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Directorate: Curriculum FET

Learner revision material

Technical Mathematics

GRADE 12

TERM 1

2024

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ALGEBRAIC EXPRESSIONS**QUESTION 1**

Simplify the following the expressions:

$$1 \quad \frac{(3x)^{-2}}{3x^{-3}} \quad (3)$$

$$2 \quad \sqrt[3]{343x^{12}} + \sqrt[3]{64x^{12}} \quad (3)$$

$$3 \quad \sqrt{3}(\sqrt{3} + \sqrt{6}) + \sqrt{2} \quad (3)$$

$$4 \quad \log_{\frac{1}{2}} 9 \times \log_{\frac{1}{3}} 8 \quad (4)$$

QUESTION 2

Simplify the following expressions:

$$1 \quad \frac{2^{2x} - 2^x}{2^x - 1} \quad (2)$$

$$2 \quad (3 - \sqrt{2})(3 + \sqrt{2}) \quad (2)$$

$$3 \quad \frac{3(x^4 \cdot y^{-4})^2}{2(x \cdot y^2)^2} \div \frac{(xy)^{-3}}{(3x^{-2} \cdot y^4)^2} \quad (5)$$

$$4 \quad \frac{\sqrt{75} - \sqrt{27}}{\sqrt{12}} \quad (5)$$

$$5 \quad \frac{\log 9}{\log \frac{1}{3}} \quad (3)$$

QUESTION 3

Simplify the following expressions:

$$1 \quad \left(\frac{\sqrt{b^3}}{b^{-\frac{1}{2}}} \right)^{-1} \quad (3)$$

$$2 \quad \frac{9}{\sqrt{48} - \sqrt{3}} \quad (3)$$

$$3 \quad \frac{\sqrt{45x^5} - \sqrt{20x^3}}{\sqrt{125x^3}} \quad (4)$$

$$4 \quad 2\log_3 5 - \log_3 50 + \log_3 6 \quad (4)$$

QUESTION 4

Simplify the following expressions:

$$1 \quad \left(2\frac{1}{4} \right)^{-\frac{3}{2}} \quad (3)$$

$$2 \quad \frac{3^{x+1} - 3^{x-1}}{2 \cdot 3^x} \quad (3)$$

$$3 \quad \sqrt{8x^2} - 4\sqrt{72x^2} + 5\sqrt{98x^2} \quad (4)$$

$$4 \quad \log_3 27 - \log_7 \sqrt{7} \quad (5)$$

EQUATIONS AND INEQUALITIES**QUESTION 1**

1.1 Solve for x :

1.1.1 $x(x - 4) = 21$ (4)

1.1.2 $(2x - 1)(x + 3) = 12x + 1$ (5)

1.1.3 $\log_5(x + 3) + \log_5(x - 1) = 1$ (5)

1.1.4 $x^2 - 2x \geq 3$ (4)

1.2 Solve for x and y :

1.2.1 $2y + x - 1 = 0$ and $3x^2 - x - y = 3$ (7)

1.2.2 $x + 3i = (1 + yi)(2 - 3i)$ (5)

1.3 Convert 111001_2 to a decimal number. (3)

1.4 The formula for calculating the distance travelled by an object at time t seconds is:

$$s = ut + \frac{1}{2}at^2$$

where u is the initial velocity and a is the acceleration of the object.

1.4.1 Make u the subject of the formula. (2)

1.4.2 Hence, calculate the value of u if acceleration, $a = 3 \text{ m/s}^2$ and the distance travelled after 4 seconds is 36 m. (2)

QUESTION 2

2.1 Solve for x :

2.1.1 $(x+2)(3x-6)=0$ (2)

2.1.2 $x^2 - 5x = 2$ (Correct to TWO decimal places) (4)

2.1.3 $2x^2 - 7x - 4 \leq 0$ (4)

2.1.4 $2x^{\frac{3}{4}} = 16$ (3)

2.1.5 $3^x - 3^{x-2} = 24$ (4)

2.1.6 $\log(x+1) + \log x = 1$ (5)

2.2 Solve for x and y :

2.2.1 $y - x + 2 = 0$ and $x^2 + 2x + y^2 - 6y = 40$ (7)

2.2.2 $2x + i = (4 - y)(1 + i)$ (5)

2.3 Covert 100 to a binary number. (3)

2.4 Given $i = \frac{nE}{R + \frac{nr}{m}}$

2.4.1 Make R the subject of the formula. (3)

2.4.2 Determine the value of R if $i = 2,3$; $E = 2,4$; $n = 5,6$; $m = 2$
and $r = 0,4$ (2)

QUESTION 3

3.1.1 Solve for x :

$$3.1.1 \quad (x + 2)(x - 5) = -6 \quad (4)$$

$$3.1.2 \quad 2x^2 = 7x - 4. \text{ Round your answer to 2 decimal digits.} \quad (4)$$

$$3.1.3 \quad -x^2 + 5x - 4 < 0 \quad (4)$$

$$3.1.4 \quad 2\log x = \log(2x + 3) \quad (4)$$

3.2 Solve for x and y :

$$3.2.1 \quad 2x - y = 1 \text{ and } x^2 - 2xy + 2x - y^2 = 0 \quad (7)$$

$$3.2.2 \quad \frac{1}{2}x - 3i = (1 + yi)(1 - 5i) \quad (5)$$

3.3 Given the decimal numbers 11 and 23

$$3.3.1 \quad \text{Convert the given decimal numbers to binary numbers.} \quad (4)$$

$$3.3.2 \quad \text{Hence, add the binary numbers calculated in 1.3.1} \quad (1)$$

CALCULUS**AVERAGE GRADIENT****QUESTION 1**

- 1.1 Determine the average gradient of $f(x) = 2x^2 + x$ between the points where $x = 1$ and $x = 4$
- 1.2 Average gradient of $g(x) = 1 - x^2$ between the points where $x = -1$ and $x = p$ is -6 . Determine the value (s) of p .

QUESTION 2

Differentiate the following from the **FIRST PRINCIPLES**:

- 2.1 $f(x) = 3$
- 2.2 $g(x) = -2x$
- 2.3 $p(x) = -x - 1$
- 2.4 $f(x) = 4x + 3$

RULES OF DIFFERENTIATIONS**QUESTION 3**

Differentiate the following:

- 3.1 $y = (3x - 1)(x - 3)$
- 3.2 $D_x [x(\sqrt{x} - x)]$
- 3.3 $f(x) = \frac{2}{x} - \sqrt[4]{x}$
- 3.4 $g(x) = \frac{4x^3 - x^4}{x}$
- 3.5 $y = \frac{8x^3 - 27}{3 - 2x}$

CUBIC FUNCTION**QUESTION 4**

Given $f(x) = 2x^3 - 2x^2 - 10x - 6$

- 4.1 Factorize f completely.
- 4.2 Hence write down the coordinates of the x -intercepts of f .
- 4.3 Determine the coordinates of the turning points of f .
- 4.4 Draw sketch graph of f . Clearly indicate all the intercepts with the axes and the turning points.
- 4.5 Determine the equation of a tangent to f at the point where $x = 1$

QUESTION 5

Given $f(x) = -x^3 - x^2 + 16x + 16$

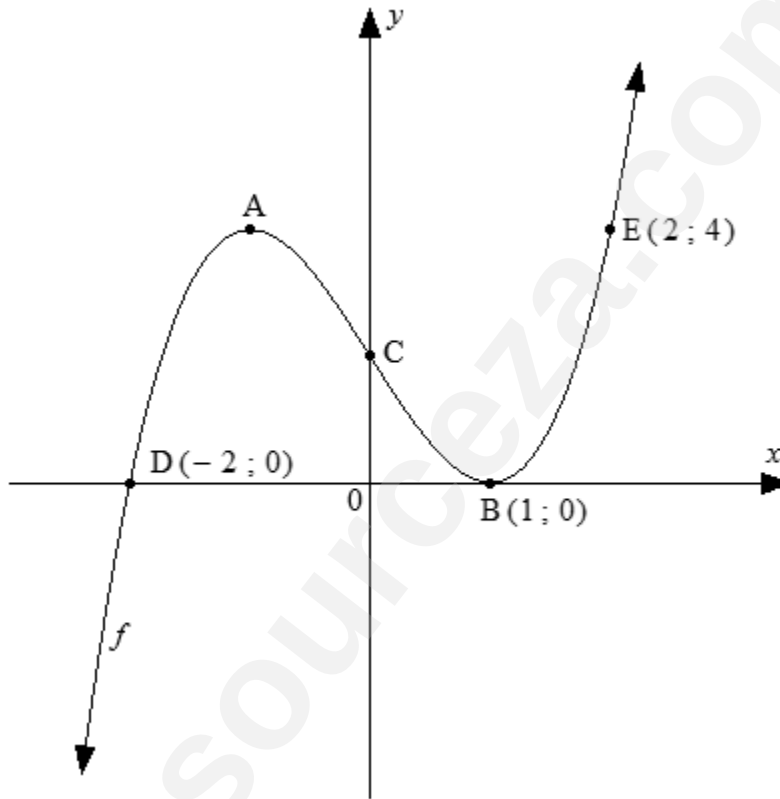
- 5.1 Factorize f completely.
- 5.2 Hence write down the coordinates of the x -intercepts of f .
- 5.3 Determine the coordinates of the turning points of f .
- 5.4 Draw sketch graph of f . Clearly indicate all the intercepts with the axes and the turning points.
- 5.5 For what values of x is $f(x) \geq 0$?
- 5.6 Determine the equation of a tangent to f at the point $(-1; -2)$

QUESTION 6

The sketch below represents the graph of f defined by $f(x) = ax^3 + bx + c$

A and B are the turning points of f . D is the x -intercept and B is the x -intercept of f .

E(2 ; 4) is a point on f .

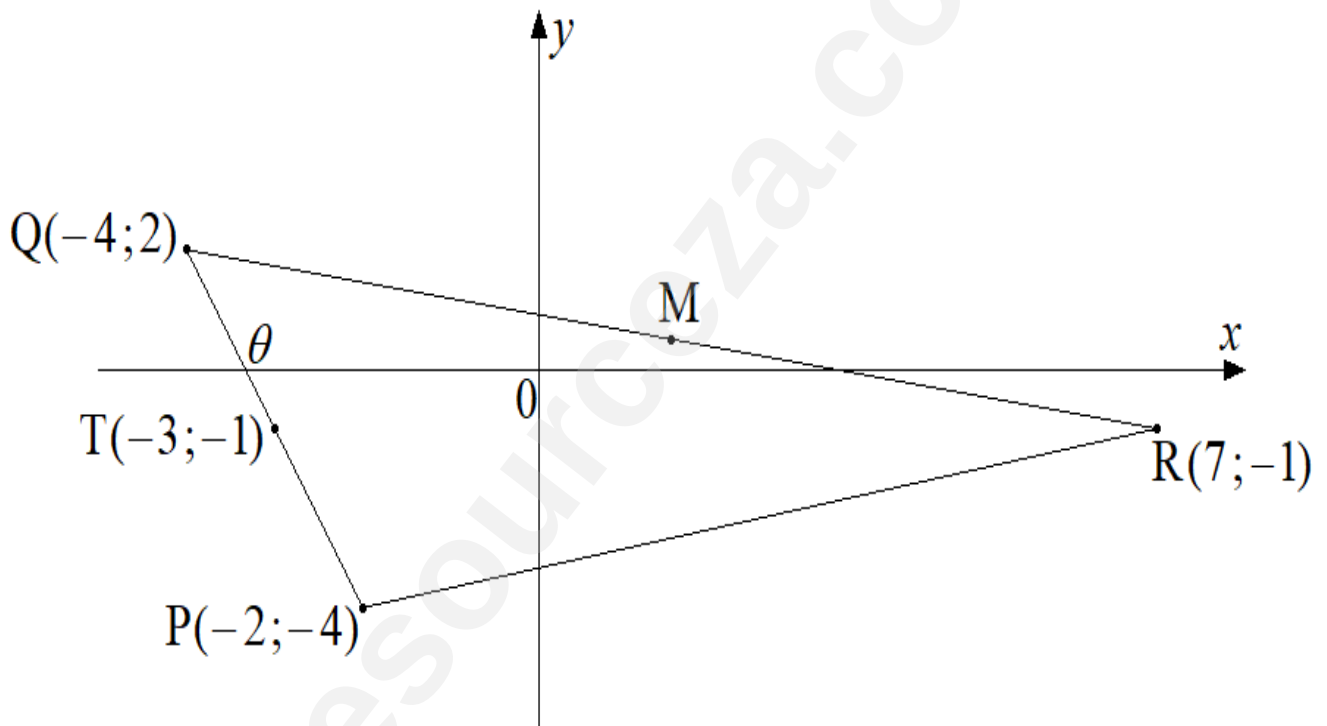


- 6.1 Show that the equation of f is $f(x) = x^3 - 3x + 2$
- 6.2 Determine the coordinates of A.
- 6.3 Write down the coordinates of C.
- 6.4 The gradient of a tangent to f at the point $(r ; t)$ is 24.
Determine the values of r and t if $r > 0$

ANALYTICAL GEOMETRY

QUESTION 1

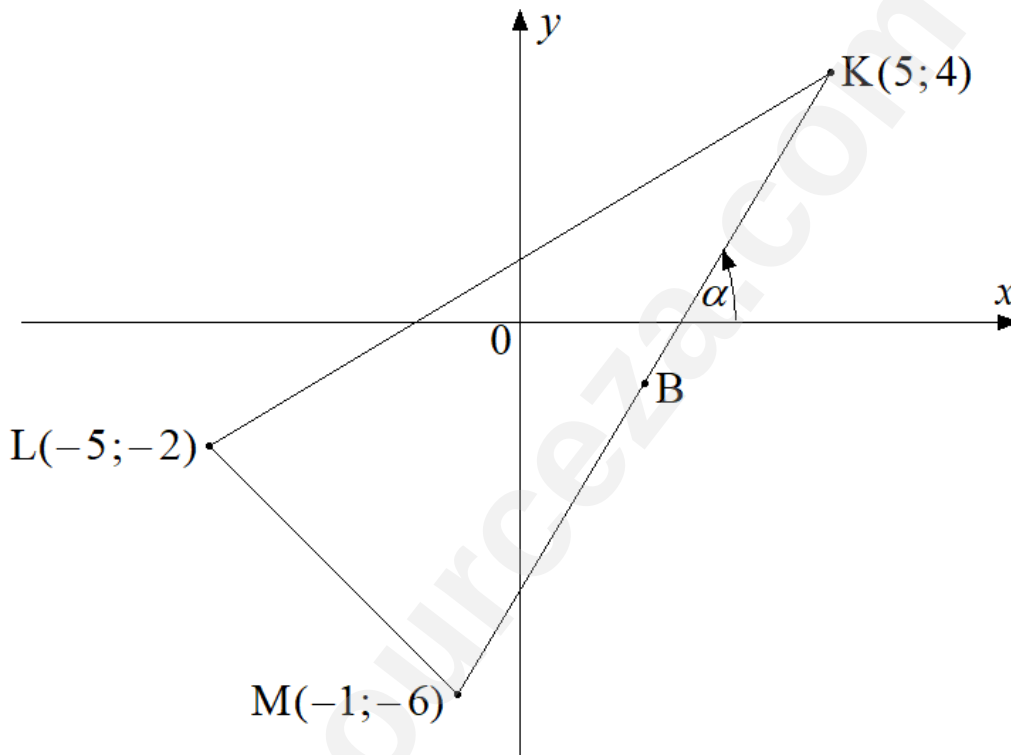
$P(-2; -4)$, $Q(-4; 2)$ and $R(7; -1)$ are the vertices of $\triangle PQR$ in a Cartesian plane as shown in the diagram below. θ is an angle of inclination of PQ . M and T are the midpoints of QR and QP respectively.



- 1.1 Determine the gradients of QP and PR . (4)
- 1.2 What can you conclude about QP and PR ? (2)
- 1.3 Calculate the value of θ , to the nearest degree. (3)
- 1.4 Determine the coordinates of M . (2)
- 1.5 Hence, determine the equation of the line MN , passing through M and parallel to PR . (5)
- 1.6 Determine whether the point of PQ lies on MN . (2)

QUESTION 2

In the diagram below, points $L(-5; -2)$; $M(-1; -6)$ and $K(5; 4)$ are the vertices of ΔKLM . B is the midpoint of MK and α is the angle of inclination of MK with the x -axis.



Determine:

- 2.1 Length of KL. (2)
- 2.2 Coordinates of B. (2)
- 2.3 Gradient of LM. (2)
- 2.4 The equation of the line perpendicular to LM passing through M. (3)
- 2.5 The size of α . (4)

QUESTION 3

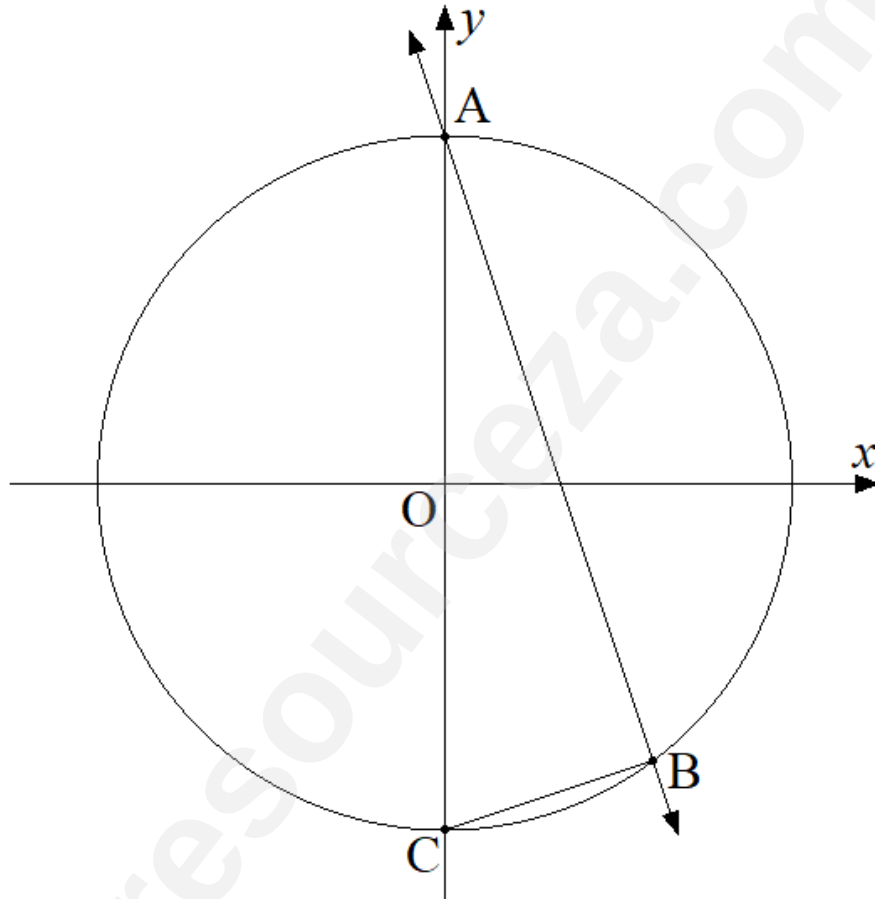
$M(4; 4)$, $N(p; 3)$, $R(1; 1)$ and $T(-1; q)$ are points in a Cartesian plane. Calculate:

3.1.1 The value of p if the gradient of MN is equal to $\frac{1}{2}$ (3)

3.1.2 The value of q if $RM \perp MT$ (4)

QUESTION 4

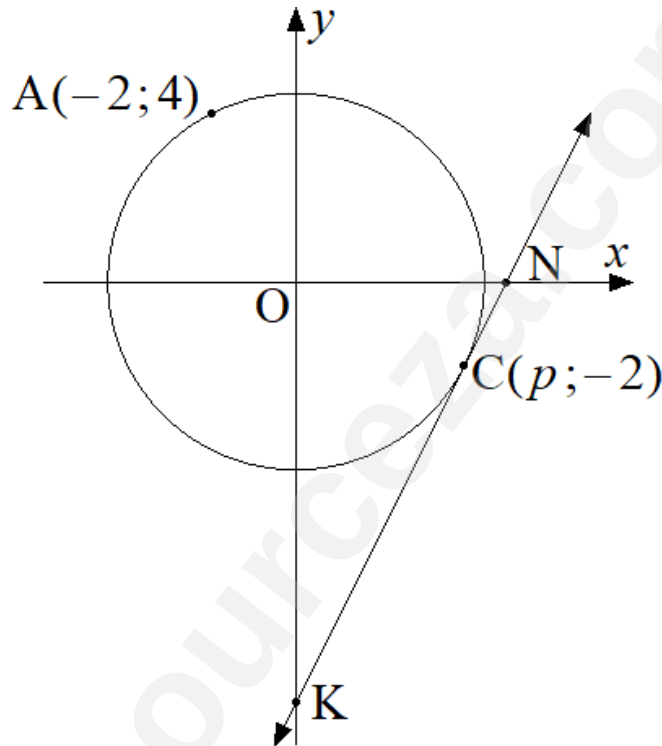
In the diagram below, circle ABC with centre O is given. Points A and C are on the y-axis. The equation of AB is given by $y = -3x + 8$



- 4.1 Determine the coordinates of A. (2)
- 4.2 Hence, determine the equation of the circle. (3)
- 4.3 If the coordinates of B are $\left(\frac{24}{5}; -\frac{32}{5}\right)$, determine the equation of a tangent to the circle at the point B. (5)

QUESTION 5

In diagram below, circle with centre O at the origin passes through $A(-2; 4)$.
The tangent to the circle at $C(p; 2)$ cuts the y -axis at K and the x -axis at N .

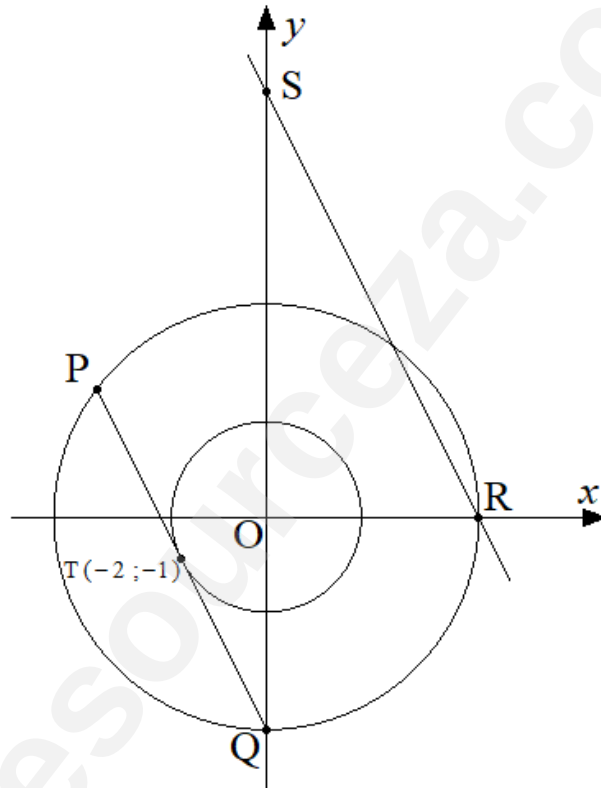


- 5.1 Calculate the length of the radius OA . Leave your answer in surd form. (2)
- 5.2 Hence write down the equation of the circle. (2)
- 5.3 Show that $p = 4$ (3)
- 5.4 Write down the gradient of OC . (1)
- 5.5 Hence, determine the equation of CK , tangent to the circle at C . (4)
- 5.6 Determine the coordinates of N . (2)
- 5.7 Determine the equation of the diameter which is parallel to tangent CK . (3)

QUESTION 6

In the figure, the origin O is the centre of two concentric circles. The chord PQ of the larger circle touches the smaller circle at $T(-2; -1)$ and intersect the y -axis at point Q .

PQ is a tangent to the smaller circle at point T . P is a point on the larger circle, while Q and R are the y and x -intercepts of the larger circle respectively. S is the y -intercept of the straight line SR .



- 6.1 Determine the equation of the smaller circle. (3)
- 6.2 Given that $PT = 2\sqrt{5}$ units, show that the equation of the larger circle is defined by $x^2 + y^2 = 25$ (4)
- 6.3 Write down the coordinates of point R . (1)
- 6.4 Given that $PQ \parallel SR$, determine the equation of SR . (4)
- [12]**