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Sequences and series



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Lesson 1: Sequences and series – Introduction

$$\text{Area of } n \text{ rectangles} = \frac{1}{n^3} (1^2 + 2^2 + 3^2 + \dots + n^2)$$

Lesson 2: Sequences and series – Sigma notation

$$1. \sum_{n=5}^{15} n^2 - n = 20 + 30 + 42 + 56 + 72 + 90 + 110 + 132 + 156 + 182 + 210$$

$$2. 5 + 10 + 15 + \dots + 5\,000 = \sum_{i=1}^{1000} 5i$$

Lesson 3: Sequences and Series – Arithmetic Progressions

1. See lesson notes
2. a. $T_n = a + (n-1)d = 55 + (n-1)(-5) = 60 - 5n$
 b. $S_n = \frac{n \times (2a + (n-1)d)}{2} = \frac{n \times (2 \times 55 + (n-1)(-5))}{2} = \frac{n(115 - 5n)}{2}$
 c. $T_n = 60 - 5n = 0$
 $\therefore n = 12$
 d. $S_n = \frac{n(115 - 5n)}{2} = 0$
 $\therefore n(115 - 5n) = 0$
 $\therefore n = 0 \text{ or } 115 - 5n = 0$
 $\therefore n = 23$

Lesson 4: Sequences and series – Geometric progressions

1. See lesson notes
2. a. $T_n = a \times r^{n-1} = 81 \times \left(-\frac{1}{3}\right)^{n-1}$
 b. $S_n = \frac{a(1 - r^n)}{(1 - r)} = \frac{81 \left(1 - \left(-\frac{1}{3}\right)^n\right)}{1 + \frac{1}{3}} = \frac{243}{4} \left(1 - \left(-\frac{1}{3}\right)^n\right)$
 c. $T_n = 81 \times \left(-\frac{1}{3}\right)^{n-1} = 1$
 $\therefore \left(-\frac{1}{3}\right)^{n-1} = \frac{1}{81}$
 $\therefore \left(-\frac{1}{3}\right)^{n-1} = \left(-\frac{1}{3}\right)^4$
 $\therefore n - 1 = 4$
 $\therefore n = 5$
 d. $S_n = \frac{243}{4} \left(1 - \left(-\frac{1}{3}\right)^n\right) = 61$
 $\therefore \frac{243}{4} \left(1 - \left(-\frac{1}{3}\right)^n\right) = \frac{244}{243}$
 $\therefore -\left(-\frac{1}{3}\right)^n = \frac{244}{243} - 1$

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$$\therefore -\left(-\frac{1}{3}\right)^n = -\frac{1}{243} = -\left(-\frac{1}{3}\right)^5$$

$$\therefore n = 5$$

Lesson 5: Sequences and series – Sum to infinity of geometric progressions

1. See lesson notes
2. For the sequence to be geometric:

$$\frac{m}{m+2} = \frac{2m-3}{m}$$

$$\therefore m^2 = 2m^2 + m - 6$$

$$\therefore 0 = m^2 + m - 6$$

$$\therefore 0 = (m+3)(m-2)$$

$$\therefore m = -3 \text{ or } m = 2$$

When $m = -3$, $T_1 = -1$, $T_2 = -3$, $T_3 = -9$ and $r = 3$ so the series is divergent

When $m = 2$, $T_1 = 4$, $T_2 = 2$, $T_3 = 1$ and $r = \frac{1}{2}$ so the series is convergent and

$$S_{\infty} = \frac{4}{1 - \frac{1}{2}} = 8$$

