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Matric Security Blanket for Paper 1

Sequences

Suppose that we have a sequence $T_1 ; T_2 ; T_3 ; .....$

The first term is also known as a. n is the term number and $n \in \mathbb{N}$.

If $T_2 - T_1 = T_3 - T_2$ then we have an **arithmetic sequence**. The **first differences** are common and we call this the common difference, d.

If $\frac{T_2}{T_1} = \frac{T_3}{T_2}$ then we have a **geometric sequence**. The **common ratio** is called r.

The following formulae apply to arithmetic and geometric sequences:

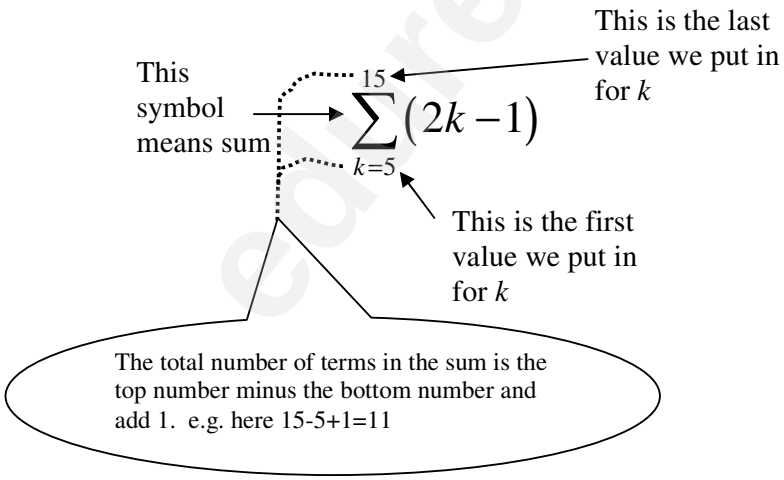
	Term	Sum
Arithmetic	$T_n = a + (n-1)d$	$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}\left[a + T_n\right]$
Geometric	$T_n = ar^{n-1}$	$S_n = \frac{a(r^n - 1)}{r-1} = \frac{a(1-r^n)}{1-r}$ If $-1 < r < 1$ *then* S_n *converges and* $S_\infty = \frac{a}{1-r}$

If the second differences are common then the n^{th} term will be of the form $T_n = an^2 + bn + c$ and $a = \frac{2^{nd}\ difference}{2}$. We can work out T_0 (the term before the first term) to get c. Then we can then solve for b by substituting a term for T_n and its position for n.

Alternatively we can use the following formula directly:

$T_n = T_1 + (n-1)f + \frac{(n-1)(n-2)}{2}s$ where f is the 1st term of the 1st difference and s is the 2nd difference

Sigma notation



The standard strategy with sigma notation is to write out three terms in order to establish whether we are dealing with an arithmetic progression or a geometric progression.

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Financial Maths

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Appreciation occurs when something gains value:

$A = P(1+in)$ simple interest, only really used in calculating interest for HP agreements

$A = P(1+i)^n$ compound interest

Inflation is always calculated with the compound interest formula and is calculated annually.

Depreciation occurs when something loses value:

$A = P(1-in)$ straight line depreciation

$A = P(1-i)^n$ reducing balance depreciation

Reducing Balance depreciation is always calculated annually.

For compound interest, i refers to the interest rate per compounding period and n to the number of compounding periods.

Nominal and effective rates

12% p.a. compounded quarterly is called a nominal rate. It can be written in symbols as: $i^{(4)} = 12\%$ and it means that you will receive 12% interest over the year but your interest will be calculated 4 times per year, 3% each time. The result will be higher than 12% and is called the effective rate.

The easiest way to convert between nominal rates and effective rates is to consider investing R1 for one year:

$$1\left(1+\frac{i_{eff}}{1}\right)^1 = 1\left(1+\frac{i_{nom}}{m}\right)^m$$

Time lines are useful when there are multiple withdrawals, deposits or changing interest rates.

Annuities

An annuity is a series of regular (almost always monthly), fixed payments for the purposes of investing or repaying a loan.

One can calculate the Future value of an annuity (investment) or the Present vale of an annuity (loan) by writing down a few terms and then using the sum formula for a geometric progression.

Alternatively you can make use of the following two formulae where x is the regular payment.

$$F_v = x\left[\frac{(1+i)^n - 1}{i}\right] \qquad P_v = x\left[\frac{1-(1+i)^{-n}}{i}\right]$$

A **sinking fund** is a fund which is used to invest money (normally by means of an annuity) for the replacement of capital equipment.

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One can work out the **outstanding balance** of a loan at any stage by:

Calculating the value of the loan with interest less the value of all the payments made up to that point with interest	OR	Calculating the present value of all the outstanding payments

Since the method on the right requires only one calculation it is preferred.

An **amortization schedule** is a schedule that shows the reducing balance on a loan. Providing one knows the interest rate, the repayment and the outstanding balance at a given time, it is a simple matter to generate a few rows of the table. A portion of each payment goes to paying off interest and the balance capital. Initially the major share goes to interest but the portion paying off capital gradually increases.

For example, suppose a loan of R100 000 is taken out over a twenty year period with monthly repayments, interest charged at 12% per annum (compounded monthly). Then it can be shown that the monthly repayment will be R1 101.09. It can also be shown that the outstanding balance after 60 payments will be R91 744.33. The next three rows of the amortization schedule are shown:

Payment number	Balance immediately after payment	Interest charged	Balance immediately before next payment	Percentage of payment going to interest	Percentage of payment going to reducing capital
60	91 744.33	$\frac{0.12}{12}\times 91\,744.33$ $=917.44$	$91744.33+917.44$ $=9\,2661.77$	$\frac{917.44}{1101.09}\times 100$ $=83.3\%$	$100-83.3$ $=16.7\%$
61	$92\,661.77-1101.09$ $=91\,560.69$	etc.	etc.	etc.	etc.

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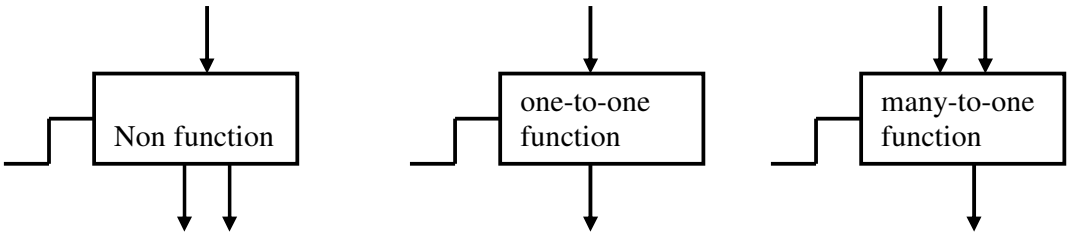
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Functions

A **function** is a rule such that each input (x) uniquely defines an output (y).



It is possible to pass a vertical line over the graph of a function without it ever touching the graph at more than one point at a time.

The **domain** of a function is the set of permissible inputs. We usually write, $Domain: x \in \mathbb{R}$, followed by any restrictions that may exist.
The **range** of a function is the set of possible outputs. We usually write, $Range: y \in \mathbb{R}$, followed by any restrictions that may exist.

Things to look out for when considering domain and range are:

- Fractions: cannot have zero on the denominator and cannot be zero unless the numerator can be zero.
- Squares: cannot be negative.
- Square roots, cannot square root a negative number

An **asymptote** is a line which a function approaches without ever touching.

A function whose **outputs** are **increasing** as the inputs increase is said to be **increasing**. The graph would slope uphill from left to right.
A function whose **outputs** are **decreasing** as the inputs increase is said to be **decreasing**. The graph would slope downhill from left to right.
A **stationary point** is a point where the graph is neither increasing nor decreasing.

The following steps are useful in sketching ANY function:

1. Determine the $y-intercept$ by letting $x=0$
2. Determine the $x-intercept(s)$ by letting $y=0$
3. Determine the $x-coordinates$ of any stationary points by letting $\frac{dy}{dx}=0$

The **inverse** of a function is the function that "undoes" the function. We use the notation f^{-1} to denote the inverse of the function f. If the function is one-to-one then the inverse will be one-to-one and will also be a function.

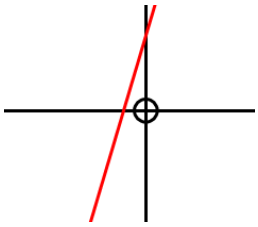
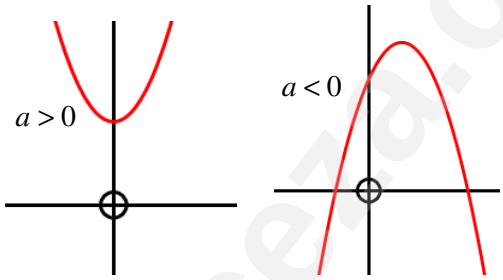
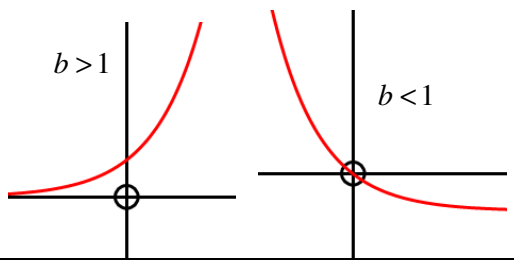
However, if the function is many-to-one then the inverse will be one-to-many and will not be a function. This can be overcome by restricting the domain on the original function so as to make it one-to-one.

We determine the inverse of a function as follows:

- Interchange *x* and *y*.
- Write the function in the form $y=....$ then the function can be written as $f^{-1}(x)=....$ where the right hand side of each is the same.

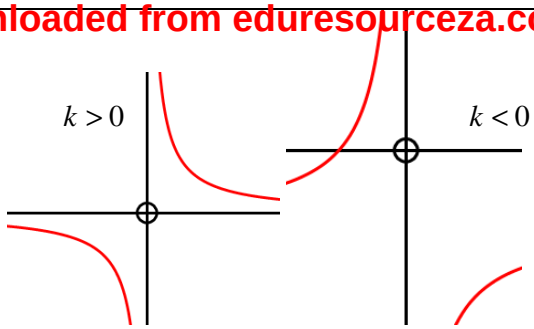
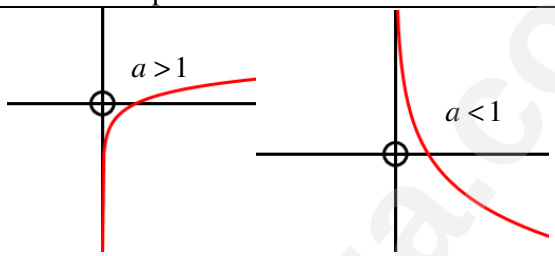
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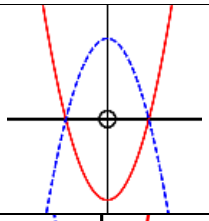
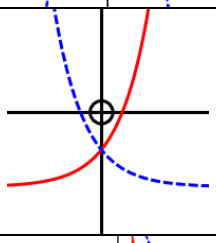
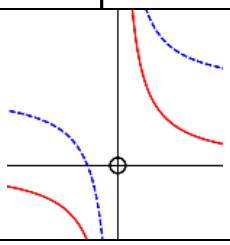
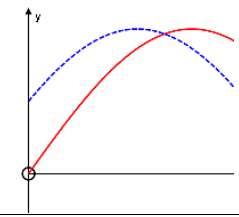
Name	Equation	Picture	Finding the equation
Linear	$y = mx + c$	m tells us the gradient and c the y–intercept. Remember $y = k$ is a horizontal line and $x = k$ is a vertical line. 	Two methods: Find m, the slope. Substitute m and a point into the equation $y = mx + c$ and solve for c. OR Substitute 2 different points into the equation $y = mx + c$ and solve simultaneously for m and c.
Quadratic Function	$y = ax^2 + bx + c$		Given two x– intercepts $(\alpha \text{ and } \beta)$ and another point: Substitute $(\alpha \text{ and } \beta)$ and other point into equation $y = a(x-\alpha)(x-\beta)$ and solve for a. Given the turning point $(p;q)$ and another point. Substitute p and q and other point into equation $y = a(x-p)^2 + q$ and solve for a. In either case you can simply substitute three points into the equation $y = ax^2 + bx + c$ and solve simultaneously for a, b and c.
Exponential Function	$y = ab^x + c$ By definition $b > 0$ *and* $b \neq 1$.	The line $y = c$ will be a horizontal asymptote. 	First fill in c by considering the position of the asymptote. Then substitute the y–intercept to find a. Finally, substitute another point on the curve to find b.

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Hyperbolic Function	$y = \frac{k}{x+a} + b$	 Easiest to draw the graph and then move the axes to compensate for a and b.	First put in a and b according to the position of the asymptotes. Finally substitute in another point to calculate k.
Logarithmic Function	$y = \log_a x$ By definition $a > 0$ *and* $a \neq 1$ Domain: $x > 0$		

The following transformations of functions are important:

Putting a –'ve in front of y reflects the graph in the x–axis	$y = -f(x)$	
Putting a –'ve in front of every x reflects the graph in the y–axis	$y = f(-x)$	
Adding a number to y after calculating the function moves the graph up (positive) or down (negative)	$y = f(x) + a$	
Adding a number to x before calculating the function moves the graph left (positive) or right (negative)	$y = f(x+a)$	

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Logarithms

$\log_a x$ is only defined for $a > 0, a \neq 1$ *and* $x > 0$.

$\log_a x$ is the power we must put onto a in order to get x.

If the base is omitted it is understood to be 10.

$\log_c c = 1$ *and* $\log_c 1 = 0$

Log Laws

1. $\log_a b + \log_a c = \log_a bc$
2. $\log_a b - \log_a c = \log_a \frac{b}{c}$
3. $\log_a b^c = c \log_a b$
4. $\log_a b = \frac{\log_c b}{\log_c a}$ Note that: $\log_a b = \frac{1}{\log_b a}$

It is very important to be able to change from log form to exponent form and back:

$\log_a b = c \Leftrightarrow a^c = b$

We solve a log equation by using log laws to get a single log on each side (with the same base). Then we can "drop the logs" since the log function is "one-to-one".

$\log_a b = \log_a c \Leftrightarrow b = c$

We must always remember to check the solutions to a log equation! Remember we cannot find the log of a negative number!

A number, k can always be written in the form of a log of any base: e.g. $k = k.1 = k.\log_c c = \log_c c^k$.

Exponents and Surds

$a^b \times a^c = a^{b+c}$ $\quad a^b \div a^c = \frac{a^b}{a^c} = a^{b-c}$ $\quad (ab)^c = a^c b^c$ $\quad a^0 = 1$

$\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$ $\quad (a^b)^c = a^{bc}$ $\quad a^{\frac{b}{c}} = \sqrt[c]{a^b}$ $\quad a^{-b} = \frac{1}{a^b}$

Many problems require us to get the same bases by first prime factorizing our bases: e.g.

$75^x = (3 \times 5^2)^x = 3^x 5^{2x}$ We are generally expected to know the first few powers of the first few numbers!

2; 4; 8; 16; 32; 64; 128; 256 *and* 3; 9; 27; 81 *and* 4; 16; 64 *and* 5; 25; 125

We are expected to give answers with positive exponents, making use of the rule: $a^{-b} = \frac{1}{a^b}$ e.g.

$5x^{-2}y^3 3^{-2} = \frac{5y^3}{3^2 x^2}$ *or* $\frac{5y^3}{9x^2}$

We are expected to give answers with rational denominators. Consider the following two "tricks":

$\frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$ *and* $\frac{1}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}} = \frac{3-\sqrt{2}}{7}$

Since the exponential function is a one-to-one function, if we can manipulate an equation to have one factor and one term with the same base on each side then we can equate the exponents: e.g.

$2^{2x+1} = 2^{3x} \Leftrightarrow 2x + 1 = 3x \therefore x = 1$

Equations of the form $2^x = 7$ can be solved by changing into log form: $x = \log_2 7$

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Calculus

The derivative of a function gives its gradient!
The process of finding a derivative is called differentiation.
The process of differentiation by first principles is important.

Remember $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Remember to have limit on every line but the last.
Remember = sign goes on left of limit!
Remember to use brackets when you no longer have a fraction

The following notations are important:

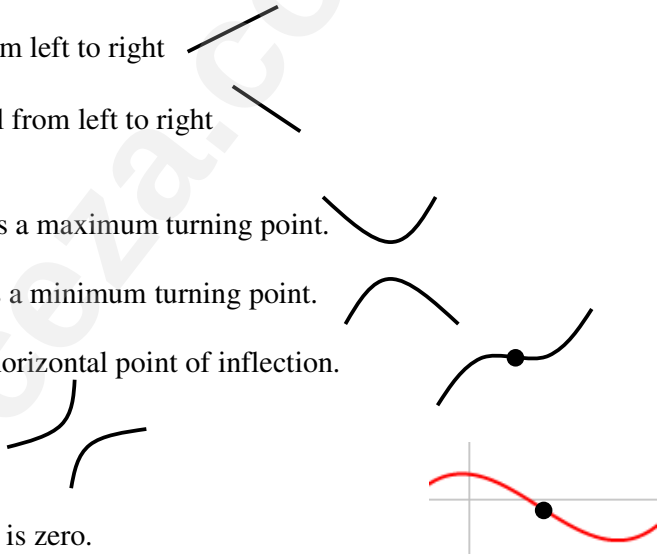
if $y = \ldots$ *then* $\frac{dy}{dx}$ *represents the first derivative and* $\frac{d^2y}{dx^2}$ *the second derivative*

if $f(x) = \ldots$ *then* $f'(x)$ *represents the first derivative and* $f''(x)$ *the second derivative*

$D_x(3x^2 + 2)$ *means the derivative of* $3x^2 + 2$ *with respect to* x

If the first derivative is:

- positive the graph is increasing, going uphill from left to right
- negative the graph is decreasing, going downhill from left to right
- zero then the graph is stationary
  - if the second derivative is negative this is a maximum turning point.
  - if the second derivative is positive this is a minimum turning point.
  - if the second derivative is zero this is a horizontal point of inflection.



If the gradient is increasing the graph is concave up.
If the gradient is decreasing the graph is concave down.

A point of inflection occurs when the second derivative is zero.
At this point the graph changes concavity. The gradient goes from increasing to decreasing or vice versa.

Differentiating displacement gives velocity and differentiating velocity gives acceleration.
To optimize (make as big / small as possible) a quantity we need to get an expression for it involving only one variable. Then we need to differentiate with respect to that variable, equate to zero and solve.

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Equations

To solve an equation means to find the value(s) of the variable(s) which make it true.

Quadratic Equations: $ax^2+bx+c=0$. Solve by:

- Factorising OR
- Completing the Square OR
- Using the quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Remember that if you have a square root in an equation then you can eliminate it by getting it on one side on its own and then squaring both sides of the equation. However, remember to square properly! $(a+b)^2=a^2+2ab+b^2$ *NOT* a^2+b^2 !!! Also, by doing this you may have introduced an extraneous, incorrect solution so you need to remember to CHECK both your answers in the original equation!

Cubic Equations: $ax^3+bx^2+cx+d=0$. Solve by:

- Determining one solution by trial and error. Say this solution is α then we know that $(x-\alpha)$ is a factor.
- Determine the other (quadratic) factor by long division or inspection.
- Then factorise the quadratic factor or use the quadratic formula.

Equations where the base is unknown. Suppose we have $x^{\frac{a}{b}}=g$ then we can raise both sides to the power $\dfrac{b}{a}$ to "get rid" of the power on x. This gives $x=g^{\frac{b}{a}}$

Simultaneous Equations

If you have two equations in two variables then you have two choices:

1. **By Substitution:** chose one of the equations and make one variable the subject of the formula, $x=....or\ y=...$ Then substitute this result for every occurrence of that variable in the other equation, giving you one equation in one unknown. Solve for that unknown. Finally substitute your answer into either equation to solve for the other unknown.

   OR

2. **By Elimination:** If either of your variables has the same size coefficient in both equations then you can eliminate that variable by adding the equations (if the coefficients have different signs) or subtracting the equations (if the coefficients have the same signs). This will give you one equation in one unknown. Solve for that unknown. Finally substitute your answer into either equation to solve for the other unknown.

Remember that you can check the solution to ANY equation by substituting it in!

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Probability

The **probability** of an event is a measure of the likelihood of it happening.

If the event is E then the probability of the event happening is denoted by $P(E)$.

$0 \le P(E) \le 1$

The **sample space** is all possible outcomes.

The probability of an event happening is calculated by dividing the number of ways the event can happen by the total number of possible outcomes.

The probability of the event not happening is denoted by $P(\bar{E})$. $\bar{E}$ is called the complement of E.

$P(E) + P(\bar{E}) = 1$

The complement of "**at least one**" is none!

It two events are **independent** it means that the happening of one does not affect the probability of the other happening.

The multiplication law for independent events:

The probability that A **and** B happen is denoted by $P(A \cap B)$ and $P(A \cap B) = P(A).P(B)$.

If two events cannot both happen then they are said to be **mutually exclusive**.

The addition law for mutually exclusive events:

The probability that A or B happens is denoted by $P(A \cup B)$ and $P(A \cup B) = P(A) + P(B)$.

If two events can both happen then they are said to be **non-mutually exclusive**.

The addition law for non-mutually exclusive events:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Counting Techniques

The number of ways we can arrange n **distinct** objects is $n!$.

If we have n objects of which a are identical to one another and b are identical to one another and c are identical to one another then we can arrange them in $\frac{n!}{a!b!c!}$ different ways.

The number of different ways we can take r objects from n objects if we replace them is n^r.

The number of ways we can take r objects from n objects if we don't replace them is $\frac{n!}{(n-r)!}$.

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Contingency Tables

If one has a table with two groups and two criteria then we can test for dependence / independence of the criteria:

FOR INDPENDENCE:
$P(A \text{ and } B) \approx P(A) \times P(B)$

If the two values are sufficiently close to each other, the events are independent.

For example

	Younger than 20	**20 yrs or older**	**Totals**
Support political candidate	40	70	110
Do not support candidate	10	80	90
Totals	50	150	200

Since

$$P(\text{Younger than 20 and supporter}) = \frac{40}{200} = 0,2$$

$$P(\text{Younger than 20}) \times P(\text{Supporter}) = \frac{50}{200} \times \frac{110}{200} = 0,137$$

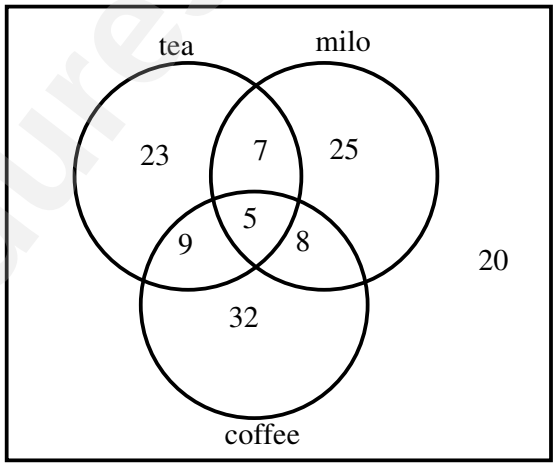
So, as $P(\text{Younger than 20 and supporter}) \neq P(\text{Younger than 20}) \times P(\text{Supporter})$

being a supporter is not independent of age being above or below 20 yrs. (i.e. It is dependent on it)

Venn Diagrams

They typically involve 3 "dimensions" e.g. 3 different drinks that people may drink. Drawing them is a bit of a logic exercise – start by filling in the middle figure!

In this case the format looks like this:



$$P(\textit{only coffee}) = \frac{32}{130}$$

$$P(\textit{coffee but not milo}) = \frac{41}{130}$$

$$P(\textit{tea or coffee}) = \frac{85}{130}$$

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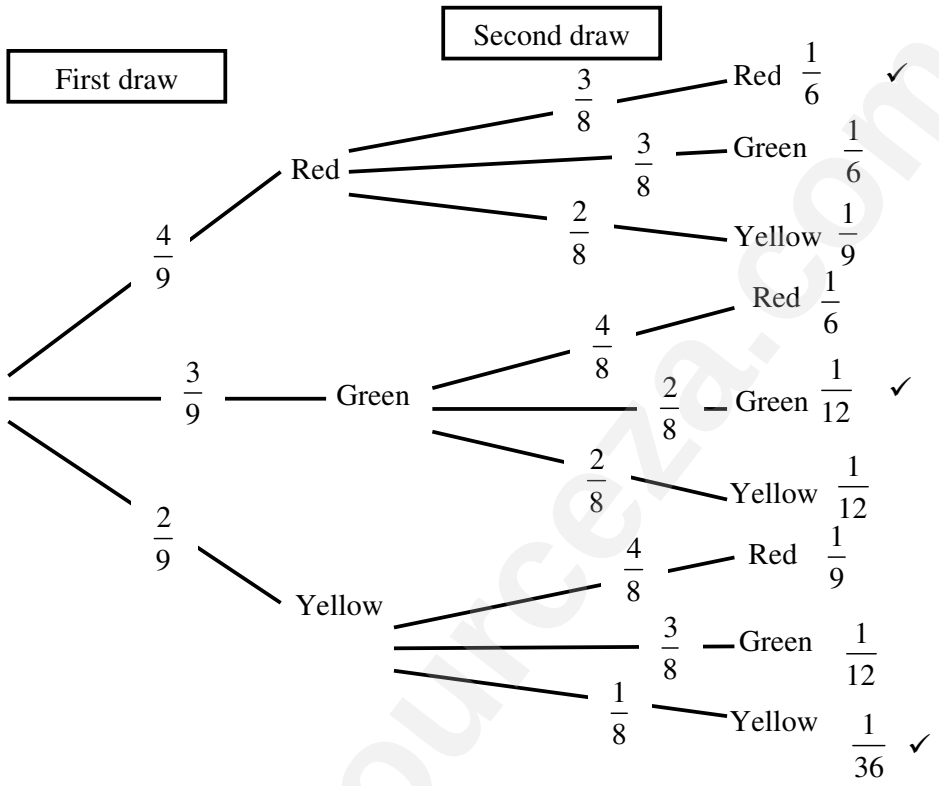
Probability Trees

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Again, best explained by means of an example:

Suppose I have a bag containing 4 red beads, 3 green beads and 2 yellow beads.
I draw out a bead, note its colour and then draw out another bead without replacing the first one:

The following tree depicts the situation with the probabilities on each of the branches:



Note that at each stage the probabilities of all the choices must add to 1!

If we want the probability of the combination of events at the end of a branch we simply multiply the probabilities

en route! e.g $P(\textit{red then green}) = \frac{4}{9} \times \frac{3}{8} = \frac{1}{6}$

To calculate composite probabilities we simply add. For example, what is the probability of getting two of the same colour? I have ticked the relevant probabilities:

$$P(2\ \textit{of the same colour}) = \frac{1}{6} + \frac{1}{12} + \frac{1}{36} = \frac{10}{36}$$

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