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NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2015

MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours

150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

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SECTION A**QUESTION 1**

(a) $M(6; 3)$ (2)

(b) $\hat{CMA} = 90^\circ$ (Opp angles of cyclic quad)

$$m_{AB} = -\frac{6}{12}$$

$$\therefore m_{MC} = 2$$

$$y = 2x + c$$

sub in $M(6; 3)$

$$3 = 2(6) + c$$

$$c = -9$$

$$y = 2x - 9$$
 (5)

(c) (1) Area of $\Delta MCB = \frac{1}{2}(b)(h)$

$$2x - 9 = 0$$

$$x = 4\frac{1}{2}$$

$$OB - OC = BC = 7,5$$

$$h = 3$$

$$\therefore \text{Area of } \Delta MCB = \frac{1}{2}(7,5)(3)$$
 (4)

(2) $\therefore \text{area of AMCO} = 36 - 11,25$
 $= 24,75 \text{ units}^2$

(2)

[13]

QUESTION 2

(a) $\frac{1}{2}x + 4 = x$

$$x + 8 = 2x$$

$$x = 8$$

$$\therefore y = x = 8$$

$$N(8; 8)$$

(3)

(b) B(16; 16)

(1)

(c) $7y = 10x$

$$\therefore 7(16) = 10x$$

$$\therefore x_D = 11,2$$

$$D(11,2; 16)$$

$$\therefore DB = 16 - 11,2$$

$$= 4,8 \text{ units}$$

(3)

[7]

QUESTION 3

$$\begin{aligned}
 \text{(a)} \quad (1) \quad &= \frac{\sin \theta \sin \theta - 1}{\cos \theta} \\
 &= \frac{\sin^2 \theta - 1}{\cos \theta} \\
 &= \frac{-\cos^2 \theta}{\cos \theta} \\
 &= -\cos \theta
 \end{aligned}
 \tag{4}$$

$$\begin{aligned}
 (2) \quad &-\cos \theta > 0 \text{ or } \cos \theta < 0 \\
 &\therefore \theta \in (90^\circ; 270^\circ)
 \end{aligned}
 \tag{2}$$

$$\begin{aligned}
 \text{(b)} \quad (1) \quad &\tan \theta \sin \theta + \cos \theta \\
 &= \frac{\sin \theta}{\cos \theta} \cdot \sin \theta + \cos \theta \\
 &= \frac{\sin^2 \theta}{\cos \theta} + \cos \theta \\
 &= \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta} \\
 &= \frac{1}{\cos \theta}
 \end{aligned}
 \tag{3}$$

$$(2) \quad \frac{1}{\cos \theta} = \frac{3}{\sin \theta}$$

$$\sin \theta = 3 \cos \theta$$

$$\tan \theta = 3$$

$$\text{Reference angle} = 71.57^\circ$$

$$\theta = 71.57^\circ + k180^\circ \quad k \in \mathbb{Z}$$

(5)
[14]

QUESTION 4

$$\hat{B} + \hat{D}_2 = 180^\circ$$

Opp angles of a cyclic quadrilateral

$$\hat{D}_1 + \hat{D}_2 + \hat{D}_3 = 180^\circ$$

Angles on straight line

$$\hat{D}_1 = \hat{C}_2$$

tan chord theorem

hence

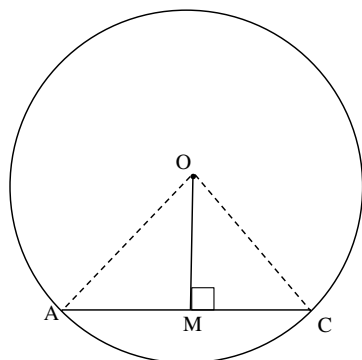
$$\hat{C}_2 + \hat{D}_3 = \hat{B}$$

(3)

[3]

QUESTION 5

(a)



Construction: Join AO and OC

RTP: $AM = MC$ Proof: In Δ 's OAM, OCM

OM is a common side

 $OA = OC$ (Radii) $\hat{M}_1 = \hat{M}_2 = 90^\circ$ (given)Therefore $\Delta AOM \equiv \Delta COM$ (R, H, S)Hence $AM = MC$

(6)

(b) (1) $\hat{B} = 90^\circ$ (Angle in semi circle)

$$BE^2 = 20^2 - 12^2 \quad (\text{Pythag})$$

$$BE = 16 \text{ units}$$

(2)

(2) $\frac{BC}{CE} = \frac{GO}{OE} = \frac{4}{10}$ or $\frac{2}{5}$; line parallel to one side of Δ /mean proportion theorem

(2)

(3) $BD = 8$ units (Line from centre $\perp$ chord)

$$\frac{BC}{16} = \frac{4}{14}$$

$$BC = 4,57 \text{ units} \quad \text{OR} \quad \frac{32}{7} = 4\frac{4}{7} \text{ units}$$

$$\therefore CD = 8 - 4,57$$

$$CD = 3,43 \text{ units} \quad \text{OR} \quad \frac{24}{7} \text{ units}$$

(4)

ORMake $BE = 1k$; $BC = \frac{2}{7}k$ and $DE = \frac{1}{2}k$ (line from centre $\perp$ chord)

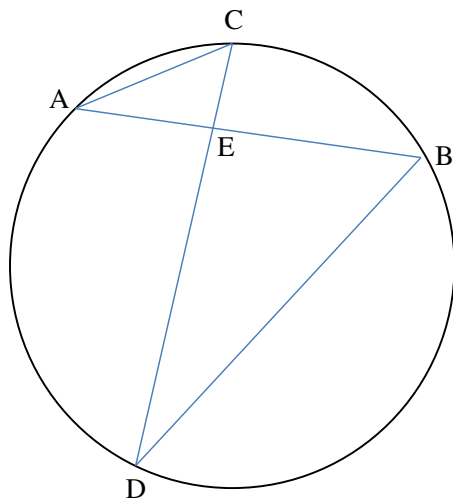
$$\text{therefore } CD = \left(1 - \frac{2}{7} - \frac{1}{2}\right) \times 16$$

$$CD = \frac{24}{7}$$

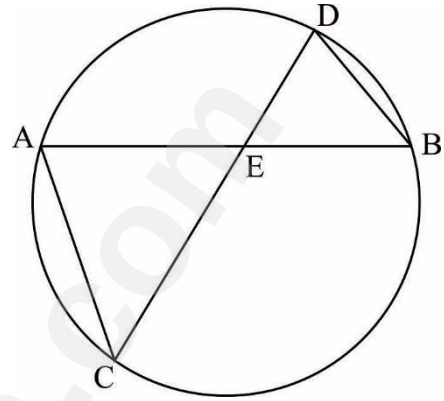
[14]

QUESTION 6

(a)



OR



(1)

- (b) $\hat{A} = \hat{D}$ (Angles in same seg)
 $\hat{C} = \hat{B}$ (Angles in same seg)
 $\triangle AEC \parallel \triangle DEB$ (AAA) or they could give the third angle

(3)

- (c) $\frac{AE}{EC} = \frac{DE}{EB}$ ($\triangle AEC \parallel \triangle DEB$)

$$\therefore AE \times EB = DE \times EC$$

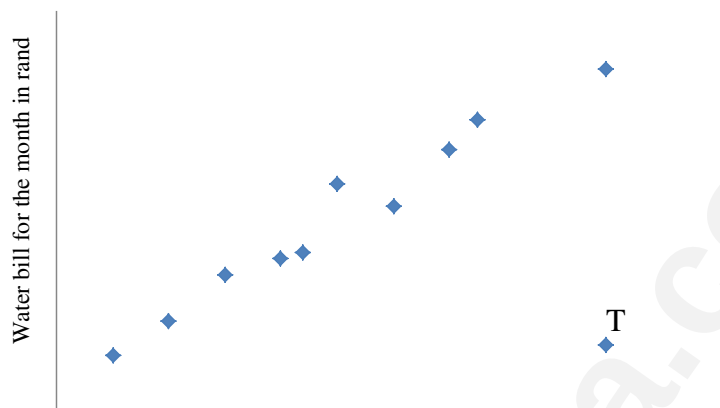
(2)

[6]

QUESTION 7

- (a) (1) $y = 4x - 2$ (1 for line equation; 1 for the gradient and 1 for the y intercept) (3)
- (2) $r = 1$ perfect correlation (2)

(b) (1)



Electricity bill for the month in rand

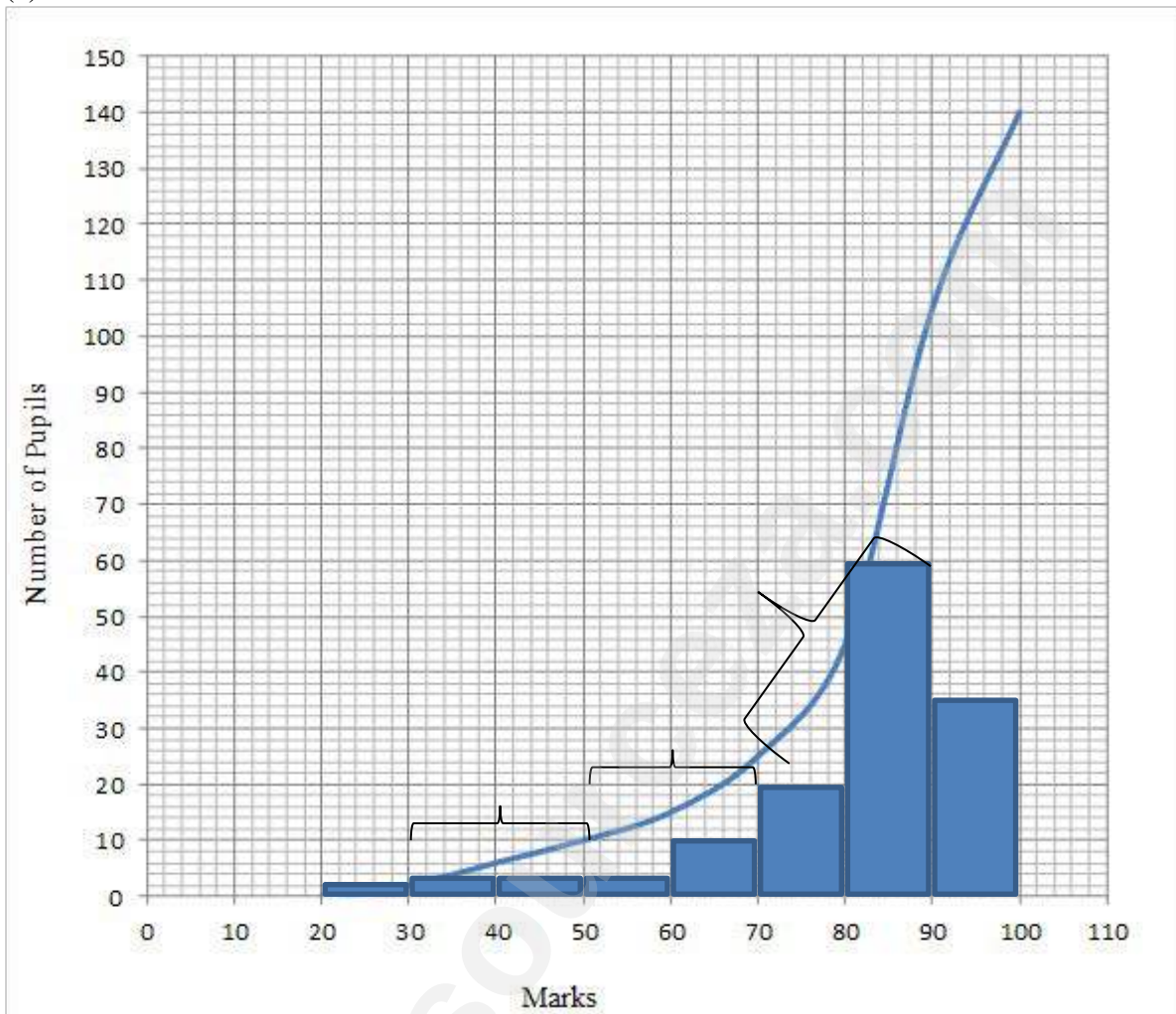
(1)

- (2) Strong/positive relationship or (as the water bill increases so does the elec bill) (1)
- (3) It would increase. (1)
- (4) B would increase. (1)
- (5) No, as you would be extrapolating.
OR: the equation is valid for values of x between 100 and 1 000
OR: Yes, but the result will be innacurate (1)

[10]

QUESTION 8

(a)



For recognising it is a histogram e.g. no spaces

(5)

(b) The mark distribution is skewed to the left or negatively skewed

(1)

(c) TRUE; Data skewed to left

(2)

Or they calculate the quartile values and make decision: 75; 85 and 90

[8]

SECTION B**QUESTION 9**

$$(a) \quad \hat{B} + \hat{C} = \hat{D} + \hat{F} \quad (1)$$

$$(b) \quad \begin{aligned} \hat{B}_1 &= \hat{D}_2 && (\text{Alternate angles } AB \parallel DC) \\ \hat{C} &= \hat{A} && (\text{Angle subtended from equal chord of equal circle}) \\ \therefore \hat{B}_2 &= \hat{D}_1 && (\text{Angles in a triangle}) \end{aligned}$$

But these are alternate angles

BC // AD and ABCD is a parallelogram (6)

OR:

$$\begin{aligned} \hat{B}_1 &= \hat{D}_2 && (\text{alternate angles } AB \parallel DC) \checkmark \\ \hat{C} &= \hat{A} && (\text{angle subtended from equal chord of equal circle}) \\ BD &\text{ is a common side} \\ \therefore \triangle ABD &\equiv \triangle CBD && (\text{A.S.A}) \checkmark \\ AB &= CD \\ \therefore ABCD &\text{ is a parallelogram} && (\text{one pair of opposite sides equal and parallel}) \end{aligned}$$

OR

$$\begin{aligned} \hat{B}_1 + \hat{B}_2 &= 180 - \hat{C} && (\text{co-int angles // lines}) \\ \hat{D}_1 + \hat{D}_2 &= 180 - \hat{A} && (\text{co-int // lines}) \\ \text{but} \\ \hat{A} &= \hat{C} && (\text{Angle subtended by same chord equal circles}) \\ \therefore \hat{B}_1 + \hat{B}_2 &= \hat{D}_1 + \hat{D}_2 \\ \therefore \text{parm} &(\text{Opp angles are equal}) \end{aligned}$$

[7]

QUESTION 10

(a) $\hat{C} = \hat{B} = 55^\circ$ (tan chord theorem) OR (angles in same segment)
 $\hat{D}_2 = 18^\circ$ (tan chord theorem)
 $\hat{D}_1 + \hat{D}_2 = 55^\circ$ (alt angles CD // tangent)
 $\therefore \hat{D}_1 = 37^\circ$
 $\hat{E}_2 = 180^\circ - (55^\circ + 37^\circ)$ ($\angle$'s in a Δ)
 $= 88^\circ$ (6)

(b) (1) $P\hat{D}O = 90^\circ$ (line from centre $\perp$ tangent)
 $P\hat{E}O = 90^\circ$ (line from centre $\perp$ tangent)
 $\hat{P} = 2\hat{A}$ ($\angle$ at centre = $2 \times \angle$ at circumference)
 $\hat{O}_1 = 180^\circ - 2\hat{A}$ (Opp $\angle$'s of quad) (8)

(2) $\hat{O}_2 = 180^\circ + 2\hat{A}$ ($\angle$ around a point)
 $\hat{K}_2 = 90^\circ + \hat{A}$ ($\angle$ at centre = $2 \times \angle$ at circumference)
 $\hat{K}_2 = \hat{C}_3 + \hat{E}_1$ (ext $\angle$ of Δ = sum of interior opposite angles)
 $\therefore \hat{C}_3 + \hat{E}_1 = 90^\circ + \hat{A}$

Alternate:

Construct line DE.

$\hat{C}_3 + \hat{E}_1 = \hat{K}_2$ (ext $\angle$ of Δ = sum of interior opposite angles)

$\hat{K}_2 = 180^\circ - (\hat{C}\hat{D}\hat{E} + \hat{K}\hat{E}\hat{D})$ ($\angle$'s in Δ)

but $\hat{E}_1 = \hat{C}\hat{D}\hat{E}$ (tan chord theorem)

$\therefore \hat{K}_2 = 180^\circ - (\hat{E}_1 + \hat{K}\hat{E}\hat{D})$

but $\hat{E}_1 = \hat{K}\hat{E}\hat{D} = \frac{180 - \hat{P}}{2}$ (PD = PE, tangents from common point)

$\therefore \hat{C}_3 + \hat{E}_1 = 180^\circ - \left(90 - \frac{\hat{P}}{2}\right)$

$= 90^\circ + \hat{A}$

(6)
[20]

QUESTION 11

$$(a) \quad 2 \sin 2x + 2 = 2 \cos x + 2 \quad 2 \sin 2x = 2 \cos x$$

$$2 \sin x \cos x - \cos x = 0$$

$$\cos x(2 \sin x - 1) = 0$$

$$\cos x = 0 \text{ or } \sin x = \frac{1}{2}$$

$$x = \pm 90^\circ + k360^\circ \text{ or } x = 30^\circ + k360^\circ \text{ or } 150^\circ + k360^\circ$$

$$\therefore A(150^\circ; -\sqrt{3} + 2)$$

$$\text{OR} \quad 2 \sin 2x + 2 = 2 \cos x + 2 \quad 2 \sin 2x = 2 \cos x$$

$$\sin 2x = \cos x$$

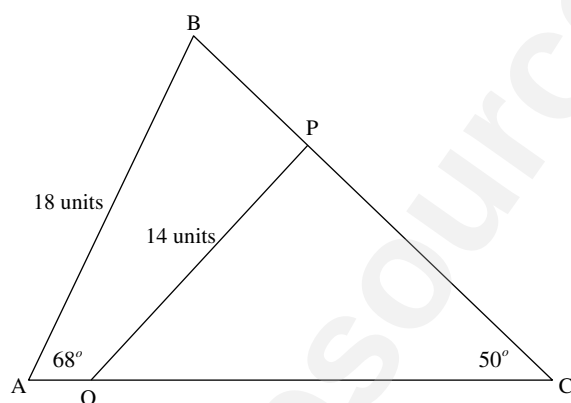
$$\sin 2x = \sin(90^\circ - x) \quad \sin 2x = \sin(90^\circ + x) \quad 2x = 90 - x$$

$$x = 30^\circ + k.120^\circ \quad x = 90^\circ + k.360^\circ$$

$$\therefore A(150^\circ; -\sqrt{3} + 2)$$

(7)

(b)



$$\frac{BC}{\sin 68^\circ} = \frac{18}{\sin 50^\circ}$$

$$BC = \frac{18 \sin 68^\circ}{\sin 50^\circ}$$

$$PC = BC \times \frac{3}{5}$$

$$\frac{\sin \hat{PQC}}{PC} = \frac{\sin 50^\circ}{14}$$

$$\sin \hat{PQC} = \frac{18 \sin 68^\circ}{\sin 50^\circ} \times \frac{3}{5} \times \sin 50^\circ \times \frac{1}{14}$$

$$\sin \hat{PQC} = \frac{54 \sin 68^\circ}{70}$$

$$\hat{PQC} = 45,66^\circ$$

(6)

$$\begin{array}{lcl}
 \text{(c)} & \frac{\cos A \cos 45^\circ + \sin A \sin 45^\circ}{\cos A \cos 45^\circ - \sin A \sin 45^\circ} & \frac{\cos^2 A + 2 \sin A \cos A + \sin^2 A}{\cos^2 A - \sin^2 A} \\
 & \frac{\frac{1}{\sqrt{2}} (\cos A + \sin A)}{\frac{1}{\sqrt{2}} (\cos A - \sin A)} & \frac{(\cos A + \sin A)^2}{(\cos A + \sin A)(\cos A - \sin A)} \\
 & \frac{(\cos A + \sin A)}{(\cos A - \sin A)} & \frac{(\cos A + \sin A)}{(\cos A - \sin A)} \\
 & \therefore &
 \end{array}$$

OR

$$\frac{\cos A \cos 45^\circ + \sin A \sin 45^\circ}{\cos A \cos 45^\circ - \sin A \sin 45^\circ}$$

$$\frac{\frac{1}{\sqrt{2}} (\cos A + \sin A)}{\frac{1}{\sqrt{2}} (\cos A - \sin A)}$$

$$\frac{(\cos A + \sin A)}{(\cos A - \sin A)}$$

$$\frac{(\cos A + \sin A)}{(\cos A - \sin A)} \times \frac{(\cos A + \sin A)}{(\cos A + \sin A)}$$

$$\frac{\cos^2 A + 2 \sin A \cos A + \sin^2 A}{\cos^2 A - \sin^2 A}$$

$$\frac{1 + \sin 2A}{\cos 2A}$$

(6)
[19]

QUESTION 12

(a) $x^2 + 10x + 25 + y^2 - 6y + 9 = 30 + 25 + 9$

$$(x+5)^2 + (y-3)^2 = 64$$

$\therefore$ Radius is 8 units

(4)

(b) $PQ = \sqrt{(7 - (-5))^2 + (-2 - 3)^2}$

$$PQ = \sqrt{169}$$

$$PQ = 13$$

(3)

(c) $P(7; -2)$

$$Q(-5; 3)$$

$$M_{PQ} = \frac{-2-3}{7-(-5)}$$

$$= \frac{-5}{12}$$

$$y = \frac{-5}{12}x + c$$

sub in $(7; -2)$

$$-2 = \frac{-5}{12}(7) + c$$

$$c = \frac{11}{12}$$

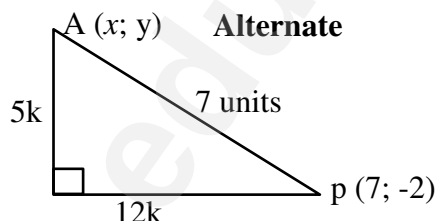
$$\therefore y = \frac{-5x}{12} + \frac{11}{12}$$

$$12y = -5x + 11$$

$$\therefore 5x + 12y = 11$$

(4)

(d) **Alternate**



$$25k^2 + 144k^2 = 49$$

$$k^2 = \frac{49}{169}$$

$$k = \frac{7}{13}$$

$$A\left(7 - 12\left(\frac{7}{13}\right); -2 + 5\left(\frac{7}{13}\right)\right)$$

$$A\left(\frac{-61}{13}; \frac{61}{13}\right)$$

Main

$$\left. \begin{array}{l} (x-7)^2 + (y+2)^2 = 49 \\ 5x + 12y = 11 \end{array} \right\} \text{Solve simultaneously}$$

$$169y^2 + 676y - 549 = 0$$

$$y = \frac{9}{13} \text{ OR } \frac{-61}{13}$$

$$5x + 12\left(\frac{9}{13}\right) = 11$$

$$x = \frac{7}{13}$$

$$A\left(\frac{7}{13}; \frac{9}{13}\right)$$

(5)

OR

$$(x-7)^2 + (y+2)^2 = 49$$

$$5x + 12y = 11 \quad (\text{Recognising line and circle})$$

Substitution str line into circle

$$169y^2 + 676y - 549 = 0$$

Substituting the correct value into the equation to get other coordinate

$$A\left(\frac{7}{13}; \frac{9}{13}\right)$$

$$(e) \quad (x-7)^2 + (y+2)^2 - 49 = x^2 + y^2 + 10x - 6y - 30$$

$$x^2 - 14x + 49 + y^2 + 4y + 4 - 49 = x^2 + y^2 + 10x - 6y - 30$$

$$10y - 24x = -34$$

$$\text{OR } y = \frac{12x}{5} - \frac{17}{5} \quad (3)$$

$$(f) \quad \frac{12}{5} \times \frac{-5}{12} = -1$$

$\therefore$ perpendicular

OR

PCQD is a kite

$\therefore$ diagonals are perpendicular to each other.

(2)
[21]

QUESTION 13

$$BC^2 = 1^2 + 1,5^2 - 2(1)(1,5) \cos 30^\circ$$

$$BC = 0,8074179764$$

$$\begin{aligned} \text{Area of triangle} &= \frac{1}{2}(1)(1,5) \sin 30^\circ \\ &= 0,375 m^2 \end{aligned}$$

$$\frac{1}{2} \times BC \times h = 0,375$$

$$h = 0,9288869234$$

Therefore

Volume of the water tank is:

$$\pi(3)^2 \times 0,9288869234$$

$$= \underline{\underline{26,26 m^3}}$$

Alternate method of finding height

$$\frac{80,7}{\sin 30} = \frac{100}{\sin \hat{B}CA}$$

$$\hat{B}CA = 38,26$$

Use trig ratio

$$\sin 38,26 = \frac{h}{150}$$

$$h = 92,89 \text{ cm}$$

[8]

Total: 150 marks