2x - 6$$

(2) $f'(-3) = 2(-3) - 6 = -12$

(b) $y = \pi x^{-1} + 3x^{\frac{1}{3}}$

$$\frac{dy}{dx} = -\pi x^{-2} + x^{-\frac{2}{3}}$$

SECTION B

QUESTION 6

(a) (1) Domain = $x \in \mathbb{R}$; $x \neq 3$

(2) Range = $y \in \mathbb{R}$; $y \neq -3$

(3) (i) 5 units

(ii) 5 units

(b) (1) $y = a.b^x$ substitute $\left(0; \frac{1}{4}\right)$

$$\frac{1}{4} = a.b^0$$

$$a = \frac{1}{4}$$

$y = \frac{1}{4}b^x$ substitute $\left(2; \frac{9}{4}\right)$

$$\frac{9}{4} = \frac{1}{4}b^2$$

$$b^2 = 9$$

$$\therefore b = \pm 3 \text{ but } b > 0 \therefore b = 3$$

(2)

f : shape
intercept
domain

f^{-1} shape
intercept
range

(3) Range = $\left[\frac{1}{4}; \infty\right)$

(4) $f(x) = \frac{1}{4}.3^x$

For f^{-1} : $x = \frac{1}{4}.3^y$; $y \geq 0$

$$4x = 3^y$$

$$y = \log_3(4x) \text{ for } x \geq \frac{1}{4}$$

(5) See graph in Question 6 (b) (2) above.

QUESTION 7

(a) $f(x) = x^2 + 6x + (3)^2 + 5 - (3)^2$

$$f(x) = (x+3)^2 - 4$$

$$\therefore \text{T.P.}(-3; -4)$$

(b) (1) $x^2 + 6x + 5 = -x - 5$

$$x^2 + 7x + 10 = 0$$

$$x = -2 \text{ or } x = -5$$

$$A(-5;0) \text{ and } B(-2;-3)$$

(2) Horizontal shift: $\therefore -5 < t < -2$

(c) (1) Length MN = $(-x - 5) - (x^2 + 6x + 5)$

$$\text{Length MN} = -x - 5 - x^2 - 6x - 5$$

$$\text{Length MN} = -x^2 - 7x - 10$$

For max. length: Let $D_x = 0$

$$-2x - 7 = 0$$

$$x = -\frac{7}{2}$$

$$\therefore \text{Max. length MN} = -\left(-\frac{7}{2}\right)^2 - 7\left(-\frac{7}{2}\right) - 10$$

$$\therefore \text{Max. length MN} = \frac{9}{4} \text{ units} \quad \text{i.e. } 2,25 \text{ units}$$

(2) Vertical shift: $\therefore k > \frac{9}{4}$

QUESTION 8

(a) (1) $\frac{3}{2}; -\frac{9}{2}; \frac{27}{2}; \dots$
 $\therefore r = -3$ and series is geometric
however, series is not convergent since $r < -1$.
 $\therefore x \neq -\frac{3}{2}$

(2) $\frac{x-3}{x+3} = \frac{12-x}{x-3}$
 $(x-3)^2 = (12-x)(x+3)$
 $x^2 - 6x + 9 = 12x + 36 - x^2 - 3x$
 $2x^2 - 15x - 27 = 0$
 $x = 9 \text{ or } x \neq -\frac{3}{2}$

(b) $S_4 = 7\frac{1}{2}; S_5 = 15\frac{1}{2}$ and $S_6 = 31\frac{1}{2}$
 $T_5 = S_5 - S_4$
 $T_5 = 8$

$$T_6 = S_6 - S_5$$
$$T_6 = 16$$

$$T_5 = ar^4 = 8$$
$$T_6 = ar^5 = 16$$

$$\frac{T_6}{T_5} = r = 2$$

$$\therefore a = \frac{1}{2}$$

$$S_n = \frac{\frac{1}{2}(2^n - 1)}{2 - 1}$$
$$= 2^{n-1} - \frac{1}{2}$$

QUESTION 9

(a) $f(x) = -x^3 + bx^2 + cx - 3$
 $f(1) = -(1)^3 + b(1)^2 + c(1) - 3 = 4$
 $b + c = 8$

$$f'(x) = -3x^2 + 2bx + c$$

$$f''(x) = -6x + 2b$$

$$f''\left(\frac{1}{2}\right) = -6\left(\frac{1}{2}\right) + 2b = 1$$

$$b = 2$$

$$\therefore c = 6$$

(b) For concave up: $f''(x) > 0$

$$-6x + 4 > 0$$

$$x < \frac{2}{3}$$

QUESTION 10

$$\begin{aligned}\frac{340}{x} - \frac{340}{x+2} &= 3 \quad \text{LCD: } x(x+2) \\ 340(x+2) - 340x &= 3x(x+2) \\ 3x^2 + 6x - 680 &= 0 \\ x &= 14,09 \quad \text{or} \quad x \neq -16,09 \\ \text{Therefore Time} &= \frac{340}{14,09} \approx 24,13 \text{ seconds}\end{aligned}$$

Alternative

Let original time taken be represented by y .

$$\therefore xy = 340 \quad \dots \text{eq. 1}$$

$$(x+2)(y-3) = 340 \quad \dots \text{eq. 2}$$

$$\text{From eq. 1} \quad y = \frac{340}{x}$$

$$\therefore (x+2)\left(\frac{340}{x} - 3\right) = 340$$

$$\therefore 3x^2 + 6x - 680 = 0$$

$$\therefore x = 14,09 \quad \text{or} \quad x \neq -16,09$$

$$\text{Therefore Time} = \frac{340}{14,09} \approx 24,13 \text{ seconds}$$

QUESTION 11

(a) (1) When $x = -2$ and $x = 0$

(2)

(b) $y = \frac{1}{5}x^3 + \frac{3}{4}x + 3$

$$\frac{dy}{dx} = \frac{3}{15}x^2 + \frac{3}{4} \text{ substitute } x = 0$$

$$\frac{dy}{dx} = \frac{3}{4}$$

Equation of tangent: $y = \frac{3}{4}x + c$ where $c = 3$

Equation of tangent: $y = \frac{3}{4}x + 3$

For point of intersection between tangent

and line BC, substitute $x = 2$ into $y = \frac{3}{4}x + 3$

$$\therefore y = 4\frac{1}{2} \therefore \text{Pt}\left(2; 4\frac{1}{2}\right)$$

$$\text{Area of Busi's region} = \frac{1}{2}\left(5 + 3\frac{1}{2}\right) \times 2$$

$$= 8\frac{1}{2} \text{ units}^2$$

$$\text{Area of Khanya's region} = \frac{1}{2}\left(3 + 4\frac{1}{2}\right) \times 2$$

$$= 7\frac{1}{2} \text{ units}^2$$

Therefore Busi's region is larger.

Total: 150 marks