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QUESTION 6

- 6.1 Derive the formula $S_n = \frac{n}{2} [2a + (n-1)d]$ for the sum to n terms of an arithmetic series, where a is the first term and d is the common difference. (5)
- 6.2 A team of workers is erecting a fence along a main road. They pitch camp at one end of the fence, and erect 0.75 km of new fencing each day. Every evening they return to their camp from where they started. (2)
- 6.2.1 How far will they travel in total in 35 days? (6)
- 6.2.2 How far will they travel on day 35? (2)
- 6.3 The 3rd term of a geometric series is 18, and the 5th term is 162. Determine the sum of the first 7 terms, where $r < 0$. (8)
- 6.4 Given: $\sum_{k=1}^{\infty} 30(3)^{k-1}r$ (2)
- 6.4.1 Write down the first 3 terms of the series. (2)
- 6.4.2 Determine the sum of the series. (3)
- 6.5 For which value(s) of p is the infinite series $2(p-5) + 2(p-5)^2 + 2(p-5)^3 + \dots$ convergent? (4)
- 6.6 Write the series $\frac{3}{1} + \frac{5}{2} + \frac{7}{4} + \frac{9}{8} + \dots$, to n terms, in Σ -notation. (4)
- QUESTION 7 [34]
- 7.1 Determine $f'(x)$ from first principles if $f(x) = -3x^2 + 4x$. (6)
- 7.2 Determine $\frac{dy}{dx}$ if: (3)
- 7.2.1 $y = (x-x^{-1})^2$ (2)
- 7.2.2 $6 = xy$ (2)
- 7.3 Given: $f(x) = x^3 - 5x^2 - 8x + 12$ (2)
- 7.3.1 Draw a neat sketch graph of f , clearly showing all the coordinates of the intercepts with the axes, as well as the coordinates of the turning points. (Show all your calculations.) (13)
- 7.3.2 For which value(s) of c will the equation, $f(x) = c$, have exactly two real roots? (4)

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QUESTION 6

- 6.1 The 1st term of an arithmetic sequence is 1 and the 9th term is 15. Calculate the middle term. (3)
- 6.2 Prove that $a + ar + ar^2 + \dots + ar^{n-1} = a \frac{1-r^n}{1-r}$. (4)
- 6.3 Given the geometric series: $6 + 3 + \frac{3}{2} + \frac{3}{4} + \dots$ (4)
- 6.3.1 find the sum of the first eleven terms of the series. (4)
- 6.3.2 find the sum to infinity (3)
- 6.3.3 write the sum of the first eleven terms of the series in Σ -notation. (2)
- 6.4 A machine has five gears which give five different velocities in geometrical sequence. The lowest velocity is 324 revolutions per minute and the highest velocity is 2 500 revolutions per minute. What is the second lowest velocity? (4)
- 6.5 The common difference of an arithmetic sequence is 4. Prove that $\sqrt{5n} - \sqrt{5n-4} = 2n$. (7)
- QUESTION 7 [27]
- 7.1 Calculate the derivative of the function $f(x) = 6x^2 - 3x - 1$ from first principles. (6)
- 7.2 Determine: (4)
- $\frac{d}{dx} \left(4\sqrt{x} - \frac{8}{\sqrt{x}} + \pi x^3 \right)$ (4)
- 7.3 If $y = (x^2 - \sqrt{x})^2$, calculate $\frac{dy}{dx}$. (5)
- 7.4 The equation of a tangent to the curve of $f(x) = ax^2 + bx$ is $y - x - 4 = 0$. If the point of contact is $(-1; 3)$, calculate the values of a and b . (7)

QUESTION 8

- 8.1 An environmental study has revealed that the water from a factory has polluted the water in a nearby dam. The pollution has an effect on the water lilies growing on the water surface of the dam. At first the polluted water caused the plants to grow profusely and later on it caused the plants to die and the area covered by the plants to decrease. (2)
- A mathematician established that the surface area, A (m^2), covered by the water lilies can be approximated by the formula: $A = -\frac{1}{2}t^3 + 12t^2$, where t is the time in months after the start of the study, with $0 \leq t \leq 24$. (22)

DEPARTMENT OF EDUCATION
NATIONAL SENIOR CERTIFICATE EXAMINATION

MATHEMATICS HIGHER GRADE
PAPER ONE: ALGEBRA

MARCH 2002

Time: 3 hours

Marks: 200

QUESTION 1

1.1 Given: $a + \frac{1}{a} = 2$

1.1.1 Solve for a . (3)

1.1.2 Hence solve for x (round off to two decimal places if necessary): $(2x^2 + 3x) + \frac{1}{(2x^2 + 3x)} = 2$ (5)

1.2 Solve for x :

1.2.1 $\frac{2-x}{x+5} \leq 0$ (5)

1.2.2 $\sqrt{4x-3} - 2x = -3$ (7)

[20]

QUESTION 2

2.1 Given: $x^2 + 2x(k+1) + (2k+1) = 0$

Show that the roots of the equation are real for all real values of k . (5)

2.2 Show that the value of the expression $2x^2 - 8x + 5$ can never be smaller than -3 , no matter what the value of x is. (4)

[9]

QUESTION 7

7.1 If $f(x) = 3x - x^2$, determine $f'(x)$ from first principles. (6)

7.2 Determine $\frac{dy}{dx}$, if

7.2.1 $xy = 5$ (2)

7.2.2 $y = \frac{1-2x+\sqrt{x}}{x^2}$ (4)

7.3 Given $f(x) = 2x^2 + x - 1$, calculate the equation of the tangent to the curve of f where the gradient equals -3 . (6)

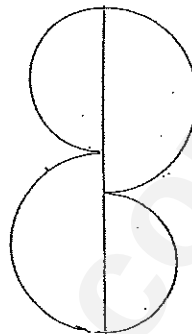
7.4 Draw a neat sketch graph of the function $y = x^3 - x^2 - 5x - 3$. Clearly show all intercepts with the axes as well as the coordinates of the turning points. Show ALL your calculations. (17)

[35]

QUESTION 8

8.1

An advertising company has asked you to design an advertising disc that consists of four semicircles and has the shape as shown in the figure below. The larger semicircles have radius R and the smaller semicircles have radius r . The values of R and r may vary but $R + r = 200$ mm. To minimise costs the company has stated that the area of the shape must be a minimum.



8.1.1 Show that the area of the figure is given by

$$A = 2\pi R^2 - 400\pi R + 40\,000\pi$$

(4)

8.1.2 Use differential calculus to determine the values of R and r if the area, A , of the figure is a minimum. (4)

8.1.3 Consider your solution in QUESTION 8.1.2 and explain why the shape suggested by the company is not possible if you want to maintain a minimum area. (2)

QUESTION 6

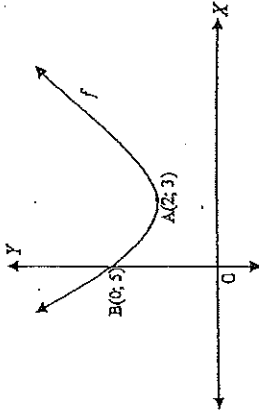
- 6.1 Derive the formula $S_n = \frac{n}{2} [2a + (n-1)d]$ for the sum to n terms of an arithmetic series, where a is the first term and d is the common difference. (5)
- 6.2 A team of workers is erecting a fence along a main road. They pitch camp at one end of the fence, and erect 0.75 km of new fencing each day. Every evening they return to their camp from where they started. (5)
- 6.2.1 How far will they travel in total in 35 days? (6)
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- 6.6 Write the series $\frac{3}{1} + \frac{5}{2} + \frac{7}{4} + \frac{9}{8} + \dots$ to n terms, in Σ -notation. (4)

QUESTION 7

- 7.1 Determine $f'(x)$ from first principles if $f(x) = -3x^2 + 4x$. (6)
- 7.2 Determine $\frac{dy}{dx}$ if (3)
- 7.2.1 $y = (x - x^{-1})^2$ (2)
- 7.2.2 $6 = xy$ (2)
- 7.3 Given: $f(x) = x^2 - 5x^2 - 8x + 12$ (13)
- 7.3.1 Draw a neat sketch graph of f , clearly showing all the coordinates of the intercepts with the axes, as well as the coordinates of the turning points. (Show all your calculations.) (13)
- 7.3.2 For which value(s) of c will the equation $f(x) = c$ have exactly two real roots? (4)

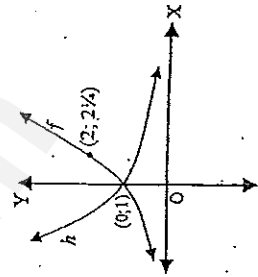
QUESTION 3

- 3.1 The sketch graph shows the parabola f with turning point $A(2; 3)$ and y -intercept $B(0; 5)$.



- 3.1.1 Show that the equation of the parabola f can be written as $y = \frac{1}{2}x^2 - 2x + 5$. (5)
- 3.1.2 Without calculating the discriminant, decide, with a reason, whether the discriminant of f is positive, negative or zero. (2)
- 3.1.3 Use the graph to determine the value(s) of k for which the equation $\frac{1}{2}x^2 - 2x + 5 = k$ has real and unequal roots. (2)
- 3.1.4 The parabola is shifted vertically until the new y -intercept is the origin. Find the equation of the new parabola. (2)
- 3.2 Given: $h(x) = x^3 - 4x^2 + 4x - 4$ (3)
- 3.2.1 Sketch the curve of h in the answer book, clearly showing the intercepts with the axes. (4)
- 3.2.2 For which values of x is $|x| - 4 > 0$? (3)
- 3.3 In the accompanying graph f represents the function $f(x) = a^x$, ($a > 0$). h is symmetrical to f about the y -axis.

The point $(2; 2\frac{1}{4})$ lies on the curve of f .



- 3.3.1 Find the value of a . (2)

- 7.3.1 What is the annual increase if, over 20 years, he would have received R143 500 altogether? (4)
- 7.3.2 His initial annual expenditure is R2 600 and increases at a rate of R400 per year. In which year will his expenses exceed his income? (6)

QUESTION 8

- 8.1 Calculate the derivative of f from first principles, if

$$f(x) = x - x^2$$

- 8.2 Determine $\frac{dy}{dx}$ if

$$8.2.1 \quad y = (x^2 - 1)^2 \quad (5)$$

$$8.2.2 \quad y = \frac{x^3 + \sqrt{x^2}}{x} \quad (4)$$

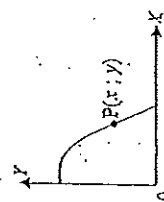
8.3 Given: $f(x) = -x^3 + 6x^2 - 9x + 4$

- 8.3.1 Draw a neat sketch graph of f , showing the coordinates of the intercepts with the axes, as well as the coordinates of the turning points. (Show all your calculations.) (17)

- 8.3.2 Determine the equation of the tangent to the curve of f at the point $(2; 2)$. (5)

QUESTION 9

- 9.1 The graph of $x^3 + y^3 = 8$, not drawn to scale, is represented alongside for the interval $0 \leq x \leq 2$. $P(x; y)$ is any point on the graph. (34)



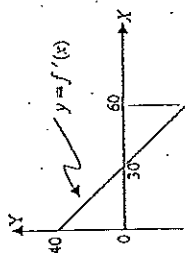
- 9.1.1 Determine OP^2 in terms of x and y . (1)

- 9.1.2 Show that $OP^2 = -x^3 + x^2 + 8$. (1)

- 9.1.3 Determine the maximum value of OP^2 . (5)
- 9.1.4 Calculate the shortest distance from the origin to the graph. (3)

- 9.2 The equation of the tangent to $y = f(x)$ at $x = -1$ is $y = -3x + 4$. Determine $f(-1)$ and $f'(-1)$. (2)

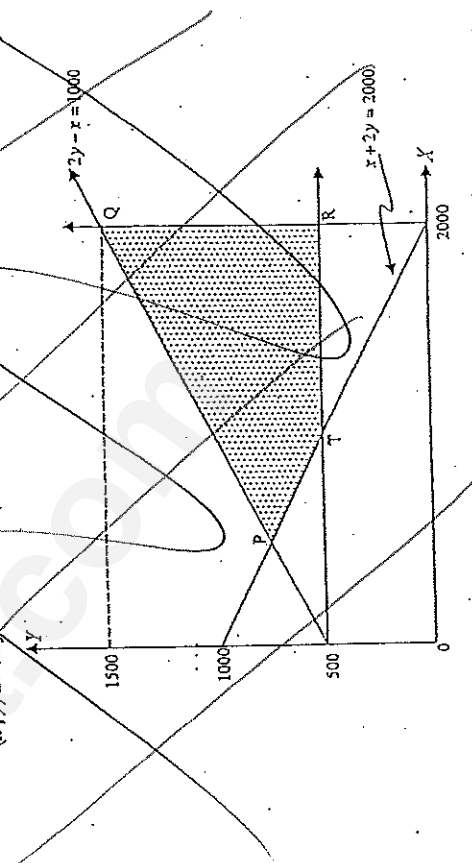
- 9.3 The accompanying sketch represents the curve of $f'(x)$.



- 9.3.1 What is the gradient of f at $x = 0$? (1)
- 9.3.2 f has a maximum at a . Determine the value of a . (2)
- 9.3.3 For which values of x is f increasing? (2)
- [17]

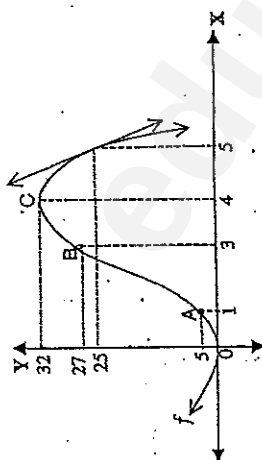
QUESTION 10

An entrepreneur manufactures two types of furniture pieces: chairs and tables. The costs are R250 per chair and R300 per table. He sells each chair for R300 and each table for R400. He makes x chairs and y tables each month, so that the points $(x; y)$ lie only in the shaded (feasible) region below.



6.3 Answer the following questions by referring to the graph of

$$f(x) = -x^3 + 6x^2$$



6.3.1 What is the average gradient of the curve of f between the points A and B? (3)

6.3.2 What is the gradient of the tangent to the curve of f at:

6.3.2.1 C, where C is the turning point of f (1)

6.3.2.2 The point on f where $x = 5$ (4)

6.3.3 Determine the equation of the tangent to the curve of f at the point $x = 5$. (3) [24]

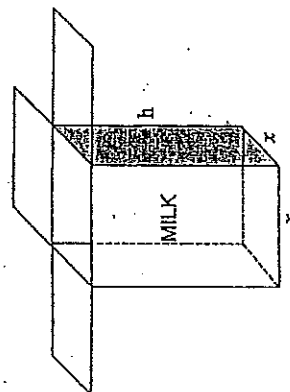
QUESTION 7

7.1 Draw a neat sketch graph of the function

$$y = -x^3 + 10x^2 - 17x - 28$$

Clearly show all the intercepts with the axes as well as the co-ordinates of the turning points. Show ALL your calculations. (17)

7.2 A rectangular container has a square base and contains 1 litre of milk. The container has a folding lid that covers 3 times the area of the base. The length of the side of the square base is x cm and the height of the container is h cm. [1 litre = 1 000 cm^3] See diagram below.



7.2.1 Show that the area, A , of cardboard used to make the container can be expressed as:

$$A = 4x^2 + \frac{4000}{x}$$

(Hint: The volume of the container = area of base $\times$ height.) (5)

7.2.2 Determine the dimensions of the container in cm, so that the quantity of cardboard used is as small as possible. (6) [28]

QUESTION 8

A local health board is producing a guide for healthy living. The guide should provide advice on healthy education, healthy lifestyles and the like. The board intends to produce the guide in two formats: one will be in the form of a short video, the other as a printed binder.

The board is currently trying to decide how many of each type to produce for sale. It has estimated that it is likely to sell no more than 10 000 copies, of both items together. At least 4 000 copies of the video and at least 2 000 copies of the binder could be sold, although sales of the binder are not expected to exceed 4 000 copies.

Let x be the number of videos sold, and y the number of printed binders sold.

8.1 Write down the constraint inequalities that can be deduced from the given information. (5)

8.2 Use the graph paper provided to represent these inequalities graphically and indicate the feasible region clearly. (7)

8.3 The board is seeking to maximise the income, I , earned from sales of the two products. Each video will sell for R50 and each binder for R30. Write down the objective function for the income. (1)

8.4 Determine graphically, by using a search line, the number of videos and binders that ought to be sold to maximise the income. (3)

8.5 What maximum income will be generated by the two guides? (2) [18]

TOTAL: 200