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GRADE	12	SUBJECT	PHYSICAL SCIENCES	Week	4
Topic	VERTICAL PROJECTILE: VERTICAL PROJECTILE MOTION				
ATP	(1week) - Explain that projectiles fall freely with gravitational acceleration downwards with a constant acceleration irrespective of whether the projectile is moving upwards or downward or is at maximum height. - Know that projectiles take the same time to reach their greatest height from the point of upward launch as the time they take to fall back to the point of launch. This is known as time symmetry. - Known that projectiles can have their motion described by a single set of equations for the upwards and downward motion.				



SABC BROADCAST -
VPM.pptx



Interpreting
Motion Graphs.mp4

Initial activities: (± 5 minutes)

In everyday life we use the term **PROJECTILE** when we refer to bullets fired by guns or other weapons however in Physics the concept of projectile is more general.

PROJECTILE is any object (body) that moves only under the action of the gravitational force.

The projectile can be projected vertically, horizontally or at any angle to the horizontal but here we are going to refer only to the vertical projectile motion.

DEVELOPMENT OF THE LESSON: (± 45 minutes)

Learner should use PHET simulation on projectile motion to support the explanation.

<https://phet.colorado.edu/en/simulation/projectile-motion>

One learner drops a ball while other learner throws the ball vertically downwards at same time. (It would be good to do this from the top of a staircases)

Two learners stand on the top of a staircase. One learner drops a ball while at the same instant the other learner throws the ball vertically upwards.

Learners observe and state their observations orally and ask questions.

A projectile move with constant acceleration irrespective of whether the projectile is moving upwards or downward or is at maximum height.

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English (United States)



-The teacher must explain that projectiles fall freely with gravitational acceleration downwards.

We know from Newton's Law of Universal Gravitation that an object in the Earth's gravitational field experiences a force pulling it towards the center of the Earth. If this is the only force acting on the object, then the object will accelerate towards the center of the Earth.

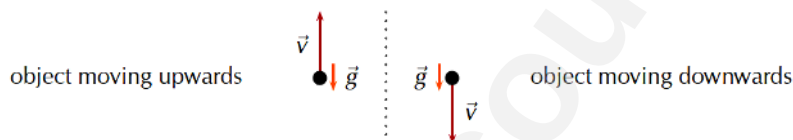
A person standing on the Earth's surface will interpret this acceleration as objects always falling downwards. In Grade 11, we showed that the value of the acceleration due to gravity can be treated as having a constant magnitude of $g = 9,8 \text{ m} \cdot \text{s}^{-2}$.

Any object that is being acted upon only by the force of gravity is said to be in a state of **free fall**. There are two important motion characteristics that are true of free-falling objects:

- ✚ Free-falling objects do not encounter air resistance. ([from CAPS document](#))
- ✚ All free-falling objects (on Earth) accelerate downwards at a rate of $9.8 \text{ m} \cdot \text{s}^{-2}$

Free fall is any motion of a body where the gravitational force is the only force acting upon it.

Projectiles moving upwards or downwards in the Earth's gravitational field always accelerate downwards with a constant acceleration $\vec{g}$. ([Siyavula Physical Science grade 12 Page 73](#))



Projectiles that have an initial velocity upwards will have zero velocity at their greatest height, h_{max} but the acceleration is still $\vec{g}$.

This means that if an object is moving upwards with some initial velocity in the vertical direction, the magnitude of the velocity in the vertical direction decreases until it stops ($v = 0 \text{ m} \cdot \text{s}^{-1}$) for an instant. The point at which the velocity is reduced to zero corresponding with the maximum height, h_{max} , that the object reaches. After this, the object starts to fall. It is very important to remember that the acceleration is constant, but the velocity vector has changed in magnitude and direction. At the maximum height where the velocity is zero the acceleration is still $\vec{g}$.

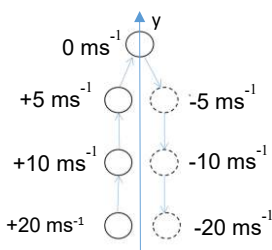
Consider an object thrown vertically upwards from an initial height (h_i). We have said that the object will travel upwards with decreasing vertical velocity until it stops, and then starts moving vertically downwards. The time that it takes for the object to fall back down to the initial height (h_i) is the same as the time taken for the object to reach its maximum height from the initial height (h_i).

This is known as **time symmetry**. This is a consequence of the constant (uniform) acceleration the projectile experiences.



This has two implications for projectiles that pass through a point on both the upward and downward part of their motion when in free-fall:

- time symmetry:** the time intervals during the upward motion and the downward motion are the same, for example it will take the same time to rise from initial position to maximum height as it will to drop back to the initial position. The time interval between the projectile passing the point and being at maximum height is the same, Δt .
- magnitude of the velocity:** the magnitude of the velocity at the same point on the upward and downward motion will be the same; the direction will be reversed, as indicated by the arrows in the figure below.



This is very useful when solving problems because if you have any information about the upward motion you can learn something about the downward motion as well and vice versa.

The motion of any projectile can be described using the EQUATIONS OF KINEMATICS (equations of motion) in the case of a vertical projectile motion the equations for a motion with constant acceleration can be used even if the object (increases velocity) or slow down (decreases velocity).

In the study of vertical projectile motion it is possible to make an analogy with the horizontal motion.

$$\vec{a} = \vec{g}$$

$$\vec{x} = \vec{y}$$

$$\Delta \vec{x} = \Delta \vec{y}$$

Physical Quantity	Motion on the horizontal plane	Vertical projectile motion
Displacement	$\Delta \vec{x} = \vec{v}_i \Delta t + \frac{1}{2} \vec{a} \Delta t^2$ $\Delta \vec{x} = \frac{\vec{v}_f^2 - \vec{v}_i^2}{2\vec{a}}$ $\Delta \vec{x} = \left(\frac{\vec{v}_i + \vec{v}_f}{2} \right) \Delta t$	$\Delta \vec{y} = \vec{v}_i \Delta t + \frac{1}{2} \vec{g} \Delta t^2$ $\Delta \vec{y} = \frac{\vec{v}_f^2 - \vec{v}_i^2}{2\vec{g}}$ $\Delta \vec{y} = \left(\frac{\vec{v}_i + \vec{v}_f}{2} \right) \Delta t$
Velocity	$\vec{v}_f = \vec{v}_i + \vec{a} \Delta t$	$\vec{v}_f = \vec{v}_i + \vec{g} \Delta t$



	$\triangleright \vec{v}_f^2 = \vec{v}_i^2 + 2\vec{a}\Delta\vec{x}$	$\triangleright \vec{v}_f^2 = \vec{v}_i^2 + 2\vec{g}\Delta\vec{y}$
Acceleration	Constant $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$	Constant $\vec{g} = 9,8 \text{ m} \cdot \text{s}^{-2}$
Position	$\vec{x}_f = \vec{x}_i + \Delta\vec{x}$	$\vec{y}_f = \vec{y}_i + \Delta\vec{y}$

POSITION ($\vec{x}$ or $\vec{y}$), **DISPLACEMENT** ($\Delta\vec{x}$ or $\Delta\vec{y}$), **VELOCITY** ($\vec{v}$) and **ACCELERATION** ($\vec{a}$ or $\vec{g}$) are vector quantities because they have both magnitude and direction; that is why it is important to determine the sign of each of them when solving problems.

Position, displacement, velocity and acceleration are measured with respect to a coordinate system (frame of reference), a grid or axis you impose on a problem to analyze the motion. We will find it convenient to use a y-axis for vertical motion. A coordinate axis has two features:

- ✚ And origin to define zero
- ✚ An x or y label to indicate the positive end of the axis.

Example

A projectile is moving upwards after being thrown.

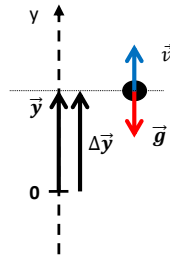
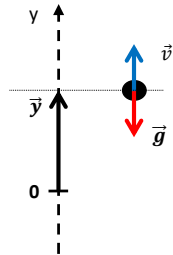
Let's select our **positive direction upwards** (vectors pointing up are positive and vectors pointing down are negative)

Position above origin (position vector pointing up) is positive ($\vec{y} > 0$). The sign of position (x or y) tells us where an object is.

Velocity vector pointing upwards is positive ($\vec{v} > 0$). The sign of velocity (v_x or v_y) tells us which direction the object is moving.

Acceleration vector ($\vec{g}$) pointing down is negative ($\vec{g} < 0$). The sign of acceleration (a_x or a_y) tells us which way the acceleration vector points, not whether the object is speeding up or slowing down.

Displacement vector pointing upwards is positive ($\Delta\vec{y} > 0$)



Meaning of minus signs

A negative value of position ($\vec{y}$) means that the object is below the chosen origin. A negative value of velocity means that the object is moving in the opposite direction of the y axis and the negative value of acceleration tells us that the acceleration vector is pointing in opposite direction of the y axis, a negative acceleration.

Note: As we are working with vector quantities and vector equations, the positive direction must be chosen before starting to solve the problems.

- The teacher must explain example 1.

Example 1

A man drops a ball and it strikes the ground with a velocity of $10 \text{ m}\cdot\text{s}^{-1}$ downwards.

1.1. Calculate the height from where the ball was dropped.

1.2. Determine the time taken for the ball to hit the ground.

Solution: taking downwards as positive direction.

Data: $V_i = 0 \text{ m}\cdot\text{s}^{-1}$

$$1.1 \quad \vec{v}_f^2 = \vec{v}_i^2 + 2\vec{a}\Delta\vec{y}$$

$$V_i = 0 \text{ m}\cdot\text{s}^{-1}$$

$$V_f = 10 \text{ m}\cdot\text{s}^{-1}$$

$$(+10)^2 = (0)^2 + 2(+9,8)\Delta y$$

$$g = 9,8 \text{ m}\cdot\text{s}^{-1}$$

$$\Delta y = \frac{100}{19,6}$$

$$\Delta y = ?$$

$$\Delta y = 5,10 \text{ m}$$

$$V_f = 10 \text{ m}\cdot\text{s}^{-1}$$

$$1.2 \quad \vec{v}_f = \vec{v}_i + \vec{g}\Delta t$$



$$(+10) = 0 + (+9.8)\Delta t$$

$$\Delta t = \frac{10}{9.8}$$

$$\Delta t = 1.02 \text{ s}$$

Example 2 (Example 1 page 77, Siyavula Physical Science, Grade 12):Adapted

A ball is thrown vertically upwards with an initial velocity of $10 \text{ m} \cdot \text{s}^{-1}$. A photograph (shown) is taken when the ball is 1,5 m above the point of release.

2.1 Determine the maximum height above the thrower's hand reached by the ball.

2.2 Determine the time it takes the ball to reach its maximum height.

2.3 Determine the time(s) when the ball will be at the position shown in the photograph.



SOLUTION

Step 1: Identify what is required and what is given

Data:

2.1

$$\vec{v}_i = 10 \text{ m} \cdot \text{s}^{-1} \text{ upwards}$$

$$\vec{v}_f = 0 \text{ m} \cdot \text{s}^{-1}$$

$$\vec{g} = 9.8 \text{ m} \cdot \text{s}^{-2} \text{ downwards}$$

$$h_{\max} = \Delta \vec{y} = ?$$

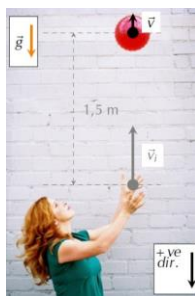
2.2

$$\Delta t = ?$$

2.3

$$\vec{y} = 1.5 \text{ m upwards}$$

$$\Delta t = ?$$





Step 2: Determine how to approach the problem.

Even though there are a number of questions to answer, the best approach is to work through them one at a time.

First, as with rectilinear motion in Grade 10, select a positive direction. We choose **down as positive** and remember to **keep this sign convention** for the whole problem.

This will allow us to analyze the information given and determine directions and signs.

Step 3: Identify the appropriate equation to determine the maximum height.

We can use $\vec{v}_f^2 = \vec{v}_i^2 + 2\vec{a}\Delta\vec{y}$ to solve for the height.

Step 4: Substitute the values into the equation and find the maximum height.

$$\vec{v}_f^2 = \vec{v}_i^2 + 2\vec{a}\Delta\vec{y}$$

$$0^2 = (-10)^2 + (2)(9,8)\Delta\vec{y}$$

$$-100 = 19,6\Delta\vec{y}$$

$$\Delta\vec{y} = -5,10 \text{ m}$$

The value for the displacement will be negative because the displacement is upward, and we have chosen downward as positive (and upward as negative).

Step 5: Identify the appropriate equation to determine the time to reach maximum height.

We can use: $\vec{v}_f = \vec{v}_i + \vec{g}\Delta t$ to solve for the time.

$$\vec{v}_f = \vec{v}_i + \vec{g}\Delta t$$

$$0 = -10 + 9,8\Delta t$$

$$\Delta t = 1,02 \text{ s}$$

Step 7: Determine the times at which the ball is in the position in the photograph.

We know the position, initial velocity and the acceleration of the ball. We also expect to get two values as we know the ball will pass through the point twice.

We know that $\vec{y}_f = \vec{y}_i + \Delta\vec{y}$ and initial position is 0 so $\vec{y}_f = \Delta\vec{y} = \vec{v}_i\Delta t + \frac{1}{2}\vec{g}\Delta t^2$, which is a quadratic equation in time, so we expect to find two different solutions if we solve for time.



$$\vec{y}_f = \vec{v}_i \Delta t + \frac{1}{2} \vec{g} \Delta t^2$$

$$\frac{1}{2} \vec{g} \Delta t^2 + \vec{v}_i \Delta t - \vec{y}_f = \vec{0}$$

$$\frac{1}{2} (9,8) \Delta t^2 + (-10) \Delta t - (-1,5) = 0$$

$$4,9 \Delta t^2 - 10 \Delta t + 1,5 = 0$$

$$\Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(4,9)(1,5)}}{2(4,9)} = \frac{10 \pm \sqrt{70,6}}{9,8} = \frac{10 \pm 8,40}{9,8}$$

$$\Delta t = \frac{10 - 8,40}{9,8} = 0,16 \text{ s} \quad \text{OR} \quad \Delta t = \frac{10 + 8,40}{9,8} = 1,88 \text{ s}$$

The ball passes through the point in the photograph at $t = 0,16 \text{ s}$ on the way up and $t = 1,88 \text{ s}$ on the way down

FINAL ACTIVITIES: (± 10 minutes)

CONCLUSION:

-PROJECTILE is any object (body) that moves only under the action of the gravitational force.

Projectiles moving upwards or downwards in the Earth's gravitational field always accelerate downwards with a constant acceleration $\vec{g} = 9,8 \text{ m} \cdot \text{s}^{-2}$.

The motion of any projectile can be described using the EQUATIONS OF KINEMATICS (equations of motion)

Homework:

Activity 1: (Exercise 3 - 1.1 page 82, Siyavula Physical Science Grade 12): (Adapted)

A man standing at the top of a building drops a ball over the edge of the building. The ball strikes the ground with a velocity of $30 \text{ m} \cdot \text{s}^{-1}$ downwards.

1.1 Calculate the height from where the ball was dropped (height of the building).

1.2 Determine the time taken for the ball to hit the ground.

1.3 At what height above the ground the ball's speed will be $20 \text{ m} \cdot \text{s}^{-1}$.

Activity 2:

N.B: For more exercises refer to other textbooks on the National Catalogue, NSC papers, etc.

**Terms:**Examples

Projectile: an object upon which the only force acting is the force of gravity.

Free fall: is any motion of a body where the gravitational force is the only force acting upon it.

✚ The teacher must look at *terms that consist of compound words* and show learners how to analyze it.

Examples

✚ Free fall (free + fall) = adjective + noun

Adjective: is a word which describes the thing for which a noun stands in the sentence.

Noun: is a word that is the name of a person, place, thing, quality, action, etc., and can be used as the subject or object of a verb or the object of a preposition.

Homework Activity 1: Solution

Note: To choose direction is compulsory, but it's your choice to make upward positive or negative. Remind learners to choose the direction of initial velocity as positive.

Positive direction downwards:

1.1 Data:

$$\vec{v}_i = 0$$

$$\vec{v}_f = 30 \text{ m} \cdot \text{s}^{-1} \text{ downwards.}$$

$$\vec{g} = 9,8 \text{ m} \cdot \text{s}^{-2} \text{ downwards.}$$

$$h = \Delta \vec{y} = ? \text{ m}$$

$$h = \Delta \vec{y} = \frac{\vec{v}_f^2 - \vec{v}_i^2}{2\vec{g}} = \frac{(30)^2 - 0}{(2)(9,8)} = \frac{900}{19,6} = 45,92 \text{ m from the floor}$$

1.2 Data:

$$\vec{v}_i = 0$$



$$\vec{v}_f = 30 \text{ m} \cdot \text{s}^{-1} \text{ downwards}$$

$$\vec{g} = 9,8 \text{ m} \cdot \text{s}^{-2} \text{ downwards}$$

$$\Delta \vec{y} = 45,92 \text{ m downwards}$$

$$\Delta t = ?$$

Option 1	Option 2	Option 3
$\vec{v}_f = \vec{v}_i + \vec{g}\Delta t$ $30 = 0 + 9,8\Delta t$ $\frac{30}{9,8} = \Delta t$ $\Delta t = 3,06 \text{ s}$	$\Delta \vec{y} = \left(\frac{\vec{v}_i + \vec{v}_f}{2}\right) \Delta t$ $45,92 = \left(\frac{0 + 30}{2}\right) \Delta t$ $45,92 = 15\Delta t$ $\Delta t = 3,06 \text{ s}$	$\Delta \vec{y} = \vec{v}_i \Delta t + \frac{1}{2} \vec{g} \Delta t^2$ $45,92 = (0)\Delta t + \frac{1}{2}(9,8)\Delta t^2$ $\frac{(45,92)(2)}{9,8} = \Delta t^2$ $\Delta t = \sqrt{9,37}$ $\Delta t = 3,06 \text{ s}$

1.3

Data:

$$\vec{v}_i = 0$$

$$\vec{v}_f = 20 \text{ m} \cdot \text{s}^{-1} \text{ downwards}$$

$$\vec{g} = 9,8 \text{ m} \cdot \text{s}^{-2} \text{ downwards}$$

$$H_{\max} = 45,92 \text{ m}$$

$$h = ?$$

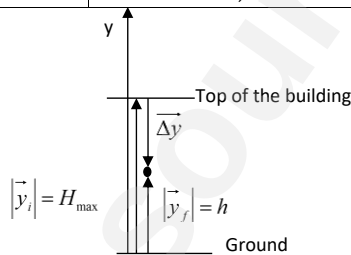
$$\vec{y}_f = \vec{y}_i + \Delta \vec{y}$$

Taking upwards as positive

$$y_f = y_i - \Delta y$$

Also

$$H_{\max} = h + \Delta y$$





$$h = H_{\max} - \Delta y$$

$$\Delta y = \frac{\vec{v}_f^2 - \vec{v}_i^2}{2\vec{g}} = \frac{(20)^2 - 0}{(2)(9,8)} = \frac{900}{19,6} = 20,41 \text{ m}$$

$$h = 45,92 - 20,41 = 25,51 \text{ m above the ground}$$



GRADE	12	SUBJECT	PHYSICAL SCIENCES	Week	5
Topic	VERTICAL PROJECTILE: GRAPHS				
ATP	(± 1 week) ➤ Draw position vs. time, velocity vs. time and acceleration vs. time graphs of 1 D projectile motion. ➤ Given x vs. t , v vs. t or a vs. t , graphs determine position, displacement, velocity, or acceleration at any time t .				

Initial activities:

POSITION ($x^{\rightarrow}$ or $y^{\rightarrow}$), **DISPLACEMENT** ($\Delta x^{\rightarrow}$ or $\Delta y^{\rightarrow}$), **VELOCITY** ($v^{\rightarrow}$) and **ACCELERATION** ($a^{\rightarrow}$ or $g^{\rightarrow}$) are vector quantities because they have both magnitude and direction; that is why it is important?

to determine the sign of each of them when solving problems.

Position, displacement, velocity, and acceleration are measured with respect to a coordinate.

system (frame of reference), a grid or axis you impose on a problem to analyse the motion. We

will find it convenient to use a y -axis for vertical motion. A coordinate axis has two features:

1. And origin to define zero.
2. An x or y label to indicate the positive end of the axis.

http://youtube.com/watch?v=7GJ_SYM8cyU OR <https://www.youtube.com/watch?v=bqf8m7xNvLg>

DEVELOPMENT OF THE LESSON:

The learner must use the demonstration of throwing a ball upwards and catch it as it comes down to differentiate between the representation of the actual path taken by the projectile and the representation of a particular concept e.g. velocity v /s time and acceleration a /s time or position x /s time on a graph. Let's take a situation where a projectile is thrown upwards and return to the thrower's hand, as the projectile goes up (actual path) the velocity is decreasing (concept under question).

(Siyavula Physical Science grade 12)

Projectiles that have an initial velocity upwards will have zero velocity at their greatest height, h_{max} . The acceleration is still $\vec{g}$. (concept). This means that if an object is moving upwards with some initial velocity in the vertical direction, the magnitude of the velocity in the vertical direction decreases until it stops ($v = 0 \text{ m} \cdot \text{s}^{-1}$) for an instant. The point at which the velocity is reduced to zero corresponding with the maximum height, h_{max} , that the object reaches. After this, the object starts to fall. It is very important to remember that the acceleration is constant, but the velocity vector has changed in magnitude and direction. At the maximum height where the velocity is zero the acceleration is still $\vec{g}$. It is imperative that when drawing graphs to represent motion of a projectile you make a choice of direction. If we select our **positive direction upwards** (vectors pointing up are positive and vectors pointing down are negative)



Position above origin (position vector pointing up) is positive ($\vec{y} > 0$). The sign of position (x or y) tells us where an object is.

Velocity vector pointing upwards is positive ($\vec{v} > 0$). The sign of velocity (v_x or v_y) tells us which direction the object is moving.

Acceleration vector ($\vec{g}$) pointing down is negative ($\vec{g} < 0$). The sign of acceleration (a_x or a_y) tells us which way the acceleration vector points, not whether the object is speeding up or slowing down.

Displacement vector pointing upwards is positive ($\Delta\vec{y} > 0$). A negative value of position ($\vec{y}$) means that the object is below the chosen origin. A negative value of velocity means that the object is moving in the opposite direction of the y axis and the negative value of acceleration tells us that the acceleration vector is pointing in opposite direction of the y axis, a negative acceleration. (note that you can also choose the upwards motion as negative) We can describe vertical projectile motion through a series of graphs: position, velocity and acceleration versus time graphs. This isn't new, in Grade 10 we learnt about the graphs for describing rectilinear motion with constant acceleration.

Remember that: $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$ and $\vec{v} = \frac{\Delta\vec{x}}{\Delta t}$. For the graphs of these quantities the **slope** and the **area** under a graph can tell us about changes in other quantities.

Graph	Slope	Area
Displacement vs time	$\vec{v}$	-
Velocity vs time	$\vec{a}$	$\Delta\vec{x}$
Acceleration vs time	-	$\vec{v}$

- The learner should use a video to explain the Graph of position, velocity and acceleration versus time for vertical projectile motion.

<http://youtube.com/watch?v=EFJvKL52TVU>

Graph of position, velocity, and acceleration versus time:

Type of motion	Position vs. time	Velocity vs. time	Acceleration vs. time
Stationary object			



Uniform motion			
Constant acceleration (Velocity increase or object moving downwards)			
Constant acceleration (Velocity decreases or object moving upwards)			

The graphs are the graphical representations of the equations of motion. This means that if you have the graph for one of position, velocity or acceleration you should be able to write down the corresponding equation and vice versa.

The type of graphs is related to the equation of motion: position is a parabola; velocity is a straight line and acceleration is a constant. The characteristic features then depend on the sign convention and the specific values of the problem.

Activity 1:(Northern Cape DE)

A man standing at the top of a building drops a ball over the edge of the building. The ball strikes the ground with a velocity of 30 m.s^{-1} downwards.

- 1.1 Calculate the height from where the ball was dropped (height of the building).
- 1.2 Determine the time taken for the ball to hit the ground.
- 1.3 Draw the position vs. time graph.
- 1.4 Draw the velocity vs. time graph.
- 1.5 Draw the acceleration time graph.



Positive direction downwards:

1.1 Data:

$$\vec{v}_i = 0$$

$$\vec{v}_f = 30 \text{ m} \cdot \text{s}^{-1} \text{ downwards.}$$

$$\vec{g} = 9,8 \text{ m} \cdot \text{s}^{-2} \text{ downwards.}$$

$$h = \Delta \vec{y} = ? \text{ m}$$

$$h = \Delta \vec{y} = \frac{\vec{v}_f^2 - \vec{v}_i^2}{2\vec{g}} = \frac{(30)^2 - 0}{(2)(9,8)} = \frac{900}{19,6} = 45,92 \text{ m from the floor}$$

1.2 Data:

$$\vec{v}_i = 0$$

$$\vec{v}_f = 30 \text{ m} \cdot \text{s}^{-1} \text{ downwards}$$

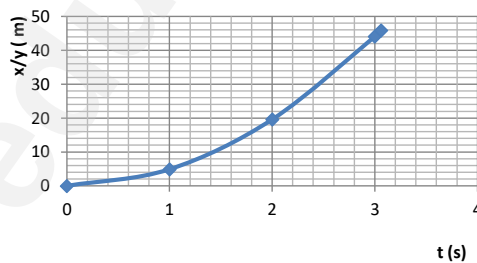
$$\vec{g} = 9,8 \text{ m} \cdot \text{s}^{-2} \text{ downwards}$$

$$\Delta \vec{y} = 45,92 \text{ m downwards}$$

$$\Delta t = ?$$

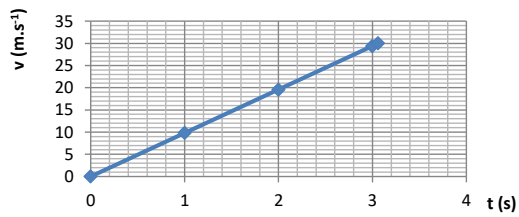
Option 1	Option 2	Option 3
$\vec{v}_f = \vec{v}_i + \vec{g}\Delta t$ $30 = 0 + 9,8\Delta t$ $\frac{30}{9,8} = \Delta t$ $\Delta t = 3,06 \text{ s}$	$\Delta \vec{y} = \left(\frac{\vec{v}_i + \vec{v}_f}{2}\right) \Delta t$ $45,92 = \left(\frac{0 + 30}{2}\right) \Delta t$ $45,92 = 15\Delta t$ $\Delta t = 3,06 \text{ s}$	$\Delta \vec{y} = \vec{v}_i \Delta t + \frac{1}{2} \vec{g} \Delta t^2$ $45,92 = (0)\Delta t + \frac{1}{2} (9,8)\Delta t^2$ $\frac{(45,92)(2)}{9,8} = \Delta t^2$ $\Delta t = \sqrt{9,37}$ $\Delta t = 3,06 \text{ s}$

1.3

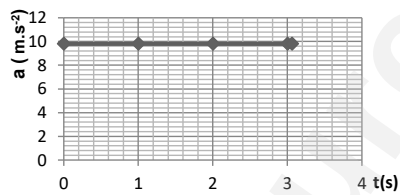




1.4



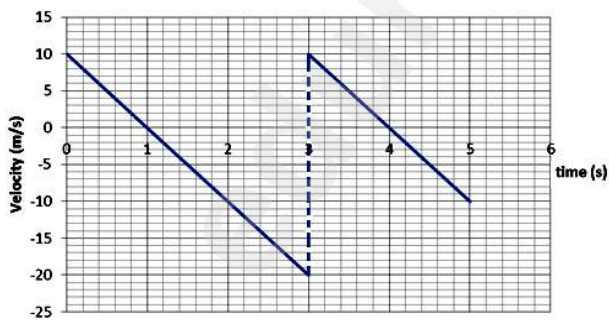
1.5



Example 2:

A boy standing on the top of a building throws a tennis ball vertically upwards. The graph below shows the velocity of the ball from the moment it was thrown until it hits the ground for the second time. Ignore air resistance.

Graph of velocity versus time



2.1 Describe the motion of the tennis ball in words.



- 2.2 What is the initial velocity of the tennis ball?
- 2.3 Calculate the height of the building.
- 2.4 Is the collision of the tennis ball with the ground elastic or inelastic? Explain.
- 2.5 Draw the position- time graph.

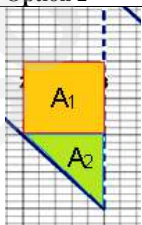
Solution:

1.1

- From 0 to 1 s the velocity of the ball decreases from $10 \text{ m}\cdot\text{s}^{-1}$ to zero, the acceleration of the ball is constant.
- From 1 to 2 s the velocity of the ball increases from zero to $(-10 \text{ m}\cdot\text{s}^{-1})$ with constant acceleration. At 2 s the ball is at the point of projection.
- From 2 to 3 s the velocity of the ball increases from $(-10 \text{ m}\cdot\text{s}^{-1})$ to $(-20 \text{ m}\cdot\text{s}^{-1})$. At 3 s the ball hits the ground and bounces back.
- From 3 to 4 s the velocity of the ball decreases to zero at the maximum height
- From 4 to 5 s the velocity of the ball increases with constant acceleration until it hits the ground for the second time.

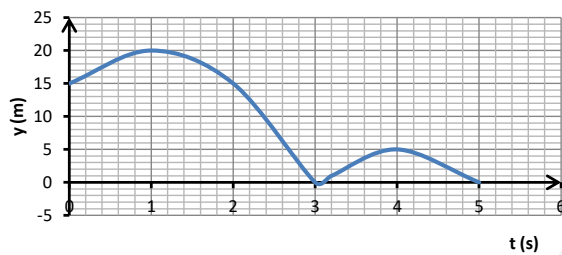
1.2 $10 \text{ m}\cdot\text{s}^{-1}$ upwards

1.3

Option 1	Option 2
$H = \Delta y = \left(\frac{\vec{v}_f + \vec{v}_i}{2} \right) \Delta t = \left(\frac{-20 + (-10)}{2} \right) (3 - 2) = 15 \text{ m}$	 $H = A = A_1 + A_2 = bh + \frac{1}{2}bh$ $H = (1)(-10) + \frac{1}{2}(1)(-10)$ $H = -10 + (-5) = 15 \text{ m}$
1.4. Inelastic because after bouncing the speed of the ball is smaller than the speed it hits the ground then kinetic energy is not conserved.	
1.5.	

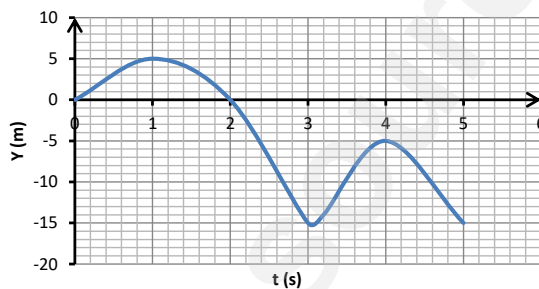


Position vs. time (taking ground as level zero)



Or

Position vs. time (taking top of building as level zero)



FINAL ACTIVITIES: (± 10 minutes)

CONCLUSION: *How will I conclude the lesson.*

✚ We can describe vertical projectile motion through a series of graphs: position, velocity, and acceleration versus time graphs.

The type of graphs is related to the equation of motion: position is a parabola; velocity is a straight line and acceleration is a constant parallel line to the t axis.

Homework: (Northern Cape DE)

A man is on the top of 40 m high building. The man throws a brick vertically down with a velocity of $4 \text{ m}\cdot\text{s}^{-1}$. Ignore the effects of air resistance.

- 1.1. Calculate the time the brick takes to reach the ground.
- 1.2. Calculate the velocity of the brick when it reaches the ground.



- 1.3. Draw the position vs. time graph.
 1.4. Draw the velocity vs. time graph.
 1.5. Draw the acceleration vs. time graph.
 1.6. The mass of the brick is 2.5 kg. Calculate the impulse of the gravitational force.

Positive direction downwards

1.1 Data:

$$h = \Delta y = 40 \text{ m downwards}$$

$$\vec{v}_i = 4 \text{ m} \cdot \text{s}^{-1} \text{ downwards}$$

$$\vec{g} = 9,8 \text{ m} \cdot \text{s}^{-2} \text{ downwards}$$

$$\Delta t = ?$$

$$\Delta y = \vec{v}_i \Delta t + \frac{1}{2} \vec{a} \Delta t^2$$

$$40 = 4\Delta t + \frac{1}{2}(9,8)\Delta t^2$$

$$4,9\Delta t^2 + 4\Delta t - 40 = 0$$

$$\Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4(4,9)(-40)}}{(2)(4,9)} = \frac{-4 \pm \sqrt{800}}{9,8}$$

$$\Delta t = \frac{-4 + \sqrt{800}}{9,8} = 2,48 \text{ s or } \Delta t = \frac{-4 - \sqrt{800}}{9,8} = -3,29 \text{ s}$$

The time is 2,48 s

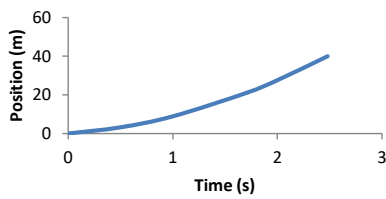
1.2.

Option 1	Option 2	Option 3
$\vec{v}_f = \vec{v}_i + \vec{g}\Delta t$ $\vec{v}_f = 4 + (9,8)(2,48)$ $\vec{v}_f = +28,30 \text{ m} \cdot \text{s}^{-1}$	$\vec{v}_f^2 = \vec{v}_i^2 + 2\vec{g}\Delta y$ $\vec{v}_f^2 = 4^2 + (2)(9,8)(40)$ $\vec{v}_f = +28,30 \text{ m} \cdot \text{s}^{-1}$	$\Delta y = \left(\frac{\vec{v}_i + \vec{v}_f}{2}\right) \Delta t$ $\frac{(2)(40)}{2,48} = 4 + \vec{v}_f$ $32,26 - 4 = \vec{v}_f$ $\vec{v}_f = 28,29 \text{ m} \cdot \text{s}^{-1} \approx 28,30 \text{ m} \cdot \text{s}^{-1}$

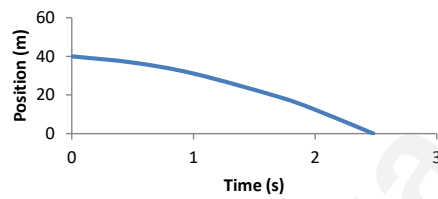
1.3



**Position vs. Time
(Downwards positive)**



**Position vs. Time
(Upwards positive)**

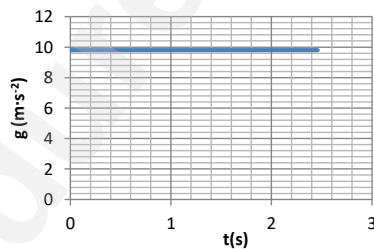


OR

1.4



1.5



1.6 Data

$m = 2.5$ kg



$$v_f = 28.30 \text{ m}\cdot\text{s}^{-1}$$

$$v_i = 4 \text{ m}\cdot\text{s}^{-1}$$

$$\Delta t = 2.48 \text{ s}$$

$$\text{Impulse} = \vec{j} = ?$$

$$\vec{j} = \vec{F}_g \Delta t$$

$$\vec{j} = m \vec{g} \Delta t$$

Let's take downwards as positive

$$\vec{j} = 2.5 \times (+9.8) \times 2.48$$

$$\vec{j} = +60.76 \text{ N}\cdot\text{s}$$

$$\vec{j} = 60.76 \text{ N}\cdot\text{s downwards}$$

Option 2

$$\vec{j} = \Delta \vec{p}$$

$$\vec{j} = m(\vec{v}_f - \vec{v}_i)$$

$$\vec{j} = 2.5(28.30 - 4)$$

$$\vec{j} = +60.76 \text{ N}\cdot\text{s} \quad \text{OR} \quad \vec{j} = 60.75 \text{ N}\cdot\text{s downwards}$$



GRADE	12	SUBJECT	PHYSICAL SCIENCES	Week	6
Topic	VERTICAL PROJECTILE: CALCULATIONS				
ATP	(± 1 week) – ➤ Draw a graph of position vs. time and velocity vs. time for a free-falling object. ➤ Determine the acceleration due to gravity using the data collected. ➤ Export data into the appropriate form of data presentation (e.g., equation, table, graph, or diagram).				

Initial activities: (±10 minutes)

You studied the vertical projectile motion, and you know that it is an example of motion with constant acceleration, a projectile is an object moving only under the action of the gravitational force then a projectile fall freely.

- PROJECTILE is any object (body) that moves only under the action of the gravitational force.
- Projectiles moving upwards or downwards in the Earth's gravitational field always accelerate downwards with a constant acceleration $\vec{g} = 9,8 \text{ m} \cdot \text{s}^{-2}$.
- Free fall is any motion of a body where the gravitational force is the only force acting upon it.
- The motion of any projectile can be described using the EQUATIONS OF KINEMATICS (equations of motion)

Physical Quantity	Motion on the horizontal plane	Vertical projectile
Displacement	➤ $\Delta \vec{x} = \vec{v}_i \Delta t + \frac{1}{2} \vec{a} \Delta t^2$ ➤ $\Delta \vec{x} = \frac{\vec{v}_f^2 - \vec{v}_i^2}{2\vec{a}}$ ➤ $\Delta \vec{x} = \left(\frac{\vec{v}_i + \vec{v}_f}{2} \right) \Delta t$	➤ $\Delta \vec{y} = \vec{v}_i \Delta t + \frac{1}{2} \vec{g} \Delta t^2$ ➤ $\Delta \vec{y} = \frac{\vec{v}_f^2 - \vec{v}_i^2}{2\vec{g}}$ ➤ $\Delta \vec{y} = \left(\frac{\vec{v}_i + \vec{v}_f}{2} \right) \Delta t$
Velocity	➤ $\vec{v}_f = \vec{v}_i + \vec{a} \Delta t$ ➤ $\vec{v}_f^2 = \vec{v}_i^2 + 2\vec{a} \Delta \vec{x}$	➤ $\vec{v}_f = \vec{v}_i + \vec{g} \Delta t$ ➤ $\vec{v}_f^2 = \vec{v}_i^2 + 2\vec{g} \Delta \vec{y}$
Acceleration	Constant $\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$	Constant $\vec{g} = 9,8 \text{ m} \cdot \text{s}^{-2}$
Position	$\vec{x}_f = \vec{x}_i + \Delta \vec{x}$	$\vec{y}_f = \vec{y}_i + \Delta \vec{y}$

- We can describe vertical projectile motion through a series of graphs: position, velocity and acceleration versus time graphs.
- The type of graphs is related to the equation of motion: position is a parabola; velocity is a straight line and acceleration is a constant parallel line to the t axis.



Type of motion	Position vs. time	Velocity vs. time	Acceleration vs. time
Constant acceleration (Velocity increase or object moving downwards)			
Constant acceleration (Velocity decreases or object moving upwards)			

DEVELOPMENT OF THE LESSON: (±40 minutes)

Experiment 1 (Experiment page 74, Siyavula book, Grade 12) Adapted.

Aim:

To measure the position and time during motion and to use that data to plot a “Position vs Time” and “Velocity vs Time” graph from which we can calculate the magnitude for gravitational acceleration, $\vec{g}$.

Apparatus:

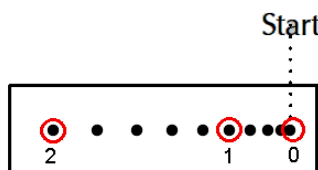
- ticker tape apparatus
- ticker timer
- tape
- graph paper
- ruler
- foam cushion

**Method:**

1. Clamp the timer to the table to allow the tape to fall through the staples on the timer such that the ticker tape will be perpendicular to the floor. Plug in the timer.
2. Thread a piece of ticker tape through the staples. Make sure the tape is under the carbon disc. The length of the tape should be the distance from the timer to the floor.



3. Using masking tape attach a 100 g mass to the end of the tape closest to the floor.
4. Position the foam cushion directly below the mass. (The cushion will protect your masses and your floor).
5. Hold the tape so that it is vertical and will easily pass through the staples.
6. Turn on the timer.
7. Let go of the tape with the mass, allowing it to drop to the floor.
8. Turn off the timer.
9. Take the tape and find the first clear dot nearest the mass end of the tape. Circle this first clear dot and label it as zero. Then count five dots and label the fifth dot as number 1. Count another 5 dots and label the fifth dot number 2. Continue this procedure.



10. On each piece of tape, measure the distance between successive dots.
11. Use the frequency of the ticker timer to work out the time intervals between successive dots.
12. If the time interval between two dots is 0,02 s so the time interval between the dots is $(\Delta t = n \times T = 5 \times 0,02 = 0,1 \text{ s})$ so the time between numbered dots is 0,1 s.
13. Use the between numbered dots and the known time interval to calculate velocity.

Results:

Measurement	Time (s)	Positions (m)	Velocity ($\text{m} \cdot \text{s}^{-1}$)
1 (5 dots)			
2 (10 dots)			
3 (15 dots)			
4 (20 dots)			
5 (25 dots)			
6 (30 dots)			

Analysis of the results:

14. Use the information to plot a graph of position vs. time into a graph paper.
15. Use the magnitude of the velocity and time values to plot a graph of "Velocity vs. Time" onto graph paper. Stick the graph paper into your workbook.
16. Draw the best straight line through your data points.



17. Determine the slope of the straight line. This is the magnitude of the acceleration.

Possible results of the Experiment:

Experiment 1:

Results:

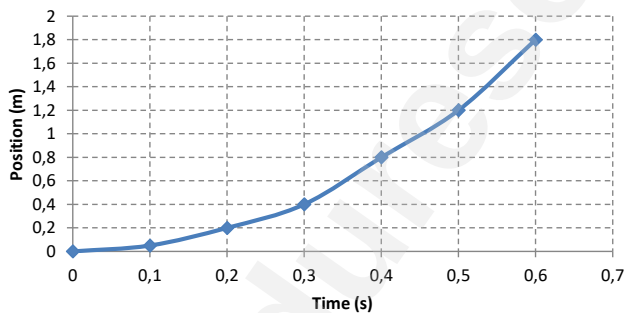
Measurement	Time (s)	Positions (m)	Velocity ($\text{m}\cdot\text{s}^{-1}$)
1 (5 dots)	0,1	0,05	0,98
2 (10 dots)	0,2	0,2	1,96
3 (15 dots)	0,3	0,4	2,94
4 (20 dots)	0,4	0,8	3,92
5 (25 dots)	0,5	1,2	4,9
6 (30 dots)	0,6	1,8	5,88

Note: The learner does not need to get the values of the table it is only an example (ideal case).

Analysis of the results:

14. Use the information to plot a graph of position vs. time into a graph paper.

Position vs. Time

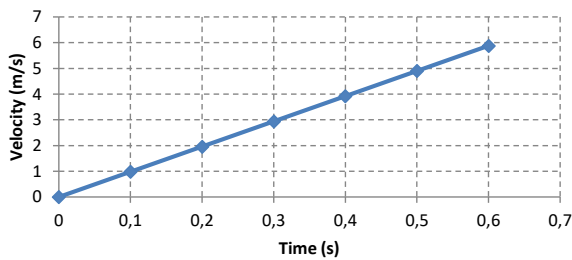


15. Use the magnitude of the velocity and time values to plot a graph of “Velocity vs. Time” onto graph paper. Stick the graph paper into your workbook.

16. Draw the best straight line through your data points.



Velocity vs. Time



17. Determine the slope of the straight line. This is the magnitude of the acceleration.

$$m = \vec{g} = \frac{\Delta y}{\Delta x} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} = \frac{5.88 - 0}{0.6 - 0} = 9.8 \text{ m} \cdot \text{s}^{-2}$$

FINAL ACTIVITIES: (±10 minutes)

CONCLUSION

- We can describe vertical projectile motion through a series of graphs: position, velocity, and acceleration versus time graphs.

The type of graphs is related to the equation of motion: position is a parabola; velocity is a straight line and acceleration is a constant parallel line to the t axis.

HOMEWORK (Northern Cape DE)

A ball of mass 100 g is dropped from a height of 2 m from the ground. After bouncing back up the ball reaches a height of 0.5 m. Ignore the air friction.

- 3.1 Define *impulse of a force* in words. (2)
- 3.2 Calculate the average force exerted by the ground on the ball if the collision lasted 2×10^{-3} s. (6)
- 3.3 Sketch the velocity time graph for the motion of the ball. (4)
- Show the following on the graph:
- (a) Velocity values
- (b) Time taken to reach the ground
- (c) Time taken to reach the height of 0.5 m after bouncing.
- 3.4 Is the collision elastic or inelastic? Explain the answer. (4)

Solution to the Homework.



1.1. *The impulse of a force* is the product of the force (resultant/net) acting on an object and the time the force (resultant/net) acts on the object. ✓✓

(2)

3.2 Option 1

$$\vec{F}_{ave} = \frac{\Delta \vec{p}}{\Delta t} \text{ OR } \vec{F}_{ave} = \frac{m(\vec{v}_f - \vec{v}_i)}{\Delta t}$$

Moving downwards

Downwards positive

$$v_f^2 = v_i^2 + 2g\Delta y \checkmark$$

$$v_f^2 = 0^2 + 2 \times 9,8 \times 2 \checkmark$$

$$v_f = +6,26 \text{ m} \cdot \text{s}^{-1}$$

Moving upwards

$$0^2 = v_i^2 + 2 \times 9,8 \times 0,5 \checkmark$$

$$v_i = -3,13 \text{ m} \cdot \text{s}^{-1}$$

$$\vec{F}_{ave} = \frac{0,1(-3,13 - 6,26)}{2 \times 10^{-3}} \checkmark$$

$$\vec{F}_{ave} = -4,695 \times 10^2 \text{ N} = -469,5 \text{ N} \checkmark$$

OR/OF

$$\vec{F}_{ave} = 4,695 \times 10^2 \text{ N Upwards} \checkmark \text{ OR } \vec{F}_{ave} = 469,5 \text{ N upwards}$$

Option 2

$$\vec{F}_{ave} = \frac{\Delta \vec{p}}{\Delta t} \text{ OR } \vec{F}_{ave} = \frac{m(\vec{v}_f - \vec{v}_i)}{\Delta t}$$

Moving downwards

Upwards positive

$$v_f^2 = v_i^2 + 2g\Delta y \checkmark$$

$$v_f^2 = 0^2 - 2 \times 9,8 \times 2 \checkmark$$

$$v_f = -6,26 \text{ m} \cdot \text{s}^{-1}$$

Moving upwards

$$0^2 = v_i^2 - 2 \times 9,8 \times 0,5 \checkmark$$

$$v_i = +3,13 \text{ m} \cdot \text{s}^{-1}$$

$$\vec{F}_{ave} = \frac{0,1(+3,13 - (-6,26))}{2 \times 10^{-3}} \checkmark$$

$$\vec{F}_{ave} = +4,695 \times 10^2 \text{ N} = +469,5 \text{ N} \checkmark$$

OR/OF

$$\vec{F}_{ave} = 4,695 \times 10^2 \text{ N upwards} \checkmark \text{ OR } \vec{F}_{ave} = 469,5 \text{ N upwards}$$

(6)

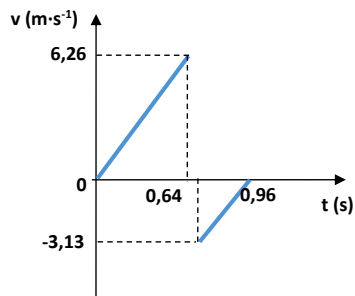


3.3

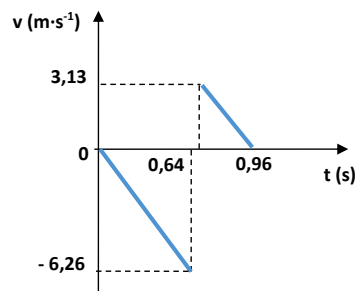
Option

1 Downwards

Positive



Option 2
Upwards positive



Moving downwards

$$v_f = v_i + g\Delta t$$

$$6,26 = 0 + 9,8\Delta t$$

$$\Delta t = 0,64 \text{ s}$$

Moving upwards

$$v_f = v_i + g\Delta t$$

$$0 = -3,13 + 9,8\Delta t$$

$$\Delta t = 0,32 \text{ s}$$

$$t_t = 0,64 + 0,002 + 0,32 = 0,96 \text{ s}$$

Criteria for graph	Marks
Correct shape.	✓
Velocity values	✓
Time taken to reach the ground.	✓
Total time taken to reach the height of 0,5 m	✓

(4)

3.4

Inelastic. ✓

Mass is constant, ✓ velocity before collision is greater than velocity after collision ✓, kinetic energy is not conserved. ✓

(4)