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DATA HANDLING REVISION (1)



Learner Note: In this session we will revise Grade 11 Data Handling, which is an extremely important part of Paper 2. Make sure you master the content of this exciting topic as it counts almost 30 marks out of 150.

SECTION A: TYPICAL EXAM QUESTIONS

Question 1

The results for Argentina for the past 14 World Cup tournaments are recorded in the table below:

World Cup tournament	Matches played	Wins	Draws	Losses	Goals for	Goals against
2006 Germany	5	3	2	0	11	3
2002 Korea/Japan	3	1	1	1	2	2
1998 France	5	3	1	1	10	4
1994 USA	4	2	0	2	8	6
1990 Italy	7	2	3	2	5	4
1986 Mexico	7	6	1	0	14	5
1982 Spain	5	2	0	3	8	7
1978 Argentina	7	5	1	1	15	4
1974 Germany	6	1	2	3	9	11
1966 England	4	2	1	1	4	2
1962 Chile	3	1	1	1	2	3
1958 Sweden	3	1	0	2	5	10
1934 Italy	1	0	0	1	2	3
1930 Uruguay	5	4	0	1	18	9

Source: www.2010 FIFA World Cup: Statistics - MediaClubSouthAfrica.com

- Determine the quartiles for the matches played. (3)
- Determine the quartiles for the wins. (3)
- Determine the quartiles for the goals scored against Argentina. (3)
- Draw box and whisker plots for the matches played by Argentina, the wins and the goals scored against Argentina. (15)
- By referring to the box and whisker plots, comment on the distribution of the data. (6)
- Calculate the mean for the number of matches played over the 14 tournaments. (2)
- By referring to the box and whisker plots, comment on the distribution of the data. (6)
- Calculate the mean for the number of matches played over the 14 tournaments. (2)

(g) Complete the following table:

Matches played (x)	$x - \bar{x}$	$(x - \bar{x})^2$
5		
3		
5		
4		
7		
7		
5		
7		
6		
4		
3		
3		
1		
5		
		$\sum (x - \bar{x})^2 =$

(h) Use the table to calculate the standard deviation for the matches played. (3)

(i) Using your calculator, calculate the standard deviation for the goals for and the goals against Argentina. Comment on your findings. (4)

SECTION B: ADDITIONAL CONTENT NOTES

Mean

The mean of a set of data is the average. To get the mean, you add the scores and divide by the number of scores.

Mode

This is the most frequently occurring score.

Quartiles

Quartiles are measures of dispersion around the median, which is a good measure of central tendency. The median divides the data into two halves. The lower and upper quartiles further subdivide the data into quarters.

There are three quartiles:

The Lower Quartile (Q_1): This is the median of the lower half of the values.

The Median (M or Q_2): This is the value that divides the data into halves.

The Upper Quartile (Q_3): This is the median of the upper half of the values.

If there is an **odd** number of data values in the data set, then the specific quartile will be a value in the data set. If there is an **even** number of data values in the data set, then the specific quartile will not be a value in the data set. A number which will serve as a quartile, will need to be inserted into the data set (the average of the two middle numbers).

Range

The range is the difference between the largest and the smallest value in the data set. The bigger the range, the more spread out the data is.

The Inter-quartile range (IQR)

The difference between the lower and upper quartile is called the inter-quartile range.

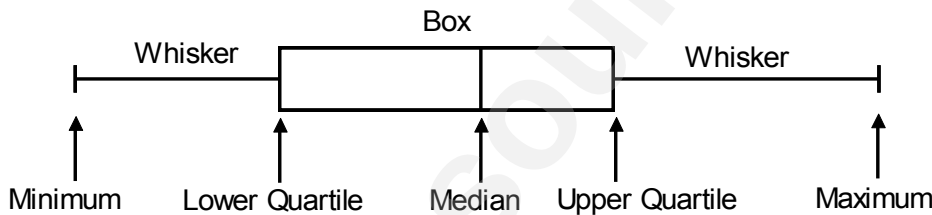
Five Number Summaries

The Five Number Summary uses the following measures of dispersion:

- Minimum: The smallest value in the data
- Lower Quartile: The median of the lower half of the values
- Median: The value that divided the data into halves
- Upper Quartile: The median of the upper half of the values
- Maximum: The largest value in the data

Box and Whisker Plots

A Box and Whisker Plot is a graphical representation of the Five Number Summary.



a table

Standard deviation using

The standard deviation (SD) can be determined by using the following formula:

$$SD = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

Calculator programmes to calculate standard deviation**CASIO fx-82ES PLUS:**

MODE

2 : STAT

1 : 1 – VAR

Enter the data points: push = after each data point

AC

SHIFT

STAT

5: VAR

3 : $\bar{x}$ σ n

push = to get standard deviation

SHARP DAL

MODE 1 =

Enter data points: push STO 2 M+ after each data point

RCL 6 to get standard deviation

SECTION C: SOLUTIONS AND HINTS**Question 1**

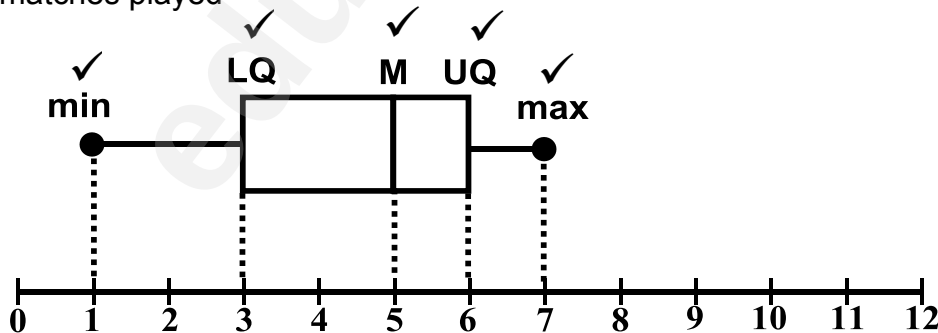
Rearrange the data from smallest to biggest:

Matches played	Wins	Draws	Losses	Goals for	Goals against
1	0	0	0	2	2
3	1	0	0	2	2
3	1	0	1	2	3
3	1	0	1	4	3
4	1	0	1	5	3
4	2	1	1	5	4
5	2	1	1	8	4
5	2	1	1	8	4
5	2	1	1	9	5
5	3	1	2	10	6
6	3	1	2	11	7
7	4	2	2	14	9
7	5	2	3	15	10
7	6	3	3	18	11

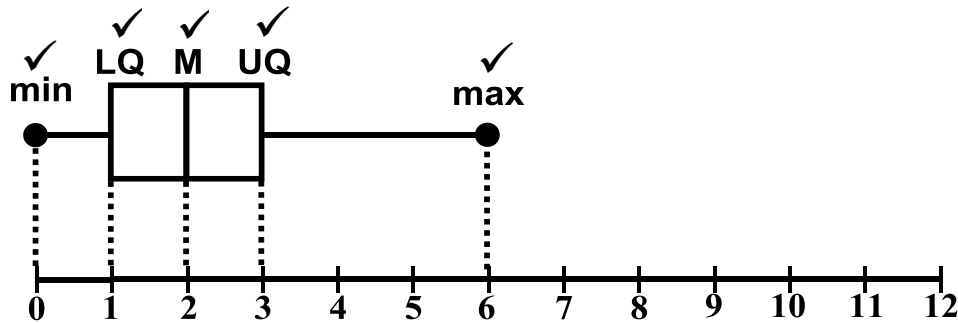
1(a)	1 3 3 3 4 4 5 5 5 5 6 7 7 7 Median = $\frac{7\text{th} + 8\text{th}}{2} = \frac{5+5}{2} = 5$ Lower quartile = 4th = 3 Upper quartile = 11th = 6	✓ M = 5 ✓ Lower quartile = 3 ✓ Upper quartile = 6 (3)
1(b)	0 1 1 1 1 2 2 2 2 3 3 4 5 6 Median = $\frac{7\text{th} + 8\text{th}}{2} = \frac{2+2}{2} = 2$ Lower quartile = 4th = 1 Upper quartile = 11th = 3	✓ M = 2 ✓ Lower quartile = 1 ✓ Upper quartile = 3 (3)
1(c)	2 2 3 3 3 4 4 4 5 6 7 9 10 11 Median = $\frac{7\text{th} + 8\text{th}}{2} = \frac{4+4}{2} = 4$ Lower quartile = 4th = 3 Upper quartile = 11th = 7	✓ M = 4 ✓ Lower quartile = 3 ✓ Upper quartile = 7 (3)

1(d)

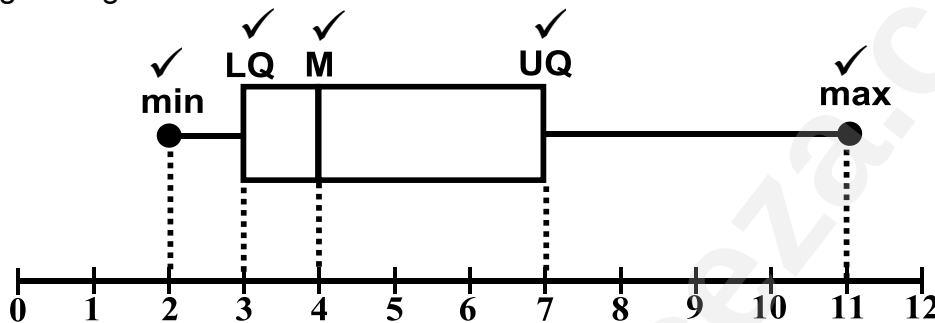
matches played



wins



goals against



1(e)	Matches played: The data is negatively skewed. For more than half of the World Cup tournaments, Argentina played 5 games or more (max of 7 games played per tournament), which is excellent. Wins: The number of wins between the lower and upper quartiles are clustered symmetrically about the median. Half of the wins are 2 or less, and half of the wins are 2 or more. Goals against: The data is positively skewed. For about half of the 14 tournaments, 4 goals were scored against Argentina per tournament.	✓ negatively skewed. ✓ 5 games or more. ✓ symmetrical about the median ✓ half of games are 2 or more or 2 or less ✓ positively skewed. ✓ 4 goal against for the 14 tournaments (6)
1(f)	Mean $= \frac{1+3+3+3+4+4+5+5+5+5+6+7+7+7}{14}$ $= \frac{65}{14}$ $= 4,6$	✓ $\frac{65}{14}$ ✓ 4,6 (2)

1(g) $\bar{x} = 4,6$

Matches played (x)	$x - \bar{x}$	$(x - \bar{x})^2$
5	0,4	0,16
3	-1,6	2,56
5	0,4	0,16
4	-0,6	0,36
7	2,4	5,76
7	2,4	5,76
5	0,4	0,16
7	2,4	5,76
6	1,4	1,96
4	-0,6	0,36
3	-1,6	2,56
3	-1,6	2,56
1	-3,6	12,96
5	0,4	0,16
		$\sum (x - \bar{x})^2 = 41,24$

1(h)	$SD = \sqrt{\frac{\sum (x - \bar{x})^2}{14}} = \sqrt{\frac{41,24}{14}} = 1,7$	✓ 41,24 ✓ $\sqrt{\frac{41,24}{14}}$ ✓ 1,7 (3)
1(i)	Goals for: 4,9 Goals against: 2,9 The standard deviation for the goals scored for Argentina is larger than for the goal scored against the team. This means that the goals scored against the team, were more closely clustered about the mean and, therefore, more consistent. The goals scored for the team were more spread out. This means that the goal scoring was less consistent.	✓ 4,9 ✓ 2,9 ✓ ✓ discussion (4)

SECTION D: HOMEWORK**Question 1**

The results for Argentina for the past 14 World Cup tournaments are recorded in the table below.

World Cup tournament	Matches played	Wins	Draws	Losses	Goals for	Goals against
2006 Germany	5	3	2	0	11	3
2002 Korea/Japan	3	1	1	1	2	2
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1990 Italy	7	2	3	2	5	4
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1930 Uruguay	5	4	0	1	18	9

Source: www.2010 FIFA World Cup:Statistics – MediaClubSouthAfrica.com

- Determine the quartiles for the goals scored for Argentina. (3)
- Determine the quartiles for the goals scored against Argentina. (3)
- Draw box and whisker plots for the goals scored for and against Argentina. (10)
- By referring to the box and whisker plots, comment on the distribution of the data. (4)
- Calculate the mean for the number of losses over the 14 tournaments. (2)

(f) Complete the following table:

Losses (x)	$x - \bar{x}$	$(x - \bar{x})^2$
0		
1		
1		
2		
2		
0		
3		
1		
3		
1		
1		
2		
1		
1		
		$\sum (x - \bar{x})^2 =$

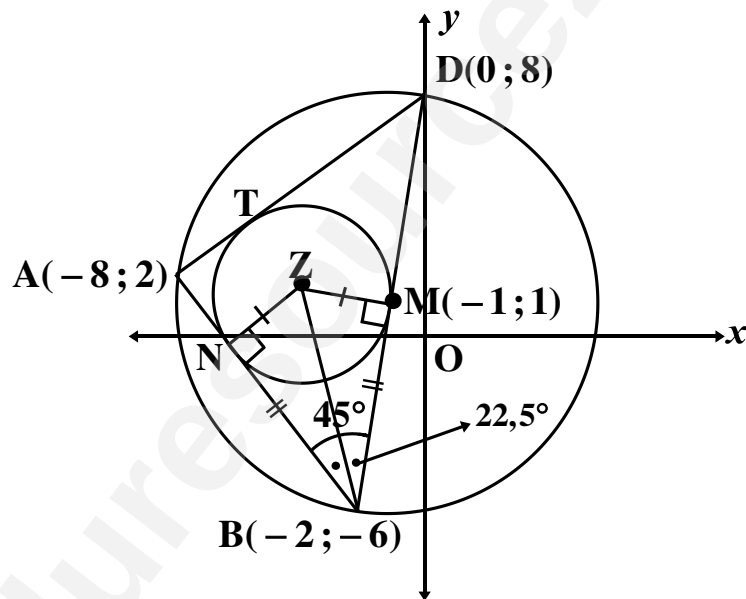
(g) Use the table to calculate the standard deviation for the number of losses. (3)

(h) Using your calculator, calculate the standard deviation for the losses and wins for Argentina. Comment on your findings. (4)

SECTION E: SOLUTIONS TO SESSION 30 HOMEWORK

(a)	$M\left(\frac{0+(-2)}{2}; \frac{8+(-6)}{2}\right)$ $= M(-1; 1)$	✓ x-coordinate ✓ y-coordinate (2)
(b)	$y = 7x + 58$ $\therefore y = 7(-8) + 58$ $\therefore y = 2$	✓ substitution ✓ answer (2)
(c)	$m_{\text{line}} = 7$ $m_{\text{AM}} = \frac{2-1}{-8-(-1)} = -\frac{1}{7}$ $m_{\text{line}} \times m_{\text{AM}} = 7 \times -\frac{1}{7} = -1$ The line is a tangent to the circle	✓ relationship ✓ $m_{\text{line}} = 7$ ✓✓ $m_{\text{AM}} = -\frac{1}{7}$ ✓ product (5)
(d)	$AD^2 = (0+8)^2 + (8-2)^2 \quad AB^2 = (-8+2)^2 + (2+6)^2$ $\therefore AD = \sqrt{(0+8)^2 + (8-2)^2} \quad \therefore AB = \sqrt{(-8+2)^2 + (2+6)^2}$ $\therefore AD = \sqrt{36+64} \quad \therefore AB = \sqrt{36+64}$ $\therefore AD = \sqrt{100} \quad \therefore AB = \sqrt{100}$ $\therefore AD = 10 \quad \therefore AB = 10$	✓ substitution ✓ answer ✓ substitution ✓ answer (4)
(e)	$m_{\text{AD}} = \frac{8-(2)}{0-(-8)} = \frac{6}{8} = \frac{3}{4}$ $m_{\text{AB}} = \frac{2-(-6)}{-8-(-2)} = \frac{8}{-6} = -\frac{4}{3}$ $m_{\text{AB}} m_{\text{AD}} = -\frac{4}{3} \times \frac{3}{4} = -1$ $\therefore \hat{DAB} = 90^\circ$	✓ gradient of AD ✓ gradient of AB ✓ product (3)

(f)	$\theta = 45^\circ$	✓ answer (1)
(g)	$BM^2 = (-1 - (-2))^2 + (1 - (-6))^2$ $\therefore BM^2 = 1 + 49$ $\therefore BM^2 = 50$ $\therefore BM = \sqrt{50}$ $\frac{ZM}{BM} = \sin 22,5^\circ$ $\therefore \frac{ZM}{\sqrt{50}} = \sin 22,5^\circ$ $\therefore ZM = \sqrt{50} \sin 22,5^\circ$ $\therefore ZM = 2,93$	✓ distance formula ✓ length of BM ✓ tan perpendicular to rad ✓ definition ✓ $22,5^\circ$ ✓ answer (6)



SECTION F: SOLUTIONS TO CONSOLIDATION EXERCISE SESSION 30

1(a)	$\frac{3}{LB} = \tan 40^\circ$ $\therefore LB = \frac{3}{\tan 40^\circ}$ $\therefore LB = 3,58 \text{ m}$	✓ definition ✓ answer (3)
1(b)	$AB^2 = AL^2 + BL^2 - 2.AL.BL.\cos 113^\circ$ $\therefore AB^2 = (5,2)^2 + (3,58)^2 - 2(5,2)(3,58)\cos 113^\circ$ $\therefore AB^2 = 54,40410138 \text{ m}^2$ $\therefore AB = 7,38 \text{ m}$	✓ cosine rule ✓ substitution ✓✓ answer (4)
1(c)	$\text{Area of } \triangle ABL = \frac{1}{2} AL.BL.\sin \hat{A}LB$ $= \frac{1}{2} (5,2)(3,58)\sin 113^\circ$ $= 8.568059176$ $= 8,57 \text{ m}$	✓ area rule ✓ substitution ✓✓ answer (4)
2.	$\text{Area } \triangle ADC = \frac{1}{2} (b)(b)\sin 60^\circ$ $\therefore \text{Area } \triangle ADC = \frac{1}{2} b^2 \frac{\sqrt{3}}{2} = \frac{\sqrt{3}b^2}{4}$ $\text{Area } \triangle ABF = \frac{\sqrt{3}c^2}{4}$ $\text{Area } \triangle CBE = \frac{\sqrt{3}a^2}{4}$ $\therefore \text{Area } \triangle ABF + \text{Area } \triangle CBE = \frac{\sqrt{3}c^2}{4} + \frac{\sqrt{3}a^2}{4}$ $\therefore \text{Area } \triangle ABF + \text{Area } \triangle CBE = \frac{\sqrt{3}}{4} (c^2 + a^2)$ $\therefore \text{Area } \triangle ABF + \text{Area } \triangle CBE = \frac{\sqrt{3}}{4} (b^2) = \frac{\sqrt{3}b^2}{4}$ $\therefore \text{Area } \triangle ADC = \text{Area } \triangle ABF + \text{Area } \triangle CBE$	✓ area rule ✓ equal sides ✓ 60° ✓ $\text{Area } \triangle ADC = \frac{\sqrt{3}b^2}{4}$ ✓ $\text{Area } \triangle ABF = \frac{\sqrt{3}c^2}{4}$ ✓ $\text{Area } \triangle CBE = \frac{\sqrt{3}a^2}{4}$ ✓ use of Pythagoras to establish identity (7)