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MATHEMATICS GRADE 12

LAST PUSH 2019



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"You have to believe in yourself, challenge yourself, and push yourself until the very end; that's the only way you'll succeed." – G-Dragon



TABLE of (mal) CONTENTS

1. ALGEBRA, EQUATIONS AND INEQUALITIES.....	3
2. SEQUENCES AND SERIES.....	5
3. FUNCTIONS AND INVERSES.....	9
4. FINANCE, GROWTH AND DECAY.....	18
5. DIFFERENTIAL CALCULUS.....	22
6. PROBABILITY.....	26
7. STATISTICS.....	30
8. ANALYTICAL GEOMETRY.....	41
9. TRIGONOMETRY.....	52
10. EUCLIDEAN GEOMETRY.....	62

TO DO LIST:

- ✓ Basic quadratic equations and inequalities
- ✓ Using the quadratic formula.
- ✓ Surd equations, **NB:** Remember to square both sides and check the solution(s) obtained!
- ✓ Practise a lot of inequalities! Use number line and parabola to write solution. *Usually* for *greater than* ($>$), there are two solutions, $x <$ smaller value of the two or $x >$ larger value. For smaller than ($<$), the solution is usually, smaller value $< x <$ bigger value.
- ✓ The substitution/ K method! Also look into the Laws of exponents!
- ✓ Exponential equations! Look for common factor in most cases.
- ✓ Simultaneous equations, you just need to be careful of your substitutions!
- ✓ NATURE OF ROOTS! Bathong! Asseblief go through this thoroughly! Most of the non-routine questions are generated from this.

CHECK YOURSELF1.1 Solve for x

$$1.1.1 \quad (2x-1)(5+x)=0 \quad (2)$$

$$1.1.2 \quad -3x^2 + 7x - 1 = 0 \quad (\text{correct to TWO decimal places}) \quad (3)$$

$$1.1.3 \quad 1 - 2\sqrt{2x-1} = 12 - x \quad (6)$$

$$1.1.4 \quad x(x-9)+14 > 0 \quad (3)$$

1.2 Given: $2^{x+1} + 2^x = 3^{y+2} - 3^y$ Solve for x and y , where x and y are integers. (4)

1.3 Simplify WITHOUT the use of a calculator:

$$\frac{1}{(a+b)^{-1}} - \left(a^{\frac{1}{2}} - b^{\frac{1}{2}} \right)^2 \quad (4)$$

1.4 Prove that $k = 2$ is the only root of the equation $k - 1 + \frac{1}{k-3} = 0$ (3)**[25]**

Here are some of the questions that have been a bit of a challenge from past papers:



2.1 Solve for x

2.1.1 $2 \cdot 3^x = 81 - 3^x$ (4)

2.1.2 $2^{2x} - 6 \cdot 2^x = 16$ (4)

2.1.3 $x(3 - x) = -3$ (correct to TWO decimal places). (4)

2.2 (a) Solve for p if $p(p + 5) = 14$ (4)

(b) Hence, or otherwise solve for x if $\sqrt{x+5} + 5 = \frac{14}{\sqrt{x+5}}$ (3)

2.3 Given $f(x) = x^2 + 8x + 16$

For which values of p will $f(x) = p$ have TWO unequal negative roots? (4)

2.4 Given $x = 999\,999\,999\,999$, determine the exact value of $\frac{x^2 - 4}{x - 2}$ (3)

2.5 The volume of a box with a rectangular base is $3\,072\text{ cm}^3$. The lengths of the sides are in the ratio $1 : 2 : 3$. Calculate the length of the shortest side. (4)

2.6 Simplify, WITHOUT the use of a calculator:

$$\left(\frac{\sqrt{y} + \sqrt{y^3}}{\sqrt{y}} - 1 \right)^2, \quad y > 0$$
 (4)

2.7 Determine for which values of x , $\frac{(x+1)^2}{x^2 + 3x - 4}$ will be undefined. (2)

2.8 If $2x^{-1} + x^{-2} = 3$ and $x < 0$, determine, WITHOUT the use of a calculator, the value of $(x^2 - 3x)^{-1}$. (4)
[36]

Are you happy with your performance from the above questions? Check with your peers or teacher for the questions you struggled with! Make it a point that you are fully comfortable with QUESTION 1 of your exam! it's worth +/- 25 marks of your paper!

There's more questions to practise from past papers!! Make sure you go through them

A very interesting and easy topic (not saying the others are boring and difficult)! Theory is very important apha! Sugowa! Just know the following:

✓ **Arithmetic sequences and series**

- Always remember that this is depicted by first differences, always think of the variables from the formula provided in the formula sheet!
- Application of the formulae! Proof of the series also examinable!

✓ **Geometric sequences and series**

- Think of common ratio all the time!
- Proof of geometric series is examinable! Apply the formulae.
- Sum to infinity! Only when series converges! The moment you see convergent or infinite, think common ratio and the formula.

✓ **Quadratic sequences**

✓ **Sigma notation:**

- think sum! Check what type of series by looking at the first three terms!
- Top minus bottom plus 1 gives you the number of terms

✓ $T_n = S_n - S_{n-1}$

✓ Simplification of exponents is key (especially for geometric sequence).

CHECK YOURSELF

1.1 The sequence $3 ; p ; 25$ is a quadratic sequence. The sequence of first differences is $9 ; q ; \dots$

1.1.1 Calculate p and q . (2)

1.1.2 Hence, or otherwise, determine the n th term of the sequence. (4)

1.2 A pendulum is set in motion. In the first swing, where both forward and backward motion is taken into account, the tip covers 72 cm. The tip covers 48 cm in the second swing and 32 cm in the third swing. It continues indefinitely in this way.

Determine the total distance covered by the tip. (3)

1.3 Explain why the series $-4 - \frac{8}{3} - \frac{16}{9} - \dots$ converges. (2)

1.4 Determine the first two terms of $\sum_{n=-1}^x 108 \times \left(\frac{2}{3}\right)^n$ (2)

[13]

QUESTION 2

- 2.1 The first three terms of an arithmetic sequence are $2c - 1$; $4c - 5$ and $3c + 3$
- 2.1.1 Determine the value of c . (2)
- 2.1.2 Write down the common difference of the sequence. (1)
- 2.1.3 Determine the 45th term of the sequence. (3)
- 2.2 The following information of a geometric pattern is given: $T_1 + T_2 = -1$ and $T_3 + T_4 = -4$
- Determine the numerical values of the first three terms of the pattern if $r > 0$. (6)
- [12]

QUESTION 3

Consider the sequence 298 ; 259 ; 222 ; 187 ; 154 ; 123 ;

The n th term for the sequence is given by $T_n = an^2 + bn + 339$.

- 3.1 Determine the values of a and b . (4)
- 3.2 Calculate the value of the 27th term of the sequence. (2)
- 3.3 Show that the sum of the first differences of the above sequence can be given by $S_n = n^2 - 40n$. (3)
- [9]

QUESTION 4

- 4.1 The first term of a geometric sequence is 15. If the second term is 10, calculate:
- 4.1.1 the common ratio of the sequence. (1)
- 4.1.2 S_9 (2)
- 4.2 Determine for which values of x will $\sum_{k=1}^{\infty} (4x - 1)^k$ exist. (3)
- 4.3 The first three terms of an arithmetic sequence are $2p + 3$; $p + 6$ and $p - 2$.
- 4.3.1 Show that $p = 11$. (2)
- 4.3.2 Calculate the smallest value of n for which $T_n < -55$. (3)

QUESTION 5

5.1 Given the arithmetic series: $18 + 24 + 30 + \dots + 300$

5.1.1 Determine the number of terms in this series. (2)

5.1.2 Calculate the sum of all the whole numbers up to and including 300 that are NOT divisible by 6. (4)

5.2 A quadratic pattern has a third term equal to 28. The general term of the first differences is given by $-6n + 5$. Determine the formula for the n th term. (4)

5.3 A water tank contains 216 litres of water at the end of day 1. Because of a leak, the tank loses one-sixth of the previous day's contents each day. How many litres of water will be in the tank by the end of:

5.3.1 the 2nd day? (2)

5.3.2 the 7th day? (3)

5.4 The second term of a geometric series is 24 and the eight term is 0,375. Determine the common ratio and the first term. (5)

5.5 6 ; $5 + x$; -6 and $6x$ are the first four terms of a quadratic sequence.

Calculate the value of x . (3)

5.6 Consider the sequence $\frac{1}{2}$; 4 ; $\frac{1}{4}$; 7 ; $\frac{1}{8}$; 10 ;

5.6.1 If the pattern continues in the same way, write down the next TWO terms in the sequence (2)

5.6.2 Calculate the sum of the first 50 terms of the sequence. (6)

5.7 Given the arithmetic series: $-7 - 3 + 1 + \dots + 173$. Write the series in sigma notation. (3)

5.8 Consider the arithmetic sequence: -8 ; -2 ; 4 ; 10 ; ...

Calculate the smallest value of n for which the sum of the first n terms of the sequence will be greater than 10 140. (5)

5.9 In a geometric series, the sum of the first n terms is given by $S_n = p \left(1 - \left(\frac{1}{2} \right)^n \right)$ and the sum to infinity of this series is 10.

5.9.1 Calculate the value of p . (4)

5.9.2 Calculate the second term of the series. (4)

[47]

QUESTION 6

6.1 The following series is a combination of an arithmetic and geometric series:

$$6 + 4 + 3 + 2 + \frac{3}{2} + 0 + \frac{3}{4} + \dots$$

6.1.1 Write down the next two terms of the series. (2)

6.1.2 Find the sum of the first 113 terms of the series. (5)

6.1.3 Give a reason why the geometric series converges. (1)

6.2 In an arithmetic sequence, the first term and the seventh terms are x^2 and $(x + 6 - 5x^2)$ respectively.

If the common difference is x , determine the possible values of x (4)

6.3 Consider: 0 ; -1 ; 1 ; x ; 14

6.3.1 Solve for x if the terms for a quadratic sequence. (3)

6.3.2 Calculate the general term of the sequence. (4)

6.4 Calculate p if $\sum_{n=1}^{\infty} 27p^k = \sum_{t=2}^5 (1-t^2)$ (6)

6.5 In a converging geometric series $T_2 = 9$ and $S_{\infty} = \frac{81}{2}$. Calculate the possible value(s) of the first term in the series (6)

6.6 The common difference of an arithmetic sequence is 4.

Prove that $\sqrt{S_{2n} - 2S_n} = 2n$ (5)

6.7 Given the quadratic sequence 3 ; 5 ; 11 ; 21 ;

Prove that the terms of this sequence will never consist of even numbers. (7)

6.8 A quadratic pattern has $T_1 = T_3 = 0$ and $T_4 = -3$.

6.8.1 Determine the value of the second difference of this pattern. (4)

6.8.2 Determine T_5 . (2)

6.9 In an arithmetic sequence, the eight term is twice the fourth term. The sum of the three terms is 12. Determine the first term of this sequence. (5)

6.10 Evaluate: $\sum_{k=-2}^{14} 3.2^{k-1}$ (3)

WHAT TO KNOW:

- ✓ Definition of a function, vertical line test.
- ✓ Domain and Range
- ✓ Inverse of a function
- ✓ How the DOMAIN of the function may need to be restricted (in order to obtain a one-to-one function).
- ✓ Intercepts with the axes.
- ✓ Turning points (stationary points)
- ✓ Minima and Maxima
- ✓ Asymptotes (Horizontal and vertical)
- ✓ Shape of the graph.
- ✓ Line of symmetry/ axes of symmetry.
- ✓ Interval on which function INCREASES/ DECREASES.

1. STRAIGHT LINE GRAPH

- Parallel lines have equal gradients whereas the product of two perpendicular lines equal -1 .
- Domain: $x \in R$ and Range $y \in R$.

2. THE PARABOLA

- For the standard form: $y = ax^2 + bx + c$:
 - ✓ **Turning point:** $x = -\frac{b}{2a}$, then substitute the value to the original graph.
 - ✓ Alternatively, calculate using the calculus method (**see calculus section**).
- For the turning point form: $y = a(x - p)^2 + q$:
 - ✓ **Turning point:** $(p; q)$
- For the intercept form: $y = a(x - x_1)(x - x_2)$, turning points are calculated the same way as the standard form or alternatively: $x = \frac{x_1 + x_2}{2}$ then substitute to the original equation.

	FOR $a < 0$	FOR $a > 0$
Domain	$x \in R$	
Range	$y \in (-\infty; q]$	$y \in [q; \infty)$

3. HYPERBOLA

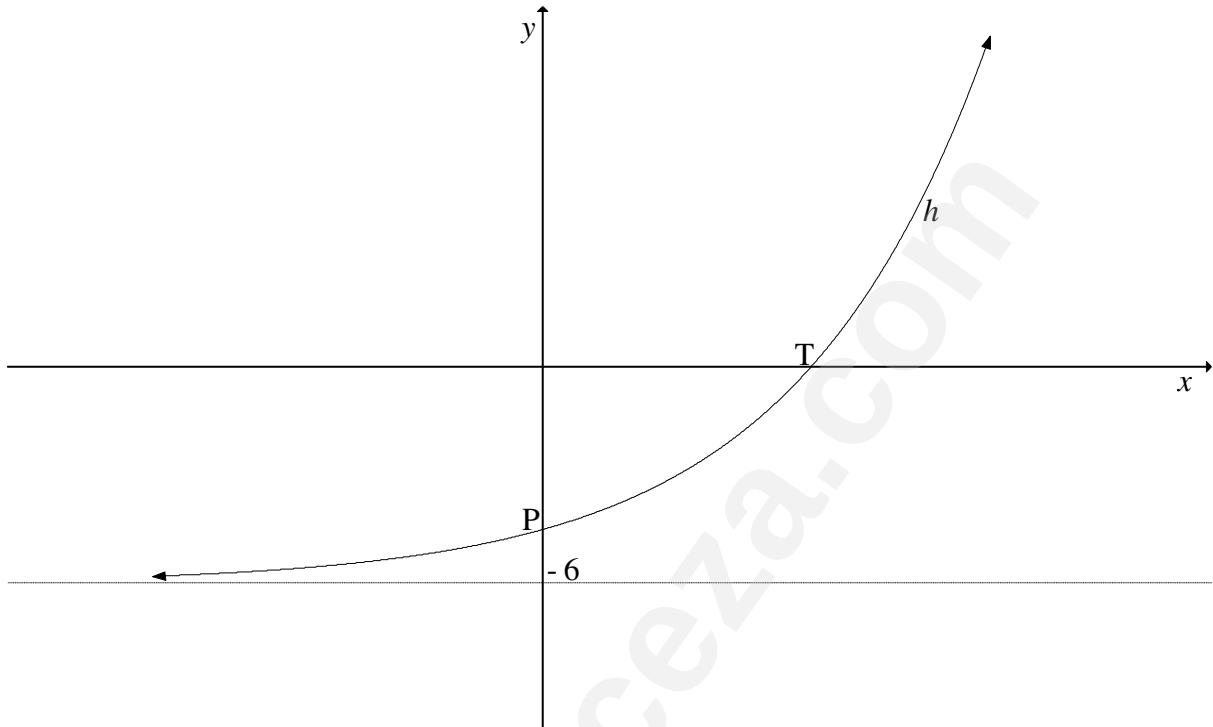
- Standard form: $y = \frac{a}{x - p} + q$
- Equations of asymptotes:
 - ✓ **Vertical asymptote:** $x = p$.
 - ✓ **Horizontal asymptote:** $y = q$
- **Axis of symmetry:** $y = x - p + q$ OR $y = -(x - p) + q$
- **Domain:** $x \in R, x \neq p$ and **Range:** $y \in R, y \neq q$.

4. LOGARITHMIC AND EXPONENTIAL FUNCTIONS

- Horizontal asymptote: $y = q$.
- **Domain:** $x \in R$ and **Range:** $y > q$.
- **For Inverse:**
 - ✓ Domain of the original function becomes the range of an inverse.
 - ✓ Range of the original functions becomes domain of the inverse.
 - ✓ Swap x and y . Then make y subject of the formula.
 - ✓ NB: There is no logarithmic of a negative number nor zero!!

QUESTION 1

Given: $h(x) = a \cdot 2^{x-1} + q$. The line $y = -6$ is an asymptote to the graph of h . P is the y -intercept of h and T is the x -intercept of h .



- 1.1 Write down the value of q . (1)
 - 1.2 If the graph of h passes through the point $\left(-1; -5\frac{1}{4}\right)$, calculate the value of a . (4)
 - 1.3 Calculate the area of $\triangle OPT$. (4)
- [9]

EXTRA

The function defined as $y = \frac{a}{x+p} + q$ has the following properties:

- The domain is $x \in R, x \neq -2$.
- $y = x + 6$ is an axis of symmetry.
- The function is increasing for all $x \in R, x \neq -2$.

Draw a neat sketch graph of this function. Your sketch must include the asymptotes, if any.

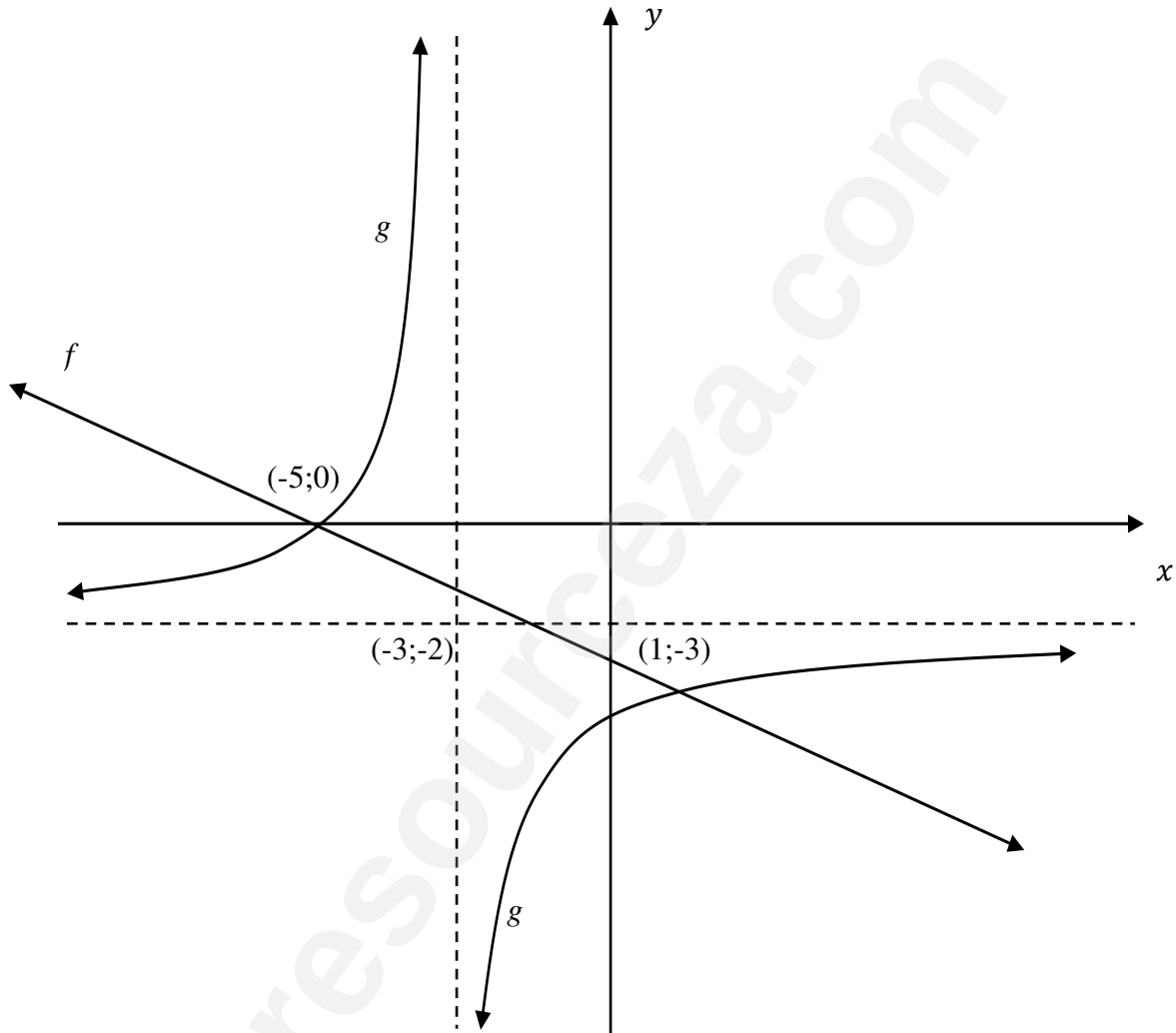
(4)

QUESTION 2

The graphs of $g(x) = \frac{a}{x-p} + q$ and the straight line f are sketched below.

The asymptotes of g intersect at $(-3; -2)$.

The points of intersection of f and g are $(-5; 0)$ and $(1; -3)$.



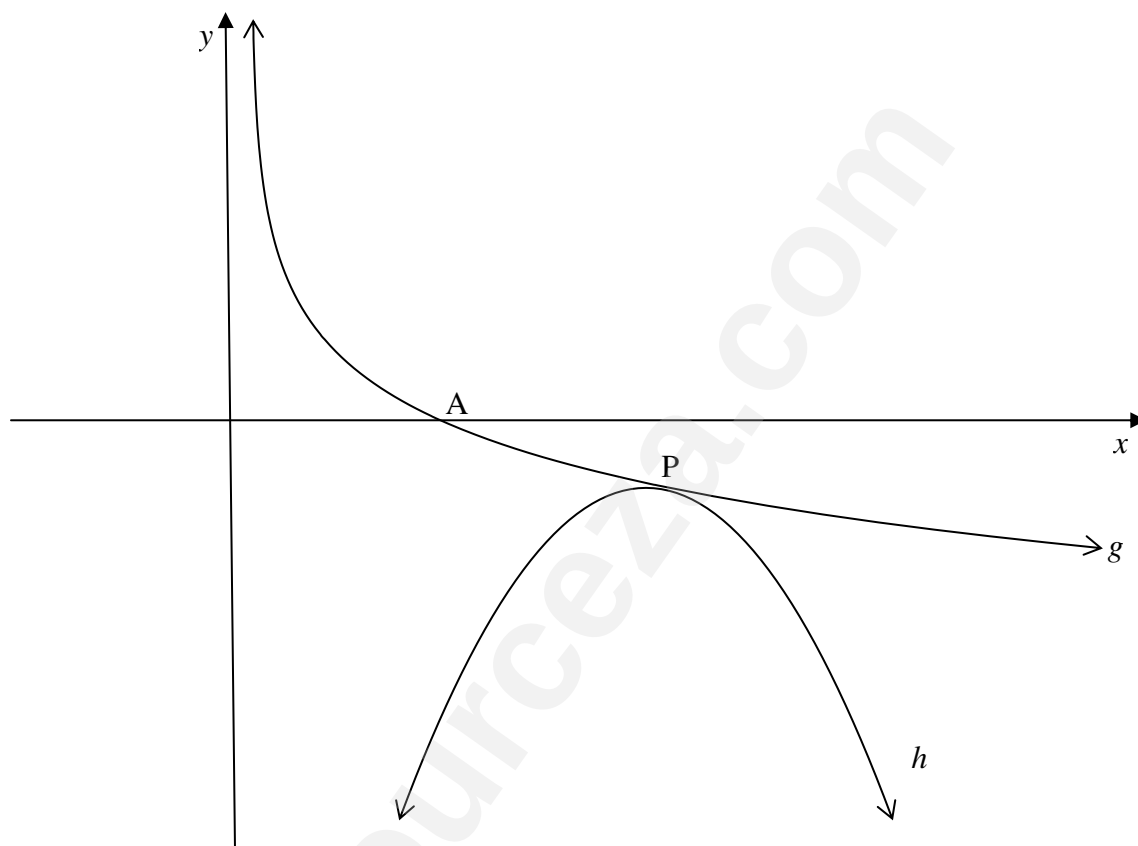
- 2.1 Determine the values of a , p and q (4)
- 2.2 Use the graphs to solve for x if $f(x) \geq g(x)$. (2)
- 2.2 The equation of one of the axis of symmetry of g is $y = x + k$. Calculate the value of k . (2)
- 2.3 For what value(s) of x will $x \cdot g'(x) \leq 0$? (2)

[10]

QUESTION 3

Sketched below are the graphs of $g(x) = \log_a x$ and $h(x) = -(x-3)^2 - 1$.

A is the x -intercept of g and P is the turning point of h . The two graphs intersect at point P.



- 3.1 Calculate the value of a . (2)
 - 3.2 Write down the coordinates of A and P. (2)
 - 3.3 Write down the equation of $g^{-1}(x)$, the inverse of g in the form $y = \dots$. (2)
 - 3.4 Determine a possible restriction that could be placed on h to ensure that $h^{-1}(x)$ is a function. (1)
 - 3.5 Determine the values of x for which $g(x) \cdot h(x) < 0$. (2)
- [9]**

QUESTION 4

Given: $h(x) = 16^x$

- 4.1 Show algebraically that $h\left(x + \frac{1}{2}\right) = 4h(x)$. (3)
- 4.2 Calculate showing all calculations: $h\left(\frac{1}{4}\right) + 2h^{-1}(4) - 4^0$. (6)

QUESTION 5

The function defined as $g(x) = ax^2 + bx + c$ has the following properties:

- The range is $y \in \left[-\frac{27}{16}; \infty\right)$
- $x = 0$ and $x = \frac{9}{4}$ are the x intercepts.
- $f'\left(\frac{9}{8}\right) = 0$

Sketch the graph of g .

[4]

QUESTION 6

Given the exponential function $h(x) = \log_a x$ where $a > 0$. $S\left(\frac{1}{3}; -1\right)$ is a point on the graph of h .

- 6.1 Show that $a = 3$. (3)
- 6.2 Write down the domain of h . (1)
- 6.3 Determine the equation of h^{-1} in the form $y = \dots$ (2)
- 6.4 Is h^{-1} a function? Justify your answer. (2)
- 6.5 S' , the image of S , lies on h^{-1} . Write down the coordinates of S' . (1)
- 6.6 Determine the values for which $0 < \log_3 x < 1$. (2)

[11]

QUESTION 7

Given: $f(x) = x^2 - 5x - 14$ and $g(x) = 2x - 14$

- 7.1 On the same set of axes, sketch the graphs of f and g . Clearly indicate all intercepts with the axes and turning points. (6)
- 7.2 Determine the equation of the tangent to f at $x = 2\frac{1}{2}$. (2)
- 7.3 Determine the value(s) of k for which $f(x) = k$ will have two unequal positive real roots. (2)
- 7.4 A new graph h is obtained by first reflecting g in the x -axis and then translating it 7 units to the left. Write down the equation of h in the form $h(x) = mx + c$. (2)

[12]

QUESTION 8

Two functions are defined by $f(x) = \log_5 x$ and $g(x) \rightarrow (x - p)(y + t) = 2$

The vertical asymptote of g is also the x -intercept of f .

The equation of the horizontal asymptote of g is $x = 1$.

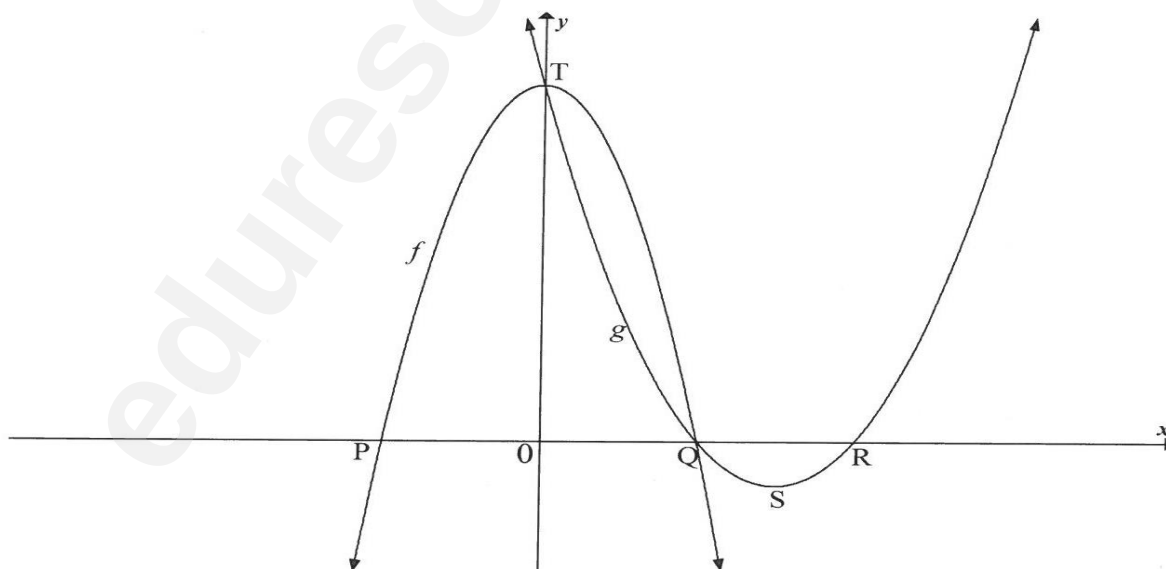
The line $y = x$ intersects the asymptotes of g at R and the graph of g at V.

- 8.1 Write down the coordinates of the x -intercepts of f . (1)
 - 8.2 Determine the values of p and t . (3)
 - 8.3 Write down the equation of h , the reflection of f in the line $y = x$, in the form $y = \dots$ (2)
 - 8.4 Determine the coordinates of R. (1)
 - 8.5 Sketch the graph of f on the DIAGRAM SHEET provided. Clearly show ALL intercepts with the axes. (2)
 - 8.6 For which values of x will $f(x) < 1$? (1)
 - 8.7 Determine the minimum value of $h(x - 1) + 2$. (1)
- [11]

QUESTION 6

- 6.1 The graphs of $f(x) = -2x^2 + 18$ and $g(x) = ax^2 + bx + c$ are sketched below.

Points P and Q are the x -intercepts of f . Points Q and R are the x -intercepts of g . S is the turning point of g . T is the y -intercept of both f and g .

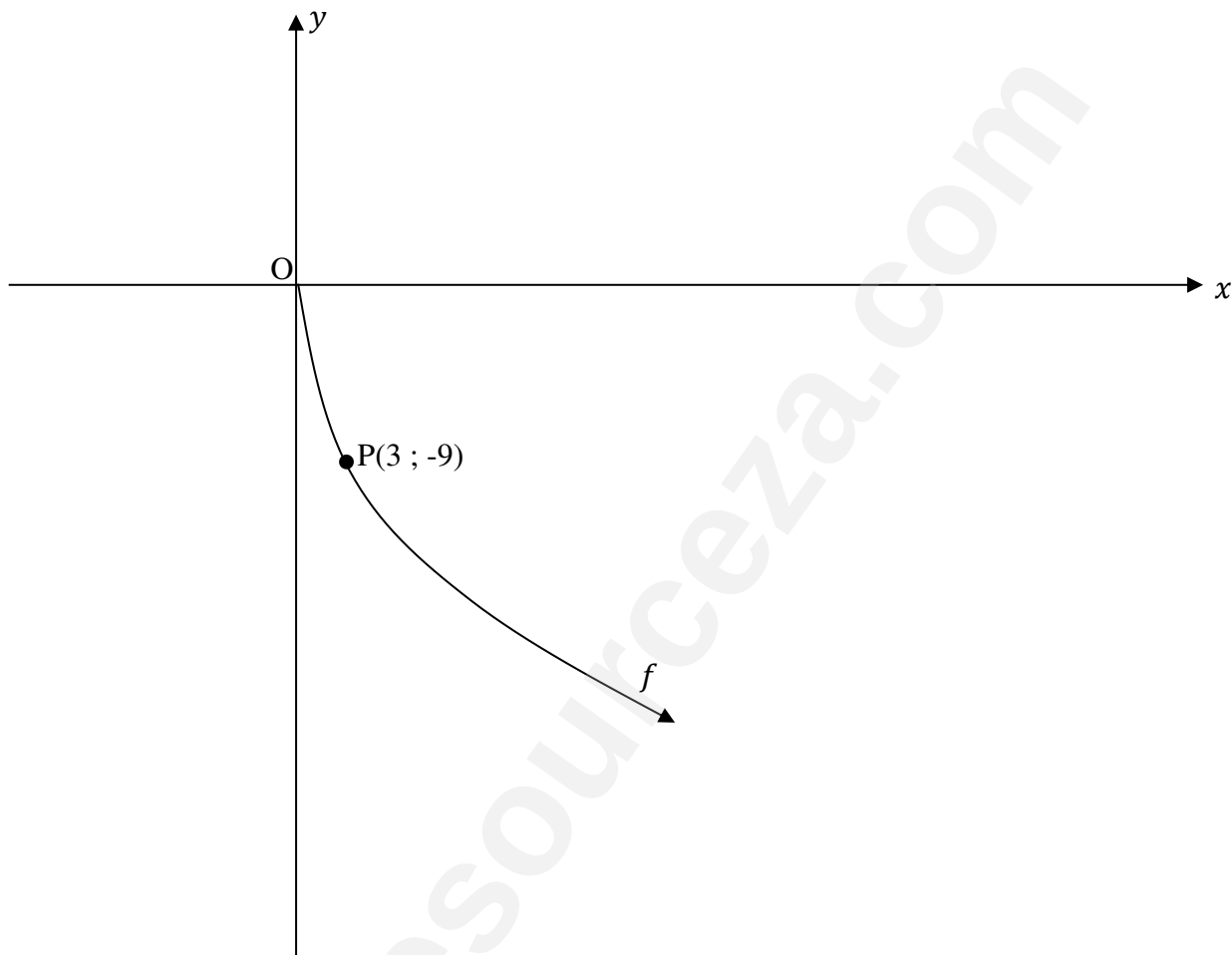


- 6.1.1 Write down the coordinates of T. (1)
- 6.1.2 Determine the coordinates of Q. (3)
- 6.1.3 Given that $x = 4, 5$ at S, determine the coordinates of R. (2)
- 6.1.4 Determine the value(s) of x for which $g''(x) > 0$. (2)

QUESTION 9

The graph of $f(x) = -\sqrt{27x}$ for $x \geq 0$ is sketched below.

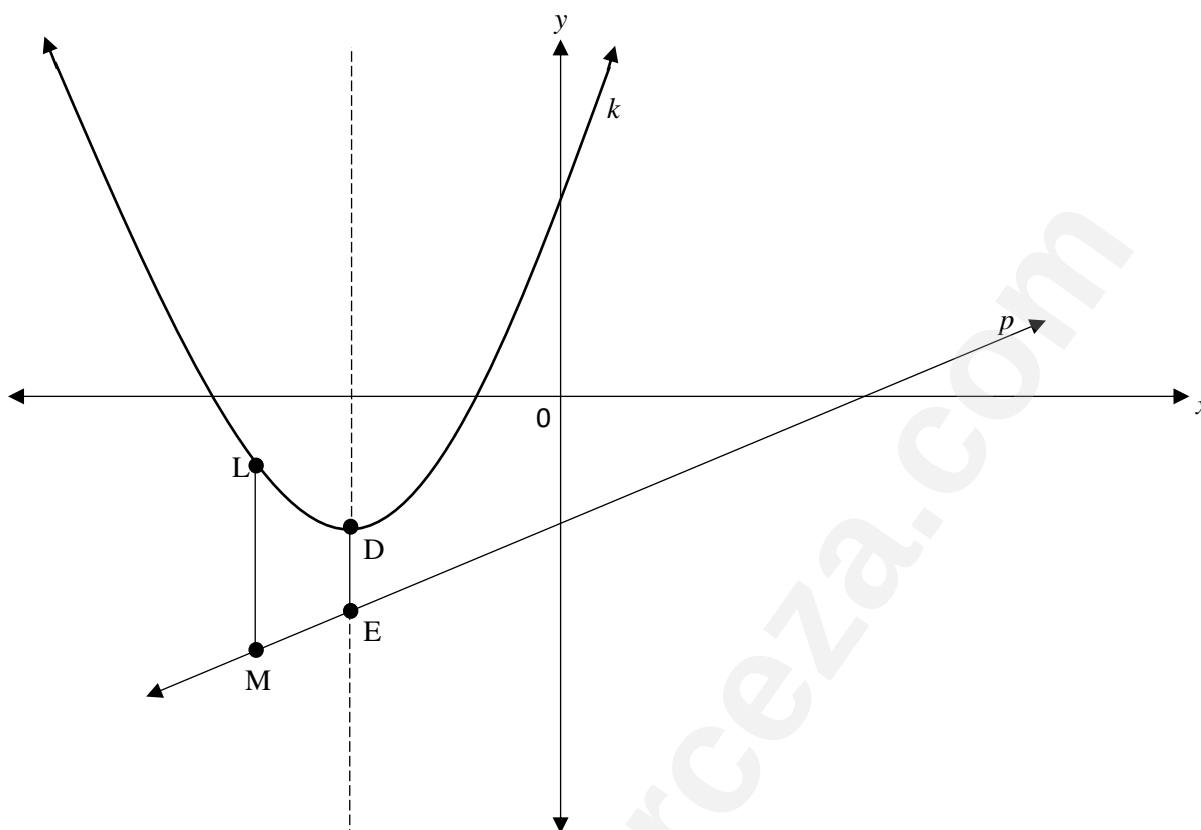
The point $P(3 ; -9)$ lies on the graph of f .



- 9.1 Use your graph to determine the values of x for which $f(x) \geq -9$. (2)
- 9.2 Write down the equation of f^{-1} in the form $y = \dots$. Indicate ALL restrictions. (3)
- 9.3 Sketch f^{-1} , the inverse of f . Indicate the intercept(s) with the axes and the coordinates of ONE other point (3)
- 9.4 Describe the transformation from f to g if $g(x) = \sqrt{27x}$, where $x \geq 0$. (1)
- [9]**

QUESTION 10

The graphs of $k(x) = 2x^2 + 8x + 3$ and $p(x) = 2x - 4$ are sketched below.



Determine:

- 10.1 the coordinates of D, the turning point of k . (3)
- 10.2 the values of x for which $k(x) \geq 3$ (2)
- 10.3 the minimum length of LM, where LM is parallel to the y -axis, with points L on k and M on p respectively. (4)
- 10.4 the average gradient between $k(-2)$ and $k(3)$. (2)
- 10.5 the value of t such that the straight line, $y = 2x + t$, touches the graph of $k(x) = 2x^2 + 8x + 3$ only ONCE. (5)

[16]

You need to be fully comfortable with sequences and series in order to better understand this chapter. It is the application of sequences and series.

- Revise grade 10 and 11 depreciation and simple together with compound interest.
- Compare investments, with the one accumulating higher amount as the best.
- Effective and nominal interest rates, converting interest rates, i.e. $1 + i_{\text{eff}} = \left(1 + \frac{i_n}{n}\right)^n$.
- Loans using the present value annuity formula. **NB:** Loans always **exclude** deposits, i.e subtract the deposit amount from the total loan value.
- Deferred loans: if you *don't pay back the loan*, the loan value *accumulates* (think compound interest).
- **Outstanding balance** of the loan uses the present value annuity formula with the period being the remaining period or outstanding period, all other variables remaining the same.
- Sinking funds: difference between the accumulated value and the depreciated value.
- Sinking funds uses the future value annuity formula.
- Know terms like, replacement value, decay (depreciation), inflation (compound interest).
- Withdrawals uses present value annuity for you are withdrawing presently!!

QUESTIONS

- Patrick opens a savings account on 1 January 2012. He makes an immediate payment of R2 000 into the account and thereafter a monthly payment of R1 200 at the end of each month.
The last payment is made on 31 December 2013. Interest is calculated at 8% per year, compounded monthly.
 - Calculate the value of Patrick's investment on 31 December 2013. (5)
 - Patrick decides to withdraw the money on 31 December 2013. He makes no further payments and the investment earns the same interest rate.
Calculate the value of the investment on 31 May 2014. (3)
- Lilly takes out a loan to the value of R150 000. She repays the loan by means of equal monthly instalments which she makes at the end of each month. The first instalment is made three months after the granting of the loan and the last instalment is eight years after the granting of the loan.
The interest rate is 15% per year, compounded monthly.
 - Calculate the value of the equal monthly instalments. (5)
 - Convert the interest rate to an effective interest rate, rounded to two decimal places. (2)

[15]

QUESTION 1

- 1.1 Determine how many years it would take for the value of a car to depreciate to 50% of its original value, if the rate of depreciation, based on the reducing balance method, is 8% per annum. (3)
- 1.2 Siyanda wants to take out a loan for a house over twenty years. He approached two financial institutions and was offered two different options. The two options are shown below:

OPTION A	OPTION B
• Loan amount: R950 000 • Deposit: 12% payable immediately. • Interest rate: 12% p.a., compounded monthly • Repayments: R x per quarter	• Loan amount: R950 000 • Interest rate: 11,8% p.a., compounded monthly. • Repayments: R10 328,16 per month • Bank Charges: R200

- 1.2.1 Determine the total cost for option 1. (9)
- 1.2.2 Which option is the best? Provide relevant calculations to justify your answer. (3)
- [14]

QUESTION 2

- 2.1 Josephine opened a savings account with a single deposit of R1 000 on the 1st of April 2018. She then makes 18 monthly deposits of R700 at the end of every month. Her first payment is made on the 30th of April 2018 and her last payment on the 30th of September 2019. The account earns interest at 15% per annum compounded monthly.
- Determine the amount that should be in her savings account immediately after her last deposit of R700 is made. (5)
- 2.2 Calculate how many years it will take for the value for the value of a truck to decrease to 50% of its original value if depreciation is calculated at 15% per annum using the reducing-balance method. (4)
- 2.3 Brenda takes out a twenty year loan of R400 000. She repays the loan by means of equal monthly payments starting one month after the granting of the loan. The interest rate is 18% per annum compounded monthly.
- 2.3.1 Calculate the monthly repayments. (4)
- 2.3.2 Calculate the amount owed after the 12th payment was made. (3)
- [16]

QUESTION 3

- 3.1 On the 2nd day of January 2015 a company bought a new printer for R150 000.
- The value of the printer decreases by 20% annually on the reducing balance method.
 - When the book value of the printer is R49 152, the company will replace the printer.
- 3.1.1 Calculate the book value of the printer on the 2nd of January 2017. (3)
- 3.1.2 At the beginning of which year will the company have to replace the printer? Show ALL calculations. (4)
- 3.1.3 The cost of a similar printer will be R280 000 at the beginning of 2020. The company will use the R49 152 that it will receive from the sale of the old printer to cover some of the costs of replacing the printer. The company set up a sinking fund to cover the balance. The fund pays interest at 8,5% per annum, compounded quarterly. The first deposit was made on 2 April 2015 and every three months thereafter until 2 January 2020. Calculate the amount that should be deposited every three months to have enough money to replace the printer on 2 January 2020. (4)
- 3.2 Lebo wishes to apply for a home loan. The bank charges interest at 11% per annum, compounded monthly. She can afford a monthly instalment of R9 000 and wants to repay the loan over a period of 15 years. She will make the first monthly repayment one month the loan is granted. Calculate, to the nearest thousand rand, the maximum amount that Lebo can borrow from the bank. (5)
- [16]**

QUESTION 4

Jake takes out a bank loan of R600 000 to pay for his new car. He repays the loan with monthly instalments of R9 000, starting one month after the granting of the loan. The interest rate is 13% per annum, compounded quarterly.

- 4.1 Show that the effective interest rate is 12,86% p.a., compounded monthly. (3)
- 4.2 How many instalments of R9 000 must be paid? (5)
- 4.3 What will the final payment be? (5)
- 4.4 What did the car cost Jake in total by the time it is paid off? (2)
- [15]**

QUESTION 5

- 5.1 The **world population** is the total number of living humans on earth. According to the U.S. Census Bureau the world population in **2011** was about **6.9 billion** people and it would grow about 76 million during the year. That is an increase of about 1,1%. In **2016** it is estimated at **7.288 billion** (7 288 000 000).

Using this information, the world population can be modelled by the formula:

$$P = 6900(1,011)^t$$

where P is in millions and t is the number of years after 2011.

- 5.1.1 Estimate the population in the year 2050 to the nearest million. (3)
- 5.1.2 By which year will the population be double what it was in 2011? (4)
- 5.2 Monthly payments of R11 192,71 are required to repay a loan of R850 000. The payments start two months after the granting of the loan and continue for 20 years. The interest rate is 15% per annum, compounded monthly. What will the balance of the loan be after 12 years? (5)
- 5.3 Abram has R150 000 to invest in two separate accounts. One account pays interest at a rate of 8,4% per annum, compounded quarterly, and the other account at a rate of 9,6% per annum compounded monthly. How much money should he invest in each account so that he will collect the same amount from each account at the end of 12 years? (6)
- [18]**

QUESTION 6

- 6.1 Lerato wants to purchase a house that costs R850 000. She is required to pay a 12% deposit and she will borrow the balance from a bank. Calculate the amount that Lerato must borrow from the bank. (2)
- 6.2 The bank charges interest at 9% per annum, compounded monthly on the loan amount. Lerato works out that the loan will carry an effective interest rate of 9,6% per annum. Is her calculation correct or not? Justify your answer with appropriate calculations. (4)
- 6.3 Lerato takes out a loan from the bank for the balance of the purchase price and agrees to pay it back over 20 years. Her repayments start one month after her loan is granted. Determine her monthly instalment if interest is charged at 9% per annum compounded monthly. (4)
- 6.4 Lerato can afford to repay R7 000 per month. How long will it take her to repay the loan amount if she chooses to pay R7 000 as a repayment every month? (4)
- [14]**

IMPORTANT!!!

- ✓ First principles and the rules of differentiations should be your best friends! Easy marks.
- ✓ Equation of a tangent. NB: First derivative is your gradient, then look for a point by substituting to the original function.
- ✓ Parallel lines have equal gradients, perpendicular lines, the products of their gradients equals a negative 1.
- ✓ **Curve sketching:** turning points $f'(x) = 0$, the obtained x -values to be substituted to the original equation.
- ✓ **Point of Inflection (think second derivative):** graph changes concavity, i.e. concave up: $f''(x) > 0$ and concave down: $f''(x) < 0$.
- ✓ **Function increases for** $f'(x) > 0$ **and decreases for** $f'(x) < 0$.
- ✓ **Application:** Know your volume and surface area formulae as well and general understanding of calculus.

QUESTION 1

1.1 Determine $f'(x)$ from first principles if $f(x) = -3x^2$. (5)

1.2 Evaluate $\frac{dr}{dp}$ if $r = \sqrt[3]{p^2} - \frac{p^2 - 2p}{p^2}$ (5)

1.3 Given a function $f(x) = ax^3 + bx^2 - 3$ and $f''(-2) = 0$. If it is further given that the point $(-3; 6)$ lies on the graph of f .

Show that $a = \frac{1}{3}$ and $b = 2$. (6)

[16]**QUESTION 2**

2.1 Determine $f'(x)$ from first principles if $f(x) = 3x - x^2$ (5)

2.2 Determine $\frac{dp}{dr}$ if $p = (3r)^2 - \frac{1}{2r}$ (3)

2.3 Given $f(x) = ax^3 + bx^2 + 3x + 3$ and $g(x) = f''(x)$ where $g(x) = 12x + 4$

2.3.1 Show that $a = 2$ and $b = 2$. (4)

2.3.2 Prove that f will NEVER decrease for any real value of x (5)

2.3.3 Explain the concavity of f for all values of x where $g(x) < 0$. (1)

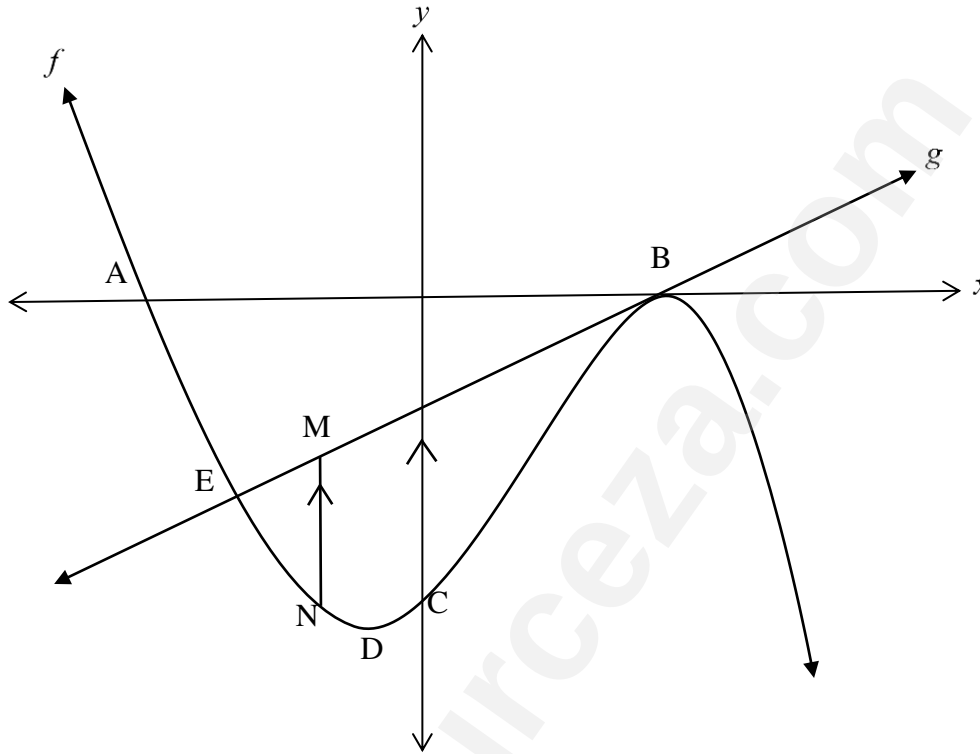
[18]

QUESTION 3

The graphs of $f(x) = -x^3 + 3x^2 + 9x - 27$ and $g(x) = tx + q$ are sketched below.

A(-3;0) and B(3;0) are the x -intercepts and C the y -intercept of f .

The turning points of f are D and B. g and f intersect at E (-2;-10) and B.



- 3.1 Determine the equation of g . (2)
 - 3.2 Determine the coordinates of D, the turning point of f . (4)
 - 3.3 For which value(s) of x is f concave up. (2)
 - 3.4 Determine for which value(s) of x is MN a maximum, if M is a point on g between E and B, N is a point on f and MN is parallel to the y -axis. (5)
 - 3.5 Write down the coordinates of the turning points of $h(x) = f(x) - 3$. (2)
 - 3.5 Determine the equation of the tangent to f at $x = 1$. (4)
 - 3.6 Explain the significance of the point of inflection. (1)
 - 3.7 Determine the value(s) of p for which $f(x) = p$ will have only one real roots. (2)
- [22]

QUESTION 4

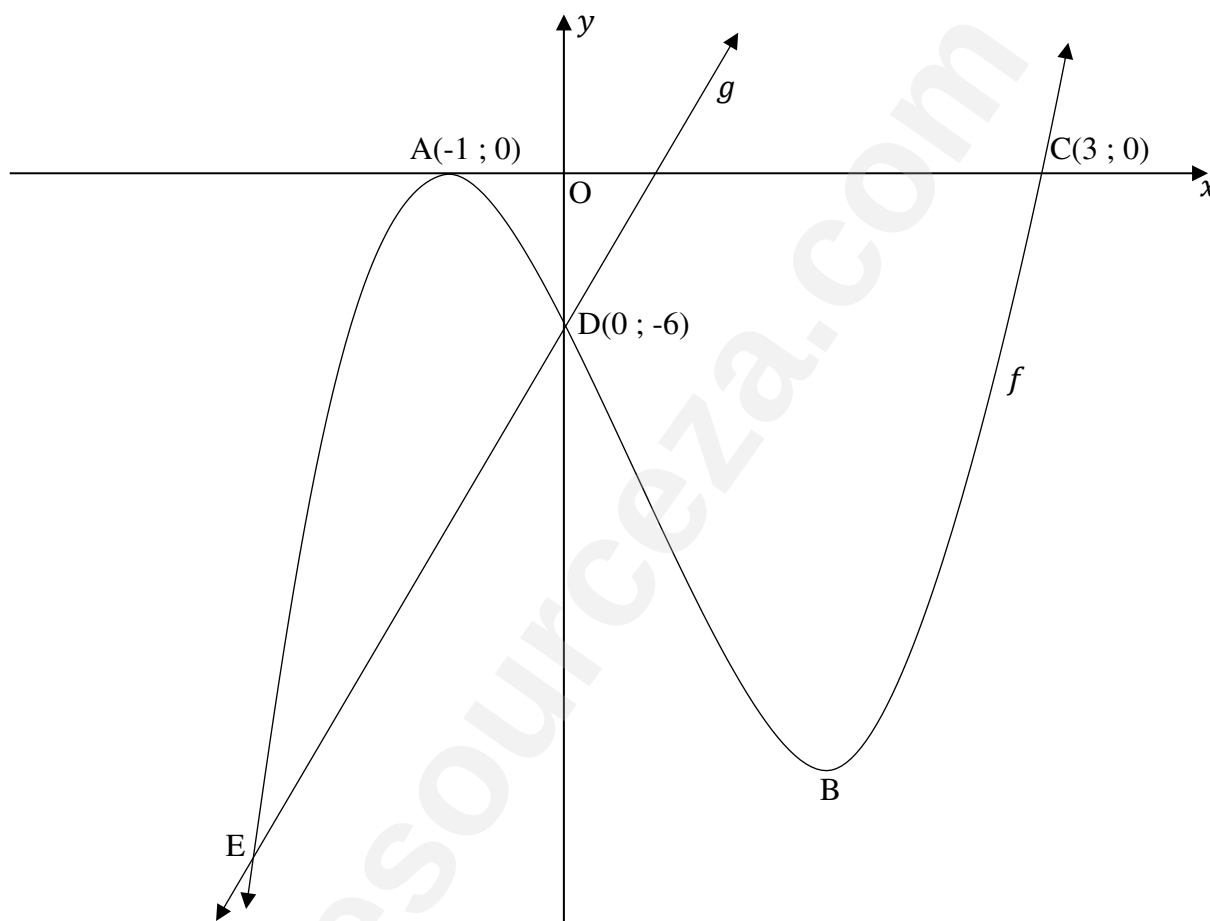
The graphs of $f(x) = ax^3 + bx^2 + cx + d$ and $g(x) = 6x - 6$ are sketched below.

$A(-1; 0)$ and $C(3; 0)$ are the x -intercepts of f .

The graph of f has turning points at A and B.

$D(0; -6)$ is the y -intercept of f .

E and D are points of intersection of the graphs of f and g .



4.1 Show that $a = 2$; $b = -2$; $c = -10$ and $d = -6$. (5)

4.2 Calculate the coordinates of the turning point B. (5)

4.3 $h(x)$ is the vertical distance between $f(x)$ and $g(x)$, that is $h(x) = f(x) - g(x)$. Calculate x such that $h(x)$ is a maximum, where $x < 0$. (5)

4.4 Calculate the x - coordinate of the point at which $f'(x)$ is a maximum. (3)

[18]

QUESTION 5

Given $f(x) = -x^3 + x^2 + 8x + 12$

- 5.1 Calculate the y -intercept of f . (1)
- 5.2 Calculate the x -intercepts of f . (5)
- 5.3 Determine the turning points of f . (5)
- 5.4 Sketch the graph of f , showing clearly ALL the intercepts with the axes and turning points. (3)
- 5.5 Write down the x -coordinate of the point of inflection of f . (2)
- 5.6 Write down the coordinates of the turning points of $h(x) = f(x) - 3$. (2)

[18]**QUESTION 6**

- 6.1 Determine the equation of the tangent to the curve $f(x) = -x^2 - 6x - 8$ that is parallel to the straight line $g(x) = -2x - 1$. (5)
- 6.2 Use differentiation rules to differentiate the following:
- 6.2.1 $y = \frac{2}{5\sqrt{x}} - \sqrt[3]{x}$ (4)
- 6.2.2 $y = \frac{x^4 - 3x^2 + 7}{x}$ (4)
- 6.2.3 $D_x \left[\sqrt{x} \left(2x - \frac{7}{\sqrt[3]{x}} \right) \right]$ (5)
- 6.3 A linear function satisfies the following $h(-2) = 7$ and $h'(x) = 2$.
Determine the inverse of h in the form $y = \dots$. (4)
- 6.4 Given $h(x) = x^3 + 2$ and $g(x) = ax^3 + bx^2 + cx + d$ where $g(x)$ is the function achieved when $y = h(x)$ is shifted 3 units to the right and 4 units upwards.
Determine the values of a , b and c . (4)
- 6.5 Andre is required in a test to find the derivative of a function $f(x)$. However, by mistake, he finds the inverse instead. He finds that: $f^{-1}(x) = \sqrt{\frac{x}{2}}$.
Determine the correct answer to the original question. (4)

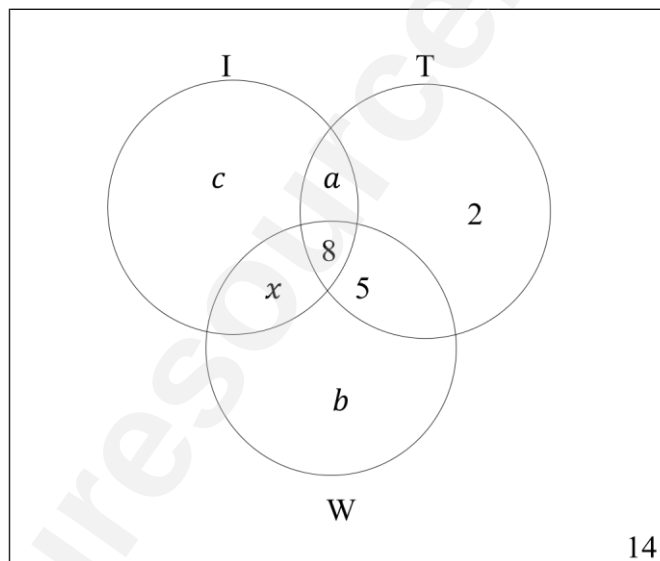
[30]

*Addition Rule, Independent events and fundamental counting principles***QUESTION 1**

A survey was conducted among 100 grade 12 learners about their use of Instagram (I), Twitter (T) and WhatsApp (W) on their cell phones. The survey revealed the following:

- 8 use all three.
- 12 use Instagram and Twitter.
- 5 use Twitter and WhatsApp, but not Instagram.
- x use Instagram and WhatsApp, but not Twitter.
- 61 use Instagram.
- 19 use Twitter.
- 73 use WhatsApp.
- 14 use none of these applications.

The Venn diagram below shows the information above.



- 1.1 Calculate the value of x . (3)
- 1.2 How many learners use Instagram only? (1)
- 1.3 Calculate the probability that a learner chosen randomly, uses exactly two of these applications. (2)

[6]

QUESTION 2

2.1 One coin is randomly picked from a savings jar. The probability of picking a R5 coin out of the savings jar is 0,36 whereas the probability of picking a R2 coin out of the same jar is 0,47.

2.1.1 Explain why the events "picking a R5 coin" and "picking a R2 coin" are mutually exclusive events. (1)

2.1.2 Determine the probability of picking a R2 coin or a R5 coin from the jar. (2)

2.2 How many four – digit codes can be constructed from the set of digits {1 ; 3 ; 4 ; 6 ; 7 ; 8 ; 0 }, if the last digit is 0 and repetition of digits is not allowed? (2)

2.3 Six players of a volleyball team stand at random positions in a row before the game begins. P and Q are two players in this team.

Determine the probability that P and Q will not stand to each other. (3)
[8]

QUESTION 3

3.1 The probability of events A and B occurring are denoted by $P(A)$ and $P(B)$ respectively. For any two events A and B it is given that:

- $P(\text{not } A) = \frac{5}{8}$
- $P(B) = P(A) - \frac{1}{8}$
- A and B are independent.

Determine $P(A \text{ or } B)$. (4)

3.2 A bag contains 3 red and 2 white balls. A second bag contains 5 red and 4 white balls. A bag is chosen at random and a ball is taken from it.

3.2.1 Draw a tree diagram to show ALL the possible outcomes. (4)

3.2.2 Determine the probability that a ball chosen at random is red. (3)

3.3 A group of 10 000 people were tested for colour-blindness and the results are shown in the table below:

	Male	Female	TOTAL
Colour-blind	393	15	408
Normal	j	5 075	9 592
TOTAL	4 910	k	10 000

3.3.1 Calculate the values of j and k . (2)

3.3.2 Determine the probability that a person chosen at random is a colour-blind male. (2)

- 4.1 Consider the digits 1, 2, 3, 4, 5, 6, 7 and 8 and answer the following questions:
- 4.1.1 How many 2-digit numbers can be formed if repetition is allowed? (2)
 - 4.1.2 How many 4-digit numbers can be formed if repetition is NOT allowed? (3)
 - 4.1.3 How many numbers between 4 000 and 5 000 can be formed? (3)
- 4.2 There are 7 different shirts and 4 different pairs of trousers in a cupboard. The clothes have to be hung on the rail.
- 4.2.1 In how many different ways can the clothes be arranged on the rail? (2)
 - 4.2.2 In how many different ways can the clothes be arranged if all the shirts are to be hung next to each another and the pairs of trousers are to be hung next to each another on the rail? (3)
 - 4.2.3 What is the probability that a pair of trousers will hang at the beginning of the rail and a shirt will hang at the end of the rail? (4)
- 4.3 Consider the word: PRODUCT.
- 4.3.1 How many different arrangements are possible if all the letters are used? (2)
 - 4.3.2 How many different arrangements can be made if the first letter is T and the fifth letter is C? (2)
 - 4.3.3 How many different arrangements can be made if the letters R, O and D must follow each other, in any order? (3)
- 4.4 Every client of CASHSAVE Bank has a personal identity number (PIN) which is made up of 5 digits chosen from the digits 0 to 9.
- 4.4.1 How many personal identity numbers (PINs) can be made if:
 - (a) Digits can be repeated (2)
 - (b) Digits cannot be repeated (2)
 - 4.4.2 Suppose that a PIN can be made up by selecting digits at random and that the digits can be repeated. What is the probability that such a PIN will contain at least one 9? (4)

QUESTION 5

- 5.1 The Matric Dance Committee has decided on the menu below for the 2008 Matric Dance. A person attending the dance must choose only ONE item from each category, that is starters, main course and dessert.

MENU		
STARTERS	MAIN COURSE	DESSERT
Crumbed Mushrooms	Fried Chicken	Ice-cream
Garlic Bread	Beef Bolognaise	Malva Pudding
Fish	Chicken Curry	
	Vegetable Curry	

- 5.1.1 How many different meal combinations can be chosen? (2)
- 5.1.2 A particular person wishes to have chicken as his main course. How many different meal combinations does he have? (2)
- 5.2 A photographer has placed six chairs in the front row of a studio. Three boys and three girls are to be seated in these chairs.
- In how many different ways can they be seated if:
- 5.2.1 Any learner may be seated in any chair (2)
- 5.2.2 Two particular learners wish to be seated next to each other (3)
- 5.3 Eight learners are seated on eight chairs in the front row at an assembly.
- 5.3.1 In how many different ways can these 8 learners be seated? (2)
- 5.3.2 In how many different ways can the 8 learners be seated if 3 of the learners must sit together? (3)
- 5.3.3 In how many different ways can the 8 learners be seated if 2 particular learners refuse to sit next to each other? (3)

1 MEASURES OF CENTRAL TENDENCY AND SPREAD

Five Number Summary and Box and Whisker Plot	Ogive
- Minimum - Lower Quartile, i.e. Q_1 - Median, i.e. Q_2 - Upper Quartile, i.e. Q_3 - Maximum - Range = Max – Min - Interquartile Range(IQR) = $Q_3 - Q_1$	- Cumulative Frequency curve. - Used to estimate quartiles in grouped data. - Ground. - Smooth curve – No Ruler - Use upper limit of interval. - To Calculate positions of the quartiles: ✓ $Q_1 = \frac{1}{4}(n+1)$ ✓ $Q_2 = \frac{1}{2}(n+1)$ ✓ $Q_3 = \frac{3}{4}(n+1)$

2 MEAN AND STANDARD DEVIATION

- Mean = $\frac{\text{sum of values}}{\text{number of values}}$
- For **grouped data**, use midpoint of each interval and then Calculator with Frequency mode ON(Mode setup, Stat).
- Standard Deviation**
 - ✓ A measure of spread about the mean.
 - ✓ Calculator: Stat Mode : Var
 - ✓ Variance : σ^2
 - ✓ Data within n standard deviation of the mean : $(\bar{x} \pm n\sigma)$

3 REGRESSION LINE

- Least-squares regression line: $\hat{y} = a + bx$
- Correlation Coefficient (r) : Indicates strength of relationship between variables.

4 CALCULATOR SKILLS

1. Mean

E.g. 4; 7; 1; 9; 4; 8; 11; 10; 19; 2; 5; 7; 19; 3

CASIO	SHARP
[MODE] [2: STAT] [1: 1 – VAR] 4 [=] 7 [=] 1 [=] 9 [=] 4 [=] 8 [=] 11 [=] 10 [=] 19 [=] 2 [=] 5 [=] 7 [=] 19 [=] 3 [=] [AC] [SHIFT] [1: STAT] [4: VAR] [2: $\bar{x}$] [=]	[MODE] [1: STAT] [0: SD] [2ndF] [CA] 4 [DATA] 7 [DATA] 1 [DATA] 9 [DATA] 4 [DATA] 8 [DATA] 11 [DATA] 10 [DATA] 19 [DATA] 2 [DATA] 5 [DATA] 7 [DATA] 19 [DATA] 3 [DATA] [ALPHA] [=] [$\bar{x}$]

Finding the mean of data in a frequency table:

E.g.

No. of learners (x)	Frequency (f)
24	3
25	4
26	2
27	4

CASIO	SHARP
- First add in a frequency column: [SHIFT] [SETUP] [$\downarrow$] [3:STAT] [1:ON] - Then enter the data [SETUP] [2:STAT] [1:1-VAR] 24 [=] 25 [=] 26 [=] 27[=] [$\downarrow$] [$\rightarrow$] 3 [=] 4 [=] 2 [=] 4[=] [AC] [SHIFT] [STAT] [1] [4:VAR] [2:$\bar{x}$]	[MODE] [1: STAT] [0: SD] [2ndF] [MODE] [CA] 24 [(x;y)] 3 [DATA] 25 [(x;y)] 4 [DATA] 26 [(x;y)] 2 [DATA] 27 [(x;y)] 4 [DATA] [ALPHA] [4] [$\bar{x}$]

2. Standard Deviation

E.g. 9; 7; 11; 10; 13; 7

CASIO	SHARP
Press the following keys [MODE] [2:STAT] [1:1-VAR] 9 [=] 7 [=] 11 [=] 10 [=] 13 [=] 7 [=] [AC] [SHIFT]:1 [STAT] [4:VAR] [3: σx] [=]	Press the following keys: [MODE] [1:STAT] [O:SD] 9 [CHANGE] 7 [CHANGE] 11 [CHANGE] 10 [CHANGE] 13 [CHANGE] 7 [CHANGE] [ALPHA] [6: σx] [=]

3. Correlation coefficient (r)

E.g.

No. of rooms	1	2	3	4	5	6	7	8	9	10
% of households	20,6	13,9	10,9	17,9	13,1	9,8	6,1	3,8	2,6	1,3

CASIO	SHARP
- First get the calculator in STAT mode: [MODE] [2: STAT] [2: A + BX] - Enter the x-values: 1 [=] 2 [=] 3 [=] 4 [=] 5 [=] 6 [=] 7 [=] 8 [=] 9[=] 10 [=] - Enter the y-values:	- First get the calculator in STAT mode: <MODE> [1: STAT] [1: LINE] - Enter x-values and y-values together: 1 [(x;y)] 20,6 [CHANGE] 2 [(x;y)] 13,9 [CHANGE] 3 [(x;y)] 10,9 [CHANGE] 4 [(x;y)] 17,9 [CHANGE]

$\downarrow$ [$\rightarrow$] 20,6 [=] 13,9 [=] 10,9 [=] 17,9 [=] 13,1 [=] 9,8 [=] 6,1 [=] 3,8 [=] 2,6 [=] 1,3 [=] [AC]	5 [(x;y)] 13,1 [CHANGE] 6 [(x;y)] 9,8 [CHANGE] 7 [(x;y)] 6,1 [CHANGE] 8 [(x;y)] 3,8 [CHANGE] 9 [(x;y)] 2,6 [CHANGE] 10 [(x;y)] 1,3 [CHANGE]
Find the value of r [SHIFT] [1: STAT] [5: Reg] [3: r] [=] $r = -0,9179521043 \approx -0,9$	Find the value of r, the y-intercept [ALPHA] [$\div$] [=] $r = -0,9179521043 \approx -0,9$

4. Equation of Regression Line

E.g.

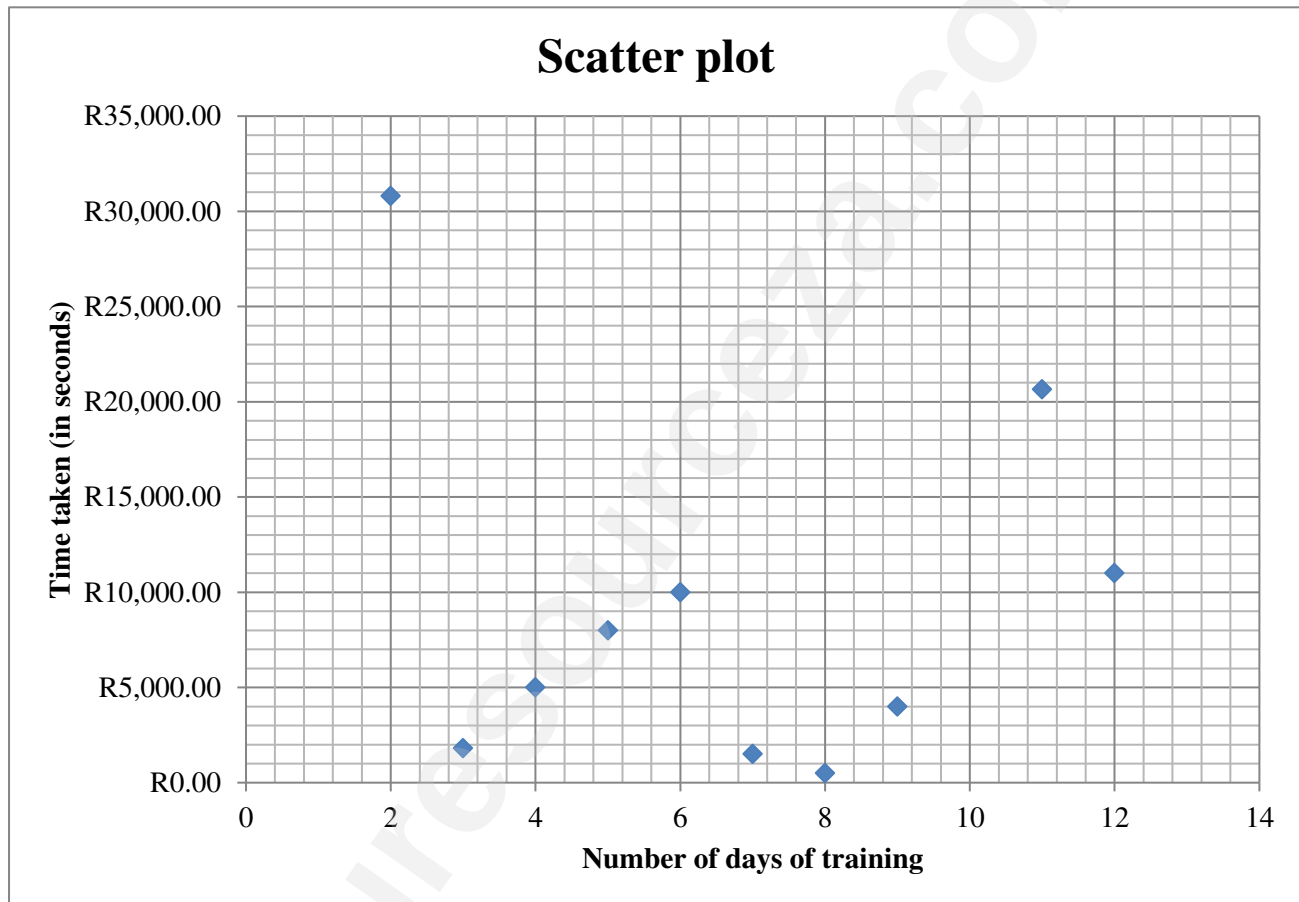
Mass (kg)	49	65	82	60	65	94	88
Height (cm)	156	176	183	153	163	192	180

CASIO	SHARP
- First get the calculator in STAT mode: [MODE] [2: STAT] [2: A + BX] - Enter the x-values: 49 [=] 65 [=] 82 [=] 60 [=] 65 [=] 94 [=] 88 [=] - Enter the y-values: $\downarrow$ [$\rightarrow$] 156 [=] 176 [=] 183 [=] 153 [=] 163 [=] 192 [=] 180 [=] [AC] - Find the value of a, the y-intercept [SHIFT] [1: STAT] [5: Reg] [1: A] [=] $a = 113,4716211 \approx 113,47$ - Get the value of b, the gradient [SHIFT] [1: STAT] [5: Reg] [2: B] [=] $b = 0,812522171 \approx 0,81$	- First get the calculator in STAT mode: <MODE> [1: STAT] [1: LINE] - Enter x-values and y-values together: 49 [(x;y)] 156 [CHANGE] 65 [(x;y)] 176 [CHANGE] 82 [(x;y)] 183 [CHANGE] 60 [(x;y)] 153 [CHANGE] 65 [(x;y)] 163 [CHANGE] 94 [(x;y)] 192 [CHANGE] 88 [(x;y)] 180 [CHANGE] - Find the value of a, the y-intercept [ALPHA] [()] [=] $a = 113,4716211 \approx 113,47$ - Find the value of b, the gradient [ALPHA] [)] [=] $b = 0,812522171 \approx 0,81$

QUESTION 1

Twelve athletes trained to run the 100 m sprint event at the local athletics club trials. Some of them took their training more seriously than others. The following table and scatter plot shows the number of days that an athlete trained and the time taken to run the event. The time taken, in seconds, is rounded to one decimal place.

Number of days of training	50	70	10	60	60	20	50	90	100	60	30	30
Time taken (in seconds)	12,9	13,1	17,0	11,3	18,1	16,5	14,3	11,7	10,2	12,7	17,2	14,3



- 2.1 Discuss the trend of the data collected. (1)
- 2.2 Identify any outlier(s) in the data. (1)
- 2.3 Calculate the equation of the least squares regression line. (3)
- 2.4 Predict the time taken to run the 100 m sprint for an athlete training for 45 days. (2)
- 2.5 Calculate the correlation coefficient. (2)
- 2.6 Comment on the strength of the relationship between the variables. (1)

[10]

QUESTION 2

Maths Literacy teachers are always complaining about their learners' language and reading skills.

Below we have the June results for some matric learners in English and Mathematical Literacy.

MATHS LIT	25	38	40	46	47	12	49	49	54	55	59	64
ENGLISH	34	53	62	40	44	20	50	55	61	63	54	59

- 2.1 Use the grid provided on **DIAGRAM SHEET 1** to draw a scatter plot for the data. (3)
- 2.2 Using the least squares method, show by calculation (Calculator) that the relationship between the English and Math Literacy results is given by $\hat{y} = 17,32 + 0,72x$. (4)
- 2.3 Draw the least squares line for the data on the scatter plot diagram drawn in QUESTION 1.2 (**DIAGRAM SHEET 1**). (2)
- 2.4 Estimate the Math Literacy results for a learner who gets 58% for English. (2)
- 2.5 Estimate the English results for a learner who gets 69% for Math Literacy. (2)
- 2.6 Determine the correlation between the English and Math Literacy results. (2)
- 2.7 Use the correlation coefficient to comment on the relationship between the English and Math Literacy results. (2)
- 2.8 Determine the standard deviation of the Math Literacy results. (2)

[19]**QUESTION 3**

The data in the table below represents the marks obtained by 10 Grade 12 learners for English Home Language (HL) and Afrikaans First Additional Language (FAL).

English HL	42	54	85	32	63	71	92	62	58	66
Afrikaans FAL	50	58	80	45	60	65	98	75	71	58

- 3.1 Draw a scatter plot of the data above by making use of the grid provided in the SPECIAL ANSWER BOOK. (4)
- 3.2 Calculate the equation of the least squares regression line for this data. (3)
- 3.3 Calculate the correlation coefficient. (2)
- 3.4 Describe the correlation between English Home Language and Afrikaans First Additional Language. (1)
- 3.5 Predict the final English Home Language mark for the learner who obtained 74 marks in Afrikaans First Additional Language. (2)

QUESTION 4

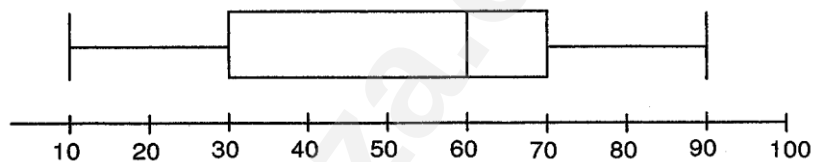
Two schools, Mshiye High School and Bee Vee High School are in a competition to see which school performed better in mathematics in the June Examination.

The marks of the learners at Mshiye High School are recorded below. The box and whisker diagram illustrates the results of Bee Vee High School. Both schools have 25 learners. (Marks are given in %).

Marks for Mshiye High School learners

The box and whisker diagram for the Bee Vee High School learners

9	14	14	19	21
23	33	35	37	37
42	45	55	56	57
59	68	75	75	75
77	78	80	81	92



- 4.1 Write down the five number summary of Mshiye High School. (4)
- 4.2 Draw the box and whisker diagram that represents Mshiye High School marks. Clearly indicate ALL relevant values. (3)
- 4.3 Comment on the skewness of the data of Mshiye High School. (1)
- 4.4 Calculate the mean mark of Mshiye High School. (2)
- 4.5 Determine which school performed better in the June Examination and give reasons for your conclusion. (3)
- [13]**

QUESTION 5

The table below shows the results from a survey of cellphone expenditure for 100 learners from a secondary school in Rustenburg.

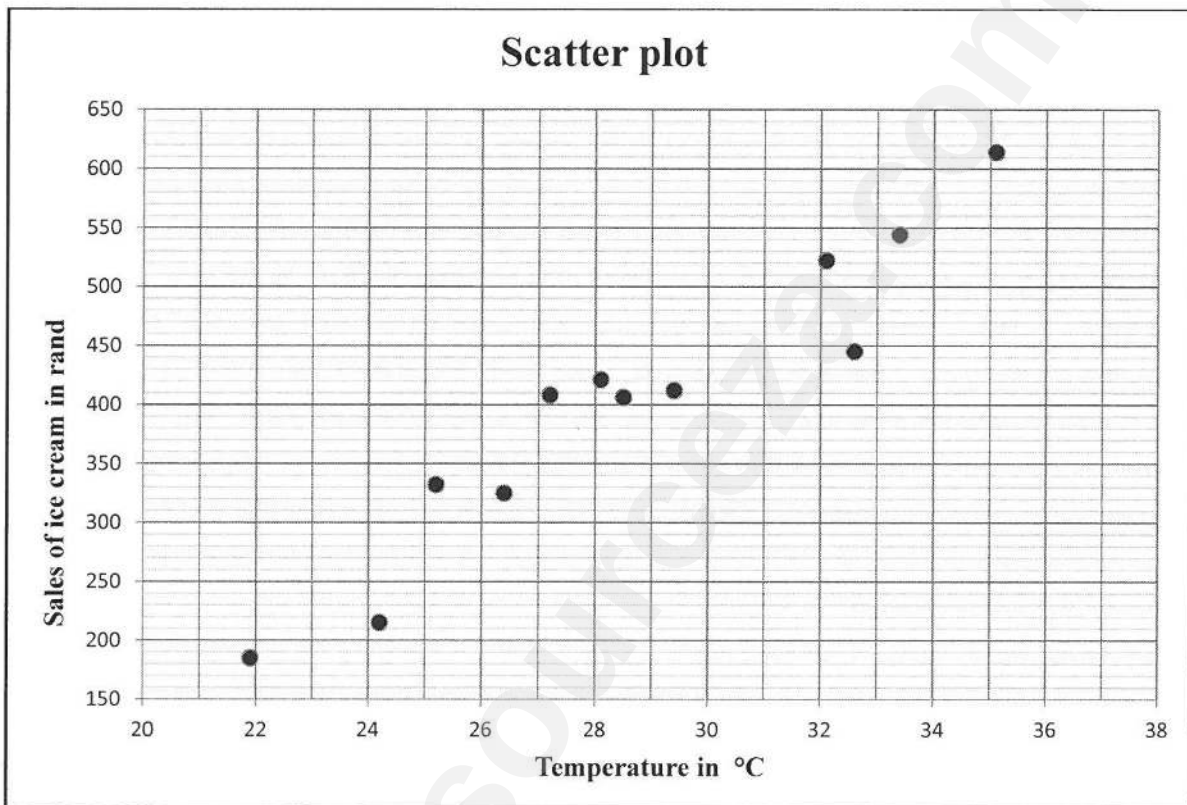
Expenditure (in rand)	Frequency	Cumulative frequency
$50 \leq x < 100$	24	
$100 \leq x < 150$	52	
$150 \leq x < 200$	14	
$200 \leq x < 250$	<i>a</i>	
$250 \leq x < 300$	4	

- 5.1 Determine the value of a . (1)
- 5.2 Complete the cumulative frequency table in the ANSWER BOOK. (2)
- 5.3 Draw an ogive (cumulative frequency) for the data. (3)
- 5.4 What is the modal class for the data. (1)
- [7]**

QUESTION 7

An ice-cream shop recorded the sales of ice cream, in rand, and the maximum temperature in °C, for 12 days in a certain month. The data that they collected is represented in the table and scatter plot below.

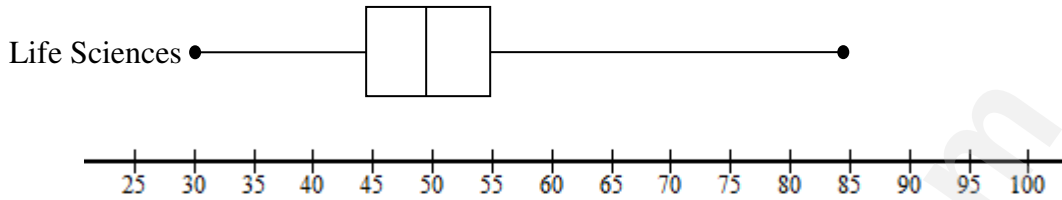
Temperature in °C	24,2	26,4	21,9	25,2	28,5	32,1	29,4	35,1	33,4	28,1	32,6	27,2
Sales of ice cream in rand	215	325	185	332	406	522	412	614	544	421	445	408



- 7.1 Describe the influence of temperature on the sales of ice cream in the scatter plot. (1)
- 7.2 Give a reason why this trend cannot continue indefinitely. (1)
- 7.3 Calculate an equation for the least squares regression line. (3)
- 7.4 Calculate the correlation coefficient. (1)
- 7.5 Comment on the strength of the relationship between the variables. (1)
- [7]**

QUESTION 8

In a certain school 60 learners wrote examinations in Mathematics and Life Sciences. The box-and-whisker diagram below shows the marks (out of 100) that these learners scored in the Life Sciences examination.



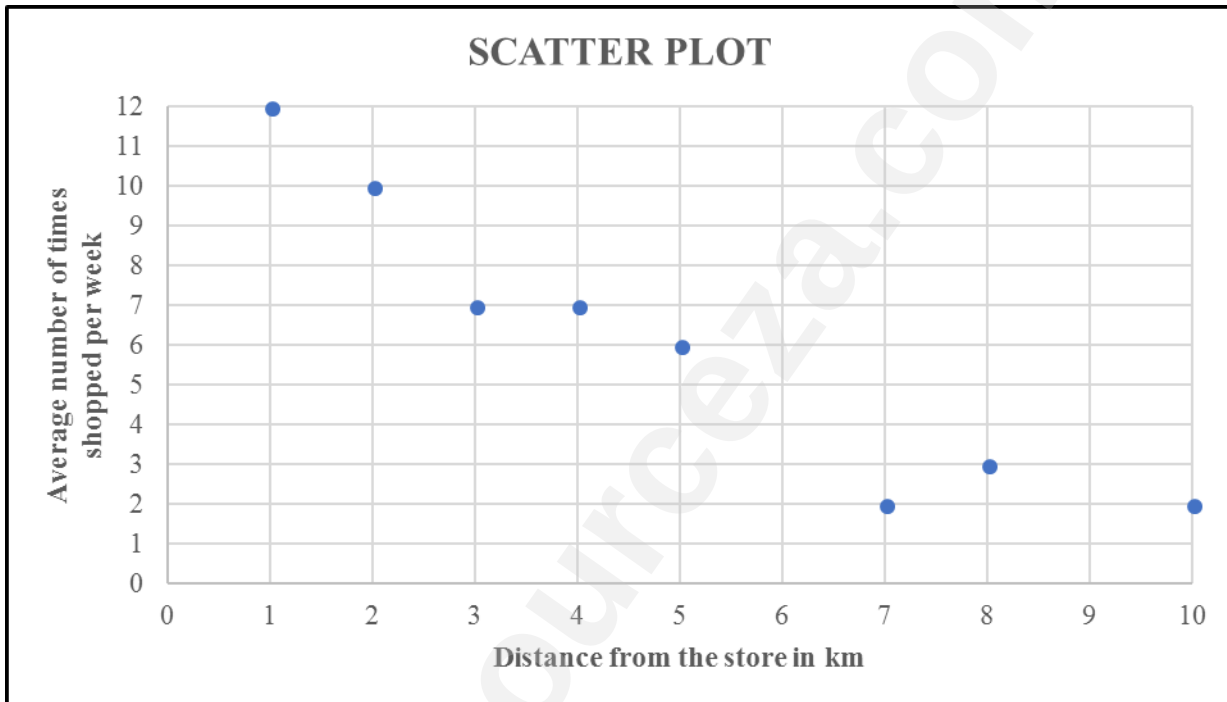
- 8.1 Write down the range of the marks scored in the Life Sciences examination (1)
- 8.2 Use the information below to draw the box-and-whisker diagram for the Mathematics results on the ANSWER BOOK provided. (4)
- Minimum mark = 30
 - Range = 55
 - Upper quartile = 70
 - Interquartile range = 30
 - Median = 55
- 8.3 How many learners scored less than 70% in the Mathematics examination? (2)
- 8.4 Joe claims that the number of learners who scored between 30 and 45 in Life Sciences is smaller than the number of learners who scored between 30 and 55 in Mathematics. Is Joe's claim valid? Justify your answer. (2)

[9]

QUESTION 9

A survey was conducted at a local supermarket relating the distance that shoppers lived from the store to the average number of times they shopped at the store in a week. The results are shown in the table below:

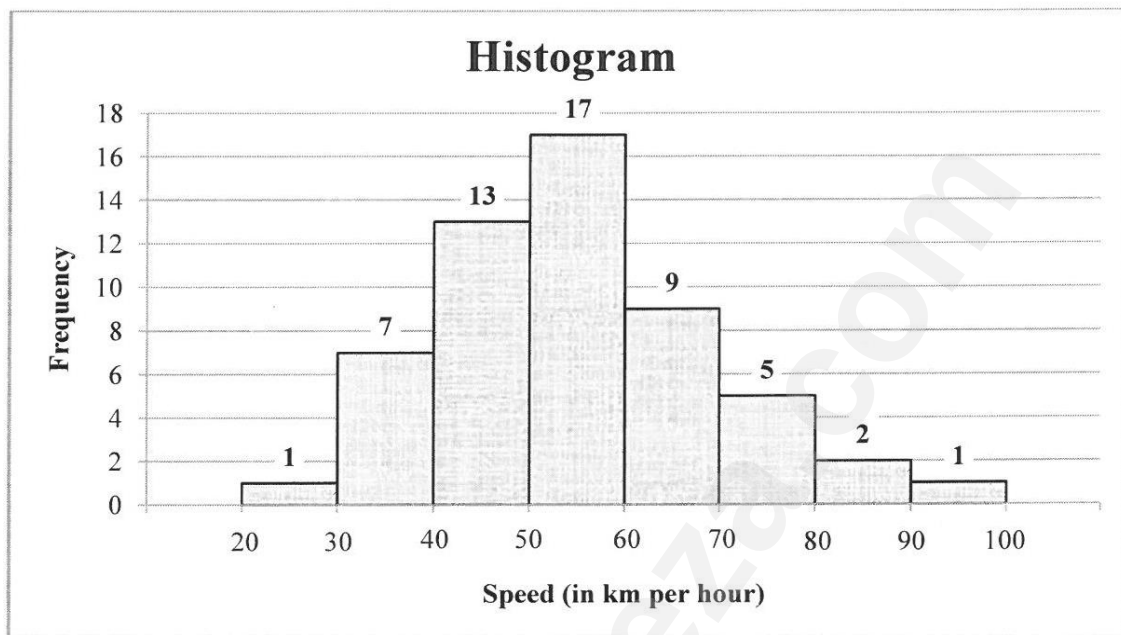
Distance from the store in km	1	2	3	4	5	7	8	10
Average number of times shopped per week	12	10	7	7	6	2	3	2



- 9.1 Use the scatter plot to comment on the strength of the relationship between the distance a shopper lived from the store and the average number of times she/he shopped at the store in a week. (1)
- 9.2 Calculate the correlation coefficient of the data. (1)
- 9.3 Calculate the equation of the least squares regression line of the data. (3)
- 9.4 Use your answer at QUESTION 9.3 to estimate the average number of times that a shopper living 6 km from the supermarket will visit the store in a week. (2)
- 9.5 Sketch the least squares regression line on the scatter plot provided in the ANSWER BOOK. (2)
- [9]**

QUESTION 10

The speeds of 55 cars passing through a certain section of a road are monitored for one hour. The speed limit on this section of road is 60 km per hour. A histogram is drawn to represent this data.



- 10.1 Identify the modal class of the data. (1)
- 10.2 Use the histogram to:
- 10.2.1 Complete the cumulative frequency column in the table on ANSWER BOOK provided. (2)
- 10.2.2 Draw an ogive (cumulative frequency graph) of the above data on the grid in the ANSWER BOOK. (3)
- 10.3 The traffic department sends speeding fines to all motorists whose speed exceeds 66 km per hour. Estimate the number of motorists who will receive a speeding fine. (2)
- [8]**

Important Notes

- Gradient of a line:

- The gradient shows how steep the line is.
- The formula of a finding a gradient of a line is denoted as

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

- If two lines are Parallel to each other, that means they have equal gradients

$$AB \parallel CD \Rightarrow m_{AB} = m_{CD}$$

- If two lines are perpendicular($\perp$) to each other, that means their products equals to -1

$$AB \perp CD \Rightarrow m_{AB} \times m_{CD} = -1$$

- The distance formula:

$$\text{Length of line AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- The midpoint of a line:

- If you are given two endpoints of a line you can find the coordinates of the midpoint of these two endpoints.
- The formula is

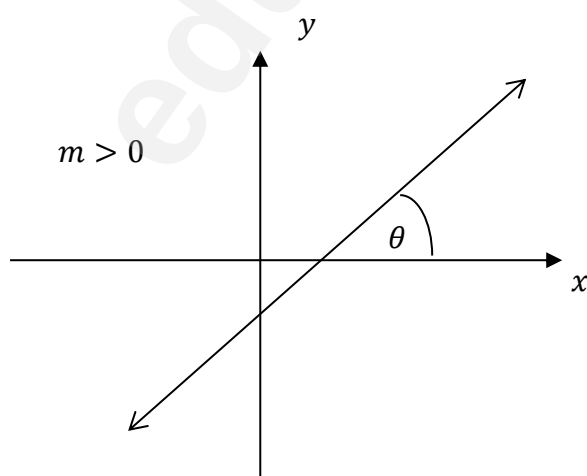
$$M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$$

- The equation of a line:

- You can find the equation of a straight line using $y = mx + c$, if you are given the gradient m and the y-intercept c .
- You can also use the equation $y - y_1 = m(x - x_1)$ to find the equation of a straight line if you are given the gradient m and any point on the line

- The inclination of a line:

- To find the inclination of a line, or the angle it makes with the x -axis, we use $\tan \theta$.
- A line with a positive gradient ($m > 0$) has an acute angle of inclination ($0^\circ < \theta < 90^\circ$)

**Example:**

Determine the angle of inclination if the gradient of a line is $\frac{1}{5}$.

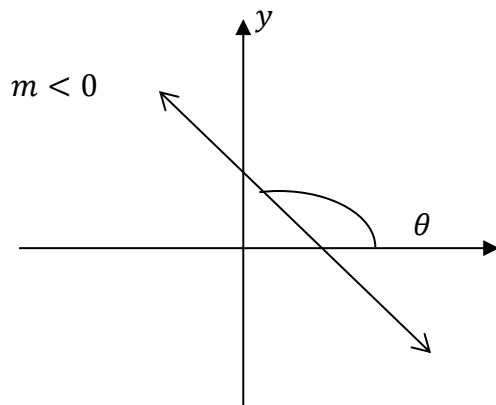
Solution: $\tan \theta = m$

$$= \frac{1}{5}$$

$$\therefore \theta = \tan^{-1}\left(\frac{1}{5}\right)$$

$$= 11.3^\circ$$

- If we are calculating the angle of inclination of a line with a negative gradient ($m < 0$), then we add 180° to change the negative angle to an obtuse angle ($90^\circ < \theta < 180^\circ$).



θ is an obtuse angle

Example:

Determine the angle of inclination if the gradient of a line is -0.7

Solution: $\tan \theta = m$

$$= -0.7$$

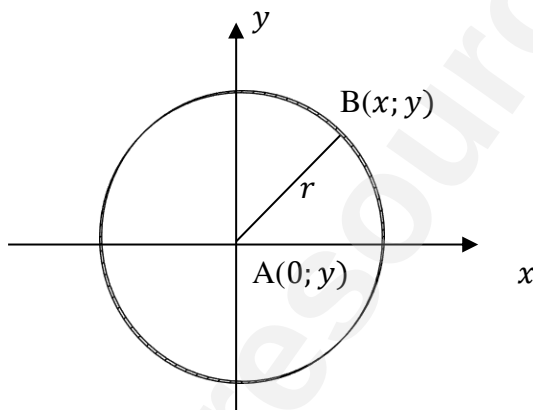
$$\therefore \theta = \tan^{-1}(-0.7)$$

$$= -35.0^\circ$$

$$\text{Obtuse angle } \theta = -35.0^\circ + 180^\circ$$

$$= 145^\circ$$

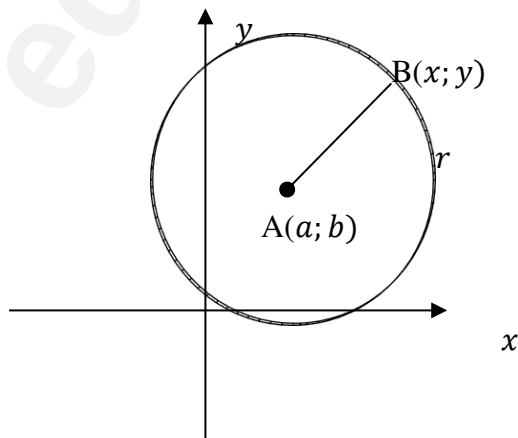
- The equation of a circle:
 - Circle with centre at an origin



Use the following equation to find the equation of a circle with a centre at the origin:

$$x^2 + y^2 = r^2$$

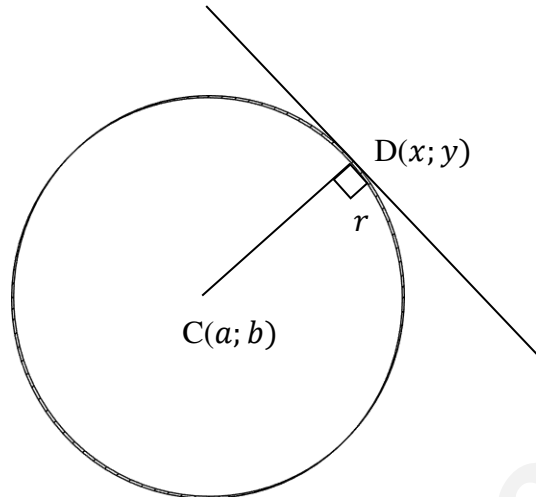
- Circle not centred at the origin



Use the following equation to find the equation of a circle with a centre at a point on the Cartesian plane:

$$(x - a)^2 + (y - b)^2 = r^2$$

- The equation of a tangent to a circle:



Steps:

- * Determine the equation of the circle and write it in the form of $(x - a)^2 + (y - b)^2 = r^2$
- * From the equation, determine the coordinates of the centre of the circle $(a; b)$
- * Determine the gradient of the radius.
- * The radius is perpendicular to the tangent of the circle at a point D so:

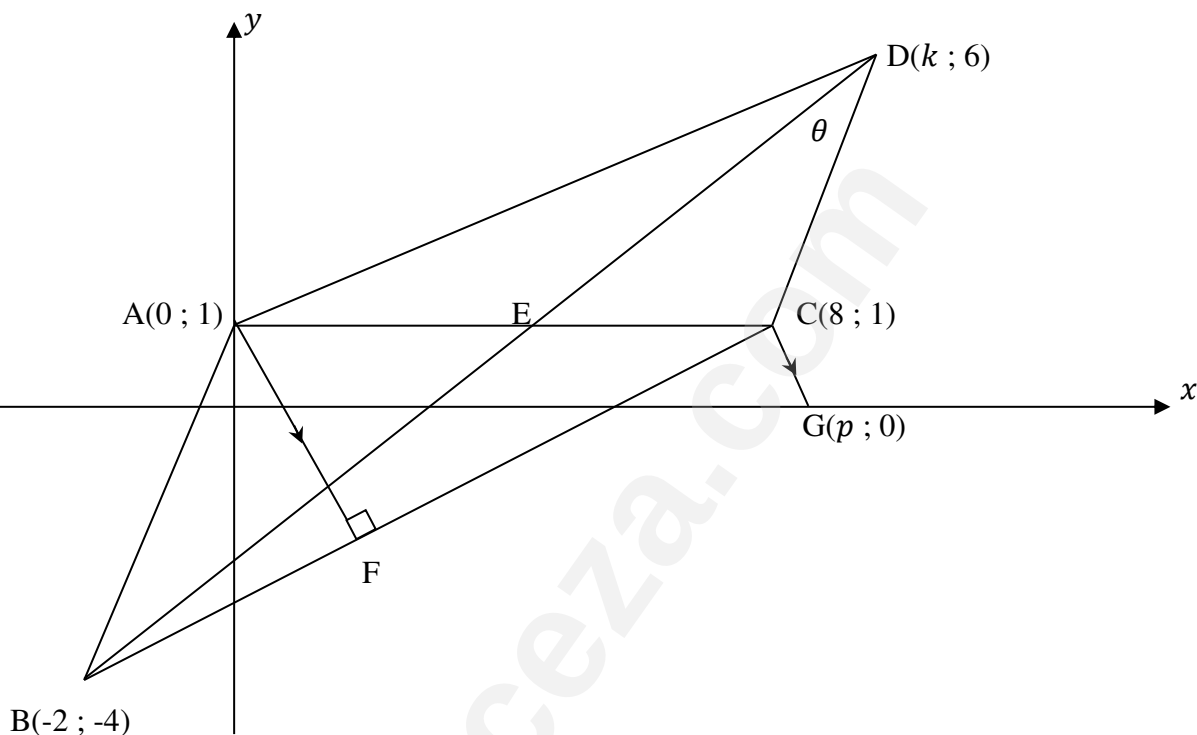
$$m_{AB} \times m_{CD} = -1$$

$$\therefore m_{AB} = -\frac{1}{m_{CD}}$$
- * Write down the gradient-point of a straight line equation and substitute m_{AB} and the coordinates of D. Make y the subject of the equation:

$$y - y_1 = m(x - x_1)$$

QUESTION 1

In the figure below, ABCD is a parallelogram with vertices $A(0 ; 1)$, $B(-2 ; -4)$, $C(8 ; 1)$ and $D(k ; 6)$. AF is perpendicular to BC and parallel to CG . E is a point of intersection of the diagonals of ABCD.



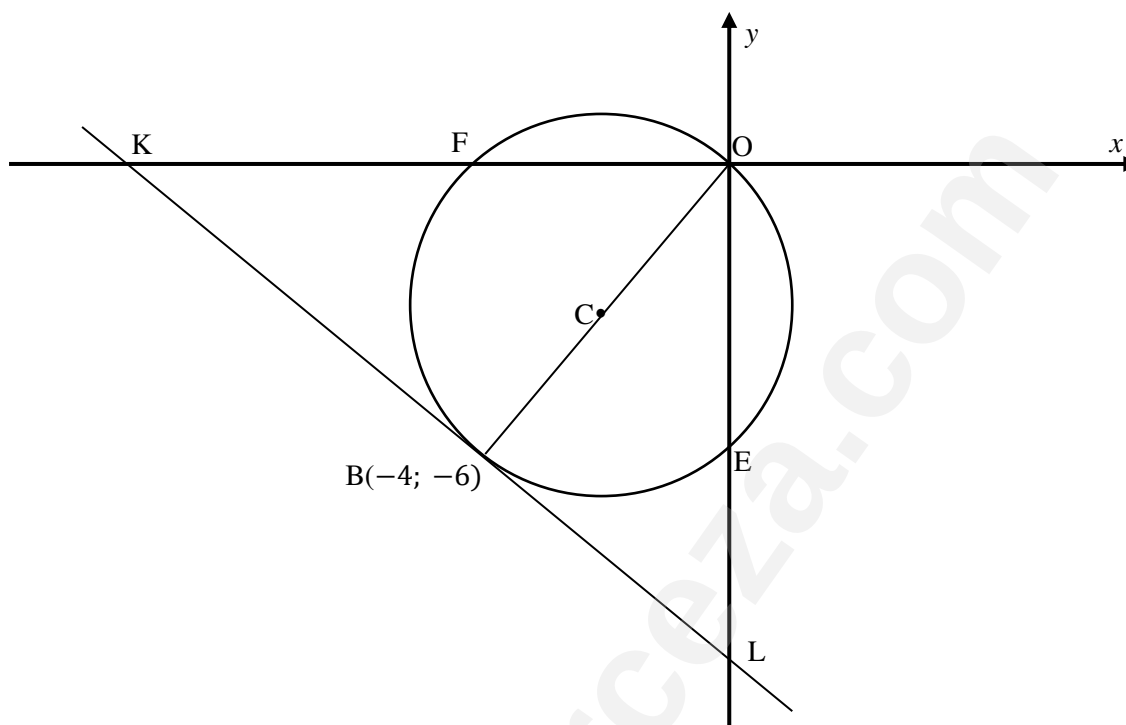
Determine:

- 1.1 the length of BC in simplest surd form. (2)
- 1.2 the gradient of BC. (1)
- 1.3 the equation of AF. (2)
- 1.4 the coordinates of E. (2)
- 1.5 the value of k . (2)
- 1.6 the value of p . (2)
- 1.7 the size of θ rounded off to one decimal place. (6)

[17]

QUESTION 2

In the diagram, the circle, having centre at C, passes through the origin, O, and intersects the x -axis at F and the y -axis at E. The tangent to the circle at B(-4 ; -6) intersects the x -axis at K and the y -axis at L.

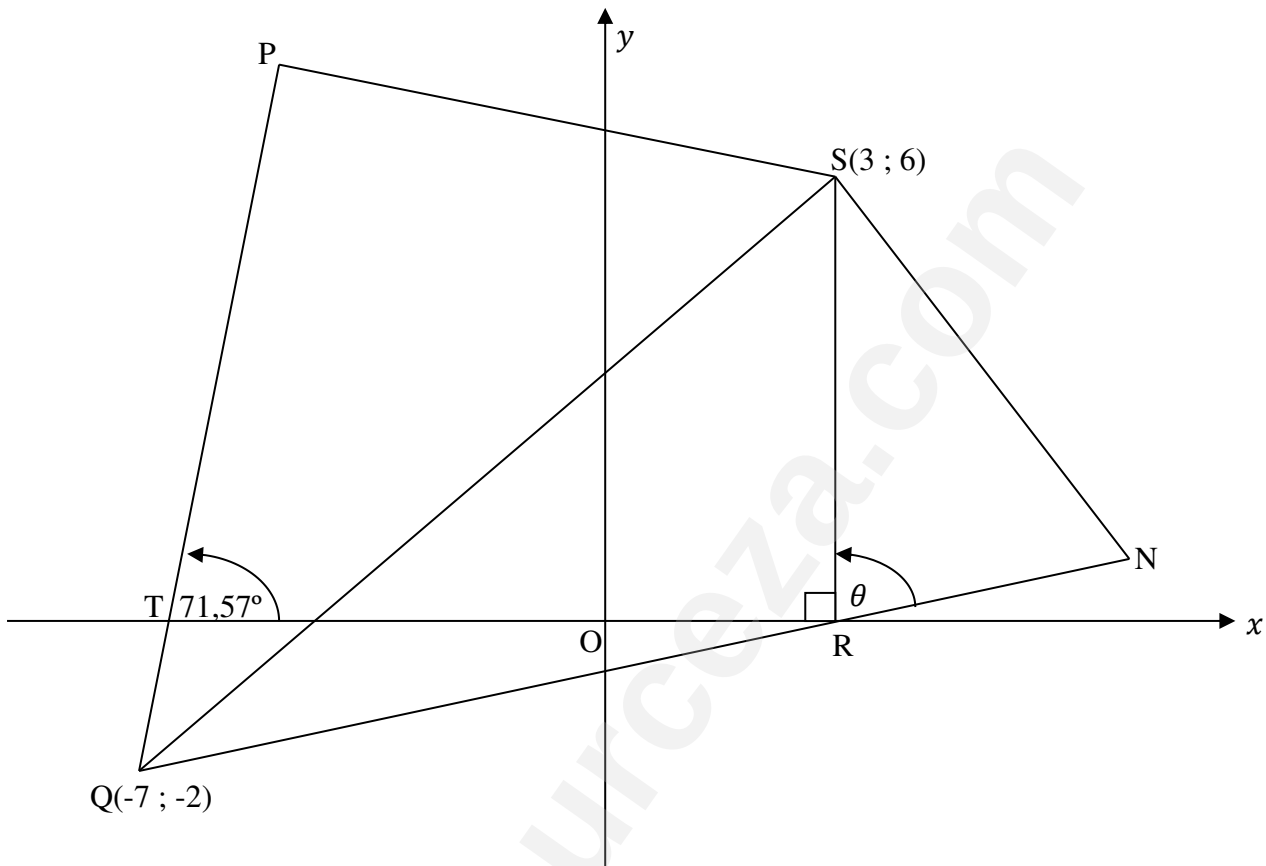


- 2.1 Calculate the coordinates of C. (2)
- 2.2 Calculate the length of the radius of the circle. (2)
- 2.3 Determine the equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$ (2)
- 2.4 Why is $\triangle OBL$ a right angled triangle? (1)
- 2.5 Determine the equation of the tangent KL in the form $ax^2 + bx + c = 0$ (4)
- 2.6 Calculate the coordinates of E. (4)
- 2.7 Determine the ratio of the following in its simplest form: $\frac{\text{area } \triangle OBE}{\text{area } \triangle OBL}$ (4)

[19]

QUESTION 3

In the diagram, P, Q(-7 ; -2), R and S(3 ; 6) are vertices of a quadrilateral. R is a point on the x -axis. QR is produced to N such that $QR = 2RN$. SN is drawn. $\hat{PTO} = 71,57^\circ$ and $\hat{SRN} = \theta$.



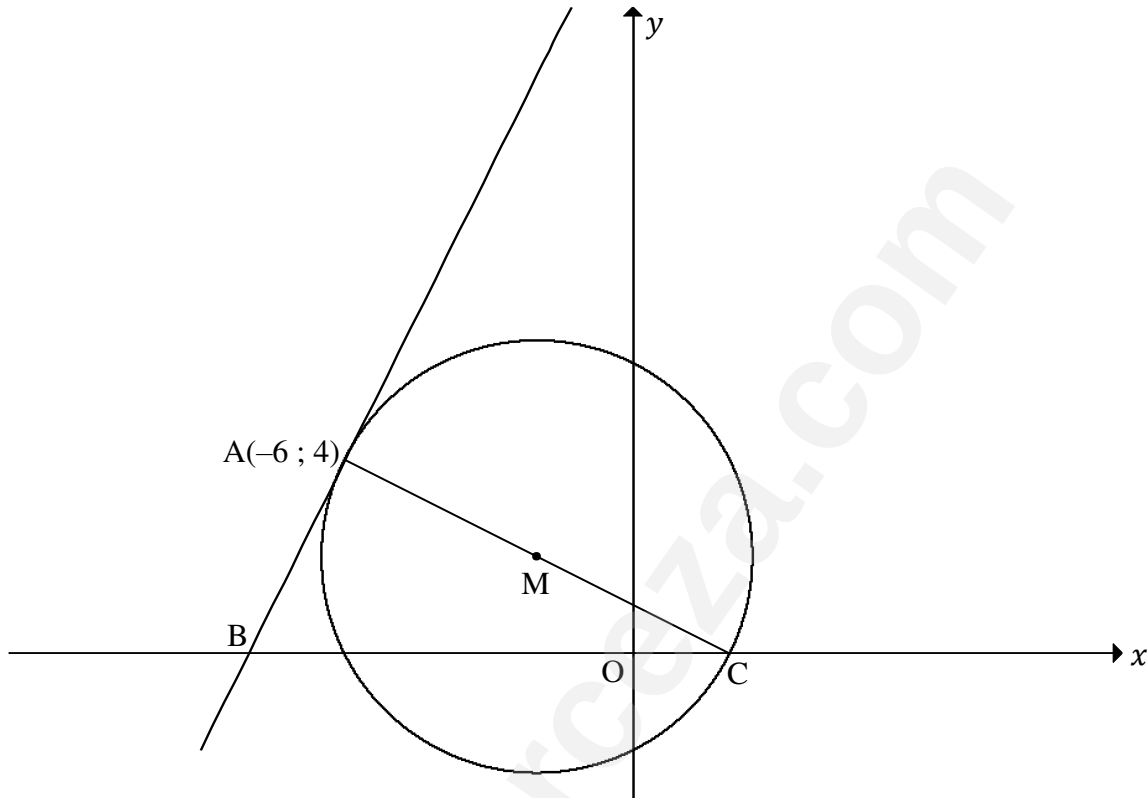
Determine:

- 3.1 The equation of SR. (1)
- 3.2 The gradient of QP to the nearest integer. (2)
- 3.3 The equation of QP in the form $y = mx + c$ (2)
- 3.4 The length of QR. Leave your answer **in surd form**. (2)
- 3.5 $\tan(90^\circ - \theta)$ (3)
- 3.6 The area of $\triangle RSN$, **without using a calculator**. (6)

[16]

QUESTION 4

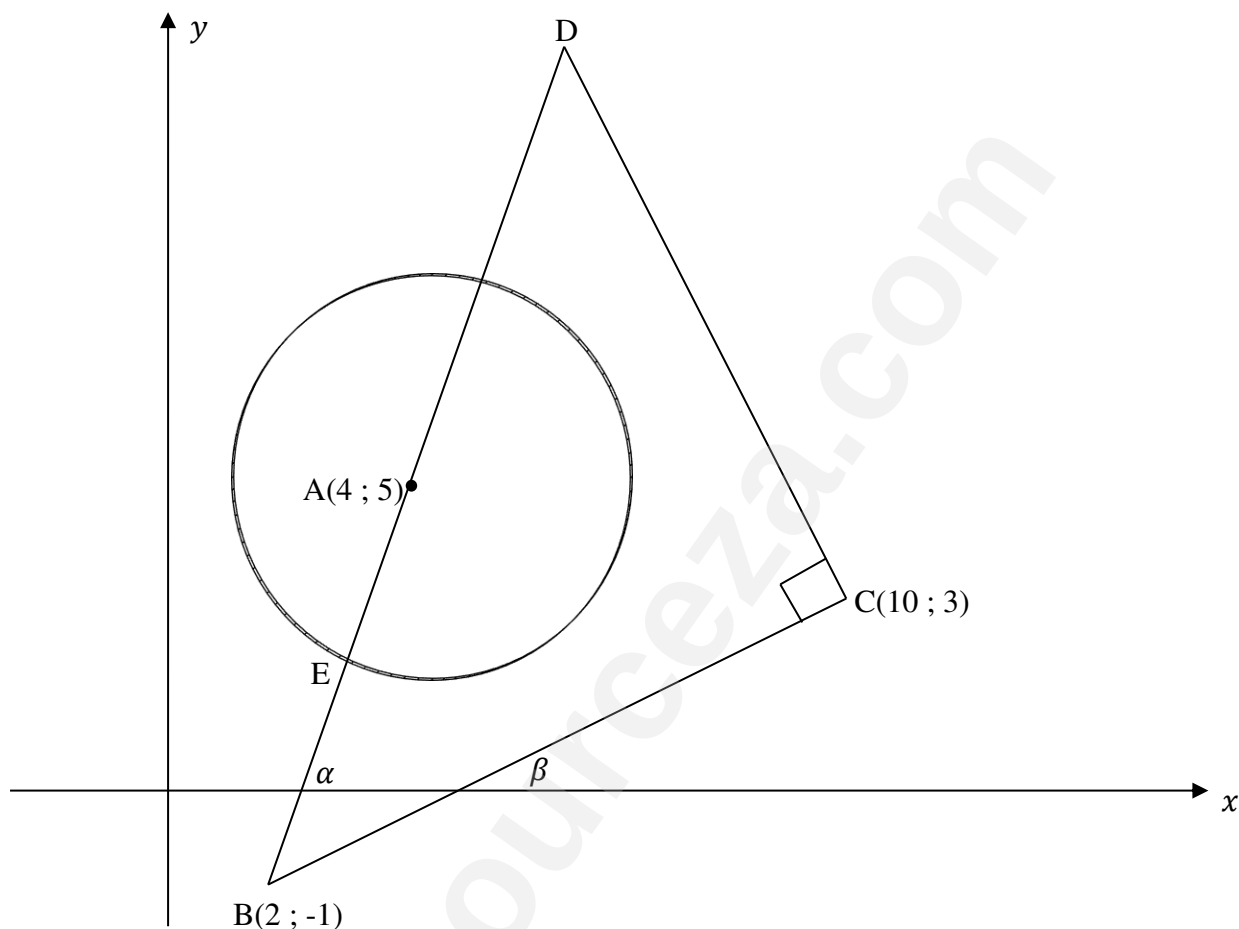
In the diagram, the circle with centre M and equation $x^2 + y^2 + 4x - 4y - 12 = 0$ is drawn. C is the x -intercept of the circle. The tangent AB touches the circle at A($-6 ; 4$) and cuts the x -axis at B.



- 4.1 Calculate the:
- 4.1.1 Coordinates of M. (2)
- 4.1.2 Coordinates of C. (3)
- 4.2 Determine, giving reasons, the equation of the tangent AB in the form $y = mx + c$ if it is given that the gradient of MC is $-\frac{1}{2}$. (4)
- 4.3 Calculate the area of $\triangle ABC$. (5)
- 4.4 Determine for which values of k the line $y = 2x + k$ will intersect the circle at two points. (5)
- [19]**

QUESTION 5

In the diagram below, $A(4 ; 5)$ is the centre of the circle passing through E , the midpoint of AB . The points $B(2 ; -1)$, $C(10 ; 3)$ and D are given such that $BC \perp DC$. The equation of the line through BA is given by $y = 3x - 7$.

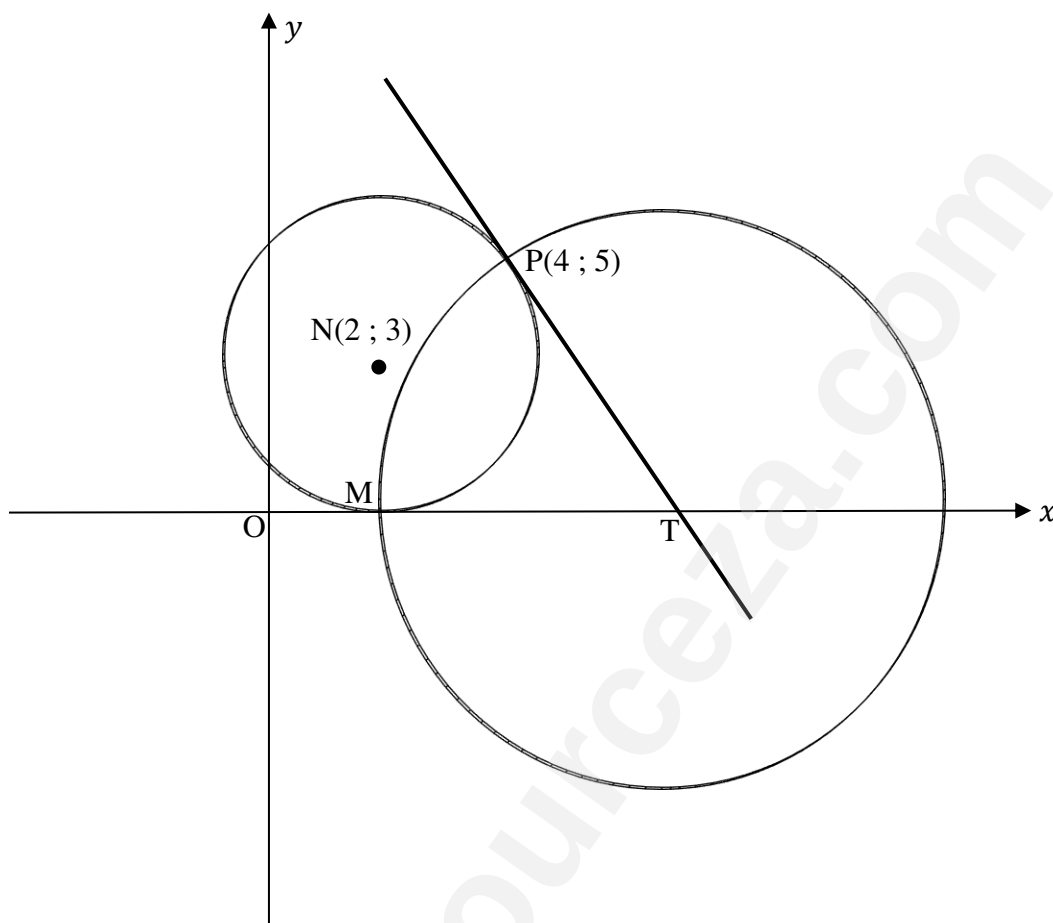


Determine:

- 5.1 the coordinates of E , the midpoint of AB . (2)
 - 5.2 the equation of the circle with centre A passing through E in the form $(x - a)^2 + (y - b)^2 = r^2$. (3)
 - 5.3 the gradient of BC . (2)
 - 5.4 the equation of line DC . (3)
 - 5.5 Calculate the coordinates of D . (4)
 - 5.6 Calculate the size of $\widehat{EBC}$. (4)
- [18]**

QUESTION 6

In the figure below, T is a point on the x -axis. A circle having T as its centre intersects a circle having N(2 ; 3) as its centre at P(4 ; 5) and M. TP is a tangent to the circle centre N at P.

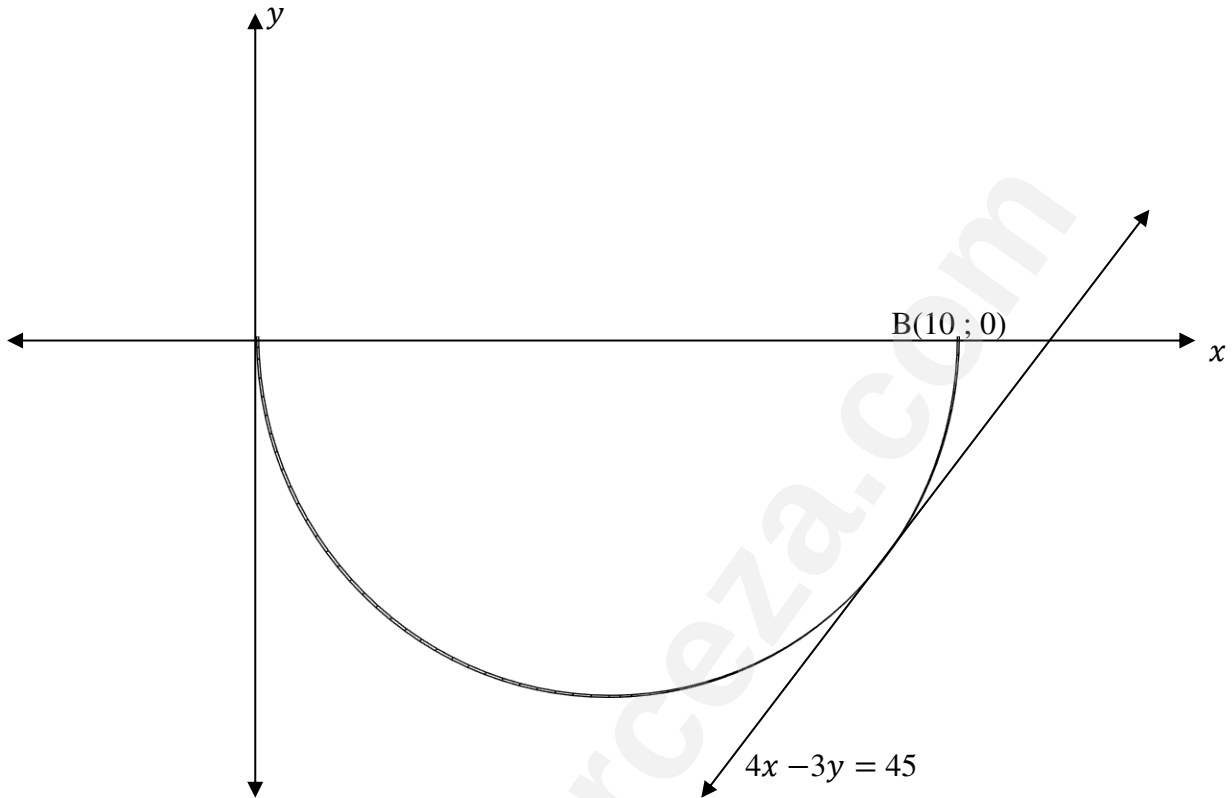


- 6.1 Determine the equation of the circle centre N in the form: $(x-a)^2 + (y-b)^2 = r^2$ (4)
- 6.2 Calculate the coordinates of T, the x -intercept of PT. (6)
- 6.3 Calculate the length of PT. Leave your answer in surd form. (2)
- 6.4 Calculate the area of circle centre T. Give your answer rounded off to the nearest integer. (2)
- 6.5 Calculate the size of $\hat{NTP}$, correct to ONE decimal place. (2)
- 6.6 Prove that MNPT is a kite. (3)
- 6.7 Calculate the size of $\hat{MNP}$, correct to ONE decimal place. (4)

[23]

QUESTION 7

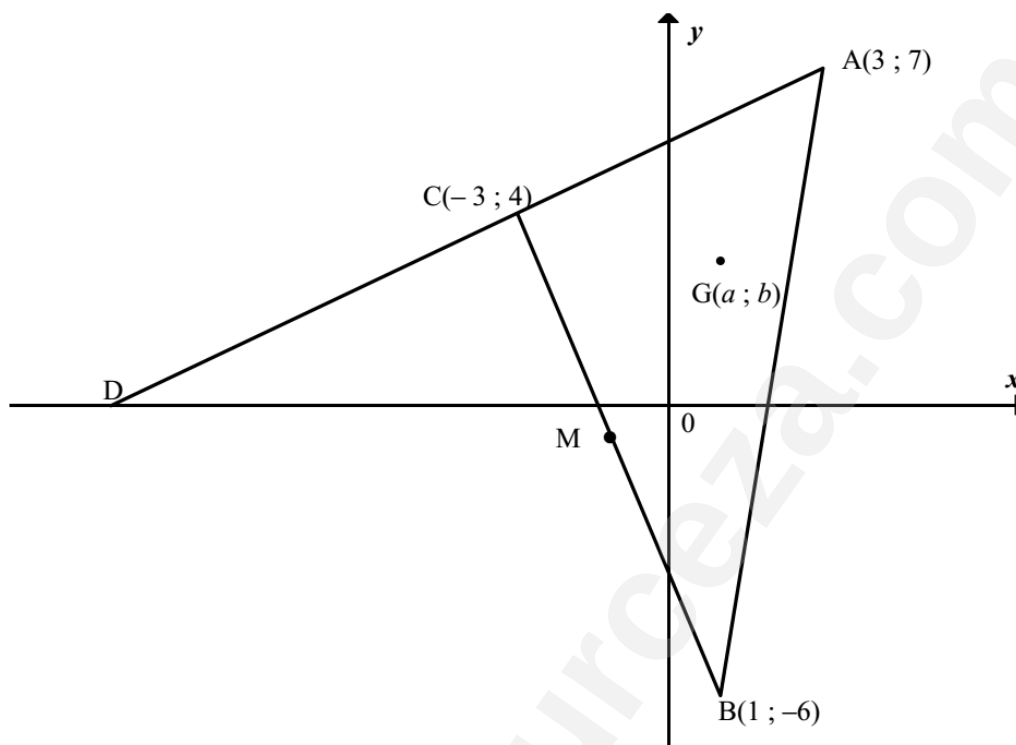
The diagram below shows a semi-circle with diameter OB along with the line $4x - 3y = 45$.



- 7.1 Determine the equation of the semi-circle. (2)
- 7.2 Prove that the line $4x - 3y = 45$ is a tangent to the circle. (5)
- [7]

QUESTION 8

In the diagram below, A, B and C are vertices of a triangle. AC is extended to cut the x -axis at D.



- 8.1 Calculate the gradient of:
- 8.1.1 AD (1)
- 8.1.2 BC (2)
- 8.2 Calculate the size of $\angle D\hat{C}B$. (3)
- 8.3 Write down the equation of the straight line AD. (2)
- 8.4 Determine the coordinates of M, the midpoint of BC. (2)
- 8.5 If $G(a; b)$ is a point such that A, G and M lie on the same straight line, show that $b = 2a + 1$. (4)
- 8.6 Hence calculate TWO possible values of b if $GC = \sqrt{17}$ (6)
- [20]**

(ALL RELEVANT FORMULAE PROVIDED IN THE FORMULA SHEET)

QUESTION 1

1.1 If $\sin 24^\circ = p$, express the following in terms of p :

1.1.1 $\cos 24^\circ$ (3)

1.1.2 $\sin 48^\circ$ (3)

1.1.3 $\sin 12^\circ \cos 12^\circ - \sin(-66^\circ) \tan 204^\circ$ (5)

1.2 Simplify fully without using a calculator:

$$\frac{\tan(180^\circ - \theta) \cdot \cos(-\theta) \cdot \sin 390^\circ}{(\cos 300^\circ \cdot \sin \theta) - \cos 450^\circ}$$
 (6)

1.3 Determine, without using a calculator, the general solution of the equation: (Mind Action gr12)

$$\cos(90^\circ + 2x) = \tan^2 495^\circ \cdot \cos(-210^\circ)$$
 (6)

1.4 Prove the identity:
$$\frac{\cos 2x}{(\cos x + \sin x)^3} = \frac{\cos x - \sin x}{1 + \sin 2x}$$
 (5)

1.5 Given that $\sin \theta = \frac{1}{3}$, calculate the numerical value of $\sin 3\theta$, WITHOUT using a calculator. (5)
[33]

QUESTION 2

2.1 If $\sin \theta = \frac{2k}{1+k^2}$, $0 < k < 1$ and $90^\circ \leq \theta \leq 270^\circ$ Determine with the aid of a diagram, the value of $\frac{1}{\sin \theta} + \frac{1}{\tan \theta}$ in terms of k . (6)2.2 If $\sin(\theta - 45^\circ) = m \sin(\theta + 45^\circ)$, $m \neq 1$, determine $\tan \theta$ in terms of m . (6)

2.3 Simplify each of the following without the use of a calculator:

2.3.1
$$\frac{\tan(180^\circ + x) \cdot \cos(360^\circ - x)}{\sin(180^\circ - x) \cdot \cos(90^\circ + x) + \cos(540^\circ + x) \cdot \cos(-x)}$$
 (8)

2.3.2
$$\frac{1 - \cos 2x - \sin x}{\sin 2x - \cos x}$$
 (5)

2.4 Find the value of x if $\cos x$; $\sin x$; $\sqrt{3} \sin x$ is a geometric sequence. (3)

[28]

QUESTION 3

3.1 If $\cos 18^\circ = k$, express the following in terms of k , without the use of a calculator:

3.1.1 $\sin 108^\circ$ (2)

3.1.2 $\cos(-36^\circ)$ (3)

3.2 Given that $\tan \theta = p$ in any right-angled triangle:

3.2.1 Show that $\sin 2\theta = \frac{2p}{p^2 + 1}$ (2)

3.2.2 Hence, or otherwise, calculate the maximum value of $\frac{(p+1)^2}{p^2 + 1}$ (3)

3.3 If $\sin \theta = \frac{2n}{n^2 + 1}$; $n > 1$ and $0^\circ < \theta < 90^\circ$, prove that $\frac{1 + \sin \theta}{\cos \theta} = \frac{n + 1}{n - 1}$. (7)

3.4 If $\sin 37^\circ = p$, express the following in terms of p :

3.4.1 $\sin 577^\circ$ (2)

3.4.2 $\cos 16^\circ$ (4)

3.5 If $4 \tan A = 3$ and $3 \sin B - 1 = 0$, use a sketch and determine the value of:
 $5 \sin A - 6 \cos^2 B$ where $180^\circ \leq A \leq 360^\circ$ and $0^\circ \leq B \leq 90^\circ$. (5)

3.6 If $\cos \beta + \sin \beta = T$, express $\frac{\cos 2\beta}{\sin(\beta - 45^\circ)}$ in terms of T (5)

3.7 If $\sin 18^\circ = t$, express $\cos 72^\circ \sin 198^\circ$ in terms of t . (3)

3.8 If $\sin K = \frac{5}{13}$ and $\cos K = \frac{a}{39}$, determine, without the use of a calculator, the value of a if:

3.8.1 K is acute. (4)

3.8.2 K is obtuse. (2)

QUESTION 4

4.1 Consider: $\sin(2x + 40^\circ)\cos(x + 30^\circ) - \cos(2x + 40^\circ)\sin(x + 30^\circ)$

4.1.1 Write as single trigonometric term in its simplest form. (2)

4.1.2 Determine the general solution of the following equation:
 $\sin(2x + 40^\circ)\cos(x + 30^\circ) - \cos(2x + 40^\circ)\sin(x + 30^\circ) = \cos(2x - 20^\circ)$ (7)

4.2 Simplify, the following expression to one trigonometric ratio of θ :

$$\frac{\sin(180^\circ - \theta)\sin(540^\circ - \theta)\cos(\theta - 90^\circ)}{\tan(-\theta)\sin^2(360^\circ - \theta)} \quad (7)$$

4.3 Simplify each of the following without the use of a calculator:

4.3.1
$$\frac{8\cos A \cos 2A}{2\cos^3 A - \sin 2A \sin A} \quad (4)$$

4.3.2
$$\frac{2\cos(90^\circ - x)}{\sin(180^\circ - 2x)} \times \frac{\cos(60^\circ - x)\cos x - \sin(60^\circ - x)\sin x}{\tan(-x)} \quad (7)$$

4.3.3
$$\sqrt{2}\cos(-45^\circ) + \cos 210^\circ - \tan 840^\circ \quad (5)$$

4.3.4
$$\cos(x + 65^\circ)\cos(x + 20^\circ) - \sin(x + 245^\circ)\sin(x + 20^\circ) \quad (4)$$

4.4.5
$$\sin 20^\circ \cdot \cos 320^\circ + \cos(-20^\circ)\sin 400^\circ \quad (5)$$

4.4.6
$$\frac{\tan(180^\circ - \theta)\cos(-\theta)\sin 390^\circ}{(\cos 300^\circ \cdot \sin \theta) - \cos 450^\circ} \quad (5)$$

QUESTION 5

5.1 If $\tan \theta = p$, express $\frac{1 - \sin 2\theta}{1 - \sin 2\theta + \cos 2\theta}$ in terms of p . (6)

5.2 5.2.1 If $\cos \beta - \sqrt{3} \sin \beta = 2\sqrt{3} \cos(\beta - 30^\circ)$, show that $\tan \beta = -\frac{1}{\sqrt{3}}$ (4)

5.2.2 Hence, or otherwise, determine the general solution to $\cos \beta - \sqrt{3} \sin \beta = 2\sqrt{3} \cos(\beta - 30^\circ)$ (3)

5.3 (a) Prove the following identity: $\tan \theta \sin \theta + \cos \theta = \frac{1}{\cos \theta}$ (3)

(b) Hence, or otherwise, determine the general solution to: $\tan \theta \sin \theta + \cos \theta = \frac{3}{\cos \theta}$ (5)

5.4 Prove that $\frac{\cos(A - 45^\circ)}{\cos(A + 45^\circ)} = \frac{1 + \sin 2A}{\cos 2A}$ (6)

5.5 (a) Prove the identity: $\frac{\sin 3\alpha}{\sin \alpha} = 3 - 4 \sin^2 \alpha$ (6)

(b) Hence, determine the general solution to $\frac{\sin 3\alpha}{\sin \alpha} = 2$. (4)

5.6 Prove that $\frac{\cos \theta}{\sin 2\theta} - \frac{\cos 2\theta}{2 \sin \theta} = \sin \theta$ (6)

5.7 Calculate the general solution for θ if $3 \sin^2 \theta = 2 \sin \theta$. Give answer correct to one decimal place. (6)

5.8 Determine the general solution for θ if $\sin 2\theta + \cos \theta = 0$ (8)

5.9 Prove that $\frac{\sin \theta}{1 + \sin(90^\circ + \theta)} - \frac{\sin(-\theta)}{1 - \cos(-\theta)} = \frac{2}{\sin \theta}$ (6)

5.10 If $M = \frac{2 \sin^2(180^\circ - \theta) + \sin 2\theta}{\cos 2\theta}$ and $P = \frac{2 \sin \theta}{\cos \theta - \sin \theta}$

Prove that $M = P$. (5)

QUESTION 6

6.1 (a) Prove that $\cos(\alpha - 30^\circ) - \cos(\alpha + 30^\circ) = \sin \alpha$ (4)

(b) Hence, determine the general solution to the equation: $\cos(\alpha - 30^\circ) - \cos(\alpha + 30^\circ) = \cos 3\alpha$ (6)

6.2 Simplify $\frac{-\cos(90^\circ + \theta) - \sin^3 \theta}{\sin 2\theta}$ (5)

6.3 Simplify $\frac{\sqrt{1 - \cos^2 2A}}{\cos(-A) \cdot \cos(90^\circ + A)}$ (5)

6.4 If $\sin \theta = \frac{1}{4}$, find the value of $\sin 3\theta$ without the use of a calculator. (5)

6.5 Solve the following equation $\sin 2x = \tan 225^\circ \cdot \cos 210^\circ$ for $x \in [0^\circ; 180^\circ]$ (7)

QUESTION 7

7.1 Simplify:

$$\frac{\sin(180^\circ - 2x) \cos x}{2 \cos(90^\circ - x)} - \tan(180^\circ - x) \cos(180^\circ + x) \sin(-x - 720^\circ)$$
 (7)

7.2 Without using a calculator, prove that:

$$\frac{\sin 234^\circ}{\cos 36^\circ} - \frac{\sin(x - 90^\circ) \cos(90^\circ - 2x)}{\sin(x - 360^\circ)} = \cos 2x$$
 (7)

7.3 Given: $\sin 40^\circ = m$

Express the following in terms of m without the use of a calculator:

$$\frac{\cos 310^\circ}{\sin 150^\circ} + 4 \sin 140^\circ \tan 135^\circ$$
 (7)

7.4 Consider: $f(x) = -\frac{1}{2} \sin(x + 30^\circ)$

7.4.1 Determine the general solution of $f(x) = 0$. (4)

7.4.2 Write down the minimum value of $f(x)$. (1)

7.4.3 Determine the range of $f(x) + 1$. (2)

7.4.4 The graph of f is shifted 60° to the left and then reflected about the x -axis to form a new graph h . Determine the equation of h in its simplest form. (3)

QUESTION 8

8.1 If $\sin 54^\circ = m$, express the following in terms of m , **without using a calculator**:

8.1.1 $\sin 594^\circ$ (3)

8.1.2 $\cos 18^\circ$ (5)

8.2 If $\cos(A - B) = \cos A \cos B + \sin A \sin B$, derive a formula for $\sin(A + B)$. (3)

8.3 8.3.1 For which value(s) of x will $\frac{2 \tan x - \sin 2x}{2 \sin^2 x}$ be undefined in the interval $0^\circ \leq x \leq 180^\circ$? (3)

8.3.2 Prove the identity: $\frac{2 \tan x - \sin 2x}{2 \sin^2 x} = \tan x$ (6)

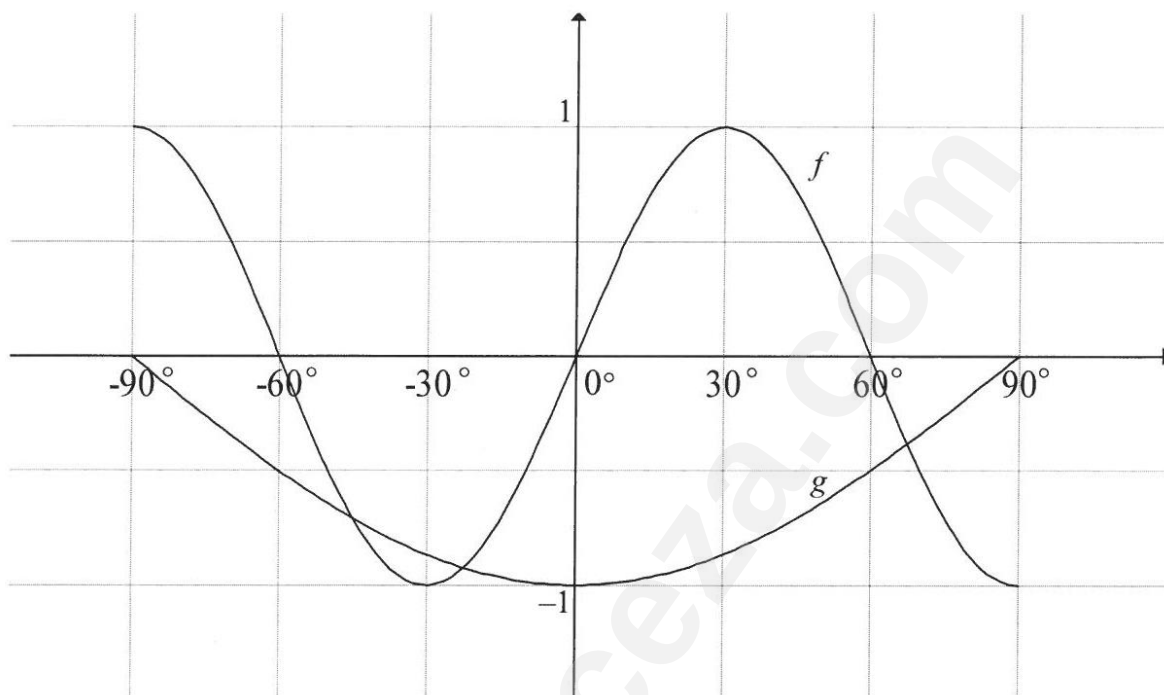
8.4 Simplify the following expression to one trigonometric ratio of x :

$$\frac{\sin(180^\circ - x) \cdot \sin(540^\circ - x) \cdot \cos(x - 90^\circ)}{\tan(-x) \cdot \sin^2(360^\circ - x)} \quad (7)$$

8.5 Given: $4 \tan \theta + 5 = 0$ and $\theta \in [0^\circ; 180^\circ]$. Determine, **without the use of a calculator**, the value of: $\sqrt{41} \cos \theta - 4 \sin(-150^\circ) \cdot \cos 180^\circ$ (6)
[33]

QUESTION 9

In the diagram below the graphs of $f(x) = \sin bx$ and $g(x) = -\cos x$ are drawn for $-90^\circ \leq x \leq 90^\circ$. Use the diagram to answer the following questions.



- 9.1 Write down the period of f . (1)
- 9.2 Determine the value of b . (1)
- 9.3 The general solutions of the equation $\sin bx = -\cos x$ are $x = 67,5^\circ + k.90^\circ$ or $x = 135^\circ + k.180^\circ$ where $k \in \mathbb{Z}$. Determine the x -values of the points of intersection of f and g for the given domain. (3)
- 9.4 Write down the values of x for which $\sin bx + \cos x < 0$ for the given domain. (4)

[9]

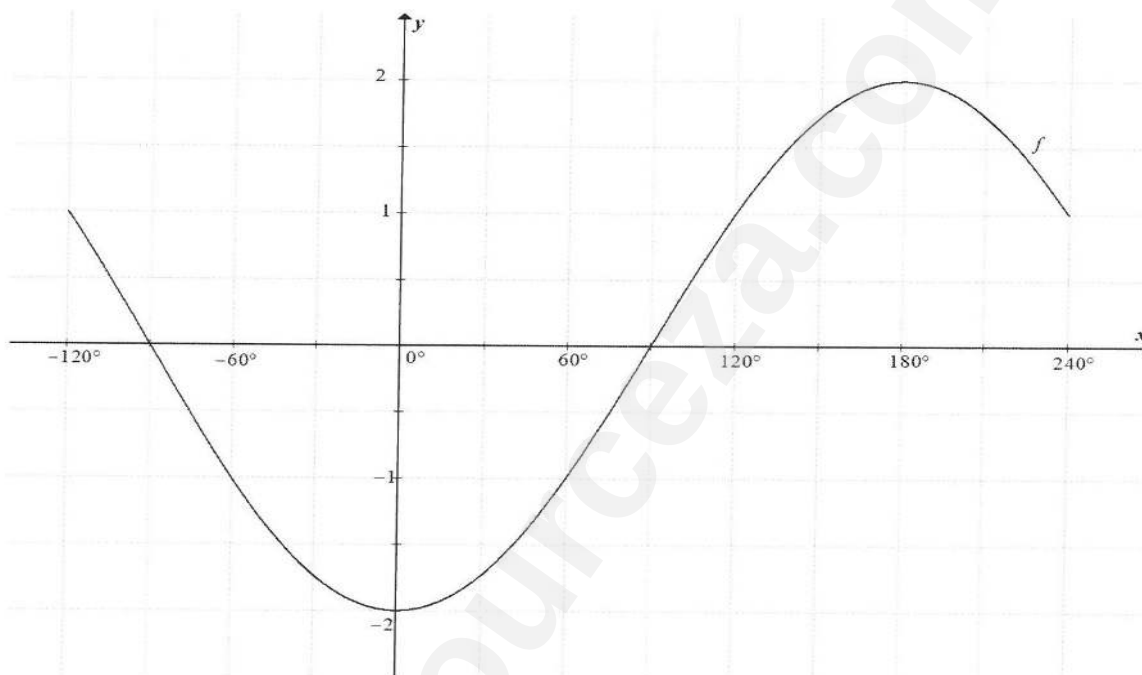
QUESTION 10

Given the equation: $\sin(x + 60^\circ) + 2\cos x = 0$

10.1 Show that the equation can be written as $\tan x = -4 - \sqrt{3}$. (4)

10.2 Determine the solutions of the equation $\sin(x + 60^\circ) + 2\cos x = 0$ in the interval $-180^\circ \leq x \leq 180^\circ$. (3)

10.3 In the diagram below, the graph of $f(x) = -2\cos x$ is drawn for $-120^\circ \leq x \leq 240^\circ$.



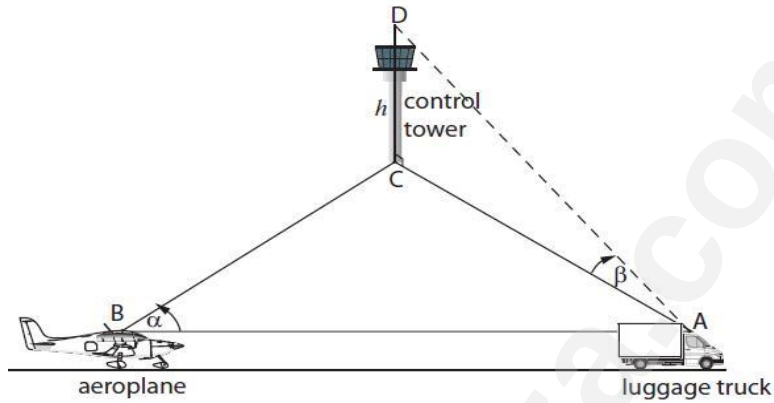
10.3.1 Draw the graph of $g(x) = \sin(x + 60^\circ)$ for $-120^\circ \leq x \leq 240^\circ$ on the grid provided in the ANSWER BOOK. (3)

10.3.2 Determine the values of x in the interval $-120^\circ \leq x \leq 240^\circ$ for which $\sin(x + 60^\circ) + 2\cos x > 0$. (3)
[13]

QUESTION 11

An aeroplane and a luggage truck are in the same horizontal plane on the runway and are equidistant from the foot of the airport traffic control tower.

The angle made between the aeroplane, the runway and the foot of the tower is α . The height of the tower, above the ground, is h and the angle of inclination from the luggage truck to the top of the tower is β .



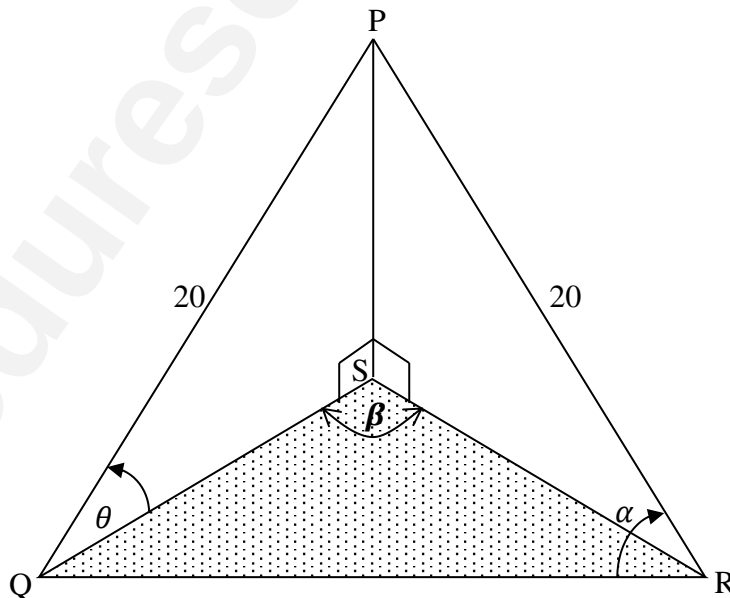
Show that the distance between the aeroplane and the luggage truck can be expressed as:

$$AB = \frac{2h \cdot \cos \alpha}{\tan \beta}$$

[7]

QUESTION 12

In the figure, Q, S and R lie in the same horizontal plane and SP represents a vertical tower. $\hat{SQP} = \theta$, $\hat{PRQ} = \alpha$ and $\hat{RSQ} = \beta$



12.1 Show that the area of $\Delta RQS = 200 \sin \beta \cdot \cos^2 \theta$

(5)

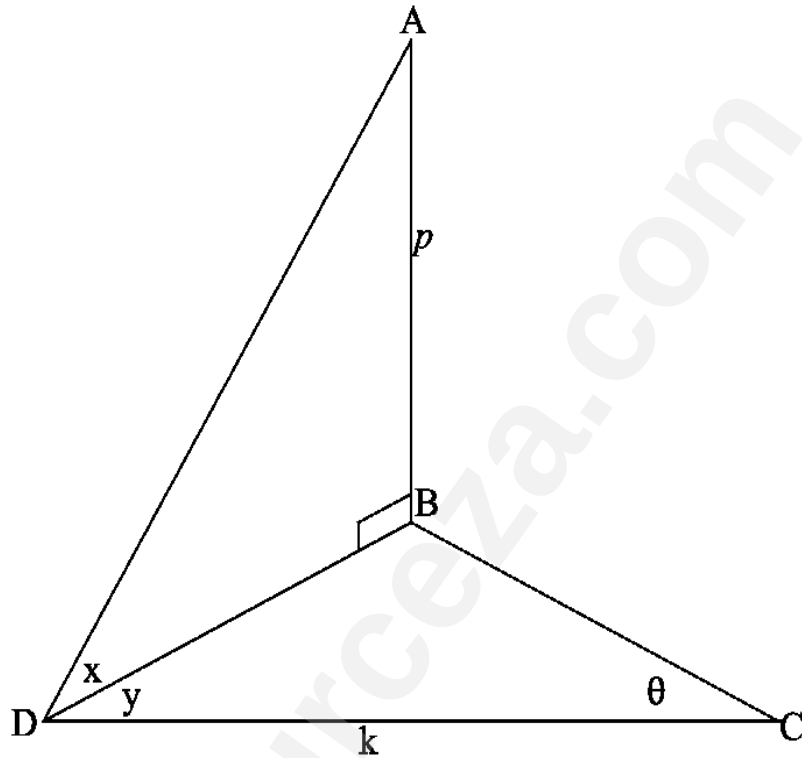
12.2 Show that $QR = 20\sqrt{2 + 2 \cos 2\alpha}$

(5)

[10]

QUESTION 13

AB is a vertical tower of p units high. D and C are in the same horizontal plane as B, the foot of the tower. The angle of elevation of A from D is x . $\hat{BDC} = y$ and $\hat{DCB} = \theta$. The distance between D and C is k units.



13.1 Prove that $p = \frac{k \sin \theta \tan x}{\sin y \cos \theta + \cos y \sin \theta}$ (5)

13.2 Find BC to the nearest meter if $x = 51,7^\circ$, $y = 62,5^\circ$, $p = 80\text{m}$ and $k = 95\text{m}$. (4)
[9]

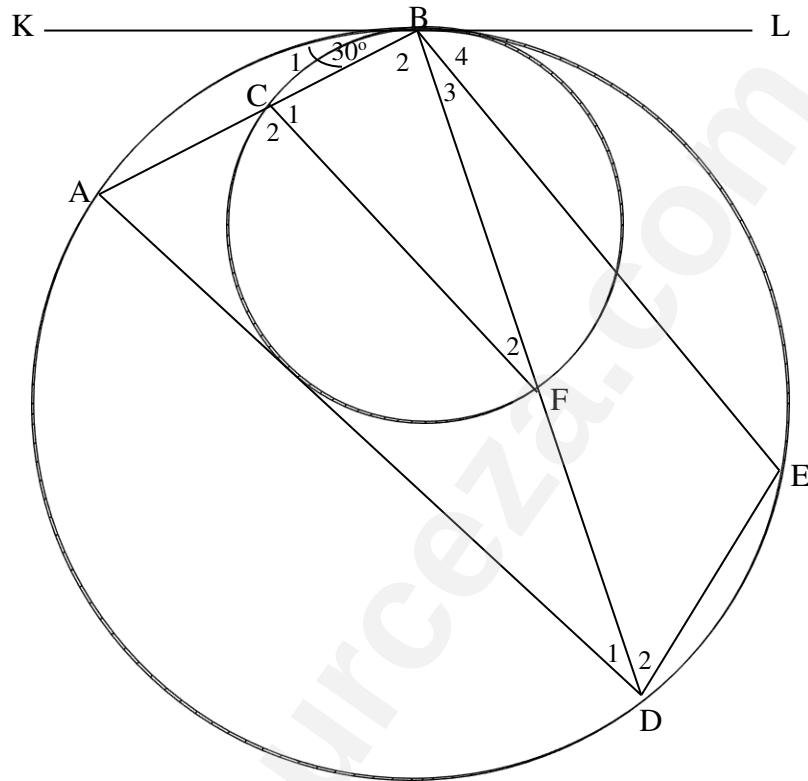
Tips:

- READ the information next to the diagram thoroughly.
- Look for KEYWORDS and underline them, e.g.
 - * Tangents: try to draw small diagrams which relates to tangents and then look at the diagram to see which one will be useful.
 - * Cyclic quad: think of all the properties of cyclic quad and draw theorems which relates to them.
- Fill all info in on sketch as you work.
 - * NB: Read carefully on the information provided and immediately indicate the following on your diagram
 1. Centre of the circle
 2. Radii that are equal =
 3. Angle in semi-circle
 4. Tangent $\rightarrow$ indicate that the angles are equal
 5. Tangent $\perp$ radius $\rightarrow$ indicate
 6. Any other angle that you see that are equal
- NEVER ASSUME something!!!!!!
 - * Don't assume that a certain line is a diameter of a circle unless it is clearly stated or unless you can prove it.
 - * Don't assume that a point is the centre of a circle unless it is clearly stated.
- When asked to prove that $AB = BC$, you must prove that $\hat{A} = \hat{C}$ (and vice versa).
- When asked to prove that line AC is a tangent to the circle, first prove that the line is perpendicular to radius OB.
- Three ways to prove that a quadrilateral is a cyclic quad.
Prove that:
 - * One pair of opposite angles are supplementary (opp $\angle$'s sup)
 - * The exterior angle is equal to the opposite interior angle (ext $\angle =$ int opp $\angle$)
 - * Two angles subtended by a line segment at two other vertices of the quadrilateral, are equal (line subt = $\angle$'s)
- When asked to prove that line AB is parallel to DC ($AB \parallel DC$), you must prove one of the following properties:
 - * Corresponding angles
 - * Alternating angles
 - * Co-interior angles
- Prove that $\triangle RST \sim \triangle PQT$
 - * T is found on both Δ s, then it is common
 - * Then first letter in both Δ 's: R in $\triangle RST$ and P in $\triangle PQT$ are both equal (provide reason)
 - * S in $\triangle RST$ and Q in $\triangle PQT$ is the remaining angle which are equal
 - * $\therefore \triangle RST \sim \triangle PQT$ ($\angle \angle \angle$)

QUESTION 1

In the diagram below ABED is a cyclic quadrilateral. KBL is a tangent to both circles at B. AD is the diameter of the larger circle.

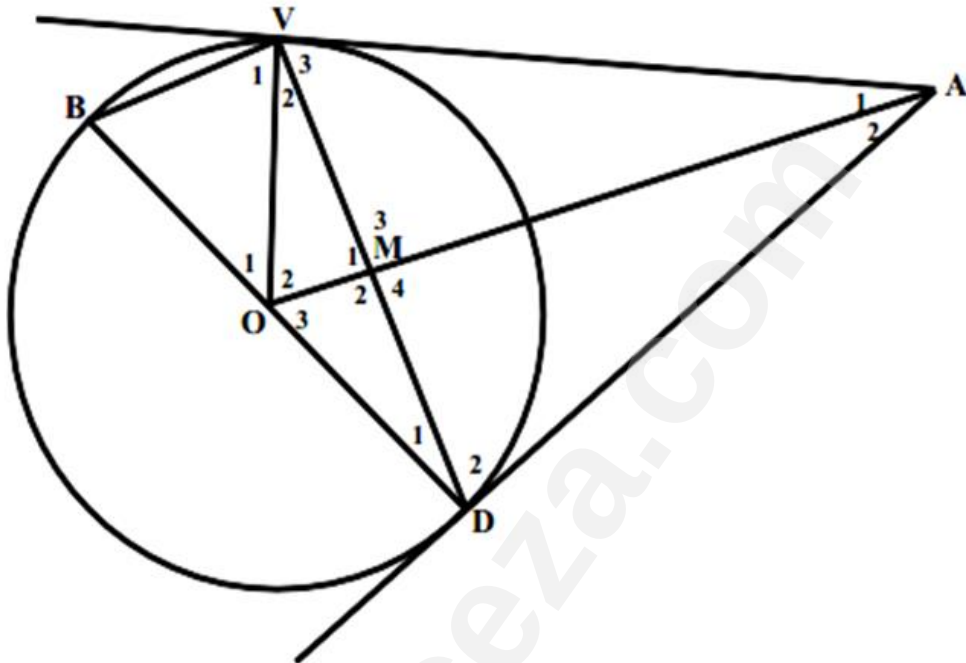
$$\hat{B}_1 = 30^\circ$$



- 1.1 Why is $\hat{B}_2 = 90^\circ$? (1)
 - 1.2 Calculate the size of $\hat{F}_2$? (2)
 - 1.3 Prove that $CF \parallel AD$ (3)
 - 1.4 Calculate the size of $\hat{E}$. (3)
- [9]**

QUESTION 2

From a point outside the circle, centre O, two tangents AD and AV are drawn. AO and VD meet in M. BOD is a diameter of the circle. BV and VO are drawn and $\hat{A}_1 + \hat{A}_2 = 40^\circ$.



2.1 Complete the following table by filling in the reasons:

	Statement	Reason
2.1.1	$\hat{A}\hat{D}O = 90^\circ$	
2.1.2	$\hat{B}\hat{V}D = 90^\circ$	

(2)

2.2 Calculate, with reasons, the size of:

2.2.1 $\hat{D}$ (4)

2.2.2 $\hat{O}_1$ (2)

2.3 Prove, with reasons, that BV is parallel to OA. (3)
[11]

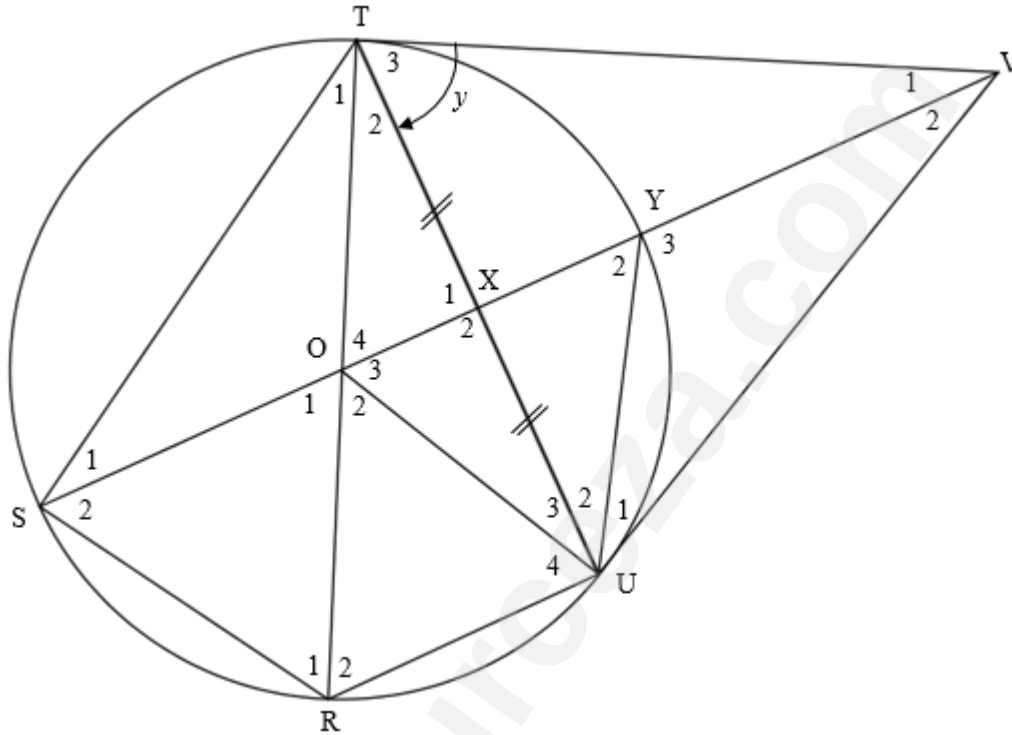
QUESTION 3

TV and VU are tangents to the circle with centre O at T and U respectively.

TSRUY are points on the circle such that RT is the diameter.

X is the midpoint of chord TU.

$\hat{T}_3 = y$.



Prove that:

3.1 $RU \parallel SY$ (5)

3.2 $\hat{T}_1 = \frac{1}{2}y$ (5)

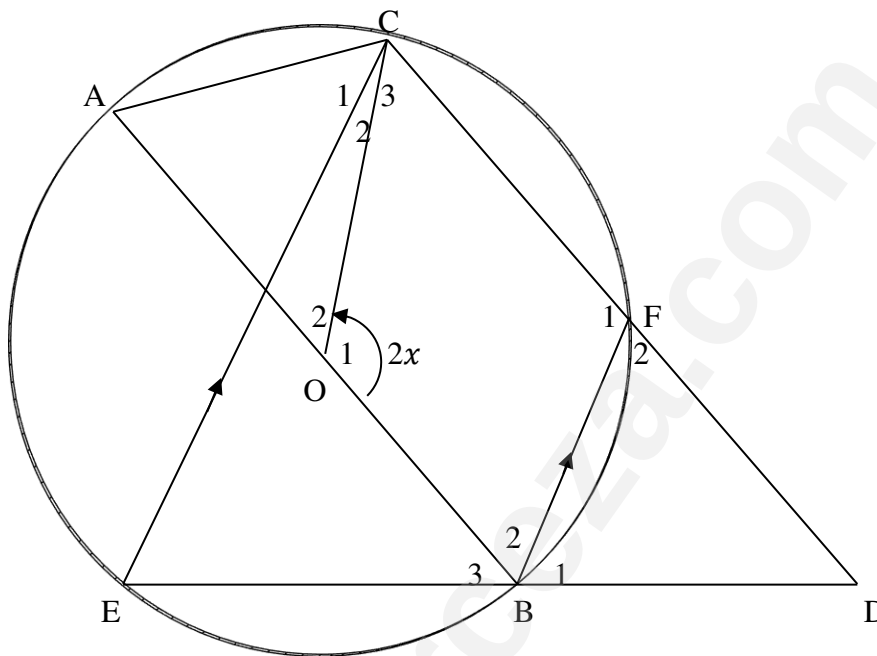
3.3 TOUV is a cyclic quadrilateral. (5)
[15]

QUESTION 4

In the diagram below, O is the centre of the circle. AB is a diameter of the circle. Chord CF produced meets chord EB produced at D. Chord EC is parallel to chord BF. CO and AC are produced.

(Feb/March 2012)

Let $\hat{O}_1 = 2x$

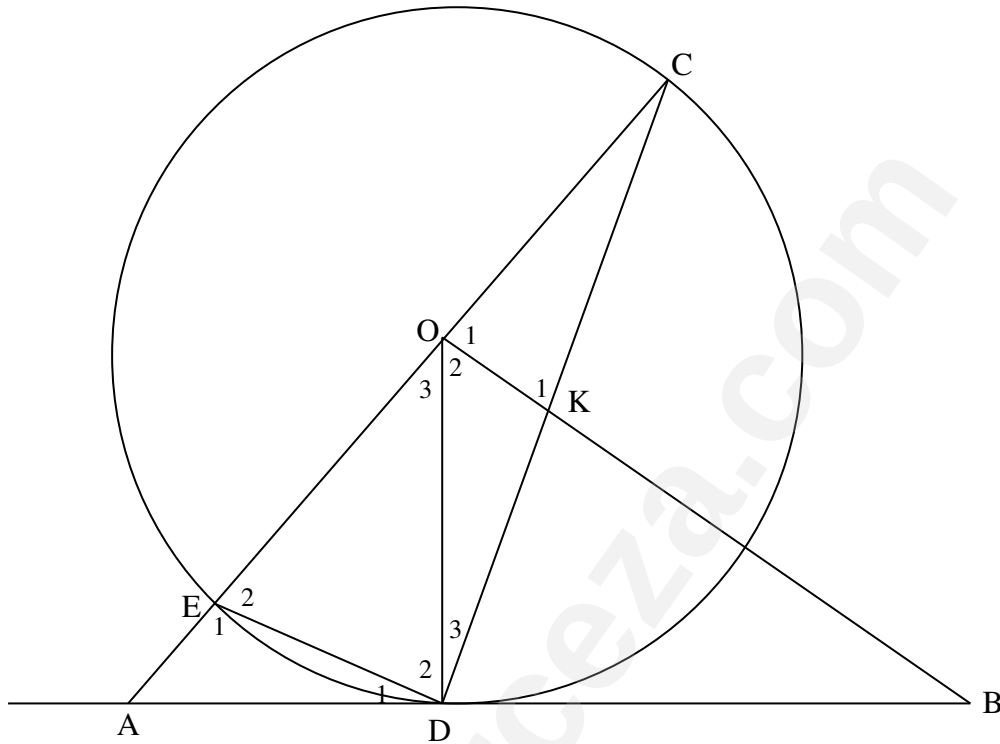


- 4.1 Determine, in terms of x , the size of $\hat{F}_1$. (4)
- 4.2 Prove that $DF = BD$. (4)
- 4.3 Show that $\hat{C}_1 = \hat{C}_3$. (4)
- 4.4 If $DF = 5$ cm and $OA = 6$ cm, calculate area $\triangle BFD$: area $\triangle AOC$. (4)

[16]

QUESTION 5

In the diagram, the circle with centre O is drawn. Points C, D and E lie on the circle. AB is a tangent to the circle at D. OB // ED and $\hat{D}_3 = x$.



5.1 Determine, with reasons, the size of EACH of the following in terms of x :

5.1.1 $\hat{C}$ (1)

5.1.2 $\hat{D}_1$ (1)

5.1.3 $\hat{B}$ (1)

5.2 Determine the size of:

5.2.1 $\hat{K}_1$ (3)

5.2.2 $\hat{ADC}$ in terms of x (3)

5.3 Prove that:

5.3.1 $DK = KC$ (1)

5.3.2 $\triangle ACD \parallel \triangle ADE$ (4)

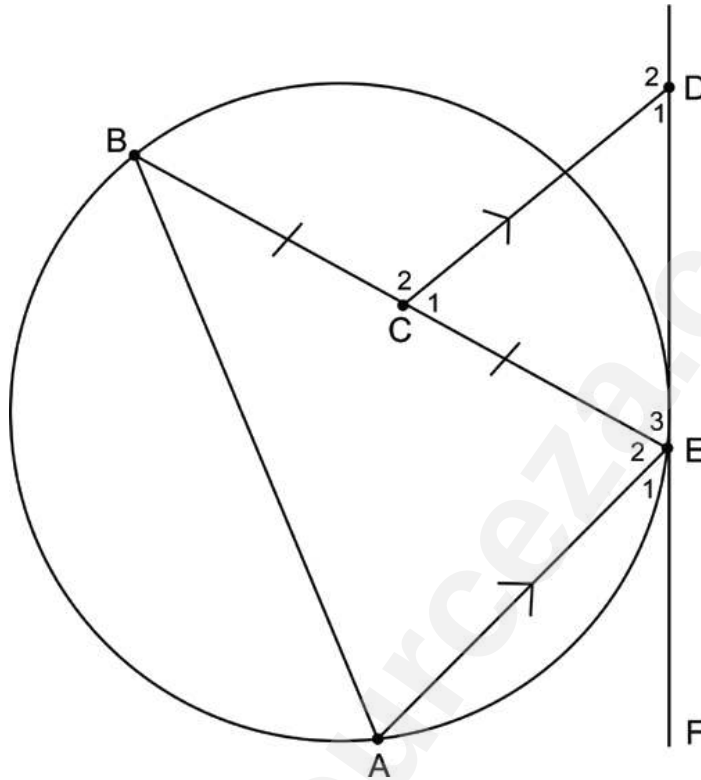
5.3.3 $CK \cdot AD = \frac{AC \cdot ED}{2}$ (4)

[18]

QUESTION 6

In the diagram below:

- FD is a tangent to the circle at point E.
- $AE \parallel CD$.
- A, B and E lie on the circle.
- $BC = CE$.



- 6.1 Prove that $\hat{D}_1 = \hat{B}$. (4)
- 6.2 Prove that $\triangle ABE \sim \triangle EDC$. (3)
- 6.3 Show that $2EC^2 = AE \cdot DC$. (4)
- [11]

QUESTION 7

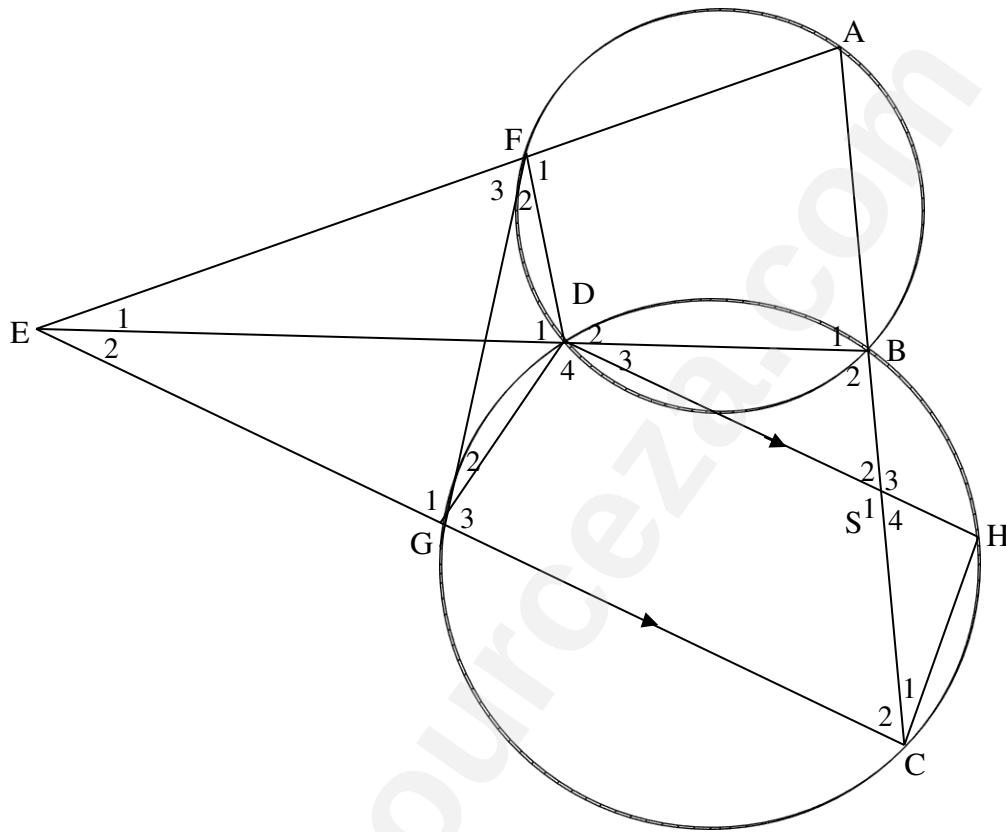
In the diagram below, the two circles intersect at B and D.

ABC is a double chord.

Chord AF of the smaller circle and chord CG of the bigger circle are produced to meet BD produced at E.

Chord DH of the bigger circle is drawn parallel to EC to intersect AC at S.

HC, FG, FD and DG are drawn.



Prove that:

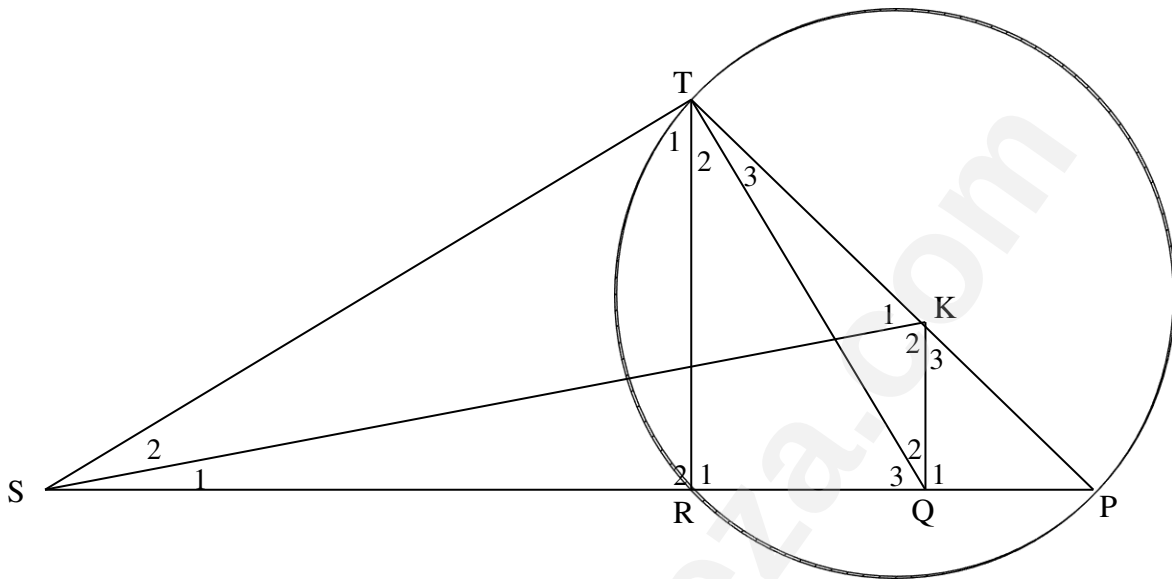
- 7.1 FDGE is a cyclic quadrilateral. (4)
 - 7.2 HC is a tangent to circle BCE. (4)
 - 7.3 $\hat{F}_3 = \hat{S}_2$ (4)
 - 7.4 HS. DE = SC. GD (5)
- [17]

QUESTION 8

In the diagram below, ST is a tangent to circle TRP.

PT is a diameter.

SRQP is a secant, K is a point on PT such that $PK : KT = 1 : 2$, $PR = \sqrt{18}$ units, $PQ = \sqrt{2}$ units.



Prove that:

8.1 $RT \parallel QK$. (4)

8.2 TKQS is a cyclic quadrilateral. (5)

8.3 $\Delta QRT \sim \Delta KTS$ (4)

8.4 If $PS = \sqrt{32}$ units, calculate stating reasons the length of:

8.4.1 ST (5)

8.4.2 KT (4)

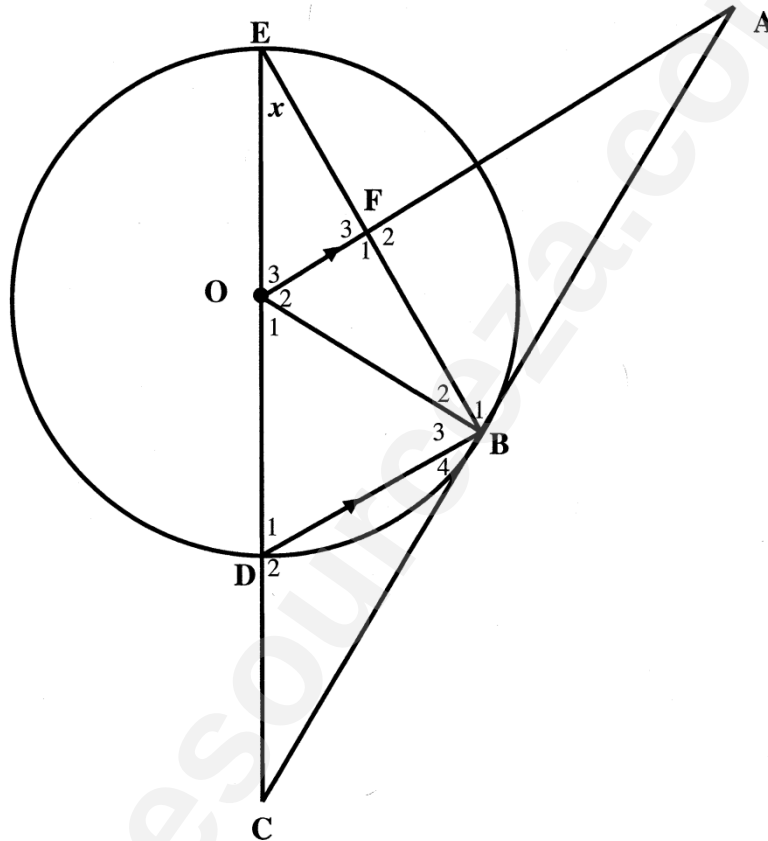
[22]

QUESTION 9

In the diagram below, ED is a diameter of the circle with centre O.

ED is produced to C and CA is a tangent to the circle at B. AO intersects BE at F. $BD \parallel AO$.

Let $\hat{E} = x$



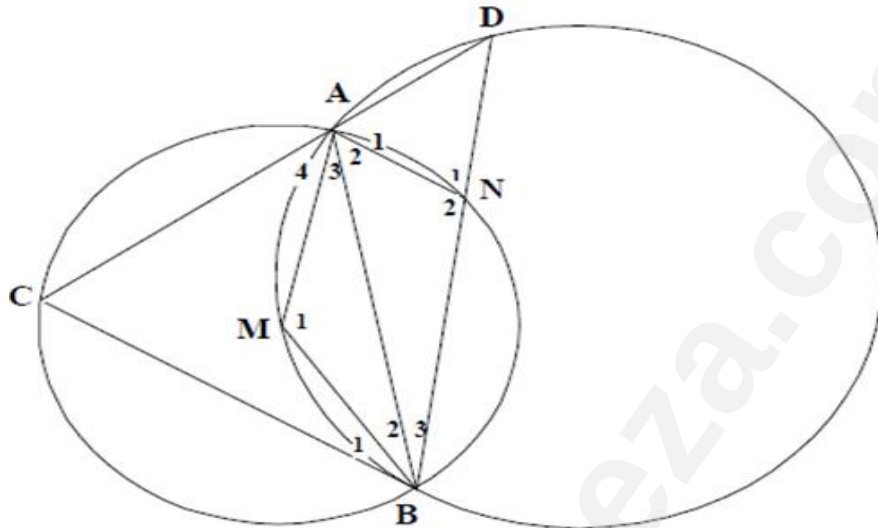
- 9.1 Write down, with reasons, THREE other angles each equal to x . (6)
- 9.2 Determine, with reasons, $\hat{CBE}$ in terms of x . (3)
- 9.3 Prove that F is the midpoint of BE. (5)
- 9.4 Calculate the length of the diameter if it is further given that $EB = 8\text{cm}$ and $OF = 3\text{ cm}$ (4)

[18]

QUESTION 10

In the diagram alongside, circles ACBN and AMBD intersect at A and B.
 CB is a tangent to the larger circle at B.
 M is the centre of the smaller circle.
 CAD and BND are straight lines.

Let $\hat{A}_3 = x$



10.1 Determine the size of $\hat{D}$ in terms of x . (3)

10.2 Prove that:

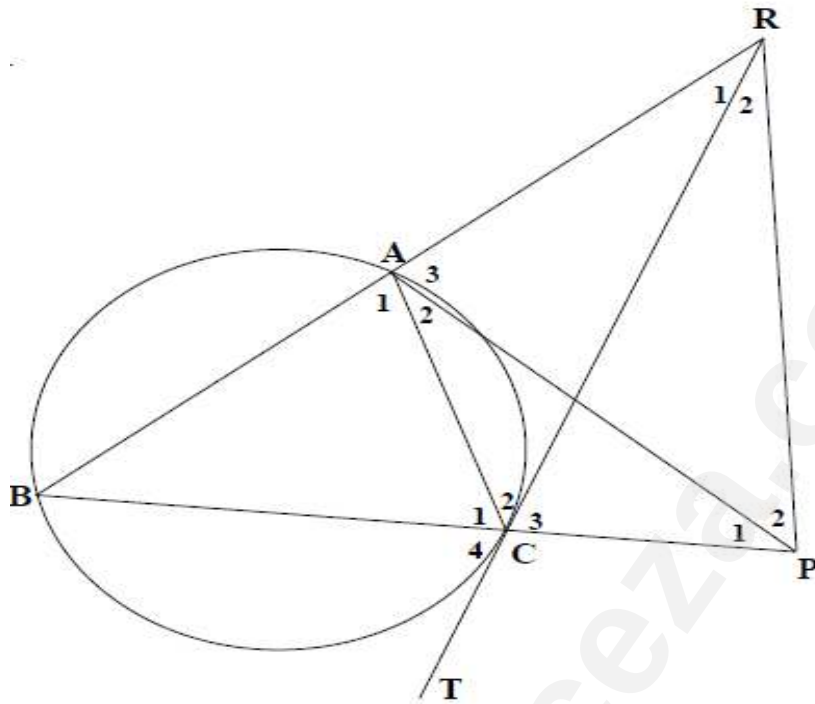
10.2.1 $CB \parallel AN$ (4)

10.2.2 AB is a tangent to circle ADN. (3)

[10]

QUESTION 11

In the diagram below, chord BA and tangent TC of circle ABC are produced to meet at R. BC is produced to P with $RC=RP$. AP is not a tangent.



11.1 ACPR is a cyclic quadrilateral. (4)

11.2 $\triangle CBA \parallel \triangle RPA$ (3)

11.3 $RC = \frac{CB \cdot RA}{AC}$ (3)

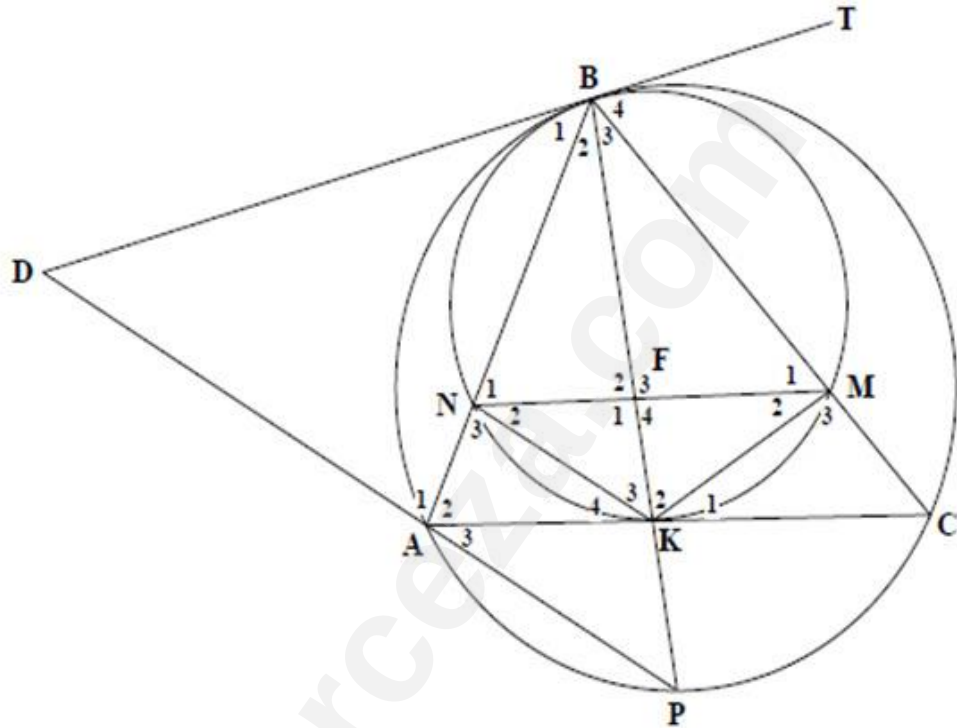
11.4 $RB \cdot AC = RC \cdot CB$ (4)

11.5 Hence, prove that $RC^2 = RA \cdot RB$ (3)

[17]

QUESTION 12

In the diagram, TBD is a tangent to circles BAPC and BNKM at B.
 AKC is a chord of the larger circle and is also a tangent to the smaller circle at K.
 Chords MN and BK intersect at F. PA is produced to D.
 BMC, BNA and BFKP are straight lines.



- 12.1 $MN \parallel CA$ (4)
- 12.2 $\triangle KMN$ is isosceles (3)
- 12.3 $\frac{BK}{KP} = \frac{BM}{MC}$ (6)
- 12.4 DA is a tangent to the circle passing through points A, B and K. (4)

[17]

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} ; r \neq 1$$

$$S_\infty = \frac{a}{1 - r} ; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \Delta ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{area } \Delta ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cos \alpha$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$