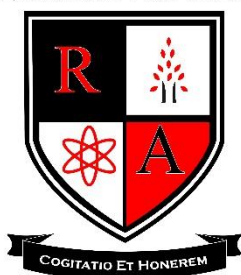




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GDE PREPARATORY PAST PAPERS 2014 TO 2020

REDWOOD ACADEMY



GRADE 12

MATHEMATICS PAPER 1

BY NAEEM MOHAMED

NAME: _____

GAUTENG DEPARTMENT OF EDUCATION
PREPARATORY EXAMINATION

PROOFREAD	☐	DATE	_____
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SIGNATURE			_____

MATHEMATICS
(First Paper)

TIME: 3 hours

MARKS: 150

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. The question paper consists of 13 questions.
 2. Answer ALL the questions.
 3. Show ALL calculations, diagrams, graphs, etc. that you have used in obtaining your answer.
 4. Answers only will not necessarily be awarded full marks.
 5. An approved scientific calculator (non-programmable, non-graphic) may be used, unless stated otherwise.
 6. Where necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
 7. Diagrams are not necessarily drawn to scale.
 8. A formula sheet is attached at the end of the question paper.
 9. Number your answers exactly as the questions are numbered.
 10. It is in your own interest to write neatly and legibly.
-
-

QUESTION 1

- 1.1 Calculate the values of x :

$$x^2 + 2x = 0$$

(3)

- 1.2 Hence, determine the positive values of x for which

$$x^2 \geq -2x.$$

(3)

- 1.3 Solve for x :

1.3.1 $2x^2 - 3x - 7 = 0$ (correct to TWO decimal places).

(3)

1.3.2 $2^0 + 2^{x-2} + 2^{x+1} + 2^x = 53$

(5)
[14]**QUESTION 2**

- 2.1 Given:

$$k = \sqrt{(x+1)^2 - 4}, \text{ where } k \text{ is a real number.}$$

2.1.1 Solve for x if $k = 4$. (leave the answer in the simplest surd form).

(5)

2.1.2 Write down the minimum value of k .

(1)

- 2.2 Calculate the values of k , for which the equation $3x^2 + 2x - k + 1 = 0$ has real roots.

(4)
[10]

QUESTION 3

Given:

$$\sum_{p=1}^n (101 - 2p)$$

3.1 Write down the first THREE terms of the series. (3)

3.2 Calculate n for which $\sum_{p=1}^n (101 - 2p) = 0$ (5)
[8]

QUESTION 4

4.1 The infinite number sequence below is a combination of two sequences:

1; 1; 1; 5; 1; 9; 1; 13; ...

Determine:

4.1.1 T_{31} , the thirty-first term of the sequence. (1)

4.1.2 S_{32} , the sum of the first thirty two terms of the sequence. (4)

4.1.3 the smallest value of the even number n for which $T_n > 97$. (3)

4.2 A plant is infected with bacteria. The number pattern below represents the number of bacterial cells in the plant at the end of every hour after infection.

3; 10; 19; 30 ...

4.2.1 Determine a general formula for the n th term (T_n) of this sequence. (4)

4.2.2 Calculate the number of bacterial cells in the plant at the end of the first day (24 hours). (2)

- 4.2.3 After the first day of infection, the plant undergoes treatment, which reduces the formation of new bacterial cells. The sequence below represents the number of new bacterial cells formed in the plant at the end of every day after infection:

670 ; 536 ; 428,8 ; ...

The plant will perish if infected by more than 3 500 bacterial cells. If the treatment continues indefinitely, prove that the plant will survive this bacterial infection.

Show all calculations used in your answer.

(3)
[17]

QUESTION 5

- 5.1 Convert an interest rate of 10% per annum, compounded monthly, to an annual interest rate, compounded semi-annually. (3)
- 5.2 Gavin purchases a house for R1 200 000. He pays a deposit of 10% of the value of the house. The bank grants him a loan for the outstanding amount, at an interest rate of 8,4% per annum compounded monthly, payable over a period of 20 years
- 5.2.1 Calculate the deposit Gavin pays on the house. (2)
- 5.2.2 Calculate Gavin's monthly repayments. (4)
- 5.2.3 Calculate the total amount Gavin would have paid for the house at the end of 20 years (3)
- 5.3 For two years, equipment worth R20 000, depreciates by 20% per annum according to the reducing-balance method. Each year thereafter, the annual depreciation is 2% less than that of the previous year. Determine the depreciated value of the equipment after 4 years. (4)
- [16]

QUESTION 6

The function $f(x) = \frac{3}{x-1} - 1$ is given.

- 6.1 Draw a neat sketch graph of f in your answer book.
Clearly show all intercepts with the axes as well as all the asymptotes. (4)
- 6.2 Determine the equation(s) of the axis of symmetry of f . (2)
- 6.3 Determine the range of f . (1)
- [7]

QUESTION 7

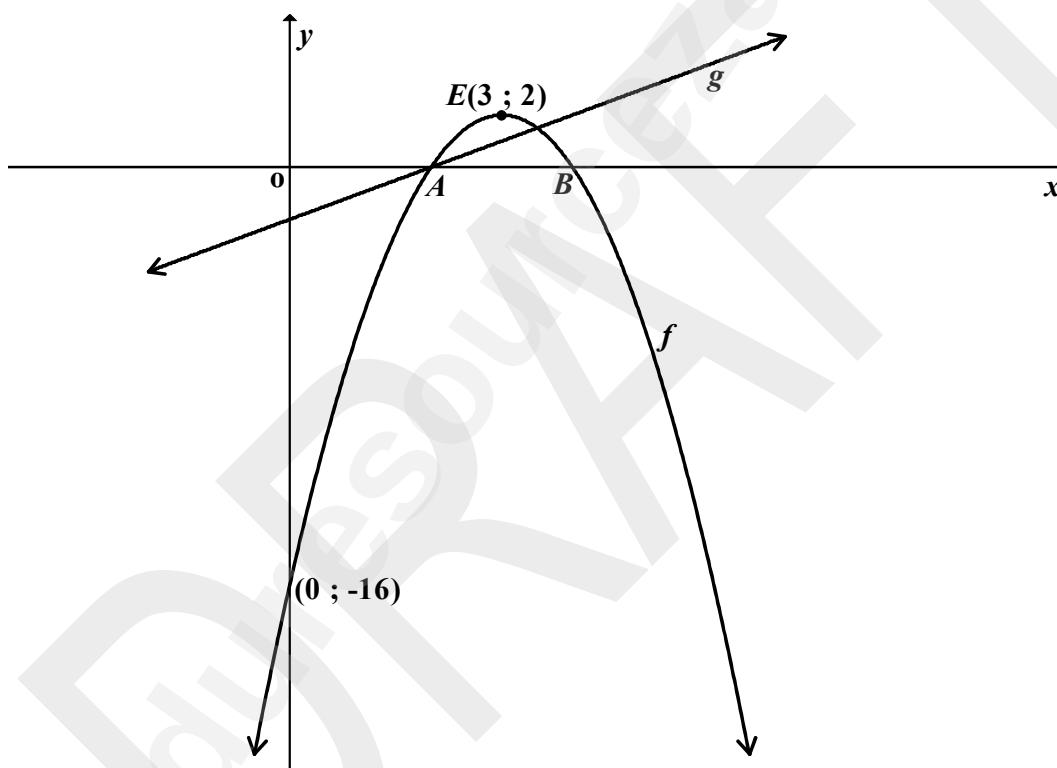
Sketched below are the graphs of $f(x) = a(x + p)^2 + q$ and $g(x) = mx + c$, where a, p, q, m and c are real constants.

$E(3; 2)$ is the turning point and $(0; -16)$ is the y -intercept of f .

The graph of f intersects the x -axis at A and B .

C , a point on the y -axis, is symmetrical to A about the line $y = -x$.

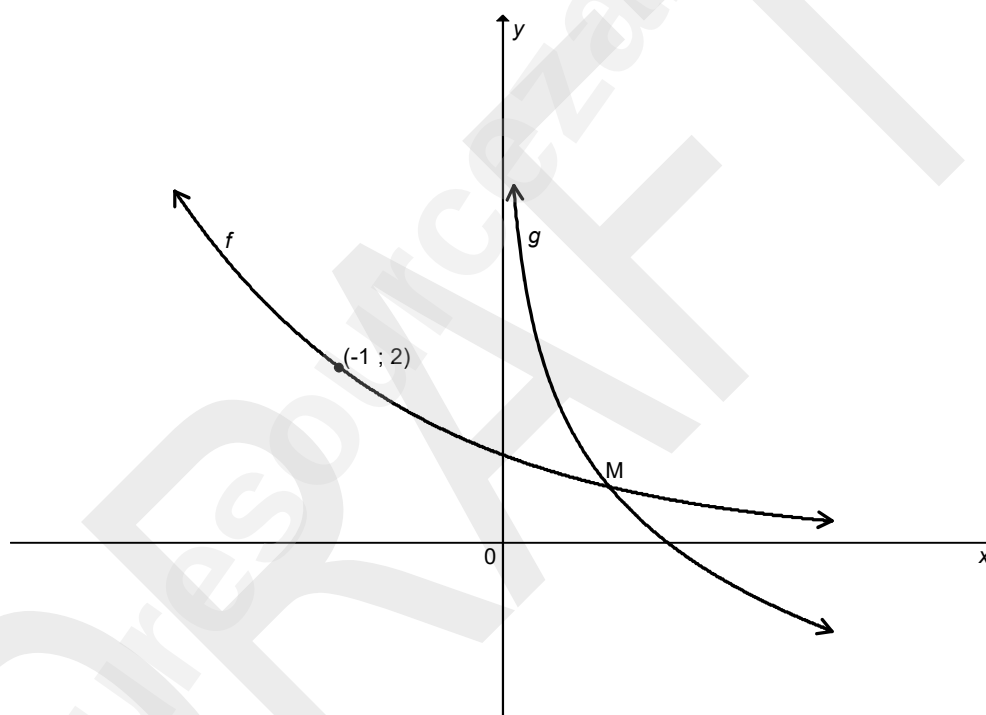
The graph of g passes through points C and A .



- 7.1 Show that the equation of f can be written as $f(x) = -2x^2 + 12x - 16$ (3)
- 7.2 Determine the coordinates of points A , B and C . (4)
- 7.3 Determine the equation of g . (3)
- 7.4 If $h(x) = \frac{-96}{x}$, calculate the point of intersection of h and f . (4)
- 7.5 Write down the turning point of p , if $p(x) = -f\left(\frac{x}{2}\right)$. (2)

QUESTION 8

The graphs of $f(x) = a^x$, $a > 0$ and $g(x) = f^{-1}(x)$, where g is the inverse of f , are sketched below: The point $(-1; 2)$ lies on f . M is the point of intersection of f and g .



- 8.1 Show that $a = \frac{1}{2}$. (1)
- 8.2 Determine the equation of g in the form $g(x) = \dots$. (2)
- 8.3 Write down the equation of the line which divides f and g symmetrically. (1)
- 8.4 If M is the point of intersection of f and the line $y = x$, show that $x \log \frac{1}{2} - \log x = 0$. (3)

[7]

QUESTION 9

9.1 Given $f(x) = 1 - 3x^2$.

9.1.1 Differentiate f from first principles (5)

9.1.2 Determine the equation of the tangent of f at $x = -2$. (3)

9.2 Evaluate:

9.2.1 $Dx \left[\left(1 - x^{\frac{1}{2}} \right) \left(1 - 2x^{-\frac{1}{2}} \right) \right]$ (4)

9.2.2 $\frac{dy}{dp}$ if $y = \frac{p^3 + 1}{p + 1}$ (3)
[15]

QUESTION 10

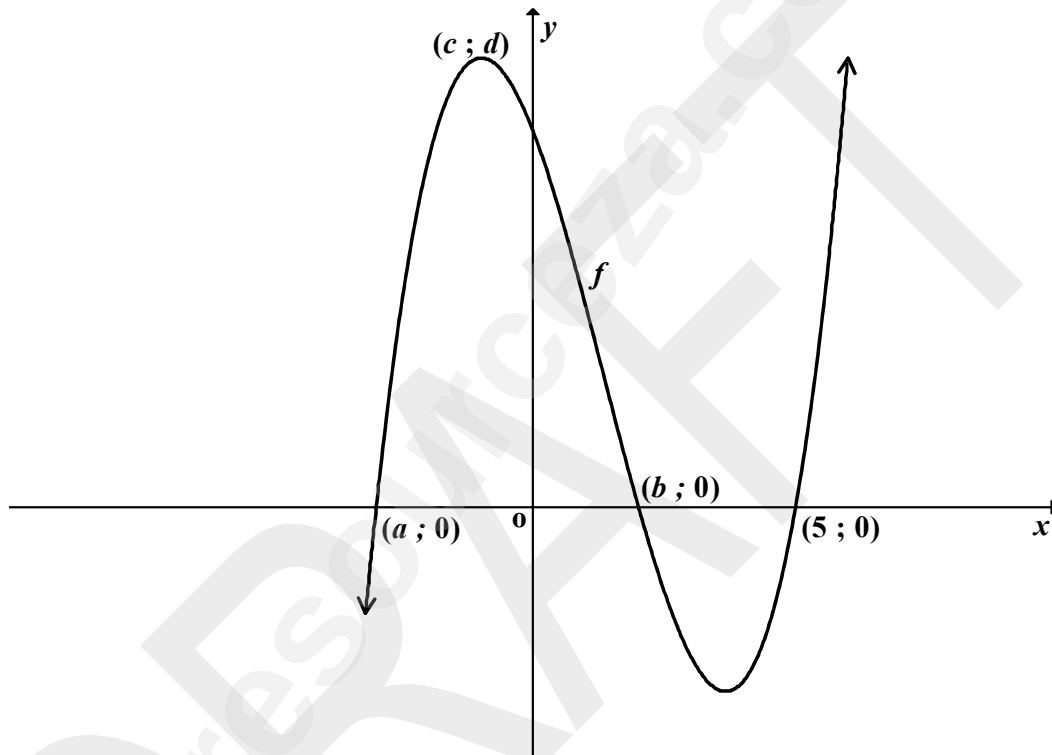
The displacement, (s) (distance from a fixed point), of a car moving towards a fixed point after t seconds, is given by

$$s = t^3 - 9t^2 + 24t + 8.$$

10.1 Determine an expression for the velocity (v) of the car.
(rate of change of distance with respect to time) (1)

10.2 Calculate the time t for which the car was reversing. (5)
[6]

QUESTION 11



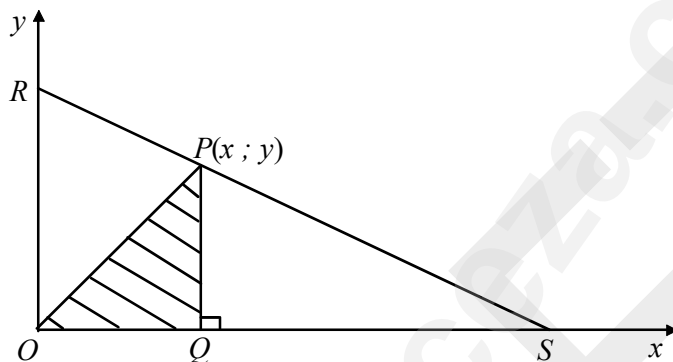
Determine

- 11.1.1 the values of a and b . (4)
- 11.1.2 the values of c and d , where $(c; d)$ is a turning point of f . (4)
- 11.1.3 the values of x for which $f'(x) < 0$ and $f(x) \geq 0$. (2)
- 11.2 The turning points of a cubic graph $y = g(x)$ are $(-1; 6)$ and $(5; 4)$.
In your answer book, draw a sketch graph of y' (the derivative function of $y = g(x)$). Label only the x -intercepts clearly. (3)

[13]

QUESTION 12

In the diagram below, $P(x; y)$ is a point on line segment PS with equation $x + 2y = 18$, where $0 \leq x \leq 18$. Q is a point on the x -axis such that $\triangle POQ$ forms a right-angled triangle. $PQ \perp OS$



12.1 Write the y -coordinate of point P in terms of x . (1)

12.2 Calculate the coordinates of P so that the area of $\triangle OPQ$ is at its maximum. (5)
[6]

QUESTION 13

13.1 Tebo writes an Art and a Music examination. He has 40% chance of passing the Music examination, 60% chance of passing the Art examination and 30% chance of passing both the Music and Art examination.

Calculate the probability that Tebo will pass the Music or Art examination. (3)

13.2 A survey was conducted asking 60 people with which hand they write and what colour hair they have. The results are summarized in the table below.

		HAND USED TO WRITE WITH		
		Right	Left	Total
	Light	a	b	20
	Dark	c	d	40
	Total	48	12	60

The survey concluded that the 'hand used for writing' and 'hair colour' are independent events.

Calculate the values of a , b , and c .

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(5)

13.3 The digits from 1 to 9 are used to make 5-digit codes.

- 13.3.1 Determine the number of 5-digit codes possible, if the digits are arranged in any order without repetition. (2)
- 13.3.2 Determine the number of 5-digit codes possible, if the code formed has to be an even number and the digits may not be repeated (3)
- 13.3.3 Determine the number of 5-digit codes possible, if the code formed only uses even digits and repetition of digits is allowed. (2)
- [15]

TOTAL: 150

5. INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$



GAUTENG PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

**GAUTENG DEPARTMENT OF EDUCATION
PREPARATORY EXAMINATION**

2015

10611

MATHEMATICS

FIRST PAPER

**MARKS: 150
TIME: 3 hours**

11 Pages and 1 Information sheet

MATHEMATICS: Paper 1
10611E



10611E

X10



GAUTENG DEPARTMENT OF EDUCATION
PREPARATORY EXAMINATION

MATHEMATICS
(First Paper)

TIME: 3 hours

MARKS: 150

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-

QUESTION 1

1.1 Solve for x . Leave the answer in the simplest surd form where necessary:

1.1.1 $(2x + 5)(x^2 - 2) = 0$ (3)

1.1.2 $x^2 - 4 \geq 5$ (4)

1.1.3 $12^{2x} = 8.36^x$ (4)

1.2 Solve for x , correct to two decimal places:

$2(x+1)^2 = 9$ (4)

1.3 Solve for x and y simultaneously:

$y = -2x + 7$ and $\frac{y+5}{x-1} = \frac{1}{2}$ (4)

1.4 Determine the values of p , for which the equation

$3^x = 1 - p$ will have real solutions. (2)

[21]

QUESTION 2

The first two terms of an arithmetic series, A, and an infinite geometric series, B, are the same.

A: $-2 + x + \dots$ and

B: $-2 + x + \dots$ are given.

2.1 Write down in terms of x

2.1.1 The third term of the geometric series, B. (2)

2.1.2 The third term of the arithmetic series, A. (2)

2.2 If the sum of the first three terms in the arithmetic series A is equal to the third term of the geometric series B, then calculate the value of x . (5)

2.3 If $x = -6$, does the geometric series B converge?

Show calculations to support your answer. (3)

[12]

QUESTION 3

Given: $\sum_{k=1}^n T_k = n^2 + 4n$, where T_k is the general term of a series.

3.1 Calculate $\sum_{k=1}^{250} T_k$ (2)

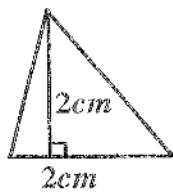
3.2 Calculate T_{100} (3)

3.3 How many terms of the sequence must be added to give a sum of 1 440? (3)
[8]

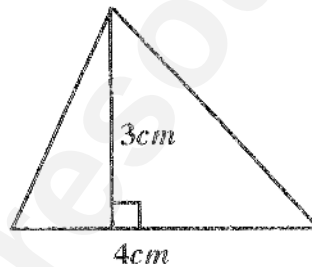
QUESTION 4

A pattern of triangles is formed by increasing the base of the triangle by 2 cm and the perpendicular height by 1 cm, in each successive triangle.

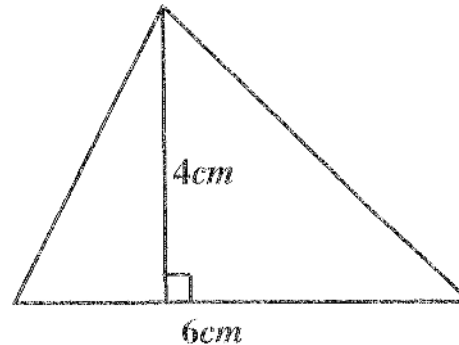
The first triangle has a base of 2 cm and a height of 2 cm. The pattern continues in this manner.



TRIANGLE 1



TRIANGLE 2



TRIANGLE 3

4.1 Calculate the areas of the first four triangles. (2)

4.2 Calculate the area of the hundredth triangle in the pattern. (4)
[6]

QUESTION 5

- 5.1 5.1.1 Dumisani establishes a trucking company. The first truck he buys costs R650 000 and it depreciates at 30% per annum on a reducing balance method.
- Calculate the depreciated value of the truck after 4 years. (2)
- 5.1.2 The price of a new truck appreciates, as a result of inflation, at a rate of 15% per annum.
- Calculate the value of a new truck in 4 years' time. (2)
- 5.1.3 Dumisani plans to replace the original truck in 4 years' time. He plans to use the current truck as a trade-in and pay the remaining amount in cash.
- To save up for the required cash amount, he sets up a sinking fund, making equal monthly instalments into a fund earning interest of 9,5% per annum, compounded monthly.
- The first instalment of the sinking fund is paid at the end of the first month of the 4-year period and the last instalment is made at the end of the 4 years.
- Calculate the monthly instalment Dumisani pays into the sinking fund. (4)
- 5.2 5.2.1 Harry buys a property for R1 500 000. After paying a deposit, he takes a loan for the remaining amount of R1 275 000, at an interest rate of 9,2% per annum, compounded monthly over a period of 20 years. The monthly instalment on the loan is R11 636,92.
- Calculate the outstanding balance of the loan after 7 years. (3)
- 5.2.2 After 7 years, due to financial difficulty, Harry misses 5 consecutive payments. Thereafter he continues making monthly payments into the loan account until the end of the 20 year period.
- Calculate the value of the new monthly instalment to settle the loan. (4)
- [15]

QUESTION 6

In your ANSWER BOOK, draw a clear sketch graph of function h defined by the equation $h(x) = ab^x + q$, where $a < 0$; $b > 1$ and $q < 0$; a , b and q are real numbers.

Indicate all intercepts with the axes and asymptotes. (3)

[3]

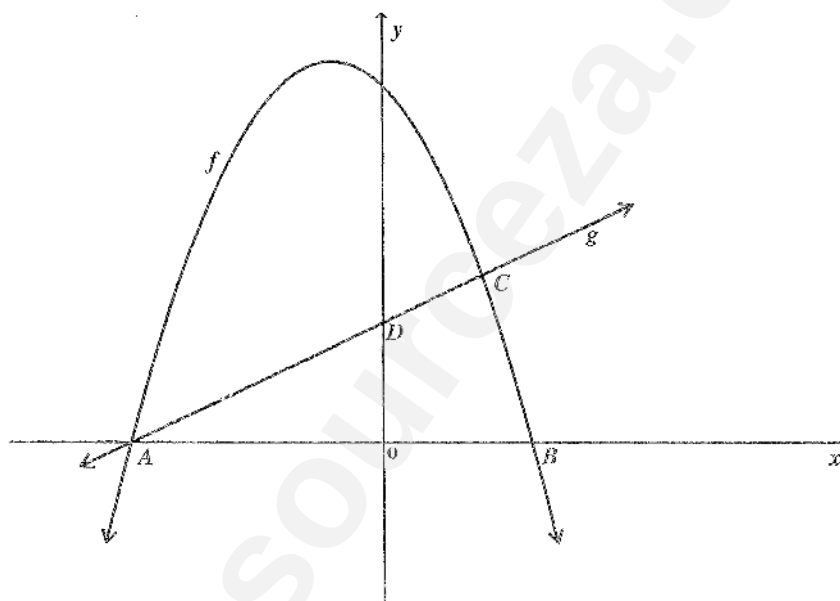
QUESTION 7

The figure below represents the sketch graphs of f and g , defined by

$$f(x) = -2x^2 - 4x + 30 \quad \text{and} \quad g(x) = 2x + 10.$$

A and B are the x -intercepts of f . The graph of g passes through A .

C is the point of intersection of f and g . The graph of g intersects the y -axis at D .



Referring to the sketch above:

- 7.1 Determine the coordinates of A and B . (4)
- 7.2 Write the function of f in the form $f(x) = a(x - p)^2 + q$ and hence write down the coordinates of the turning point. (5)
- 7.3 Determine whether $(1;12)$ are the coordinates of C . Show ALL calculations. (2)
- 7.4 The straight line with equation $y = mx + 32$ is a tangent to the graph of f . Calculate the possible values of m . (6)
- 7.5 Determine the values of x for which $f(x), g(x) > 0$. (2)

[19]

QUESTION 8

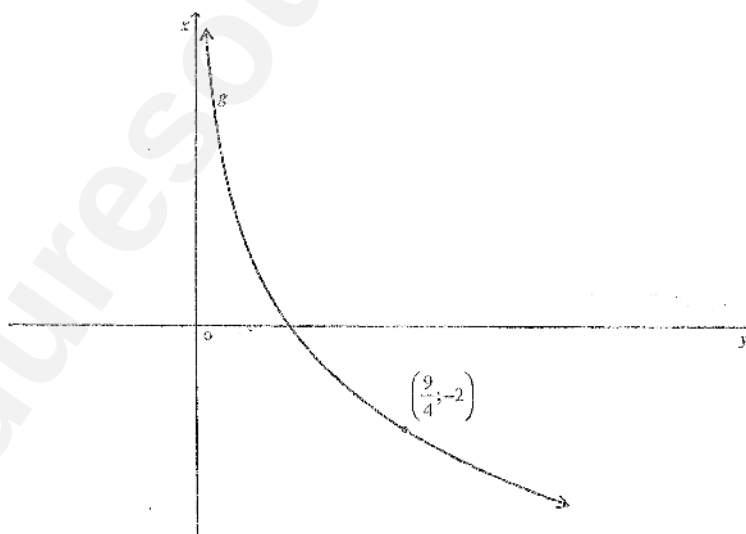
Given: $f(x) = \frac{1}{x-2} - 1$

- 8.1 Write down the equation of the horizontal and vertical asymptotes of f . (2)
- 8.2 Determine the x - and y -intercepts of f . (4)
- 8.3 Draw a neat sketch of f in your ANSWER BOOK. Show ALL asymptotes and intercepts with the axes. (3)
- 8.4 Calculate value of k if $(k; 2)$ is a point on f . (2)
- [11]

QUESTION 9

The sketch shows the graph of $g(x) = \log_a x$, where a is a positive real number.

The graph passes through $\left(\frac{9}{4}; -2\right)$.



- 9.1 Calculate the value of a . (3)
- 9.2 Determine the equation of the function h , the reflection of g in the x -axis. (1)
- 9.3 Use the graph to determine the values of $g(x)$ for which $x < \frac{9}{4}$. (1)
- 9.4 Write down the domain of g^{-1} . (1)

[6]

QUESTION 10

10.1 Given: $f(x) = -2x^2 + 1$.

10.1.1 Show that the average gradient of the graph of f between the points where $x = 3$ and $x = 3 + h$, ($h \neq 0$), is $-12 - 2h$. (4)

10.1.2 Use your answer in QUESTION 10.1.1 to calculate $f'(3)$ from first principles. (2)

10.1.3 Determine the numerical value of the gradient of the graph of f at $x = 0$. (1)

10.2 Differentiate y with respect to x . Leave your answer with positive exponents.

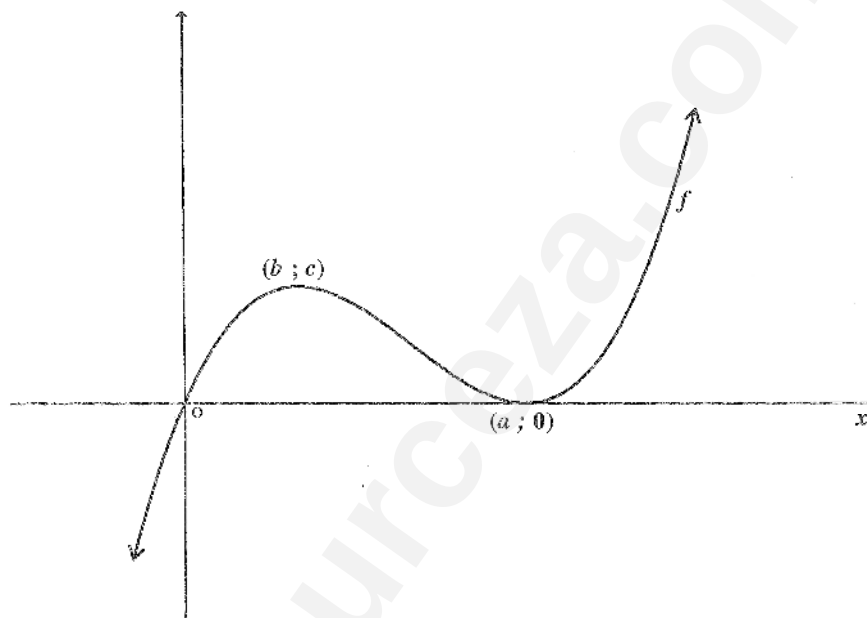
10.2.1 $y = (4 - 2x)^2$ (3)

10.2.2 $y = \sqrt{x}(\sqrt{x} - a) + a$ (4)

[14]

QUESTION 11

The sketch graph below represents a cubic function f with equation $f(x) = x^3 - 4x^2 + 4x + k$.
The graph passes through the origin, has a local maximum at $(b; c)$ and a local minimum at $(a; 0)$.



- 11.1 Explain why $k = 0$. (1)
- 11.2 Using this value of k , determine the values of
- 11.2.1 a . (2)
- 11.2.2 b . (4)
- 11.3 The graph of g with equation $g(x) = mx$ is a tangent to f at the point $(0; 0)$.
Calculate the value of m . (2)
- 11.4 Make use of the graph, or any other way, to determine the value of p for which
 $x^3 - 4x^2 + 4x + 2 = p$ will have only one negative solution. (2)

[11]

QUESTION 12

The profit P in rand/day which the owner of a taxi makes when the taxi is driven at an average speed of x km/hour, is given by the formula

$$P = x^2 \left(40 - \frac{x}{3} \right)$$

- 12.1 Calculate the average speed at which the owner will suffer a loss in revenue. (3)
- 12.2 Calculate the most economical average speed of the taxi and the corresponding daily profit. (6)
- [9]

QUESTION 13

- 13.1 A bag contains 3 purple and 6 white balls. A second bag contains 7 purple and 1 white balls. A ball is taken randomly from one of the bags. Calculate the probability of the ball being purple. (4)
- 13.2 For a sample space S and events A and B , it is given that:

$$P([\text{not } A] \text{ and } B) = P(A' \text{ and } B) = \frac{5}{12}$$

$$P(A \text{ and } B) = \frac{1}{6}$$

$$P(1 - [A \text{ or } B]) = \frac{1}{3}$$

- 13.2.1 Represent this information on a Venn diagram. (3)
- 13.2.2 Calculate $P(A)$. (3)
- 13.3 A team of class representatives, consisting of 4 learners, needs to be elected at the beginning of the school year. The learners are chosen from Grades 11A, 11B, 11C and 11D. One learner from each class is randomly chosen and each learner has an equal chance to be chosen.

The number of learners in each class is:

Grade 11A – 20 learners;
Grade 11B – 15 learners;
Grade 11C – 12 learners;
Grade 11D – 10 learners.

Calculate in how many different ways this team of representatives can be elected. (1)

- 13.4 Two friends, Albert and James, take part in a swimming gala. There are in total 10 competitors and ten lanes. Only one swimmer is allowed in each lane.

If all swimmers take part, calculate the possible arrangements in the starting line-up under the following conditions:

13.4.1 Albert and James are placed next to each other. (2)

13.4.2 Albert is placed in the first lane and James is NOT placed next to Albert. (2)
[15]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A \quad \text{area } \triangle ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$(x; y) \rightarrow (x \cos \theta - y \sin \theta; y \cos \theta + x \sin \theta)$$

$$\bar{x} = \frac{\sum fx}{n}$$

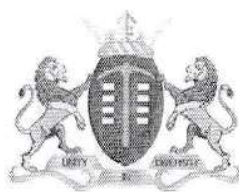
$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ of } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$



GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

**GAUTENG DEPARTMENT OF EDUCATION
PREPARATORY EXAMINATION**

2016

10611

MATHEMATICS

FIRST PAPER

TIME: 3 hours

MARKS: 150

11 pages + 1 information sheet and 1 answer sheet

MATHEMATICS: Paper 1
1061E



10611E

X10



GAUTENG DEPARTMENT OF EDUCATION
PREPARATORY EXAMINATION – 2016**MATHEMATICS**
(First Paper)**TIME: 3 hours****MARKS: 150**

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, etc. which were used in determining the answers.
4. Answers only will not necessarily be awarded full marks.
5. Use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. Where necessary, answers should be rounded-off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An ANSWER SHEET for answering Question 5.3 and Question 6.5 is located at the end of the question paper. This page must be submitted together with your ANSWER BOOK.
9. An INFORMATION SHEET appears on Page 12 of the question paper.
10. Number the answers correctly according to the numbering system used in the question paper.
11. Write neatly and legibly.

QUESTION 1

1.1 Solve for x :

1.1.1 $3x^2 + 5x = 2$

1.1.2 $\sqrt{x+7} - 1 = x$ (3)

1.1.3 $x^2 - 8x = 10$ (Round the answer off to ONE decimal place.) (5)

1.1.4 $3^x + 3^{-x+1} \cdot 5 = 8$ (4)

1.2 Simplify without the use of a calculator: (5)

$$\frac{2^{2015} + 2^{2013}}{4^{1006}}$$

(3)

[20]

QUESTION 2

2.1 Calculate the possible values of k if the roots of $kx^2 + kx + 1 = 0$ are non-real. (4)

2.2 Solve for x and y in the following simultaneous equations:

$$2^{x+1} = 4^y$$

$$x^2 + 2y = 3$$

(5)

[9]

QUESTION 3

3.1 Given the sequence:

$-8; -2; 8; 22; \dots$

3.1.1 Write down the next term. (1)

3.1.2 Determine the formula for the general term of the sequence. (4)

3.2 The first term of an arithmetic sequence is 5 and the sixth term is 10 times the third term.

3.2.1 Calculate the constant first difference. (4)

3.2.2 If $d = -3$, calculate the sum of the first 20 terms. (3)

3.3 Calculate:

3.3.1 $\sum_{k=1}^{\infty} 2\left(\frac{1}{2}\right)^k$ (3)

3.3.2 $\sum_{k=0}^7 2\left(\frac{1}{2}\right)^k$ (3)

3.3.3 the value of T if:

$$T = \sum_{k=1}^{\infty} 2\left(\frac{1}{2}\right)^k - \sum_{k=0}^7 2\left(\frac{1}{2}\right)^k \quad (1)$$

3.4 Given the arithmetic sequence:

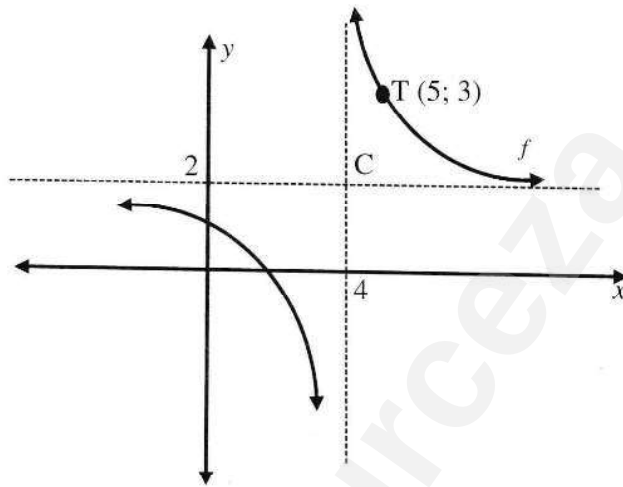
$19; 18\frac{1}{5}; 17\frac{2}{5}; \dots$

Determine which term in this sequence will be the FIRST to be negative. (6)
[25]

QUESTION 4

The graph of $f(x) = \frac{a}{x+p} + q$ is sketched below with asymptotes $x = 4$ and $y = 2$.

$T(5; 3)$ is a point on f and C is the point of intersection of the asymptotes.



- 4.1 Determine the values of a , p and q . (3)
- 4.2 Give the equation of h , the reflection of f in the y -axis. (1)
- 4.3 If the graph of f is symmetrical about the line $y = -x + c$, determine the value of c . (2)
- [6]

QUESTION 5

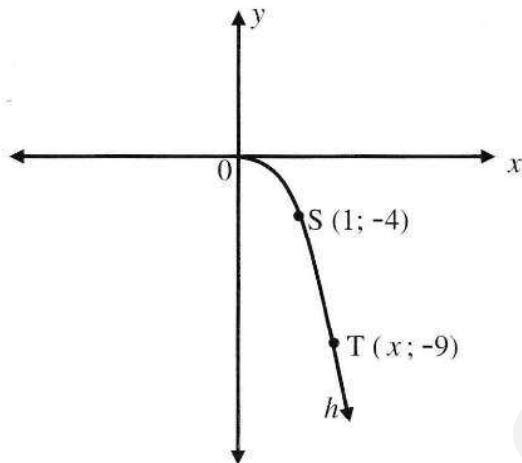
Given that $f(x) = a^x$ with $a > 0$, and point $Q \left(-2 ; 1\frac{9}{16}\right)$ lies on the graph of f .

- 5.1 Show that the value of $a = \frac{4}{5}$. (2)
- 5.2 Write down the equation of f^{-1} , the inverse graph of f .
Leave your answer in the form $y = \dots$ (2)
- 5.3 Sketch the graphs of f and f^{-1} on the axes provided on the ANSWER SHEET. (3)
- 5.4 For which values of x is $f(x) < 1$? (1)
- 5.5 Write down the equation of h if h is a reflection of f about the x -axis. (1)
- 5.6 Write down the equation of a line that f has to be reflected about in order to obtain the graph of f^{-1} . (1)
- 5.7 Determine the range of the graph of $f(x) - 1$. (1)

[11]

QUESTION 6

The graph of h is sketched below, with $x \geq 0$.



- 6.1 Determine the equation of h . (3)
- 6.2 Determine the x -coordinate of point T if the equation of $h(x) = -4x^2$. (2)
- 6.3 Calculate the average gradient of ST if the coordinates of point T are $\left(\frac{3}{2}; -9\right)$. (2)
- 6.4 Determine the equation of $h^{-1}(x)$ in the form $y = \dots$ (4)
- 6.5 Sketch the graph of h^{-1} on the axes provided on the ANSWER SHEET. (3)
Clearly indicate the axis of symmetry of h and h^{-1} [14]

QUESTION 7

- 7.1 Johan purchased a new car. The bank offered him a loan at an effective interest rate of 16,4% p.a. Determine the nominal interest rate, compounded monthly, that he is required to pay. (3)
- 7.2 A farmer sets up a sinking fund. He plans to accumulate R2,3 million in the fund at the end of 8 years by making equal quarterly payments into the fund. The interest rate on money accumulated is 12% p.a. compounded quarterly. Calculate the quarterly payments into the fund if his first payment is made in 3 months' time. (4)
- 7.3 Emily plans to attend the Summer Olympics in Rio de Janeiro. She books an all-inclusive Olympic Package through a travel agency at the price of R135 000. She acquires a personal loan to cover the costs at an interest rate of 14,75% p.a. compounded monthly and wishes to pay back the loan in 18 equal monthly instalments starting three months after receiving the loan.
- 7.3.1 Calculate the total amount owing, two months after the loan is granted. (2)
- 7.3.2 Calculate the monthly instalments on the loan. (4)
- [13]**

QUESTION 8

- 8.1 Given $f(x) = 4 - 3x^2$, determine $f'(x)$, using FIRST PRINCIPLES. (5)
- 8.2 Determine:
- 8.2.1 $D_x \left[x^4 - 2x + \frac{1}{x^2} \right]$ (3)
- 8.2.2 $\frac{dy}{dx}$ if $y = \frac{2x-3}{\sqrt[4]{x}}$ (2)
- 8.3 Calculate the x -coordinates of the points on the graph of $f(x) = x^3 - 7x^2$ at which the gradient of the tangent to f is equal to 5. (4)
- [14]**

QUESTION 9

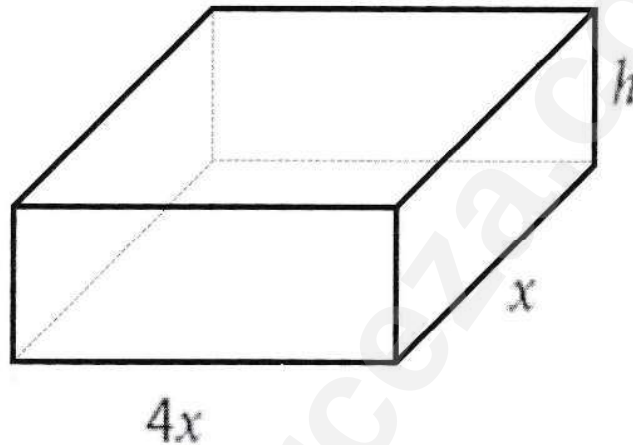
A cubic graph f is defined by the following properties:

- $f(-2) = f(1) = f(6) = 0$
- $f(0) = 12$
- $a > 0$

- 9.1 By using the given information above, sketch a basic graph clearly indicating the intercepts with the axes. (3)
- 9.2 Show that $f(x) = x^3 - 5x^2 - 8x + 12$. (4)
- 9.3 Calculate the coordinates of the turning points of f . (5)
- 9.4 Determine the values of x for which $f'(x) < 0$. (1)
- 9.5 Calculate the value of x for which f is concave up. (3)
- [16]**

QUESTION 10

An open rectangular box is made of a very thin sheet of metal. The volume is 128 cm^3 and the base of the box has a width of $x \text{ cm}$ and a length of $(4x) \text{ cm}$.



- 10.1 Determine an expression for the height of the box in terms of x . (2)
- 10.2 Show that the total surface area of the box can be written as $\left(4x^2 + \frac{320}{x}\right) \text{ cm}^2$. (3)
- 10.3 Calculate the height of the box for which the surface area is a minimum. (4)
- [9]

QUESTION 11

A survey of 80 participants at the 2016 Olympic Games was taken. Athletes' participation was recorded as follows:

- 44 participated in swimming;
- 33 participated in gymnastics;
- 39 participated in athletics;
- 23 participated in both swimming and athletics;
- 19 participated in both gymnastics and athletics;
- 9 participated in all three events;
- 69 participated in at least one event and
- 11 participants remained as reserves for the events (did not participate).

- 11.1 Let the number of participants in both swimming and gymnastics, but not in athletics, be represented by x .
Draw a Venn diagram to represent the survey. (4)
- 11.2 Hence show that $x = 5$. (2)
- 11.3 What is the probability that a participant chosen at random will participate in at least two of the three events. (3)
[9]

QUESTION 12

Given the word: EDUCATION

- 12.1 In how many unique ways can all the letters in the word above be arranged? (1)
- 12.2 If 5 letters are randomly chosen from the word "EDUCATION", determine how many unique 5-letter arrangements can be formulated? (3)
[4]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} ; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r} ; \quad -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A \quad \text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

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$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$(x; y) \rightarrow (x \cos \theta - y \sin \theta; y \cos \theta + x \sin \theta)$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ of } B) = P(A) + P(B) - P(A \text{ en } B)$$

$$\hat{y} = a + bx$$

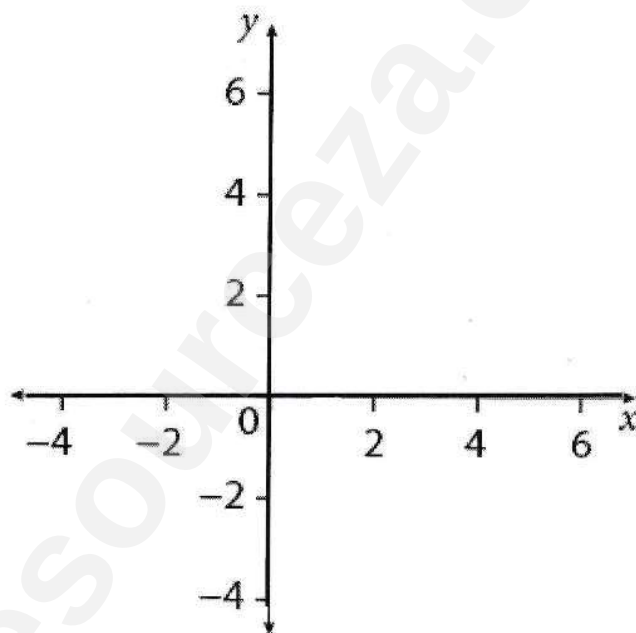
$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

NAME OF LEARNER: _____

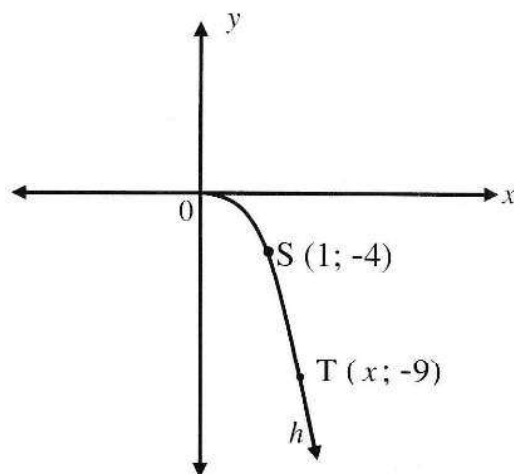
GRADE: _____

ANSWER SHEET

QUESTION 5.3



QUESTION 6.5



**GAUTENG DEPARTMENT OF EDUCATION
PREPARATORY EXAMINATION
2017**

**10611
MATHEMATICS
FIRST PAPER**

TIME: 3 hours

MARKS: 150

11 pages + 1 information sheet and 1 answer sheet

MATHEMATICS: Paper 1

1061E



10611E

X05



**GAUTENG DEPARTMENT OF EDUCATION
PREPARATORY EXAMINATION – 2017**

**MATHEMATICS
(First Paper)**

TIME: 3 hours

MARKS: 150

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, etc. which were used in determining the answers.
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7. Diagrams are NOT necessarily drawn to scale.
8. An ANSWER SHEET for Question 4.2 is located on Page 13 of the question paper.
This page must be submitted together with your ANSWER BOOK.
9. An information sheet is included on Page 12 of the question paper.
10. Number the questions correctly according to the numbering system used in the question paper.
11. Write neatly and legibly.

QUESTION 1

1.1 Solve for x :

1.1.1 $x(x-1) = 12$ (3)

1.1.2 $2x^2 + 3 = 8x$ (Correct to TWO decimal places) (4)

1.1.3 $(2x+3)(3-x) > 4$ (5)

1.1.4 $2^x - 5.2^{x+1} = -144$ (3)

1.2 Solve for x and y simultaneously:

$y + 2x - 3 = 0$ and $y = x^2 + 2x + 3$ (5)
[20]

QUESTION 2

2.1 If $a + 2$; $4a$; $6a + 4$ are the first three terms of an arithmetic progression, determine the value of a . (2)

2.2 Given the sequence: 5; 6; 12; 12; 19; 24; 26; 48; ...

2.2.1 Write down the next 2 terms. (2)

2.2.2 Determine the value of the 45th term. (3)

2.3 The eighth term of an arithmetic pattern is 33 and the eleventh term is 45. Determine the 15th term. (5)

2.4 Consider the sequence: 6; 6; 2; -6; -18; ... Determine the n^{th} term of the sequence. (4)

2.5 Given: $A = \sum_{r=1}^{\infty} 3\left(\frac{1}{2}\right)^{r-1}$

$$B = \sum_{k=1}^n 3\left(\frac{1}{2}\right)^{k-1}$$

2.5.1 Calculate the value of A . (2)

2.5.2 Determine B in terms of n . (2)

2.5.3 For which values of n is $A - B < \frac{1}{36}$? (5)

[25]

QUESTION 3

3.1 Joseph opens a savings account for 5 years at an interest rate of 6% per annum, compounded quarterly. Determine the effective interest rate. (3)

3.2 A loan is required to purchase equipment for a gym. The owner of the gym can pay R2 000 per month, starting one month after the loan is granted. The repayments continue for 2 years at 24% p.a. compounded monthly. Calculate how much money the owner intends to borrow. (4)

3.3 On 1 August 2017, a company purchased a new vehicle which they will replace in exactly 5 years' time. The company needs R200 000 to cover the replacement cost of this vehicle and sets up a sinking fund for this purpose. Calculate the monthly deposits that must be made into the sinking fund at an interest rate of 6,25% p.a. compounded monthly. The first deposit is made 4 months after the original vehicle was purchased. (6)
[13]

QUESTION 4

Given: $f(x) = 4 \cdot 2^{x+1} - 2$

4.1 Determine:

4.1.1 $f(-1)$ (1)

4.1.2 The x -intercept (3)

4.1.3 The y -intercept (2)

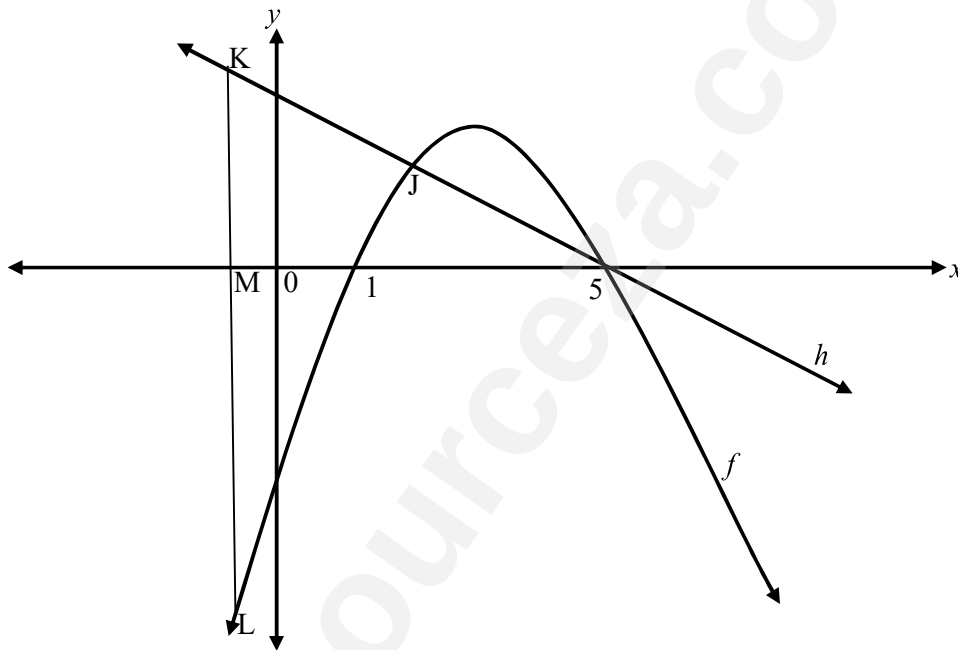
4.1.4 The equation of the asymptote (1)

4.2 Sketch the graph of f on the ANSWER SHEET provided on Page 13.
Indicate the intercepts with the axes, asymptote and coordinates of any ONE point on the graph which is not on an axis. (3)

4.3 Write down the equation of graph g , if g represents the graph of f shifted 2 units to the right. (1)
[11]

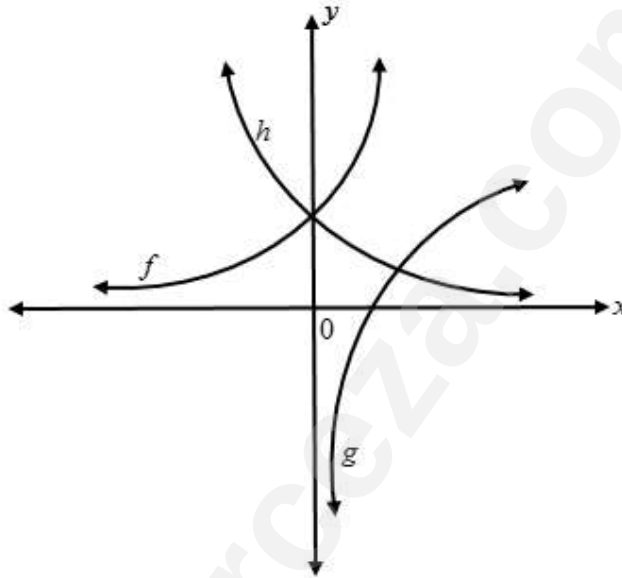
QUESTION 5

- 5.1 The graphs of $f(x) = -(x-3)^2 + 4$ and $h(x) = -x + 5$ are sketched below. The points (1; 0) and (5; 0) are the x -intercepts of f . The point J (2 ; 3) is a point of intersection of f with h . KML is parallel to the y -axis.



- 5.1.1 Write down the coordinates of the turning point of f . (2)
- 5.1.2 Write down the range of g , the reflection of f in the x -axis. (2)
- 5.1.3 Determine the average gradient of the curve of f between $x = 2$ and $x = 5$. (2)
- 5.1.4 If $OM = 1$ unit, determine the length of KL . (3)
- 5.1.5 For which values of x is $f(x) > h(x)$? (2)

5.2 The diagram below shows the graphs of g , h and $f(x) = \left(\frac{4}{3}\right)^x$.



5.2.1 Write down the equation of g if $g(x) = f^{-1}(x)$. (2)

5.2.2 Write down the equation of h , the reflection of f in the y -axis. (2)

5.2.3 Determine, with the aid of the sketch above, the x -value(s) for which :

a) $f(x) - h(x) = 0$ (1)

b) $\log_{\frac{4}{3}} x \leq 0$ (2)

[18]

QUESTION 6

Given the equation: $f(x) = -\frac{4}{x} + 7$; $x \neq 0$

6.1 Show that the gradient of all tangents to the curve f will ALWAYS be positive. (3)

6.2 Determine the equation of a tangent to the curve f which is perpendicular to the line $y = -4x$ for values of $x < 0$. (5)

[8]

QUESTION 7

7.1 Given: $f(x) = 1 - 4x^2$

7.1.1 Determine $f'(x)$ from first principles. (5)

7.1.2 Hence, calculate the gradient of the tangent to f at $x = 2$. (2)

7.2 Determine:

7.2.1 $\frac{dy}{dx}$ if $y = (2 - x)^2$ (3)

7.2.2 $f'(x)$ if $f(x) = \sqrt[3]{x^2} + \frac{1}{4x^4}$ (4)

[14]

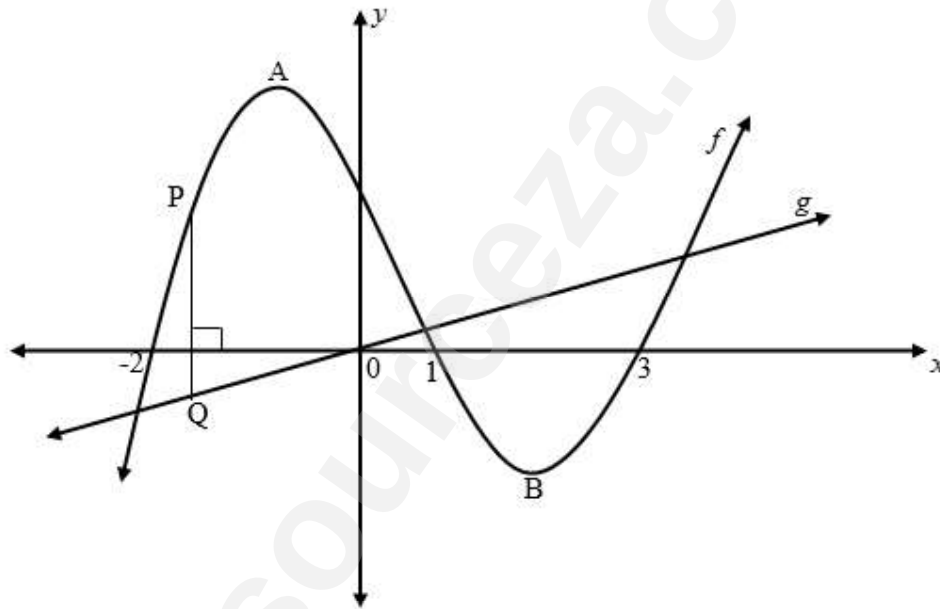
QUESTION 8

Given: $f(x) = x^3 + bx^2 + cx + d$ and $g(x) = 2x$

The graph of f intersects the x -axis at $x = -2$; $x = 1$ and $x = 3$.

The turning points of f are at points A and B respectively, where $x_B > x_A$.

Line PQ is perpendicular to the x -axis, with point P on f and point Q on g .



8.1 Show that the equation of f can be given as:

$$f(x) = x^3 - 2x^2 - 5x + 6. \quad (3)$$

8.2 Calculate the coordinates of points A and B. (4)

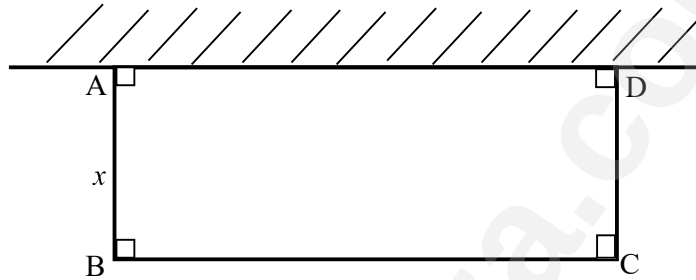
8.3 Calculate the maximum length of line PQ, for the interval $-2 < x < 3$. (6)

8.4 The graph of f is concave down for $x < k$. Calculate the value(s) of k . (3)

[16]

QUESTION 9

A 50m long fence is used to enclose a rectangular garden ABCD, using the wall (AD) as one of its boundaries. Let $AB = x$.



- 9.1 Express the length of BC in terms of x . (1)
- 9.2 Hence determine the value of x such that the enclosed area will be maximum. (4)
- [5]

QUESTION 10

A packet of sweets has 3 pink, 2 green and 5 blue sweets. Two sweets are removed from the packet, one at a time, without replacement.

- 10.1 Draw a tree diagram to determine ALL possible outcomes. Indicate on your diagram the probability associated with each branch of the tree diagram. (5)
- 10.2 Determine the probability that ...
- 10.2.1 both sweets are blue. (2)
- 10.2.2 a green and a pink sweet are selected. (3)
- [10]

QUESTION 11

- 11.1 A school requires a new set of codes to classify library books uniquely. All combinations in the current system have been exhausted. Vowels (A; E; I; O; U) and the letter Q may NOT be used. All digits from 0 to 9 can be used. Repetition of letters and numbers is allowed.

BCD 012
CURRENT

BCDF 0123
NEW

How many different combinations are possible if 4 letters and 4 digits are used to generate new codes? (3)

- 11.2 If $P(A) = 0,45$ and $P(B) = 0,35$. Calculate $P(A \text{ or } B)$ if ...

11.2.1 A and B are mutually exclusive events. (3)

11.2.2 A and B are independent events. (4)
[10]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A \quad \text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cdot \cos \alpha$$

$$(x; y) \rightarrow (x \cos \theta - y \sin \theta; y \cos \theta + x \sin \theta)$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ of } B) = P(A) + P(B) - P(A \text{ en } B)$$

$$\hat{y} = a + bx$$

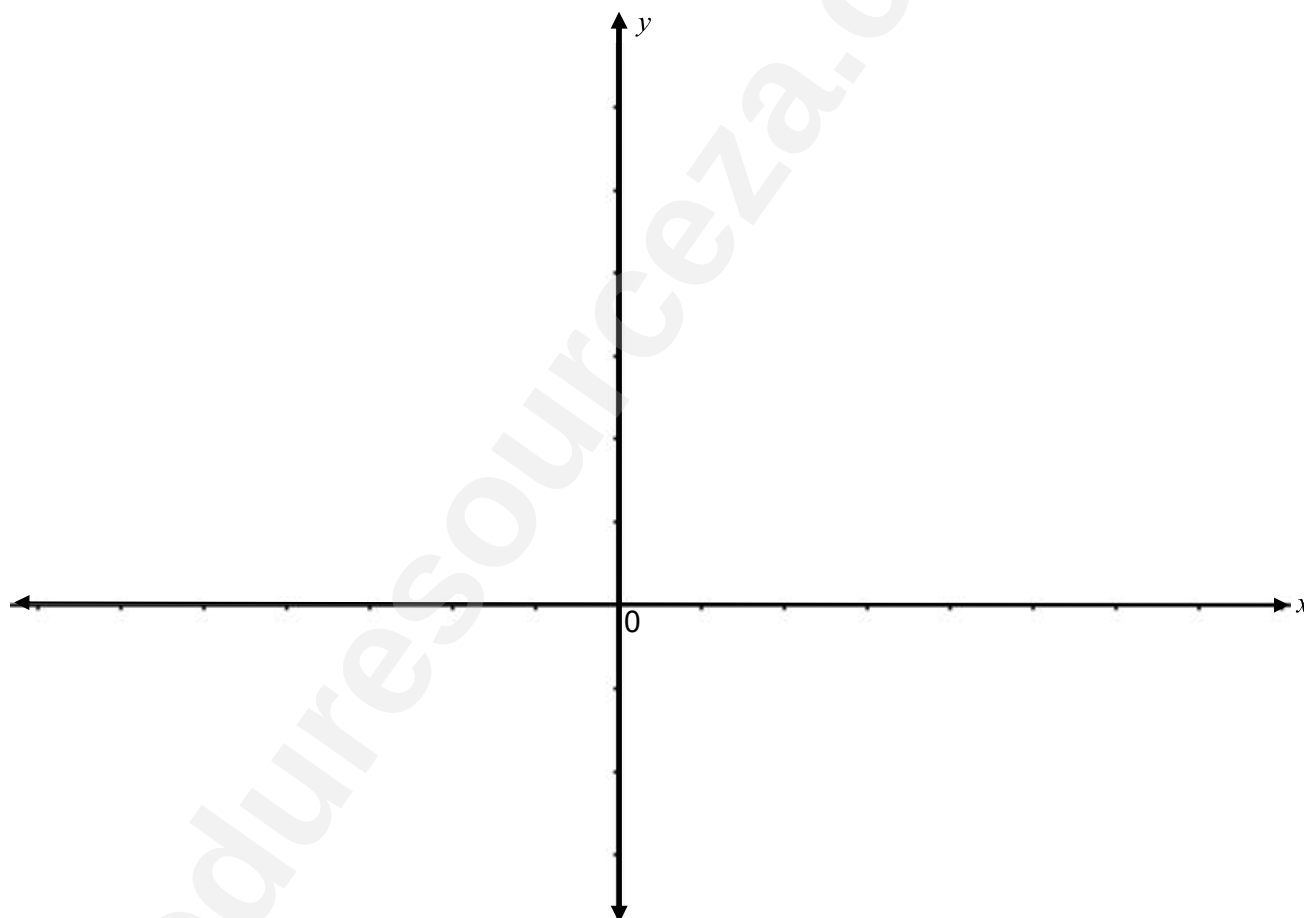
$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

ANSWER SHEET

NAME: _____

GRADE 12: _____

QUESTION 4.2





GAUTENG PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

**GAUTENG DEPARTMENT OF EDUCATION
PREPARATORY EXAMINATION
2018**

10611

MATHEMATICS

PAPER 1

TIME: 3 hours

MARKS: 150

9 pages and 1 information sheet

MATHEMATICS: Paper 1

1061E



10611E

X05



GAUTENG DEPARTMENT OF EDUCATION
PREPARATORY EXAMINATION – 2018

MATHEMATICS
(Paper 1)

TIME: 3 hours

MARKS: 150

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 13 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, etc. which were used in determining the answers.
4. Answers only will not necessarily be awarded full marks.
5. Use an approved scientific calculator (non-programmable and non-graphical).
6. Where necessary, answers should be rounded-off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet is included on Page 10 of the question paper.
9. Number the questions correctly according to the numbering system used in this question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x :

1.1.1 $x^2 - x - 30 = 0$ (2)

1.1.2 $3x^2 - 8x = 4$ (correct to TWO decimal places) (4)

1.1.3 $\sqrt{5-x} - x = 1$ (5)

1.1.4 $\frac{6x^2 - 3x}{3} \leq 3x^2$ (5)

1.1.5 $2^{x+2} + 7\sqrt{2^x} = 2$ (5)

1.2 Prove that the equation $6x^2 + 2px - 3x - p = 0$ has rational roots for all rational values of p . (4)
[25]

QUESTION 2

2.1 Calculate the number of terms in the following arithmetic sequence:

6; 1; -4; -9; ... ; -239 (3)

2.2 The 3rd term of a geometric series is 18 and the 5th term is 162.
Determine the sum of the first 7 terms, where $r < 0$. (6)

2.3 The following terms form a quadratic sequence:
3; x ; 11; 21; 35; ...
Determine the value of x . (3)

2.4 The first term of a geometric sequence is 9. The ratio of the sum of the first eight terms to the sum of the first four terms is 97 : 81.
Determine the first THREE terms of the sequence, if all terms are positive. (6)

2.5 Consider the infinite geometric series:
 $2(p-5) + 2(p-5)^2 + 2(p-5)^3 + \dots$

2.5.1 For which value(s) of p is the series convergent? (3)

2.5.2 If $p = 4\frac{1}{2}$, calculate S_{∞} . (3)
[24]

QUESTION 3

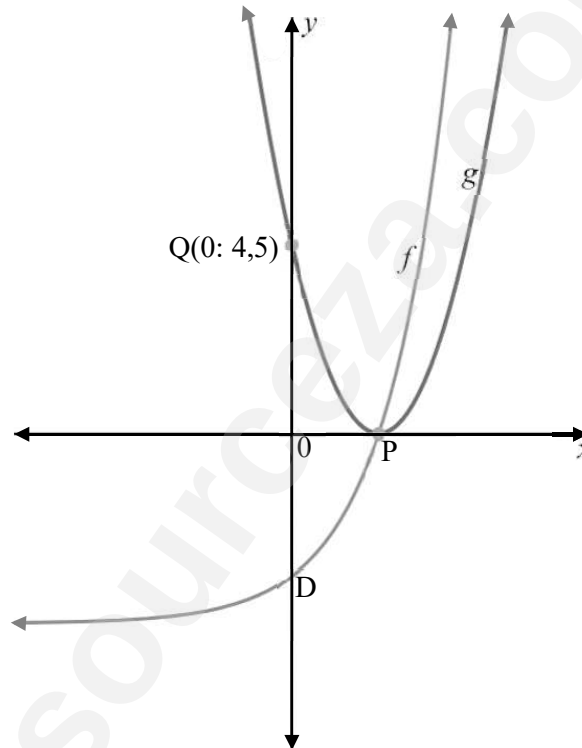
- 3.1 Lungile bought a car for R134 000. It depreciates on a reducing balance method at a rate of 6,8% per annum.
After how many years will its value be R100 000? (4)
- 3.2 A bank granted Clive a loan of R150 000 at an interest rate of 15,25% per annum, compounded monthly. Clive will repay the loan in 24 equal monthly payments. Payments will start 3 months after the loan was granted.
- 3.2.1 Calculate his monthly payment. (5)
- 3.2.2 Calculate the balance outstanding immediately after Clive makes his 18th payment. (4)
- [13]**

QUESTION 4

The graphs of $f(x) = 2^x - 8$ and $g(x) = ax^2 + bx + c$ are sketched below.

Point Q (0 ; 4,5) and point D are the y – intercepts of graphs g and f respectively.

The graphs intersect at point P , which is the turning point of graph g and the common x – intercept of f and g .

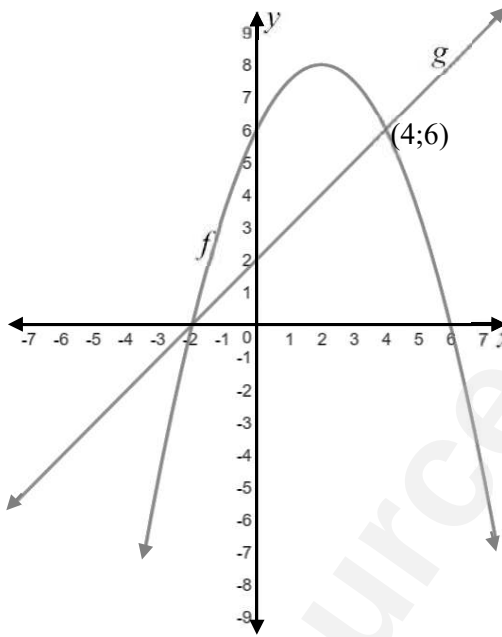


- 4.1 Write down the equation of the asymptote of graph f . (1)
- 4.2 Determine the coordinates of point P and point D. (4)
- 4.3 Determine the equation of h if $h(x) = f(2x) + 8$. (2)
- 4.4 Determine the equation of h^{-1} in the form $y = \dots$ (2)
- 4.5 Write down the range of h^{-1} . (1)
- 4.6 Determine the equation of g. (3)
- 4.7 Calculate: $\sum_{k=0}^3 g(k) - \sum_{k=4}^5 g(k)$ (3)
- 4.8 Describe the transformation that should be applied to graph g so that the new graph obtained will have non-real roots? (1)

[17]

QUESTION 5

The graphs of $f(x) = -\frac{1}{2}x^2 + 2x + 6$ and $g(x) = x + 2$ are sketched below. The graphs intersect at $(-2; 0)$ and $(4; 6)$.



Use the graphs to determine the values of x for which:

5.1 $f(x) = g(x)$ (2)

5.2 $\frac{f(x)}{g(x)} \geq 0$ (2)

5.3 $f'(x) \cdot g(x) \geq 0$ (2)
[6]

QUESTION 6

Given: $f(x) = \frac{1}{4}x^2$

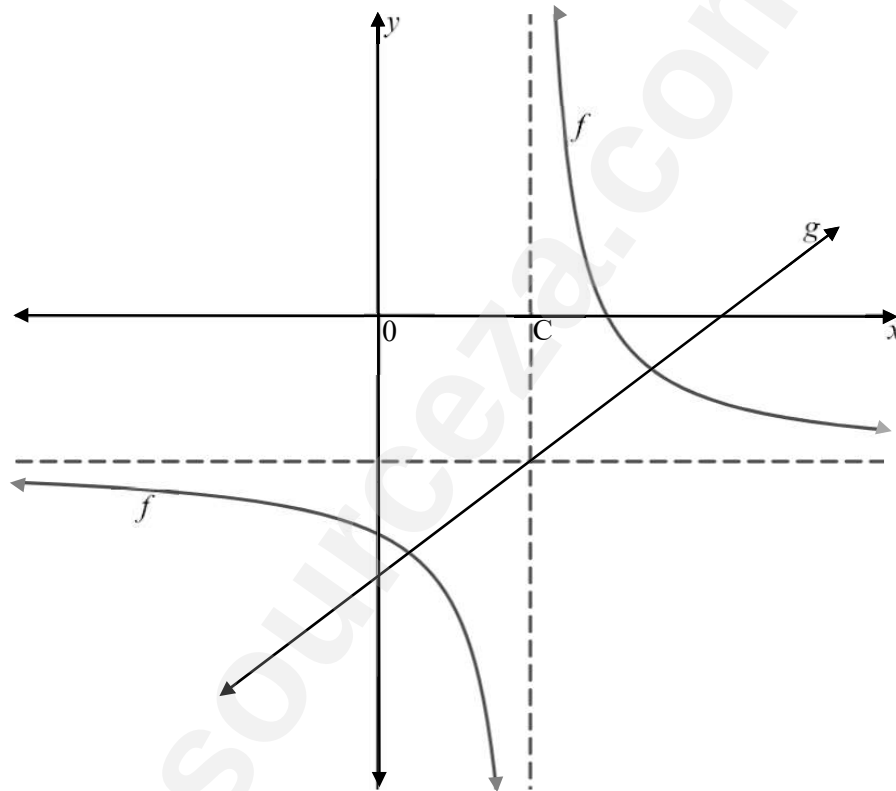
6.1 Write down the equation of g if g is the reflection of f about the y -axis. (1)

6.2 Write down the equation of h if f is translated TWO units down to obtain h . (1)

6.3 Write down the range of h . (1)
[3]

QUESTION 7

The graphs of $f(x) = \frac{3}{x-2} - 3$ and g , an axis of symmetry of f , are sketched below. The vertical asymptote cuts the x -axis at C.



- 7.1 Write down the equation of the vertical asymptote of f . (1)
- 7.2 Describe how the graph of $h(x) = \frac{3}{x}$ was transformed to obtain f . (2)
- 7.3 Write down the domain of $f(x-1)$. (1)
- 7.4 Determine the equation of the line, parallel to g (an axis of symmetry of f) passing through point C. (3)
- [7]

QUESTION 8

Given: $f(x) = 1 - 3x^2$

- 8.1 Determine $f'(x)$ from FIRST principles. (5)
- 8.2 Hence, calculate the gradient of a tangent to f at $x = 2$. (2)
- [7]

QUESTION 9

Determine the following:

9.1 $\frac{d}{dt}[(t-2)(t+3)]$ (3)

9.2 $D_x \left[\frac{5x^3 - 4}{x} \right]$ (3)
[6]

QUESTION 10

The gradient of a tangent to the curve $f(x) = ax^3 + bx^2$ at point C (1 ; 7) is 17.

10.1 Calculate the values of a and b . (6)

10.2 If it is given that $a = 3$ and $b = 4$, determine the coordinate of one other point on the curve where the gradient of the curve is also equal to 17. (6)

10.3 Sketch the graph of $f(x) = 3x^3 + 4x^2$, indicating all intercepts with the axes as well as the turning points. (4)

10.4 Calculate the values of x for which $f(x) = 3x^3 + 4x^2$ is concave up. (3)
[19]

QUESTION 11

The path travelled by a meteor can be tracked using the formula:

$s(t) = 6000 - 600t - 0,2t^3 + 2 \times 10^{-3}t^5$, where $s(t)$ is the distance (in meters) that the meteor is from the earth, t seconds after being detected.

11.1 Determine the velocity at which the meteor approaches the earth when FIRST detected. (3)

11.2 Show that the meteor will collide with the earth at $t = 10s$. (2)

11.3 Determine the acceleration (rate of change of velocity) of the meteor at $t = 5s$. (3)
[8]

QUESTION 12

Events A, B and C occur as follows where A and B are independent events.

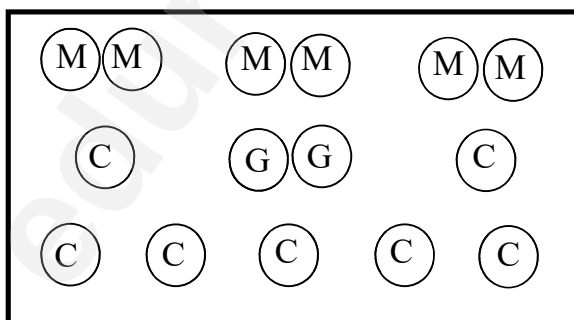
- $P(A) = 0,38$
- $P(B) = 0,42$
- $P(A \cap B) = 0,1596$
- $P(C) = 0,28$
- There are 456 people in event A.

- 12.1 Are A and B mutually exclusive events? Motivate your answer. (2)
- 12.2 By using an appropriate formula, show that the value of $P(A \cup B) = 0,64$. (2)
- 12.3 Calculate the number of people in the sample space. (2)
- 12.4 Determine $n(C')$. (2)
- [8]**

QUESTION 13

- 13.1 The letters in the word JOHAN are arranged in any order WITHOUT repetition. What is the probability that the word JOHAN will start with the letter J and end with the letter A? (3)
- 13.2 The Lauwrens family takes family photos. The photographer arranges three married couples, seven children and two grandparents as follows:

The couples stand husband and wife together at the back, the grandparents in the middle and the children in the other positions as shown in the diagram below.



M	Married Couples
G	Grandparents
C	Children

- How many different ways can the Lauwrens family be arranged for the photo? (4)
- [7]**

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A \quad \text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cdot \cos \alpha$$

$$(x; y) \rightarrow (x \cos \theta - y \sin \theta; y \cos \theta + x \sin \theta)$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$



GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

**GAUTENG DEPARTMENT OF EDUCATION
PREPARATORY EXAMINATION
2019**

**10611
MATHEMATICS
PAPER 1**

TIME: 3 hours

MARKS: 150

12 pages + 1 information sheet

MATHEMATICS: Paper 1



10611E

X10



GAUTENG DEPARTMENT OF EDUCATION
PREPARATORY EXAMINATION**MATHEMATICS**
(Paper 1)**TIME: 3 hours****MARKS: 150**

INSTRUCTIONS AND INFORMATION

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10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x :

1.1.1 $2x^2 + 3 = 8x$ (correct to TWO decimal places) (4)

1.1.2 $4x - 2x(x - 3) \leq 0$ (4)

1.1.3 $2^x - 5 \cdot 2^{x+1} = -144$ (4)

1.2 If $f(2) = 0$ and $f(-6) = 0$, determine an equation for $f(x)$ in the form $f(x) = x^2 + bx + c$. (2)

1.3 Solve for x and y simultaneously:

$2x + y = 17$ and $xy = 8$ (6)

1.4 Given: $2mx^2 = 3x - 8$ where $m \neq 0$.
Determine the value(s) of m for which the roots of the equation are non-real. (4)

[24]

QUESTION 2

Given the quadratic sequence: $-\frac{1}{2}; 2; \frac{11}{2}; 10; \dots$

2.1 Show that the n th term of this sequence can be written as $T_n = \frac{1}{2}(n^2 + 2n - 4)$. (4)

2.2 Determine the value of $T_{75} - T_{74}$. (2)

2.3 The first differences of the given sequence above forms another number sequence.

2.3.1 Is the sequence of the first differences arithmetic or geometric?
Give a reason for your answer. (2)

2.3.2 Which term in the sequence of the first differences will be equal to $\frac{151}{2}$? (1)

2.3.3 Calculate the value of the 30th first difference. (2)

2.3.4 Calculate the number of terms in the quadratic sequence if the sum of the first n first differences is 2 176. (4)

[15]

QUESTION 3

- 3.1 The following geometric series is given:

$$2(3x-1) + 2(3x-1)^2 + 2(3x-1)^3 \dots$$

Determine the value(s) of x for which the series converges. (3)

- 3.2 The first two terms of a convergent geometric series are k and 6 respectively where $k \neq 0$.
The sum of the infinite series is 25.

Calculate the value(s) of k . (5)

- 3.3 Given the series:

$$(1 \times 2) + (5 \times 6) + (9 \times 10) + (13 \times 14) + \dots + (81 \times 82)$$

Write the series in sigma notation. (It is not necessary to calculate the value of the series). (4)

[12]

QUESTION 4

- 4.1 How many years will it take for an investment of R3 000 to accumulate to R4 500, if it is invested at 8% p.a. compounded monthly? (4)

- 4.2 Bongani paid off a 20-year loan of R40 000. During the period of the loan the interest rate changed from 24% p.a. compounded monthly for the first five years to 18% p.a. compounded monthly for the remaining years.

4.2.1 Calculate the initial monthly payment before the interest rate changed. (4)

4.2.2 What is the outstanding balance of the loan after the FIRST five years? (4)

4.2.3 Determine the monthly payment after the interest rate changed. (4)

[16]

QUESTION 5

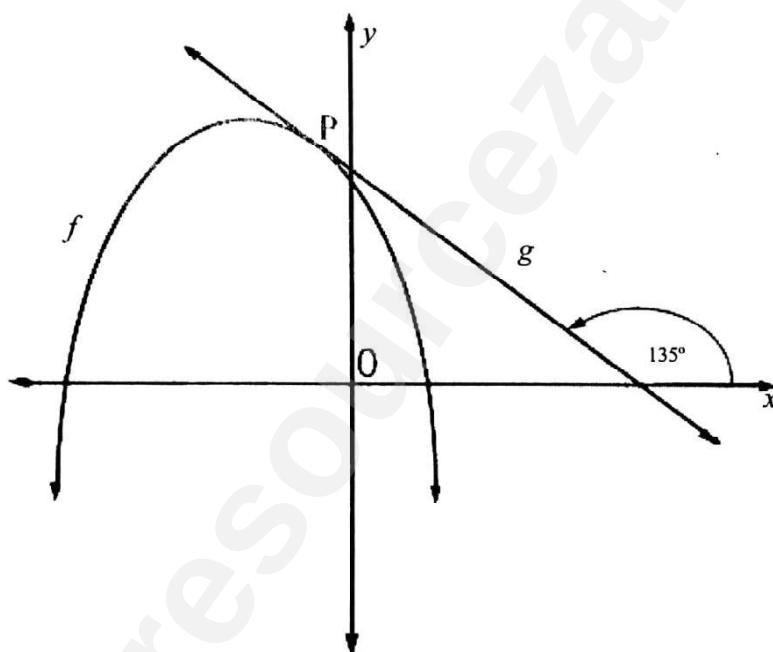
Given: $f(x) = \frac{3}{x-1} - 2$

- 5.1 Calculate the coordinates of the x -intercept of f . (2)
- 5.2 Calculate the coordinates of the y -intercept of f . (1)
- 5.3 Sketch the graph of f in your ANSWER BOOK. Clearly show the asymptotes and the intercepts with the axes. (3)
- 5.4 ONE of the axes of symmetry of f is a decreasing function. (3)
Write down the equation of this axis of symmetry.

[9]

QUESTION 6

The graphs of $f(x) = -2x^2 - 5x + 3$ and $g(x) = ax + q$ are sketched below.
The angle of inclination of g is 135° . Graph g is a tangent to f at point P.



- 6.1 Calculate the coordinates of the turning point of f . (3)
- 6.2 Write down the range of f . (1)
- 6.3 Calculate the coordinates of point P, the point of contact of f and g . (4)
- 6.4 Determine the value(s) of k for which the straight line $y = k$ is NOT a tangent to $y = 2x^2 + 5x - 3$. (2)

[10]

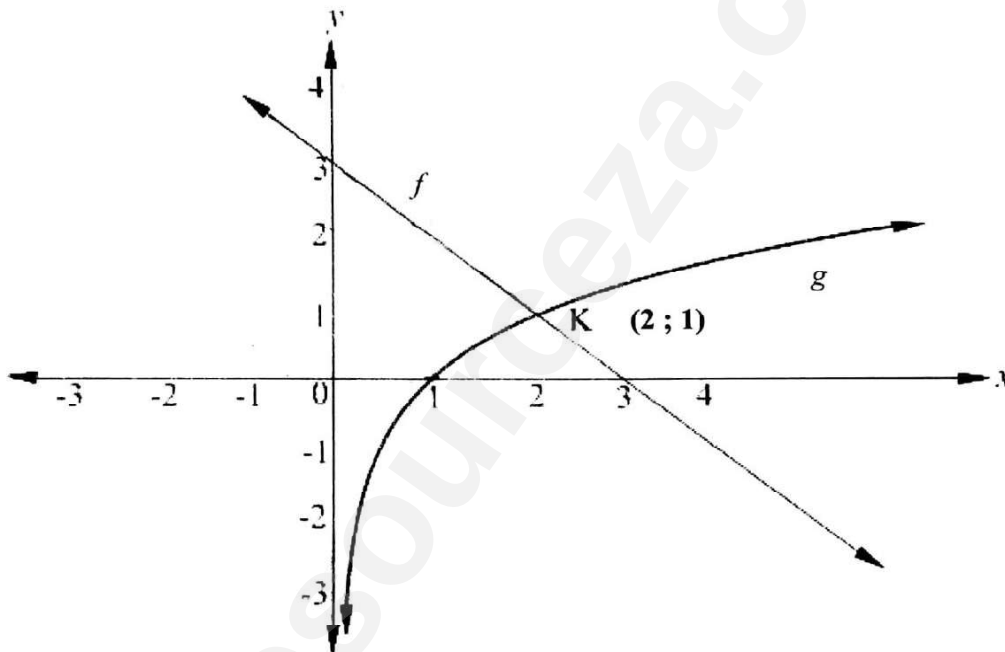
QUESTION 7

Given $f(x) = a^x$, where $a > 0$, passing through the point $(2; \frac{1}{4})$ and $g(x) = 4x^2$.

- 7.1 Prove that $a = \frac{1}{2}$. (2)
- 7.2 Determine the equation of $y = f^{-1}(x)$ in the form $y = \dots$ (2)
- 7.3 Determine the equation of $y = h(x)$ where h is the reflection of f in the x -axis. (1)
- 7.4 How must the domain of $g(x)$ be restricted so that $g^{-1}(x)$ will be a function? (2)
- [7]**

QUESTION 8

The graphs of $f(x) = -x + 3$ and $g(x) = \log_2 x$ are drawn below.
Graphs f and g intersect at point $K(2; 1)$.



8.1 Write down value(s) of x for which:

8.1.1 $f(x) - g(x) > 0$ (2)

8.1.2 $g(x) \cdot g^{-1}(x) \leq 0$ (2)

8.2 8.2.1 Write down the equation of g^{-1} in the form $y = \dots$ (2)

8.2.2 Explain how you could use the given sketch to solve the equation $\log_2(3-x) = x$. (2)

8.2.3 Write down the solution to $\log_2(3-x) = x$. (1)

[9]

QUESTION 9

9.1 Given: $f(x) = 3x - x^2$

9.1.1 Determine $f'(x)$ from FIRST principles. (5)

9.1.2 Determine the average gradient of f between $x = 1$ and $x = 3$. (3)

9.2 Determine:

9.2.1 $\frac{dy}{dx}$ if $y = \frac{8-3x^6}{8x^5}$ (3)

9.2.2 $D_x \left[\sqrt[3]{x^2} + \frac{1}{x} + 2x \right]$ (4)

[15]

QUESTION 10

A cubic function has the following essential properties:

- $f(0) = 8$
- $f(4) = f(1) = 0$
- $f'(3) = f'(1) = 0$
- $f(3) = 8$

10.1 Sketch the graph of f in your ANSWER BOOK clearly indicating the turning point(s) and the points of intersection of the graph with the axes. (3)

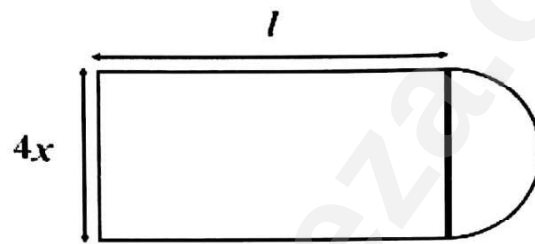
10.2 Show that the defining equation of f is $f(x) = -2x^3 + 12x^2 - 18x + 8$. (4)

10.3 Determine the value(s) of x for which graph of f is concave down. (3)

[10]

QUESTION 11

The diagram below shows a garden in the form of a rectangle and a semi-circle.



The rectangular section of the garden has the dimensions length (l) and width ($4x$).

The perimeter of the garden is $32m$.

11.1 Express the length (l) in terms of x . (3)

11.2 Show that the area of the garden can be written as $A(x) = -8x^2 - 2\pi x^2 + 64x$. (2)

11.3 Determine the value of x for which the area of the garden is minimum. (3)

[8]

QUESTION 12

12.1 For two events, A and B, it is given that:

- $P(A) = 0,3$
- $P(B) = 0,4$
- $P(A \text{ or } B) = 0,6$

Calculate $P(A \text{ and } B)$.

(2)

12.2 A survey was completed among Grade 11 and Grade 12 learners at a certain school to establish the type of cell phone that each learner uses. Some of the results are shown in the table below.

	Gr. 11	Gr. 12	Total
Android	<i>A</i>	33	65
iPhone	53	<i>B</i>	101
Total	85	81	166

12.2.1 Calculate values for *A* and *B* in the table.

(2)

12.2.2 If a learner from this group is selected at random, what is the probability that he/she will use an iPhone?

(2)

12.2.3 All these Grade 11 and Grade 12 learners attend Mathematics lessons in a particular class at the school. At the end of the day, the Mathematics educator found an iPhone in this class.

What is the probability that the phone belongs to a Grade 12 learner?

(2)

[8]

QUESTION 13

Cindy has the following books to arrange on a bookshelf:

- 4 Mathematics books
- 3 Physical Sciences books
- 2 Life Sciences books

- 13.1 Determine the number of different ways that ALL the books can be arranged. (2)
- 13.2 Determine the number of different ways that the books can be arranged in order that the books in each learning area are NEXT to each other. (3)
- 13.3 In how many different ways can all the books be arranged in order of descending height? (2)

[7]

TOTAL: 150



GAUTENG PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

**GAUTENG DEPARTMENT OF EDUCATION
PREPARATORY EXAMINATION**

2020

10611

MATHEMATICS

PAPER 1

TIME: 3 hours

MARKS: 150

11 pages and 1 information sheet

Name	
Surname	
Current Teacher	OVN

P.T.O.

INSTRUCTIONS AND INFORMATION

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1. Answer ALL the questions.
2. This question paper consists of 11 questions.
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11. Write neatly and legibly.

QUESTION 1

1.1 Solve for x :

1.1.1 $3x^2 + 5x = 7$ (correct to TWO decimal places) (4)

1.1.2 $2x^2 = 9x + 5$ (3)

1.1.3 $x^2 - 5x > -4$ (4)

1.1.4 $x - 3x^{\frac{1}{2}} = 4$ (6)

1.2 Show that the equation $2^{2x+1} + 7.2^x - 4 = 0$ has only ONE solution. (4)

1.3 Solve for x and y simultaneously:

$x = y - 13$ and $\sqrt{2-x} = y - 3$ (6)

[27]

QUESTION 2

2.1 Given the following quadratic sequence:

1 ; 7 ; 15 ; 25 ; x ; ...

2.1.1 Write down the value of x in the sequence. (1)

2.1.2 Determine the expression for the n^{th} term of this sequence. (4)

2.1.3 W_n represents the general term of a sequence for the first differences.

Determine the value of the n^{th} term of the quadratic sequence if $W_n = 50$. (5)

2.2 Consider the following:

$0 ; -\frac{1}{2} ; 0 ; \frac{1}{2} ; 0 ; \frac{3}{2} ; 0 ; \frac{5}{2} ; 0 ; \frac{7}{2} ; 0 ; \dots$

Assume that the pattern continues consistently.

2.2.1 Write down the value of the 191st term of this sequence. (1)

2.2.2 Determine the sum of the first 500 terms of this sequence. (4)

P.T.O.

- 2.3 Consider the following geometric series:

$$4\left(\frac{1-k}{5}\right) + 8\left(\frac{1-k}{5}\right)^2 + 16\left(\frac{1-k}{5}\right)^3 \dots$$

Determine the values of k .

(4)
[19]

QUESTION 3

The first THREE terms of an infinite geometric sequence are 16, 8 and 4 respectively.

- 3.1 Determine ALL possible values of n for which the sum of the first n terms of this sequence is greater than 31.
- 3.2 Calculate the sum to infinity of this sequence.

(3)

(2)
[5]

QUESTION 4

Given: $f(x) = \frac{6}{x+2} - 1$

- 4.1 Write down the equations of the asymptotes of f . (2)
- 4.2 Calculate:
- 4.2.1 the y -intercept of f . (2)
- 4.2.2 the x -intercept of f . (2)
- 4.3 Sketch the graph of f showing clearly the asymptotes and the intercepts of the axes. (3)
- 4.4 Determine the equation of the line of symmetry of f that has a negative gradient.
Leave your answer in the form $y = \dots$ (3)
- [12]

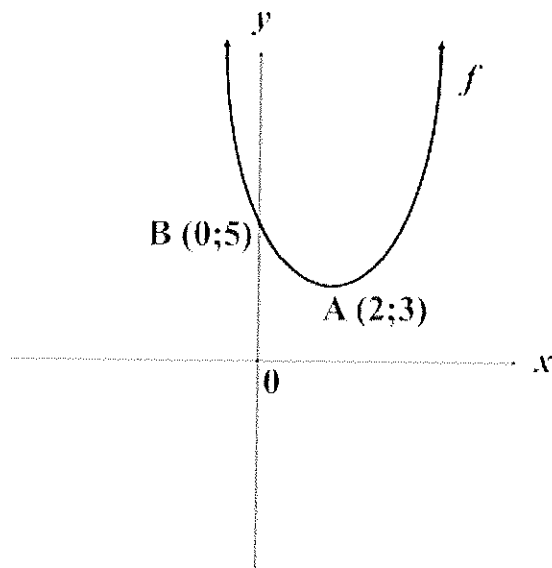
QUESTION 5

Given: $p(x) = \log_3 x$

- 5.1 Write down the equation of p^{-1} , the inverse of p , in the form $y = \dots$ (2)
- 5.2 Sketch in your ANSWER BOOK the graphs of p and p^{-1} on the same system of axis. Show clearly the intercepts of the axes and ONE other point on each graph. (6)
- 5.3 Determine the values of x for which $p(x) \leq 2$. (3)
- 5.4 Write down the x -intercept of h if $h(x) = p(-x)$. (2)
- [13]

QUESTION 6

The sketch below shows the parabola f with turning point $A(2; 3)$ and y -intercept $B(0; 5)$.



- 6.1 Show that the equation of the parabola f can be written as $y = \frac{1}{2}x^2 - 2x + 5$. (5)
- 6.2 WITHOUT calculating the discriminant, give an appropriate reason whether the discriminant of f is positive, negative or zero. (2)
- 6.3 Use the graph to determine the value(s) of k for which the equation $\frac{1}{2}x^2 - 2x + 5 = k$ has real and un-equal roots. (2)
- 6.4 The parabola is shifted vertically until the new y -intercept is the origin. Determine the equation of the NEW parabola. (1)
- [10]

QUESTION 7

- 7.1 Sarah's investment earns interest at 11% p.a. compounded semi-annually.
Mary's investment earns an effective interest of 11,42% *p.a.*
Whose investment, Sarah's or Mary's, earns a higher rate of interest per annum. (3)
- 7.2 Buhle decided to start saving before retirement. She makes payments
of R10'000 monthly into an account yielding 7,72% *p.a.* compounded monthly,
starting on 1 November 2016 with a final payment on 1 April 2026.
- 7.2.1 Calculate how much will be in the savings account immediately after
the last deposit is made. (4)
- 7.2.2 At the end of the investment period Buhle re-invested the full amount in order
for her to be able to draw a monthly pension from the fund.

She re-invested the money at an interest rate of 10% p.a. compounded monthly.
If she draws an amount of R30 000 per month from this investment, for how
many full months will she be able to receive R30 000? (4)
- 7.2.3 After withdrawing R30 000 for 20 months Buhle requires R1 500 000.
Determine whether she can access this amount of money from this annuity. (4)
- [15]

QUESTION 8

8.1 Determine $f'(x)$ from FIRST principles if

$$f(x) = -2x^2 + 6x \quad (4)$$

8.2 Evaluate:

$$8.2.1 \quad \frac{dy}{dx} \text{ if } y = 2x^2 + \frac{1}{2}x^4 - 3 \quad (2)$$

$$8.2.2 \quad f'(x) \text{ if } f(x) = \frac{x^3 - 5x^2 + 4x}{x - 4} \quad (4)$$

8.3 The tangent to the curve of $y = 2x^2 - 3x - 5$ is drawn at the point $(2; -3)$.

A straight line parallel to the tangent passing through the y -intercept of the curve is then drawn. Determine the equation of the straight line.

(4)

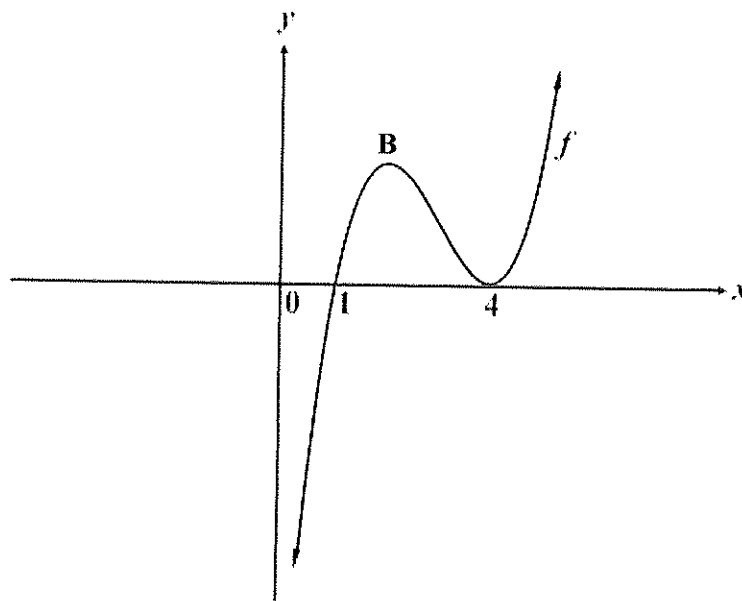
[14]

QUESTION 9

9.1 Determine the points on the curve of $y = \frac{4}{x}$ where the gradient of the tangent to the curve is -1 . (5)

9.2 The graph of the cubic function with equation $y = x^3 + ax^2 + bx + c$ is drawn below.

- $f(1) = f(4) = 0$
- f has a local maximum at point B and a local minimum at $x = 4$.



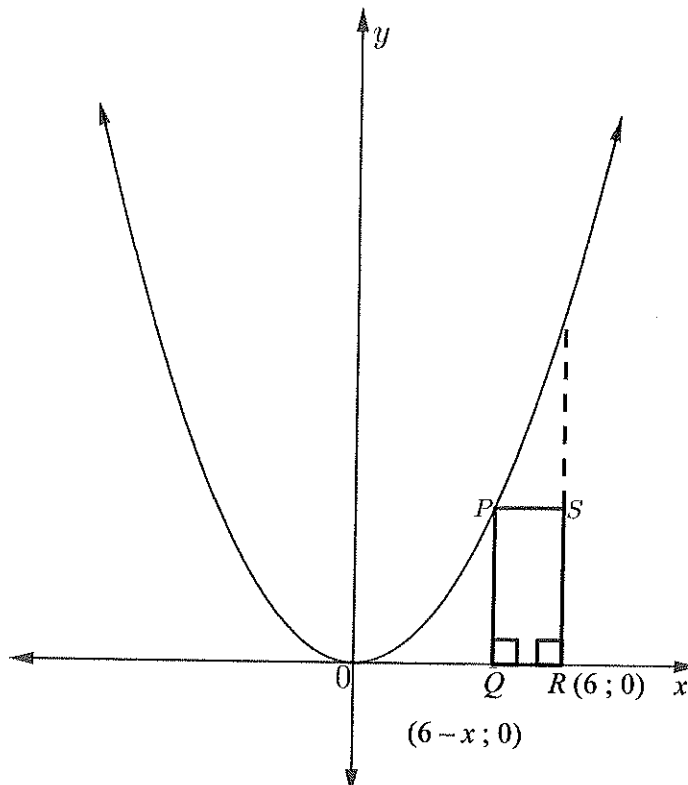
- 9.2.1 Show that $a = -9$, $b = 24$ and $c = -16$. (2)
- 9.2.2 Calculate the coordinates of point B. (4)
- 9.2.3 Determine the value(s) of k for which $f(x) = k$ has ONLY negative roots. (2)
- 9.2.4 Determine the value(s) of x for which f is concave up. (2)

[15]

QUESTION 10

The diagram below represents the graph of $y = x^2$ and rectangle PQRS.

- Point P is a common point on both the graph and the rectangle.
- The graphs of $x = 6$ and $y = 0$ represent TWO adjacent infinite lines of rectangle PQRS.
- The coordinates of point R(6 ; 0) and point Q(6 - x ; 0) are given.



10.1 Write down the coordinates of point P in terms of x .

(1)

10.2 Determine the maximum area of rectangle PQRS.

(5)

[6]

P.T.O.

QUESTION 11

11.1 Given that A and B are independent events.

- $P(B \text{ only}) = 0,3$
- $P(A \text{ and } B) = 0,2$
- $P(A \text{ only}) = x$
- $P(\text{not } A \text{ or } B) = y$

Determine the values of x and y .

(4)

11.2 Six players of a volleyball team stand at random positions in a row before the game begins. X and Y are two players in this team.

Determine the probability that X and Y will NOT stand next to each other.

(3)

11.3 Determine how many 4-digit numbers can be formed from 10 digits, 0 to 9, if:

11.3.1 repetition of digits is allowed

(2)

11.3.2 repetition of digits is NOT allowed

(3)

11.3.3 the last digit must be 0 and repetition of digits is allowed

(2)

[14]

TOTAL: 150

END

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 + i)^n$$

$$A = P(1 + i)^n$$

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1-r}; \quad -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A \quad \text{area } \triangle ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cos \alpha$$

$$(x; y) \rightarrow (x \cos \theta - y \sin \theta; y \cos \theta + x \sin \theta)$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ of } B) = P(A) + P(B) - P(A \text{ en } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

GAUTENG DEPARTMENT OF EDUCATION
PREPARATORY EXAMINATION

PROOFREAD ☐ DATE _____
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MATHEMATICS
(First Paper)

MEMORANDUM

QUESTION 1 [14]

1.1 $x(x+2) = 0$
 $x = 0$ or $x = -2$

1A factorizing
2 CA for each answer

[3]

1.2 $x \leq -2$ or $x \geq 0$; with $x > 0$
 $\therefore x > 0$

2CA for each inequality
(-1) if ' $<$ ' is used
1CA for selecting correct inequality
Full marks for answer only.

[3]

1.3 $x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-7)}}{2(2)}$

1A substitution into correct formula

$x = \frac{3 \pm \sqrt{65}}{4}$

$\frac{2}{3}$ if left in surd form

2CA for each correct answer

[3]

1.4 $1 + 2^{n-2} + 2^{n-1} + 2^n = 53$
 $2^n(2^{-2} + 2^{-1} + 1) = 52$
 $2^n = 16$
 $n = 4$

1A for $2^0 = 1$
1A for factorizing
1A for RHS
1CA

[5]

QUESTION 2 [10]

2.1.1 $(x+1)^2 - 4 = 16$
 $(x+1)^2 = 20$
 $x+1 = \pm\sqrt{20}$
 $x = \pm 2\sqrt{5} - 1$

1A squaring both sides
1CA for isolating complete square
1M for taking square root
2CA for each answer

OR

$(x+1)^2 - 4 = 16$
 $x^2 + 2x - 19 = 0$
 $x = \frac{-2 \pm \sqrt{(2)^2 - 4(-19)}}{2}$
 $x = \pm 2\sqrt{5} - 1$

1A squaring both sides
1CA simplification
1CA substitution into correct formula

2CA for each answer

[5]

2.1.2 $k = 0$

1A

[1]

1

QUESTION 5 [16]

5.1 $\left(1 + \frac{0.1}{12}\right)^{12} = \left(1 + \frac{r^2}{2}\right)^2$
 $i^{(2)} = 2 \left[\sqrt{\left(1 + \frac{0.1}{12}\right)^{12}} - 1 \right]$
 $i^{(2)} = 0.1021$ or $r = 10.21\%$

1A substitution into correct formula

1CA simplification
1CA

[3]

5.2.1 0.1×1200000
 $= R120\,000$

1M
1A

[2]

5.2.2 $1\,080\,000 = \frac{x \left(1 - \left(1 + \frac{0.084}{12} \right)^{-240} \right)}{\frac{0.084}{12}}$
 $x = R9\,304.25$

1A for 1080000
1M correct formula
1A correct substitution

1CA

[4]

5.2.3 Total amount paid = $9\,304.25 \times 240 + 120\,000$
 $= R2\,353\,020$

2CA
1CA

[3]

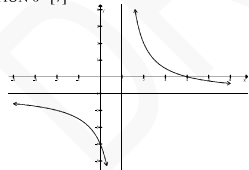
5.3 $A = 20\,000(1 - 0.2)^2(1 - 0.18)(1 - 0.16)$
 $A = R8\,816.64$

1M correct formula
2A
1CA
(-1 for incorrect rounding)

[4]

QUESTION 6 [7]

6.1



1A for shape
1A x-intercept (4; 0)
1A y-intercept (0; -4)
1A both asymptotes $y = -1$ and $x = 1$

[4]

6.2 $y = -x$ and $(m = -1$ passing through (1; -1))
 $y = x - 2$ ($m = 1$; passing through (1; -1))

1A
1A

[2]

6.3 Range: $\mathbb{R} \setminus \{-1\}$

1A

[1]

All real values excluding -1.

2.2 $\Delta \geq 0$

$(2)^2 - 4(3)(-k+1) \geq 0$

$k \geq \frac{2}{3}$

1M realizing to use $b^2 - 4ac$
1A correct inequality
1A substitution into correct formula

1CA

[4]

QUESTION 3 [8]

3.1 $99 + 97 + 95 + \dots + 101 - 2n$

1M for series (+ signs)
1A for first term
1A for last term

[3]

3.2 $\frac{n}{2}[2(99) + (n-1)(-2)] = 0$

$200n - 2n^2 = 0$ or $100n - n^2 = 0$
 $n(100 - n) = 0$
 $n = 100$ (or $n \neq 0$)

1M use of correct formula
1CA correct substitution and equation
1CA for correct simplification
1CA correct factorization
1CA for choosing correct answer

[5]

QUESTION 4 [17]

4.1.1 $T_{31} = 1$

1A

[1]

4.1.2 Sequence: 1; 5; 9;
 $S_{16} = \frac{16}{2}[2(1) + 15(4)]$

1M realizing sum of 16 terms.
1CA substitution into correct formula

$= 496$
 $S_{32} = 496 + 16$
 $= 512$

1CA
1CA addition of sums

[4]

4.1.3 $2n - 3 > 97$
 $n > 50$
 $\therefore n = 52$

1CA
1CA
1CA

[3]

4.2.1 second difference: 2
 $a = 1$; $b = 4$; $c = -2$

1A finding second difference
1A value of a
1CA value of b
1CA value of c
Any valid method may be used

[4]

$T_n = n^2 + 4n - 2$

4.2.2 $T_{24} = (24)^2 + 4(24) - 2$
 $T_{24} = 670$

1 CA correct substitution
1CA

[2]

4.2.3 $S_n = \frac{670}{1 - 0.8}$
 $= 3\,350$

1M
1A substitution
1CA

[3]

2

QUESTION 7 [16]

7.1 $y = a(x-3)^2 + 2$
 $-16 = a(0-3)^2 + 2$
 $a = -2$
 $\therefore y = -2x^2 + 12x - 16$

1A substitution of turning point
1CA substitution of (0; -16)
1CA

[3]

7.2 $0 = -2(x-3)^2 + 2$ or $0 = -2x^2 + 12x - 16$
 $(x-4)(x+2) = 0$ (or formula)
 $\therefore A(2; 0)$ $B(4; 0)$
 $C(0; -2)$

1 M substituting $y = 0$
1CA
1CA for both A and B
(Full marks for answer only)
1CA
(-1 if not in coordinate form)

[4]

7.3 $y - 0 = 1(x - 2)$

1A for gradient
1CA for correct substitution
1CA

$x - y = 2$ or $-x + y = -2$
or

$y = (1)x + c$
 $0 = (1)(1) + c$
 $x - y = 2$ or $-x + y = -2$

1A for gradient
1CA for substitution
1CA

[3]

7.4 $\frac{-96}{x} = -2x^2 + 12x - 16$

1M

$x^3 - 6x^2 + 8x - 48 = 0$
 $(x-6)(x^2 + 8) = 0$

1CA simplification
1CA factorization

$x = 6$
 $y = -16$
 $(6; -16)$

1CA

7.5 $(6; -2)$

2A

[2]

QUESTION 8 [7]

8.1 $2 = a^{-1}$

1A for substitution

[1]

8.2 $y = \log_{\frac{1}{2}} x$ or $y = -\log_2 x$

1M use of logs
1A

[2]

8.3 $y = x$

1A

[1]

8.4

at M $\left(\frac{1}{2}\right)^x = x$

1M

$\log\left(\frac{1}{2}\right)^x = \log x$

1M introducing logs

$x \log\left(\frac{1}{2}\right) - \log x = 0$

1CA applying log law.

OR

at M $\left(\frac{1}{2}\right)^x = x$

1M

$\log_{\frac{1}{2}} x = x$

1CA into log form

$\frac{\log x}{\log \frac{1}{2}} = x$

1CA applying log law

[3]

5

QUESTION 11 [13]

11.1.1 $x^3 - 4x^2 - 11x + 30 = 0$
 $(x-5)(x^2 + x - 6) = 0$
 $(x-5)(x+3)(x-2) = 0$
 $a = -3$ and $b = 2$

1M = 0
1A factors

2CA

If only solved for $x: \frac{2}{4}$

[4]

11.1.2 $3x^2 - 8x - 11 = 0$
 $(3x+11)(x-1) = 0$
 $c = -1$
 $d = 36$

1A derivative = 0
1CA
1CA
1CA

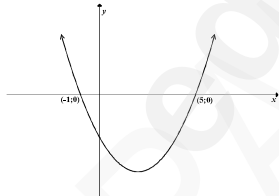
[4]

11.1.3 $x \in (-1; 2)$ or $-1 < x < 2$

1A for values
1A for correct inequality

[2]

11.2

1A shape
1A x-intercepts (-1 and 5)
1M turning point on correct side

[3]

QUESTION 12 [6]

7

QUESTION 9 [15]

9.1.1 $\lim_{h \rightarrow 0} \frac{1-3(x+h)^2 - (1-3x^2)}{h}$
 $= \lim_{h \rightarrow 0} \frac{-6xh - 3h^2}{h}$
 $= \lim_{h \rightarrow 0} (-6x - 3h)$
 $= -6x$

1A for $f(x+h)$
1A correct substitution into correct formula1CA simplification
1CA simplification

1CA

- (-1) for incorrect notation
- Using Rule $-\frac{0}{5}$

[5]

9.1.2 $f(-2) = -6(-2)$

1A

$= 12$

$f(-2) = 1 - 3(-2)^2$

1A

$= -11$

$y+11 = 12(x+2)$

1CA for finding equation
standard form not needed

$y = 12x + 13$

[3]

9.2.1 $D_x \left(1 - 2x^{\frac{1}{2}} - x^{\frac{1}{2}} + 2 \right)$
 $= x^{-\frac{1}{2}} - \frac{1}{2} x^{-\frac{1}{2}}$

2A simplification

2CA

[4]

9.2.2 $y = \frac{(p+1)(p^2 - p + 1)}{p+1}$

1A factorizing

$\frac{dy}{dp} = 2p - 1$

2CA for each part of final answer

[3]

QUESTION 10 [6]

10.1 $v = 3t^2 - 18t + 24$

1A

[1]

10.2 $\frac{dv}{dt} < 0$

1M

$3t^2 - 18t + 24 < 0$

$(t-4)(t-2) < 0$

$2 < t < 4$

1A for derivative
1CA factorizing
2CA

[5]

6

12.1 $y = -\frac{x}{2} + 9$

1A

[1]

12.2 $A = \frac{1}{2}xy$
 $= \frac{1}{2}y(18-2y)$
 $= 9y - y^2$
 $\frac{dA}{dy} = 9 - 2y = 0$
 $y = 4\frac{1}{2}$
 $x = 9$ $P(9; 4\frac{1}{2})$

1CA

1CA

1CA

Full marks if not in coordinate form

OR

$A = \frac{1}{2}xy$
 $A = \frac{1}{2}x(-\frac{x}{2} + 9)$
 $= -\frac{x^2}{4} + \frac{9x}{2}$
 $\frac{dA}{dx} = -\frac{1}{2}x + \frac{9}{2} = 0$
 $x = 9$
 $y = 4\frac{1}{2}$ $P(9; 4\frac{1}{2})$

1A

1A substituting for y

1CA

1CA

1CA

Full marks if not in coordinate form. [5]

8

QUESTION 13 [15]

- 13.1 $P(\text{pass Music or Art})$
 $= P(\text{pass Music}) + P(\text{pass Art}) - P(\text{pass Music and Art})$ IM for use of formula
 $= 0,4 + 0,6 - 0,3$ 1A substitution
 $= 0,7$ or 70% ICA
Answer only – Full Marks [3]
- 13.2 $P(\text{Right}) \times P(\text{Light}) = P(\text{Right and Light})$ IM
 $\frac{48}{60} \times \frac{20}{50} = \frac{a}{60}$
 $\frac{4}{15} = \frac{a}{60}$ 1A
 $a = \frac{4}{15} \times 60$
 $= 16$ ICA
 $b = 4$ ICA
 $c = 32$ ICA
- OR
 $P(\text{Right}) \times P(\text{Dark}) = P(\text{Right and Dark})$ IM
 $\frac{48}{60} \times \frac{40}{50} = \frac{c}{60}$
 $\frac{8}{15} = \frac{c}{60}$ 1A
 $c = 32$ ICA
 $a = 16$ ICA
 $b = 4$ ICA
- OR
 $P(\text{Left}) \times P(\text{Light}) = P(\text{Left and Light})$ IM
 $\frac{12}{60} \times \frac{20}{50} = \frac{b}{60}$
 $\frac{1}{15} = \frac{b}{60}$ ICA
 $b = 4$ ICA
 $a = 16$ ICA
 $c = 32$ ICA
- 13.3.1 $\frac{9!}{(9-5)!}$ or $9 \times 8 \times 7 \times 6 \times 5$ 1A
 $= 15120$ 1A [2]
- 13.3.2 $8 \times 7 \times 6 \times 5 \times 4$ 1A for 4 at the end
 $= 6720$ 1A for $8 \times 7 \times 6 \times 5$ ICA [3]
- 13.3.3 4^5 1A
 $= 1024$ 1A [2]



PREPARATORY EXAMINATION

2015

MEMORANDUM

SUBJECT:	MATHEMATICS P1 (10611)
----------	------------------------

MEMORANDUM

QUESTION 1		[21 MARKS]
1.1	1.1.1	$x = -\frac{5}{2}$ or $x = \sqrt{2}$ or $x = -\sqrt{2}$
		3A
	1.1.2	$x^2 \geq 9$ or $x^2 - 9 \geq 0$ $x \geq 3$ or $x \leq -3$
		[2] 1A 1A for critical value of 3 2A for each inequality Max 2/3 if 'and' is used Full marks if 'or' is omitted
	1.1.3	$6^{2x} \cdot 2^{2x} = 8 \cdot 6^{2x}$ or $2^{4x} \cdot 3^{2x} = 8 \cdot 3^{2x} \cdot 2^{2x}$ or $2^{4x} \cdot 3^{2x} = 2^{3+2x} \cdot 3^{2x}$ $2^{2x} = 8$ $2x = 3$ $x = \frac{3}{2}$
		[4] 1A for factorizing bases 1CA 1CA 1CA
	1.2	$(x+1)^2 = \frac{9}{2}$ $x+1 = \pm \sqrt{\frac{9}{2}}$ $x = 1, 12$ or $x = -3, 12$
		[4] 1A 1CA 2CA for each solution OR 1A standard form 1CA substitution into correct formula 2CA for each solution [4]

1

10611/15

1.3	$\frac{-2x+7}{x-1} + 5 = \frac{1}{2}$ $2(-2x+7) = x-1$ $x=5$ Substitute $x=5$: $y = -2(5) + 7$ $y = -3$ OR $\frac{y+5}{2} = \frac{1}{2}$ $\frac{y}{2} + \frac{5}{2} = \frac{1}{2}$ $2(y+5) = -\frac{y}{2} + \frac{5}{2}$ $y = -3$ substitute $y = -3$: $x = -\frac{-3}{2} + \frac{7}{2}$ $x = 5$	1A correct substitution 1CA simplification 1CA 1CA OR 1A correct substitution 1CA simplification 1CA 1CA 1CA [4] 1CA
1.4	$1-p > 0$ $p < 1$	1A 1CA [2]

QUESTION 2 [12 MARKS]

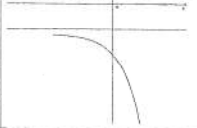
2.1	2.1.1	$x \left(-\frac{x}{2} \right)$ $= -\frac{x^2}{2}$	1A for $-\frac{x}{2}$ 1CA for multiplication with x [2]
	2.1.2	$x+x+2$ $= 2x+2$	1A for $x+2$ 1CA for adding to x [2]
2.2		$-2+x+2x+2 = -\frac{x^2}{2}$ $x^2+6x=0$ $x(x+6)=0$ $x=-6$	1M 1CA 1CA for simplified form 1CA factorizing 1CA choosing correct x-value [5]
2.3		The series does NOT converge. $r=3$, to converge, $-1 < r < 1$	1CA 2M for any logical explanation. [3]

10611/15

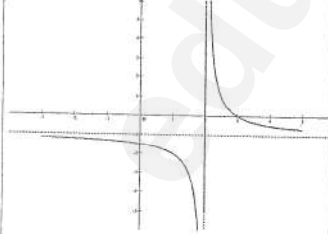
QUESTION 3		[8 MARKS]
3.1	$(250)^2 + 4(250)$ $= 63500$	1A substitution 1CA [2]
3.2	$T_{100} = S_{100} - S_{99}$ $= 100^2 + 4(100) - [99^2 + 4(99)]$ $= 203$ OR $T_1 = 5$; $T_2 = 7$; $T_3 = 9$ $T_{100} = 5 + 99(2)$ $= 203$	1M 1A correct substitution 1CA OR 1A for first three terms 1M for correct formula 1CA [3]
3.3	$n^2 + 4n = 1440$ $(n+40)(n-36) = 0$ $n = 36$	1A 1CA correct factors or correct substitution into correct formula. 1CA choice of correct n. Full marks for answer only [3]

QUESTION 4 [6 MARKS]

4.1	$2; 6; 12; 20$	1A for first three terms 1A for the 4 th term [2]
4.2	$a=1$; $b=1$; $c=0$ $T_n = n^2 + n$ or $T_n = n(n+1)$ $T_{100} = 100(100+1)$ $= 10100cm^2$	3A finding a, b, c using any correct method. 1CA [4]

QUESTION 5 [15 MARKS]		
5.1	5.1.1	$A = 650000(1 - 0,3)^4$ $= R156\,065$
		1A substitution into correct formula 1CA
	5.1.2	$A = 650000(1 + 0,15)^4$ $= R1\,136\,854,06$
		1A substitution into correct formula 1CA
	5.1.3	$1136854,06 - 156065 = 980789,06$ $980789,06 = \frac{x \left(1 + \frac{0,095}{12}\right)^{48} - 1}{\frac{0,095}{12}}$ $= R16\,875,92$
		1M for use of correct formula 1CA us of correct formula 1CA
5.2	5.2.1	$OB = 1275000 \left(1 + \frac{0,092}{12}\right)^{11036,02} \left(1 + \frac{0,092}{12}\right)^{11} - 1$ $= R1056675,39$
		1M 1A correct substitution 1CA
	5.2.2	$1056675,39 \left(1 + \frac{0,092}{12}\right)^5$ $= R1097807,15$
		1CA substitution into correct formula 1CA
		$1097807,15 = \frac{x \left(1 + \frac{0,092}{12}\right)^{111} - 1}{\frac{0,092}{12}}$ $= R12297,82$
		1CA substitution into correct formula 1CA [4]
QUESTION 6 [3 MARKS]		
		
		1M for correct shape 1A for horizontal asymptote 1A for negative y-value [3]

QUESTION 7 [19 MARKS]		
7.1		$-2x^2 - 4x + 30 = 0$ $(x+5)(x-3) = 0$ $A(-5;0) \quad B(3;0)$
		1M equating to 0 1A correct factors 2CA for both intercepts -1 if not in coordinate form [4]
7.2		$y = -2(x^2 + 2x - 15)$ $y = -2(x^2 + 2x + 1 - 15 - 1)$ $y = -2[(x+1)^2 - 16]$ $y = -2(x+1)^2 + 32$ Tpt $(-1;32)$
		1A for taking out -2 1CA $\left(\frac{b}{2}\right)^2$ added and deducted 1CA in complete square form 1CA OR 1A substitute into correct formula
		$x = \frac{-b}{2a}$ $x = -1$ $y = 32$ $y = -2(x+1)^2 + 32$ Tpt $(-1;32)$
		1CA 1CA 1CA in correct form 1CA
7.3		$y = -2(1)^2 - 4(1) + 30$ $y = 24$ - No
		OR $-2x^2 - 4x + 30 = 2x + 10$ $x^2 + 3x - 10 = 0$ $x = 2$ or $x = -5$ - No
		1M substitution into equation 1CA value and conclusion OR 1M finding point of intersection 1CA for values with conclusion [2]
7.4		$-2x^2 - 4x + 30 = mx + 32$ $-2x^2 + (-4-m)x - 2 = 0$ $A = 0$ $(-4-m)^2 - 4(-2)(-2) = 0$ $m = 0$ or $m = -8$
		1M for equating 1CA for standard form 1M 1CA correct substitution 2CA for each value of m
7.5		$(-\infty;3) \cup (-5)$ or $x < 3$; $x \neq -5$
		[6] 2A [2]

QUESTION 8 [11 MARKS]		
8.1		VA: $x = 2$ HA: $y = -1$
		1A 1A [2]
8.2		y-intercept: $y = \frac{1}{0-2} - 1$ $= -\frac{3}{2}$ x-intercept: $\frac{1}{x-2} - 1 = 0$ $x = 3$
		1M 1CA [4]
8.3		
		1A intercepts with axes 1A asymptotes 1A Shape [3]
8.4		$2 = \frac{1}{k-2} - 1$ $k = \frac{4}{3}$
		1A correct substitution 1CA [2]

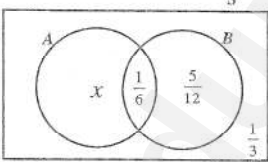
QUESTION 9 [6 MARKS]		
9.1		$-2 = \log_4 \left(\frac{9}{4}\right)$ $a^2 = \left(\frac{9}{4}\right)$ $a = \frac{3}{2}$
		1A correct substitution 1CA exponential form 1CA [3]
9.2		$y = -\log_3 x$ or $y = \log_3 x$
		1A [1]
9.3		$g(x) > -2$
		1A [1]
9.4		$(-\infty; \infty)$
		1A [1]
QUESTION 10 [14 MARKS]		
10.1	10.1.1	$\frac{f(3+h) - f(3)}{h}$ $= \frac{-2(3+h)^2 + 1 - [-2(3)^2 + 1]}{h}$ $= \frac{-12h - 2h^2}{h}$ $= \frac{h(-12-2h)}{h}$ $= -12 - 2h$ OR $\frac{f(x+h) - f(x)}{h}$ $= \frac{-2(x+h)^2 + 1 - [-2x^2 + 1]}{h}$ $= \frac{-4xh - 2h^2}{h}$ $= \frac{h(-4x-2h)}{h}$ $= -4x - 2h$
		1M correct formula 1A correct substitution 1CA correct simplification 1CA Max 1/3 if "limit" is used OR 1M correct formula 1A correct substitution 1CA correct simplification 1CA substitution of $x=3$ Max 1/3 if "limit" is used [4]
	10.1.2	$\lim_{h \rightarrow 0} (-12 - 2h)$ $= -12$
		1M using limit 1CA [2] 1A [1]

10.2	10.2.1	$y = 16 - 16x + 4x^2$ y' or $\frac{dy}{dx} = -16 + 8x$	1A for simplification 2CA (-1 for incorrect notation) [3]
	10.2.2	$y = x - ax^{\frac{1}{2}} + a$ $y' = 1 - \frac{1}{2}ax^{-\frac{1}{2}}$ $y' = 1 - \frac{a}{2x^{\frac{1}{2}}}$	1A for simplification 2CA 1CA positive exponents [4]
QUESTION 11 [11 MARKS]			
11.1		The graph passes through the origin. Or $f(x) = (x-0)^2(x-a)^2$	1M any logical explanation [1]
11.2	11.2.1	$f(x) = x(x-2)^2$ $a = 2$	1A factorization 1CA [2]
	11.2.2	$f'(x) = 0$ $3x^2 - 8x + 4 = 0$ $(3x-2)(x-2) = 0$ $b = \frac{2}{3}$	1M 1A correct differentiation 1CA factors 1CA [4]
11.3		$f'(0) = 3(0)^2 - 8(0) + 4$ $m = 4$	1CA substitution into derivative 1CA [2]
11.4		$x^3 - 4x^2 + 4x = p - 2$ $p < 2$	1M taking 2 to RHS 1CA (full marks for answer only) [2]

8

QUESTION 12 [9 MARKS]			
12.1		$x^2 \left(40 - \frac{x}{3}\right) < 0$ $x > 120 \text{ km/hour}$	1M for inequality 1A for inequality sign 1CA for 120km/hour Full marks if units are omitted. [3]
12.2		$P = 40x^2 - \frac{x^3}{3}$ $P'(x) = 0$ $80x - x^2 = 0$ $x(80-x) = 0$ $x = 80 \text{ km/hour}$ $P = 10(80)^2 - \frac{80^3}{3}$ $P = 21333.33 \text{ Rand/day}$ $R \leq 333.33$	1M 1CA 1CA correct factorization 1CA 1CA 1CA Full marks if units are omitted. [6]

9

QUESTION 13 [15 MARKS]			
13.1		$\frac{1}{6} + \frac{7}{16}$ $= \frac{29}{48}$	2A for both outcome 1M for addition 1CA for simplification [4]
13.2	13.2.1		3A for each section [3]
	13.2.2	$x + \frac{1}{6} + \frac{5}{12} + \frac{1}{3} = 1$ $x = \frac{1}{12}$ $P(A) = \frac{1}{6} + \frac{1}{12} = \frac{1}{4}$	1M 1A 1CA [3]
13.3		36 000	1A [1]
13.4	13.4.1	2.9!	2A [2]
	13.4.2	8.8!	2A [2]

PREPARATORY EXAMINATION

2017

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MATHEMATICS (FIRST PAPER) (10611)

14 pages

QUESTION 1		
1.1		
1.1.1	$x(x-1)=12$ $x^2-x=12$ $x^2-x-12=0$ $(x-4)(x+3)=0$ $x=4$ OR $x=-3$	✓ standard form ✓ both factors ✓ both answers (3)
1.1.2	$2x^2+3=8x$ $2x^2-8x+3=0$ $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ $= \frac{8 \pm \sqrt{(-8)^2-4(2)(3)}}{2(2)}$ $= \frac{8 \pm \sqrt{64-24}}{4}$ $= \frac{8 \pm \sqrt{40}}{4}$ $x=3,58$ OR $x=0,42$	✓ standard form ✓ substitution into correct formulae ✓ ✓ each x answer (4)
1.1.3	$(2x+3)(3-x) > 4$ $6x-2x^2+9-3x-4 > 0$ $-2x^2+3x+5 > 0$ $2x^2-3x-5 < 0$ $(2x-5)(x+1) < 0$ $-1 < x < \frac{5}{2}$	✓ simplification ✓ standard form $2x^2-3x-5 < 0$ ✓ factors ✓ critical values ✓ inequalities (5)
1.1.4	$2^x = 5 \cdot 2^{x+1} = -144$ $2^x(1-5 \cdot 2) = -144$ $2^x(-9) = -144$ $2^x = 16$ $2^x = 2^4$ $x=4$	✓ common factor ✓ simplification $2^x = 2^4$ ✓ answer (3)

2

10611/17

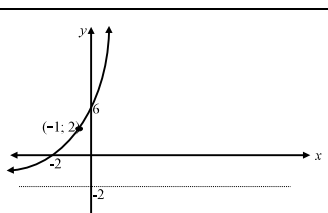
1.2	$y+2x-3=0$ (1) $y=x^2+2x+3$ (2) From (1): $y=-2x+3$ (3) Substitute (3) into (2): $-2x+3=x^2+2x+3$ $x^2+4x=0$ $x(x+4)=0$ $x=0$ OR $x=-4$ Substitute $x=0$ into (3): $y=-2(0)+3$ $y=3$ OR Substitute $x=-4$ into (3): $y=-2(-4)+3$ $y=11$	✓ $y=-2x+3$ ✓ equating $-2x+3=x^2+2x+3$ ✓ factorisation ✓ both x-values ✓ both y-values (5)
QUESTION 2		
2.1	$4a-a-2=6a+4-4a$ $a=6$	✓ equating ✓ answer (2)
2.2		
2.2.1		✓ 33 ✓ 96 (2)
2.2.2	45 – Term odd 23^{rd} term of the odd sequence: $T_{23} = 5 + (23-1)(7)$ $T_{23} = 159$	✓ 45 – Term odd ✓ verifying T_{23} ✓ answer (3)

10611/17

2.3	$T_8=33=a+7d$ (1) $T_{11}=45=a+10d$ (2) From equation (2): $a+10d=45$ (2) $a+7d=33$ (1) $a=45-10d$ (3) OR $(2)-(1)$: $3d=12$ $d=4$ Substitute (3) into (1): $33=45-10d+7d$ $-12=-3d$ $d=4$ Substitute into (2): $a=45-10(4)$ $a=5$ $T_{15}=5+14(4)$ $T_{15}=61$ OR $T_8+3d=T_{11}$ $33+3d=45$ $3d=12$ $d=4$ $T_{15}=T_{11}+4d$ $T_{15}=45+4(4)$ $T_{15}=61$	✓ equation 1 ✓ equation 2 ✓ value of d ✓ value of a ✓ $T_{15}=61$ ✓ $T_8+3d=T_{11}$ ✓ substitution ✓ value of d ✓ $T_{15}=T_{11}+4d$ ✓ answer (5)
2.4	$2a=-4$ $a=-2$ $3a+b=0$ $3(-2)+b=0$ $b=6$ $a+b+c=6$ $-2+6+c=6$ $c=2$ $T_n = -2n^2 + 6n + 2$	✓ second difference ✓ value of a ✓ value of b ✓ value of c (4)

2.5		
2.5.1	$3; \frac{3}{2}; \frac{3}{4}; \dots$ $S_n = \frac{a}{1-r}$ $S_n = \frac{3}{1-\frac{1}{2}}$ $= 6$	✓ substitute into the correct formula ✓ answer (2)
2.5.2	$S_n = \frac{a(1-r^n)}{1-r}$ $S_n = \frac{3\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\frac{1}{2}}$ $= 6\left(1-\left(\frac{1}{2}\right)^n\right)$	✓ substitute into the correct formula ✓ answer (2)
2.5.3	$A-B < \frac{1}{36}$ $6-6\left(1-\left(\frac{1}{2}\right)^n\right) < \frac{1}{36}$ $6\left(1-1+\left(\frac{1}{2}\right)^n\right) < \frac{1}{36}$ $1-\left(\frac{1}{2}\right)^n < \frac{1}{216}$ $\left(\frac{1}{2}\right)^n < \frac{1}{216}$ $n > \log_{\frac{1}{2}} \frac{1}{216}$ $n > 7,75$ $\therefore n=8, 9, \dots$	✓ factorisation ✓ $\frac{1}{216}$ ✓ $\left(\frac{1}{2}\right)^n < \frac{1}{216}$ ✓ $\log_{\frac{1}{2}} \frac{1}{216}$ ✓ answer (5)
		[25]

	QUESTION 3	
3.1	$(1+i_{eff}) = \left(1 + \frac{i_{nom}}{m}\right)^m$ $(1+i_{eff}) = \left(1 + \frac{0,06}{4}\right)^4$ $i_{eff} = 1,06 - 1$ $i_{eff} = 6,14\%$	✓ correct formula ✓ correct substitution ✓ answer (3)
3.2	$P = \frac{x[1-(1+i)^{-n}]}{i}$ $P = \frac{2000[1-(1+0,02)^{-24}]}{0,02}$ $= R37\,827,85$ The gym borrowed R 37 827,85.	✓ substitution of x into correct formula ✓ $n = 24$ ✓ $i = 0,02$ ✓ answer (4)
3.3	$F = \frac{x[(1+i)^n - 1]}{i}$ $200000 = \frac{x\left[\left(1 + \frac{0,0625}{12}\right)^{37} - 1\right]}{\frac{0,0625}{12}}$ $200000 \times \frac{0,0625}{12} = x\left[\left(1 + \frac{0,0625}{12}\right)^{37} - 1\right]$ $x = 3022,73$ $\therefore$ The monthly payments are R 3 022,73 per month.	✓ formula ✓ value of F ✓ $n = 37$ ✓ $i = \frac{0,0625}{12}$ ✓ making x the subject ✓ answer (6)
		[13]

	QUESTION 4	
4.1	$f(-1) = 4 \cdot 2^{-(-1)+1} - 2$	
4.1.1	$y = 4 - 2$ OR $f(-1) = 2$ $y = 2$	✓ $y = 2$ (1)
4.1.2	$f(x) = 4 \cdot 2^{x+1} - 2$ x -intercept $f(x) = 0$ $4 \cdot 2^{x+1} - 2 = 0$ $4 \cdot 2^{x+1} = 2$ $2^{x+3} = 2^1$ $x+3 = 1$ $x = -2$ $(-2; 0)$ OR $4 \cdot 2^{x+1} - 2 = 0$ $4 \cdot 2^{x+1} = 2$ $2^{x+3} = \frac{2}{4}$ $2^{x+3} = 2^{-1}$ $x+3 = -1$ $x = -2$ $(-2; 0)$	✓ simplification $(2^{x+3} = 2^1)$ ✓ $x+3 = 1$ ✓ answer ✓ simplification $(2^{x+3} = 2^{-1})$ ✓ $x+3 = -1$ ✓ answer (3)
4.1.3	y -intercept $x = 0$ $y = 4 \cdot 2^{0+1} - 2$ $y = 8 - 2$ $y = 6$ $(0; 6)$	✓ substitute $x = 0$ ✓ answer (2)
4.1.4	$y = -2$	✓ $y = -2$ (1)
4.2		✓ asymptote ✓ shape with any one correct coordinate ✓ both intercepts with axes (3)

4.3	$g(x) = 4 \cdot 2^{x-2+1} - 2$ $g(x) = 4 \cdot 2^{x-1} - 2$ OR $g(x) = 2 \cdot 2^x - 2$ OR $g(x) = 2^{x+1} - 2$	✓ answer ✓ answer ✓ answer (1)
	QUESTION 5	
5.1		✓ 3
5.1.1	$(3; 4)$	✓ 4 (2)
5.1.2	$y \geq -4$ OR $y \in [-4; \infty)$	✓ critical values ✓ inequality ✓ critical values ✓ correct brackets (2)
5.1.3	$(2; 3)$ $(5; 0)$ $m = \frac{0-3}{5-2}$ $m = -1$	✓ substitute with correct coordinates ✓ answer (2)
5.1.4	$y = -(-1) + 5$ $y = 6$ $KM = 6$ $y = -(-1-3)^2 + 4$ $y = -12$ $LM = 12$ $KL = 12 + 6$ $= 18$	✓ $KM = 6$ OR $y = 6$ ✓ $LM = 12$ OR $y = -12$ ✓ $KL = 18$ (3)
5.1.5	$x \in (2; 5)$ OR $2 < x < 5$	✓ critical values ✓ correct brackets ✓ critical values ✓ correct inequality (2)

5.2		
5.2.1	$y = \left(\frac{4}{3}\right)^x$ $x = \left(\frac{4}{3}\right)^y$ $y = \log_{\frac{4}{3}} x$ $g(x) = \log_{\frac{4}{3}} x$	✓ interchange x and y ✓ $g(x) = \log_{\frac{4}{3}} x$ (2)
5.2.2	$h(x) = \left(\frac{4}{3}\right)^{-x}$ $= \left(\frac{3}{4}\right)^x$	✓ $-x$ ✓ answer (2)
5.2.3		
a)	$x = 0$	✓ answer (1)
b)	$0 < x \leq 1$ OR $x \in (0;1]$	✓ critical values ✓ inequalities ✓ critical values ✓ correct brackets (2)
		[18]

	QUESTION 6	
6.1	$f'(x) = \frac{4}{x^2}$ $\therefore x^2 \geq 0$ $\therefore \frac{4}{x^2} > 0$	✓ $f'(x) = \frac{4}{x^2}$ ✓ $x^2 \geq 0$ ✓ $\frac{4}{x^2} > 0$ (3)
6.2	$m = -4$ <i>Perpendicular lines</i> $m = \frac{1}{4}$ $f'(x) = \frac{4}{x^2}$ $\frac{1}{4} = \frac{4}{x^2}$ $16 = x^2$ $\therefore x = -4 \quad y = -\frac{4}{-4} + 7$ $y = 8$ Substitute $(-4; 8)$ $y = \frac{1}{4}x + c \quad \text{OR} \quad y - 8 = \frac{1}{4}(x + 4)$ $8 = \frac{1}{4}(-4) + c \quad y = \frac{1}{4}x + 9$ $c = 9$ $y = \frac{1}{4}x + 9$	✓ $m = \frac{1}{4}$ ✓ $\frac{1}{4} = \frac{4}{x^2}$ ✓ selecting $x = -4$ ✓ $y = 8$ ✓ answer (5)
		[8]

	QUESTION 7	
7.1	$f(x+h) = 1 - 4(x+h)^2$	✓ $f(x+h) = 1 - 4x^2 - 8xh - 4h^2$
7.1.1	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{1 - 4x^2 - 8xh - 4h^2 - (1 - 4x^2)}{h}$ $= \lim_{h \rightarrow 0} \frac{-8xh - 4h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(-8x - 4h)}{h}$ $= -8x$	✓ correct substitution into correct formula ✓ simplify $(-8xh - 4h^2)$ ✓ common factor h ✓ $-8x$ (5)
7.1.2	$f'(2) = -8(2)$ $= -16$	✓ substitute $x = 2$ into $f'(x)$ ✓ answer (2)
7.2		
7.2.1	$y = x^2 - 4x + 4$ $\frac{dy}{dx} = 2x - 4$	✓ expansion ✓ $2x$ ✓ -4 (3)
7.2.2	$f(x) = x^{\frac{2}{3}} + \frac{1}{4}x^{-4}$ $f'(x) = \frac{2}{3}x^{-\frac{1}{3}} - x^{-5}$	✓ $x^{\frac{2}{3}}$ ✓ $\frac{1}{4}x^{-4}$ ✓ $\frac{2}{3}x^{-\frac{1}{3}}$ ✓ $-x^{-5}$ (4)
		[14]

	QUESTION 8	
8.1	$f(x) = (x+2)(x-1)(x-3)$ $= x^3 - 2x^2 - 5x + 6$	✓ factors $(x+2)(x-1)(x-3)$ ✓ product of any two factors ✓ product of any binomial and trinomial $x^3 + x^2 - 2x - 3x^2 - 3x + 6$ (3)
8.2	$f'(x) = 3x^2 - 4x - 5$ $0 = 3x^2 - 4x - 5$ $x = \frac{4 \pm \sqrt{16 + 60}}{6}$ $x = 2,12 \quad \text{OR} \quad x = -0,7$ $y = 8,21 \quad \text{OR} \quad y = -4,06$ $A(-0,79; 8,21) \quad B(2,12; -4,06)$	✓ $f'(x) = 0$ ✓ substitute into quadratic formula ✓ x -values ✓ y -values (4)
8.3	$y = x^3 - 2x^2 - 5x + 6 - (2x)$ $= x^3 - 2x^2 - 7x + 6$ $\frac{dy}{dx} = 3x^2 - 4x - 7$ $= (3x - 7)(x + 1)$ <i>for max</i> $0 = (3x - 7)(x + 1)$ $x = \frac{7}{3} \quad \text{OR} \quad x = -1$ $y = (-1)^3 - 2(-1)^2 - 7(-1) + 6$ $PQ = 10$	✓ simplification $x^3 - 2x^2 - 7x + 6$ ✓ differentiation ✓ factors ✓ $\frac{dy}{dx} = 0$ ✓ x -values ✓ answer (6)
8.4	$f'(x) = 3x^2 - 4x - 5$ $f''(x) = 6x - 4$ $6x - 4 < 0$ $x < \frac{2}{3}$ $k = \frac{2}{3}$	✓ $f''(x)$ ✓ $f''(x) < 0$ ✓ answer (3)
		[16]

QUESTION 9		
9.1	$BC = 50 - 2x$	✓ answer (1)
9.2	$\text{Area} = AB \times BC$ $= x(50 - 2x)$ $= 50x - 2x^2$ $\text{Max } A'(x) = 50 - 4x$ $0 = 50 - 4x$ $x = 12,5m$	✓ $x(50 - 2x)$ ✓ simplification $(50x - 2x^2)$ ✓ differentiate ✓ answer (4)
QUESTION 10		
10.1		✓ first branches $\checkmark \frac{2}{9}; \frac{2}{9}; \frac{5}{9}$ $\checkmark \frac{3}{9}; \frac{1}{9}; \frac{5}{9}$ $\checkmark \frac{3}{9}; \frac{2}{9}; \frac{4}{9}$ ✓ all outcomes (5)
10.2		
10.2.1	$P(B, B) = \frac{5}{10} \times \frac{4}{9}$ $= 0,22 \quad \text{OR} \quad \frac{2}{9}$	✓ $\frac{5}{10} \times \frac{4}{9}$ ✓ answer (2)
10.2.2	$P(G, P) = \frac{3}{10} \times \frac{2}{9} + \frac{2}{10} \times \frac{3}{9}$ $= 0,13 \quad \text{OR} \quad \frac{12}{90} \quad \text{OR} \quad \frac{2}{15}$	$\checkmark \frac{3}{10} \times \frac{2}{9} \text{ OR } \frac{6}{90}$ $\checkmark \frac{2}{10} \times \frac{3}{9} \text{ OR } \frac{6}{90}$ ✓ answer (3)
		10

QUESTION 11		
11.1	Letters : $26 - 6 = 20$ Digits : $0 - 9 = 10$ $20^4 \times 10^4$ $= 1\,600\,000\,000$	✓ 20^4 ✓ 10^4 ✓ answer (3)
11.2		
11.2.1	$P'(A) = 1 - P(A)$ $0,45 = 1 - P(A)$ $P(A) = 0,55$ $P(A \text{ or } B) = P(A) + P(B)$ $= 0,55 + 0,35$ $= 0,9$	✓ $P(A)$ ✓ substitute in correct formula ✓ answer (3)
11.2.2	$P(A) \times P(B)$ $= 0,55 \times 0,35$ $= 0,1975 \quad \text{OR} \quad \frac{77}{400}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$ $= 0,55 + 0,35 - 0,1975$ $= 0,71 \quad \text{OR} \quad \frac{283}{400} \quad \text{OR} \quad \frac{71}{100}$	✓ $0,55 \times 0,35$ ✓ $0,1975 \text{ OR } \frac{77}{400}$ ✓ substitute in correct formula ✓ answer (4)
		10
		TOTAL: 150

PREPARATORY EXAMINATION

2018

MARKING GUIDELINES

10611 MATHEMATICS (PAPER 1)

14 pages

QUESTION 1		
1.1		
1.1.1	$x^2 - x - 30 = 0$ $(x-6)(x+5) = 0$ $x = 6$ or $x = -5$	✓ correct factors ✓ both answers correct (2)
1.1.2	$3x^2 - 8x - 4 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(-4)}}{2(3)}$ $x = 3,10$ or $x = -0,43$	✓ standard form ✓ correct subst. in the correct formula ✓ $x = 3,10$ ✓ $x = -0,43$ Penalise for incorrect rounding off here only (4)
1.1.3	$\sqrt{5-x} - x = 1$ $\sqrt{5-x} = 1+x$ $(\sqrt{5-x})^2 = (1+x)^2$ $x^2 + 3x - 4 = 0$ $(x+4)(x-1) = 0$ $x = -4$ or $x = 1$ NA	✓ isolate square root ✓ square both sides ✓ standard form ✓ both answers correct ✓ choose correct answer (5)
1.1.4	$\frac{6x^2 - 3x}{3} \leq 3x^2$ $2x^2 - x \leq 3x^2$ $2x^2 - 3x^2 - x \leq 0$ $-x^2 - x \leq 0$ $-x(x+1) \leq 0$ $x \geq 0$ or $x \leq -1$ OR	✓ simplify ✓ standard form ✓ correct factors ✓ critical values ✓ correct inequalities

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	$\frac{6x^2 - 3x}{3} \leq 3x^2$ $6x^2 - 3x \leq 9x^2$ $6x^2 - 9x^2 - 3x \leq 0$ $-3x^2 - 3x \leq 0$ $3x(x+1) \geq 0$ $x \geq 0$ or $x \leq -1$	✓ simplify ✓ standard form ✓ correct factors ✓ critical values ✓ correct inequalities (5)
1.1.5	$2x^2 + 7\sqrt{2}x = 2$ $2^2 \cdot 2^x + 7 \cdot 2^{\frac{x}{2}} - 2 = 0$ $4 \cdot 2^x + 7 \cdot 2^{\frac{x}{2}} - 2 = 0$ $(4 \cdot 2^{\frac{x}{2}} - 1)(2^{\frac{x}{2}} + 2) = 0$ $2^{\frac{x}{2}} = \frac{1}{4}$ or $2^{\frac{x}{2}} = -2$ $2^{\frac{x}{2}} = 2^{-2}$ NA $\frac{x}{2} = -2$ $x = -4$	✓ standard form ✓ factors ✓ $2^{\frac{x}{2}} = 2^{-2}$ ✓ NA ✓ $x = -4$ (5)
1.2	$6x^2 + 2px - 3x - p = 0$ $6x^2 + x(2p-3) - p = 0$ $\Delta = b^2 - 4ac$ $\Delta = (2p-3)^2 - 4(6)(-p)$ $= 4p^2 + 12p + 9$ $= (2p+3)^2$ $\Delta = \text{perfect square}$ $\therefore$ rational roots	✓ coefficient of x ✓ correct substitution in correct formula ✓ factors ✓ $\therefore \Delta = \text{perfect square}$ (conclusion) (4)

[25]

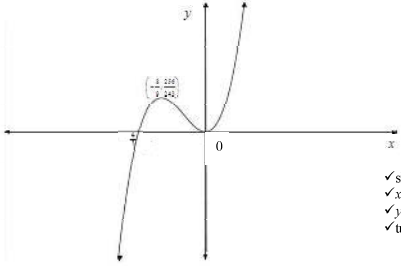
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QUESTION 2		
2.1	$T_n = a + (n-1)d$ $-239 = 6 + (n-1)(-5)$ $-239 = 6 - 5n + 5$ $250 = 5n$ $n = 50$	✓ $T_n = -239$ ✓ $d = -5$ ✓ answer (3)
2.2	$T_3 = ar^2 = 18 \dots \textcircled{1}$ $T_5 = ar^4 = 162 \dots \textcircled{2}$ $\textcircled{2} \div \textcircled{1}: r^2 = 9 \ (r < 0)$ $r = -3$ $ar^2 = 18$ $a(9) = 18$ $a = 2$ $S_r = \frac{a(r^r - 1)}{r - 1}$ $S_r = \frac{2((-3)^r - 1)}{-3 - 1}$ $= \frac{2(-2187 - 1)}{-4}$ $= 1094$	✓ eq① ✓ eq② ✓ $r = -3$ ✓ $a = 2$ ✓ correct substitution in correct formula ✓ answer (6)
2.3	3; x; 11; 21; 35 x-3; 11-x; 10; 14 $-2x = 4 - 14$ $-2x = -10$ $x = 5$	✓ first difference d_1 ✓ equating second difference ✓ answer (3)

4.8	Any upwards translation will result in non-real roots.	✓ answer (1)
QUESTION 5		
5.1	$x = -2$ or $x = 4$	✓ $x = -2$ ✓ $x = 4$ (2)
5.2	$x < -2$ or $-2 < x \leq 6$ OR $x \leq 6; x \neq -2$	✓ $x < -2$ ✓ $-2 < x \leq 6$ OR ✓ $x \leq 6$ ✓ $x \neq -2$ (2)
5.3	$-2 \leq x \leq 2$	✓ correct critical values ✓ correct inequalities (2)
QUESTION 6		
6.1	$g(x) = \frac{1}{4}x^2$	✓ answer (1)
6.2	$h(x) = \frac{1}{4}(x)^2 - 2$	✓ answer (1)
6.3	$y \geq -2$ OR $y \in [-2; \infty)$	✓ answer (correct inequality / bracket) (1)
QUESTION 7		
7.1	$x = 2$	✓ answer (1)
7.2	Translation of 2 units to the right and 3 units downwards.	✓ 2 units right ✓ 3 units down (2)
7.3	$x \in R; x \neq 3$	✓ answer (1)
7.4	$y = x + c$ $0 = 2 + c$ $-2 = c$ $y = x - 2$	✓ correct gradient (m=1) ✓ substitution C(2; 0) ✓ equation (3)

QUESTION 8		
8.1	$f(x+h) = 1 - 3(x+h)^2$ $= 1 - 3(x^2 + 2xh + h^2)$ $= 1 - 3x^2 - 6xh - 3h^2$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{1 - 3x^2 - 6xh - 3h^2 - (1 - 3x^2)}{h}$ $= \lim_{h \rightarrow 0} \frac{1 - 3x^2 - 6xh - 3h^2 - 1 + 3x^2}{h}$ $= \lim_{h \rightarrow 0} \frac{-6xh - 3h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(-6x - 3h)}{h}$ $= -6x$	✓ $f(x+h) = 1 - 3x^2 - 6xh - 3h^2$ ✓ correct substitution into correct formula ✓ simplify $(-6xh - 3h^2)$ ✓ common factor h ✓ $-6x$ (5)
8.2	$f'(2) = -6(2)$ $= -12$	✓ substitute $x = 2$ ✓ answer (2)
QUESTION 9		
In this question, penalise learners ONCE for incorrect notation.		
9.1.	$\frac{d}{dt} [(t-2)(t+3)]$ $= \frac{d}{dt} [t^2 + t - 6]$ $= 2t + 1$	✓ correct quadratic expression ✓ 2t ✓ 1 (3)
9.2	$D_x \left[\frac{5x^3 - 4}{x} \right]$ $= D_x \left[5x^2 - \frac{4}{x} \right]$ $= D_x [5x^2 - 4x^{-1}]$ $= 10x + 4x^{-2}$	✓ simplification ✓ 10x ✓ $4x^{-2}$ (3)

QUESTION 10		
10.1	$f(x) = ax^3 + bx^2$ $7 = a + b \dots (1)$ $f'(x) = 3ax^2 + 2bx$ $17 = 3a(1)^2 + 2b(1)$ $17 = 3a + 2b \dots (2)$ from (1) $b = 7 - a$ Substitute in (2) $3a + 2(7 - a) = 17$ $3a + 14 - 2a = 17$ $a = 3$ $b = 4$ OR $f(x) = ax^3 + bx^2$ $7 = a + b$ $f'(x) = 3ax^2 + 2bx$ $17 = 3a(1)^2 + 2b(1)$ $17 = 3a + 2b$ from (1) $a = 7 - b$ Substitute in (2) $3(7 - b) + 2b = 17$ $21 - 3b + 2b = 17$ $-b = -4$ $b = 4$ $a = 3$	✓ $7 = a + b$ ✓ derivative ✓ $17 = 3a + 2b$ ✓ $b = 7 - a$ ✓ $a = 3$ (must show calculation) ✓ $b = 4$ (must show calculation) ✓ $7 = a + b$ ✓ derivative ✓ $17 = 3a + 2b$ ✓ $a = 7 - b$ ✓ $b = 4$ (must show calculation) ✓ $a = 3$ (must show calculation) (6)
10.2	$f(x) = 3x^2 + 4x^2$ $f'(x) = 9x^2 + 8x$ $9x^2 + 8x = 17$ $9x^2 + 8x - 17 = 0$ $(9x + 17)(x - 1) = 0$ $x = -\frac{17}{9}$ or $x = 1$ $y = 3\left(-\frac{17}{9}\right)^2 + 4\left(-\frac{17}{9}\right)^2$ $= -5,95$ The other point is $(-1,89; -5,95)$	✓ derivative ✓ equate to 17 ✓ standard form ✓ factors ✓ x-value ✓ y-value (6)

10.3	 OR $x\text{-intercepts}$ $0 = 3x^3 + 4x^2$ $0 = x^2(3x + 4)$ $x = 0 \text{ or } x = -\frac{4}{3}$ $y\text{-intercepts: } (0; 0)$ Turning point $0 = 9x^2 + 8x$ $x(9x + 8) = 0$ $x = 0 \text{ or } x = -\frac{8}{9}$ $y = 3\left(-\frac{8}{9}\right)^2 + 4\left(-\frac{8}{9}\right)^2$ $= \frac{256}{243} \text{ or } 1,05$ $(0; 0) \text{ or } \left(-\frac{8}{9}; \frac{256}{243}\right)$ (If no sketch – award 3 marks for correct calculations)	✓ shape ✓ x-intercepts ✓ y-intercept ✓ turning points ✓ x-intercepts ✓ y-intercepts ✓ turning points (4)
10.4	$f(x) = 3x^3 + 4x^2$ $f'(x) = 9x^2 + 8x$ $f''(x) = 18x + 8$ $18x + 8 > 0$ $x > -\frac{8}{18}$ OR $x > -\frac{4}{9}$	✓ $f''(x) = 18x + 8$ ✓ $18x + 8 > 0$ ✓ answer (3)

QUESTION 11		
11.1	$s(t) = 6000 - 600t - 0,2t^3 + 2 \times 10^{-3}t^5$ $s'(t) = -600 - 0,6t^2 + 10^{-2}t^4$ $s'(0) = -600 - 0,6(0)^2 + 10^{-2}(0)^4$ $s'(0) = -600$ Thus meteorite is approaching at 600 m/s	✓ derivative ✓ substitute with 0 ✓ answer (600 m/s) (3)
11.2	$s(t) = 6000 - 600t - 0,2t^3 + 2 \times 10^{-3}t^5$ $s(10) = 6000 - 600(10) - 0,2(10)^3 + 2 \times 10^{-3}(10)^5$ $= 0$ The meteor will be 0 m away from the planet, thus hitting it.	✓ substitution ✓ answer (2)
11.3	$s''(t) = -1,2t + 4 \times 10^{-2}t^3$ $s''(5) = -1,2(5) + 4 \times 10^{-2}(5)^3$ $= -1 \text{ m/s}^2$	✓ second derivative ✓ substitution ✓ answer (3)
[8]		
QUESTION 12		
12.1	No. $P(A \cap B) \neq 0$ OR $P(A \cap B) = 0,1596$	✓ No ✓ $P(A \cap B) \neq 0$ OR ✓ $P(A \cap B) = 0,1596$ (2)
12.2	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $= 0,38 + 0,42 - 0,16$ $= 0,64$	✓ substitution ✓ answer (2)
12.3	$n(S) = \frac{456}{0,38}$ $= 1200$	✓ substitution ✓ answer (2)
12.4	$n(C') = 1200(0,72)$ $= 864$	✓ substitution ✓ answer (2)
[8]		

QUESTION 13		
13.1	$P(E) = \frac{n(E)}{n(S)}$ $= \frac{3!}{5!}$ $= \frac{1}{20}$ $= 0,05$	✓ 3! ✓ 5! ✓ answer (3)
13.2	Children $7!$ Grandparents $2!$ Parents $2! \times 2! \times 2! \times 3! = 48$ Number of arrangements $7! \times 2! \times 48$ $= 483840$	✓ 7! ✓ 2! ✓ 48 ✓ answer (4)
[7]		
TOTAL: 150		

PREPARATORY EXAMINATION

2018

MARKING GUIDELINES

10611 MATHEMATICS (PAPER 1)

14 pages

QUESTION 1		
1.1		
1.1.1	$x^2 - x - 30 = 0$ $(x-6)(x+5) = 0$ $x = 6$ or $x = -5$	✓ correct factors ✓ both answers correct (2)
1.1.2	$3x^2 - 8x - 4 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(-4)}}{2(3)}$ $x = 3,10$ or $x = -0,43$	✓ standard form ✓ correct subst. in the correct formula ✓ $x = 3,10$ ✓ $x = -0,43$ Penalise for incorrect rounding off here only (4)
1.1.3	$\sqrt{5-x} - x = 1$ $\sqrt{5-x} = 1+x$ $(\sqrt{5-x})^2 = (1+x)^2$ $x^2 + 3x - 4 = 0$ $(x+4)(x-1) = 0$ $x = -4$ or $x = 1$ NA	✓ isolate square root ✓ square both sides ✓ standard form ✓ both answers correct ✓ choose correct answer (5)
1.1.4	$\frac{6x^2 - 3x}{3} \leq 3x^2$ $2x^2 - x \leq 3x^2$ $2x^2 - 3x^2 - x \leq 0$ $-x^2 - x \leq 0$ $-x(x+1) \leq 0$ $x \geq 0$ or $x \leq -1$ OR	✓ simplify ✓ standard form ✓ correct factors ✓ critical values ✓ correct inequalities

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	$\frac{6x^2 - 3x}{3} \leq 3x^2$ $6x^2 - 3x \leq 9x^2$ $6x^2 - 9x^2 - 3x \leq 0$ $-3x^2 - 3x \leq 0$ $3x(x+1) \geq 0$ $x \geq 0$ or $x \leq -1$	✓ simplify ✓ standard form ✓ correct factors ✓ critical values ✓ correct inequalities (5)
1.1.5	$2x^2 + 7\sqrt{2}x = 2$ $2^2 \cdot 2^x + 7 \cdot 2^{\frac{x}{2}} - 2 = 0$ $4 \cdot 2^x + 7 \cdot 2^{\frac{x}{2}} - 2 = 0$ $(4 \cdot 2^{\frac{x}{2}} - 1)(2^{\frac{x}{2}} + 2) = 0$ $2^{\frac{x}{2}} = \frac{1}{4}$ or $2^{\frac{x}{2}} = -2$ $2^{\frac{x}{2}} = 2^{-2}$ NA $\frac{x}{2} = -2$ $x = -4$	✓ standard form ✓ factors ✓ $2^{\frac{x}{2}} = 2^{-2}$ ✓ NA ✓ $x = -4$ (5)
1.2	$6x^2 + 2px - 3x - p = 0$ $6x^2 + x(2p-3) - p = 0$ $\Delta = b^2 - 4ac$ $\Delta = (2p-3)^2 - 4(6)(-p)$ $= 4p^2 + 12p + 9$ $= (2p+3)^2$ $\Delta = \text{perfect square}$ $\therefore \text{rational roots}$	✓ coefficient of x ✓ correct substitution in correct formula ✓ factors ✓ $\therefore \Delta = \text{perfect square}$ (conclusion) (4)

[25]

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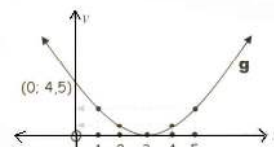
QUESTION 2		
2.1	$T_n = a + (n-1)d$ $-239 = 6 + (n-1)(-5)$ $-239 = 6 - 5n + 5$ $250 = 5n$ $n = 50$	✓ $T_n = -239$ ✓ $d = -5$ ✓ answer (3)
2.2	$T_3 = ar^2 = 18 \dots \textcircled{1}$ $T_5 = ar^4 = 162 \dots \textcircled{2}$ $\textcircled{2} \div \textcircled{1}: r^2 = 9 \ (r < 0)$ $r = -3$ $ar^2 = 18$ $a(9) = 18$ $a = 2$ $S_r = \frac{a(r^r - 1)}{r - 1}$ $S_r = \frac{2((-3)^r - 1)}{-3 - 1}$ $= \frac{2(-2187 - 1)}{-4}$ $= 1094$	✓ eq① ✓ eq② ✓ $r = -3$ ✓ $a = 2$ ✓ correct substitution in correct formula ✓ answer (6)
2.3	3; x; 11; 21; 35 x-3; 11-x; 10; 14 $-2x = 4 - 14$ $-2x = -10$ $x = 5$	✓ first difference d_1 ✓ equating second difference ✓ answer (3)

2.4	$S_8 : S_4 = 97:81$ $\frac{S_8}{S_4} = \frac{97}{81}$ $\frac{a(1-r^8)}{1-r} \div \frac{a(1-r^4)}{1-r} = \frac{97}{81}$ $\frac{a(1-r^8)}{1-r} \times \frac{1-r}{a(1-r^4)} = \frac{97}{81}$ $\frac{(1-r^8)}{(1-r^4)} = \frac{97}{81}$ $\frac{(1-r^4)(1+r^4)}{(1-r^4)} = \frac{97}{81}$ $1+r^4 = \frac{97}{81}$ $r^4 = \frac{16}{81}$ $r^4 = \left(\frac{2}{3}\right)^4$ $r = \left(\frac{2}{3}\right)$ $\therefore r = \left(\frac{2}{3}\right)$ First three terms: 9; 6; 4	$\checkmark$ $\frac{a(1-r^8)}{1-r} \div \frac{a(1-r^4)}{1-r} = \frac{97}{81}$ $\checkmark$ simplify $\frac{(1-r^8)}{(1-r^4)} = \frac{97}{81}$ $\checkmark$ factors $\checkmark r^4 = \left(\frac{2}{3}\right)^4$ $\checkmark r = \left(\frac{2}{3}\right)$ $\checkmark$ first three terms	(6)
2.5.1	$2(p-5)+2(p-5)^2+2(p-5)^3+\dots$ $r=p-5$ Convergent: $-1 < r < 1$ $-1 < p-5 < 1$ $4 < p < 6$	$\checkmark r = p-5$ $\checkmark -1 < p-5 < 1$ $\checkmark 4 < p < 6$	(3)
2.5.2	$S_\infty = \frac{a}{1-r}$ $= \frac{-1}{1-(-\frac{1}{3})}$ $= -\frac{2}{3}$	$\checkmark$ substitute $a = -1$ $\checkmark r = -\frac{1}{3}$ $\checkmark$ answer	(3)
[24]			

QUESTION 3			
3.1	$A = P(1-i)^n$ $100\,000 = 134\,000(1-0,068)^n$ $\frac{50}{67} = (0,932)^n$ $n = \log_{0,932}\left(\frac{50}{67}\right)$ $n = 4,16 \text{ years}$	$\checkmark$ correct formula $\checkmark$ correct substitution $\checkmark n = \log_{0,932}\left(\frac{50}{67}\right)$ $\checkmark$ answer	(4)
3.2.1	$A = P(1+i)^n$ $A = 150\,000\left(1 + \frac{0,1525}{12}\right)^2$ $= R153\,836,73$ $P = \frac{x[1-(1+i)^{-n}]}{i}$ $153\,836,73 = \frac{x\left[1 - \left(1 + \frac{0,1525}{12}\right)^{-24}\right]}{\frac{0,1525}{12}}$ $x = R7\,477,31$ $\therefore$ The monthly payments are R7 477,31 per month	$\checkmark n = 2$ $\checkmark i = \frac{0,1525}{12}$ $\checkmark$ correct formula and substitute for P $\checkmark n = -24$ $\checkmark$ answer	(5)
3.2.2	$P = \frac{x[1-(1+i)^{-n}]}{i}$ $7\,477,31 \left[1 - \left(1 + \frac{0,1525}{12}\right)^{-6}\right]$ $P = \frac{0,1525}{12}$ $= R42\,934,09$ OR	$\checkmark$ substitute in correct formula $\checkmark i = \frac{0,1525}{12}$ $\checkmark n = -6$ $\checkmark$ answer	

	$F = \frac{x[1-(1+i)^{-n}]}{i}$ $7\,477,31 \left[1 - \left(1 + \frac{0,1525}{12}\right)^{-18}\right]$ $F = \frac{0,1525}{12}$ $F = R150\,164,35$ $A = P(1+i)^n$ $A = 153\,836,73 \left(1 + \frac{0,1525}{12}\right)^{18}$ $A = R193\,098,52$ $\therefore$ The outstanding balance is R42 934,17	$\checkmark n = -18$ $\checkmark i = \frac{0,1525}{12}$ $\checkmark 153\,836,73 \left(1 + \frac{0,1525}{12}\right)^{18}$ $\checkmark$ answer	(4)
[13]			
QUESTION 4			
4.1	$y = -8$	$\checkmark$ answer	(1)
4.2	$f(x) = 0$ $2^x - 8 = 0$ At P: $2^x = 8$ $2^x = 2^3$ $x = 3$ $P(3; 0)$ $x = 0$ At D: $f(x) = 2^x - 8$ $= 1 - 8$ $= -7$ $D(0; -7)$	$\checkmark 2^x = 2^3$ $\checkmark P(3;0)$ $\checkmark 2^0 = 1$ $\checkmark$ answer D(0; -7)	(4)
4.3	$h(x) = f(2x) + 8$ $= 2^{2x} - 8 + 8$ $= 2^{2x}$ $h(x) = 2^{2x}$ OR $h(x) = 4^x$	$\checkmark$ substitution $\checkmark$ answer	(2)
4.4	$y = 4^x$ $h^{-1}: x = 4^y$ $y = \log_4 x$ OR	$\checkmark$ interchange x and y $\checkmark$ answer	

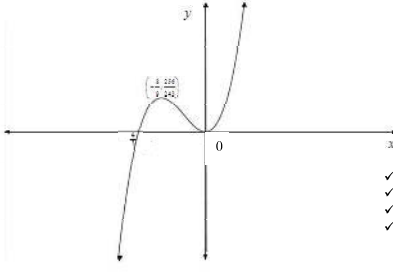
	$x = 2^{2y}$ $2y = \log_2 x$ $y = \frac{1}{2} \log_2 x$ $\log_2 \sqrt{x}$	$\checkmark$ interchange x and y $\checkmark$ answer	(2)
4.5	$y \in R$ OR $y \in (-\infty; \infty)$	$\checkmark$ answer	(1)
4.6	$y = a(x-p)^2 + q$ $y = a(x-3)^2$ $4,5 = a(0-3)^2$ $\frac{1}{2} = a$ $y = \frac{1}{2}(x^2 - 6x + 9)$ OR $y = \frac{1}{2}(x-3)^2$ OR $y = \frac{1}{2}x^2 - 3x + 4,5$	$\checkmark$ substitution of points D and P $\checkmark$ value of a $\checkmark$ equation (in any form)	(3)
4.7	By symmetry: $g(1) = g(5)$ and $g(2) = g(4)$ $\sum_{k=1}^5 g(k) - \sum_{k=1}^5 g(k)$ $= [g(0) + g(1) + g(2) + g(3)] - [g(4) + g(5)]$ $= g(0) + [g(1) - g(5)] + [g(2) - g(4)] + g(3)$ $= 4,5 + 0 + 0 + 0$ $= 4,5$	$\checkmark$ correct deduction $\checkmark$ expansion $\checkmark$ answer	(3)



4.8	Any upwards translation will result in non-real roots.	✓ answer (1)
QUESTION 5		
5.1	$x = -2$ or $x = 4$	✓ $x = -2$ ✓ $x = 4$ (2)
5.2	$x < -2$ or $-2 < x \leq 6$ OR $x \leq 6; x \neq -2$	✓ $x < -2$ ✓ $-2 < x \leq 6$ OR ✓ $x \leq 6$ ✓ $x \neq -2$ (2)
5.3	$-2 \leq x \leq 2$	✓ correct critical values ✓ correct inequalities (2)
QUESTION 6		
6.1	$g(x) = \frac{1}{4}x^2$	✓ answer (1)
6.2	$h(x) = \frac{1}{4}(x)^2 - 2$	✓ answer (1)
6.3	$y \geq -2$ OR $y \in [-2; \infty)$	✓ answer (correct inequality / bracket) (1)
QUESTION 7		
7.1	$x = 2$	✓ answer (1)
7.2	Translation of 2 units to the right and 3 units downwards.	✓ 2 units right ✓ 3 units down (2)
7.3	$x \in R; x \neq 3$	✓ answer (1)
7.4	$y = x + c$ $0 = 2 + c$ $-2 = c$ $y = x - 2$	✓ correct gradient (m=1) ✓ substitution C(2; 0) ✓ equation (3)

QUESTION 8		
8.1	$f(x+h) = 1 - 3(x+h)^2$ $= 1 - 3(x^2 + 2xh + h^2)$ $= 1 - 3x^2 - 6xh - 3h^2$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{1 - 3x^2 - 6xh - 3h^2 - (1 - 3x^2)}{h}$ $= \lim_{h \rightarrow 0} \frac{1 - 3x^2 - 6xh - 3h^2 - 1 + 3x^2}{h}$ $= \lim_{h \rightarrow 0} \frac{-6xh - 3h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(-6x - 3h)}{h}$ $= -6x$	✓ $f(x+h) = 1 - 3x^2 - 6xh - 3h^2$ ✓ correct substitution into correct formula ✓ simplify $(-6xh - 3h^2)$ ✓ common factor h ✓ $-6x$ (5)
8.2	$f'(2) = -6(2)$ $= -12$	✓ substitute $x = 2$ ✓ answer (2)
QUESTION 9		
In this question, penalise learners ONCE for incorrect notation.		
9.1.	$\frac{d}{dt} [(t-2)(t+3)]$ $= \frac{d}{dt} [t^2 + t - 6]$ $= 2t + 1$	✓ correct quadratic expression ✓ 2t ✓ 1 (3)
9.2	$D_x \left[\frac{5x^3 - 4}{x} \right]$ $= D_x \left[5x^2 - \frac{4}{x} \right]$ $= D_x [5x^2 - 4x^{-1}]$ $= 10x + 4x^{-2}$	✓ simplification ✓ 10x ✓ $4x^{-2}$ (3)

QUESTION 10		
10.1	$f(x) = ax^3 + bx^2$ $7 = a + b \dots (1)$ $f'(x) = 3ax^2 + 2bx$ $17 = 3a(1)^2 + 2b(1)$ $17 = 3a + 2b \dots (2)$ from (1) $b = 7 - a$ Substitute in (2) $3a + 2(7 - a) = 17$ $3a + 14 - 2a = 17$ $a = 3$ $b = 4$ OR $f(x) = ax^3 + bx^2$ $7 = a + b$ $f'(x) = 3ax^2 + 2bx$ $17 = 3a(1)^2 + 2b(1)$ $17 = 3a + 2b$ from (1) $a = 7 - b$ Substitute in (2) $3(7 - b) + 2b = 17$ $21 - 3b + 2b = 17$ $-b = -4$ $b = 4$ $a = 3$	✓ $7 = a + b$ ✓ derivative ✓ $17 = 3a + 2b$ ✓ $b = 7 - a$ ✓ $a = 3$ (must show calculation) ✓ $b = 4$ (must show calculation) ✓ $7 = a + b$ ✓ derivative ✓ $17 = 3a + 2b$ ✓ $a = 7 - b$ ✓ $b = 4$ (must show calculation) ✓ $a = 3$ (must show calculation) (6)
10.2	$f(x) = 3x^2 + 4x^2$ $f'(x) = 9x^2 + 8x$ $9x^2 + 8x = 17$ $9x^2 + 8x - 17 = 0$ $(9x + 17)(x - 1) = 0$ $x = -\frac{17}{9}$ or $x = 1$ $y = 3\left(-\frac{17}{9}\right)^2 + 4\left(-\frac{17}{9}\right)^2$ $= -5,95$ The other point is $(-1,89; -5,95)$	✓ derivative ✓ equate to 17 ✓ standard form ✓ factors ✓ x-value ✓ y-value (6)

10.3	 OR $x\text{-intercepts}$ $0 = 3x^3 + 4x^2$ $0 = x^2(3x + 4)$ $x = 0 \text{ or } x = -\frac{4}{3}$ $y\text{-intercepts: } (0; 0)$ Turning point $0 = 9x^2 + 8x$ $x(9x + 8) = 0$ $x = 0 \text{ or } x = -\frac{8}{9}$ $y = 3\left(-\frac{8}{9}\right)^2 + 4\left(-\frac{8}{9}\right)^2$ $= \frac{256}{243} \text{ or } 1,05$ $(0; 0) \text{ or } \left(-\frac{8}{9}; \frac{256}{243}\right)$ (If no sketch – award 3 marks for correct calculations)	✓ shape ✓ x-intercepts ✓ y-intercept ✓ turning points ✓ x-intercepts ✓ y-intercepts ✓ turning points (4)
10.4	$f(x) = 3x^3 + 4x^2$ $f'(x) = 9x^2 + 8x$ $f''(x) = 18x + 8$ $18x + 8 > 0$ $x > -\frac{8}{18}$ OR $\frac{4}{9}$	✓ $f''(x) = 18x + 8$ ✓ $18x + 8 > 0$ ✓ answer (3)

QUESTION 11		
11.1	$s(t) = 6000 - 600t - 0,2t^3 + 2 \times 10^{-3}t^5$ $s'(t) = -600 - 0,6t^2 + 10^{-2}t^4$ $s'(0) = -600 - 0,6(0)^2 + 10^{-2}(0)^4$ $s'(0) = -600$ Thus meteorite is approaching at 600 m/s	✓ derivative ✓ substitute with 0 ✓ answer (600 m/s) (3)
11.2	$s(t) = 6000 - 600t - 0,2t^3 + 2 \times 10^{-3}t^5$ $s(10) = 6000 - 600(10) - 0,2(10)^3 + 2 \times 10^{-3}(10)^5$ $= 0$ The meteor will be 0 m away from the planet, thus hitting it.	✓ substitution ✓ answer (2)
11.3	$s''(t) = -1,2t + 4 \times 10^{-2}t^3$ $s''(5) = -1,2(5) + 4 \times 10^{-2}(5)^3$ $= -1 \text{ m/s}^2$	✓ second derivative ✓ substitution ✓ answer (3)
[8]		
QUESTION 12		
12.1	No. $P(A \cap B) \neq 0$ OR $P(A \cap B) = 0,1596$	✓ No ✓ $P(A \cap B) \neq 0$ OR ✓ $P(A \cap B) = 0,1596$ (2)
12.2	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $= 0,38 + 0,42 - 0,16$ $= 0,64$	✓ substitution ✓ answer (2)
12.3	$n(S) = \frac{456}{0,38}$ $= 1\,200$	✓ substitution ✓ answer (2)
12.4	$n(C') = 1\,200(0,72)$ $= 864$	✓ substitution ✓ answer (2)
[8]		

QUESTION 13		
13.1	$P(E) = \frac{n(E)}{n(S)}$ $= \frac{3!}{5!}$ $= \frac{1}{20}$ $= 0,05$	✓ 3! ✓ 5! ✓ answer (3)
13.2	Children $7!$ Grandparents $2!$ Parents $2! \times 2! \times 2! \times 3! = 48$ Number of arrangements $7! \times 2! \times 48$ $= 483\,840$	✓ 7! ✓ 2! ✓ 48 ✓ answer (4)
[7]		
TOTAL: 150		

PREPARATORY EXAMINATION

2019

MARKING GUIDELINES

MATHEMATICS (PAPER 1) (10611)

20 pages

QUESTION 1		
1.1.1	$2x^2 + 3 = 8x$ $2x^2 - 8x + 3 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4(ac)}}{2(a)}$ $x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(3)}}{2(2)}$ $x = 3,58 \quad \text{or} \quad x = 0,42$ Penalise for rounding-off in this question ONLY.	✓ standard form ✓ correct substitution into correct formula ✓✓ answers (4)
1.1.2	$4x - 2x(x-3) \leq 0$ $4x - 2x^2 + 6x \leq 0$ $2x^2 - 10x \geq 0$ $2x(x-5) \geq 0$ $x \geq 5 \quad \text{or} \quad x \leq 0$ Both inequalities MUST be correct for allocating the marks.	✓ standard form ✓ factors ✓✓ answers (4)

2

10611/19

1.1.3	$2^x - 5 \cdot 2^{x+1} = -144$ $2^x(1 - 5 \cdot 2) = -144$ $2^x(1 - 10) = -144$ $2^x(-9) = -144$ $2^x = 16$ OR $x = \log_2 16$ $2^x = 2^4$ $x = 4$	✓ factorise ✓ simplification (-9) ✓ simplification (2^4) OR correct use of logs ✓ answer (4)
1.2	$f(x) = (x-2)(x+6)$ $f(x) = x^2 + 4x - 12$	✓ both correct factors ✓ answer (2)
1.3	$2x + y = 17 \dots\dots\dots(1)$ $xy = 8 \dots\dots\dots(2)$ from (1) $2x + y = 17$ $y = 17 - 2x \dots\dots\dots(3)$ substitute (3) into (2) $x(17 - 2x) = 8$ $17x - 2x^2 = 8$ $-2x^2 + 17x - 8 = 0$ $2x^2 - 17x + 8 = 0$ $(2x-1)(x-8) = 0$ $x = \frac{1}{2}$ OR $x = 8$ substitute into (3)	✓ solve for y in equ. 1 ✓ substitute for y in equ. 2 ✓ standard form ✓ factors ✓ both x answers

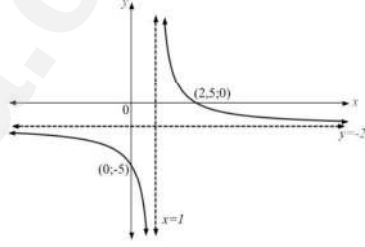
3

10611/19

$y = 17 - 2\left(\frac{1}{2}\right)$ $y = 16$ OR $y = 17 - 2(8)$ $y = 1$ OR	✓ both y answers OR
$2x + y = 17 \dots\dots\dots(1)$ $xy = 8 \dots\dots\dots(2)$ from (2) $xy = 8$ $y = \frac{8}{x} \dots\dots\dots(3)$ substitute (3) into (1) $2x + \frac{8}{x} = 17$ $2x^2 - 17x + 8 = 0$ $(2x-1)(x-8) = 0$ $x = \frac{1}{2}$ OR $x = 8$ substitute into (2) $\frac{1}{2}y = 8$ OR $8y = 8$ $y = 16$ $y = 1$	✓ solve for y in equ. 2 ✓ substitute for y in equation 1 ✓ standard form ✓ factors ✓ both x answers ✓ both y answers

4

	= R39 206,20	
4.2.3	$P_n = \frac{x[1-(1+i)^{-n}]}{i}$ $39205,67 = \frac{x[1-(1+\frac{0,18}{12})^{-180}]}{\frac{0,18}{12}}$ $x = R631,38$ NOTE: If the candidate uses R39 206,20 then the answer is R631,39.	✓ $n = 180$ ✓ $i = \frac{0,18}{12}$ ✓ correct substitution into correct formula ✓ correct answer (4)
		[16]

QUESTION 5		
5.1	$f(x) = \frac{3}{x-1} - 2$ $0 = \frac{3}{x-1} - 2$ $2(x-1) = 3$ $2x - 2 = 3$ $2x = 5$ $x = \frac{5}{2}$ $\therefore (\frac{5}{2}; 0)$ NOTE: Must be in coordinate form.	✓ equate to 0 ✓ answer (2)
5.2	$f(0) = \frac{3}{0-1} - 2$ $y = -5$ $\therefore (0; -5)$ NOTE: Not necessarily in coordinate form.	✓ answer (1)
5.3		✓ shape ✓ x- and y-intercepts ✓ correct asymptotes (3)

5.4	$f(x) = \frac{3}{x-1} - 2$ Point of intersection (1; -2) $y = -x + q$ $-2 = -(1) + q$ $q = -1$ $\therefore y = -x - 1$	✓ $m = -1$ ✓ substitute point (1; -2) ✓ answer (3)
		[9]

QUESTION 6		
6.1	$f(x) = -2x^2 - 5x + 3$ $x = \frac{-(-5)}{2(-2)}$ $x = -\frac{5}{4}$ $f(-\frac{5}{4}) = -2(-\frac{5}{4})^2 - 5(-\frac{5}{4}) + 3$ $f(-\frac{5}{4}) = -\frac{49}{8}$ $\therefore TP(-\frac{5}{4}; -\frac{49}{8})$ OR $f'(x) = 0$ $-4x - 5 = 0$ $x = -\frac{5}{4}$ $f(-\frac{5}{4}) = -2(-\frac{5}{4})^2 - 5(-\frac{5}{4}) + 3$ $f(-\frac{5}{4}) = -\frac{49}{8}$ $\therefore TP(-\frac{5}{4}; -\frac{49}{8})$ OR $f(x) = -2x^2 - 5x + 3$ $f(x) = -2[x^2 + \frac{5}{2}x - \frac{3}{2}]$ $f(x) = -2[x^2 + \frac{5}{2}x + (\frac{5}{4})^2 - (\frac{5}{4})^2 - \frac{3}{2}]$ $f(x) = -2[(x + \frac{5}{4})^2 - \frac{49}{16}]$ $f(x) = -2(x + \frac{5}{4})^2 + \frac{49}{8}$ $\therefore TP(-\frac{5}{4}; -\frac{49}{8})$	✓ correct substitution into $x = \frac{-b}{2a}$ ✓ x-value ✓ y-value OR ✓ derivative ✓ equate to zero ✓ y-value OR ✓ determine $\frac{25}{16}$ or $(\frac{5}{4})^2$ ✓ factorisation $(x + \frac{5}{4})^2$ ✓ y-value (3)

6.2	$y \in (-\infty; \frac{49}{8}]$ OR $y \leq \frac{49}{8}$	✓ answer (1)
6.3	$\tan 135^\circ = -1$ $m = -1$ $f'(x) = -1$ $-4x - 5 = -1$ $\therefore x = -1$ $f(-1) = -2(-1)^2 - 5(-1) + 3$ $y = 6$ $\therefore P(-1; 6)$	✓ $m = -1$ ✓ $f'(x) = -1$ ✓ $x = -1$ ✓ $y = 6$ (4)
6.4	$k < -\frac{49}{8}$ OR $k > -\frac{49}{8}$	✓ answer ✓ answer (2) [10]

QUESTION 7		
7.1	$f(x) = a^x$ $\frac{1}{4} = a^2$ $\sqrt{\frac{1}{4}} = a$ $\frac{1}{2} = a$	✓ correct substitution ✓ square root (2)
7.2	$y = \left(\frac{1}{2}\right)^x$ $x = \left(\frac{1}{2}\right)^y$ $y = \log_{\frac{1}{2}} x$	✓ interchange x and y ✓ answer (2)
7.3	$y = \left(\frac{1}{2}\right)^x$ $-y = \left(\frac{1}{2}\right)^x$ $h(x) = -\left(\frac{1}{2}\right)^x$ OR $h(x) = -a^x$ OR $h(x) = -f(x)$	✓ answer (1)
7.4	$x \leq 0$ OR $x > 0$ OR $x < 0$ OR $x \geq 0$	✓✓ answer (2)
		[7]

QUESTION 8			
8.1	8.1.1	$x < 2$	✓✓ answer (2)
	8.1.2	$0 < x \leq 1$	✓ critical values ✓ answer in correct notation (2)
8.2	8.2.1	$g(x) = \log_2 x$ $y = \log_2 x$ $\therefore x = 2^y$ $\therefore y = 2^x$ $\therefore g^{-1}(x) = 2^x$	✓ interchange x and y ✓ answer (2)
	8.2.2	$\log_2(3-x) = x$ $2^x = 3-x$ therefore point of intersection of g^{-1} and f	✓ g^{-1} (inverse) ✓ point of intersection (2)
	8.2.3	$x = 1$	✓ answer (1)
			[9]

QUESTION 9			
9.1	9.1.1	$f(x) = 3x - x^2$ $f(x+h) = 3(x+h) - (x+h)^2$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{3(x+h) - (x+h)^2 - (3x - x^2)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{3x + 3h - x^2 - 2xh + h^2 - 3x + x^2}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{3h - 2xh + h^2}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{h(3 - 2x + h)}{h}$ $f'(x) = 3 - 2x$ NOTE: Answer only $\left(\frac{0}{5}\right)$	✓ $f(x+h)$ ✓ correct substitution ✓ simplification ✓ common factor h ✓ correct answer (5)
	9.1.2	$f(1) = 2$ $f(3) = 0$ $\frac{f(3) - f(1)}{2}$ $= \frac{0 - 2}{2}$ $= -1$	✓ $f(1)$ and $f(3)$ both correct ✓ substitution ✓ answer (3)
9.2	9.2.1	$y = \frac{8 - 3x^6}{8x^5}$ $y = x^{-6} - \frac{3}{8}x$ $\frac{dy}{dx} = -5x^{-6} - \frac{3}{8}$	✓ simplification ✓ $-5x^{-6}$ ✓ $-\frac{3}{8}$ (3)

9.2.2	$D_x \left[\sqrt{x^2 + \frac{1}{x}} + 2x \right]$ $D_x \left[x^{\frac{1}{2}} + x^{-1} + 2x \right]$ $D_x \left[\frac{2}{3}x^{\frac{1}{3}} - x^{-1} + 2 \right]$	✓ simplification ✓ $\frac{2}{3}x^{-\frac{1}{3}}$ ✓ $-x^{-2}$ ✓ 2	(4)
[15]			

QUESTION 10			
10.1		✓ shape ✓ x and y-intercepts ✓ turning points	(3)
10.2	$f(x) = a(x-1)^2(x-4)$ $8 = a(3-1)^2(3-4)$ $8 = a(-4)$ $a = -2$ $f(x) = -2(x-1)^2(x-4)$ $f(x) = -2(x^2 - 2x + 1)(x-4)$ $f(x) = -2x^3 + 12x^2 - 18x + 8$	✓ substitution ✓ $a = -2$ ✓ substitution ✓ simplification	(4)
10.3	$f(x) = -2x^3 + 12x^2 - 18x + 8$ $f'(x) = -6x^2 + 24x - 18$ $f''(x) = -12x + 24$ $f''(x) < 0$ $-12x + 24 < 0$ $-x < -2$ $x > 2$	✓ second derivative ✓ $f''(x) < 0$ ✓ answer	(3)
[10]			

QUESTION 11			
11.1	$P = \pi r + 4x + 2l$ $32 = \pi \cdot 2x + 4x + 2l$ $16 = \pi x + 2x + l$ $l = 16 - \pi x - 2x$	✓ correct formula ✓ correct substitution ✓ answer	(3)
11.2	$A = 4x/2 + \frac{1}{2}\pi(2x)^2$ $A = 4x[16 - \pi x - 2x] + 2\pi x^2$ $= -8x^2 - 2\pi x^2 + 64x$	✓ correct formula ✓ correct substitution and simplification	(2)
11.3	$A'(x) = 0$ $-16x - 4\pi x + 64 = 0$ $4x + \pi x - 16 = 0$ $x(4 + \pi) = 16$ $x = \frac{16}{4 + \pi}$ $x = 2,24m$ OR $x = \frac{-64}{2(-8 - 2\pi)}$ $x = 2,24$	✓ derivative ✓ derivative = 0 ✓ answer OR ✓ sub. a and b into A.o.S. formula ✓ answer	(3)
[8]			

QUESTION 12				
12.1		$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $0,6 = 0,3 + 0,4 - P(A \text{ and } B)$ $P(A \text{ and } B) = 0,1$	✓ substitution ✓ answer	(2)
12.2	12.2.1	$A = 32$ $B = 48$	✓ answer ✓ answer	(2)
	12.2.2	$P(\text{iPhone}) = \frac{101}{166}$ $= 0,61$	✓✓ $\frac{101}{166}$	(2)
	12.2.3	$P(\text{iPhone/Gr 12}) = \frac{48}{101}$ $= 0,48$	✓✓ $\frac{48}{101}$	(2)
				[8]

QUESTION 13			
13.1	$9! = 362\,880$ OR $9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 362\,880$	✓✓ 9! OR ✓✓ $9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$	(2)
13.2	$4! \times 3! \times 2! \times 3! = 1\,728$	✓✓ $4! \times 3! \times 2! \times 3!$ ✓ answer	(3)
13.3	From left to right and from right to left Therefore 2 ways NOTE: If answer is 1 way, award 1 mark ONLY	✓✓ answer	(2)
[7]			

PREPARATORY EXAMINATION 2020 MARKING GUIDELINES

MATHEMATICS P1 (10611)

17 pages

1

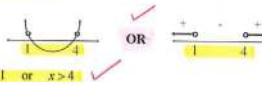
10611/20

1.1.4	$x - 3x^2 = 4$ $x - 4 = 3x^2$ $x^2 - 8x + 16 = 9$ $x^2 - 17x + 16 = 0$ $(x-16)(x-1) = 0$ $x = 16 \text{ OR } x = 1$ N/A Check solution b/c of the original eqn	- isolating $3\sqrt{x}$ - squaring both sides - standard form - factors - critical values - selection/rejection	(6)	
1.2	$2^{2x+1} + 7 \cdot 2^x - 4 = 0$ let $2^x = k$ $2k^2 + 7k - 4 = 0$ $(2k-1)(k+4) = 0$ $k = \frac{1}{2} \text{ OR } k = -4$ $\therefore 2^x = \frac{1}{2} \quad 2^x = -4$ $x = -1 \quad \text{Not a solution}$	$2^{2x+1} + 7 \cdot 2^x - 4 = 0$ $(2 \cdot 2^x + 7)(2^x - 1) = 0$ $2 \cdot 2^x + 7 = 0 \text{ OR } 2^x - 1 = 0$ $2^x = -\frac{7}{2} \quad 2^x = 1$ $x = -1$	- standard form - factors - critical values - rejection of 1 answer	(4)
1.3	$x = y - 13 \dots (1) \text{ and } \sqrt{2-x} = y - 3 \dots (2)$ sub (1) into (2) $\sqrt{2-(y-13)} = y - 3$ $\sqrt{15-y} = y - 3$ $15 - y = y^2 - 6y + 9$ $0 = y^2 - 5y - 6$ $0 = (y-6)(y+1)$ $y = 6 \text{ OR } y = -1$ $x = -7$ check solutions b/c of the original eqn	- substitution - square both sides - standard form - factors - y-values - x-value	OR	

3

10611/20

QUESTION 1

1.1.1	$3x^2 + 5x = 7$ $3x^2 + 5x - 7 = 0$ $x = \frac{-5 \pm \sqrt{5^2 - 4(3)(-7)}}{2(3)}$ $x = \frac{-5 \pm \sqrt{109}}{6}$ $x = 0,91$ or $x = -2,57$	standard form subst. into correct formula answers	(4)
1.1.2	$2x^2 = 9x + 5$ $2x^2 - 9x - 5 = 0$ $(2x+1)(x-5) = 0$ $x = -\frac{1}{2}$ OR $x = 5$ ANY OTHER VALID METHOD	standard form factors both answers correct	(3)
1.1.3	$x^2 - 5x + 4 > 0$ $x^2 - 5x + 4 > 0$ $(x-4)(x-1) > 0$ 	standard form factors critical values both correct answers	(4)

1.1.4	$x - 3x^2 = 4$ $x - 4 = 3x^2$ $(x-4)(x-1) = 0$ $x = 4$ OR $x = 1$ circle solution	standard form standard form factors/ formula critical values/ test rejection/ verifying squaring/ bracket answer statement	(8)
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2

10611/20

$x = y - 13$ and $\sqrt{2-x} = y - 3$ $\therefore y = x + 13 \dots (1)$ $\sqrt{2-x} = y - 3 \dots (2)$ Sub for y into equation 2 $\sqrt{2-x} = x + 13 - 3$ $\sqrt{2-x} = x + 10$ $2 - x = x^2 + 20x + 100$ $0 = x^2 + 21x + 98$ $0 = (x+14)(x+7)$ $x = -14$ OR $x = -7$ $y = -1$ OR $y = 6$	substitution square both sides standard form factors x-values y-values	(6)
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[27]

4

QUESTION 2

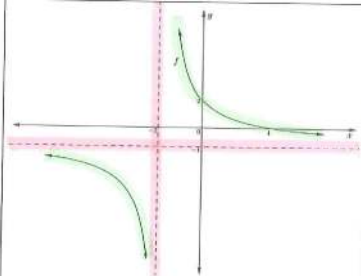
2.1.1	37 ✓	✓ answer	(1)
2.1.2	$\begin{array}{c} 1 \quad 7 \quad 15 \\ \quad \swarrow \quad \searrow \\ 6 \quad 8 \\ \quad \swarrow \quad \searrow \\ 2 \end{array}$ $2a = 2$ ✓ $a = 1$ ✓ $3a + b = 6$ ✓ $3(1) + b = 6$ ✓ $b = 3$ ✓ $a + b + c = 1$ ✓ $1 + 3 + c = 1$ ✓ $c = -3$ ✓ $T_n = n^2 + 3n - 3$ ✓	✓ second difference ✓ $a = 1$ ✓ $b = 3$ ✓ $c = -3$	(4)
2.1.3	$W_n = 2n + 4$ ✓ $50 = 2n + 4$ ✓ $46 = 2n$ ✓ $n = 23$ ✓ $T_{23} = 23^2 + 3(23) - 3$ ✓ $= 595$ ✓	✓ $W_n = 2n + 4$ ✓ equating to 50 ✓ $n = 23$ ✓ substitution ✓ answer	(5)
2.2.1	$T_{10} = 0$ ✓	✓ answer	(1)
2.2.2	$\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \dots$ 250 terms $d = 1$ ✓ $S_{250} = \frac{250}{2} \left[2 \left(\frac{1}{2} \right) + (250 - 1)(1) \right]$ ✓ $S_{250} = 31000$ ✓ $\therefore 0 + \frac{1}{2} + 0 + \frac{1}{2} + 0 + \frac{3}{2} \dots$ to 500 terms $S_{500} = 31000$ ✓	✓ value of d ✓ correct sub. into correct formula ✓ $S_{250} = 31\,000$ ✓ answer	(4)

QUESTION 3

3.1	$S_n > 31$ $16 \left(1 - \left(\frac{1}{2} \right)^n \right) > 31$ ✓ $1 - \left(\frac{1}{2} \right)^n > \frac{31}{16}$ ✓ $1 - \left(\frac{1}{2} \right)^n > \frac{31}{32}$ ✓ $-\left(\frac{1}{2} \right)^n > \frac{1}{32}$ ✓ $-\left(\frac{1}{2} \right)^n > -\left(\frac{1}{2} \right)^5$ ✓ $\therefore n > 5$ or $n \geq 6$ ✓ <i>n can't be 5 because then $S_5 = 31$ and S_n must be > 31 and $n \in \mathbb{N} \therefore n \geq 6$</i>	✓ correct substitution into correct formula ✓ simplification $\left(-\frac{1}{32} \right)$ ✓ correct answers	(3)
3.2	$S_n = \frac{a}{1-r}$ $S_n = \frac{16}{1 - \frac{1}{2}}$ ✓ $S_n = 32$ ✓	✓ correct substitution into correct formula ✓ answer	(2)

2.3	$r = 2 \left(\frac{1-k}{5} \right)$ ✓ $\therefore -1 < 2 \left(\frac{1-k}{5} \right) < 1$ ✓ $-\frac{1}{2} < \left(\frac{1-k}{5} \right) < \frac{1}{2}$ ✓ $-\frac{5}{2} < 1-k < \frac{5}{2}$ ✓ $-\frac{7}{2} < -k < \frac{3}{2}$ ✓ $\frac{3}{2} < k < \frac{7}{2}$ ✓ $\frac{1-k}{5} = \frac{1-k}{5}$ ✓ $1-k = 1-k$ ✓ $0 = 0$ ✓ $k \in \mathbb{R}$ ✓	✓ r ✓ $-1 < r < 1$ ✓ correct critical values ✓ answer	(4)
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QUESTION 4

4.1	$x = -2$ ✓ $y = 1$ ✓	✓ answer ✓ answer	(2)
4.2.1	$y = \frac{6}{x+2} - 1$ ✓ $y = 2$ ✓	✓ $x = 0$ ✓ answer	(2)
4.2.2	$0 = \frac{6}{x+2} - 1$ $1 = \frac{6}{x+2}$ $x+2 = 6$ $x = 4$ ✓	✓ $y = 0$ ✓ answer	(2)
4.3		✓ shape ✓ asymptotes ✓ x- and y- intercepts	(3)

4.4	$y = -(x+2) + 1$ $y = -x - 3$ OR $y = mx + c$ $-1 = m(-2) + c$ $-3 = c$ $\therefore y = -x - 3$	$m = -1$ $\checkmark$ correct subst. of point $(-2; -1)$ $\checkmark$ answer OR $m = -1$ $\checkmark$ correct subst. of point $(-2; -1)$ $\checkmark$ answer
	ANSWER ONLY NOT ACCEPTABLE	(3)
		[12]

QUESTION 6

6.1	$y = a(x-p)^2 + q$ Use the turning point $5 = a(0-2)^2 + 3$ for m $5 = 4a + 3$ $2 = 4a$ $\therefore a = \frac{1}{2}$ $y = \frac{1}{2}(x-2)^2 + 3$ $y = \frac{1}{2}(x^2 - 4x + 4) + 3$ Expand $y = \frac{1}{2}x^2 - 2x + 2 + 3$ $y = \frac{1}{2}x^2 - 2x + 5$	$\checkmark \checkmark$ sub. point A and B correctly $\checkmark$ value of a $\checkmark$ sub. a, p and q $\checkmark$ simplification
6.2	The graph does NOT cut the x-axis. $\therefore$ No real roots $\therefore \Delta < 0$	$\checkmark$ explanation $\checkmark \Delta < 0$
6.3	$\frac{1}{2}x^2 - 2x + 5 = k$ $\therefore k > 3$	$\checkmark \checkmark$ answer $\checkmark$ (Real)
6.4	$y = \frac{1}{2}x^2 - 2x + 5$ $y = \frac{1}{2}x^2 - 2x$	$\checkmark$ answer Shift point B $(0; 5)$ 5 units down. $\downarrow$ old y-int must become the new y-int.
		(1)
		[10]

QUESTION 5

5.1	$p(x) = \log_3 x$ $x = \log_3 y$ $\therefore p^{-1}: y = 3^x$	$\checkmark \checkmark$ answers
5.2		$\checkmark \checkmark$ shape p $\checkmark \checkmark$ shape p^{-1} $\checkmark \checkmark$ point on p^{-1}
5.3	$2 = \log_3 x$ $x = 9$ $0 < x \leq 9$	$\checkmark$ value of x $\checkmark$ answer
5.4	x -intercept of p is $(1; 0)$ x -intercept of h is $(-1; 0)$ When the input changed its a reflection about the y-axis.	$\checkmark \checkmark$ answer
		(2)
		[13]

$$\begin{aligned}
 y &= \log_3 x \\
 y &= \log_3 (-x) \\
 0 &= \log_3 (-x) \quad (x\text{-int: } y=0) \\
 3^0 &= -x \\
 1 &= -x \\
 \therefore x &= -1
 \end{aligned}$$

QUESTION 7

7.1	$1 + i_{\text{eff}} = \left(1 + \frac{0.11}{2}\right)^2$ We are charging compound s.h. to compound annually $i_{\text{eff}} = \left(1 + \frac{0.11}{2}\right)^2 - 1$ $i_{\text{eff}} = 11.305\%$ $\therefore$ Mary has secured the better rate.	$\checkmark$ correct sub. into correct formula $\checkmark$ answer $\checkmark$ conclusion
7.2.1	$10\,000 \left[\frac{1 - \left(1 + \frac{0.0772}{12}\right)^{144}}{\frac{0.0772}{12}} \right] - 1$ $= R1\,674\,501.44$	$\checkmark$ value of i $\checkmark$ value of n $\checkmark$ correct sub. into correct formula $\checkmark$ answer
7.2.2	$R1\,674\,501.44 = \frac{30\,000}{h} \left[1 - \left(1 + \frac{0.1}{12}\right)^n \right]$ Use Rv formula. $0.46513... = \left[1 - \left(1 + \frac{0.1}{12}\right)^n \right]$ $0.53486... = \left(1 + \frac{0.1}{12}\right)^n$ $\log_{\left(1 + \frac{0.1}{12}\right)} 0.53486... = n$ $n = 75.4$ Must be rounded down b/c 76 months will go beyond the amount. She will be able to receive the money in 75 full months.	$\checkmark$ subst. of P, x and i into correct formula $\checkmark$ simplification $\checkmark$ correct use of logs $\checkmark$ answer at P
7.2.3	$30\,000 \left[1 - \left(1 + \frac{0.1}{12}\right)^{75} \right] = \frac{0.1}{12}$ $Pv = R1\,319\,260.60$ $\therefore$ No	$\checkmark$ subst. x and i into correct formula $\checkmark$ correct value of n $\checkmark$ answer conclusion
		(4)
		[15]

1 Nov 2016 $\rightarrow$ 1 Oct 2025 = 12 payments per year $\rightarrow$ 9 years.
 1 Nov $\rightarrow$ 1 Apr = 6 payments $\rightarrow$ 1 Nov 2025 - 1 Apr 2026 = 6 months.

QUESTION 8

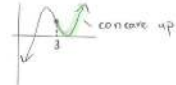
8.1	$f(x) = -2x^2 + 6x$ $f(x+h) = -2(x+h)^2 + 6(x+h)$ $f(x+h) = -2(x^2 + 2xh + h^2) + 6x + 6h$ $f(x+h) = -2x^2 - 4xh - 2h^2 + 6x + 6h$ ✓ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-2x^2 - 4xh - 2h^2 + 6x + 6h - (-2x^2 + 6x)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-4xh - 2h^2 + 6h}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{-4x - 2h + 6}{1}$ $f'(x) = -4x - 2h + 6$ $f'(x) = -4x + 6$ ✓ <i>You cannot use short hand at all</i>	✓ value of $f(x+h)$ ✓ correct sub. into correct formula ✓ factorise ✓ answer	(4)
8.2.1	$f(x) = 2x^2 + \frac{1}{2}x^{-1}$ $f'(x) = 4x + 2x^{-2}$ $f'(x) = 4x + \frac{2}{x^2}$ ✓	✓ $4x$ ✓ $2x^{-2}$	(2)
8.2.2	$f(x) = \frac{x^3 - 5x^2 + 4x}{x-4}$ $f(x) = \frac{x(x^2 - 5x + 4)}{x-4}$ ✓ $f(x) = \frac{x(x-4)(x-1)}{x-4}$ ✓ $f(x) = x(x-1)$ ✓ $f'(x) = 2x - 1$ ✓	✓ factorising ✓ factors ✓ simplification of f ✓ answer	(4)

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QUESTION 9

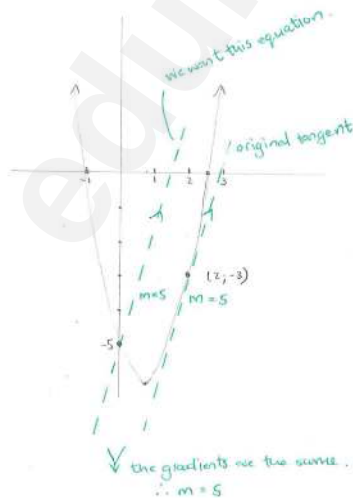
9.1	For $y = \frac{4}{x}$ the gradient of the tangent to the curve is - 1. $y = 4x^{-1}$ ✓ $\frac{dy}{dx} = -4x^{-2} = -\frac{4}{x^2}$ ✓ $-\frac{4}{x^2} = -1$ ✓ $4 = x^2$ $\therefore x = 2 \text{ OR } x = -2$ $\therefore y = -2 \text{ OR } y = 2$ $(-2, -2) \quad (2, 2)$	✓ exponential form ✓ derivative ✓ derivative = -1 ✓ x-values ✓ y-values	(5)
9.2.1	$y = (x-1)(x-4)$ ✓ $y = (x-1)(x^2 - 4x + 16)$ ✓ $y = x^3 - 9x^2 + 24x - 16$ ✓ <i>NB! $f'(x) = f'(u) = 0$ Are there roots $\therefore (1;0)$ and $(4;0)$ it's also a T.F.</i>	✓ $(x-1)(x-4)^2$ ✓ squaring binomial	(2)
9.2.2	$y = x^3 - 9x^2 + 24x - 16$ $\frac{dy}{dx} = 3x^2 - 18x + 24 = 0$ ✓ $x^2 - 6x + 8 = 0$ ✓ $(x-2)(x-4) = 0$ ✓ $x = 2 \text{ OR } x = 4$ ✓ $y = 4 \text{ OR } y = 0$ ✓ $B(2; 4)$ ✓	✓ derivative = 0 ✓ factors ✓ y-values ✓ coordinates of B	(4)
9.2.3	$k < -16$ ✓ <i>At the moment the function has 2 positive roots and $k < 0$. BUT when k goes through the y-int of the graph will have only one root. $k < -16$ i.e. less than the y-int graph will cut on the y-axis.</i>	✓ answer	(2)
9.2.4	$f''(x) = 6x - 18$ $6x - 18 > 0$ $x > 3$ ✓ <i>Second derivative = 0 $6x - 18 = 0$ $6x = 18$ $x = 3$ This is the point of inflection.</i>	✓ $6x - 18$ ✓ answer	(2)

Extending to the right of this point it will be concave up.



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8.3	$y = 2x^2 - 3x - 5$ $\frac{dy}{dx} = 4x - 3$ ✓ gradient at $x = 2$ $y = 2x^2 - 3x - 5$ $m = 4(2) - 3$ $m = 5$ $y = 5x - 5$ ✓ <i>NB! we have the y-int from original equation.</i>	✓ derivative ✓ substitution ✓ value of m ✓ answer	(4)
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QUESTION 10

10.1	$P(6-x; (6-x)^2)$ ✓ <i>Replace $(6-x)$ into $y = x^2$ $\therefore y = (6-x)^2$</i>	✓ answer	(1)
10.2	 $P(6-x; (6-x)^2)$ $Q(6-x; 0)$ $R(6; 0)$ $S(6; (6-x)^2)$ $A = l \times b$ $A = x[(6-x)^2]$ ✓ $A = x(36 - 12x + x^2)$ $A = x^3 - 12x^2 + 36x$ ✓ $\frac{dA}{dx} = 3x^2 - 24x + 36$ ✓ $x^2 - 8x + 12 = 0$ ✓ $(x-6)(x-2) = 0$ ✓ $x = 6 \text{ OR } x = 2$ ✓ <i>because the output is 0 and that point lies on the x-axis and not on the parabola.</i> $A_{\min} = 16 \times 2 = 32 \text{ units}^2$ ✓ <i>OR sub $x=2$ into $A = (2)^3 - 12(2)^2 + 36(2)$ $A = 32 \text{ units}^2$</i>	✓ correct substitution into area formula ✓ formula for area in terms of x ✓ derivative = 0 ✓ choice of x-value to determine y-value ✓ answer	(5)

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QUESTION 11

11.1	$(x+0,2) \times 0,5 = 0,2$ $x+0,2 = 0,4$ $x = 0,2$ $0,2 + 0,2 + 0,3 + y = 1$ $y = 0,3$	$P(A \text{ and } B) = P(A) \times P(B)$ $0,2 = (0,2 \times 0,5) \times (0,2 + 0,3)$ $0,2 = (0,1) \times (0,5)$ $0,2 = 0,05$ $0,2 - 0,1 = 0,1$ $0,1 = 0,5x$ $x = 0,2$ $0,2 = 0,2$	$\checkmark P(A) \times P(B) = P(A \text{ and } B)$ $\checkmark x$ -value $\checkmark$ sum of probabilities = 1 $\checkmark y$ -value	(4)
11.2	$61 = 720$ total combos of 111 people $P(X \text{ and } Y \text{ together}) = \frac{2151}{61}$ $1 + 4 = 5$ groups $\frac{2151}{5} = 430,2$ $P(X \text{ and } Y \text{ not together}) = 1 - \frac{1}{3}$ $\frac{2}{3}$	$61 = 720$ $2151 = 2151$ $2151 - 240 = 1911$ $1911 - 240 = 1671$ $1671 - 240 = 1431$ $1431 - 240 = 1191$ $1191 - 240 = 951$ $951 - 240 = 711$ $711 - 240 = 471$ $471 - 240 = 231$ $231 - 240 = -9$ $720 - 240 = 480$ $480/720 = 2/3$	$\checkmark 2151/61$ $\checkmark 1/3$	(3)
11.3.1	4 digit numbers (with repetition) the first digit cannot be zero because then it will be 3 digit numbers $= 9 \times 10 \times 10 \times 10$ $= 9000$	$9 = 9$ $10 = 10$ $10 = 10$ $10 = 10$	$\checkmark$ product $\checkmark$ answer	(2)
11.3.2	4 digit numbers (without repetition) $= 9 \times 9 \times 8 \times 7$ $= 4536$	$9 = 9$ $9 = 9$ $8 = 8$ $7 = 7$	$\checkmark 9 \times 9$ $\checkmark 8 \times 7$ $\checkmark$ answer	(3)
11.3.3	4 digit numbers (with repetition and last digit = 0) $= 9 \times 8 \times 7 \times 1$ $= 4536$	$9 = 9$ $8 = 8$ $7 = 7$ $1 = 1$	$\checkmark$ product $\checkmark$ answer	(2)
11.3.3	4 digit numbers (with repetition and last digit = 0) $10 \times 10 \times 10 \times 1 = 1000$ OR/OF $9 \times 10 \times 10 \times 1 = 900$ $The first no. should not be 0$	$10 \times 10 \times 10 \times 1 = 1000$ OR/OF $9 \times 10 \times 10 \times 1 = 900$ $2400 - 900 = 1500$	$\checkmark$ product/probability $\checkmark$ answer $\checkmark$ product/probability $\checkmark$ answer $\checkmark$ answer	(2)
TOTAL: 150				

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11.1	$(x+0,2) \times 0,5 = 0,2$ $x+0,2 = 0,4$ $x = 0,2$ $0,2 + 0,2 + 0,3 + y = 1$ $y = 0,3$	$P(A \text{ and } B) = P(A) \times P(B)$ $0,2 = (0,2 \times 0,5) \times (0,2 + 0,3)$ $0,2 = (0,1) \times (0,5)$ $0,2 = 0,05$ $0,2 - 0,1 = 0,1$ $0,1 = 0,5x$ $x = 0,2$	$\checkmark P(A) \times P(B) = P(A \text{ and } B)$ $\checkmark x$ -value $\checkmark$ sum of probabilities = 1 $\checkmark y$ -value	(4)
11.2	$61 = 720$ total combos of 111 people $P(X \text{ and } Y \text{ together}) = \frac{2151}{61}$ $1 + 4 = 5$ groups $\frac{2151}{5} = 430,2$ $P(X \text{ and } Y \text{ not together}) = 1 - \frac{1}{3}$ $\frac{2}{3}$	$61 = 720$ $2151 = 2151$ $2151 - 240 = 1911$ $1911 - 240 = 1671$ $1671 - 240 = 1431$ $1431 - 240 = 1191$ $1191 - 240 = 951$ $951 - 240 = 711$ $711 - 240 = 471$ $471 - 240 = 231$ $231 - 240 = -9$ $720 - 240 = 480$ $480/720 = 2/3$	$\checkmark 2151/61$ $\checkmark 1/3$	(3)
11.3.1	4 digit numbers (with repetition) the first digit cannot be zero because then it will be 3 digit numbers $= 9 \times 10 \times 10 \times 10$ $= 9000$	$9 = 9$ $10 = 10$ $10 = 10$ $10 = 10$	$\checkmark$ product $\checkmark$ answer	(2)
11.3.2	4 digit numbers (without repetition) $= 9 \times 9 \times 8 \times 7$ $= 4536$	$9 = 9$ $9 = 9$ $8 = 8$ $7 = 7$	$\checkmark 9 \times 9$ $\checkmark 8 \times 7$ $\checkmark$ answer	(3)
11.3.3	4 digit numbers (with repetition and last digit = 0) $= 9 \times 8 \times 7 \times 1$ $= 4536$	$9 = 9$ $8 = 8$ $7 = 7$ $1 = 1$	$\checkmark$ product $\checkmark$ answer	(2)
11.3.3	4 digit numbers (with repetition and last digit = 0) $10 \times 10 \times 10 \times 1 = 1000$ OR/OF $9 \times 10 \times 10 \times 1 = 900$ $The first no. should not be 0$	$10 \times 10 \times 10 \times 1 = 1000$ OR/OF $9 \times 10 \times 10 \times 1 = 900$ $2400 - 900 = 1500$	$\checkmark$ product/probability $\checkmark$ answer $\checkmark$ product/probability $\checkmark$ answer $\checkmark$ answer	(2)

DIRECTOR: EXAMINATIONS MANAGEMENT
19/09/2020

P.T.O.

RESPONSES TO MATHEMATICS/ PAPER 1/ PREPARATORY EXAMINATION 2020

Q1.1.2 The marking guideline is correct. The allocation of the second mark is been queried. A candidate has an option to either factorise a quadratic equation or to use the quadratic formula to do so. A mark can therefore be allocated to any of the two processes. We have also indicated that a candidate may use another valid method. Therefore this suggestion had been catered for. The marking guideline is correct.

Q1.1.3 The critical values are clearly identified as 1 and 4. The marking guideline is correct.

Q1.1.4 An alternative correct solution has now been provided. The marking guideline is correct, however, a different view in regard to the factor mark is been provided. This request is consistent with Q1.1.2 in that a candidate can obtain this mark by either factorising or using substitution into the quadratic formula. The marking guideline is correct.

Q1.2 An alternative solution is been provided. The marking guideline is correct.

Q1.3 An alternative partially in-correct solution is suggested, that $y = 1$ when in fact $y \neq -1$. The marking guideline is correct.

Q2.3 The question is correctly stated in the Question Paper and the answer presented in the marking guideline is correct. The argument used is partially valid. The question is of a higher order and requires the candidate to reason that this is a converging series. The hint is indicated by the word values. Any application of any other geometric series formula would generate a value and not values. This question has been tested in a past NSC examination by the DBE.

This question can additionally be declared valid based the following explanation:

Let us begin with an explanation on a converging series. The modulus of the Geometric series ratio must be less than 1 to declare the series converging. This means that there is a limit to a converging series. Most educators want to see the word converging in such a problem, but an argument can be formulated that this simplifies the question and hence impacts on the cognitive level of the Question. This Question confirms that the values of k

exist which then implies that there is a limit to the solution therefore declaring the word convergence redundant. Bear also in mind, that if the sum of a series gets closer to a particular value as a result of the increase in the number of terms the series can only converge, hence the concept that a converging series has a limit. This Question stipulates that k has a limit.

Now, let us consider why anyone should not consider this series to be divergent. When the modulus of the Geometric series ratio is more and equal to 1, a divergent series is defined. Therefore, the conclusion is that a divergent series does not have a limit, which suggests infinity. This Question clearly commands that k has values.

So, the conclusion that one reaches is that the debate is based on the omission of the word convergence, but this has now been dealt with above. So, what this omission also influences is the cognitive level of the Question which was the intent of the examiners and both the internal and external moderator. As examiners and moderators, let's raise the bar to influence the quality of teaching of this concept hence improving the quality of understanding and learning. Let us look to protect the finer principles of Mathematics in conjunction with the curriculum and the cognitive level at which such a question can be pitched.

Q4.3 The marking guideline is correct. The value of a is given in the question and to request a mark for a given fact is unfeasible. It has become standard to issue the mark for the shape and this can be verified in all DBE examinations over the past 10 years.

Q4.4 The process of allocating total marks for a candidate presenting an answer only must be in line with instruction 5 on the INSTRUCTION AND INFORMATION page to candidates and should additionally be in line with the cognitive level of the question. This problem required an understanding of gradient, but additionally required a calculation as a candidate had to make a substitution to derive the answer. Answer only is therefore not applicable. The marking guideline is correct.

Q6.2 This comment is valid, but the marking guideline is most correct in that the answer is presented in a more defined mathematical manner. Marking guidelines must ultimately influence the way educators teach in the classroom and this process must define clearer presented answers from candidates.

Q7.1 A proposed alternative marking style is been requested based on a candidate totally mis-understanding the question. The comment is valid, but the focus of a marking guideline should concentrate on providing the correct solutions and not focus on a misconception in evaluating candidates answers. It should be reasonably understood that marks cannot be awarded for irrelevant solutions. The marking guideline is correct and presents the only possible manner in which this question should be answered, so the comment should be regarded as additional information to a marker and such comments should be placed in the notes to a marker.

Q7.2.1 An alternative method is been presented to calculate the value of 114 and additionally a marking style proposed on a candidate's misconception of the question has already been dealt with in Q7.1. The marking guideline is correct.

Q7.2.2 The comment has value and is generally added to the notes accompanying a marking guideline. The marking guideline is correct.

Q8.1 The comment has value and is generally added to the notes accompanying a marking guideline. The marking guideline is correct.

Q9.2.4 The question requires two processes. A candidate must determine both the first derivative and the second derivative. Evidence of this must be presented by a candidate to be awarded full marks. Refer to instruction 5 of the INSTRUCTION and INFORMATION page.

Q10.1 The comment has no bearing on the answer and is therefore irrelevant. The marking guideline is correct.

Q11.1 Advise is purely been given to alternatively evaluate this question. The advice has merit but does not render the marking guideline incorrect. The marking guideline is correct.

Q11.3.1 This is a comment that usually is inserted in the notes accompanying a marking guideline. The marking guideline is correct.

Q11.3.3 THERE IS AN ERROR ON THE MARKING GUIDELINE.

THE ANSWER SHOULD BE:

$$10 \times 10 \times 10 \times 1 = 1000 \quad \text{OR} \quad 10 \times 10 \times 10 = 1000$$

AND NOT

$$\begin{aligned} &= 9 \times 8 \times 7 \times 1 \\ &= 504 \end{aligned}$$

The recommendation is therefore valid and after cross referencing with a previous version a typing error was forwarded to the Examinations Directorate and as Internal Moderator except responsibility for the error.

The TOTAL for this question paper must be 150 not 146 as the issue around Q2.3 has now clearly been explained.

Kind Regards.

G. Frank