



**[eduresourceza.com](http://eduresourceza.com)**

CLINIC (2) MEMO

$$5.3.4 \log_2(5-x) + \log_2 x \leq 2$$

$$\log_2 x(5-x) \leq 2 \log_2 2$$

$$\log_2 x(5-x) \leq \log_2 4$$

$$\therefore x(5-x) \leq 4 \quad \checkmark$$

$$\therefore -x^2 + 5x - 4 \leq 0 \quad \checkmark$$

$$x^2 - 5x + 4 \geq 0$$

$$(x-4)(x-1) \geq 0 \quad \checkmark$$

$$\begin{array}{ccccccc} + & 0 & - & 0 & + & & \\ & 1 & & 4 & & & \\ & & & & & & x \end{array}$$

$$\therefore x \leq 1 \text{ or } x \geq 4 \quad \checkmark$$

Check domain:

$$5-x > 0 \text{ and } x > 0 \quad \checkmark$$

$$x < 5 \text{ and } x > 0$$

$$\therefore 0 < x < 5 \quad \checkmark$$

Solution:

$$0 < x \leq 1 \quad \checkmark \text{ or } 4 \leq x < 5 \quad \checkmark$$

QUESTION 6

$$6.1 \quad T_1 = 1$$

$$a = 1$$

$$T_9 = 15$$

$$a + 8d = 15 \quad \checkmark$$

$$\therefore 1 + 8d = 15$$

$$8d = 14$$

$$d = \frac{14}{8} = 1\frac{3}{4} \quad \checkmark$$

Middle term is  $T_5$

$$\therefore T_5 = a + 4d$$

$$= 1 + 4\left(\frac{7}{4}\right)$$

$$= 1 + 7$$

$$= 8 \quad \checkmark$$

6.2 Theory. Sum of a finite geometric series.

$$6.3 \quad 6 + 3 + \frac{3}{2} + \frac{3}{4} + \dots$$

$$6.3.1 \quad a = 6, \quad r = \frac{1}{2} \quad \checkmark$$

$$S_{11} = \frac{6 \left(1 - \left(\frac{1}{2}\right)^{11}\right)}{1 - \frac{1}{2}} \quad \checkmark$$

$$= \frac{6 \left(1 - \frac{1}{2048}\right)}{\frac{1}{2}}$$

$$= 12 \left(\frac{2047}{2048}\right)$$

$$= 11,99 \quad \checkmark$$

$$6.3.2 \quad S_{\infty} = \frac{a}{1-r}$$

$$= \frac{6}{1 - \frac{1}{2}} \quad \checkmark$$

$$= 12 \quad \checkmark$$

$$6.3.3 \sum_{k=1}^n 6 \left(\frac{1}{2}\right)^{k-1}$$

$$6.4 \quad T_1 = 324 \quad T_5 = 2500$$

$$\therefore ar^4 = 2500 \checkmark$$

$$\therefore (324)r^4 = 2500$$

$$r^4 = 7,716$$

$$r = 1,67 \checkmark$$

$$\therefore T_4 = ar^3$$

$$= (324)(1,67)^3$$

$$= 1509,02 \checkmark$$

$$\therefore \text{2nd lowest velocity is } 1509,02 \text{ rev.}$$

$$6.5 \quad d = 4$$

$$\sqrt{S_{2n} - 2S_n}$$

$$= \sqrt{\frac{2n}{2} [2a + (2n-1)4] - 2 \left[ \frac{n}{2} (2a + (n-1)4) \right]}$$

$$= \sqrt{n[2a + 8n - 4] - n(2a + 4n - 4)} \checkmark$$

$$= \sqrt{2an + 8n^2 - 4n - 2an - 4n^2 + 4n} \checkmark$$

$$= \sqrt{4n^2} \checkmark$$

$$= 2n \checkmark$$

### QUESTION 7

$$7.1 \quad f(x) = 6x^2 - 3x - 1$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{6(x+h)^2 - 3(x+h) - 1 - 6x^2 + 3x + 1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{6x^2 + 12xh + 6h^2 - 3x - 3h - 6x^2 + 3x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(12x + 6h - 3)}{h}$$

$$= \lim_{h \rightarrow 0} (12x + 6h - 3) \checkmark$$

$$= 12x - 3 \checkmark$$

7.2

$$\frac{d}{dx} \left( 4\sqrt{x} - \frac{8}{\sqrt{x}} + \pi x^3 \right)$$

$$= \frac{d}{dx} \left( 4x^{\frac{1}{2}} - 8x^{-\frac{1}{2}} + \pi x^3 \right)$$

$$= 2x^{-\frac{1}{2}} + 4x^{-\frac{3}{2}} + 3\pi x^2$$

$$7.3 \quad y = (x^2 - \sqrt{x})^2$$

$$= x^4 - 2x^{\frac{1}{2}} \cdot x^2 + x \checkmark$$

$$= x^4 - 2x^{\frac{5}{2}} + x$$

$$\frac{dy}{dx} = 4x^3 - 5x^{\frac{3}{2}} + 1 \checkmark$$

$$7.4 \quad f(x) = ax^3 + bx$$

Tangent:  $y - x - 4 = 0$

$$\therefore y = x + 4$$

$$\therefore f'(x) = m_T \text{ at } (-1; 3)$$

$$\therefore f'(x) = 1 \checkmark \text{ when } x = -1$$

$$\therefore 3ax^2 + b = 1$$

Subs.  $x = -1 \checkmark$

$$\therefore 3a + b = 1 \text{ ----- } \textcircled{1}$$

Also,  $(-1; 3)$  is a point on  $f$ .

$$\therefore 3 = a(-1)^3 + b(-1) \checkmark$$

$$\therefore 3 = -a - b \text{ ----- } \textcircled{2}$$

Solve simultaneously:

$$\textcircled{1} + \textcircled{2}: 4 = 2a$$

$$\therefore 2 = a \checkmark$$

Subs.  $a$  into  $\textcircled{1}$ :

$$6 + b = 1$$

$$\therefore b = -5 \checkmark$$

### QUESTION 8

$$8.1.1 \quad A = -\frac{1}{2}t^3 + 12t^2$$

$$\frac{dA}{dt} = -\frac{3}{2}t^2 + 24t \checkmark$$

For max:  $0 = -\frac{3}{2}t^2 + 24t \checkmark$

$$\therefore 0 = -3t^2 + 48t$$

$$\therefore 0 = -3t(t - 16) \checkmark$$

$$\therefore t = 0 \text{ or } t = 16$$

$\therefore$  area reached a max.  
after 16 months.

8.1.2

$$\begin{aligned} \text{Max area} &= -\frac{1}{2}(16)^3 + 12(16)^2 \checkmark \\ &= -2048 + 3072 \\ &= 1024 \text{ m}^2 \checkmark \end{aligned}$$

8.1.3 Rate of increase when

$$t = 1.$$

$$\frac{dA}{dt} = -\frac{3}{2}t^2 + 24t$$

$$= -\frac{3}{2}(1)^2 + 24(1) \checkmark$$

$$= -\frac{3}{2} + 24 \checkmark$$

$$= 22\frac{1}{2} \checkmark$$

8.1.4

$$A = -\frac{1}{2}t^3 + 12t^2$$

X-axis intercepts:

$$0 = -\frac{1}{2}t^3 + 12t^2 \checkmark$$

$$0 = -t^3 + 24t^2$$

$$0 = -t^2(t - 24) \checkmark$$

$$\begin{aligned}
 5.4 \quad & \log \frac{11}{10} + \log \frac{12}{11} + \log \frac{13}{12} + \dots + \log \frac{100}{99} \\
 &= \log \frac{11}{10} \cdot \frac{12}{11} \cdot \frac{13}{12} \cdot \dots \cdot \frac{100}{99} \checkmark \\
 &= \log \frac{100}{10} \checkmark \\
 &= \log 10 \checkmark \\
 &= 1 \checkmark
 \end{aligned}$$

### QUESTION 6

$$\begin{aligned}
 6.1 \quad & \text{Let } l = \text{last term} \\
 S_n &= a + a + d + \dots + l - d + l \\
 + S_n &= l + l - d + \dots + a + d + a \checkmark \\
 \hline
 \checkmark 2S_n &= (a+l) + (a+l) + \dots + (a+l) \\
 \therefore 2S_n &= n(a+l) \checkmark \\
 S_n &= \frac{n}{2}(a+l)
 \end{aligned}$$

$$\begin{aligned}
 \text{But } l &= a + (n-1)d \checkmark \\
 \therefore S_n &= \frac{n}{2} [2a + (n-1)d] \checkmark
 \end{aligned}$$

6.2.1 Distances travelled:

$$\text{Day 1: } 2 \times 0,75 = 1,5 \text{ km}$$

$$\text{Day 2: } 2 \times (2 \times 0,75) = 3 \text{ km}$$

$$\text{Day 3: } 2 \times (3 \times 0,75) = 4,5 \text{ km}$$

$$\text{Day 4: } 2 \times (4 \times 0,75) = 6 \text{ km}$$

$$\begin{aligned}
 \therefore \text{A.S. with } a &= 1,5 \text{ km} \checkmark \\
 d &= 1,5 \text{ km} \checkmark
 \end{aligned}$$

$$\begin{aligned}
 S_{35} \checkmark &= \frac{35}{2} [2(1,5) + (34)(1,5)] \checkmark \\
 &= 94,5 \text{ km} \checkmark
 \end{aligned}$$

$\therefore$  Travel a total of: 94,5 km

$$\begin{aligned}
 6.2.2 \quad T_{35} &= 1,5 + 34(1,5) \checkmark \\
 &= 52,5 \text{ km.} \checkmark
 \end{aligned}$$

On day 35 travel 52,5 km.

$$\begin{aligned}
 6.3 \quad T_3 &= 18 \quad T_5 = 162 \\
 ar^2 &= 18 \quad \textcircled{1} \quad ar^4 = 162 \quad \textcircled{2} \\
 \therefore \frac{T_2}{T_1} &\Rightarrow r^2 = 9 \checkmark
 \end{aligned}$$

$$\begin{aligned}
 \therefore r &= -3 \checkmark \quad r < 0. \\
 \text{and } a &= 2 \checkmark
 \end{aligned}$$

$$\begin{aligned}
 S_7 \checkmark &= \frac{2((-3)^7 - 1)}{-3 - 1} \checkmark \\
 &= \frac{-4376}{-4}
 \end{aligned}$$

$$= 1094 \checkmark$$

$$6.4 \sum_{r=1}^8 30(3)^{1-r}$$

$$6.4.1 \quad T_1 = 30 \quad \checkmark$$

$$T_2 = 30\left(\frac{1}{3}\right) = 10$$

$$T_3 = 30\left(\frac{1}{3}\right)^2 = \frac{30}{9} \quad \checkmark$$

$$6.4.2 \quad a = 30$$

$$r = \frac{1}{3} \quad \checkmark$$

$$S_{\infty} = \frac{30}{1 - \frac{1}{3}} \quad \checkmark$$

$$= \frac{30}{\frac{2}{3}}$$

$$= 45 \quad \checkmark$$

$$6.5 \quad r = p - 5 \quad \checkmark$$

$$\therefore -1 < p - 5 < 1 \quad \checkmark$$

$$\therefore 4 < p < 6 \quad \checkmark$$

$$6.6 \quad \frac{3}{1} + \frac{5}{2} + \frac{7}{4} + \frac{9}{8} + \dots$$

$$= \sum_{k=1}^n \frac{2k+1}{2^{k-1}} \quad \checkmark$$

### QUESTION 7

$$7.1 \quad f(x) = -3x^2 + 4x$$

$$f'(x) = \lim_{h \rightarrow 0} \left[ \frac{f(x+h) - f(x)}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[ \frac{-3(x+h)^2 + 4(x+h) - (-3x^2 + 4x)}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[ \frac{-3(x^2 + 2xh + h^2) + 4x + 4h - (-3x^2 + 4x)}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[ \frac{-3x^2 - 6xh - 3h^2 + 4x + 4h + 3x^2 - 4x}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[ \frac{h(-6x - 3h + 4)}{h} \right]$$

$$= \lim_{h \rightarrow 0} (-6x - 3h + 4) \quad \checkmark$$

$$= -6x + 4 \quad \checkmark$$

$$7.2.1 \quad y = (x - x^{-1})^2$$

$$y = x^2 - 2 + x^{-2} \quad \checkmark$$

$$\therefore \frac{dy}{dx} = 2x - 2x^{-3}$$

$$7.2.2 \quad 6 = xy$$

$$\therefore y = \frac{6}{x} = 6x^{-1} \quad \checkmark$$

$$\therefore \frac{dy}{dx} = -6x^{-2} \quad \checkmark$$

$$= \frac{-6}{x^2}$$

$$7.3 \quad f(x) = x^3 - 5x^2 - 8x + 12$$

7.3.1 x-int:

$$f(1) = 1 - 5 - 8 + 12 = 0$$

$\therefore (x-1)$  is a factor

$$\begin{array}{r} x^2 - 4x - 12 \\ x-1 \overline{) x^3 - 5x^2 - 8x + 12} \\ \underline{-x^3 + x^2} \phantom{+ 12} \\ -4x^2 - 8x \phantom{+ 12} \\ \underline{+4x^2 + 4x} \phantom{+ 12} \\ -12x + 12 \\ \underline{-12x + 12} \\ 0 \end{array}$$

$$\begin{aligned} f(x) &= (x-1)(x^2 - 4x - 12) \\ \therefore 0 &= (x-1)(x-6)(x+2) \\ \therefore x &= 1 \text{ or } x = 6 \text{ or } x = -2 \\ \text{x-int: } &(1; 0), (6; 0), (-2; 0) \end{aligned}$$

$$\text{y-int: } (0; 12) \quad \checkmark$$

turning points:

$$f'(x) = 3x^2 - 10x - 8 \quad \checkmark$$

$$\therefore 0 = 3x^2 - 10x - 8$$

$$0 = (3x + 2)(x - 4)$$

$$\therefore x = -\frac{2}{3} \quad \checkmark \text{ or } x = 4 \quad \checkmark$$

$$f\left(-\frac{2}{3}\right) = -\frac{8}{27} - 5\left(\frac{4}{9}\right) - 8\left(-\frac{2}{3}\right) + 12$$

$$\therefore f\left(-\frac{2}{3}\right) = -\frac{8}{27} - \frac{20}{9} + \frac{16}{3} + 12$$

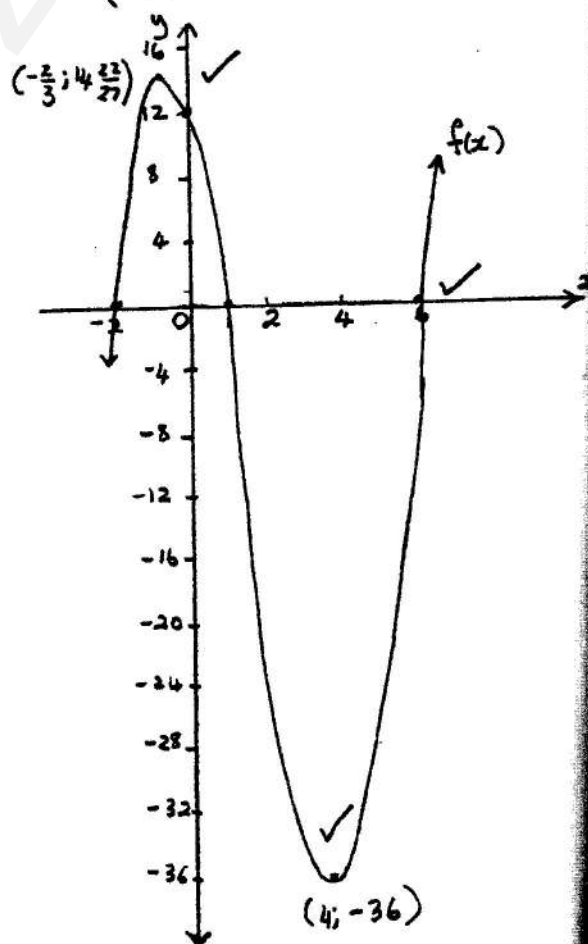
$$= -8 - 60 + 144 + 324$$

$$= 14 \frac{22}{27}$$

$$\text{T.P. } \left(-\frac{2}{3}; 14 \frac{22}{27}\right) \text{ local max}$$

$$\begin{aligned} f(4) &= 64 - 80 - 32 + 12 \\ &= -36 \quad \checkmark \end{aligned}$$

$$\text{T.P. } (4; -36) \text{ local min}$$



7.3.2 When  $c = -36 \quad \checkmark \checkmark \checkmark$

$$\text{i.e. } f(x) = x^3 - 5x^2 - 8x + 48$$

will have exactly 2 real roots

DEPARTMENT OF EDUCATION  
MATHEMATICS PAPER 1  
(HIGHER GRADE)  
MARCH 2002

QUESTION 1

$$1.1.1 \quad a + \frac{1}{a} = 2$$

$$a^2 + 1 = 2a \quad \checkmark$$

$$a^2 - 2a + 1 = 0$$

$$(a-1)^2 = 0 \quad \checkmark$$

$$\therefore a = 1 \quad \checkmark$$

$$1.1.2 \quad (2x^2 + 3x) + \frac{1}{(2x^2 + 3x)} = 2$$

$$\text{Let } a = 2x^2 + 3x \quad \checkmark$$

$$\therefore 1 = 2x^2 + 3x$$

$$\therefore 0 = 2x^2 + 3x - 1 \quad \checkmark$$

$$x = \frac{-3 \pm \sqrt{9 - 4(2)(-1)}}{2(2)}$$

$$x = \frac{-3 \pm \sqrt{17}}{4} \quad \checkmark$$

$$\therefore x = \frac{-3 + 4,12}{4} \text{ or } \frac{-3 - 4,12}{4}$$

$$\therefore x = 0,28 \quad \checkmark \text{ or } x = -1,78 \quad \checkmark$$

$$1.2.1 \quad \frac{2-x}{x+5} \leq 0$$

$$\frac{-11}{-5} + \frac{0}{2} \quad \checkmark$$

$$\therefore x < -5 \text{ or } x > 2 \quad \checkmark \checkmark$$

$$1.2.2 \quad \sqrt{4x-3} - 2x = -3$$

$$\therefore (\sqrt{4x-3})^2 = (2x-3)^2 \quad \checkmark$$

$$\therefore 4x-3 = 4x^2 - 12x + 9 \quad \checkmark$$

$$\therefore 0 = 4x^2 - 16x + 12 \quad \checkmark$$

$$0 = x^2 - 4x + 3$$

$$0 = (x-3)(x-1) \quad \checkmark$$

$$\therefore x = 3 \text{ or } x = 1 \text{ (not valid)} \quad \checkmark$$

QUESTION 2

$$2.1 \quad x^2 + 2x(k+1) + (2k+1) =$$

$$\Delta = (2k+2)^2 - 4(2k+1)$$

$$= 4k^2 + 8k + 4 - 8k - 4$$

$$= 4k^2 \quad \checkmark$$

$$\therefore \Delta \geq 0 \quad \checkmark$$

$$\therefore \text{Roots are real.} \quad \checkmark$$

$$\begin{aligned}
 2.2 \quad & 2x^2 - 8x + 5 \\
 &= 2 \left[ x^2 - 4x + \frac{5}{2} \right] \\
 &= 2 \left[ (x-2)^2 - 4 + \frac{5}{2} \right] \\
 &= 2 \left[ (x-2)^2 - \frac{3}{2} \right] \\
 &= 2(x-2)^2 - 3
 \end{aligned}$$

$\therefore$  the expression has a minimum value of  $-3$ , since  $2(x-2)^2 \geq 0$ .

### QUESTION 3

$$\begin{aligned}
 3.1.1 \quad & y = a(x-p)^2 + q \\
 \therefore & y = a(x-2)^2 + 3 \quad \checkmark \\
 \text{Subs } (0; 5) & \\
 5 &= a(0-2)^2 + 3 \quad \checkmark \\
 5 &= 4a + 3 \\
 \frac{1}{2} &= a \quad \checkmark \\
 \therefore & y = \frac{1}{2}(x-2)^2 + 3 \quad \checkmark \\
 y &= \frac{1}{2}(x^2 - 4x + 4) + 3 \\
 &= \frac{1}{2}x^2 - 2x + 2 + 3 \\
 &= \frac{1}{2}x^2 - 2x + 5 \quad \checkmark
 \end{aligned}$$

3.1.2  $\Delta$  is negative, because there are no  $x$ -intercepts.

3.1.3 Parabola must cut the  $x$ -axis twice.

$\therefore$  turning point must move down more than 3 units  
 $\therefore$   $y$ -intercept  $< 2$   
 $\therefore k > 3 \quad \checkmark$

3.1.4 Turning point will be  $(2; -2) \quad \checkmark$

$$\begin{aligned}
 \therefore & y = a(x-2)^2 - 2 \\
 \text{Subs } (0; 0) &
 \end{aligned}$$

$$\begin{aligned}
 \therefore & 0 = a(-2)^2 - 2 \\
 0 &= 4a - 2
 \end{aligned}$$

$$\frac{1}{2} = a$$

$$\begin{aligned}
 \therefore & y = \frac{1}{2}(x-2)^2 - 2 \\
 y &= \frac{1}{2}(x^2 - 4x + 4) - 2
 \end{aligned}$$

$$y = \frac{1}{2}x^2 - 2x + 2 - 2$$

$$y = \frac{1}{2}x^2 - 2x \quad \checkmark$$

(or, can be done by inspection with  $c=0$ ).

$$6.3 \quad a+1; a-1; 2a-5$$

$$6.3.1 \quad \frac{a-1}{a+1} = \frac{2a-5}{a-1} \quad \checkmark$$

$$\therefore (a-1)^2 = (a+1)(2a-5) \quad \checkmark$$

$$a^2 - 2a + 1 = 2a^2 - 3a - 5$$

$$0 = a^2 - a - 6 \quad \checkmark$$

$$0 = (a-3)(a+2) \quad \checkmark$$

$$\therefore a = 3 \quad \text{or} \quad a = -2$$

$$\therefore r = \frac{3-1}{3+1} \quad \text{or} \quad r = \frac{-2-1}{-2+1}$$

$$r = \frac{1}{2} \quad \checkmark \quad \text{or} \quad r = 3$$

But if series convergent  
then  $|r| < 1 \therefore r \neq 3 \quad \checkmark$

$$\therefore a \neq -2$$

$$\therefore a = 3 \quad \checkmark$$

$$6.3.2 \quad T_1 = 3+1 = 4 \quad \checkmark$$

$$r = \frac{1}{2} \quad \checkmark$$

$$\therefore S_{\infty} = \frac{4}{1 - \frac{1}{2}} \quad \checkmark$$

$$= 8 \quad \checkmark$$

### QUESTION 7

$$7.1 \quad f(x) = 3x - x^2$$

$$f'(x) = \lim_{h \rightarrow 0} \left[ \frac{3(x+h) - (x+h)^2 - (3x - x^2)}{h} \right] \quad \checkmark$$

$$= \lim_{h \rightarrow 0} \left[ \frac{3x + 3h - x^2 - 2xh - h^2 - 3x + x^2}{h} \right] \quad \checkmark$$

$$= \lim_{h \rightarrow 0} \left[ \frac{h(3 - 2x - h)}{h} \right] \quad \checkmark$$

$$= \lim_{h \rightarrow 0} (3 - 2x - h) \quad \checkmark$$

$$= 3 - 2x \quad \checkmark$$

$$7.2.1 \quad xy = 5$$

$$y = 5x^{-1} \quad \checkmark$$

$$\therefore \frac{dy}{dx} = -5x^{-2}$$

$$= -\frac{5}{x^2} \quad \checkmark$$

$$7.2.2 \quad y = \frac{1 - 2x + \sqrt{x}}{x^2}$$

$$y = (1 - 2x + x^{\frac{1}{2}}) \cdot x^{-2} \quad \checkmark$$

$$= x^{-2} - 2x^{-1} + x^{-\frac{3}{2}} \quad \checkmark$$

$$\therefore \frac{dy}{dx} = -2x^{-3} + 2x^{-2} - \frac{3}{2}x^{-\frac{5}{2}}$$

$$= -\frac{2}{x^3} + \frac{2}{x^2} - \frac{3}{2x^{\frac{5}{2}}}$$

$$7.3 \quad f(x) = 2x^2 + x - 1$$

$$f'(x) = 4x + 1 \quad \checkmark$$

$$m_T = -3 \quad \checkmark$$

$$\therefore -3 = 4x + 1$$

$$\therefore -4 = 4x$$

$$\therefore -1 = x \quad \checkmark$$

$$\therefore f(-1) = 2 - 1 - 1 = 0 \quad \checkmark$$

$\therefore$  Tangent at point  $(-1; 0)$

$\therefore$  Eqn. of tangent:

$$y = -3x + c$$

Subs.  $(-1; 0)$

$$0 = 3 + c$$

$$\therefore c = -3 \quad \checkmark$$

$$\therefore y = -3x - 3 \quad \checkmark$$

$$7.4 \quad y = x^3 - x^2 - 5x - 3$$

y-int:  $(0; -3) \quad \checkmark$

x-int:  $0 = x^3 - x^2 - 5x - 3$

By remainder thm.  $(x+1)$  is a factor.

$$\begin{array}{r} x^2 - 2x - 3 \quad \checkmark \checkmark \\ x+1 \overline{) x^3 - x^2 - 5x - 3} \\ \underline{-x^3 + x^2} \phantom{-3} \\ -2x^2 - 5x \phantom{-3} \\ \underline{+2x^2 + 2x} \phantom{-3} \\ -3x - 3 \phantom{-3} \\ \underline{-3x - 3} \\ 0 \end{array}$$

$$\begin{aligned} \therefore y &= (x+1)(x^2 - 2x - 3) \\ &= (x+1)(x-3)(x+1) \quad \checkmark \\ \therefore x &= -1 \quad \checkmark \text{ or } x = 3 \quad \checkmark \end{aligned}$$

$$\frac{dy}{dx} = 3x^2 - 2x - 5 \quad \checkmark$$

For stationary pts.

$$0 = 3x^2 - 2x - 5$$

$$0 = (3x-5)(x+1)$$

$$\therefore x = \frac{5}{3} \quad \checkmark \text{ or } x = -1 \quad \checkmark$$

$\therefore$  At  $x = -1$ ,  $y = 0 \therefore (-1; 0) \quad \checkmark$

$$\text{At } x = \frac{5}{3},$$

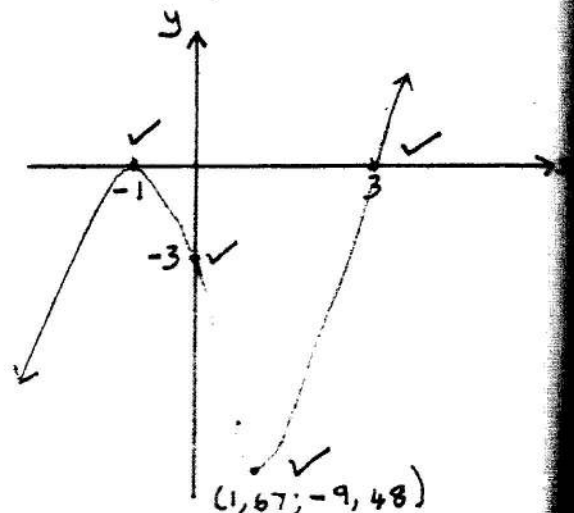
$$y = \left(\frac{5}{3}\right)^3 - \left(\frac{5}{3}\right)^2 - 5\left(\frac{5}{3}\right) - 3$$

$$= \frac{125}{27} - \frac{25}{9} - \frac{25}{3} - 3$$

$$= \frac{125 - 75 - 225 - 81}{27}$$

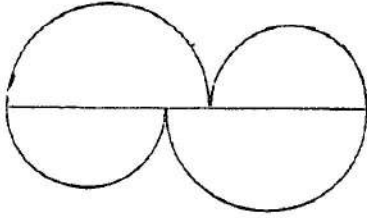
$$= \frac{-256}{27} = -9,48 \quad \checkmark$$

$$\therefore (1,67; -9,48)$$



QUESTION 8

8.1



$$8.1.1 \quad A = \pi R^2 + \pi r^2 \quad \checkmark$$

$$\text{but } r = 200 - R$$

$$\begin{aligned} \therefore A &= \pi R^2 + \pi (200 - R)^2 \\ &= \pi R^2 + \pi (40000 - 400R + R^2) \\ &= \pi R^2 + 40000\pi - 400\pi R + \pi R^2 \\ &= 2\pi R^2 - 400\pi R + 40000\pi \quad \checkmark \end{aligned}$$

$$8.1.2 \quad \frac{dA}{dR} = 4\pi R - 400\pi \quad \checkmark$$

$$\text{For minimum, } 4\pi R - 400\pi = 0 \quad \checkmark$$

$$\therefore R = \frac{400\pi}{4\pi}$$

$$\therefore R = 100 \text{ mm} \quad \checkmark$$

$$\begin{aligned} \therefore r &= 200 - 100 \\ &= 100 \text{ mm} \quad \checkmark \end{aligned}$$

8.1.3 For minimum area all four semi-circles have the same radius = 100 mm. The shape would then be two identical touching circles, not as the shape required.

8.2.1

$$P = -\frac{3}{80}x^2 + 6x - 180$$

For no profit,  $P = 0$ 

$$\therefore 0 = -\frac{3}{80}x^2 + 6x - 180$$

$$\therefore 0 = -3x^2 + 480x - 14400$$

$$0 = x^2 - 160x + 4800 \quad \checkmark$$

$$0 = (x - 120)(x - 40) \quad \checkmark$$

$$\therefore x = 120 \text{ or } x = 40$$

$\therefore$  No profit at 120 km/h or 40 km/h.

$$8.2.2 \quad \frac{dP}{dx} = -\frac{3}{40}x + 6 \quad \checkmark$$

$\therefore$  For most economical,

$$0 = -\frac{3}{40}x + 6$$

$$x = 80 \text{ km/h.} \quad \checkmark$$

$$\therefore P = -\frac{3}{80}(80)^2 + 6(80) - 180 \quad \checkmark$$

$$= -3(80) + 480 - 180$$

$$= -240 + 480 - 180$$

$$= 60$$

$\therefore$  Profit is R60 per hour.

$$8.2.3 \quad P < 0$$

$$\therefore (x - 120)(x - 40) < 0$$

$$\therefore 30 \leq x < 40 \text{ or } x > 120 \quad \checkmark$$

$$2x^2 - 8x + 5$$

$$= 2 \left[ x^2 - 4x + \frac{5}{2} \right]$$

$$= 2 \left[ (x-2)^2 - 4 + \frac{5}{2} \right]$$

$$= 2 \left[ (x-2)^2 - \frac{3}{2} \right]$$

$$= 2(x-2)^2 - 3$$

$\therefore$  the expression has a minimum value of  $-3$ , since  $2(x-2)^2 \geq 0$ .

### QUESTION 3

3.1.1  $y = a(x-p)^2 + q$   
 $\therefore y = a(x-2)^2 + 3$  ✓  
 Subs (0; 5)

$$5 = a(0-2)^2 + 3$$

$$5 = 4a + 3$$

$$\frac{1}{2} = a$$

$$\therefore y = \frac{1}{2}(x-2)^2 + 3$$

$$y = \frac{1}{2}(x^2 - 4x + 4) + 3$$

$$= \frac{1}{2}x^2 - 2x + 2 + 3$$

$$= \frac{1}{2}x^2 - 2x + 5$$

3.1.2  $\Delta$  is negative, because there are no  $x$ -intercepts.

3.1.3 Parabola must cut the  $x$ -axis twice.

$\therefore$  turning point must move down more than 3 units

$\therefore y$ -intercept  $< 2$

$\therefore k > 3$  ✓

3.1.4 Turning point will be (2; -2) ✓

$$\therefore y = a(x-2)^2 - 2$$

Subs (0; 0)

$$\therefore 0 = a(-2)^2 - 2$$

$$0 = 4a - 2$$

$$\frac{1}{2} = a$$

$$\therefore y = \frac{1}{2}(x-2)^2 - 2$$

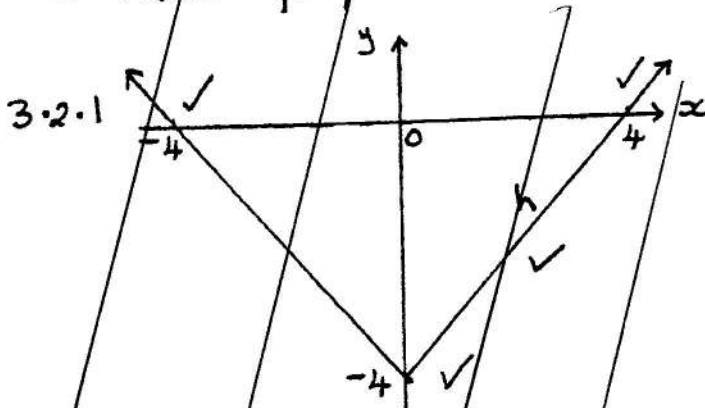
$$y = \frac{1}{2}(x^2 - 4x + 4) - 2$$

$$y = \frac{1}{2}x^2 - 2x + 2 - 2$$

$$y = \frac{1}{2}x^2 - 2x$$

(or, can be done by inspection with  $c=0$ ).

3.2  $h(x) = |x| - 4$



3.2.2  $x > 4$  or  $x < -4$

3.3.1  $f(x) = a^x$

$\therefore y = a^x$

$\therefore$  Subs  $(2; 2\frac{1}{4})$

$2\frac{1}{4} = a^2 \checkmark$

$\therefore \frac{9}{4} = a$

$\therefore a = \pm \frac{3}{2}$

but  $a > 0$

$\therefore a = \frac{3}{2} \checkmark$

3.3.2  $g(x) = \log_{\frac{3}{2}} x \checkmark$

3.3.3  $x > 0 \checkmark$

3.3.4  $h(x) = (\frac{2}{3})^x \checkmark$

3.3.5  $y > 0 \checkmark$

### QUESTION 4

$f(x) = x^3 + 3x^2 + b$

4.1  $y = x^3 + 3x^2 + b$

Subs  $(-2; 0) \checkmark$

$0 = -8 + 12 + b \checkmark$

$0 = 4 + b$

$\therefore b = -4 \checkmark$

4.2  $f(-2) = -8 + 12 - 4$   
 $= 0$

$\therefore$  remainder is 0.  $\checkmark$

4.3  $f(x) = x^3 + 3x^2 - 4$

$(x+2)$  is a factor.

$$\begin{array}{r} x^2 + x - 2 \\ x+2 \overline{) x^3 + 3x^2 - 4} \\ \underline{-x^3 + 2x^2} \phantom{-4} \\ x^2 \phantom{-4} \\ \underline{-x^2 + 2x} \phantom{-4} \\ -2x \phantom{-4} \\ \underline{-2x} \phantom{-4} \\ 0 \end{array}$$

$\therefore f(x) = (x+2)(x^2 + x - 2) \checkmark$   
 $= (x+2)(x+2)(x-1) \checkmark$

$$\therefore 150n > 2350$$

$$n > 15,67$$

$\therefore$  Expenses will exceed income in 16<sup>th</sup> year.

### QUESTION 8

8.1  $f(x) = x - x^2$

$$f'(x) = \lim_{h \rightarrow 0} \left[ \frac{f(x+h) - f(x)}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[ \frac{(x+h) - (x+h)^2 - (x - x^2)}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[ \frac{x+h - x^2 - 2xh - h^2 - x + x^2}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[ \frac{h - 2xh - h^2}{h} \right]$$

$$= \lim_{h \rightarrow 0} \frac{h(1 - 2x - h)}{h}$$

$$= \lim_{h \rightarrow 0} (1 - 2x - h)$$

$$= 1 - 2x \quad \checkmark$$

8.2.1  $y = (x^3 - 1)^2$

$$= x^6 - 2x^3 + 1 \quad \checkmark$$

$$\therefore \frac{dy}{dx} = 6x^5 - 6x^2 \quad \checkmark$$

8.2.2  $y = \frac{x^3 + \sqrt{x^3}}{x}$

$$\therefore y = \frac{x^3}{x} + \frac{x^{\frac{3}{2}}}{x} \quad \checkmark$$

$$y = x^2 + x^{\frac{1}{2}} \quad \checkmark$$

$$\therefore \frac{dy}{dx} = 2x + \frac{1}{2}x^{-\frac{1}{2}}$$

$$= 2x + \frac{1}{2\sqrt{x}} \quad \checkmark$$

8.3  $f(x) = -x^3 + 6x^2 - 9x + 4$

8.3.1 y-int:  $(0; 4) \quad \checkmark$

x-int:  $0 = -x^3 + 6x^2 - 9x + 4$

$f(1) = 0$  by remainder thm.

$\therefore (x-1)$  is a factor  $\checkmark$

$$\begin{array}{r} -x^2 + 5x - 4 \\ x-1 \overline{) -x^3 + 6x^2 - 9x + 4} \\ \underline{+x^3 + x^2} \phantom{+ 4} \\ 5x^2 - 9x \phantom{+ 4} \\ \underline{-5x^2 + 5x} \phantom{+ 4} \\ -4x + 4 \\ \underline{-4x + 4} \\ 0 \end{array}$$

$$\therefore (x-1)(-x^2 + 5x - 4) = 0 \quad \checkmark$$

$$-(x-1)(x^2 - 5x + 4) = 0$$

$$-(x-1)(x-1)(x-4) = 0$$

$$\therefore \text{x-int: } (1; 0), (4; 0) \quad \checkmark$$

$$f'(x) = -3x^2 + 12x - 9 \quad \checkmark$$

For turning points:

$$f'(x) = 0$$

$$\therefore 0 = -3x^2 + 12x - 9$$

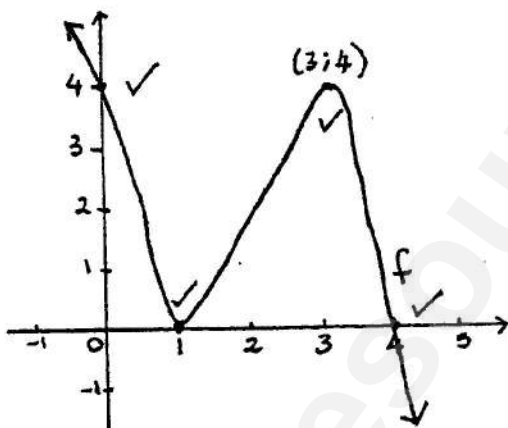
$$0 = x^2 - 4x + 3$$

$$0 = (x-1)(x-3) \quad \checkmark$$

$$f(1) = 0$$

$$f(3) = -27 + 54 - 27 + 4 \\ = 4$$

$\therefore$  min. turning pt (1; 0)  $\checkmark$   
max turning pt (3; 4)  $\checkmark$



$$8.3.2 \quad f'(x) = -3x^2 + 12x - 9 \\ f'(2) = -3(4) + 24 - 9 \\ = 3$$

$$\therefore m_T = 3 \quad \checkmark$$

$\therefore$  Equation of tangent:

$$y = 3x + c$$

Subs. (2; 2)

$$2 = 6 + c \quad \checkmark$$

$$-4 = c$$

$$\therefore y = 3x - 4 \quad \checkmark$$

### QUESTION 9

$$9.1.1 \quad OP^2 = x^2 + y^2 \quad \checkmark$$

$$9.1.2 \quad x^3 + y^2 = 8$$

$$\therefore y^2 = 8 - x^3$$

$$\therefore OP^2 = x^2 + (8 - x^3)$$

$$OP^2 = -x^3 + x^2 + 8$$

9.1.3 For maximum

$$\frac{d(OP^2)}{dx} = -3x^2 + 2x \quad \checkmark$$

$$\therefore 0 = -3x^2 + 2x$$

$$\therefore 0 = -x(3x - 2)$$

$$\therefore x = 0 \quad \checkmark \text{ or } x = \frac{2}{3} \quad \checkmark$$

If  $x = 0$ ,

$$OP^2 = 8$$

If  $x = \frac{2}{3}$ ,

$$OP^2 = -\left(\frac{8}{27}\right) + \frac{4}{9} + 8 \quad \checkmark$$

$$= \frac{-8 + 12 + 216}{27}$$

$$= \frac{220}{27} = 8,15$$

$\therefore$  Max value of  $OP^2$  is 8,15.  $\checkmark$

9.1.4 From 9.1.3,

when  $x=0$ ,  $OP^2 = 8 \therefore OP = \sqrt{8}$   
 $\therefore OP = 2\sqrt{2}$

oc-int. of graph:  $x^3 + 0^2 = 8$   
 $\therefore x^3 = 8$

$\therefore x = 2$   
 $\therefore$  Shortest distance is 2. ✓

9.2 Equation of tangent:

$$y = -3x + 4$$

$$\therefore m_T = -3 \text{ at } x = -1$$

$$\therefore f'(-1) = -3 \quad \checkmark$$

Equation of tangent:

$$y = -3x + 4$$

Subs. point  $(-1; y)$

$$\therefore y = 3 + 4$$

$$\therefore y = 7$$

$$\therefore f(-1) = 7 \quad \checkmark$$

9.3.1  $y = f'(x)$

at  $x=0$ ,  $y = f'(0) = 40$  ✓

$\therefore$  gradient of  $f$  is 40. ✓

9.3.2  $f'(x) = 0$  ✓ at maximum

$$\therefore f'(a) = 0$$

$$\therefore a = 30. \quad \checkmark$$

9.3.3  $0 < x < 30$  ✓

### QUESTION 10

10.1  $x \leq 2000$  ✓

$$y \geq 500 \quad \checkmark$$

$$y \leq \frac{1}{2}x + 500 \quad \checkmark$$

$$y \geq -\frac{1}{2}x + 1000 \quad \checkmark$$

10.2 At P: solve simultaneously

$$2y - x = 1000$$

$$2y + x = 2000 \quad \checkmark \text{ (adding)}$$

$$\therefore 4y = 3000$$

$$y = 750 \quad \checkmark$$

$\therefore$  Subs  $y$  into  $2y - x = 1000$

$$\therefore 1500 - x = 1000$$

$$500 = x \quad \checkmark$$

$\therefore$  P is the point  $(500; 750)$  ✓

At T: subs  $y = 500$  into

$$x + 2y = 2000$$

$$\therefore x + 1000 = 2000$$

$$\therefore x = 1000 \quad \checkmark$$

$\therefore$  T is the point  $(1000; 500)$  ✓

10.3 Min. total cost:

$$C = 250x + 200y$$

$$\therefore y = -\frac{5}{4}x + \frac{C}{200} \quad \checkmark$$

$\therefore$  Min. at P  $(500; 750)$  ✓

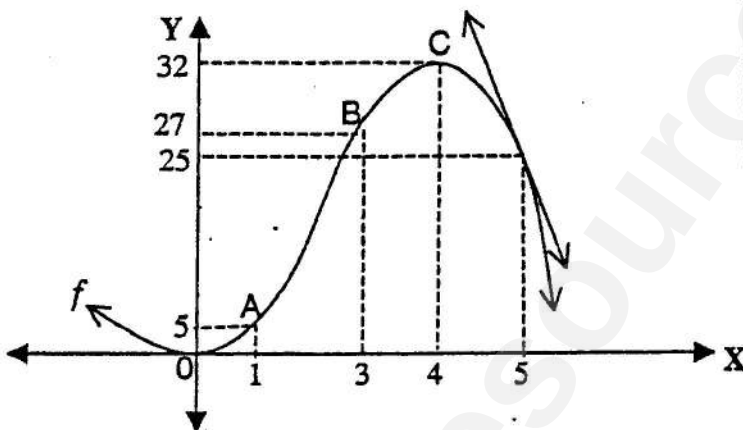
$$6.2.2 \quad y = \frac{x^2 - 2x - 3}{x+1}, \quad x \neq -1$$

$$y = \frac{(x-3)(x+1)}{x+1} \checkmark$$

$$y = (x-3) \checkmark$$

$$\therefore \frac{dy}{dx} = 1 \checkmark$$

$$6.3 \quad f(x) = -x^3 + 6x^2$$



$$6.3.1 \quad \left. \begin{array}{l} \text{At A, } x=1, f(x)=5 \\ \text{At B, } x=3, f(x+h)=27, \\ \text{and } h=2. \end{array} \right\} \checkmark$$

$$\begin{aligned} \text{Ave. gradient} &= \frac{f(x+h) - f(x)}{h} \\ &= \frac{27 - 5}{2} \checkmark \\ &= 11 \checkmark \end{aligned}$$

$$6.3.2.1 \quad \text{At C,} \\ \text{tangent gradient} = 0 \checkmark$$

$$6.3.2.2$$

$$f'(x) = -3x^2 + 12x \checkmark$$

$$f'(5) = -3(5)^2 + 12(5)$$

$$= -75 + 60$$

$$= -15 \checkmark$$

$$6.3.3 \quad \text{Eqn. of tangent.}$$

$$y = -15x + C$$

$$\text{Subs. } (5, 25)$$

$$25 = -15(5) + C \checkmark$$

$$25 = -75 + C$$

$$100 = C$$

$$\therefore y = -15x + 100 \checkmark$$

### QUESTION 7

$$y = -x^3 + 10x^2 - 17x - 28$$

$$\therefore f(x) = -x^3 + 10x^2 - 17x - 28$$

$$y\text{-int: } (0, -28) \checkmark$$

$$x\text{-int:}$$

$$0 = -x^3 + 10x^2 - 17x - 28$$

$$f(-1) = 1 + 10 + 17 - 28$$

$$= 0$$

$$\therefore (x+1) \checkmark \text{ is a factor}$$

$$\begin{array}{r}
 -x^2 + 11x - 28 \quad \checkmark \\
 x+1 \overline{) -x^3 + 10x^2 - 17x - 28} \\
 \underline{+x^3 + x^2} \phantom{-17x - 28} \\
 11x^2 - 17x \phantom{- 28} \\
 \underline{-11x^2 + 11x} \phantom{- 28} \\
 -28x - 28 \\
 \underline{-28x - 28} \\
 0
 \end{array}$$

$$\begin{aligned}
 \therefore 0 &= (x+1)(-x^2 + 11x - 28) \\
 0 &= -(x+1)(x^2 - 11x + 28) \\
 0 &= -(x+1)(x-7)(x-4) \\
 \therefore x &= -1 \text{ or } 7 \text{ or } 4 \\
 \therefore (-1; 0), (7; 0), (4; 0) \quad \checkmark
 \end{aligned}$$

$$\frac{dy}{dx} = -3x^2 + 20x - 17 \quad \checkmark$$

For stationary points:

$$0 = -3x^2 + 20x - 17$$

$$0 = 3x^2 - 20x + 17$$

$$0 = (3x - 17)(x - 1)$$

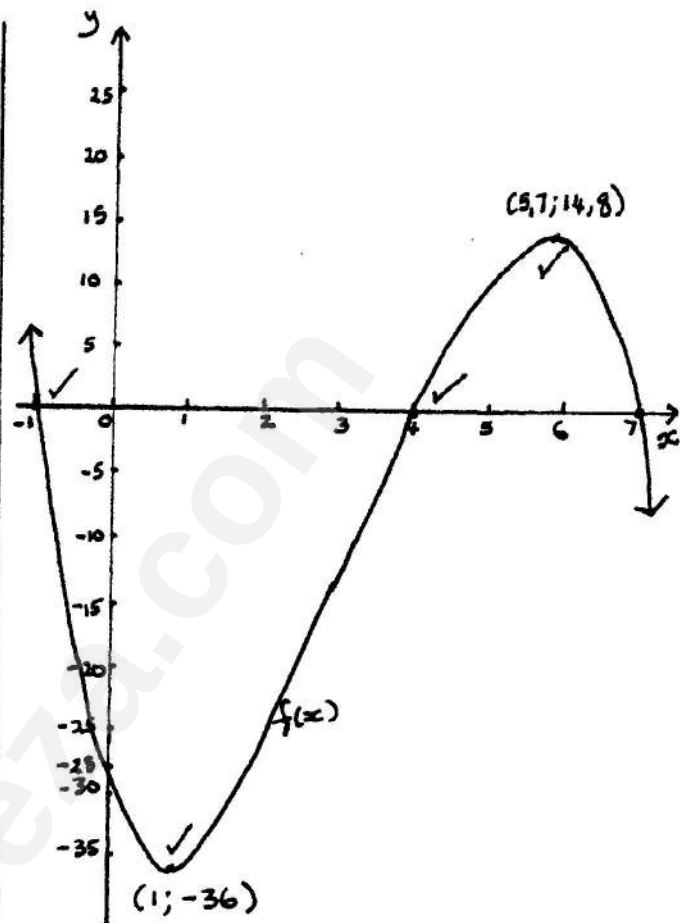
$$x = \frac{17}{3} \text{ or } x = 1 \quad \checkmark$$

$$\begin{aligned}
 f\left(\frac{17}{3}\right) &= -\left(\frac{17}{3}\right)^3 + 10\left(\frac{17}{3}\right)^2 - 17\left(\frac{17}{3}\right) - 28 \\
 &= 14,8 \quad \checkmark
 \end{aligned}$$

$\therefore$  Local max at  $(5,7; 14,8)$

$$f(1) = -1 + 10 - 17 - 28 = -36 \quad \checkmark$$

$\therefore$  Local min at  $(1; -36)$



$$7 \cdot 2 \cdot 1 \text{ Volume} = x^2 \cdot h$$

$$\therefore 1000 = x^2 \cdot h$$

$$\therefore \frac{1000}{x^2} = h \quad \checkmark$$

$$\therefore A = 4x^2 + 4xh \quad \checkmark$$

$$= 4x^2 + 4x \left( \frac{1000}{x^2} \right)$$

$$= 4x^2 + \frac{4000}{x} \quad \checkmark$$

7.2.2  $A = 4x^2 + 4000x^{-1}$   
 $\therefore \frac{dA}{dx} = 8x - 4000x^{-2}$  ✓

For minimum,  
 $0 = 8x - \frac{4000}{x^2}$

$\therefore 4000 = 8x^3$  ✓

$\therefore 500 = x^3$

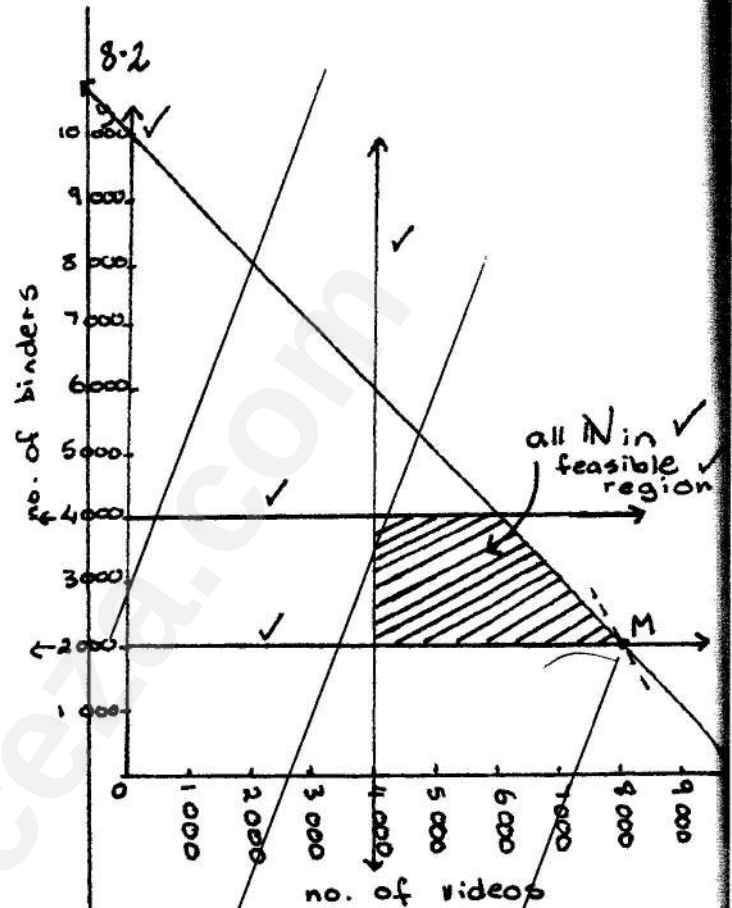
$\therefore 7,94 \text{ cm} = x$  ✓

and,  $h = \frac{1000}{(7,94)^2}$   
 $= 15,87 \text{ cm}$  ✓

### QUESTION 8

let  $x$  = no. of videos sold  
 $y$  = no. of binders sold.

8.1  $x + y \leq 10\,000$  ✓  
 $x \geq 4\,000$  ✓  
 $2\,000 \leq y \leq 4\,000$  ✓



8.3  $I = 50x + 30y$  ✓

8.4  $y = -\frac{5}{3}x + \frac{I}{30}$  ✓

For max. income  $x = 8\,000$  ✓  
 $y = 2\,000$  ✓

8.5  $I = 50(8\,000) + 30(2\,000)$  ✓  
 $= 400\,000 + 60\,000$   
 $= 460\,000$

Max. income will be R 460 000. ✓