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Data Handling

The **mean** is the average of the data.

The **mode** is the most commonly occurring data item.

The **median** is the middle data item in a ranked list. If there is an even number of items then we average the middle two.

The **range** is the biggest data value minus the smallest.

The **1st quartile** (Q_1) is the data item in the $\frac{n+1}{4}$ position in a ranked list. If this is, for example 17.25 then the first quartile is 0.25 of the way between the 17th and 18th items.

The **3rd quartile** (Q_3) is the data item in the $3\left(\frac{n+1}{4}\right)$ position in a ranked list. If this is, for example 19.75 then the third quartile is 0.75 of the way between the 19th and 20th items.

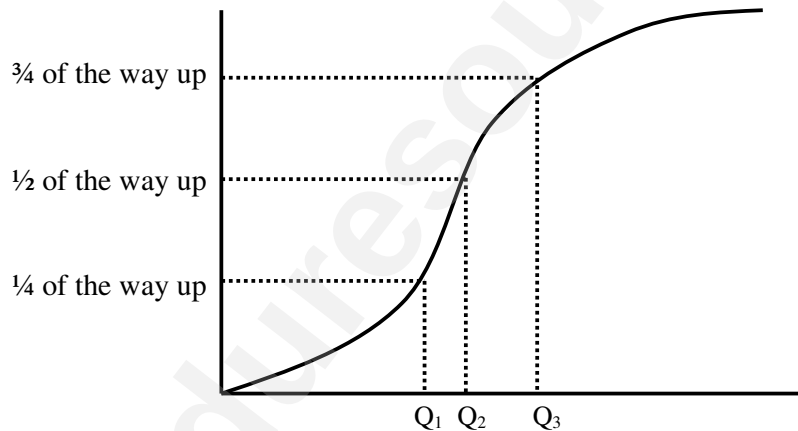
The **inter-quartile** range is $Q_3 - Q_1$.

The **five number summary** of some data is: min, Q_1 , Q_2 , Q_3 , max

A **box and whisker plot** depicts the five number summary as follows:



A **cumulative frequency plot** is also called an ogive curve. It is an S shaped curve. We can read off the quartiles as follows:



The **standard deviation** is a measure of spread of the data around the mean. It is calculated as follows:

1. From each data item subtract the mean.
2. Square your results from step 1.
3. Add up your results from step 2.
4. Divide your answer from step 3 by n (for a population) or $n-1$ (for a sample). This is the variance.
5. Square root the variance to get the standard deviation, σ .

As a general rule:

- 1 standard deviation either side of the mean will include 67% of the population.
- 2 standard deviations either side of the mean will include 95% of the population.
- 3 standard deviations either side of the mean will include 99.9% of the population.

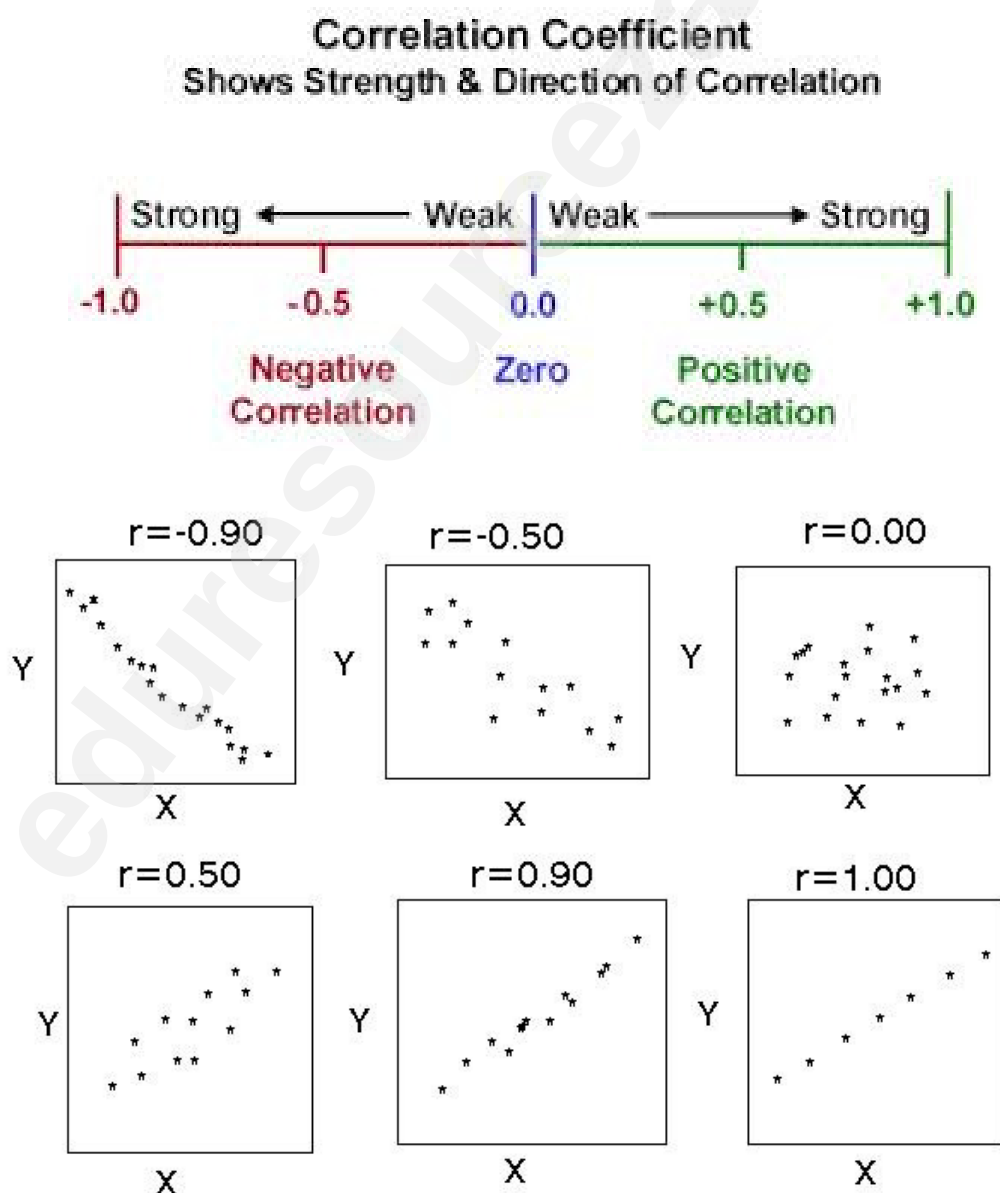
You may be required to plot data and then decide on a function which best matches it.

You will also be required to calculate the **equation of the least squares regression** line using your calculator.

Remember:

- that it is of the form $y = a + bx$.
- that the point $(\bar{x}; \bar{y})$ always lies on the line of best fit.
- It is safe to use your least squares regression line to predict a data value **within** the given data (**interpolation**). However, it is risky to use it to predict a data value **outside** of the given data (**extrapolation**).

You also need to be able to calculate a correlation coefficient and interpret it according to:



Consider two points: (x_1, y_1) and (x_2, y_2)

The **distance** between them: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

The **midpoint**: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

The **gradient** of the line joining them is: $\frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$

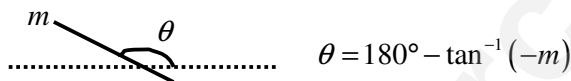
Remember that lines which slope **uphill** have positive gradients while lines which slope **downhill** have negative gradients!

Remember that **parallel** lines have equal gradients while **perpendicular** lines have gradients that are the negative reciprocal of one another! Eg. $\frac{2}{5}$ and $-\frac{5}{2}$ or 3 and $-\frac{1}{3}$.

Points which are **collinear** lie on the same straight line. The gradient calculated using any two of the points will be the same.

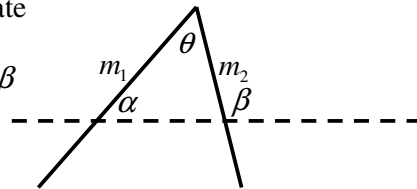
You need to be able to find the **equation of a straight line** $y = mx + c$

Remember also that the gradient is equal to the tan of the **angle of inclination**. The angle of inclination is measured anti-clockwise from the positive x -axis. Remember that lines which slope downhill have an angle of inclination greater than 90° .



One can work out angles using this in conjunction with other geometric facts. Consider the following example where we are asked to find θ if we know or can calculate the gradients of the two lines.

Then If we construct the dotted line we can calculate α and β using gradients. Then $\theta = \beta - \alpha$ (ext. $\angle$ of Δ)



Circles

$(x - a)^2 + (y - b)^2 = r^2$ is the equation of a circle, centre $(a; b)$ and radius r .

It is vital that we know how to **complete the square** to get a circle from this form:

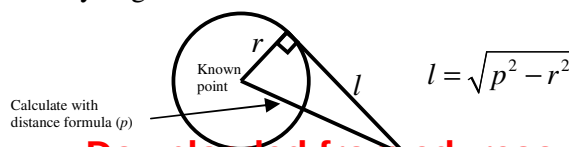
$x^2 - 8x + y^2 + 2y - 8 = 0$ to this form:

$(x - 4)^2 + (y + 1)^2 = 5^2$ which is a circle centre $(4; -1)$ radius 5

To find the **equation of a tangent to a circle** at a given point we use the fact that the tangent is perpendicular to the radius at the point of contact. We can typically find the gradient of the radius and then use this fact to find the gradient of the tangent.

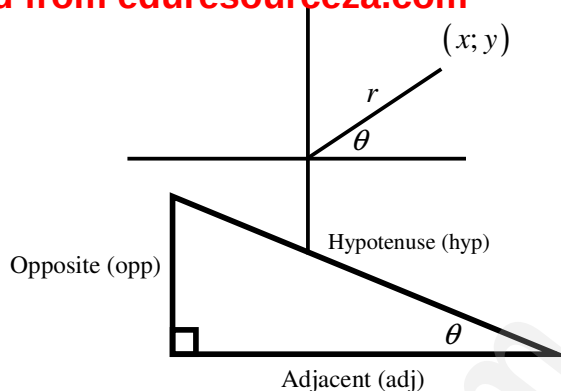
We can show whether **circles cut, touch or intersect** by considering the distance between their centers relative to the sum / difference of their radii!

We can also find the **length of a tangent** to a given circle from a given point without knowing the point of contact. We simply use Pythagoras!



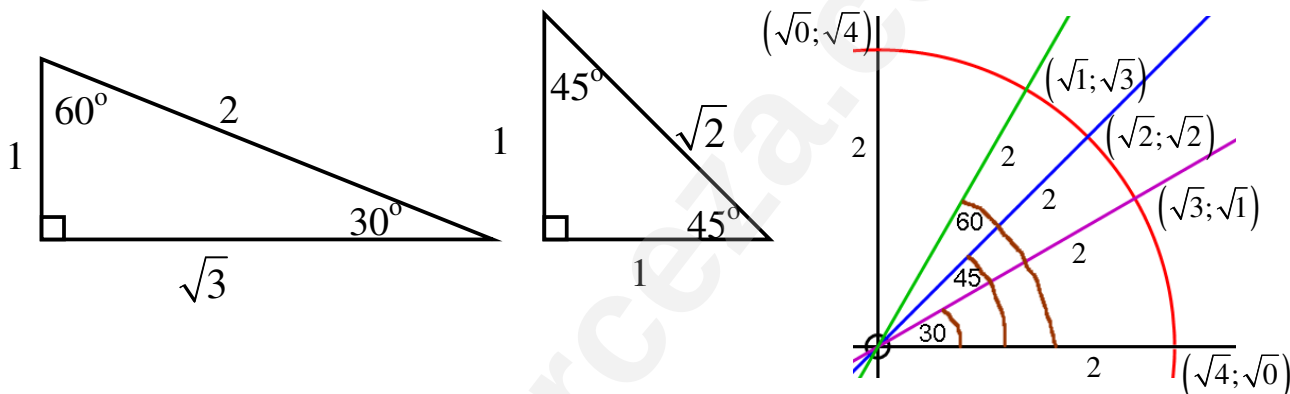
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$\sin \theta = \frac{y}{r}$	$\sin \theta = \frac{\text{opp}}{\text{hyp}}$
$\cos \theta = \frac{x}{r}$	$\cos \theta = \frac{\text{adj}}{\text{hyp}}$
$\tan \theta = \frac{y}{x}$	$\tan \theta = \frac{\text{opp}}{\text{adj}}$



Calculating ratios by means of special triangles and reduction formulae

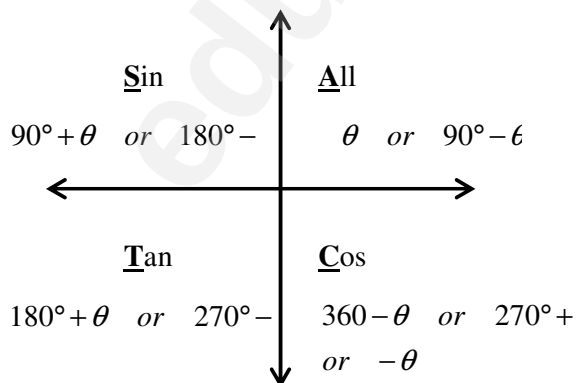
We can use the following two “special” triangles or the fan to calculate trig. ratios of the following angles: 30° ; 45° and 60° .



Note that we cannot use the triangles to calculate ratios of 90° (or any multiple of 90°). We can either do these on a calculator or by visualizing the graph or by visualizing the angle in the Cartesian plane with a chosen value for r and calculated values for x and y . For example, if we are talking about an angle of 180° with a radius of 8 cm then our x -value will be -8 and our y -value will be zero. Providing we know our x, y and r definitions we can then calculate any trig. ratio of 180° .

If we are dealing with an angle greater than 90° then there is a good chance that we can **reduce** it to a special angle using our reduction formulae.

Reduction formulae



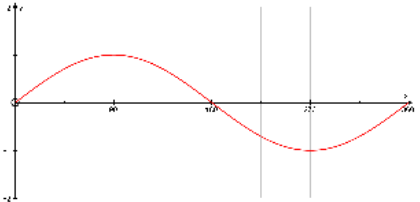
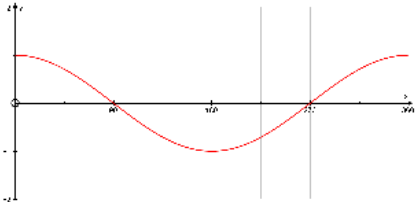
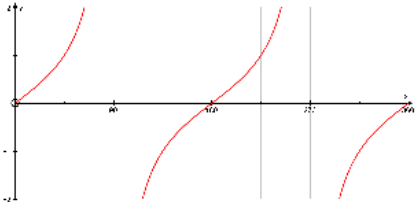
Remember that when we move off the **horizontal axis**
 $0^\circ \pm \theta$, $180^\circ \pm \theta$ or $360^\circ \pm \theta$

where θ is a positive, acute angle then **the ratio name stays the same** but the sign is obtained from the CAST diagram.

When we move off the **vertical axis**
 $90^\circ \pm \theta$, $270^\circ \pm \theta$

where θ is a positive, acute angle then the ratio name changes to the **co-ratio** and the sign is again obtained from the CAST diagram.

The co-ratio of sin is **cos**(ine) and the co-ratio of cos is **sin**.

Graph	How it looks	Period	Amplitude	Domain	Range
$y = \sin x$		360°	1	eg. $[0^\circ, 360^\circ]$ or $-360^\circ \leq x \leq 720$	$-1 \leq y \leq 1$ or $y \in [-1, 1]$
$y = \cos x$		360°	1	eg. $[0^\circ, 360^\circ]$ or $-360^\circ \leq x \leq 720$	$-1 \leq y \leq 1$ or $y \in [-1, 1]$
$y = \tan x$		180°	undefined	eg. $[0^\circ, 360^\circ]$ or $-360^\circ \leq x \leq 720$	undefined

The **period** is the number of degrees that the graph takes to complete a cycle.

The **amplitude** is the furthest the curve gets from its mean position or half the height of the wave. A definition on the Net reads: *"The magnitude of the displacement of a wave from a mean value."*

Amplitude and period are represented by a single number while domain (input) and range (output) are represented using interval notation or inequality notation.

You need to be able to draw all of them, showing intercepts with the axes as well as any turning points.

Remember that when you draw a tan graph you should always show the point $(45^\circ, 1)$ to give some idea of the scale on the vertical axis. You should also indicate the asymptotes by means of dotted vertical lines.

A change of period

In general the curve $y = \sin a\theta$ does a waves per 360° and has a period of $\frac{360^\circ}{a}$.

A change of amplitude

In general if we draw $y = a \sin x$ the amplitude changes to a .

A shift up or down

In general if we draw the graph of $y = \sin x + a$ then the graph is moved up a units – down if $a < 0$.

A shift left or right

In general if we draw the graph of $y = \sin(x + a)$ then the graph is moved left a units – right if $a < 0$.

Identities

An identity is an equation which is equal or true for all values for which it is defined.

The following need to be learnt and then applied to proving other identities.

When we prove an identity we work with the two sides (left and right) **independently**. We manipulate either (or both) until they are equal.

The Squared Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

The Quotient Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

The Compound Angle Identities

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

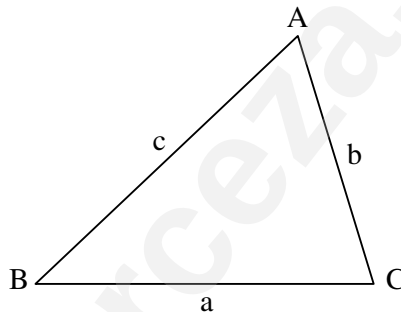
The Double Angle Identities

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = 2 \cos^2 x - 1$$

$$= 1 - 2 \sin^2 x$$

$$= \cos^2 x - \sin^2 x$$



Solving non right-angled triangles

Sine Rule	Cosine Rule
When we are given two sides and an angle or two angles and a side but, in either case, one of the sides must be opposite an angle The ambiguous case! When we are given two sides and an angle and the shorter of the two given sides is opposite the given angle then there are two possible triangles. You need to solve for both of them!	When we are given two sides and an included angle or three sides
$\frac{a}{\sin \hat{A}} = \frac{b}{\sin \hat{B}} = \frac{c}{\sin \hat{C}} \quad \text{or} \quad \frac{a}{\sin \hat{A}} = \frac{b}{\sin \hat{B}} = \frac{c}{\sin \hat{C}}$	$a^2 = b^2 + c^2 - 2bc \cos \hat{A} \quad \text{or}$ $\cos \hat{A} = \frac{b^2 + c^2 - a^2}{2bc}$
The Area Rule $\text{Area } \triangle ABC = \frac{1}{2} ab \sin \hat{C} = \frac{1}{2} ac \sin \hat{B} = \frac{1}{2} bc \sin \hat{A}$	

Remember that the biggest side will always be opposite the biggest angle, the smallest side opposite the smallest angle etc.

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Angles on a straight line add to 180° ($\angle$'s on a str. line)

The angles round a point add to 360° ($\angle$'s round a pt)

If lines parallel then the corresponding angles are equal (*corresp. $\angle$'s on $\parallel$ lines*)

If lines are parallel then the alternate angles are equal (*alt. $\angle$'s on $\parallel$ lines*)

If lines are parallel then the co-interior angles are supplementary (*co-int. $\angle$'s on $\parallel$ lines*)

If corresponding angles are equal then the lines are parallel

If alternate angles are equal then the lines are parallel

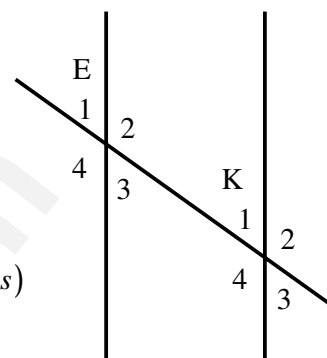
The angles of a triangle add to 180° ($\angle$'s of Δ)

The exterior angle of a triangle is equal to the sum of the opposite interior angles (*ext. $\angle$ of Δ*)

If co-interior angles are supplementary then the lines are parallel

An isosceles triangle has two equal sides and the angles opposite them equal.

An equilateral triangle has all three sides equal and each angle equal to 60°



$\hat{E}_1$ and $\hat{K}_1$ are corresponding

$\hat{E}_3$ and $\hat{K}_1$ are alternate

$\hat{E}_3$ and $\hat{K}_4$ are co-interior

Congruency

The four conditions for **congruency** are *SSS, SAS, RHS and AAS*

The symbol for congruency is $\equiv$

We always list congruent triangles in corresponding order.

If congruent triangles are properly listed then one can extract information as follows:

$$\Delta ABC \equiv \Delta GKR$$

then $\hat{BAC} = \hat{KGR}$, $KG = BA$ etc.

Similarity

One can prove **similarity** of triangles in one of two ways:

1. Prove them equiangular (*AAA*)
2. Prove that the corresponding sides are in proportion.

In fact both of these are necessary to prove shapes similar but, with triangles the one guarantees the other.

The symbol for similarity is $\parallel$

Shapes that are similar with sides in the ratio $a:b$ will have areas in the ratio $a^2:b^2$.

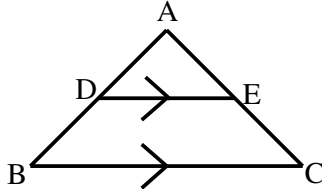
We always list similar triangles in corresponding order.

$$\triangle ABC \sim \triangle GKR$$

$$\text{then } \frac{AB}{BC} = \frac{GK}{KR} \text{ and } \frac{AC}{GR} = \frac{BC}{KR}$$

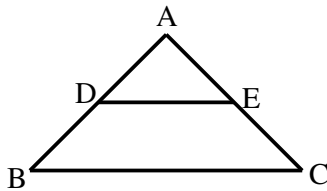
The proportional intercept theorem

A line drawn parallel to one side of a triangle cuts the other two sides in the same proportion (*Prop. int.*)



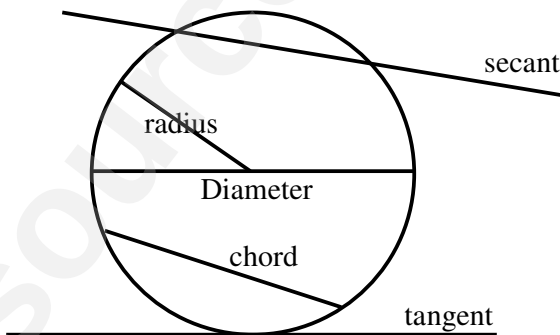
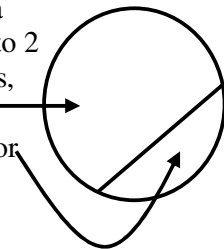
$$\text{If } DE \parallel BC \text{ then } \frac{AD}{DB} = \frac{AE}{EC} \text{ and } \frac{AD}{AB} = \frac{AE}{AC}$$

The converse is true. In other words if a line cuts two sides of a triangle in the same proportion then it will be parallel to the third side. (*Conv. Prop. int.*)

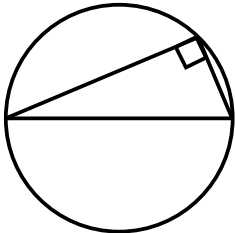
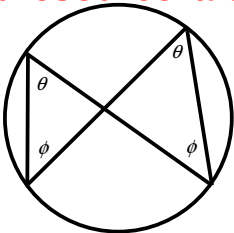
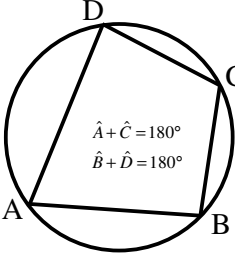
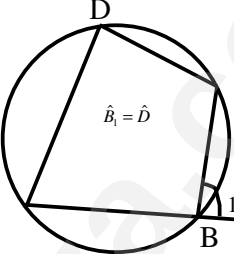
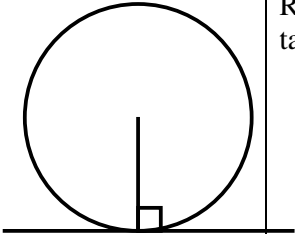
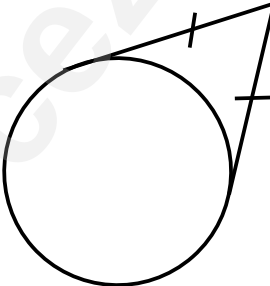
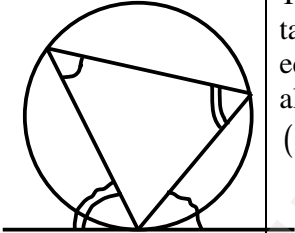
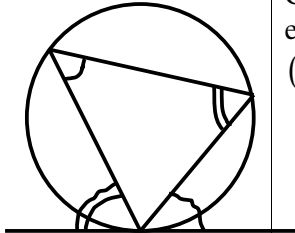


$$\text{If } \frac{AD}{DB} = \frac{AE}{EC} \text{ then } DE \parallel BC$$

A chord divides a circle into 2 segments, major and minor



	If a line is drawn from the centre of a circle perpendicular to a chord then it bisects the chord. ($\perp$ bisector of chord)		If a line is drawn from the centre of a circle to the mid-point of a chord then it will be perpendicular to the chord. ($\perp$ bisector of chord)
			The angle subtended at the centre of a circle is double the angle subtended at the circumference. ($\angle$ at center)

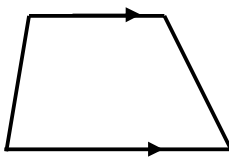
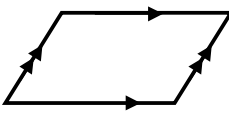
	A diameter subtends an angle of 90° at the circumference. ($\angle$ in semi-$\odot$)		Angles subtended in the same segment of a circle are equal. ($\angle$'s in same segment) The converse is true! If a line joining two points subtends equal angles at two other points on the same side of it then all 4 points are concyclic – i.e. they are the corners of a cyclic quad. (conv. $\angle$'s same seg.)
	The opposite angles of a cyclic quadrilateral are supplementary. (opp. $\angle$'s cyc. quad.) Converse is true! i.e. if opp. Angles supplementary then cyclic (conv. opp. $\angle$'s cyc. quad.)		The exterior angle of a cyclic quad is equal to the opposite interior angle. (ext. $\angle$ cyc. quad.) Converse is true! i.e. if ext. angle equals opp interior angle then cyclic (conv. ext. $\angle$ cyc. quad.)
	Radius is perpendicular to tangent.		Tangents from a common point are equal in length (tangents from common pt.)
	The angle between a tangent and a chord is equal to the angle in the alternate segment (tan - chord)		Converse is true. If angles equal then a tangent! (converse tan - chord)

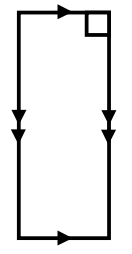
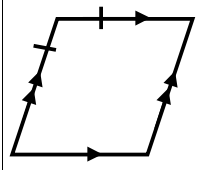
One can prove a quadrilateral is cyclic by proving:

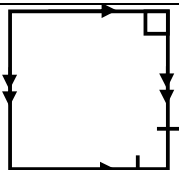
- Opposite angles supplementary (opp. $\angle$'s supplementary)
- Exterior angle equal to the opposite interior angle (ext. $\angle$ = opp. int. $\angle$)
- Converse angles in same segment. (conv. $\angle$'s same seg.)

One can prove a tangent using the converse of the tan-chord theorem.

Note that this table is in a hierarchy. Each quadrilateral is a special case of the ones above it so only the new properties are listed. For example, a square has all of the properties of the shapes listed above it.

Picture	Name	How to prove it	Properties
	Trapezium	One pair of parallel sides	Co-interior angles are supplementary
	Parallelogram (parm)	Two pairs of parallel sides OR Opposite sides equal OR One pair of equal and parallel sides OR Opposite angles equal OR diagonals bisect one another	Opposite sides equal Opposite angles equal Diagonals bisect one another

Picture	Name	How to prove it	Properties
	Rectangle	A parm with a 90° angle OR Diagonals equal in length	Diagonals are equal in length
	Rhombus	A parm with adjacent sides equal OR Diagonals bisect at 90°	Diagonals bisect at 90° Diagonals bisect the angles

Picture	Name	How to prove it	Properties
	Square	A rhombus with a 90° angle OR A rectangle with adjacent sides equal	All of the above!

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