



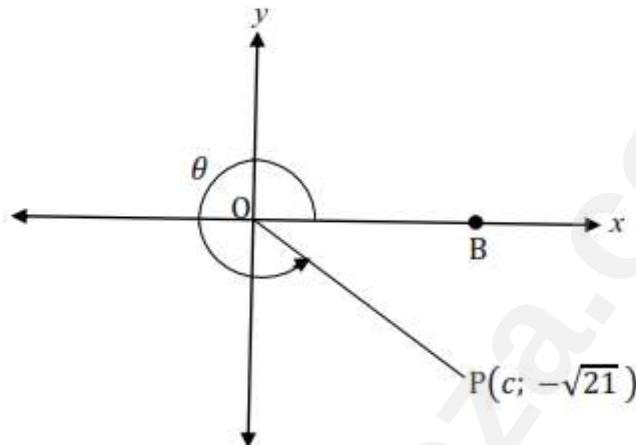
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TOPIC REVISION _TRIGONOMETRY

PREP 2020_FREE STATE

QUESTION 5

- 5.1 In the diagram, P is the point $(c; \sqrt{21})$ such that $OP = 5$ units.
 $\hat{BOP} = \theta$ as indicated.



5.1.1 Calculate the numerical value of c . (2)

5.1.2 Determine (**without the use of a calculator**), the numerical value of the following:

a) $\cos \theta$ (1)

b) $\tan \theta + \sin^2 \theta$ (2)

c) $\sin 2\theta$ (2)

5.2 Simplify (**without the use of a calculator**):

$$\frac{\sin(x-180^\circ) \cdot \tan x \cdot \cos 690^\circ}{\cos^2(x-90^\circ)}$$

(5)
[12]

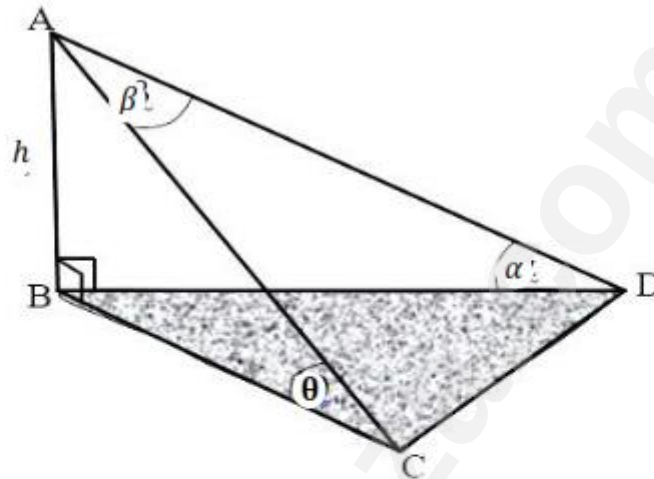
QUESTION 6

6.1 Prove the identity: $\frac{2\sin^2 x}{2\tan x - \sin 2x} = \frac{\cos x}{\sin x}$ (5)

6.2 Show that: $\sin^2 20^\circ + \sin^2 40^\circ + \sin^2 80^\circ = \frac{3}{2}$
 (Hint: $40^\circ = 60^\circ - 20^\circ$ and $80^\circ = 60^\circ + 20^\circ$) (7)
 [12]

QUESTION 7

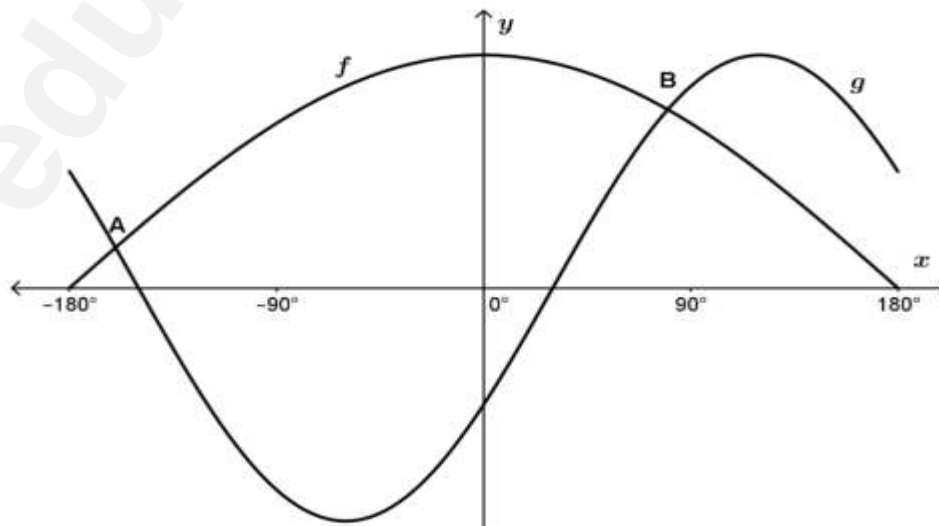
In the figure below Thabo is standing at a point A on top of building AB that is h (m) high. He observes two cars, C and D that are in the same horizontal plane as B. The angle of elevation from C to A is θ and the angle of elevation from D to A is α . $\angle CAD = \beta$.



- 7.1 Calculate the length of AC in terms of h and θ . (2)
 - 7.2 Calculate the length of AD in terms of h and α . (2)
 - 7.3 Determine the distance between the two cars, which is the length of CD in terms of α , θ , β and h . (3)
- [7]

QUESTION 8

In the diagram are the graphs of the functions $f(x) = \cos \frac{x}{2}$ and $g(x) = \sin(x - 30^\circ)$ for $x \in [-180^\circ; 180^\circ]$. The curves intersect at A and B.



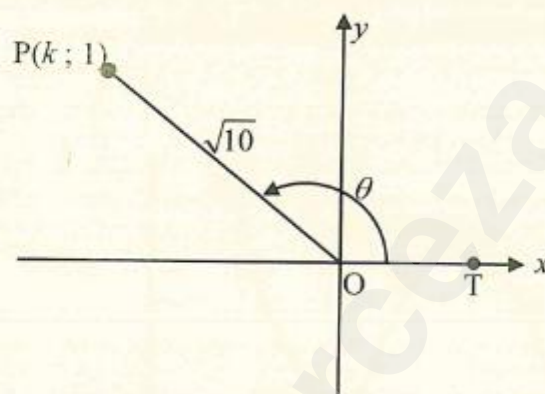
GN (DI) STRATEGY FOR LEARNERS ATTAINMENT

- 8.1 Calculate the x -coordinates of the points A and B. (6)
- 8.2 Determine the values of $x \in [-90^\circ; 180^\circ]$, for which $g(x) \cdot f(x) < 0$. (3)
- [9]

PREP 2020_NORTH WEST

QUESTION 5

- 5.1 In the diagram, $P(k; 1)$ is a point in the 2nd quadrant and is $\sqrt{10}$ units from the origin. T is a point on the positive x -axis and obtuse $\hat{POT} = \theta$.



- 5.1.1 Calculate the value of k . (2)

- 5.1.2 **Without using a calculator**, calculate the value of:

(a) $-\cos \theta$ (1)

(b) $\sqrt{1 - \sin(180^\circ + \theta) \cdot \sin(360^\circ - \theta)}$ (4)

- 5.1.3 Show, **without using a calculator**, that

$$\sin\left(\frac{\theta}{2} - 15^\circ\right) \cdot \cos\left(\frac{\theta}{2} - 15^\circ\right) = \frac{\sqrt{3} + 3}{4\sqrt{10}} \quad (5)$$

- 5.2 Given: $\sin(\theta + 60^\circ) = d$

- 5.2.1 Write down the values of d such that $\sin(\theta + 60^\circ)$ is defined. (2)

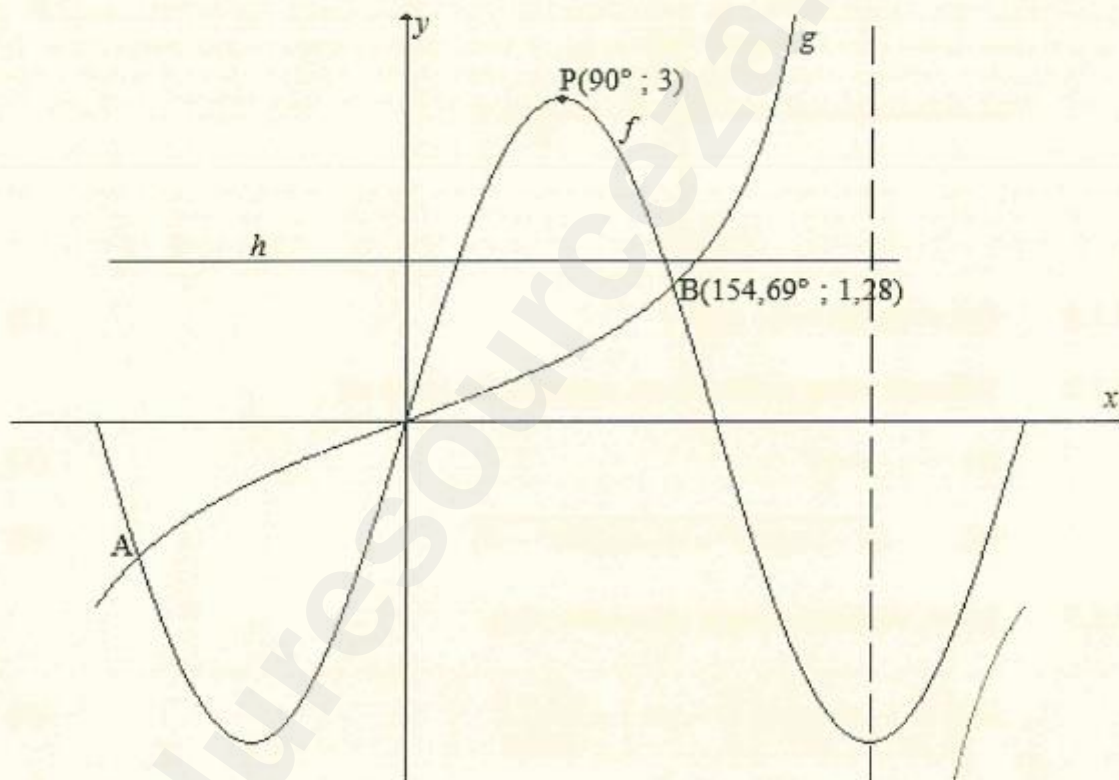
- 5.2.2 If $d = \sin \theta \cdot \cos 60^\circ + \sqrt{\frac{3}{8}}$, the general solution of $\sin(\theta + 60^\circ) = d$, is $\theta = \pm m^\circ + n \cdot 360^\circ$. Calculate, **without the use of a calculator**, the value of m . (5)

- 5.3 It is given that $A = 2^{\sin(a-m) \cdot \sin(b-m) \cdot \sin(c-m) \dots \sin(x-m)}$
Calculate the value of A.

(3)
[22]

QUESTION 6

In the diagram below, the graphs of $f(x) = a \sin x$ and $g(x) = \tan bx$ are drawn for the interval $x \in [-180^\circ; 360^\circ]$. The point $P(90^\circ; 3)$ is on the graph of f . The function $h(x) = \frac{3}{2}$ intersects the graphs of f and g . The asymptote of g goes through one of the turning points of f . A and B($154,69^\circ; 1,28$) are points of intersections of the graphs f and g .

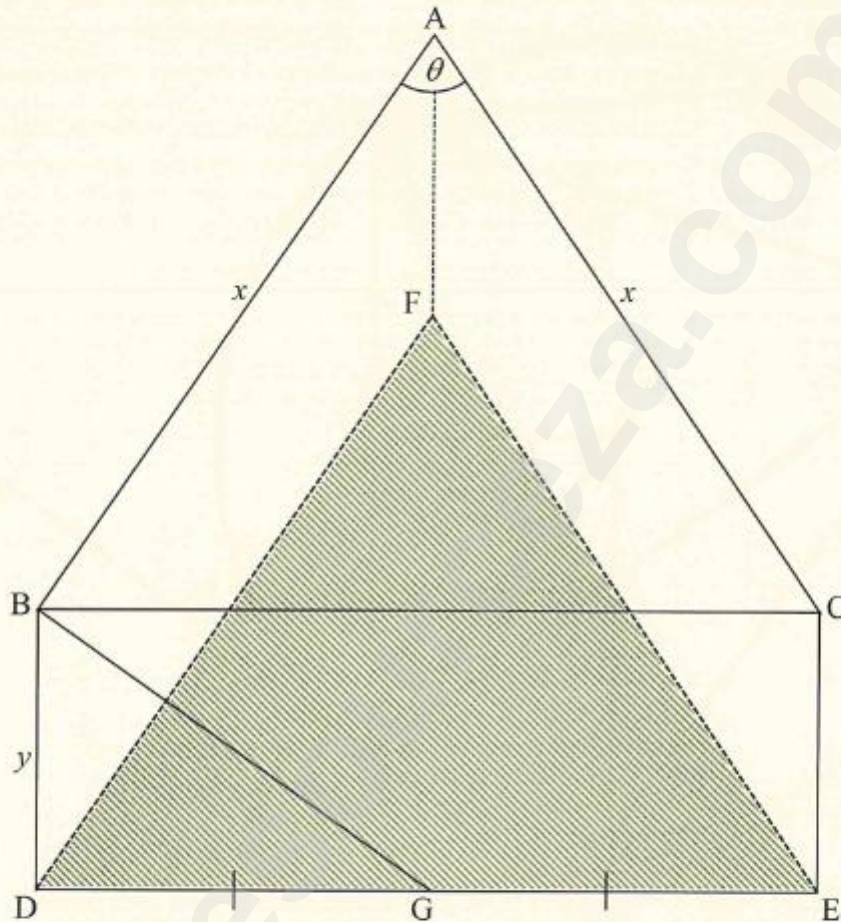


- 6.1 Write down the values of a and b . (2)
- 6.2 Write down the period of g . (1)
- 6.3 Write down the coordinates of A. (2)
- 6.4 It is given that $2 \tan 56,31^\circ - 6 \sin 168,93^\circ = t + 5$. Calculate in terms of t , the vertical distance between f and g , at the point where h intersects g . (4)

[9]

QUESTION 7

In the diagram below, steel poles AF, BD and CE are equal in length and vertical to the horizontal plane FDE. The steel poles hold $\triangle ABC$ that forms the roof of a storage area. G is the midpoint of DE. BCED is in the same vertical plane. $AB = AC = x$, $BD = y$ and $\hat{BAC} = \theta$.



- 7.1 Show that $BC = x\sqrt{2(1 - \cos \theta)}$ (3)
- 7.2 Write down the size of $\hat{BDG}$. (1)
- 7.3 Show that $BG = \sqrt{\frac{2y^2 + x^2(1 - \cos \theta)}{2}}$ (3)
- 7.4 It is given that $y = \frac{8}{3}$ metres and $x = \frac{15}{2}$ metres .
Calculate the maximum length of BG. (3)
- [10]**

PREP 2020_E. CAPE

QUESTION 5

5.1 If $\cos 22^\circ = p$; determine the following in terms of p :

5.1.1 $\cos 158^\circ$ (2)

5.1.2 $\sin 112^\circ$ (2)

5.1.3 $\sin 38^\circ$ (4)

5.2 Determine all the values of P in the interval $[0^\circ; 360^\circ]$ which satisfy the equation:
 $\sin P = \sin 2P$. (4)

5.3 If $\triangle ABC$ is a scalene triangle, show that: $\cos(A + B) = -\cos C$. (2)

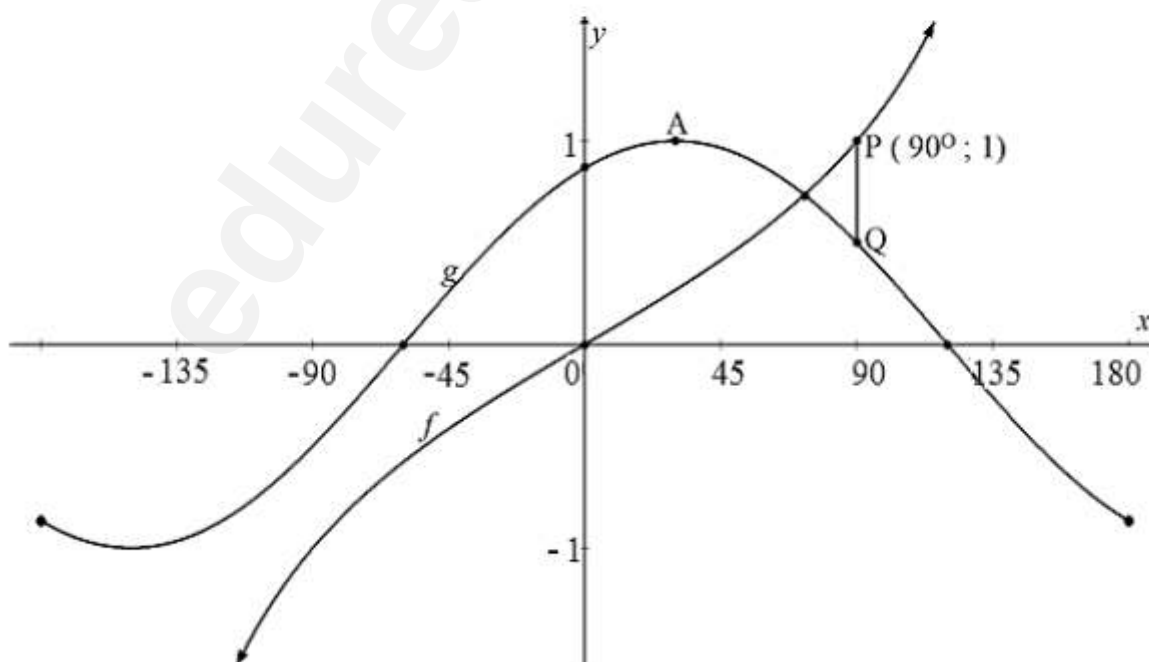
5.4 Prove the following identity:

$$\frac{\cos^2 x - \cos x - \sin^2 x}{2 \sin x \cdot \cos x + \sin x} = \frac{1}{\tan x} - \frac{1}{\sin x} \quad (5)$$

5.5 Determine the general solution of: $4 + 7 \cos \theta + \cos 2\theta = 0$. (6)
[25]

QUESTION 6

In the diagram below, the graphs of $f(x) = \tan bx$ and $g(x) = \cos(x - 30^\circ)$ are drawn on the same set of axes for $-180^\circ \leq x \leq 180^\circ$. The points $P(90^\circ; 1)$ and Q lie on f and g respectively. Use the diagram to answer the following questions.

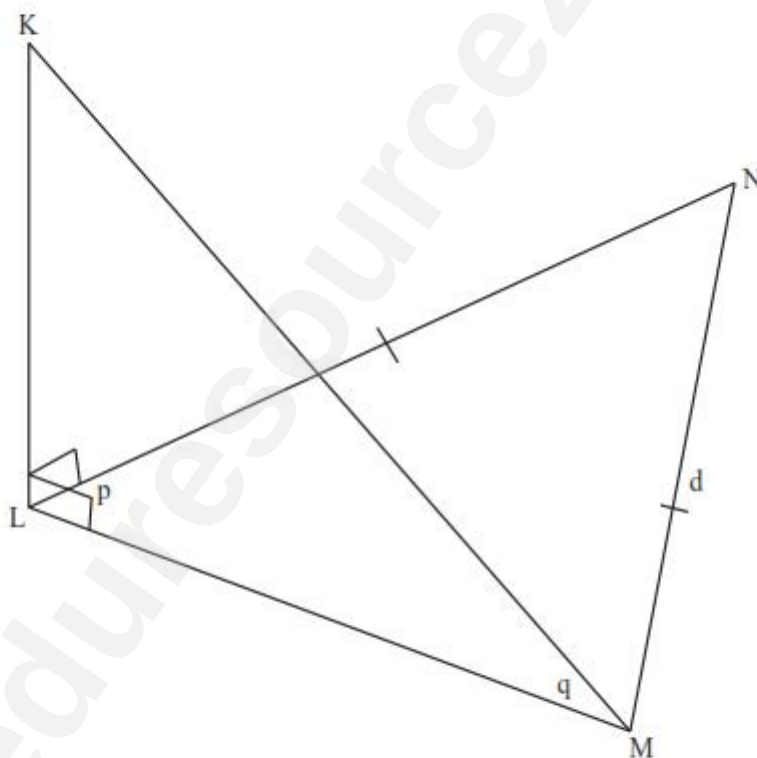


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- 6.1 Determine the value of b . (1)
- 6.2 Write down the coordinates of A, the turning point of g . (2)
- 6.3 If PQ is parallel to the y -axis, determine the coordinates of Q. (2)
- 6.4 Write down the equation of the asymptote(s) of $y = \tan b(x + 20^\circ)$ for $x \in [-180^\circ; 180^\circ]$. (1)
- 6.5 Determine the range of h if $h(x) = 2g(x) + 1$. (2)
- [8]

QUESTION 7

Points L, M and N are in the same horizontal plane. KL is a vertical tower. The angle of elevation of K from M is q° . $\widehat{NLM} = p^\circ$; $NL = NM = d$ and $KL = h$.



- 7.1 Determine the size of $\widehat{LNM}$ in terms of p . (2)
- 7.2 Prove that $LM = \frac{d \sin 2p}{\sin p}$. (2)
- 7.3 Hence, show that $h = 2d \cos p \tan q$. (3)
- [7]

PREP 202_LIMPOPO

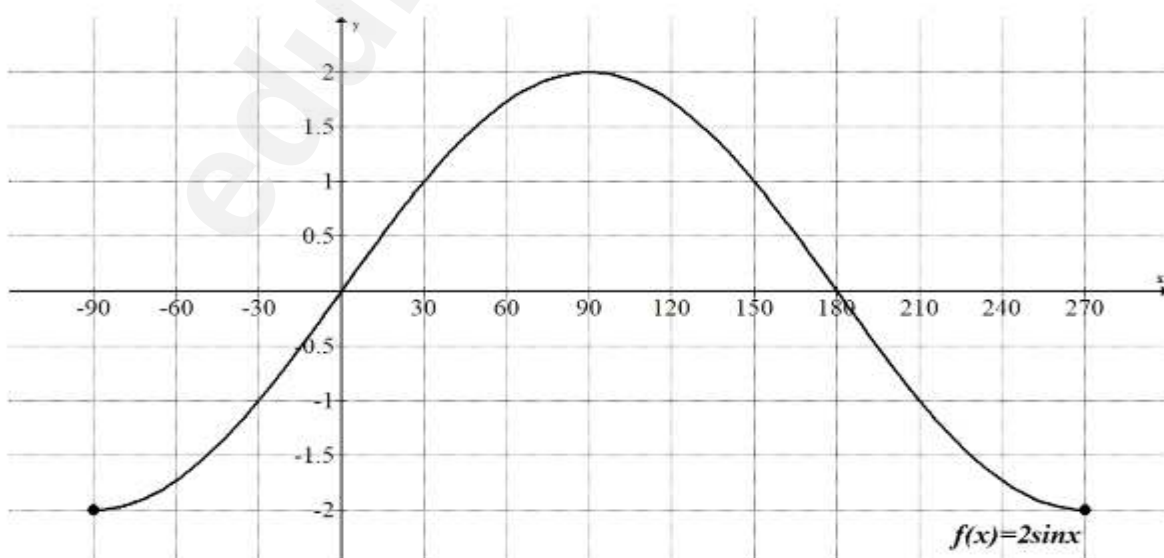
QUESTION 5

NO CALCULATORS ALLOWED IN THIS QUESTION

- 5.1 A and B are complementary angles and $A > 0$ and $B > 0$.
If $12\tan A - 5 = 0$, calculate, by using a diagram, the value of $13\sin A - 3\tan B$ (5)
- 5.2 Simplify the following expression:
$$\frac{\tan 300^\circ + \cos(90^\circ + x)}{\sin x + 2\cos(-30^\circ)}$$
 (6)
- 5.3.1 Given: $\sin(x + y) = \sin x \cos y + \cos x \sin y$.
Use this identity and derive an identity for $\cos(x + y)$ (3)
- 5.3.2 Prove that $\cos(x - y) - \cos(x + y) = 2 \sin x \sin y$ (3)
- 5.3.3 Hence, or otherwise, calculate the numerical value of $2 \sin 195^\circ \sin 45^\circ$ (5)
- [22]**

QUESTION 6

In the diagram the graph of $f(x) = 2 \sin x$ is drawn for the interval $x \in [-90^\circ; 270^\circ]$.



GN (DI) STRATEGY FOR LEARNERS ATTAINMENT

6.1 Draw the graph of g if $g(x) = \cos(x + 30^\circ)$, for the interval $x \in [-90^\circ; 270^\circ]$, in your answer book. Show all the intercepts with the axes, as well as the turning points. (3)

6.2 Show that $2 \sin x = \cos(x + 30^\circ)$ can also be expressed as $\tan x = \frac{\sqrt{3}}{5}$. (4)

6.3 Hence, determine the values of x , to ONE decimal, where $f(x) = g(x)$, for $x \in [-90^\circ; 270^\circ]$. (2)

6.4 Use the solutions obtained in Question 6.3 and the graph drawn in Question 6.1 to determine for which values of x , $x \in [0^\circ; 270^\circ]$ is:

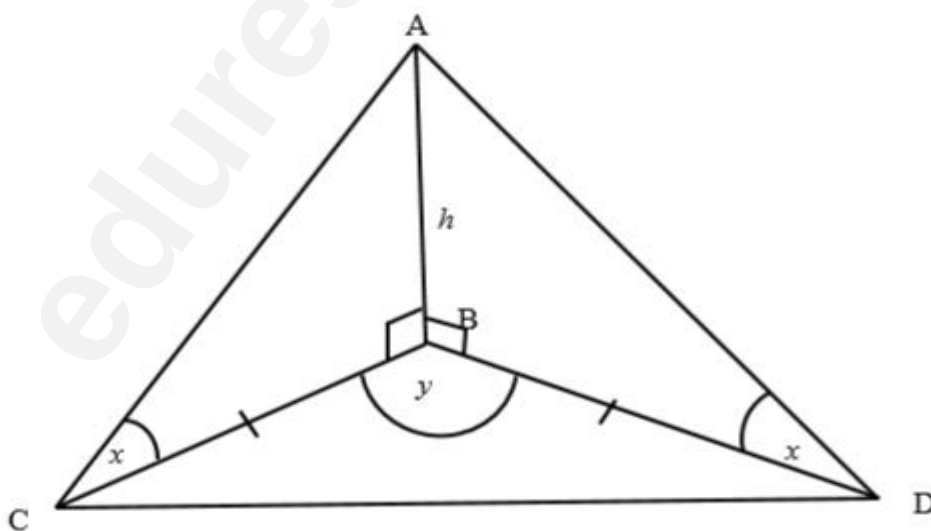
6.4.1 $f(x), g(x) \leq 0$ (3)

6.4.2 $f(x) \geq g(x)$ (2)

[14]

QUESTION 7

In the diagram C and D represent two boats in the same horizontal plane as B, the base of a lighthouse AB, which is h metres high. Boat C and D are equidistant from B. The angle of elevation from C and D to A, are both x . $\angle CBD = y$.



Prove that $CD = \frac{h}{\tan x} \sqrt{2 - 2 \cos y}$

[6]

PREP 2020_MPUMALANGA

QUESTION 5

5.1 Simplify without using a calculator: $\frac{\cos 140^\circ - \sin(90^\circ - \theta)}{\sin 410^\circ + \cos(-\theta)}$ (5)

5.2 If $\tan 36^\circ = t$, express the following in terms of t :

5.2.1 $\tan(-36^\circ)$ (1)

5.2.2 $(\sin 684)^\circ$ (4)

5.2.3 $\cos 81^\circ$ (4)

5.3 $\triangle PQR$ is right-angled at P. It is not isosceles and its smallest angle is Q.



5.3.1 What is the range of possible values of $\hat{Q}$ and $\hat{R}$? (2)

5.3.2 Given the infinite geometric series:

$$1 - \tan^2 Q + \tan^4 Q - \tan^6 Q$$

Write down the sum of this series in its simplest form. (4)

5.3.3 Evaluate the sum of the infinite series:

$$1 - \tan^2 43^\circ + \tan^4 43^\circ - \tan^6 43^\circ \text{ (correct to 3 decimal places).} \quad (2)$$

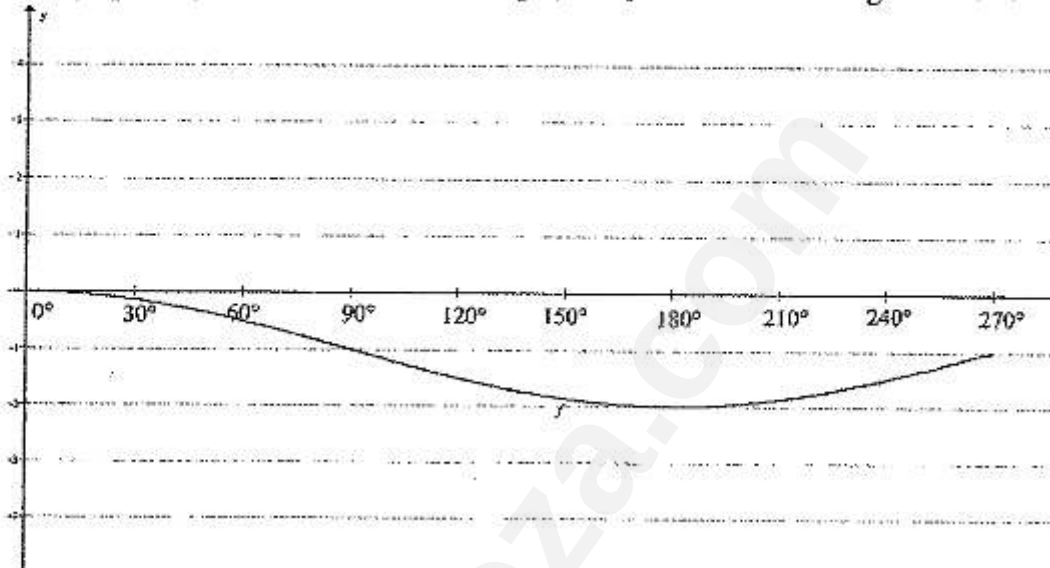
[22]

GN (DI) STRATEGY FOR LEARNERS ATTAINMENT

QUESTION 6

6.1

The graph of $f(x) = -1 + \cos x$ for $x \in [0^\circ; 270^\circ]$ is shown in the figure below.



6.1.1 Write down the period of f . (1)

6.1.2 Write down the range of f . (1)

6.2 Use the set of axes provided in the ANSWER BOOK (on which the graph of f also appears) to sketch the curve of $g(x) = \cos 2x$ for $x \in [0^\circ; 270^\circ]$ (3)

6.3 Use the graphs in Question 6.2 to answer the following questions:

6.3.1 For which value of x is $g(x) - f(x)$ a maximum? (1)

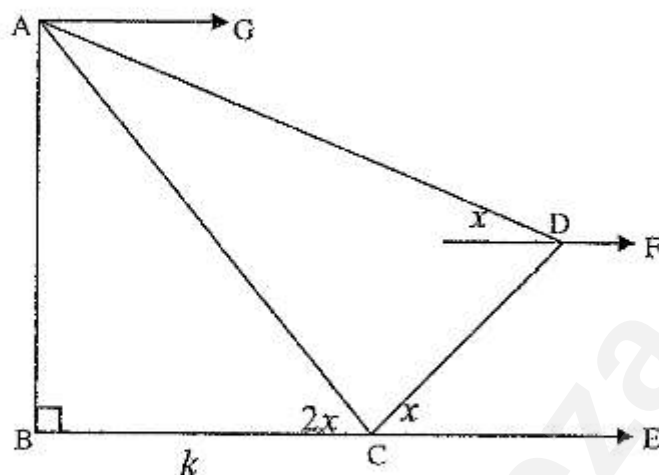
6.3.2 For which value of x is $f(x) \cdot g(x) \geq 0$? (2)

6.4 Solve for $g(x) = f(x)$ for $x \in [0^\circ; 270^\circ]$. (4)
[12]

GN (DI) STRATEGY FOR LEARNERS ATTAINMENT

QUESTION 7

In the sketch A, B, C, D, E and F are points in the same horizontal plane. $BCE \parallel DF \parallel AG$.
 $AB \perp BC$ and $BC = k$.



7.1 Express the magnitude of $\hat{CAD}$ in terms of x . (2)

7.2 Show that $CD = \frac{2k \sin x}{\sin 4x}$. (6)

[8]

PREP 2020_GAUTENG

QUESTION 5

5.1 Calculate the value of $1 - 4\sin^2 15^\circ$ without the use of a calculator. (5)

5.2 Simplify without the use of a calculator:

$$\frac{\sqrt{3} \sin x \cdot \sin^2 72^\circ + \sin^2 198^\circ \cdot \sqrt{3} \cos(x - 90^\circ)}{\tan 120^\circ \cdot \sin x} \quad (6)$$

5.3 Determine the general solution of the following:

$$6\sin x \cdot \cos x + 3\cos x - 4\sin^2 x - 2\sin x = 0 \quad (7)$$

5.4 Prove that:

$$(1 - \tan A) \left(\frac{\cos A}{\cos 2A} \right) = \frac{1}{\cos A + \sin A} \quad (4)$$

GN (DI) STRATEGY FOR LEARNERS ATTAINMENT

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5.5 If $\sin 2\theta = k$ and $0^\circ < 2\theta < 90^\circ$, determine in terms of k :

5.5.1 $\cos 2\theta$ (2)

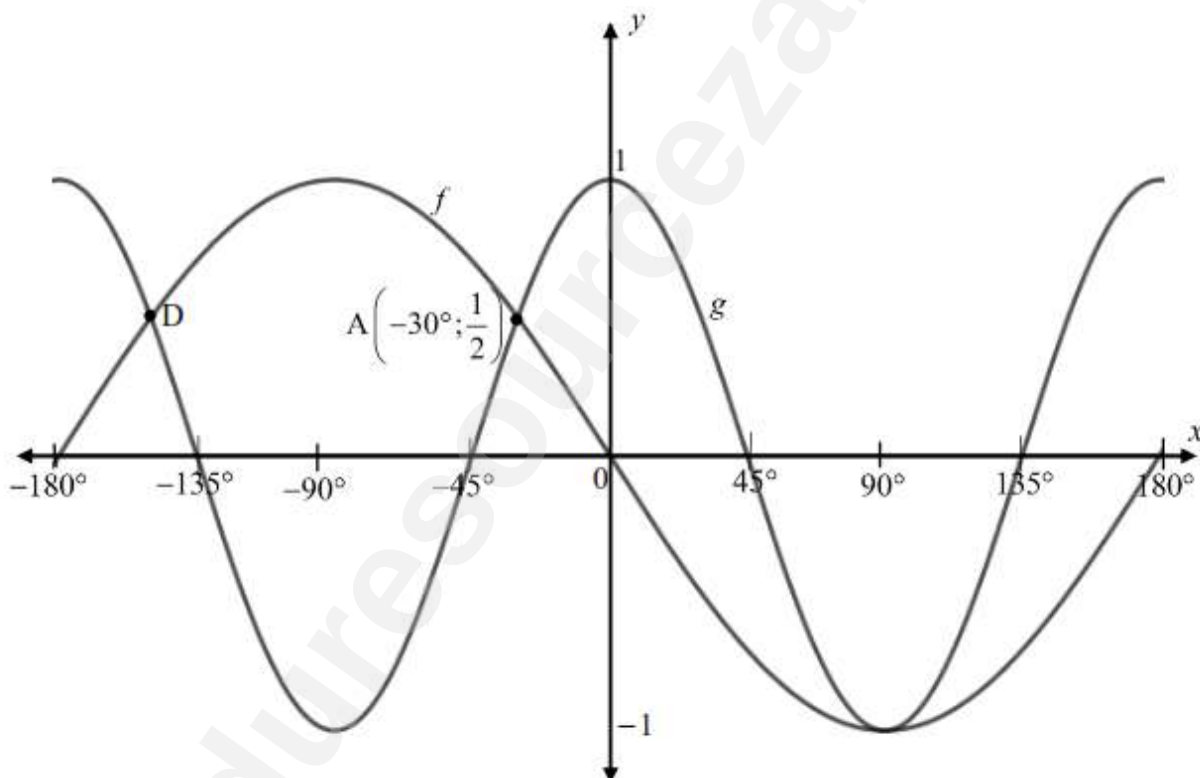
5.5.2 $\frac{\sin 2\theta}{\tan \theta}$ (5)

[29]

QUESTION 6

The sketch below shows the graphs of $f(x) = a \sin x$ and $g(x) = \cos dx$ for $x \in [-180^\circ; 180^\circ]$.

$A\left(-30^\circ; \frac{1}{2}\right)$ is a point of intersection of f and g .



6.1 Write down the values of a and d . (2)

6.2 Determine the coordinates of D. (1)

6.3 For which value(s) of x is:

6.3.1 f decreasing for $x \in [-180^\circ; 180^\circ]$? (2)

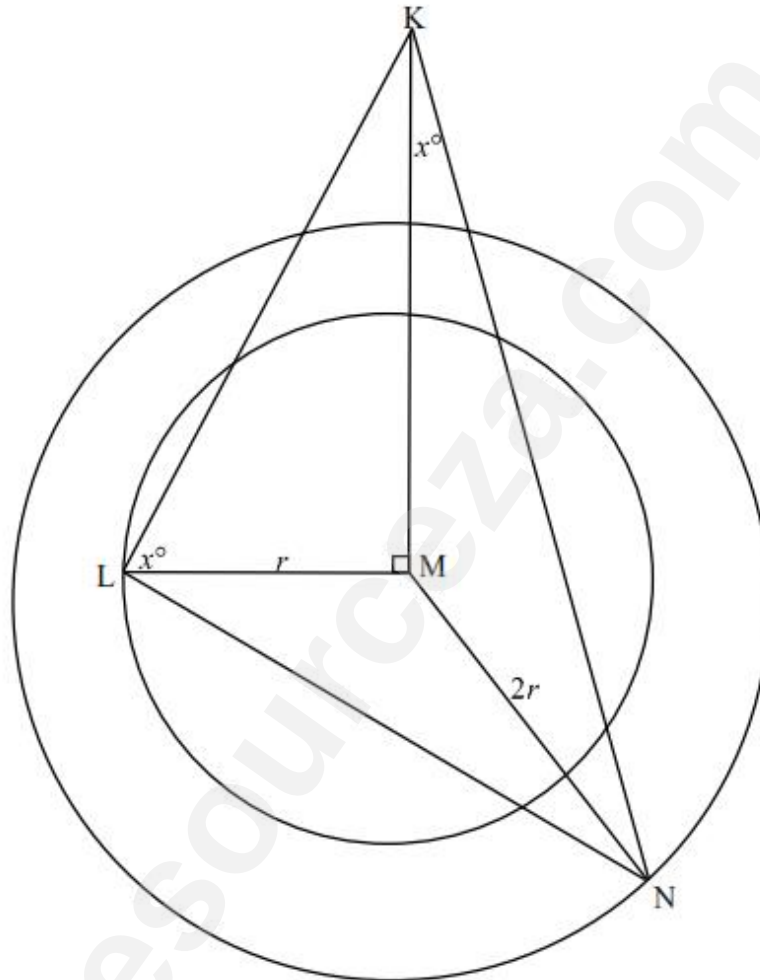
6.3.2 $f(x) \cdot g(x) < 0$ for $x \in [-180^\circ; 0^\circ]$? (2)

[7]

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QUESTION 7

In the figure below, KM is a vertical flag post set in the centre of two circles which lie on the same horizontal plane. $\hat{MKN} = \hat{MLK} = x^\circ$. The radius of the inner circle $ML = r$ units and the radius of the outer circle $MN = 2r$ units.



- 7.1 Calculate the value of x . (6)
- 7.2 If $r = 5m$ and $\hat{LMN} = 110^\circ$, calculate the length of LN . (2)
- [8]**

PREP 2020_KZN

QUESTION 5

- 5.1 Without using a calculator, evaluate
 $\cos 79^\circ \cos 311^\circ + \sin 101^\circ \sin 49^\circ$ (4)

GN (DI) STRATEGY FOR LEARNERS ATTAINMENT

5.2 Given: $\sin(x+y) = 3 \sin(x-y)$

Prove that: $\tan x = 2 \tan y$ (4)

5.3 Given: $\frac{\cos x}{\sin 2x} - \frac{\cos 2x}{2 \sin x} = \sin x$

5.3.1 Prove that $\frac{\cos x}{\sin 2x} - \frac{\cos 2x}{2 \sin x} = \sin x$ (4)

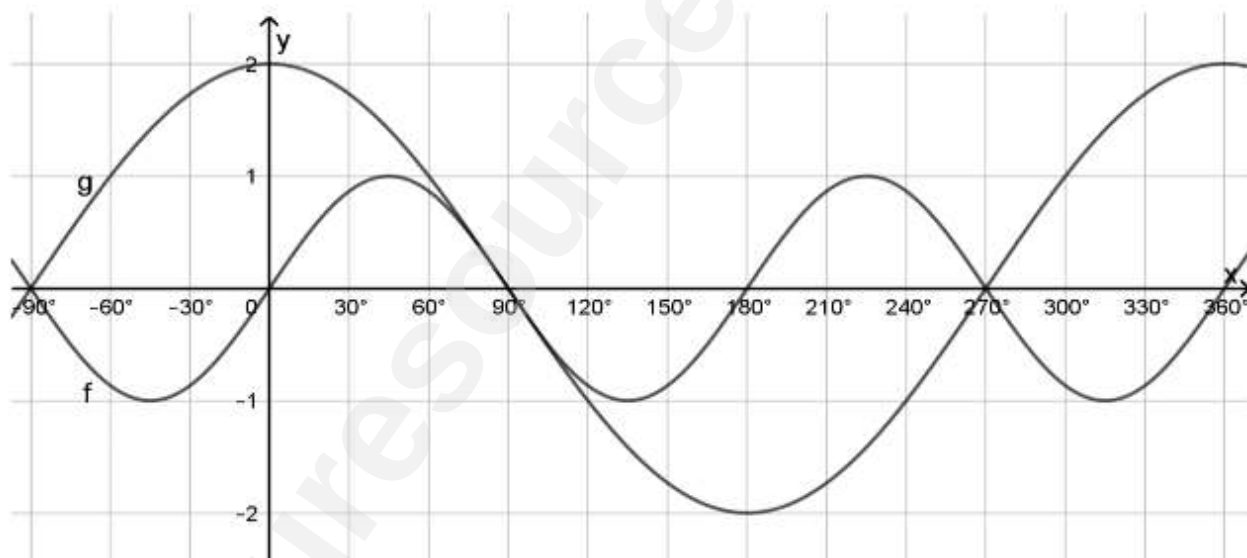
5.3.2 Hence, solve for x where $x \in [0^\circ; 360^\circ]$:

$$1 + 2 \cos 2x = \frac{\cos 2x}{2 \sin x} - \frac{\cos x}{\sin 2x} \quad (6)$$

[18]

QUESTION 6

In the diagram, the graphs of $f(x) = a \sin bx$ and $g(x) = c \cos dx$ are drawn for the interval $x \in [-90^\circ; 360^\circ]$



6.1 Determine the values of a , b , c and d . (4)

6.2 Write down the period of g . (1)

6.3 Determine the value(s) of x in the interval $x \in [-90^\circ; 360^\circ]$, for which

6.3.1 $f(x) \leq g(x)$ (2)

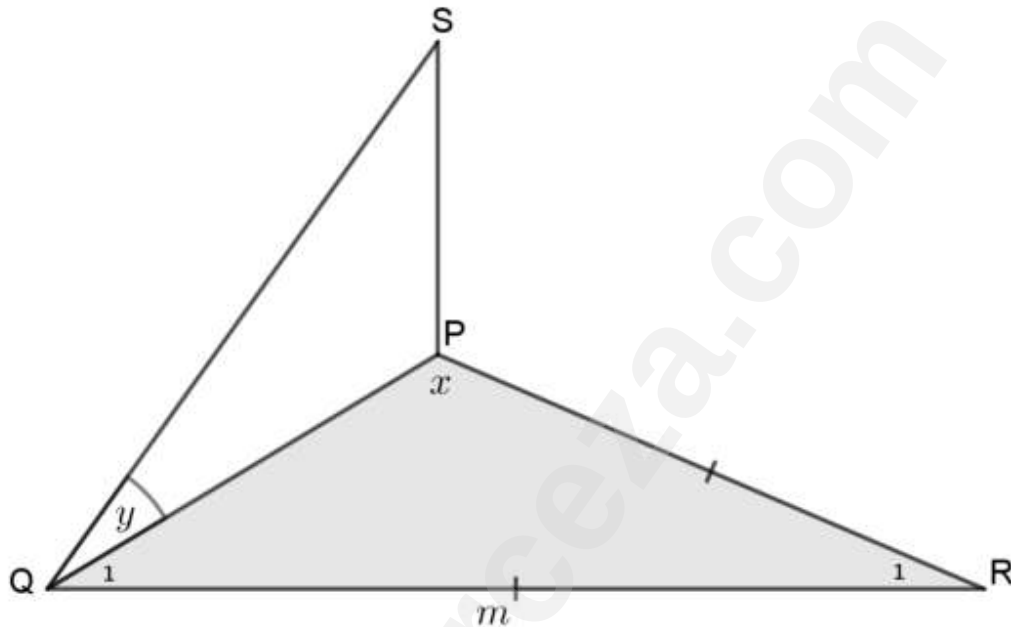
6.3.2 $f'(x) \times g'(x) > 0$ where $g(x) > 0$ (3)

GN (DI) STRATEGY FOR LEARNERS ATTAINMENT

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QUESTION 7

In the diagram P, Q and R are three points in the same horizontal plane. $PR = QR = m$, $\hat{QPR} = x$. SP is perpendicular to PQ. The angle of elevation of S from Q is y .



- 7.1 Express the area of ΔPQR in terms of x and m . (5)
- 7.2 Show that $PQ = 2m \cos x$ (4)
- 7.3 Hence, prove that $SP = 2m \cos x \tan y$ (2)

[11]