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SECONDARY SCHOOL IMPROVEMENT PROGRAMME (SSIP) 2016



GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

GRADE 12

SUBJECT: MATHEMATICS

LEARNER SOLUTIONS

(Page 1 of 47)

SESSION 6

TOPIC: CALCULUS

SECTION A: SOLUTIONS TO HOMEWORK QUESTIONS

1(a)(1)	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} \frac{1 - \frac{1}{4}(x+h)^2 - \left(1 - \frac{1}{4}x^2\right)}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} \frac{1 - \frac{1}{4}(x^2 + 2xh + h^2) - 1 + \frac{1}{4}x^2}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} \frac{1 - \frac{1}{4}x^2 - \frac{1}{2}xh - \frac{1}{4}h^2 - 1 + \frac{1}{4}x^2}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} \frac{-\frac{1}{2}xh - \frac{1}{4}h^2}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} \frac{h\left(-\frac{1}{2}x - \frac{1}{4}h\right)}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} \left(-\frac{1}{2}x - \frac{1}{4}h\right)$ $\therefore f'(x) = -\frac{1}{2}x$	$\checkmark 1 - \frac{1}{4}(x+h)^2$ $\checkmark -1 + \frac{1}{4}x^2$ $\checkmark -\frac{1}{4}x^2 - \frac{1}{2}xh - \frac{1}{4}h^2$ $\checkmark -\frac{1}{2}xh - \frac{1}{4}h^2$ $\checkmark \left(-\frac{1}{2}x - \frac{1}{4}h\right)$ $\checkmark -\frac{1}{2}x \quad (6)$
1(a)(2)	$f'(-4) = -\frac{1}{2}(-4) = 2$	$\checkmark \text{ answer} \quad (1)$
1(a)(3)	$f(x) = 1 - \frac{1}{4}x^2$ $\therefore f(-2) = 1 - \frac{1}{4}(-2)^2$ $\therefore f(-2) = 0$ The answer represents the y-value corresponding to $x = -2$	$\checkmark f(-2) = 0$ $\checkmark \text{ interpretation} \quad (2)$

1(a)(4)	$f(x) = 1 - \frac{1}{4}x^2$ $f(-2) = 0$ $f(4) = 1 - \frac{1}{4}(4)^2$ $\therefore f(4) = -3$ $(-2; 0) \text{ and } (4; -3)$ Average gradient $= \frac{-3 - 0}{4 - (-2)} = \frac{-3}{6} = -\frac{1}{2}$	$\checkmark f(-2) = -7$ $\checkmark f(4) = -31$ $\checkmark \frac{-31 - (-7)}{4 - (-2)}$ $\checkmark -4$ (4)
1(b)	$f'(x) = \lim_{h \rightarrow 0} \frac{-\frac{3}{x+h} - \left(-\frac{3}{x}\right)}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} \frac{-\frac{3}{x+h} + \frac{3}{x}}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} \frac{\frac{-3x+3(x+h)}{x(x+h)}}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} \frac{\frac{-3x+3x+3h}{x(x+h)}}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} \frac{\frac{3h}{x(x+h)}}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} \frac{3h}{x(x+h)} \times \frac{1}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} \frac{3}{x(x+h)}$ $\therefore f'(x) = \frac{3}{x(x+0)}$ $\therefore f'(x) = \frac{3}{x^2}$	$\checkmark -\frac{3}{x+h} - \left(-\frac{3}{x}\right)$ $\checkmark -\frac{3}{x+h} + \frac{3}{x}$ $\checkmark \frac{\frac{3h}{x(x+h)}}{h}$ $\checkmark \frac{3}{x(x+h)}$ $\checkmark \frac{3}{x^2}$ (5)

2(a)	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{-2(x+h) - (-2x)}{h}$ $= \lim_{h \rightarrow 0} \frac{-2x - 2h + 2x}{h}$ $= \lim_{h \rightarrow 0} \frac{-2h}{h}$ $= \lim_{h \rightarrow 0} (-2)$ $= -2$	$\checkmark -2(x+h) - (-2x)$ $\checkmark -x^2 + 2x$ $\checkmark \frac{-2h}{h}$ $\checkmark -2 \quad (4)$
2(b)	$g'(x) = \lim_{h \rightarrow 0} \frac{-2(x+h)^3 - (-2x^3)}{h}$ $\therefore g'(x) = \lim_{h \rightarrow 0} \frac{-2(x+h)(x+h)^2 + 2x^3}{h}$ $\therefore g'(x) = \lim_{h \rightarrow 0} \frac{-2(x+h)(x^2 + 2xh + h^2) + 2x^3}{h}$ $\therefore g'(x) = \lim_{h \rightarrow 0} \frac{-2(x^3 + 2x^2h + xh^2 + x^2h + 2xh^2 + h^3) + 2x^3}{h}$ $\therefore g'(x) = \lim_{h \rightarrow 0} \frac{-2(x^3 + 3x^2h + 3xh^2 + h^3) + 2x^3}{h}$ $\therefore g'(x) = \lim_{h \rightarrow 0} \frac{-2x^3 - 6x^2h - 6xh^2 - 2h^3 + 2x^3}{h}$ $\therefore g'(x) = \lim_{h \rightarrow 0} \frac{-6x^2h - 6xh^2 - 2h^3}{h}$ $\therefore g'(x) = \lim_{h \rightarrow 0} \frac{h(-6x^2 - 6xh - 2h)}{h}$ $\therefore g'(x) = \lim_{h \rightarrow 0} (-6x^2 - 6xh - 2h)$ $\therefore g'(x) = -6x^2$ $\therefore g'(2) = -6(2)^2 = -24$	$\checkmark -2(x+h)^3 - (-2x^3)$ $\checkmark \checkmark -6x^2h - 6xh^2 - 2h^3$ $\checkmark (-6x^2 - 6xh - 2h)$ $\checkmark -6x^2$ $\checkmark -24 \quad (6)$

3(a)	$f(x) = (4x-3)^2$ $\therefore f(x) = 16x^2 - 24x + 9$ $\therefore f'(x) = 16 \times 2x^{2-1} - 24 + 0$ $\therefore f'(x) = 32x - 24$	$\checkmark 16x^2 - 24x + 9$ $\checkmark 32x - 24$ (2)
3(b)	$D_x \left[\sqrt[3]{x} + \frac{1}{\sqrt{x}} \right]$ $= D_x \left[x^{\frac{1}{3}} + x^{-\frac{1}{2}} \right]$ $= \frac{1}{3} x^{-\frac{2}{3}} - \frac{1}{2} x^{-\frac{3}{2}}$ $= \frac{1}{3x^{\frac{2}{3}}} - \frac{1}{2x^{\frac{3}{2}}}$	$\checkmark x^{\frac{1}{3}} + x^{-\frac{1}{2}}$ $\checkmark \frac{1}{3} x^{-\frac{2}{3}} - \frac{1}{2} x^{-\frac{3}{2}}$ $\checkmark \frac{1}{3x^{\frac{2}{3}}} - \frac{1}{2x^{\frac{3}{2}}}$ (3)
3(c)	$D_x \left[(x^2 - \sqrt{x})^2 \right]$ $= D_x \left[x^4 - 2x^2\sqrt{x} + x \right]$ $= D_x \left[x^4 - 2x^2x^{\frac{1}{2}} + x \right]$ $= D_x \left[x^4 - 2x^{\frac{5}{2}} + x \right]$ $= 4x^3 - 5x^{\frac{3}{2}} + 1$	$\checkmark\checkmark x^4 - 2x^{\frac{5}{2}} + x$ $\checkmark\checkmark\checkmark 4x^3 - 5x^{\frac{3}{2}} + 1$ (5)
3(d)	$y = \frac{2x^2 - \sqrt{x} + 5}{\sqrt{x}}$ $\therefore y = \frac{2x^2}{x^{\frac{1}{2}}} - 1 + \frac{5}{x^{\frac{1}{2}}}$ $\therefore y = 2x^{\frac{3}{2}} - 1 + 5x^{-\frac{1}{2}}$ $\therefore \frac{dy}{dx} = 3x^{\frac{1}{2}} - \frac{5}{2}x^{-\frac{3}{2}}$ $\therefore \frac{dy}{dx} = 3x^{\frac{1}{2}} - \frac{5}{2x^{\frac{3}{2}}}$	$\checkmark x^{\frac{1}{2}}$ $\checkmark 2x^{\frac{3}{2}} - 1 + 5x^{-\frac{1}{2}}$ $\checkmark 3x^{\frac{1}{2}} - \frac{5}{2}x^{-\frac{3}{2}}$ $\checkmark 3x^{\frac{1}{2}} - \frac{5}{2x^{\frac{3}{2}}}$ (4)

4(a)	$y = f(x) = 3x^2 - 2x + 2$ $x_T = -4$ $\therefore f(-4) = 3(-4)^2 - 2(-4) + 2$ $\therefore f(-4) = 58$ $\therefore y_T = 58$ $m_T = f'(x) = 6x - 2$ $\therefore f'(-4) = 6(-4) - 2$ $\therefore f'(5) = -26$ $y - y_T = m_t(x - x_T)$ $\therefore y - 58 = -26(x - (-4))$ $\therefore y - 58 = -26x - 104$ $\therefore y = -26x - 46$	$\checkmark f(-4) = 58$ $\checkmark f'(x) = 6x - 2$ $\checkmark f'(5) = -26$ $\checkmark y - 58 = -26(x - (-4))$ $\checkmark y = -26x - 46 \quad (5)$
4(b)	$y = a(x - 0)(x - 4)$ Substitute the point (3; 6) $\therefore 6 = a(3 - 0)(3 - 4)$ $\therefore 6 = -3a$ $\therefore a = -2$ $\therefore y = -2(x - 0)(x - 4)$ $\therefore y = -2x(x - 4)$ $\therefore y = -2x^2 + 8x$ $f(x) = -2x^2 + 8x$ $\therefore f'(x) = -4x + 8$ $\therefore f'(3) = -4(3) + 8 = -4$ $y - y_T = m_t(x - x_T)$ $\therefore y - 6 = -4(x - 3)$ $\therefore y - 6 = -4x + 12$ $\therefore y = -4x + 18$	$\checkmark y = a(x - 0)(x - 4)$ $\checkmark a = -2$ $\checkmark f(x) = -2x^2 + 8x$ $\checkmark f'(x) = -4x + 8$ $\checkmark f'(3) = -4(3) + 8 = -4$ $\checkmark y = -4x + 18 \quad (6)$

6(a)	$m = \frac{0-b}{a-0} = -\frac{b}{a}$ $\therefore y = -\frac{b}{a}x + b$	$\checkmark -\frac{b}{a}$ $\checkmark y = -\frac{b}{a}x + b \quad (2)$
6(b)	$A = xy$ $\therefore A = x\left(-\frac{b}{a}x + b\right)$ $\therefore A(x) = -\frac{b}{a}x^2 + bx$ $\therefore A'(x) = -\frac{2b}{a}x + b$ $\therefore 0 = -\frac{2b}{a}x + b$ $\therefore \frac{2b}{a}x = b$ $\therefore 2bx = ab$ $\therefore x = \frac{a}{2}$ $\therefore y = -\frac{b}{a}\left(\frac{a}{2}\right) + b = \frac{-b}{2} + b = \frac{b}{2}$ $\therefore P\left(\frac{a}{2}; \frac{b}{2}\right) \text{ is the midpoint of MN.}$	$\checkmark A = xy$ $\checkmark A = x\left(-\frac{b}{a}x + b\right)$ $\checkmark A'(x) = -\frac{2b}{a}x + b$ $\checkmark 0 = -\frac{2b}{a}x + b$ $\checkmark x = \frac{a}{2}$ $\checkmark y = \frac{b}{2} \quad (6)$
7(a)	$h'(x) = -3x^2 + 2ax + b$ $h'(-1) = -3(-1)^2 + 2a(-1) + b$ $0 = -3 - 2a + b$ $2a - b = -3 \quad \dots (i)$ $h'(2) = -3(2)^2 + 2a(2) + b$ $0 = -12 + 4a + b$ $4a + b = 12 \quad \dots (ii)$ $6a = 9 \quad (i) + (ii)$ $\therefore a = \frac{3}{2}$ $\therefore 2\left(\frac{3}{2}\right) - b = -3$ $b = 6$	$\checkmark h'(x) = -3x^2 + 2ax + b$ $\checkmark h'(-1) = -3(-1)^2 + 2a(-1) + b$ $\checkmark 2a - b = -3$ $\checkmark h'(2) = -3(2)^2 + 2a(2) + b$ $\checkmark 4a + b = 12$ $\checkmark a = \frac{3}{2}$ $\checkmark b = 6 \quad (7)$

7(b)	Average gradient $= \frac{10 - (-3, 5)}{2 - (-1)} = \frac{9}{2}$	$\checkmark \frac{10 - (-3, 5)}{2 - (-1)}$ $\checkmark \frac{9}{2} \quad (2)$
7(c)	$h(x) = -x^3 + \frac{3}{2}x^2 + 6x$ $\therefore h'(x) = -3x^2 + 3x + 6$ $h'(-2) = -3(-2)^2 + 3(-2) + 6$ $h'(-2) = -12$ Point of contact $(-2 ; 2)$ $y - 2 = -12(x + 2)$ $y = -12x - 22$	$\checkmark h(x) = -x^3 + \frac{3}{2}x^2 + 6x$ $\checkmark h'(x) = -3x^2 + 3x + 6$ $\checkmark h'(-2) = -12$ $\checkmark y = -12x - 22$ $\checkmark h'(-2) = -12 \quad (5)$
7(d)	$h''(x) = -6x + 3$ $-6x + 3 = 0$ $x = \frac{1}{2}$	$\checkmark h''(x) = -6x + 3$ $\checkmark -6x + 3 = 0$ $\checkmark x = \frac{1}{2} \quad (3)$
8(a)	$A = (2x)(3x) + 2(y)(3x) + 2(y)(2x)$ $\therefore 200 = 6x^2 + 6xy + 4xy$ $\therefore 200 = 6x^2 + 10xy$ $\therefore 100 = 3x^2 + 5xy$ $\therefore 100 - 3x^2 = 5xy$ $\therefore \frac{100}{5x} - \frac{3x^2}{5x} = y$ $\therefore y = \frac{20}{x} - \frac{3x}{5}$	$\checkmark 6x^2 + 6xy + 4xy$ $\checkmark 200 =$ $\checkmark \text{arriving at answer} \quad (3)$
8(b)	$V = (2x)(3x)(y)$ $\therefore V = (2x)(3x)\left(\frac{20}{x} - \frac{3x}{5}\right)$ $\therefore V = (6x^2)\left(\frac{20}{x} - \frac{3x}{5}\right)$ $\therefore V = 120x - \frac{18x^3}{5}$	$\checkmark V = (2x)(3x)(y)$ $\checkmark V = 120x - \frac{18x^3}{5} \quad (2)$

8(c)	$V(x) = 120x - \frac{18}{5}x^3$ $\therefore V'(x) = 120 - \frac{18}{5} \times 3x^2$ $\therefore 0 = 120 - \frac{54}{5}x^2$ $\therefore 0 = 600 - 54x^2$ $\therefore 54x^2 = 600$ $\therefore x^2 = \frac{600}{54}$ $\therefore x^2 = \frac{100}{9}$ $\therefore x = \frac{10}{3}$	$\checkmark V'(x)$ $\checkmark V'(x) = 0$ $\checkmark x = \frac{10}{3}$ (3)
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SECTION B: SOLUTIONS TO TYPICAL EXAMINATION QUESTIONS

1(a)(1)	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} \frac{-2(x+h)^2 + 1 - (-2x^2 + 1)}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} \frac{-2(x^2 + 2xh + h^2) + 1 + 2x^2 - 1}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} \frac{-2x^2 - 4xh - 2h^2 + 1 + 2x^2 - 1}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} \frac{h(-4x - 2h)}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} (-4x - 2h)$ $\therefore f'(x) = -4x$	$\checkmark -2(x+h)^2 + 1$ $\checkmark -(-2x^2 + 1)$ $\checkmark -2x^2 - 4xh - 2h^2$ $\checkmark \frac{h(-4x - 2h)}{h}$ $\checkmark -4x$ (5)
1(a)(2)	$f'(x) = -4x$ $\therefore f'(-2) = -4(-2) = 8$	$\checkmark$ answer (1)

1(a)(3)	$f(x) = -2x^2 + 1$ $\therefore f(-2) = -2(-2)^2 + 1$ $\therefore f(-2) = -7$ The answer represents the y-value corresponding to $x = -2$	$\checkmark f(-2) = -7$ $\checkmark$ interpretation (2)
1(a)(4)	$f(x) = -2x^2 + 1$ $f(-2) = -2(-2)^2 + 1$ $\therefore f(-2) = -7$ $f(4) = -2(4)^2 + 1$ $\therefore f(4) = -31$ $(-2; -7)$ and $(4; -31)$ Average gradient $= \frac{-31 - (-7)}{4 - (-2)} = \frac{-24}{6} = -4$	$\checkmark f(-2) = -7$ $\checkmark f(4) = -31$ $\checkmark \frac{-31 - (-7)}{4 - (-2)}$ $\checkmark -4$ (4)
1(b)	$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} \frac{\frac{x-(x+h)}{x(x+h)}}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} \frac{\frac{x-x-h}{x(x+h)}}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} \frac{\frac{-h}{x(x+h)}}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} \frac{-h}{x(x+h)} \times \frac{1}{h}$ $\therefore f'(x) = \lim_{h \rightarrow 0} \frac{-1}{x(x+h)}$ $\therefore f'(x) = -\frac{1}{x^2}$	$\checkmark \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$ $\checkmark \frac{x-(x+h)}{x(x+h)}$ $\checkmark \frac{\frac{-h}{x(x+h)}}{h}$ $\checkmark \frac{-1}{x(x+h)}$ $\checkmark -\frac{1}{x^2}$ (5)

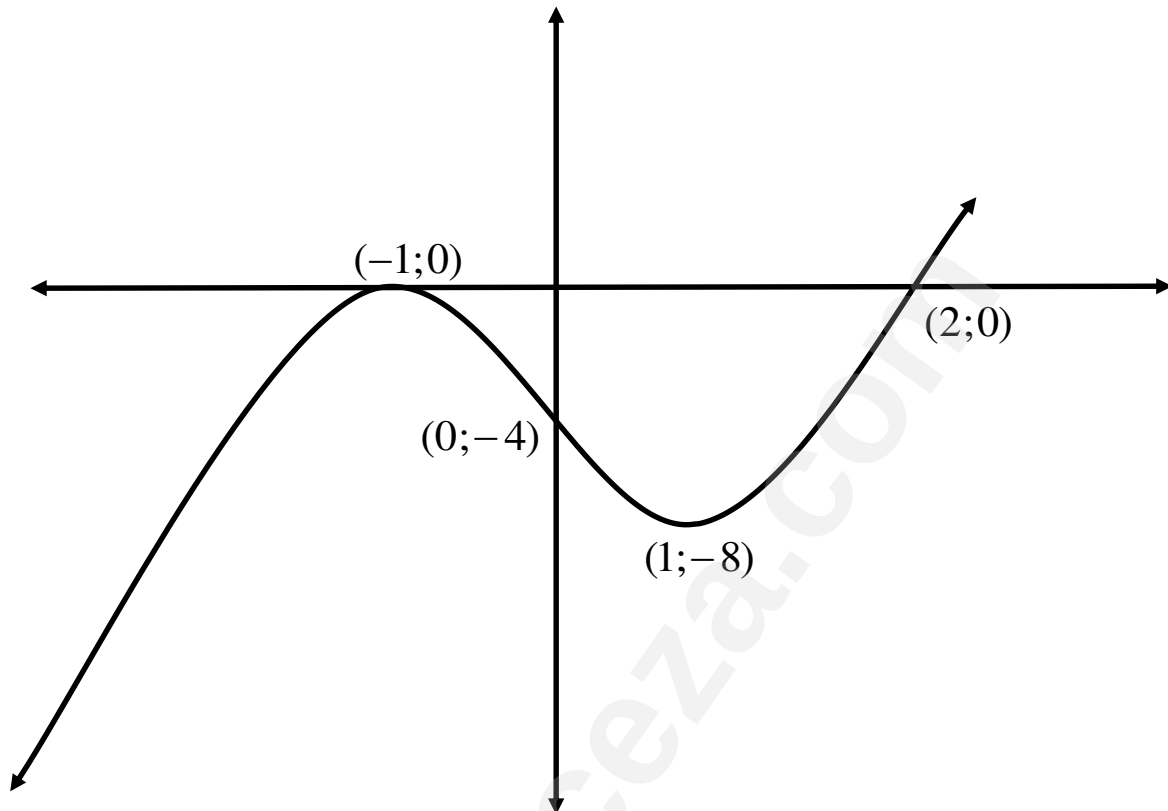
2(a)	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{(x+h)^2 - 2(x+h) - x^2 + 2x}{h}$ $= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - 2x - 2h - x^2 + 2x}{h}$ $= \lim_{h \rightarrow 0} \frac{2xh + h^2 - 2h}{h}$ $= \lim_{h \rightarrow 0} \frac{h(2x + h - 2)}{h}$ $= \lim_{h \rightarrow 0} (2x - 2 + h)$ $= 2x - 2$	$\checkmark (x+h)^2 - 2(x+h)$ $\checkmark -x^2 + 2x$ $\checkmark \frac{2xh + h^2 - 2h}{h}$ $\checkmark (2x - 2 + h)$ $\checkmark 2x - 2$ (5)
2(b)	$g'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h}$ $\therefore g'(x) = \lim_{h \rightarrow 0} \frac{(x+h)(x+h)^2 - x^3}{h}$ $\therefore g'(x) = \lim_{h \rightarrow 0} \frac{(x+h)(x^2 + 2xh + h^2) - x^3}{h}$ $\therefore g'(x) = \lim_{h \rightarrow 0} \frac{x^3 + 2x^2h + xh^2 + x^2h + 2xh^2 + h^3 - x^3}{h}$ $\therefore g'(x) = \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h}$ $\therefore g'(x) = \lim_{h \rightarrow 0} \frac{h(3x^2 + 3xh + h^2)}{h}$ $\therefore g'(x) = \lim_{h \rightarrow 0} (3x^2 + 3xh + h^2)$ $\therefore g'(x) = 3x^2$ $\therefore g'(3) = 3(3)^2 = 27$	$\checkmark (x+h)^3 - x^3$ $\checkmark \checkmark 3x^2h + 3xh^2 + h^3$ $\checkmark (3x^2 + 3xh + h^2)$ $\checkmark 3x^2$ $\checkmark 27$ (6)
3(a)	$D_x [(2x-3)(x+4)]$ $= D_x [2x^2 + 5x - 12]$ $= 4x + 5$	$\checkmark 2x^2 + 5x - 12$ $\checkmark 4x + 5$ (2)

3(b)	$f(x) = \frac{1}{2\sqrt[4]{x^3}}$ $\therefore f(x) = \frac{1}{2x^{\frac{3}{4}}}$ $\therefore f(x) = \frac{1}{2}x^{-\frac{3}{4}}$ $\therefore f'(x) = \frac{1}{2} \times -\frac{3}{4}x^{-\frac{3}{4}-1}$ $\therefore f'(x) = -\frac{3}{8}x^{-\frac{7}{4}}$ $\therefore f'(x) = -\frac{3}{8x^{\frac{7}{4}}}$	$\checkmark \frac{1}{2}x^{-\frac{3}{4}}$ $\checkmark -\frac{3}{8}x^{-\frac{7}{4}}$ $\checkmark -\frac{3}{8x^{\frac{7}{4}}}$ (3)
3(c)	$y = \left(2\sqrt{x} - \frac{1}{3x}\right)^2$ $\therefore y = 4x - \frac{4\sqrt{x}}{3x} + \frac{1}{9x^2}$ $\therefore y = 4x - \frac{4x^{\frac{1}{2}}}{3x} + \frac{1}{9}x^{-2}$ $\therefore y = 4x - \frac{4}{3}x^{-\frac{1}{2}} + \frac{1}{9}x^{-2}$ $\therefore \frac{dy}{dx} = 4 - \frac{4}{3} \times -\frac{1}{2}x^{-\frac{3}{2}} + \frac{1}{9} \times -2x^{-3}$ $\therefore \frac{dy}{dx} = 4 + \frac{2}{3}x^{-\frac{3}{2}} - \frac{2}{9}x^{-3}$ $\therefore \frac{dy}{dx} = 4 + \frac{2}{3x^{\frac{3}{2}}} - \frac{2}{9x^3}$	$\checkmark \text{squaring}$ $\checkmark 4x - \frac{4}{3}x^{-\frac{1}{2}} + \frac{1}{9}x^{-2}$ $\checkmark \checkmark \checkmark 4 + \frac{2}{3x^{\frac{3}{2}}} - \frac{2}{9x^3}$ (5)

3(d)	$D_x \left[\frac{1}{\sqrt{x}} (x^3 - 2x^2 + 3x) \right]$ $= D_x \left[\frac{x^3}{x^{\frac{1}{2}}} - \frac{2x^2}{x^{\frac{1}{2}}} + \frac{3x}{x^{\frac{1}{2}}} \right]$ $= D_x \left[x^{\frac{5}{2}} - 2x^{\frac{3}{2}} + 3x^{\frac{1}{2}} \right]$ $= \frac{5}{2} x^{\frac{3}{2}} - 3x^{\frac{1}{2}} + \frac{3}{2} x^{-\frac{1}{2}}$ $= \frac{5}{2} x^{\frac{3}{2}} - 3x^{\frac{1}{2}} + \frac{3}{2x^{\frac{1}{2}}}$	$\checkmark x^{\frac{1}{2}}$ $\checkmark D_x \left[x^{\frac{5}{2}} - 2x^{\frac{3}{2}} + 3x^{\frac{1}{2}} \right]$ $\checkmark \frac{5}{2} x^{\frac{3}{2}} - 3x^{\frac{1}{2}} + \frac{3}{2} x^{-\frac{1}{2}}$ $\checkmark \frac{5}{2} x^{\frac{3}{2}} - 3x^{\frac{1}{2}} + \frac{3}{2x^{\frac{1}{2}}} \quad (4)$
4(a)	$y = (x^6 - 1)^2$ $\therefore y = x^{12} - 2x^6 + 1$ $\therefore \frac{dy}{dx} = 12x^{11} - 12x^5 = 12x^5(x^6 - 1)$ Now $y = (x^6 - 1)^2$ $\therefore \sqrt{y} = x^6 - 1$ $\therefore \frac{dy}{dx} = 12x^5 \cdot \sqrt{y}$	$\checkmark y = x^{12} - 2x^6 + 1$ $\checkmark \sqrt{y} = x^6 - 1$ $\checkmark \frac{dy}{dx} = 12x^5 \cdot \sqrt{y} \quad (3)$
4(b)	$D_x \left[\frac{x^3 - 1}{x - 1} \right]$ $= D_x \left[\frac{(x - 1)(x^2 + x + 1)}{(x - 1)} \right]$ $= D_x \left[(x^2 + x + 1) \right]$ $= 2x + 1$	$\checkmark (x - 1)(x^2 + x + 1)$ $\checkmark (x^2 + x + 1)$ $\checkmark 2x + 1 \quad (3)$
5(a)	$x_T = 2$ $y_T = f(2) = (2)^2 - 6(2) + 5$ $\therefore y_T = -3$ $m_T = f'(x) = 2x - 6$ $\therefore f'(2) = 2(2) - 6 = -2$	$\checkmark f(2) = -2$ $\checkmark f'(x) = 2x - 6$ $\checkmark f'(2) = 2(2) - 6 = -2$ $\checkmark y - (-3) = -2(x - 2)$ $\checkmark y = -2x + 1 \quad (5)$

	$y - y_T = m_t(x - x_T)$ $\therefore y - (-3) = -2(x - 2)$ $\therefore y + 3 = -2x + 4$ $\therefore y = -2x + 1$	
5(b)	$f(x) = 3x^2 - 5x + 1$ $\therefore f'(x) = 6x - 5$ $y = 7x - 4$ $\therefore 6x - 5 = 7$ $\therefore 6x = 12$ $\therefore x = 2$ $f(2) = 3(2)^2 - 5(2) + 1$ $\therefore f(2) = 3$ $\therefore y - 3 = 7(x - 2)$ $\therefore y - 3 = 7x - 14$ $\therefore y = 7x - 11$	$\checkmark f'(x) = 6x - 5$ $\checkmark 6x - 5 = 7$ $\checkmark x = 2$ $\checkmark f(2) = 3$ $\checkmark y - 3 = 7(x - 2)$ $\checkmark y = 7x - 11$ (6)

[illegible]



	The graph is represented above	✓ intercepts with the axes ✓ turning points ✓ shape ✓ point of inflection	(15)
6(b)	$f(x) = 2x^3 - 2x^2 + 4x - 1$ $\therefore f'(x) = 6x^2 - 4x$ $\therefore f''(x) = 12x - 4$ $\therefore 12x - 4 > 0$ $\therefore 12x > 4$ $\therefore x > \frac{4}{12}$ $\therefore x > \frac{1}{3}$	✓ $f'(x) = 6x^2 - 4x$ ✓ $f''(x) = 12x - 4$ ✓ $12x - 4 > 0$ ✓ $x > \frac{1}{3}$	(4)

7	At the turning point (2 ; 9) , we know that $f'(2) = 0$ $f(x) = ax^3 + 5x^2 + 4x + b$ $\therefore f'(x) = 3ax^2 + 10x + 4$ $\therefore f'(2) = 3a(2)^2 + 10(2) + 4$ $\therefore f'(2) = 12a + 24$ $\therefore 0 = 12a + 24$ $\therefore -12a = 24$ $\therefore a = -2$ We can now substitute $a = -2$ into the original equation: $y = -2x^3 + 5x^2 + 4x + b$ In order to get the value of b, substitute the point (2 ; 9) into this equation: $\therefore 9 = -2(2)^3 + 5(2)^2 + 4(2) + b$ $\therefore 9 = -16 + 20 + 8 + b$ $\therefore 9 = 12 + b$ $\therefore -b = 3$ $\therefore b = -3$ The equation of the cubic function is therefore $f(x) = -x^3 + 5x^2 + 4x - 3$	$\checkmark f'(x) = 3ax^2 + 10x + 4$ $\checkmark f'(2) = 12a + 24$ $\checkmark a = -2$ $\checkmark y = -2x^3 + 5x^2 + 4x + b$ $\checkmark 9 = -2(2)^3 + 5(2)^2 + 4(2) + b$ $\checkmark b = -3$ $\checkmark f(x) = -x^3 + 5x^2 + 4x - 3 \quad (7)$
8(a)	$V = \pi r^2 h$ $200 = \pi r^2 h$ $h = \frac{200}{\pi r^2}$	$\checkmark V = \pi r^2 h$ $\checkmark 200 = \pi r^2 h \quad (2)$
8(b)	Surface Area = $2\pi rh + \pi r^2$ $S(r) = \pi r^2 + \frac{200}{\pi r^2} \cdot 2\pi r$ $S(r) = \pi r^2 + \frac{400}{r}$	$\checkmark S = 2\pi rh + \pi r^2$ $\checkmark S(r) = \pi r^2 + \frac{200}{\pi r^2} \cdot 2\pi r \quad (2)$

8(c)	$S(r) = \pi r^2 + 400r^{-1}$ $\frac{dS}{dr} = 2\pi r - 400r^{-2}$ At minimum: $\frac{dS}{dr} = 0$ $2\pi r - \frac{400}{r^2} = 0$ $\pi r^3 - 200 = 0$ $r^3 = \frac{200}{\pi}$ $r = 3,99 \text{ cm}$	$\checkmark S(r) = \pi r^2 + 400r^{-1}$ $\checkmark \frac{dS}{dr} = 2\pi r - 400r^{-2}$ $\checkmark 2\pi r - \frac{400}{r^2} = 0$ $\checkmark r^3 = \frac{200}{\pi}$ $\checkmark r = 3,99 \text{ cm}$
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(5)

SESSION 7

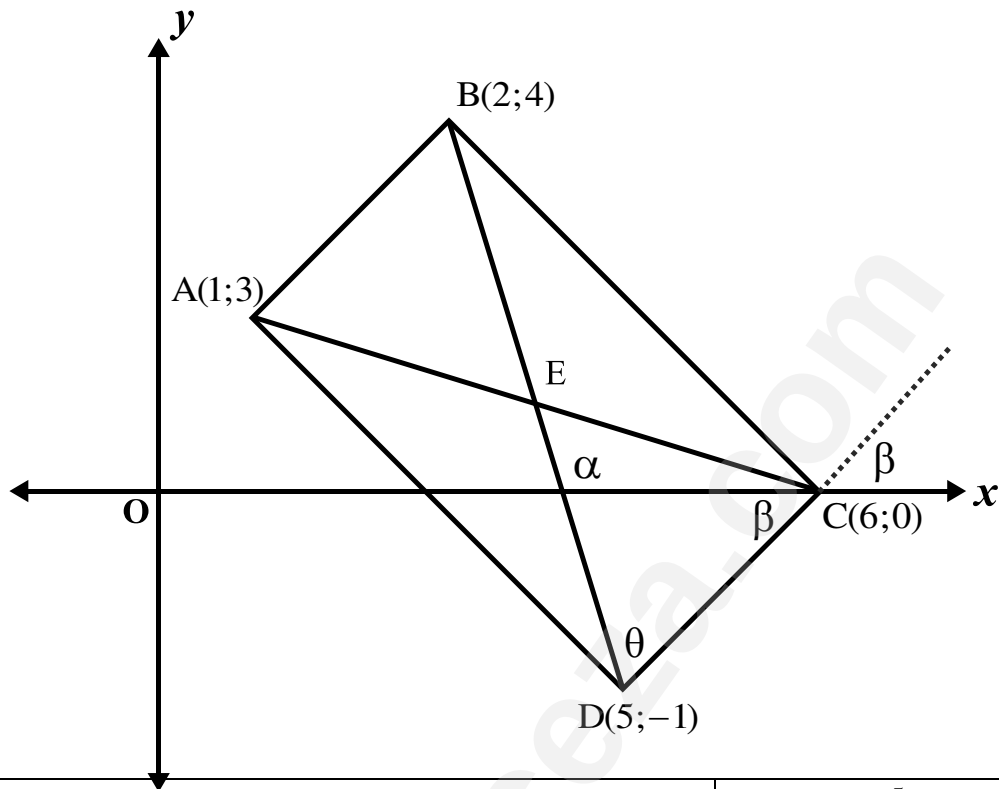
TOPIC: ANALYTICAL GEOMETRY

SECTION A: SOLUTIONS TO HOMEWORK QUESTIONS

1(a)	$E\left(\frac{2+5}{2}; \frac{4+(-1)}{2}\right)$ $= E\left(\frac{7}{2}; \frac{3}{2}\right)$	$\checkmark\checkmark E\left(\frac{7}{2}; \frac{3}{2}\right)$ (2)
1(b)	$E\left(\frac{7}{2}; \frac{3}{2}\right) = \left(\frac{x_A + x_C}{2}; \frac{y_A + y_C}{2}\right)$ $\therefore E\left(\frac{7}{2}; \frac{3}{2}\right) = \left(\frac{1+x_C}{2}; \frac{3+y_C}{2}\right)$ $\therefore \frac{7}{2} = \frac{1+x_C}{2} \quad \text{or} \quad \frac{3}{2} = \frac{3+y_C}{2}$ $\therefore 7 = 1+x_C \quad \text{or} \quad 3 = 3+y_C$ $\therefore x_C = 6 \quad \text{or} \quad y_C = 0$ $\therefore C(6; 0)$	$\checkmark \frac{7}{2} = \frac{1+x_C}{2}$ $\checkmark \frac{3}{2} = \frac{3+y_C}{2}$ $\checkmark x_C = 6$ $\checkmark y_C = 0$ $\checkmark C(6; 0)$

(5)

1(c)	$m_{AB} = \frac{4-3}{2-1} = \frac{1}{1} = 1$ $m_{CD} = \frac{0-(-1)}{6-5} = \frac{1}{1} = 1$ $\therefore m_{AB} = m_{CD}$ $\therefore AB \parallel CD$ $m_{AD} = \frac{3-(-1)}{1-5} = \frac{4}{-4} = -1$ $m_{BC} = \frac{4-0}{2-6} = \frac{4}{-4} = -1$ $\therefore m_{AD} = m_{BC}$ $\therefore AD \parallel BC$ $\therefore ABCD \text{ is a parallelogram}$	$\checkmark m_{AB}$ $\checkmark m_{CD}$ $\checkmark AB \parallel CD$ $\checkmark m_{AD}$ $\checkmark m_{BC}$ $\checkmark AD \parallel BC$ $\checkmark ABCD \text{ is a parallelogram}$ $\checkmark m_{AB} \times m_{AD} = -1$ $\checkmark \hat{A} = 90^\circ$ $\checkmark ABCD \text{ is a rectangle} \quad (10)$
	Now $m_{AB} \times m_{AD} = (1) \times (-1) = -1$ $\therefore AB \perp AD$ $\therefore \hat{A} = 90^\circ$ $\therefore ABCD \text{ is a rectangle}$ (since one interior angle of parallelogram ABCD is 90°)	
1(d)	$AB^2 = (2-1)^2 + (4-3)^2$ $\therefore AB^2 = 1+1$ $\therefore AB^2 = 2$ $\therefore AB = \sqrt{2}$ $AD^2 = (5-1)^2 + (-1-3)^2$ $\therefore AD^2 = 16+16$ $\therefore AD = \sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}$ $\text{Area } ABCD = AD \times AB$ $\therefore \text{Area } ABCD = (4\sqrt{2})(\sqrt{2}) = 8 \text{ units}^2$	$\checkmark AB^2 = (2-1)^2 + (4-3)^2$ $\checkmark AB = \sqrt{2}$ $\checkmark AD^2 = (5-1)^2 + (-1-3)^2$ $\checkmark AD = \sqrt{32}$ $\checkmark \text{Area } ABCD = (4\sqrt{2})(\sqrt{2})$ $\checkmark \text{Area } ABCD = 8 \text{ units}^2 \quad (6)$



1(e)	$\tan \alpha = m_{BD}$ $\therefore \tan \alpha = \frac{4 - (-1)}{2 - 5}$ $\therefore \tan \alpha = -\frac{5}{3}$ $\therefore \alpha = 180^\circ - 59^\circ$ $\therefore \alpha = 121^\circ$ $\tan \beta = m_{DC}$ $\therefore \tan \beta = \frac{0 - (-1)}{6 - 5} = \frac{1}{1} = 1$ $\therefore \tan \beta = 1$ $\therefore \beta = 45^\circ$ $\therefore \hat{OCD} = \beta = 45^\circ$ $\alpha = \theta + \beta$ $\therefore 121^\circ = \theta + 45^\circ$ $\therefore \theta = 76^\circ$	$\checkmark \tan \alpha = -\frac{5}{3}$ $\checkmark \alpha = 121^\circ$ $\checkmark \tan \beta = 1$ $\checkmark \beta = 45^\circ$ $\checkmark \theta = 18, 43^\circ$ $\checkmark \hat{OCD} = \beta = 45^\circ$ $\checkmark 121^\circ = \theta + 45^\circ$ $\checkmark \theta = 76^\circ$
2(a)	$3y = -px + 6$	$\checkmark y = -\frac{p}{3}x + 2$

	$\therefore y = -\frac{p}{3}x + 2$ $\therefore \tan 135^\circ = -\frac{p}{3}$ $\therefore -1 = -\frac{p}{3}$ $\therefore p = 3$	$\checkmark \tan 135^\circ = -\frac{p}{3}$ $\checkmark -1 = -\frac{p}{3}$ $\checkmark p = 3$	(4)
2(b)	$m_{AB} = m_{BC}$ $\therefore \frac{2-5}{3-6} = \frac{k+4-2}{2k-3}$ $\therefore 1 = \frac{k+2}{2k-3}$ $\therefore 2k-3 = k+2$ $\therefore k = 5$	$\checkmark m_{AB} = m_{BC}$ $\checkmark$ working out gradients $\checkmark k = 5$	(3)

3(a)	$x^2 - 6x + (-3)^2 + y^2 + 2y + (1)^2 = p + (-3)^2 + (1)^2$ $\therefore (x-3)^2 + (y+1)^2 = p+10$ centre = (3; -1)	$\checkmark (-3)^2$ $\checkmark (1)^2$ $\checkmark (x-3)^2 + (y+1)^2 = p+10$ $\checkmark$ centre	(4)
3(b)	$(3\sqrt{2})^2 = p+10$ $\therefore 18 = p+10$ $\therefore p = 8$	$\checkmark (3\sqrt{2})^2 = p+10$ $\checkmark p = 8$	(2)

4(a)	$3x + 4y = -7$ $\therefore 4y = -3x - 7$ $\therefore y = -\frac{3}{4}x - \frac{7}{4}$ $\therefore m_{CB} = -\frac{3}{4}$ $\therefore m_{\text{tangent}} = \frac{4}{3}$ $y - (-1) = \frac{4}{3}(x - (-1))$ $\therefore y + 1 = \frac{4}{3}(x + 1)$ $\therefore y + 1 = \frac{4}{3}x + \frac{4}{3}$ $\therefore y = \frac{4}{3}x + \frac{1}{3}$	$\checkmark y = -\frac{3}{4}x - \frac{7}{4}$ $\checkmark m_{CB} = -\frac{3}{4}$ $\checkmark m_{\text{tangent}} = \frac{4}{3}$ $\checkmark y - (-1) = \frac{4}{3}(x - (-1))$ $\checkmark y = \frac{4}{3}x + \frac{1}{3} \quad (5)$
4(b)	$C(x; 2)$ Substitute $y = 2$ into $3x + 4y = -7$ $3x + 4(2) = -7$ $\therefore 3x = -15$ $\therefore x = -5$ $\therefore C(-5; 2)$ $\therefore (x + 5)^2 + (y - 2)^2 = r^2$ Now $r = 5$ $\therefore (x + 5)^2 + (y - 2)^2 = 25$	$\checkmark 3x + 4(2) = -7$ $\checkmark x = -5$ $\checkmark (x + 5)^2 + (y - 2)^2 = r^2$ $\checkmark (x + 5)^2 + (y - 2)^2 = 25 \quad (4)$

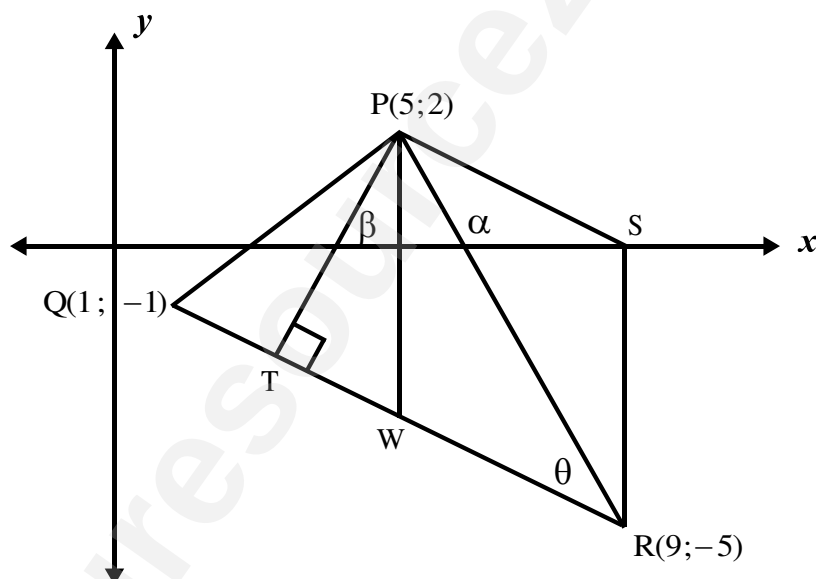
4(c)	$C(-5;2) = \left(\frac{x_D + x_B}{2} ; \frac{y_D + y_B}{2} \right)$ $\therefore C(-5;2) = \left(\frac{x_D + (-1)}{2} ; \frac{y_D + (-1)}{2} \right)$ $\therefore -5 = \frac{x_D - 1}{2} \quad \text{and} \quad 2 = \frac{y_D - 1}{2}$ $\therefore -10 = x_D - 1 \quad \text{and} \quad 4 = y_D - 1$ $\therefore x_D = -9 \quad \text{and} \quad y_D = 5$ $\therefore D(-9;5)$	$\checkmark -5 = \frac{x_D - 1}{2}$ $\checkmark 2 = \frac{y_D - 1}{2}$ $\checkmark \checkmark D(-9;5)$ (4)
5(a)	$0 = 3x + 8$ $\therefore -3x = 8$ $\therefore x = -\frac{8}{3}$ $E\left(-\frac{8}{3}; 0\right)$	$\checkmark 0 = 3x + 8$ $\checkmark E\left(-\frac{8}{3}; 0\right)$ (2)
5(b)	$\tan \hat{FEO} = m_{EF}$ $\therefore \tan \hat{FEO} = 3$ $\therefore \hat{FEO} = 71,56505118^\circ$ $\hat{ADE} = 45^\circ$ $\hat{DAE} + 45^\circ = 71,56505118^\circ$ $\therefore \hat{DAE} = 26,57^\circ$	$\checkmark \tan \hat{FEO} = 3$ $\checkmark \hat{FEO} = 71,56505118^\circ$ $\checkmark \hat{DAE} = 26,57^\circ$ (3)
5(c)	$m_{AB} = \tan \hat{DAE}$ $\therefore m_{AB} = \tan 26,57^\circ$ $\therefore m_{AB} = \frac{1}{2}$ $\therefore y = \frac{1}{2}x + c$ Substitute B(1; 5) $\therefore 5 = \frac{1}{2}(1) + c$ $\therefore c = 4\frac{1}{2}$ $\therefore y = \frac{1}{2}x + 4\frac{1}{2}$	$\checkmark m_{AB} = \tan 26,57^\circ$ $\checkmark m_{AB} = \frac{1}{2}$ $\checkmark c = 4\frac{1}{2}$ $\checkmark y = \frac{1}{2}x + 4\frac{1}{2}$ (4)

5(d)	$x - 2(3x + 8) + 9 = 0$ $\therefore x - 6x - 16 + 9 = 0$ $\therefore -5x = 7$ $\therefore x = -\frac{7}{5}$ $\therefore y = 3\left(-\frac{7}{5}\right) + 8 = \frac{19}{5}$ $D\left(-\frac{7}{5}; \frac{19}{5}\right)$	$\checkmark x - 2(3x + 8) + 9 = 0$ $\checkmark x = -\frac{7}{5}$ $\checkmark y = \frac{19}{5}$ $\checkmark D\left(-\frac{7}{5}; \frac{19}{5}\right)$ (4)
5(e)	$D\left(-\frac{7}{5}; \frac{19}{5}\right)$ and $F(0; 8)$ $DF^2 = \left(0 - \left(-\frac{7}{5}\right)\right)^2 + \left(8 - \frac{19}{5}\right)^2$ $\therefore DF^2 = \frac{49}{25} + \frac{441}{25}$ $\therefore DF^2 = \frac{490}{25}$ $\therefore DF = \sqrt{\frac{98}{5}}$ $D\left(-\frac{7}{5}; \frac{19}{5}\right)$ and $M\left(0; \frac{9}{2}\right)$ $DM^2 = \left(0 - \left(-\frac{7}{5}\right)\right)^2 + \left(\frac{9}{2} - \frac{19}{5}\right)^2$ $\therefore DM^2 = \frac{49}{25} + \frac{49}{100}$ $\therefore DM^2 = \frac{49}{20}$ $\therefore DM = \sqrt{\frac{49}{20}}$ $\text{Area of } \triangle FDM = \frac{1}{2} \left(\sqrt{\frac{49}{20}} \right) \left(\sqrt{\frac{98}{5}} \right) \sin 45^\circ$ $\text{Area of } \triangle FEO = \frac{1}{2} \left(\frac{8}{3} \right) (8) = \frac{32}{3}$ Area of quad DEOM $= \frac{32}{3} - \frac{1}{2} \left(\sqrt{\frac{49}{20}} \right) \left(\sqrt{\frac{98}{5}} \right) \sin 45^\circ = 8,21$	$\checkmark DF = \sqrt{\frac{98}{5}}$ $\checkmark DM = \sqrt{\frac{49}{20}}$ $\checkmark \frac{1}{2} \left(\sqrt{\frac{49}{20}} \right) \left(\sqrt{\frac{98}{5}} \right) \sin 45^\circ$ $\checkmark \frac{1}{2} \left(\frac{8}{3} \right) (8) = \frac{32}{3}$ $\checkmark \frac{32}{3} - \frac{1}{2} \left(\sqrt{\frac{49}{20}} \right) \left(\sqrt{\frac{98}{5}} \right) \sin 45^\circ$ $\checkmark 8,21$ (6)
6	$(x+2)^2 + (y-4)^2 = 25$ (circle A) Centre: A(-2; 4) radius of A: $r_A = 5$	$\checkmark r_A = 5$ $\checkmark r_B = 3$

$(x-5)^2 + (y+1)^2 = 9$ (circle B) Centre: B(5; -1) radius of A: $r_B = 3$ $r_A + r_B = 5 + 3 = 8$ $AB^2 = (5 - (-2))^2 + (-1 - 4)^2$ $\therefore AB^2 = 49 + 25$ $\therefore AB^2 = 74$ $\therefore AB = \sqrt{74} = 8,6$ $\therefore r_A + r_B < AB$ The two circle don't intersect.	✓ $r_A + r_B = 5 + 3 = 8$ ✓ $AB = \sqrt{74} = 8,6$ ✓ $r_A + r_B < AB$ ✓ answer (6)
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SECTION B: SOLUTIONS TO TYPICAL EXAMINATION QUESTIONS

QUESTION 1



1(a)	$W\left(\frac{1+(9)}{2}; \frac{-1+(-5)}{2}\right)$ $= W(5; -3)$ The equation of PW is $x = 5$	✓ midpoint ✓ $x = 5$ (2)
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1(b)	$m_{QR} = \frac{-5 - (-1)}{9 - 1} = \frac{-4}{8} = -\frac{1}{2}$ $\therefore m_{PS} = -\frac{1}{2} \quad (PS \parallel QR)$ $y - 2 = -\frac{1}{2}(x - 5)$ $\therefore y - 2 = -\frac{1}{2}x + \frac{5}{2}$ $\therefore y = -\frac{1}{2}x + \frac{9}{2}$	$\checkmark m_{QR}$ $\checkmark m_{PS}$ $\checkmark$ correct substitution into formula for equation $\checkmark y = -\frac{1}{2}x + \frac{9}{2}$ (4)
1(c)	$m_{PT} = 2 \quad (PT \perp QR)$ $y - 2 = 2(x - 5)$ $\therefore y - 2 = 2x - 10$ $\therefore y = 2x - 8$	$\checkmark m_{PT}$ $\checkmark$ correct substitution into formula for equation $\checkmark y = 2x - 8$ (3)
1(d)	$m_{QR} = -\frac{1}{2}$ $y - (-1) = -\frac{1}{2}(x - 1)$ $\therefore y + 1 = -\frac{1}{2}x + \frac{1}{2}$ $\therefore y = -\frac{1}{2}x - \frac{1}{2}$ $\therefore -\frac{1}{2}x - \frac{1}{2} = 2x - 8$ $\therefore -x - 1 = 4x - 16$ $\therefore -5x = -15$ $\therefore x = 3$ $\therefore y = 2(3) - 8 = -2$ $\therefore T(3; -2)$	$\checkmark$ correct substitution into formula for equation $\checkmark y = -\frac{1}{2}x - \frac{1}{2}$ $\checkmark -\frac{1}{2}x - \frac{1}{2} = 2x - 8$ $\checkmark x = 3$ $\checkmark T(3; -2)$ (5)

1(e)	$QT^2 = (1-3)^2 + (-1-(-2))^2$ $\therefore QT^2 = 4+1$ $\therefore QT^2 = 5$ $\therefore QT = \sqrt{5}$ $TR^2 = (3-9)^2 + (-2-(-5))^2$ $\therefore TR^2 = 36+9$ $\therefore TR^2 = 45$ $\therefore TR = \sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$ $\therefore \frac{1}{3}TR = \sqrt{5} = QT$ $\therefore QT = \frac{1}{3}TR$	✓ correct substitution to get QT ✓ answer for QT ✓ correct substitution to get TR ✓ answer for TR ✓ establishing that $QT = \frac{1}{3}TR$ (5)
1(f)	$\tan \alpha = m_{PR}$ $\therefore \tan \alpha = \frac{2-(-5)}{5-9}$ $\therefore \tan \alpha = -1$ $\therefore \alpha = 180^\circ - 45^\circ$ $\therefore \alpha = 135^\circ$ $\tan \beta = m_{PT}$ $\therefore \tan \beta = 2$ $\therefore \beta = 63,43494882^\circ$ Now $\hat{TPR} + \beta = \alpha$ $\therefore \hat{TPR} = \alpha - \beta$ $\therefore \hat{TPR} = 135^\circ - 63,43494882^\circ$ $\therefore \hat{TPR} = 71,56505118^\circ$ $\theta + 90^\circ + 71,56505118^\circ = 180^\circ$ $\therefore \theta = 18,43^\circ$	✓ $\tan \alpha = -1$ ✓ $\alpha = 135^\circ$ ✓ $\beta = 63,43494882^\circ$ ✓ $\hat{TPR} = 71,56505118^\circ$ ✓ $\theta = 18,43^\circ$ (5)
2(a)	$(-1;3) = \left(\frac{1+(-3)}{2}; \frac{2+k}{2} \right)$ $(-1;3) = \left(-1; \frac{2+k}{2} \right)$ $\therefore 3 = \frac{2+k}{2}$ $\therefore 6 = 2+k$ $\therefore k = 4$	✓ $(-1;3) = \left(\frac{1+(-3)}{2}; \frac{2+k}{2} \right)$ ✓ $\therefore 3 = \frac{2+k}{2}$ ✓ $\therefore k = 4$ (3)

2(b)	$m_{AB} = m_{CD}$ (lines //) $\therefore \frac{2-1}{1-3} = \frac{k-(-3)}{-3-2}$ $\therefore \frac{1}{-2} = \frac{k+3}{-5}$ $\therefore -5 = -2(k+3)$ $\therefore -5 = -2k-6$ $\therefore 2k = -1$ $\therefore k = -\frac{1}{2}$	$\checkmark\checkmark \therefore \frac{2-1}{1-3} = \frac{k-(-3)}{-3-2}$ $\checkmark \therefore k = -\frac{1}{2}$ (3)
2(c)	$m_{AB} \times m_{CD} = -1$ (lines $\perp$) $\therefore \frac{1}{-2} \times \frac{k+3}{-5} = -1$ $\therefore \frac{k+3}{10} = -1$ $\therefore k+3 = -10$ $\therefore k = -13$	$\checkmark\checkmark \therefore \frac{1}{-2} \times \frac{k+3}{-5} = -1$ $\checkmark \therefore k = -13$ (3)
2(d)	$m_{AB} = m_{BC}$ (lines //) $\therefore \frac{1}{-2} = \frac{1-k}{3-(-3)}$ $\therefore \frac{1}{-2} = \frac{1-k}{6}$ $\therefore 6 = -2(1-k)$ $\therefore 6 = -2+2k$ $\therefore 8 = 2k$ $\therefore k = 4$	$\checkmark\checkmark \therefore \frac{1}{-2} = \frac{1-k}{3-(-3)}$ $\checkmark \therefore k = 4$ (3)
2(e)	$CD = 5\sqrt{2}$ And $CD^2 = (2-(-3))^2 + (-3-k)^2$ $\therefore (5\sqrt{2})^2 = 5^2 + 9 + 6k + k^2$ $\therefore 50 = 25 + 9 + 6k + k^2$ $\therefore 0 = k^2 + 6k - 16$ $\therefore 0 = (k+8)(k-2)$ $\therefore k = -8$ or $k = 2$	$\checkmark CD^2 = (2-(-3))^2 + (-3-k)^2$ $\checkmark \therefore (5\sqrt{2})^2 = 5^2 + 9 + 6k + k^2$ $\checkmark \therefore 0 = k^2 + 6k - 16$ $\checkmark \therefore 0 = (k+8)(k-2)$ $\checkmark \therefore k = -8$ or $k = 2$ (5)
3(a)	$M\left(\frac{-8+0}{2}; \frac{1+5}{2}\right) = (-4; 3)$	$\checkmark (-4; 3)$ (1)

3(b)	$m_{AM} = \frac{5-3}{0-(-4)} = \frac{2}{4} = \frac{1}{2}$ $\therefore m_{AD} = -2$ $y - y_1 = m(x - x_1)$ $\therefore y - 5 = -2(x - 0)$ $\therefore y = -2x + 5$	$\checkmark m_{AM} = \frac{1}{2}$ $\checkmark m_{AD} = -2$ $\checkmark y - 5 = -2(x - 0)$ $\checkmark y = -2x + 5$ (4)
3(c)	$AM^2 = (0+4)^2 + (5-3)^2$ $\therefore AM^2 = 4^2 + 2^2$ $\therefore AM = \sqrt{20}$	$\checkmark \text{ formula}$ $\checkmark AM^2 = (0+4)^2 + (5-3)^2$ $\checkmark AM = \sqrt{20}$ (3)
3(d)	$(x+4)^2 + (y-3)^2 = (\sqrt{20})^2$ $\therefore (x+4)^2 + (y-3)^2 = 20$ $\therefore x^2 + 8x + 16 + y^2 - 6y + 9 = 20$ $\therefore x^2 + y^2 + 8x - 6y + 5 = 0$	$\checkmark \text{ substituting centre } (-4;3)$ $\checkmark \text{ substituting radius } \sqrt{20}$ $\checkmark (x+4)^2 + (y-3)^2 = 20$ $\checkmark x^2 + y^2 + 8x - 6y + 5 = 0$ (4)
3(e)	$AT = TK = 6 \text{ and } CD \perp AK$ Therefore ACKD is a kite since diagonal CD bisects diagonal AK at right angles	$\checkmark AT = TK = 6$ $\checkmark CD \perp AK$ $\checkmark \text{ kite}$ $\checkmark \text{ reason}$ (4)
4	$x^2 + 8x + y^2 - 6y + 5 = 0$ $\therefore x^2 + 8x + y^2 - 6y = -5$ $\therefore x^2 + 8x + \left[\frac{8}{2}\right]^2 + y^2 - 6y + \left[\frac{-6}{2}\right]^2 = -5 + \left[\frac{8}{2}\right]^2 + \left[\frac{-6}{2}\right]^2$ $\therefore x^2 + 8x + [4]^2 + y^2 - 6y + [-3]^2 = -5 + [4]^2 + [-3]^2$ $\therefore (x+4)^2 + (y-3)^2 = -5 + 16 + 9$ $\therefore (x+4)^2 + (y-3)^2 = 20$ Centre = $(-4;3)$ $r^2 = 20$ $\therefore r = \sqrt{20}$	$\checkmark \left[\frac{8}{2}\right]^2$ $\checkmark \left[\frac{-6}{2}\right]^2$ $\checkmark (x+4)^2$ $\checkmark (y-3)^2$ $\checkmark 20$ $\checkmark \text{ centre}$ $\checkmark \text{ radius}$ (7)

SESSION NO: 8 (CONSOLIDATION)

TOPIC: NUMBER PATTERNS, SEQUENCES AND SERIES

SECTION A: SOLUTIONS TO HOMEWORK QUESTIONS

1(a)	$-1\frac{1}{2} + 3 + 7\frac{1}{2} + \dots + 669$ $a = -1\frac{1}{2} \quad d = 4\frac{1}{2} \quad T_n = 669$ $669 = -1\frac{1}{2} + (n-1)\left(4\frac{1}{2}\right)$ $\therefore 670\frac{1}{2} = \frac{9n}{2} - 4\frac{1}{2}$ $\therefore 675 = \frac{9n}{2}$ $\therefore n = 150$ $S_n = \frac{n}{2}[a + l]$ $\therefore S_{150} = \frac{150}{2}\left[-1\frac{1}{2} + 669\right] = 50062\frac{1}{2}$	$\checkmark$ the value of a and d $\checkmark 669 = -1\frac{1}{2} + (n-1)\left(4\frac{1}{2}\right)$ $\checkmark n = 150$ $\checkmark S_{150} = \frac{150}{2}\left[-1\frac{1}{2} + 669\right]$ $\checkmark$ answer (5)
1(b)	$-1\frac{1}{2} + 3 - 6 + \dots + 768$ $a = -1\frac{1}{2} \quad r = -2 \quad T_n = 768$ $768 = \left(-1\frac{1}{2}\right)(-2)^{n-1}$ $\therefore -512 = (-2)^{n-1}$ $\therefore (-2)^9 = (-2)^{n-1}$ $\therefore 9 = n - 1$ $\therefore n = 10$ $S_{10} = \frac{\left(-1\frac{1}{2}\right)\left((-2)^{10} - 1\right)}{-2 - 1} = \frac{1023}{2}$	$\checkmark$ the value of a and r $\checkmark 768 = \left(-1\frac{1}{2}\right)(-2)^{n-1}$ $\checkmark (-2)^9 = (-2)^{n-1}$ $\checkmark n = 10$ $\checkmark S_{10} = \frac{\left(-1\frac{1}{2}\right)\left((-2)^{10} - 1\right)}{-2 - 1}$ $\checkmark$ answer (6)
2(a)(1)	$T_9 = 80 \quad T_6 = 10$	$\checkmark a.r^8 = 80$ $\checkmark a.r^5 = 10$

	$\therefore ar^8 = 80 \quad \therefore ar^5 = 10$ $\therefore ar^8 = 80$ $\therefore ar^5 = 10$ $\therefore r^3 = 8$ $\therefore r = 2$ $\therefore a(2)^8 = 80$ $\therefore 256a = 80$ $\therefore a = \frac{5}{16}$	$\checkmark r^3 = 8$ $\checkmark r = 2$ $\checkmark a(2)^8 = 80$ $\checkmark \text{answer}$	(6)
2(a)(2)	$T_n = 5120$ $T_n = ar^{n-1}$ $\therefore 5120 = \frac{5}{16}(2)^{n-1}$ $\therefore 16384 = (2)^{n-1}$ $\therefore 2^{14} = (2)^{n-1}$ $\therefore 14 = n-1$ $\therefore n = 15$	$\checkmark 5120 = \frac{5}{16}(2)^{n-1}$ $\checkmark 16384 = (2)^{n-1}$ $\checkmark 2^{14} = (2)^{n-1}$ $\checkmark n = 15$	(4)
2(b)(1)	$8(x-3)^2; 4(x-3)^3; 2(x-3)^4; \dots \quad x \neq 3$ $\therefore r = \frac{x-3}{2}$ $-1 < \frac{x-3}{2} < 1$ $1 < x < 5 \quad x \neq 3$	$\checkmark r = \frac{x-3}{2}$ $\checkmark -1 < \frac{x-3}{2} < 1$ $\checkmark 1 < x < 5$ $\checkmark x \neq 3$	(4)
2(b)(2)	$a = 8(3,5-3)^2 = 2$ $r = \frac{x-3}{2} = \frac{3,5-3}{2} = \frac{1}{4}$ $S_\infty = \frac{2}{1-\frac{1}{4}}$ $\therefore S_\infty = \frac{8}{3}$	$\checkmark a = 2$ $\checkmark r = \frac{1}{4}$ $\checkmark S_\infty = \frac{8}{3}$	(3)

2(c)	$\sum_{p=1}^m (2p-7)$ $= -5 + (-3) + (-1) + \dots + (2m-7) = 1015$ $\therefore a = -5, d = 2 \text{ and number of terms} = m,$ $\therefore S_m = 1015$ $\therefore S_m = \frac{m}{2} [2a + (m-1)d]$ $\therefore 1015 = \frac{m}{2} [2(-5) + (m-1)(2)]$ $\therefore 1015 = \frac{m}{2} [-10 + 2m - 2]$ $\therefore 1015 = \frac{m}{2} [2m - 12]$ $\therefore 1015 = m^2 - 6m$ $\therefore 0 = m^2 - 6m - 1015$ $\therefore 0 = (m+29)(m-35)$ $\therefore m = -29 \text{ (N/A) or } m = 35$	$\checkmark$ expanding $\checkmark a = -5, d = 2$ $\checkmark S_m = 1015$ $\checkmark \therefore 1015 = \frac{m}{2} [2(-5) + (m-1)(2)]$ $\checkmark \therefore 0 = m^2 - 6m - 1015$ $\checkmark \therefore 0 = (m+29)(m-35)$ $\checkmark m = 35$ (7)
3(a)(1)	$\begin{array}{cccc} -1 & -7 & -11 & -13 \\ & -6 & -4 & -2 \\ & & 2 & 2 \end{array}$ $\therefore p = -13$	$\checkmark$ first difference row $\checkmark p = -13$ (2)
3(a)(2)	$2a = 2$ $\therefore a = 1$ $3(1) + b = -6$ $\therefore b = -9$ $(1) + (-9) + c = -1$ $\therefore c = 7$ $\therefore T_n = n^2 - 9n + 7$	$\checkmark a = 1$ $\checkmark b = -9$ $\checkmark c = 7$ $\checkmark T_n = n^2 - 9n + 7$ (4)
3(a)(3)	$T_{n+1} - T_n = 96$ $\therefore (n+1)^2 - 9(n+1) + 7 - [n^2 - 9n + 7] = 96$ $\therefore n^2 + 2n + 1 - 9n - 9 + 7 - n^2 + 9n - 7 = 96$ $\therefore 2n = 104$ $\therefore n = 52$ $T_{52} = (52)^2 - 9(52) + 7 = 2243$ $T_{53} = (53)^2 - 9(53) + 7 = 2339$	$\checkmark T_{n+1} - T_n = 96$ $\checkmark n = 52$ $\checkmark T_{52} = 2243$ $\checkmark T_{53} = 2339$ (4)

3(b)(2)	$\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right)\left(1 + \frac{1}{4}\right)\left(1 + \frac{1}{5}\right) \dots \left(1 + \frac{1}{99}\right)$ $= \left(\frac{3}{2}\right)\left(\frac{4}{3}\right)\left(\frac{5}{4}\right)\left(\frac{6}{5}\right) \dots \left(\frac{100}{99}\right)$ $= \frac{100}{2} = 50$	$\checkmark \left(1 + \frac{1}{99}\right)$ $\checkmark \left(\frac{3}{2}\right)\left(\frac{4}{3}\right)\left(\frac{5}{4}\right)\left(\frac{6}{5}\right) \dots \left(\frac{100}{99}\right)$ $\checkmark \frac{100}{2}$ $\checkmark 50$ (4)
4(a)	$T_5 - T_4 = 25$ $T_{70} - T_{69} = 7 + (69 - 1)(6) = 415$	$\checkmark T_5 - T_4 = 25$ $\checkmark \checkmark 7 + (69 - 1)(6)$ $\checkmark 415$ (4)
4(b)	$2a = 6$ $\therefore a = 3$ $3(3) + b = 7$ $\therefore b = -2$ $\therefore T_n = 3n^2 - 2n + c$ $\therefore T_{89} = 3(89)^2 - 2(89) + c$ $\therefore 23\,594 = 23\,763 - 178 + c$ $\therefore c = 9$ $\therefore T_n = 3n^2 - 2n + 9$ $\therefore T_{69} = 3(69)^2 - 2(69) + 9 = 14\,154$	$\checkmark a = 3$ $\checkmark b = -2$ $\checkmark T_n = 3n^2 - 2n + c$ $\checkmark c = 9$ $\checkmark 14\,154$ (5)
5(a)	$T_k = -3 + (k - 1)(8)$ $\therefore T_k = -3 + 8k - 8$ $\therefore T_k = 8k - 11$	$\checkmark T_k = -3 + (k - 1)(8)$ $\checkmark T_k = 8k - 11$ (2)
5(b)	$\sum_{k=1}^n (8k - 11)$	$\checkmark \sum_{k=1}^n$ $\checkmark 8k - 11$ (2)
5(c)	$S_n = \frac{n}{2} (2(-3) + (n - 1)(8))$ $\therefore S_n = \frac{n}{2} (-6 + 8n - 8)$ $\therefore S_n = \frac{n}{2} (8n - 14)$ $\therefore S_n = 4n^2 - 7n$	$\checkmark \frac{n}{2} (2(-3) + (n - 1)(8))$ $\checkmark \frac{n}{2} (8n - 14)$ $\checkmark 4n^2 - 7n$ (3)
5(d)(1)	$Q_6 = -6 - 3 + 5 + 13 + 21 + 29$	$\checkmark \checkmark \text{ answer}$ (2)
5(d)(2)	$Q_{129} = -6 + S_{128}$ $\therefore Q_{129} = -6 + 4(128)^2 - 7(128) = 64\,634$	$\checkmark Q_{129} = -6 + S_{128}$ $\checkmark \text{ substitution}$ $\checkmark \text{ answer}$ (3)
6	Area of first circle is: $4\pi \text{ cm}^2$	$\checkmark 4\pi \text{ cm}^2$ $\checkmark r = 2$

$\pi r^2 = 4\pi$ $\therefore r^2 = 4$ $\therefore r = 2$ [Diameter of first circle is 4 cm] Area of second circle is: $\frac{9}{4} \times 4\pi \text{ cm}^2 = 9\pi \text{ cm}^2$ $\pi r^2 = 9\pi$ $\therefore r^2 = 9$ $\therefore r = 3$ [Diameter of second circle is 6 cm] Area of third circle is: $\frac{9}{4} \times 9\pi \text{ cm}^2 = \frac{81\pi}{4} \text{ cm}^2$ $\pi r^2 = \frac{81\pi}{4}$ $\therefore r^2 = \frac{81}{4}$ $\therefore r = \frac{9}{2}$ [Diameter of third circle is 9 cm] The diameters form a geometric series: $4 + 6 + 9 + \dots$ with $a = 4$; $d = \frac{3}{2}$ We want to find the number of circles where the sum of the diameters is equal to $\frac{211}{4} \text{ cm}$. [Use the formula: $S_n = \frac{a(r^n - 1)}{r - 1}$] $\therefore \frac{211}{4} = \frac{4 \left[\left(\frac{3}{2} \right)^n - 1 \right]}{\frac{3}{2} - 1}$ $\therefore \frac{211}{4} = \frac{4 \left[\left(\frac{3}{2} \right)^n - 1 \right]}{\frac{1}{2}}$ $\therefore \frac{211}{4} \times \frac{1}{2} = 4 \left[\left(\frac{3}{2} \right)^n - 1 \right]$ $\therefore \frac{211}{4} \times \frac{1}{2} = 4 \left[\left(\frac{3}{2} \right)^n - 1 \right]$	$\checkmark 9\pi \text{ cm}^2$ $\checkmark r = 3$ $\checkmark \frac{81\pi}{4} \text{ cm}^2$ $\checkmark r = \frac{9}{2}$ $\checkmark a = 4$; $d = \frac{3}{2}$ $\checkmark \frac{211}{4} = \frac{4 \left[\left(\frac{3}{2} \right)^n - 1 \right]}{\frac{3}{2} - 1}$ $\checkmark \frac{243}{32} = \left(\frac{3}{2} \right)^n$ $\checkmark n = 5$
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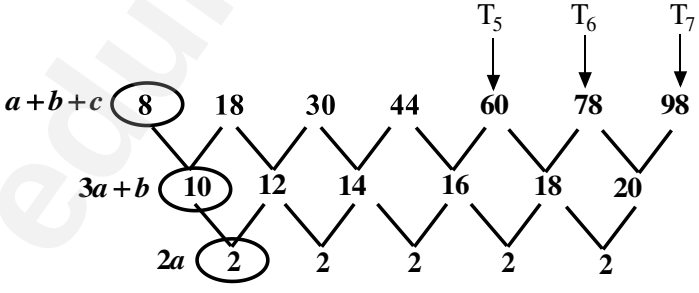
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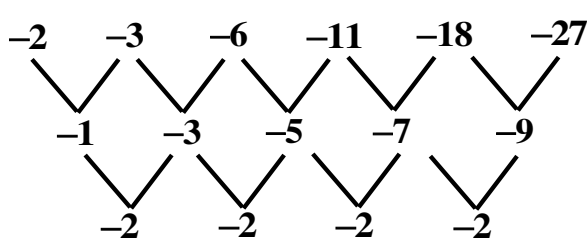
	$\therefore \frac{211}{8} = 4 \left[\left(\frac{3}{2} \right)^n - 1 \right]$ $\therefore \frac{211}{32} = \left(\frac{3}{2} \right)^n - 1$ $\therefore \frac{243}{32} = \left(\frac{3}{2} \right)^n$ $\therefore \left(\frac{3}{2} \right)^5 = \left(\frac{3}{2} \right)^n$ $\therefore 5 = n$ You would require 5 circles.	
7(a)	$3; a; 10; b; 21; \dots$ $(10 - a) - (a - 3) = 1$ $\therefore 10 - a - a + 3 = 1$ $\therefore -2a = -12$ $\therefore a = 6$ $(21 - b) - (b - 10) = 1$ $\therefore 21 - b - b + 10 = 1$ $\therefore -2b = -30$ $\therefore b = 15$	$\checkmark (10 - a) - (a - 3) = 1$ $\checkmark a = 6$ $\checkmark (21 - b) - (b - 10) = 1$ $\checkmark b = 15$ (4)
7(b)	$3; 6; 10; 15; 21; \dots$ $2a = 1$ $\therefore a = \frac{1}{2}$ $3a + b = 3$ $\therefore 3 \left(\frac{1}{2} \right) + b = 3$ $\therefore b = \frac{3}{2}$ $a + b + c = 3$ $\therefore \frac{1}{2} + \frac{3}{2} + c = 3$ $\therefore c = 1$ $\therefore T_n = \frac{1}{2}n^2 + \frac{3}{2}n + 1$	$\checkmark 3; 6; 10; 15; 21; \dots$ $\checkmark a = \frac{1}{2}$ $\checkmark b = \frac{3}{2}$ $\checkmark c = 1$ $\checkmark T_n = \frac{1}{2}n^2 + \frac{3}{2}n + 1$ (5)

7(c)	$\left(\frac{1}{2}n^2 + \frac{3}{2}n + 1\right) + \left(\frac{1}{2}(n+1)^2 + \frac{3}{2}(n+1) + 1\right)$ $= \frac{1}{2}n^2 + \frac{3}{2}n + 1 + \frac{1}{2}(n^2 + 2n + 1) + \frac{3}{2}n + \frac{3}{2} + 1$ $= \frac{1}{2}n^2 + \frac{3}{2}n + 1 + \frac{1}{2}n^2 + n + \frac{1}{2} + \frac{3}{2}n + \frac{3}{2} + 1$ $= n^2 + 4n + 4$ $= (n+2)^2$	✓ $T_n + T_{n+1}$ ✓ expanding ✓ $n^2 + 4n + 4$ ✓ $(n+2)^2$	(4)
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SECTION B: SOLUTIONS TO TYPICAL EXAMINATION QUESTIONS

1(a)(1)	Series B converges since $r = \frac{1}{3}$ which lies in the interval of convergence which is $-1 < r < 1$	✓ Series B ✓ explanation	(2)
1(a)(2)	$a = \frac{1}{2} \quad r = \frac{1}{3}$ $S_{\infty} = \frac{\frac{1}{2}}{1 - \frac{1}{3}} = \frac{3}{4}$	✓ a and r ✓ substitution ✓ answer	(3)
1(b)(1)	$T_7 = 61$ and $T_{19} = 97$ $\therefore a + 6d = 61 \dots A$ and $a + 18d = 97 \dots B$ Then $B - A \therefore 12d = 36$ and $\therefore d = 3$	✓ $a + 6d = 61$ ✓ $a + 18d = 97$ ✓ $d = 3$	(3)
1(b)(2)	$a = -6d + 61$ and $d = 3$ $\therefore a = -6(3) + 61 = 43$ $\therefore T_{15} = 43 + (15 - 1)(3)$ $\therefore T_{15} = 85$	✓ calculating a ✓ substitution ✓ $T_{15} = 85$	(3)

1(c)	$\sum_{k=1}^n (3k-4) = -1+2+5+\dots+3n-4 = 2419$ $\therefore S_n = 2419, a = -1 \text{ and } d = 3$ $\therefore 2419 = \frac{n}{2}(2(-1) + (n-1)(3))$ $\therefore 4838 = n(-2+3n-3)$ $\therefore 4838 = -5n+3n^2$ $\therefore 0 = 3n^2 - 5n - 4838$ $\therefore 0 = (3n+118)(n-41)$ $\therefore n = -\frac{118}{3} \text{ or } n = 41$ $\therefore n = 41$	$\checkmark a = -1 \quad d = 3$ $\checkmark$ substitute $S_n = 2419$ $\checkmark$ using correct formula $\checkmark 0 = 3n^2 - 5n - 4838$ $\checkmark$ factors $\checkmark n = 41$ (6)
1(d)	$a = 10368 \text{ and } r = \frac{1}{2}$ $\therefore 10368 \left(\frac{1}{2}\right)^{n-1} = 10\frac{1}{8}$ $\therefore \left(\frac{1}{2}\right)^{n-1} = \frac{10,125}{10368} \quad \left(\frac{10,125}{10368} = \frac{1}{1024}\right)$ $\therefore n-1 = \log_{\frac{1}{2}} \frac{1}{1024} \quad \text{or} \quad \left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2^{10}}\right)$ $\therefore n-1 = 10 \quad \text{or} \quad \therefore \left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^{10}$ $\therefore n = 11$ $\therefore S_{11} = \frac{10368 \left(\left(\frac{1}{2}\right)^{11} - 1\right)}{\frac{1}{2} - 1} = 20725\frac{7}{8}$	$\checkmark r = \frac{1}{2}$ $\checkmark 10368 \left(\frac{1}{2}\right)^{n-1} = 10\frac{1}{8}$ $\checkmark n-1 = \log_{\frac{1}{2}} \frac{1}{1024}$ $\checkmark n = 11$ $\checkmark$ substitution into formula $\checkmark 20725\frac{7}{8}$ (6)
2(a)		$\checkmark 60, 78, 98$ $\checkmark a = 1$ $\checkmark b = 7$ $\checkmark c = 0$ $\checkmark T_n = n^2 + 7n$ $\checkmark T_{20} = 540$ (6)

	$2a = 2$ $3a + b = 10$ $a + b + c = 8$ $a = 1$ $\therefore 3(1) + b = 10$ $\therefore 1 + 7 + c = 8$ $\therefore b = 7$ $\therefore c = 0$ $\therefore T_n = 1n^2 + 7n + 0$ $\therefore T_n = n^2 + 7n$ $T_n = n^2 + 7n$ $\therefore T_{20} = (20)^2 + 7(20)$ $\therefore T_{20} = 540$	
2(b)(1)	 Next two terms are $-18; -27$	$\checkmark -18$ $\checkmark -27$ (2)
2(b)(2)	$2a = -2$ $3a + b = -1$ $a + b + c = -2$ $a = -1$ $\therefore 3(-1) + b = -1$ $\therefore -1 + 2 + c = -2$ $\therefore b = 2$ $\therefore c = -3$ $\therefore T_n = -n^2 + 2n - 3$	$\checkmark a = -1$ $\checkmark b = 2$ $\checkmark c = -3$ $\checkmark T_n = -n^2 + 2n - 3$ (4)
2(b)(3)	$-83 = -n^2 + 2n - 3$ $\therefore n^2 - 2n - 80 = 0$ $\therefore 0 = (n - 10)(n + 8)$ $\therefore n = 10$ or $n = -8$ But $n \neq -8$ $\therefore n = 10$	$\checkmark T_n = -83$ $\checkmark$ standard form $\checkmark$ factors $\checkmark$ two values for n $\checkmark$ indicating $n = 10$ (5)
2(c)	The layers are stacked as follows: $30; 29; 28; 27; \dots; 1$ This forms an arithmetic sequence. $\therefore S_{30} = \frac{30}{2} [2(30) + (30 - 1)(-1)]$ $\therefore S_{30} = 465$ There are a maximum of 465 cans that can be stacked in this way.	$\checkmark 30; 29; 28; 27; \dots; 1$ $\checkmark$ correct formula $\checkmark$ correct substitution $\checkmark 465$ cans (4)

SESSION NO: 9 (CONSOLIDATION)

TOPIC: REVISION OF FINANCIAL MATHEMATICS

SECTION A: SOLUTIONS TO HOMEWORK QUESTIONS

1(a)(1)	$i_{eff} = \left(1 + \frac{0,08}{2}\right)^2 - 1$ $\therefore i_{eff} = 0,0816$	✓ formula ✓ 0,0816 (2)
1(a)(2)	$P = 100\,000(1,0816)^{-4}$ $\therefore P = R73\,069,02$ or $P = 100\,000\left(1 + \frac{0,08}{2}\right)^{-8}$ $\therefore P = R73\,069,02$	✓ formula ✓ answer (2)
1(b)(1)	$A = 130\,000(1 - 0,15)^3(1 - 0,16)^2$ $\therefore A = R56\,332,46$	✓ correct formula ✓ $130\,000(1 - 0,15)^3$ ✓ $(1 - 0,16)^2$ ✓ answer (4)
1(b)(2)	$56\,332,46 = 130\,000(1 - i)^5$ $\therefore \frac{56\,332,46}{130\,000} = (1 - i)^5$ $\therefore \sqrt[5]{\frac{56\,332,46}{130\,000}} = 1 - i$ $\therefore i = 1 - \sqrt[5]{\frac{56\,332,46}{130\,000}}$ $\therefore i = 0,1540142009$ $\therefore r = 15,4\%$	✓ correct formula ✓ correct substitution ✓ $i = 0,1540142009$ ✓ answer (4)
2(a)	$4000\left(1 + \frac{0,15}{12}\right)^{36} \left(1 + \frac{0,16}{4}\right)^{20} + 5000\left(1 + \frac{0,16}{4}\right)^{20} - 2000\left(1 + \frac{0,16}{4}\right)^{16}$ $\therefore A = R20916,83$	✓ correct formula ✓ correct interest rates ✓ correct values for n ✓ $4000\left(1 + \frac{0,15}{12}\right)^{36}$

		$\checkmark \left(1 + \frac{0,16}{4}\right)^{20}$ $\checkmark +5000 \left(1 + \frac{0,16}{4}\right)^{20}$ $\checkmark -2000 \left(1 + \frac{0,16}{4}\right)^{16}$ $\checkmark \text{ R20916,83} \quad (8)$
2(b)	100000 $+160000 \left(1 + \frac{0,12}{12}\right)^{-36}$ $+170000 \left(1 + \frac{0,14}{2}\right)^{-8} \left(1 + \frac{0,12}{12}\right)^{-36}$ $\therefore P = \text{R280980,71}$	$\checkmark 100\ 000$ $\checkmark \text{ correct formula}$ $\checkmark \text{ correct interest rates}$ $\checkmark \text{ correct values for } n$ $\checkmark +160000 \left(1 + \frac{0,12}{12}\right)^{-36}$ $\checkmark +170000 \left(1 + \frac{0,14}{2}\right)^{-8}$ $\checkmark \left(1 + \frac{0,12}{12}\right)^{-36}$ $\checkmark \text{ R280980,71} \quad (8)$
3(a)	$F = \frac{1000 \left[\left(1 + \frac{0,18}{12}\right)^3 - 1 \right]}{\frac{0,18}{12}} \cdot \left(1 + \frac{0,18}{12}\right)^{10} + \frac{1000 \left[\left(1 + \frac{0,18}{12}\right)^9 - 1 \right]}{\frac{0,18}{12}}$ $\therefore F = \text{R13093,44}$	$\checkmark \text{ formula}$ $\checkmark \text{ interest rate}$ $\checkmark \frac{1000 \left[\left(1 + \frac{0,18}{12}\right)^3 - 1 \right]}{\frac{0,18}{12}}$ $\checkmark \left(1 + \frac{0,18}{12}\right)^{10}$ $\checkmark \frac{1000 \left[\left(1 + \frac{0,18}{12}\right)^9 - 1 \right]}{\frac{0,18}{12}}$ $\checkmark \text{ answer} \quad (8)$
3(b)(1)	$0,35 \times 620\ 000 = 217\ 000$ $\text{Loan balance is: } \text{R620}\ 000 - \text{R217}\ 000 = \text{R403}\ 000$ $403\ 000 = \frac{x \left[1 - (1,0125)^{-240} \right]}{0,0125}$ $\therefore \frac{403\ 000 \times 0,0125}{\left[1 - (1,0125)^{-240} \right]} = x$ $\therefore x = \text{R5306,66}$	$\checkmark 217\ 000$ $\checkmark 403\ 000$ $\checkmark \text{ correct formula}$ $\checkmark \text{ correct substitution}$ $\checkmark \text{ interest rate}$ $\checkmark n = 240$ $\checkmark \text{ answer} \quad (7)$

3(b)(2)	$B = \frac{5306,66 \left[1 - (1,0125)^{-168} \right]}{0,0125} = R371\,864,91$ or $B = 403\,000(1,0125)^{72} - \frac{5306,66 \left[(1,0125)^{72} - 1 \right]}{0,0125}$ $\therefore B = R371\,865,29$	✓ correct formula ✓ $n = 168$ ✓ interest rate ✓ answer ✓ $403\,000(1,0125)^{72}$ ✓ $\frac{5306,66 \left[(1,0125)^{72} - 1 \right]}{0,0125}$ ✓ subtracting ✓ answer (4)
3(b)(3)	$371\,864,91 = \frac{x \left[1 - \left(1 + \frac{0,1225}{12} \right)^{-168} \right]}{\frac{0,1225}{12}}$ $\therefore \frac{371\,864,91 \times \frac{0,1225}{12}}{\left[1 - \left(1 + \frac{0,1225}{12} \right)^{-168} \right]} = x$ $\therefore x = R4638,10$	✓ correct formula ✓ interest rate ✓ $n = 168$ ✓ answer (4)
4(a)	$72\,500 = 145\,000(1-i)^5$ $\therefore \frac{145\,000}{72\,500} = (1-i)^5$ $\therefore \sqrt[5]{\frac{145\,000}{72\,500}} = 1-i$ $\therefore i = 1 - \sqrt[5]{\frac{145\,000}{72\,500}}$ $\therefore i = 0,12944....$ $\therefore r = 12,94\%$	✓ $72\,500 = 145\,000(1-i)^5$ ✓ $i = 1 - \sqrt[5]{\frac{145\,000}{72\,500}}$ ✓ $r = 12,94\%$ (3)
4(b)(1)	$500\,000 = \frac{x \left[1 - \left(1 + \frac{0,12}{12} \right)^{-240} \right]}{\frac{0,12}{12}}$ $\therefore x = R5\,505,43$	✓ present value formula ✓ $500\,000 =$ ✓ $n = 240$ ✓ answer (4)

4(b)(2)	$500\,000 = \frac{6\,000 \left[1 - \left(1 + \frac{0,12}{12} \right)^{-n} \right]}{\frac{0,12}{12}}$ $\therefore \frac{500\,000 \times \frac{0,12}{12}}{6\,000} = 1 - \left(1 + \frac{0,12}{12} \right)^{-n}$ $\therefore \left(1 + \frac{0,12}{12} \right)^{-n} = 1 - \frac{500\,000 \times \frac{0,12}{12}}{6\,000}$ $\therefore (1,01)^{-n} = 0,16666\dots$ $\therefore -n = \log_{1,01}(0,16666\dots)$ $\therefore n = 180,07$ 181 months	✓ present value formula ✓ $-n$ ✓ use of logs ✓ answer (4)
4(b)(3)	Samuel: $240 \times R5\,505,43 = R1\,321\,303,20$ Melissa: $180 \times R6\,000 + x \quad (x < 6\,000)$ $= R1\,080\,000 + x$ Samuel pays more interest	✓ R1 321 303,20 ✓ R1 080 000 + x ✓ answer (3)
5(a)	1 Jan 1970 to 1 Jan 2013: $2013 - 1970 = 43$ years 1 Jan 2013 to 31 Dec 2013: 1 year This is a total of 44 years. Assuming that she made her last payment on 31 Dec 2013 (one day prior to 1 Jan 2014), the total number of payments will be: $44 \times 400 \times 12 = R211\,200$	✓ 44 years ✓ answer (2)
5(b)	$F = \frac{400 \left[\left(1 + \frac{0,08}{12} \right)^{44 \times 12} - 1 \right]}{\frac{0,08}{12}}$ $\therefore F = R1\,943\,524,42$	✓ future value formula ✓✓ $44 \times 12 = 528$ ✓ $\frac{0,08}{12}$ ✓ answer (5)
5(c)	$2\,000\,000 = \frac{x \left[1 - \left(1 + \frac{0,10}{12} \right)^{-300} \right]}{\frac{0,10}{12}}$ $\therefore x = R18\,174,01$	✓ present value formula ✓ 300 ✓ $2\,000\,000 =$ ✓ answer (4)

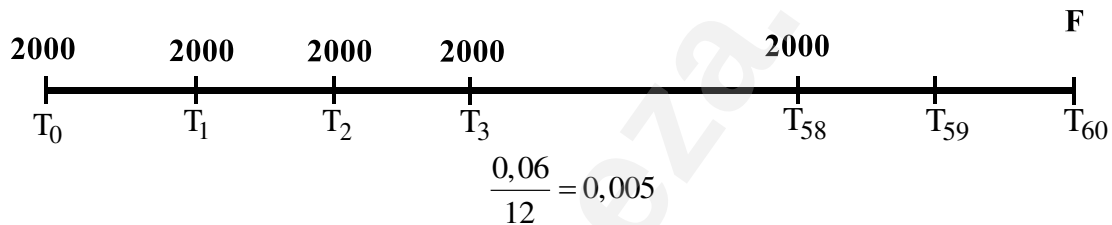
6(a)	R450 000	✓ answer (1)
6(b)	$243\,736,90 = 450\,000(1-i)^4$ $\therefore \frac{243\,736,90}{450\,000} = (1-i)^4$ $\therefore \sqrt[4]{\frac{243\,736,90}{450\,000}} = 1-i$ $\therefore i = 1 - \sqrt[4]{\frac{243\,736,90}{450\,000}}$ $\therefore i = 0,142118.....$ $\therefore r = 14,21\%$	✓ $243\,736,90 = 450\,000(1-i)^4$ ✓ $i = 1 - \sqrt[4]{\frac{243\,736,90}{450\,000}}$ ✓ $i = 0,142118.....$ ✓ $r = 14,21\%$ (4)
6(c)	$A = 450\,000(1+0,081)^4$ $\therefore A = R614\,490,66$	✓ formula ✓ substitution ✓ answer (3)
6(d)	$F = 614\,490,66 - 243\,736,90 = R370\,753,76$ $\therefore 370\,753,76 = \frac{x \left[\left(1 + \frac{0,062}{12} \right)^{36} - 1 \right]}{\frac{0,062}{12}}$ $\therefore x = R9\,397,11$	✓ R370 753,76 ✓ future value formula ✓ $n = 36$ ✓ answer (5)

SECTION A: SOLUTIONS TO TYPICAL EXAMINATION QUESTIONS

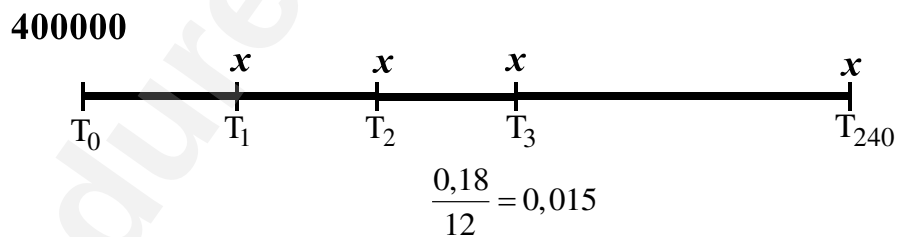
1(a)(1)	$i_{\text{eff}} = \left(1 + \frac{0,15}{12} \right)^{12} - 1$ $\therefore i_{\text{eff}} = 0,1607545177$	✓ substitution into formula ✓ answer (2)
1(a)(2)	$A = 65000(1+i_{\text{eff}})^3$ $\therefore A = R101\,656,35$ Alternatively: $A = 65000 \left(1 + \frac{0,15}{12} \right)^{36}$ $\therefore A = R101\,656,35$	✓ correct formula ✓ correct i and n ✓ answer (3)
1(a)(3)	$A = 65000(1+i_{\text{eff}})^3 + 10000(1+i_{\text{eff}})^2$ $\therefore A = R115\,129,86$ Alternatively:	✓ $65000(1+i_{\text{eff}})^3$ ✓ $10000(1+i_{\text{eff}})^2$ ✓ answer (4) or

	$A = 65000 \left(1 + \frac{0,15}{12}\right)^{36} + 10000 \left(1 + \frac{0,15}{12}\right)^{24}$ $\therefore A = R115\,129,86$ or $A = \left[65000(1 + i_{\text{eff}})^1 + 10000\right](1 + i_{\text{eff}})^2$ $\therefore A = R115\,129,86$ or $A = \left[65000 \left(1 + \frac{0,15}{12}\right)^{12} + 10000\right] \left(1 + \frac{0,15}{12}\right)^{24}$ $\therefore A = R115\,129,86$	$\checkmark 65000 \left(1 + \frac{0,15}{12}\right)^{36}$ $\checkmark\checkmark 10000 \left(1 + \frac{0,15}{12}\right)^{24}$ $\checkmark \text{ answer}$ or $\checkmark 65000(1 + i_{\text{eff}})^1$ $\checkmark +10000$ $\checkmark (1 + i_{\text{eff}})^2$ $\checkmark \text{ answer}$ or $\checkmark 65000 \left(1 + \frac{0,15}{12}\right)^{12}$ $\checkmark +10000$ $\checkmark \left(1 + \frac{0,15}{12}\right)^{24}$ $\checkmark \text{ answer}$
1(b)(1)	R100 000	$\checkmark$ answer (1)
1(b)(2)	R25 000	$\checkmark$ answer (1)
1(b)(3)	Reducing balance depreciation	$\checkmark$ answer (1)
1(b)(4)	$25000 = 100000(1 - i)^4$ $\therefore 0,25 = (1 - i)^4$ $\therefore \sqrt[4]{0,25} = 1 - i$ $\therefore i = 1 - \sqrt[4]{0,25}$ $\therefore i = 0,2928932188$ $\therefore r = 29,3\%$	$\checkmark$ correct formula $\checkmark$ correct substitution $\checkmark$ correct value for r (3)
(c)	$A = \left[100000 \left(1 + \frac{0,18}{12}\right)^{24} - 20000\right] \left(1 + \frac{0,18}{2}\right)^6$ $\therefore A = R206\,199,93$ Alternatively: $A = 100000 \left(1 + \frac{0,18}{12}\right)^{24} \left(1 + \frac{0,18}{2}\right)^6 - 20000 \left(1 + \frac{0,18}{2}\right)^6$ $\therefore A = R206\,199,93$	$\checkmark$ correct formula $\checkmark$ correct interest rates $\checkmark$ correct values for n $\checkmark\checkmark$ correct structure $\checkmark$ answer (6)
(d)	$P = 13000 \left(1 + \frac{0,09}{2}\right)^{-8} + 5000 \left(1 + \frac{0,12}{12}\right)^{-36} \left(1 + \frac{0,09}{2}\right)^{-8}$ $\therefore P = R11\,598,77$ Alternatively:	$\checkmark$ correct formula $\checkmark$ correct interest rates $\checkmark$ correct values for n $\checkmark\checkmark$ correct structure $\checkmark$ answer (6)

	$P = \left[5000 \left(1 + \frac{0,12}{12} \right)^{-36} + 13000 \right] \left(1 + \frac{0,09}{2} \right)^{-8}$ $\therefore P = R11\,598,77$	
2(a)	$A = P(1+i)^n$ $\therefore 10000 = 5000 \left(1 + \frac{0,12}{4} \right)^{4n}$ $\therefore 2 = (1,03)^{4n} \quad \text{or} \quad \log 2 = \log 1,03^{4n}$ $\therefore 4n = \log_{1,03} 2 \quad \therefore \log 2 = 4n \log 1,03$ $\therefore 4n = 23,449... \quad \therefore \frac{\log 2}{\log 1,03} = 4n$ $\therefore n = 5,86 \text{ years} \quad \therefore n = 5,86 \text{ years}$	✓ substitution into correct formula ✓ $2 = (1,03)^{4n}$ ✓ $4n = \log_{1,03} 2$ or $\log 2 = \log 1,03^{4n}$ ✓ $n = 5,86$ years (4)

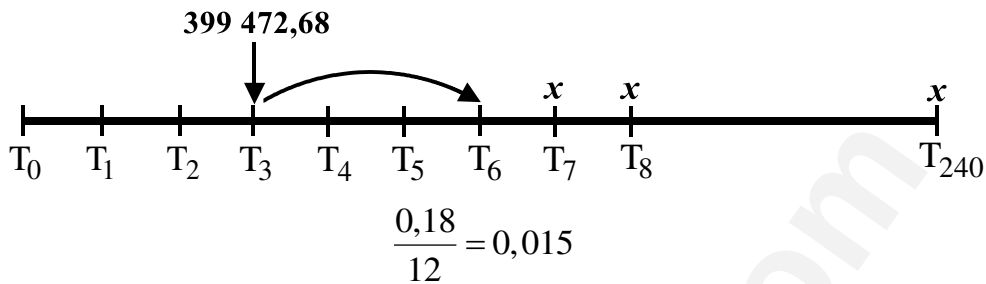


2(b)	$F = \frac{2000 \left[\left(1 + \frac{0,06}{12} \right)^{59} - 1 \right]}{\frac{0,06}{12}} \cdot \left(1 + \frac{0,06}{12} \right)^2 = R138\,227,76$	✓ $\frac{2000 \left[\left(1 + \frac{0,06}{12} \right)^{59} - 1 \right]}{\frac{0,06}{12}}$ ✓ $\left(1 + \frac{0,06}{12} \right)^2$ ✓ answer (3)
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2(c)(1)	$400\,000 = \frac{x \left[1 - \left(1 + \frac{0,18}{12} \right)^{-240} \right]}{\frac{0,18}{12}}$ $\therefore x = R6173,25$	✓ formula $400\,000 = \frac{x \left[1 - \left(1 + \frac{0,18}{12} \right)^{-240} \right]}{\frac{0,18}{12}}$ ✓ answer (3)
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2(c)(2)	$B = \frac{6173,25[1 - (1,015)^{-237}]}{0,015}$ $\therefore B = R399\,472,68$	$\checkmark B = \frac{6173,25[1 - (1,015)^{-237}]}{0,015}$ $\checkmark \text{ answer} \quad (2)$
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2(c)(3)	$399\,472,68 \left(1 + \frac{0,18}{12}\right)^3 = \frac{x \left[1 - \left(1 + \frac{0,18}{12}\right)^{-234}\right]}{\frac{0,18}{12}}$ $\therefore x = R6464,16$	$\checkmark 399\,472,68 \left(1 + \frac{0,18}{12}\right)^3$ $\checkmark \text{ present value formula}$ $\checkmark n = 234$ $\checkmark \text{ answer} \quad (4)$
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