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LIMPOPO

PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

NATIONAL
SENIOR CERTIFICATE

GRADE 12/*GRAAD 12*

MATHEMATICS/*WISKUNDE*

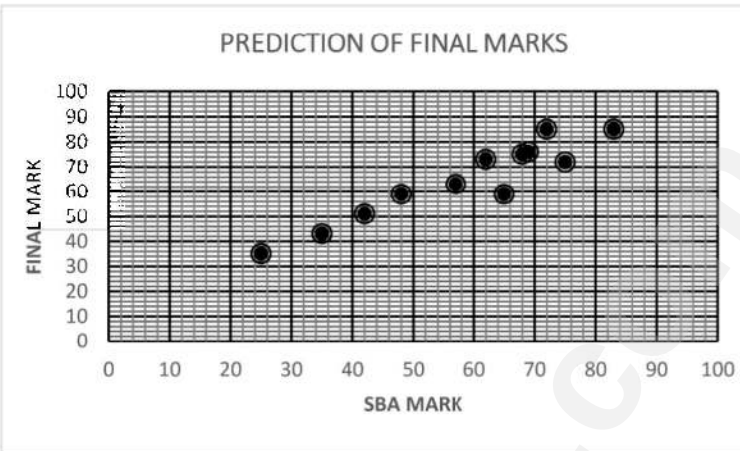
MEMORANDUM P2/*V2*

SEPTEMBER 2023

MARKS/*PUNTE*: 150

This memorandum consists of 19 pages/*Hierdie nasienriglyne bestaan uit 19 bladsye.*

QUESTION/VRAAG 1

1.1		✓✓✓✓ all dots correct/alle punte korrek ✓✓✓✓ 9 – 11 dots correct/ punte korrek ✓✓✓ 6 – 8 dots correct/ punte korrek ✓✓ 1 – 6 dots correct/ punte korrek
1.2	$r = 0,94$	✓✓ answer/antw (2)
1.3	The strong correlation between the SBA mark and the final mark implies that the points lie close to the least squares regression line. Hence the prediction is reliable.	✓ reason/rede ✓ conclusion/konklusie (2)
1.4	$a = 14,49$ $b = 0,86$ $y = 14,49 + 0,86x$	✓ value/waarde a ✓ value/waarde b ✓ equation/vgl (3)
1.5	$y = 14,49 + 0,86(66)$ $y = 71,25\%$ OR/OF $y = 71,18\%$	✓ subst/vervang ✓ answer/antw (2) ✓✓ answer/antw (2)

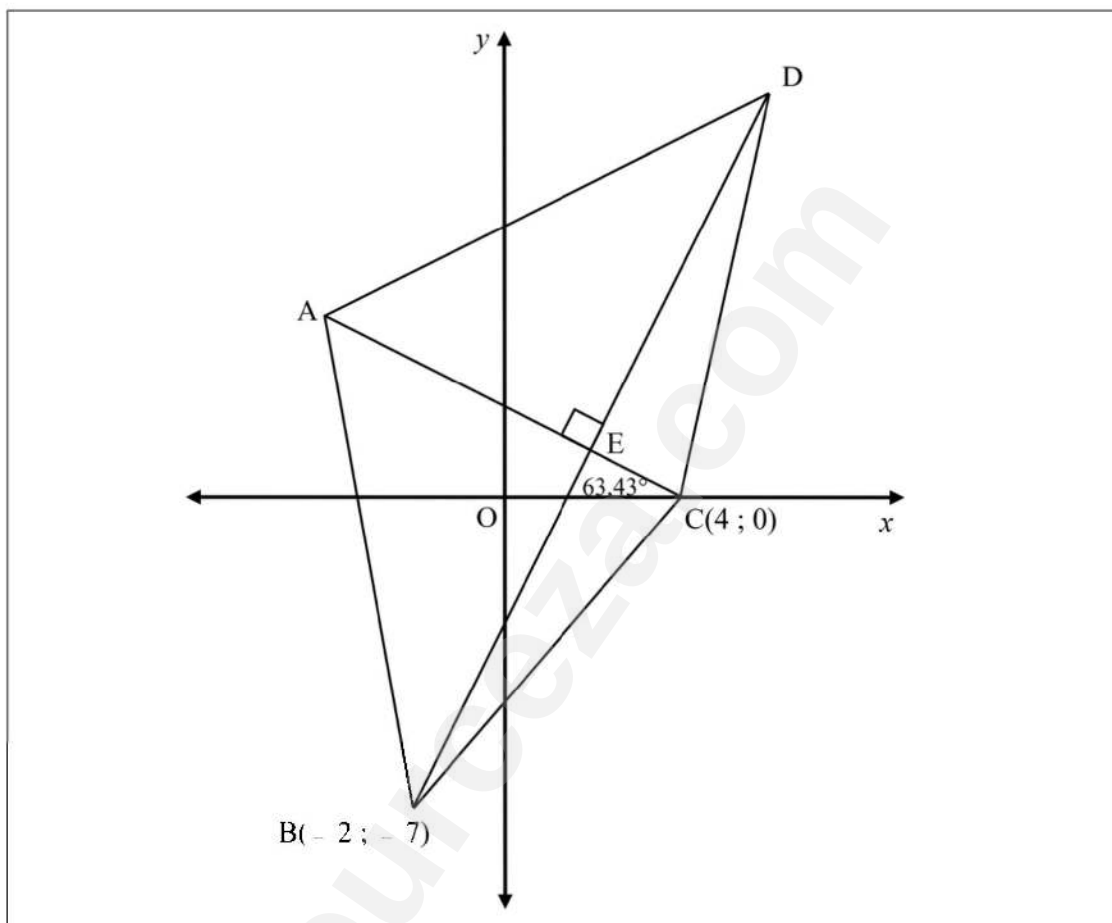
[13]

QUESTION/VRAAG 2

2.1.1	$\frac{3+4+4+6+10+12+12+4+y}{9} = 7$ $\frac{55+y}{9} = 7$ $55+y = 63$ $y = 8$	$\checkmark \frac{55+y}{9} = 7$ $\checkmark \text{value of/waarde } y$ (2)
2.1.2	Median = 6	$\checkmark 6$ (1)
2.2.1	$\bar{x} = \frac{3+4+4+4+6+8+10+12+12+7-n+7+n}{11}$ $\bar{x} = \frac{77}{11}$ $\bar{x} = 7$ OR/OF $\bar{x} = \frac{63+7-n+7+n}{11}$ $\bar{x} = \frac{77}{11}$ $\bar{x} = 7$	$\checkmark 77$ $\checkmark 7$ (2) $\checkmark 77$ $\checkmark 7$ (2)
2.2.2	$\bar{x} - \sigma_x = 3$ $7 - \sigma_x = 3$ $\sigma_x = 4$ OR/OF $\bar{x} + \sigma_x = 11$ $7 + \sigma_x = 11$ $\sigma_x = 4$	$\checkmark \text{equation/vgl}$ $\checkmark \text{answer/antw}$ (2) $\checkmark \text{equation/vgl}$ $\checkmark \text{answer/antw}$ (2)

[7]

QUESTION/VRAAG 3

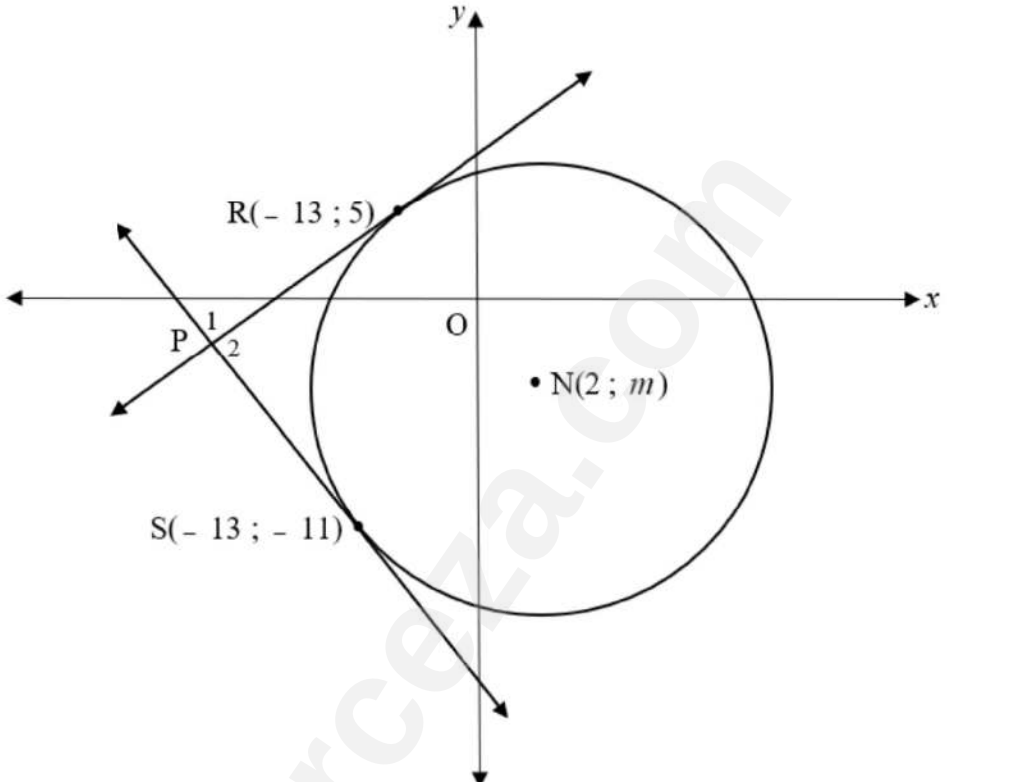


3.1	$y - y_1 = m(x - x_1)$ $y - (-7) = 2(x - (-2))$ $y + 7 = 2(x + 2)$ $BD: y = 2x - 3$ OF/OR $y = mx + c$ $-7 = 2(-2) + c$ $-7 = -4 + c$ $c = -3$ $BD: y = 2x - 3$	$m(AC) = -\frac{1}{2}$ $m(BD) = 2$ $m(AC) = -\frac{1}{2}$ $m(BD) = 2$	$\checkmark m(AC) = -\frac{1}{2}$ $\checkmark m(BD) = 2$ $\checkmark$ subt m and point B /verv m en punt B $\checkmark$ answer/antw (4) $\checkmark m(AC) = -\frac{1}{2}$ $\checkmark m(BD) = 2$ $\checkmark$ subt m and point B /verv m en punt B $\checkmark$ answer/antw (4)
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3.2	$-\frac{1}{2}x + 2 = 2x - 3$ $-x + 4 = 4x - 6$ $-5x = -10$ $x = 2$ subst into BD: $y = 2(2) - 3$ $y = 1$ E (2 ; 1)	✓ equating/vgl ✓ value of/waarde x ✓ value of/waarde y (3)
3.3	A(-4 ; 4) through translation OR/OF Find a point P so that CE = EP P(0 ; 2) Then CP = PA A(-4 ; 4)	✓ value of/waarde x ✓ value of/waarde y (2) ✓ P(0 ; 2) ✓ A(-4 ; 4) (2)
3.4	$AC = \sqrt{(-4-4)^2 + (4-0)^2} = 4\sqrt{5}$ $BE = \sqrt{(-2-2)^2 + (-7-1)^2} = 4\sqrt{5}$ Area of kite = 2 × area of $\triangle ABC$ $= 2 \times \left(\frac{1}{2} \times 4\sqrt{5} \times 4\sqrt{5} \right)$ $= 80$ Enlarged by scale factor 2: $80 \times 4 = 320$ sq unit OR/OF $AC = \sqrt{(-4-4)^2 + (4-0)^2} = 4\sqrt{5}$ $\therefore PR = 8\sqrt{5}$ $BE = \sqrt{(-2-2)^2 + (-7-1)^2} = 4\sqrt{5}$ $\therefore QS = 8\sqrt{5}$ Area of kite = 2 × area of $\triangle PQR$ $= 2 \times \left(\frac{1}{2} \times 8\sqrt{5} \times 8\sqrt{5} \right)$ $= 320$	✓ length/lengte AC ✓ length/lengte BE ✓ method/metode ✓ 80 ✓ answer/antw (5) ✓ length/lengte AC ✓ length/lengte PR ✓ length/lengte QS ✓ method/metode ✓ answer/antw (5)

[14]

QUESTION/VRAAG 4

4.1	
4.1.1 (a)	$(-13-2)^2 + (5-m)^2 = 17^2$ $225 + 25 - 10m + m^2 = 289$ $m^2 - 10m - 39 = 0$ $(m-13)(m+3) = 0$ $m = 13 \text{ or } m = -3$ NA OR/OF $(-13-2)^2 + (-11-m)^2 = 17^2$ $225 + (-11-m)^2 = 289$ $(-11-m)^2 = 64$ $-11-m = \pm 8$ $m = -19 \text{ or } m = -3$ NA ✓subst R and N/verv R en N ✓simplify/vereenv ✓std form/std vorm ✓value/value m (4) ✓subst S and N/verv S en N ✓simplify/vereenv ✓std form/std vorm ✓value/waarde m (4)

	OR/OF $(-13-2)^2 + (-11-m)^2 = (-13-2)^2 + (5-m)^2$ $225 + m^2 + 22m + 121 = 225 + m^2 - 10m + 25$ $32m = -96$ $m = -3$ OR/OF RS $\perp$ y-axis $\therefore x = -13$ same x-values Draw line NK $\perp$ RS $\therefore$ NK bisect RS at $(-13; -3)$ line from centre of circle $\perp$ to chord $\therefore N(2; -3)$	✓✓ equating ✓ simplify/vereenv ✓ value/waarde m (4) ✓ use line $x = -13$ ✓ Use NK ✓ midpt of NK ✓ $y = m = -3$ (4)
4.1.1 (b)	$(x-2)^2 + (y+3)^2 = 289$	✓ answer/antw (1)
4.1.2 (a)	$m(NR) = \frac{-3-5}{2-(-13)}$ $m(NR) = \frac{-8}{15}$	✓ subst in gradient formula/verv in gradiënt formule ✓ gradient of NR (2)
4.1.2 (b)	$m(NS) = \frac{-3-(-11)}{2-(-13)}$ $m(NS) = \frac{8}{15}$	✓ gradient of NS/ gradiënt van NS (1)
4.1.3	NR $\perp$ PR NS $\perp$ PS $\therefore m(PR) = \frac{15}{8} \quad \therefore m(PS) = -\frac{15}{8}$ $\tan \alpha = \frac{15}{8} \quad \tan \beta = -\frac{15}{8}$ $\alpha = 61,93^\circ \quad \beta = 180^\circ - 61,93^\circ$ $\beta = 118,07^\circ$ $\hat{P}_1 = 118,07^\circ - 61,93^\circ = 56,14^\circ$ $\therefore \hat{P}_2 = 180^\circ - 56,14^\circ = 123,86^\circ$	✓ $\tan \alpha$ def ✓ value/waarde α ✓ $\tan \beta$ def ✓ value/waarde β ✓ method/metode ✓ answer/antw (6)

	OF/OR Inclination of NS = $28,07^\circ$ Inclination of NR = $151,93^\circ$ $\hat{RNS} = 28,07^\circ + 28,07^\circ$ $\hat{RNS} = 56,14^\circ$ NRPS is cyclic quadrilateral Opp $\angle$'s suppl $\therefore \hat{P}_2 = 123,86^\circ$	$\checkmark 28,07^\circ$ $\checkmark 151,93^\circ$ $\checkmark \hat{RNS} = 2 \times 28,07^\circ$ $\checkmark 56,14^\circ$ $\checkmark$ Opp $\angle$'s cyclic quad/teenoorst $\angle$ 'e kvh $\checkmark$ answer/antw (6)
4.1.4	Reflection about x -axis: $(2; -3) \rightarrow (2; 3)$ Shift 2 units up: $(2; 3) \rightarrow (2; 5)$ Circle M: $(x-2)^2 + (y-5)^2 = 289$	$\checkmark \checkmark$ equation/vgl (2)
4.2		
4.2.1	Diameter = $8 + 4 + 2 + 1 \dots =$ $S_\infty = \frac{a}{1-r}$ $S_\infty = \frac{8}{1-\frac{1}{2}}$ $S_\infty = 16$ $\therefore OC = 16$	$\checkmark$ sum of diameters/som middellyne $\checkmark$ subst in sum formula/verv in som formule (2)
4.2.2	90° , radius $\perp$ tangent	$\checkmark$ answer/antw $\checkmark$ reason/rede (2)

4.2.3	$AC = 16 - 4 = 12$ $AB = 4$ $BC^2 = AC^2 - AB^2$ (Pyth) $BC^2 = 12^2 - 4^2$ $BC^2 = 128$ $BC = 8\sqrt{2}$ $\therefore \tan C = \frac{4}{8\sqrt{2}} = \frac{\sqrt{2}}{4}$	$\checkmark AC$ $\checkmark AB$ $\checkmark BC$ $\checkmark \tan \text{ratio}$
4.2.4	$y - y_1 = m(x - x_1)$ $y - 0 = \frac{\sqrt{2}}{4}(x - (-16))$ $y = \frac{\sqrt{2}}{4}x + 4\sqrt{2}$ OR/OF $y = \frac{\sqrt{2}}{4}x + c$ $0 = \frac{\sqrt{2}}{4}(-16) + c$ $c = 4\sqrt{2}$ $\therefore y = \frac{\sqrt{2}}{4}x + 4\sqrt{2}$	$\checkmark \text{subst/verv } m$ $\checkmark \text{subst point/verv } punt$ $\checkmark \text{equation/vgl}$ $\checkmark \text{subst/verv } m$ $\checkmark \text{subst point/verv } punt$ $\checkmark \text{equation/vgl}$

[27]

QUESTION/VRAAG 5

5.1		
5.1.1	$\tan \alpha = \frac{4}{3}$	✓ answer/antw (1)
5.1.2	$\sin(90^\circ + \alpha)$ $= \cos \alpha$ $= \frac{3}{5}$	✓ reduction/reduksie ✓ $r = 5$ ✓ answer/antw (3)
5.1.3	$12 + 13 \sin \beta = 0$ $\sin \beta = -\frac{12}{13}$ $m^2 = 13^2 - (-12^2)$ (Pyth) $m^2 = 25$ $m = \pm 5$ $\therefore m = -5$	✓ std form/std vorm ✓ subst into Pyth ✓ simpl/vereenv ✓ answer/antw (4)
5.1.4	$\cos(\alpha + \beta)$ $= \cos \alpha \cos \beta - \sin \alpha \sin \beta$ $= \left(\frac{3}{5}\right)\left(\frac{-5}{13}\right) - \left(\frac{4}{5}\right)\left(\frac{-12}{13}\right)$ $= -\frac{3}{13} + \frac{48}{65}$ $= \frac{33}{65}$	✓ expansion/uitbrei ✓ subst/vervang ✓ answer/antw (3)

5.2.1	$\sqrt{4^{\sin 150^\circ} \cdot 2^{3 \tan 225^\circ}}$ $= \sqrt{(2^2)^{\sin 30^\circ} \cdot 2^{3 \tan 45^\circ}}$ $= \sqrt{2^{2(\frac{1}{2})} \cdot 2^{3(1)}}$ $= \sqrt{2 \cdot 2^3}$ $= \sqrt{16}$ $= 4$	✓ sin 30° ✓ tan 45° ✓ special values/ spesiale waardes ✓ simpl/vereenv ✓ answer/antw (5)
5.2.2	$\frac{\tan(180^\circ + x) \cos x}{\sin(180^\circ + x) \cos x - \cos(540^\circ + x) \cos(90^\circ + x)}$ $= \frac{(\tan x)(\cos x)}{(-\sin x)(\cos x) - (-\cos x)(-\sin x)}$ $= \frac{\frac{\sin x}{\cos x} \cdot \cos x}{-\sin x \cos x - \cos x \sin x}$ $= \frac{\sin x}{-2 \sin x \cos x}$ $= -\frac{1}{2 \cos x}$	✓ tan x ✓ -sin x ✓ -cos x ✓ -sin x ✓ $\frac{\sin x}{\cos x}$ ✓ answer/antw (6)
5.3	$\frac{1 - \cos 2x - \sin x}{\sin 2x - \cos x}$ $= \frac{1 - (1 - 2 \sin^2 x) - \sin x}{2 \sin x \cos x - \cos x}$ $= \frac{1 - 1 + 2 \sin^2 x - \sin x}{2 \sin x \cos x - \cos x}$ $= \frac{2 \sin^2 x - \sin x}{2 \sin x \cos x - \cos x}$ $= \frac{\sin x(2 \sin x - 1)}{\cos x(2 \sin x - 1)}$ $= \frac{\sin x}{\cos x}$ $= \tan x$	✓ cos 2x expansion/ uitbrei ✓ sin 2x expansion/ uitbrei ✓ simpl/vereenv ✓ factors/faktore (4)

5.4	$\sin P \sin Q - \cos P \cos Q = \frac{1}{2}$ $\therefore \cos P \cos Q - \sin P \sin Q = -\frac{1}{2}$ $\cos(P+Q) = -\frac{1}{2}$ $P+Q = 180^\circ - 60^\circ$ $P+Q = 120^\circ \dots\dots\dots 1$ $\sin(P-Q) = \frac{1}{2}$ $P-Q = 30^\circ \dots\dots\dots 2$ $\therefore 2P = 150^\circ$ $P = 75^\circ$ $Q = 45^\circ$	✓rearrange terms/ <i>herrangskik terme</i> ✓cos identity/ <i>identiteit</i> ✓2 nd quadrant/ <i>kwadrant</i> ✓equation/vgl ✓1 st quadrant/ <i>kwadrant</i> ✓75° ✓45°
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(7)

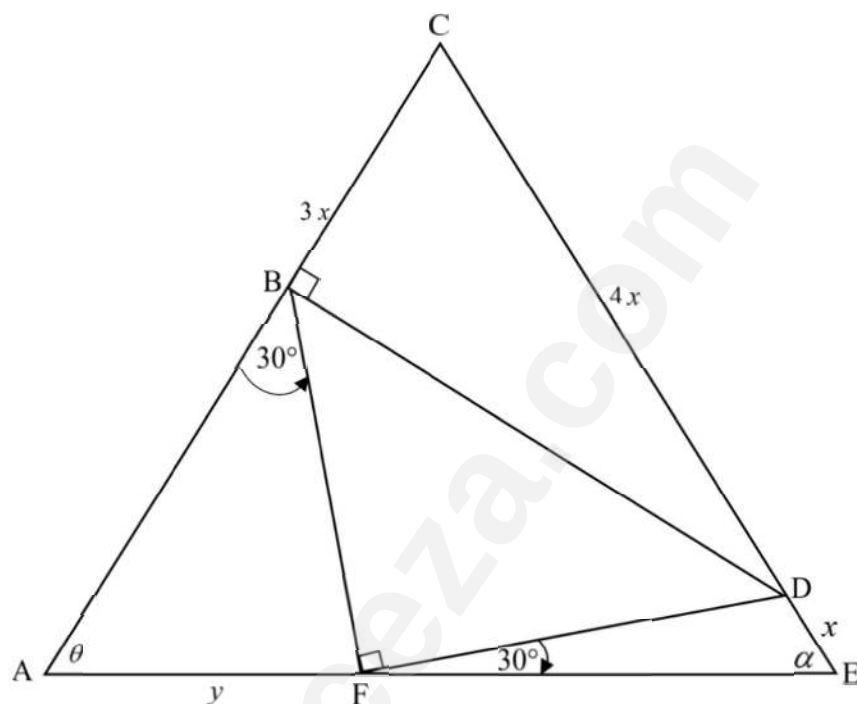
[33]

QUESTION/VRAAG 6

6.1	$a=2$ and $b=1$	✓value/waarde a ✓value/waarde b (2)
6.2.1	$2 \sin x - \cos x = 0$ $2 \sin x = \cos x$ $\frac{\sin x}{\cos x} = \frac{1}{2}$ $\tan x = \frac{1}{2}$ $x = 26,57^\circ$	✓ $\tan x = \frac{1}{2}$ ✓answer/antw (2)
6.2.2	$x \in [0^\circ ; 180^\circ]$	✓interval ✓notation/notasie (2)
6.2.3	$y = 2^{2f(x)-1}$ $y \in \left[-\frac{1}{2} ; 7\right]$	✓✓ answer/antw (2)

[8]

QUESTION/VRAAG 7



7.1	$\text{In } \triangle ABF: \frac{BF}{\sin A} = \frac{AF}{\sin B}$ $\frac{BF}{\sin \theta} = \frac{y}{\sin 30^\circ}$ $BF = \frac{y \sin \theta}{\frac{1}{2}}$ $BF = 2y \sin \theta$	✓ correct subst in sine rule/korrekte verv in sinreel ✓ subst special value/ verv spasiale waarde ✓ answer/antw (3)
7.2	$\text{In } \triangle EDF: \frac{DF}{\sin E} = \frac{DE}{\sin F}$ $\frac{DF}{\sin \alpha} = \frac{x}{\sin 30^\circ}$ $DF = \frac{x \sin \alpha}{\frac{1}{2}}$ $DF = 2x \sin \alpha$	✓ correct subst in sine rule/korrekte verv in sinreel ✓ answer/antw (2)
7.3	$BD^2 = BF^2 + FD^2 \quad (\text{Pyth})$ $BD^2 = (2y \sin \theta)^2 + (2x \sin \alpha)^2$ $BD^2 = 4y^2 \sin^2 \theta + 4x^2 \sin^2 \alpha$	✓ subst/verv in Pyth (1)

7.4	In $\triangle BDC$: $BD^2 = (4x)^2 - (3x)^2$ $BD^2 = 16x^2 - 9x^2$ $BD^2 = 7x^2$ $7x^2 = 4y^2 \sin^2 \theta + 4x^2 \sin^2 \alpha$ $7x^2 - 4x^2 \sin^2 \alpha = 4y^2 \sin^2 \theta$ $x^2(7 - 4\sin^2 \alpha) = 4y^2 \sin^2 \theta$ $x^2 = \frac{4y^2 \sin^2 \theta}{(7 - 4\sin^2 \alpha)}$ $x = \sqrt{\frac{4y^2 \sin^2 \theta}{7 - 4\sin^2 \alpha}}$ OF/OR $CD^2 = BD^2 + BC^2$ $16x^2 = 4y^2 \sin^2 \theta + 4x^2 \sin^2 \alpha + 9x^2$ $16x^2 - 9x^2 - 4x^2 \sin^2 \alpha = 4y^2 \sin^2 \theta$ $7x^2 - 4x^2 \sin^2 \alpha = 4y^2 \sin^2 \theta$ $x^2(7 - 4\sin^2 \alpha) = 4y^2 \sin^2 \theta$ $x^2 = \frac{4y^2 \sin^2 \theta}{7 - 4\sin^2 \alpha}$ $x = \sqrt{\frac{4y^2 \sin^2 \theta}{7 - 4\sin^2 \alpha}}$	✓ $BD^2 = 7x^2$ ✓ equating ✓ simpl/vereenv (3) ✓ subst/verv in Pyth ✓ simpl/vereenv ✓ i.t.o x^2 (3)
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[9]

8.1	$\hat{ABF} = \hat{E}_1 = 32^\circ$	tan-chord theorem	$\checkmark S \checkmark R$	(2)
8.2	$\hat{E}_1 = \hat{B}_1 = 32^\circ$ $\hat{F} = 116^\circ$	$\angle$'s opp equal sides sum $\angle$'s of Δ	$\checkmark S/R$ $\checkmark S/R$	(2)
8.3	$\hat{D} + \hat{F} = 180^\circ$ $\hat{D} = 64^\circ$	opp $\angle$'s of cyclic quad	$\checkmark S \checkmark R$	(2)
8.4	$\hat{O}_1 = 2 \times \hat{D}$ $\hat{O}_1 = 128^\circ$	midpt $\angle = 2 \times$ circumf $\angle$	$\checkmark S \checkmark R$	(2)
8.5	$\hat{E}_2 + \hat{O}_1 + \hat{B}_2 = 180^\circ$ $\hat{E}_2 = \hat{B}_2$ $2\hat{E}_2 + 128^\circ = 180^\circ$ $2\hat{E}_2 = 52^\circ$ $\hat{E}_2 = 26^\circ$ OR/OF $\hat{ABF} + \hat{B}_1 + \hat{B}_2 = 90^\circ$ $32^\circ + 32^\circ + \hat{B}_2 = 90^\circ$ $\hat{B}_2 = 26^\circ$ $\hat{B}_2 = \hat{E}_2 = 26^\circ$	sum $\angle$'s of Δ $\angle$'s opp equal sides $\text{rad} \perp \text{tangent}$ $\angle$'s opp equal sides	$\checkmark S/R$ $\checkmark S/R$ $\checkmark S/R$	(2)

QUESTION/VRAAG 9

9.1

Construction: Join LO and ON	✓ constr/konstr
$\hat{O}_1 = 2 \times \hat{K}$ $\angle$ at centre = $2 \times \angle$ at circumf	✓ S ✓ R
$\hat{O}_2 = 2 \times \hat{M}$ $\angle$ at centre = $2 \times \angle$ at circumf	✓ S/R
$\hat{O}_1 + \hat{O}_2 = 360^\circ$ revolution	✓ R
$2\hat{M} + 2\hat{K} = 360^\circ$ equating	
$\hat{M} + \hat{K} = 180^\circ$	
	(5)

9.2.1	$\hat{B} = \hat{M}_1 = x$ $\angle$'s in the same segment $\hat{B} = \hat{R}_3 = x$ alt $\angle$'s, TR BS $\hat{B} = \hat{R}_1 = x$ tan-chord theorem	$\checkmark S \checkmark R$ $\checkmark S \checkmark R$ $\checkmark S \checkmark R$
9.2.2	Equal chords equal circumf $\angle$'s	$\checkmark R$
9.2.3(a)	$\hat{A}_1 = x + y$ ext $\angle$ of Δ	$\checkmark S/R$
9.2.3(b)	$\hat{N}_1 = x + y$ ext $\angle$ of Δ	$\checkmark S/R$
9.2.4	In ΔSAR and ΔKNR (i) $\hat{R}_1 = \hat{R}_3$ both = x (ii) $\hat{A}_1 = \hat{N}_1$ both = $x + y$ (iii) $\hat{K}_2 = \hat{S}$ sum $\angle$'s of Δ $\therefore \Delta SAR \equiv \Delta KNR$ $\angle \angle \angle$ OR/OR In ΔSAR and ΔKNR (i) $\hat{R}_1 = \hat{R}_3$ both = x (ii) $\hat{A}_1 = \hat{N}_1$ both = $x + y$ $\therefore \Delta SAR \equiv \Delta KNR$ $\angle \angle \angle$	$\checkmark S$ $\checkmark S$ $\checkmark S$ $\checkmark S$ $\checkmark S$ $\checkmark R$
9.2.5	$\hat{K}_2 = \hat{S}$ $\Delta SAR \equiv \Delta KNR$ SAKR is a cyclic quad ext $\angle$ of quadrilateral OR converse ext $\angle$ of cyclic quad	$\checkmark S$ $\checkmark R$

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10.1	$\hat{P}_1 = C$ $\hat{C} = 180^\circ - (\hat{R}_1 + \hat{R}_2)$ $\therefore \hat{P}_1 = 180^\circ - (\hat{R}_1 + \hat{R}_2)$ $\hat{R}_1 + \hat{R}_2 = 180^\circ - (\hat{A}_1 + \hat{B}_1)$ sum $\angle$'s of Δ $\therefore \hat{P}_1 = 180^\circ - [180^\circ - (\hat{A}_1 + \hat{B}_1)]$ $\therefore \hat{P}_1 = 180^\circ - 180^\circ + (\hat{A}_1 + \hat{B}_1)$ $\therefore \hat{P}_1 = \hat{A}_1 + \hat{B}_1$	ext $\angle$ of cyclic quad opp $\angle$'s of cyclic quad 	$\checkmark S \checkmark R$ $\checkmark S \checkmark R$ $\checkmark$ method/metode
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(5)

	OR/OF $\hat{A}\hat{R}\hat{B} = 180^\circ - (\hat{A}_1 + \hat{B}_1)$ $\hat{A}\hat{R}\hat{B} = 180^\circ - \hat{C}$ $\therefore 180^\circ - (\hat{A}_1 + \hat{B}_1) = 180^\circ - \hat{C}$ $\hat{C} = \hat{A}_1 + \hat{B}_1$ $\hat{C} = \hat{P}_1$ $\therefore \hat{P}_1 = \hat{A}_1 + \hat{B}_1$	✓S ✓R ✓S ✓R ✓method/metode (5)
10.2	In $\triangle ARP$ and $\triangle ABR$ (i) $\hat{A} = \hat{A}$ common $\angle$ (ii) $\hat{P}_1 = \hat{A}_1 + \hat{B}_1$ proven $180^\circ - \hat{P}_1 = 180^\circ - (\hat{A}_1 + \hat{B}_1)$ $\therefore \hat{P}_4 = \hat{R}_1 + \hat{R}_2$ (iii) $\hat{R}_1 = \hat{B}_1$ sum $\angle$'s of $\triangle$ $\therefore \triangle ARP \parallel \triangle ABR$ $\frac{AR}{AB} = \frac{AP}{AR}$ $AR^2 = AB \cdot AP$ OR/OF In $\triangle ARP$ and $\triangle ABR$ (i) $\hat{A} = \hat{A}$ common $\angle$ (ii) $\hat{P}_1 = \hat{A}_1 + \hat{B}_1$ proven $180^\circ - \hat{P}_1 = 180^\circ - (\hat{A}_1 + \hat{B}_1)$ $\therefore \hat{P}_4 = \hat{R}_1 + \hat{R}_2$ $\therefore \triangle ARP \parallel \triangle ABR$ $\frac{AR}{AB} = \frac{AP}{AR}$ $AR^2 = AB \cdot AP$	✓ identify/ identifiseer $\triangle$'s ✓S ✓method/metode ✓R ✓S (5) ✓Identify/ identifiseer $\triangle$'s ✓S ✓method/metode ✓R ✓S (5)

[10]



education

Department of
Education
FREE STATE PROVINCE

GRADE 12

MATHEMATICS

MARKING GUIDELINE

MOCK EXAM

SEPTEMBER EXAM PAPER 2

18 AUGUST 2023

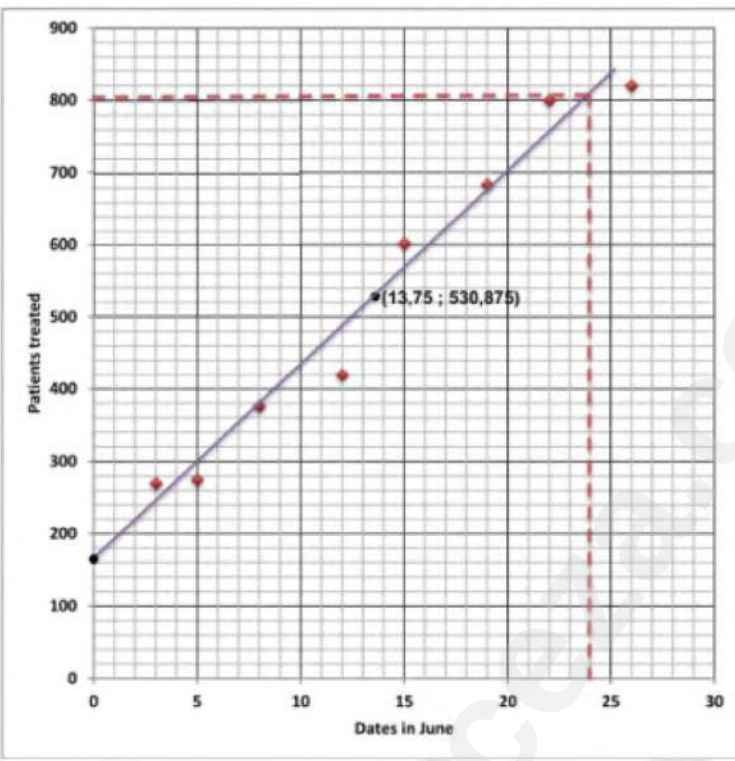
MARKS: 150

This marking guideline consists of 14 pages.

QUESTION 1

1.1	<table border="1"> <thead> <tr> <th>Speed (km/h)</th><th>Frequency (f)</th><th>Cumulative frequency</th></tr> </thead> <tbody> <tr> <td>$0 < x \leq 10$</td><td>10</td><td>10</td></tr> <tr> <td>$10 < x \leq 20$</td><td>20</td><td>30</td></tr> <tr> <td>$20 < x \leq 30$</td><td>45</td><td>75</td></tr> <tr> <td>$30 < x \leq 40$</td><td>72</td><td>147</td></tr> <tr> <td>$40 < x \leq 50$</td><td>23</td><td>170</td></tr> </tbody> </table>	Speed (km/h)	Frequency (f)	Cumulative frequency	$0 < x \leq 10$	10	10	$10 < x \leq 20$	20	30	$20 < x \leq 30$	45	75	$30 < x \leq 40$	72	147	$40 < x \leq 50$	23	170	✓ column 1 values ✓ column 2 values (2)
Speed (km/h)	Frequency (f)	Cumulative frequency																		
$0 < x \leq 10$	10	10																		
$10 < x \leq 20$	20	30																		
$20 < x \leq 30$	45	75																		
$30 < x \leq 40$	72	147																		
$40 < x \leq 50$	23	170																		
1.2 & 1.3		✓ Grounding(0; 0) ✓ Maximum (50; 170) ✓ Curve (3) ✓ Q_1 ✓ Q_2 (2)																		
1.4		✓ Q_3 ✓ Box (2)																		
1.5	164 (Accept 163 – 165)	✓ Answer (1)																		
[10]																				

QUESTION 2

2.1		✓ ✓ ✓ ALL points plotted correctly -1 for every 2 mistakes -1 if points are joined (3)
2.2	$y = 26,88x + 161,24$	✓ value of $a = 161,24$ ✓ value of $b = 26,88$ ✓ equation of the line (3)
2.3	$y = 26,88(24) + 161,24$ $y = 806,36$ 806 people treated on 24th of June	✓ substitute in formula ✓ answer as Natural Number (2)
2.4	see grid	✓ straight line ✓ graph passes through (x ; y) ✓ graph passes through y-intercept from regression line formula (3)
2.5	$r = 0,98$ This is a very strong positive, linear correlation. As the days increase the number of patients increase	✓ ✓ correct answer ✓ interpretation (3)
2.6	$(\bar{x} - \sigma; \bar{x} + \sigma)$ $(528,63 - 210,46; 528,63 + 210,46)$ $(318,17 ; 739,09)$ 4 patients.	✓ 318,17 ✓ (739,09) ✓ answer (3)

		[17]
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QUESTION 3

3.1	$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{BC} = \frac{8 - 0}{4 - 5}$ $m_{BC} = -8$	✓ correct substitution into the correct formula ✓ answer (2)
3.2	$AB = \sqrt{(-3 - 4)^2 + (k - 8)^2}$ $65 = 49 + k^2 - 16k + 64$ $k^2 - 16k + 48 = 0$ $(k - 4)(k - 12) = 0$ $k = 4 \text{ or } k = 12$ $k = 4$	✓ substitute A and B into distance formula ✓ standard form ✓ factors ✓ $k = 4$ (4)
3.3	$m_{BD} = \frac{8 - (-4)}{4 - (-2)}$ $m_{BD} = 2$ $m_{AC} = \frac{4 - 0}{-3 - 5} = -\frac{1}{2}$ $m_{AC} \times m_{BD} = 2 \times -\frac{1}{2} = -1$ $AC \perp BD$	✓ m_{BD} ✓ m_{AC} ✓ $m_{BD} \times m_{AC}$ (3)
3.4	<i>midpoint of AC = midpoint of DC</i> $\frac{x + (-3)}{2} = \frac{-2 + 5}{2} \text{ and } \frac{y + 4}{2} = \frac{-4 + 0}{2}$ $x = 6 \text{ or } y = -8$ $F(6; -8)$	✓ x-coordinate ✓ y-coordinate
3.5	$m_{AD} = m_{BC} = -8$ $\tan \alpha = -8$ $\alpha = 180^\circ - \tan^{-1}(8)$ $\alpha = 97,13^\circ$ $\tan \beta = m_{BD} = 2$ $\beta = 63,43^\circ$ $O_1 = 63,43^\circ (\text{vert opp } \angle^s)$	✓ m_{AD} ✓ $\tan \alpha = -8$ ✓ $\alpha = 97,13^\circ$ ✓ $\beta = 63,43^\circ$ ✓ $\alpha - \beta$ ✓ answer (6)

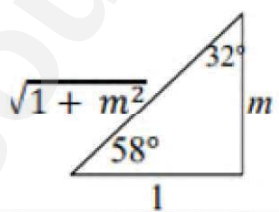
	$\hat{E}DO = \alpha - \beta$ $\hat{E}DO = 97,13^\circ - 63,43^\circ = 33,7^\circ$	
3.6	$AC = \sqrt{(5 - (-3))^2 + (0 - 4)^2} = \sqrt{80} = 4\sqrt{5}$ $DP = \sqrt{(-2 - 1)^2 + (-4 - 2)^2} = \sqrt{45} = 3\sqrt{5}$ $Area\ of\ \triangle ADC = \frac{1}{2} AC \times DP$ $Area\ of\ \triangle ADC = \frac{1}{2} (4\sqrt{5})(4\sqrt{5})$ $Area\ of\ \triangle ADC = 30\ square\ units$	✓ length of AC ✓ length of DP ✓ correct substitution into formula ✓ answer (4)
		[08]

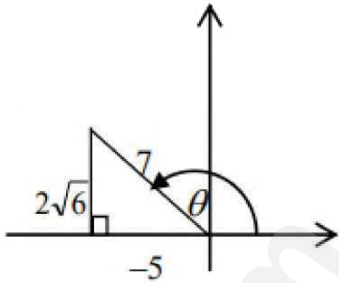
QUESTION 4

4		
4.1	$OB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $OB = \sqrt{(-4 - 0)^2 + (-6 - 0)^2}$ $OB = \sqrt{52}$ $OB = 2\sqrt{13}$ $r = \sqrt{13}$	✓ substitution ✓ $OB = \sqrt{52}$ ✓ $r = \sqrt{13}$ (3)
4.2	$C\left(\frac{-4+0}{2}; \frac{-6+0}{2}\right)$ $C(-2; -3)$ $(x+2)^2 + (y+3)^2 = 13$	
4.3	Right-angled triangle, tangent perpendicular to the radius.	✓ right angled $\triangle$ ✓ reason (2)
4.4	$m_{OB} = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{BO} = \frac{-6 - 0}{-4 - 0}$ $m_{BO} = \frac{3}{2}$ $m_{KL} = -\frac{2}{3}$ $y - y_1 = m(x - x_1)$ $y + 6 = -\frac{2}{3}(x + 4)$	✓ m_{OB} ✓ m_{KL} ✓ substitution ✓ answer (4)

	$y = -\frac{2}{3}x - \frac{26}{3}$	
4.5	E(0;-6)	✓✓ answer (2)
4.6	$(x+2)^2 + (0+3)^2 = 13$ $F(-4;0)$ $m_{FC} = \frac{-3-0}{-2+4} = -\frac{3}{2}$ $m_{CE} = \frac{-3+6}{-2+0} = -\frac{3}{2}$ Points F, C and E are collinear EF is a diameter	✓ x_F ✓ y_F ✓ m_{FC} ✓ m_{CE} ✓ Points F, C and E are collinear (5)
		[20]

QUESTION 5

5.1.1	$\tan 58^\circ = m$ $x^2 + y^2 = r^2$ $1^2 + m^2 = r^2$ $\sqrt{1+m^2} = r$ $\sin 58^\circ = \frac{m}{\sqrt{1+m^2}}$ 	✓ $\sqrt{1+m^2} = r$ ✓ answer (2)
5.1.2	$\sin 296^\circ = -\sin 64^\circ$ $\sin 296^\circ = -\sin 2(32^\circ)$ $\sin 296^\circ = -2 \sin 32^\circ \times \cos 32^\circ$ $\sin 296^\circ = -2 \times \frac{m}{\sqrt{1+m^2}} \times \frac{1}{\sqrt{1+m^2}}$ $\sin 296^\circ = -\frac{2m}{1+m^2}$	✓ $-\sin 64^\circ$ ✓ substitution ✓ answer (3)
5.2.1	$5 \tan \theta + 2\sqrt{6} = 0$	

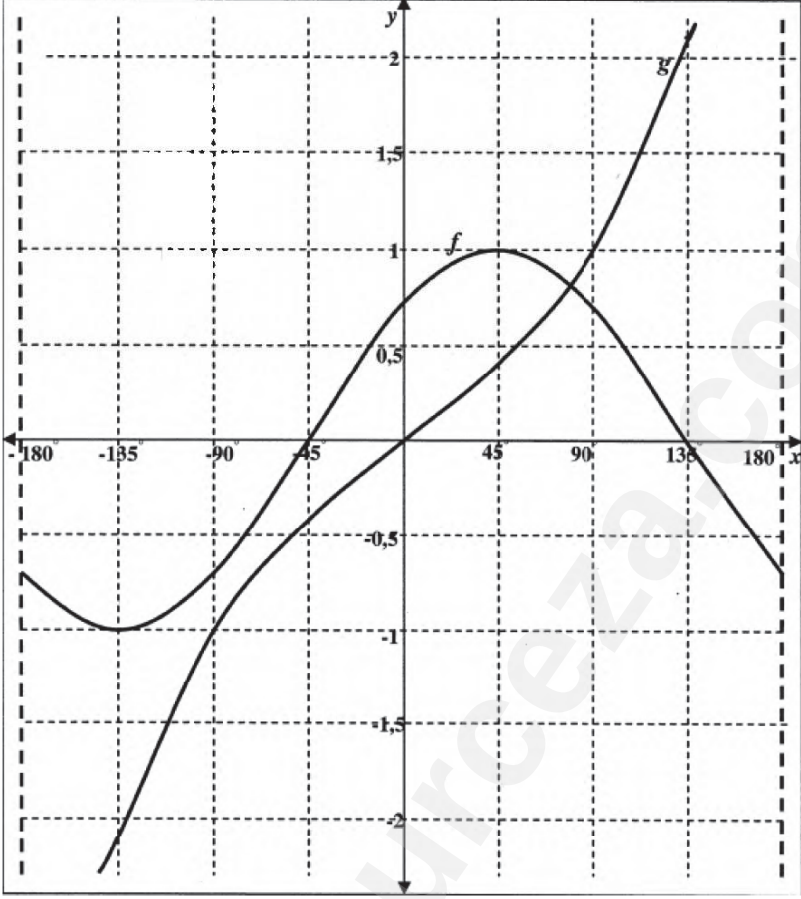
	 $\tan \theta = -\frac{2\sqrt{6}}{5}$ $0^\circ < \theta < 270^\circ$ $r^2 = y^2 + x^2$ $r^2 = (-2\sqrt{6})^2 + (5)^2$ $r^2 = 49$ $r = 7$ $\sin \theta = \frac{2\sqrt{6}}{7}$	$\checkmark r = \frac{\sqrt{649}}{5}$ $\checkmark \sin \theta = \frac{2\sqrt{6}}{\frac{\sqrt{649}}{5}}$ (2)
5.2.2	$\cos \theta = -\frac{5}{7}$	$\checkmark$ answer (1)
5.2.3	$\frac{14 \cos \theta + 7\sqrt{6} \sin \theta}{\cos(-240^\circ) \cdot \tan 225^\circ}$ $= \frac{14\left(-\frac{5}{7}\right) + 7\sqrt{6}\left(\frac{2\sqrt{6}}{7}\right)}{\cos(90^\circ + 30^\circ) \cdot \tan(180^\circ + 45^\circ)}$ $= \frac{-10 + 12}{-\sin 30^\circ \cdot \tan 45^\circ}$ $= \frac{2}{-\frac{1}{2} \times 1}$ $= -4$	$\checkmark -\sin 30^\circ$ $\checkmark \tan 45^\circ$ $\checkmark -\frac{1}{2} \times 1$ $\checkmark \text{ answer} \quad (4)$
		[15]

QUESTION 6

6.1	$\frac{\cos(180^\circ + x) \cdot \tan(360^\circ - x) \cdot \sin^2(90^\circ - x)}{\sin(180^\circ - x)} + \sin^2 x$ $= \frac{(-\cos x) \cdot (-\tan x) \cdot \cos^2 x}{\sin x} + \sin^2 x$ $= \frac{\cos x \cdot \frac{\sin x}{\cos x} \cdot \cos^2 x}{\sin x} + \sin^2 x$ $= \cos^2 x + \sin^2 x$ $= 1$	✓ $-\cos x$ ✓ $-\tan x$ ✓ $\cos^2 x$ ✓ $\sin x$ ✓ $\cos^2 x + \sin^2 x$ ✓ answer (6)
6.2.1	$\cos(A - B) - \cos(A + B)$ $= \cos A \cos B + \sin A \sin B - [\cos A \cos B - \sin A \sin B]$ $= \cos A \cos B + \sin A \sin B - \cos A \cos B + \sin A \sin B$ $= 2\sin A \sin B$	✓ ✓ ✓ (3)
6.2.2	$\cos 15^\circ - \cos 75^\circ = \cos(45^\circ - 30^\circ) - \cos(45^\circ + 30^\circ)$ $= 2\sin 45^\circ \sin 30^\circ$ $= 2 \times \frac{\sqrt{2}}{2} \times \frac{1}{2} \quad \text{or/ of} \quad 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{2}$ $= \frac{\sqrt{2}}{2} \quad \text{or/ of} \quad \frac{1}{\sqrt{2}}$ OR $\cos 15^\circ - \cos 75^\circ$ $= \cos(45^\circ - 30^\circ) - \cos(45^\circ + 30^\circ)$ $= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ - [\cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ]$ $= 2\sin 45^\circ \sin 30^\circ$ $= 2 \times \frac{\sqrt{2}}{2} \times \frac{1}{2} \quad \text{or/ of} \quad 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{2}$ $= \frac{\sqrt{2}}{2} \quad \text{or/ of} \quad \frac{1}{\sqrt{2}}$	✓ ✓ ✓ ✓ (4)

6.3	$AB^2 = (\cos \theta - 6)^2 + (\sin \theta - 7)^2$ $86 = \cos^2 \theta - 12 \cos \theta + 36 + \sin^2 \theta - 14 \sin \theta + 49$ $86 = 1 + 36 + 49 - 12 \cos \theta - 14 \sin \theta$ $0 = -12 \cos \theta - 14 \sin \theta$ $14 \sin \theta = -12 \cos \theta$ $\frac{\sin \theta}{\cos \theta} = \frac{-12}{14}$ $\tan \theta = -\frac{6}{7} / -0,86$	✓ ✓ ✓ ✓	(4)

QUESTION 7

7.1		✓ shape f ✓ intercepts ✓ turning point ✓ shape f ✓ intercepts ✓ turning point (6)
7.2.1	$x = -45^\circ$ or $x = 135^\circ$	✓ ✓ answer (2)
7.2.2	$x = 180^\circ$ or/ of $x = -180^\circ$	✓ ✓ answer (2)
7.2.3	$y \in [-1;1]$ or/ of $-1 \leq y \leq 1$	✓ answer (1)
7.2.4	1	✓ answer (1)
7.2.5	$x \in (-180^\circ; -45^\circ)$ or/ of $(0^\circ; 135^\circ)$ OR $-180^\circ < x < -45^\circ$ or/ of $0^\circ < x < 135^\circ$	✓ end points ✓ notation ✓ answer (3)
		[15]

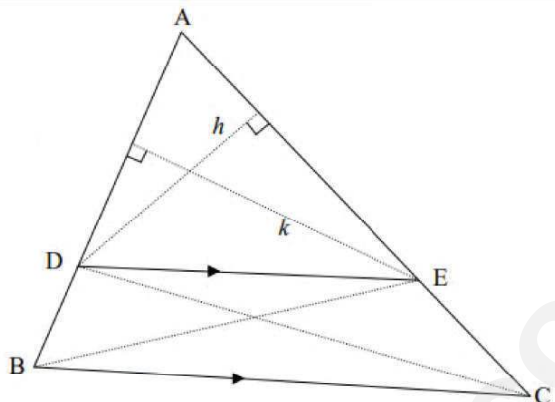
QUESTION 7

7.1	$\sin 43^\circ = \frac{45}{AC}$ $AC = 66m$ $\sin 43^\circ = \frac{45}{AC}$ $\sin 50^\circ = \frac{45}{AD}$ $AD = 58,74m$	$\checkmark \sin 43^\circ = \frac{45}{AC}$ $\checkmark AC$ $\checkmark \sin 43^\circ = \frac{45}{AC}$ $\checkmark AD$ (4)
7.2	$CD^2 = AC^2 + AD^2 - 2AC \cdot AD \cos 69^\circ$ $CD^2 = (66)^2 + (58.74)^2 - 2(66)(58.74)\cos 69^\circ$ $CD^2 = 5027,72$ $CD = 70.91m$	$\checkmark \text{ using cosine rule}$ $\checkmark \text{ substitution}$ $\checkmark \text{ answer}$ (3)

QUESTION 8

8.1.1	$\hat{D}_1 = \hat{B}_2 = 30^\circ$ (alter angles DC parallel GB)	✓ S&R (1)
8.1.2	$\hat{B}_1 = \hat{C}_1 = 60^\circ$ (tan-chord theorem)	✓ S ✓ R (2)
8.1.3	$\hat{C}_2 = 90^\circ$ (sum of angles in triangle)	✓ S&R (1)
8.1.4	$\hat{DAB} + \hat{C}_2 = 180^\circ$ $\hat{DAB} = 90^\circ$	✓ S ✓ R (2)
8.2	Yes is diameter, converse angle in a semi-circle	✓ S ✓ R (2)

QUESTION 9

9.1	 Construction: Connect DC and BE and draw the altitudes k and h $\frac{\text{Area } \triangle ADE}{\text{Area } \triangle BDE} = \frac{\frac{1}{2} \times AD \times k}{\frac{1}{2} \times BD \times k} = \frac{AD}{BD}$ $\frac{\text{Area } \triangle ADE}{\text{Area } \triangle DEC} = \frac{\frac{1}{2} \times AE \times h}{\frac{1}{2} \times EC \times h} = \frac{AE}{EC}$ but/maar: Area $\triangle BDE$ = Area $\triangle DEC$ [DE common base and DE $\parallel$ BC/ DE gemeensk basis en DE $\parallel$ BC] $\therefore \frac{\text{Area } \triangle ADE}{\text{Area } \triangle BDE} = \frac{\text{Area } \triangle ADE}{\text{Area } \triangle DEC}$ $\therefore \frac{AD}{BD} = \frac{AE}{EC}$	✓ constr ✓ S ✓ S ✓ S ✓ R ✓ conclusion (6)
9.2.1	tangent-chord theorem	✓ R (1)
9.2.2	In $\triangle ABC$ and $\triangle ADB$: $\hat{A}_1 = \hat{A}_1$ [common/gemeenskaplik] $\hat{B}_1 = \hat{D}_1$ [proven/bewys in 10.2.1] $\therefore \triangle ABC \parallel \triangle ADB$ [$\angle$; $\angle$; $\angle$] OR In $\triangle ABC$ and $\triangle ADB$: $\hat{A}_1 = \hat{A}_1$ [common/gemeenskaplik] $\hat{B}_1 = \hat{D}_1$ [proven/bewys in 10.2.1] $\hat{B}_1\hat{C}\hat{A} = \hat{B}_2$ [$\angle$s of $\Delta = 180^\circ$] $\therefore \triangle ABC \parallel \triangle ADB$	✓ S ✓ S ✓ R ✓ S ✓ S ✓ R
9.2.3	$\hat{E}_2 = \hat{F}_1$ [verwiss $\angle$e/alternate $\angle$s ; EA $\parallel$ GF] $\hat{F}_1 = \hat{D}_2$ [ext $\angle$ of cyc quad DGFC/buite $\angle$ v kdvh DGFC] $\therefore \hat{E}_2 = \hat{D}_2$	✓ S ✓ R ✓ S ✓ R

9.2.4	In $\triangle AEC$ and $\triangle ADE$: $\hat{A}_2 = \hat{A}_2 \quad [\text{common/gemeenskaplik}]$ $\hat{E}_2 = \hat{D}_2 \quad [\text{proven/bewys in 10.2.3}]$ $\therefore \triangle AEC \parallel \triangle ADE \quad [\angle ; \angle ; \angle]$ $\therefore \frac{AE}{AD} = \frac{AC}{AE}$ $\therefore AE^2 = AD \times AC$ OR In $\triangle AEC$ and $\triangle ADE$: $\hat{A}_2 = \hat{A}_2 \quad [\text{common/gemeenskaplik}]$ $\hat{E}_2 = \hat{D}_2 \quad [\text{proven/bewys in 10.2.3}]$ $\hat{ACE} = \hat{G}_1 \quad [\angle \text{ of } \Delta = 180^\circ \text{ OR ext } \angle \text{ of cyc quad DGFC/}$ buite $\angle$ v kdvh DGFC] $\therefore \triangle AEC \parallel \triangle ADE$ $\therefore \frac{AE}{AD} = \frac{AC}{AE}$ $\therefore AE^2 = AD \times AC$	✓ S ✓ S ✓ R ✓ S ✓ S ✓ S ✓ R ✓ S
9.2.5	$\frac{AB}{AD} = \frac{AC}{AB} \quad [\triangle ABC \parallel \triangle ADB]$ $AB^2 = AD \times AC$ $= AE^2 \quad [\text{from 10.2.4}]$ $\therefore AB = AE$	✓ S ✓ S ✓ S
		[21]



KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

MATHEMATICS P2

MARKING GUIDELINE

SEPTEMBER 2023

MARKS: 150

This marking guideline consists of 14 pages.

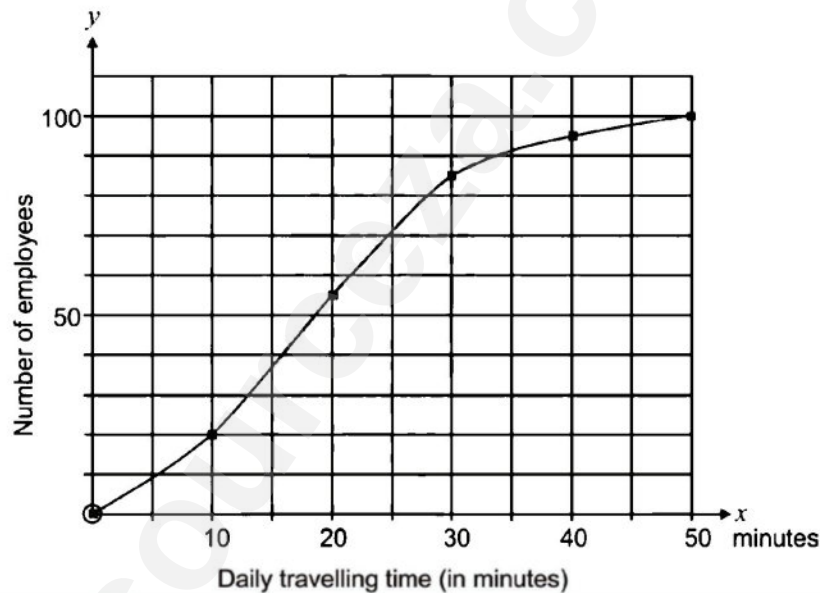
NB: CA APPLIES TO ALL SUB-QUESTIONS IN THIS MARKING GUIDELINE.

QUESTION 1

1.1	$a = 34,90$ $b = 0,08$ $y = 34,90 + 0,08x$ Answer only: Full Marks	✓A Value of a ✓A Value of b ✓CA Equation	(3)
1.2	$r = 0,88$	✓A ✓A Answer	(2)
1.3	$y = 34,90 + 0,08x$ $y = 34,90 + 0,08(560)$ $y = 79,7$ minutes = 1.33 hours Total cost = R150 + R150 = R300	✓CA Substitution ✓CA 79,7 minutes ✓CA Answer	(3)
1.4.1	Area = $100 \times 70 = 7000$ square meters $y = 34,90 + 0,08(7000)$ $y = 594,9$ minutes = 9.92 hours	✓CA Calculation	(1)
1.4.2	No. The time taken will exceed his daily 8 hour working hours.	✓CA Justification	(1)
			[10]

QUESTION 2

Daily travelling time x (in minutes)	Number of employees (f)	Midpoint of Interval (x)	$f \cdot x$
$0 \leq x < 10$	20	5	100
$10 \leq x < 20$	35	15	525
$20 \leq x < 30$	30	25	750
$30 \leq x < 40$	10	35	350
$40 \leq x < 50$	5	45	225
Total	100		1950

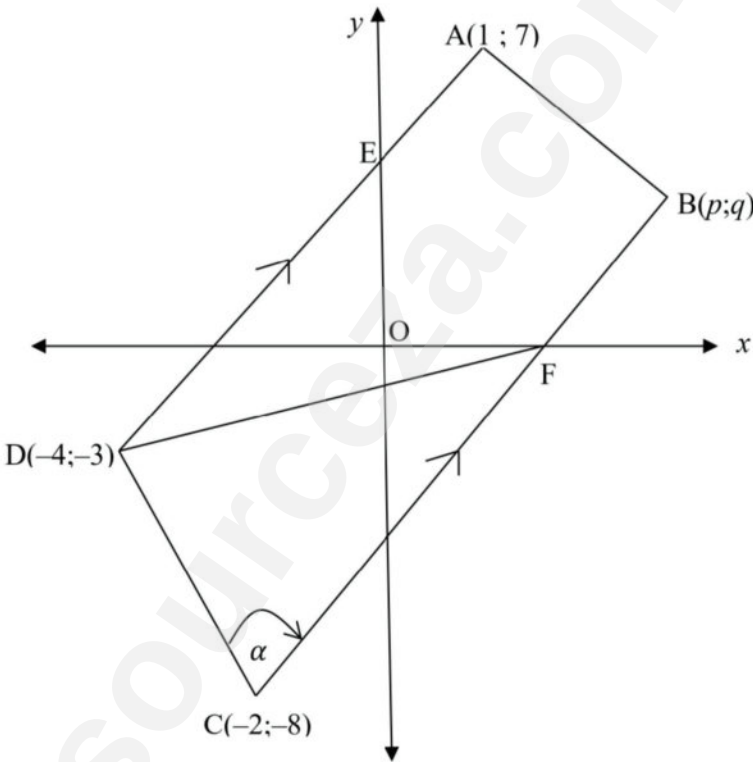


2.1	Estimated Mean = $\frac{1950}{100} = 19,5$ Answer only: Full Marks	✓A 1950 ✓A 100 ✓CA Answer	(3)
2.2	$10 \leq x < 20$	✓A ✓A Answer	(2)
2.3	See Diagram	✓A Minimum and Maximum value ✓A 1 st and 3 rd Quartiles ✓A 2 nd Quartile	(3)
Five number summary: 0 ; 12 ; 18 ; 26 ; 50 Accept: ± 1 deviation on quartiles			

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2.4.1	True	✓A	Answer	(1)
2.4.2	False	✓A	Answer	(1)
2.4.3	True	✓A	Answer	(1)
				[11]

QUESTION 3

			
3.1	$m_{AD} = \frac{7+3}{1+4} = \frac{10}{5} = 2$	✓A Substitution of points A and D ✓CA Answer	(2)
3.2	$m_{AD} = m_{BC} = 2 \dots (AD \parallel BC)$ $y = mx + c$ $-8 = 2(-2) + c$ $-4 = c$ $y = 2x - 4$	✓CA Gradient of BC ✓CA Substitution of point C and gradient ✓CA Answer	(3)

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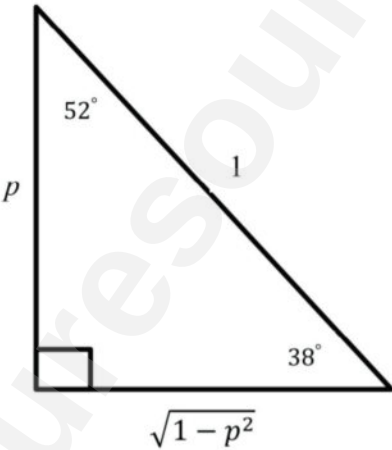
3.3	Let $y = 0$: $0 = 2x - 4$ $x = 2$ $F(2 ; 0)$	✓CA $0 = 2x - 4$ ✓CA x - value	(2)
3.4	$M(3 ; 2)$	✓A x - value ✓A y - value	(2)
3.5	$m_{CD} = \frac{-3 + 8}{-4 + 2} = \frac{5}{-2}$ Inclination of CD: $\tan \theta_1 = 180^\circ - 68,2^\circ = 111,8^\circ$ Inclination of CF: $\tan \theta_2 = 2$ $\theta_2 = 63,43^\circ$ Therefore $\alpha = 111,8^\circ - 63,43^\circ = 48,37^\circ$ OR $DF = \sqrt{(2 + 4)^2 + (0 + 3)^2} = \sqrt{45}$ $DC = \sqrt{29}$ $CF = \sqrt{80}$ $(\sqrt{45})^2 = (\sqrt{29})^2 + (\sqrt{80})^2 - 2\sqrt{29}\sqrt{80}\cos \alpha$ $\alpha = 48,37^\circ$	✓A Gradient of CD ✓A Inclination of CD ✓A Inclination of CF ✓A subtraction of the angles ✓A Distance of DF ✓A Distance of DC ✓A Distance of CF ✓A subst. into cosine rule	(4)
3.6	$DC = \sqrt{(-3 + 8)^2 + (-4 + 2)^2} = \sqrt{29}$ $FC = \sqrt{(0 + 8)^2 + (2 + 2)^2} = \sqrt{80}$ Area of $\triangle DCF = \frac{1}{2}(\sqrt{29})(\sqrt{80}) \sin 48,37^\circ$ $= 18$ square units.	✓A Length of CD ✓CA Length of FC ✓CA Substitution into area formula ✓CA Answer	(4)
			[17]

QUESTION 4

4.1.1	$x^2 + y^2 - 8x + 6y = 15$ $\text{LHS} = (2)^2 + (-9)^2 - 8(2) + 6(-9)$ $= 4 + 81 - 16 - 54$ $= 15$ $= \text{RHS}$	✓A Subst. of point ✓A Simplification	(2)
4.1.2	$(x - 4)^2 + (y + 3)^2 = 40$ Centre: C(4 ; -3) P(2 ; -9) $m_{\text{Radius}} = \frac{-3 + 9}{4 - 2} = \frac{6}{2} = 3$ $m_{\text{Tangent}} = -\frac{1}{3}$ Equation of Tangent: $y = mx + c$ $-9 = -\frac{1}{3}(2) + c$ $-\frac{25}{3} = c$ $y = -\frac{1}{3}x - \frac{25}{3}$	✓A writing the equation as $(x - 4)^2 + (y + 3)^2 = 40$ ✓CA Centre of circle ✓CA Gradient of radius ✓CA Gradient of tangent ✓CA Substitution ✓CA Answer	(6)
4.1.3	$r^2: (x - 4)^2 + (y + 3)^2 = 40$ $r^2 = 40$ $(\text{distance } Q \text{ to the centre})^2 = (-10 - 4)^2 + (12 + 3)^2$ $= 421$ $(\text{Length of tangent})^2 = 421 - 40 = 381$ $\text{Length of tangent} = \sqrt{381}$	✓CA Calculation of r^2 ✓CA distance calculation ✓CA Tangent calculation ✓CA Answer	(4)

4.2.1	$(x - 3)^2 + (y + 2)^2 = 25$ Let $x = 0$: $(0 - 3)^2 + (y + 2)^2 = 25$ $(y + 2)^2 = 16$ $y + 2 = \pm 4$ $y = -6$ or $y = 2$ $B(0; 2)$	✓A Letting $x = 0$ ✓A Simplification ✓CA y - values ✓CA Answer	(4)
4.2.2	$C(6; 2)$	✓CA x - value ✓CA y - value	(2)
4.2.3 (a)	$T(3; -2)$ and $M(12; 10)$ $TM^2 = (12 - 3)^2 + (10 + 2)^2 = 225$ $TM = 15$ units	✓A Coordinates of M ✓CA Answer	(2)
(b)	Radius, center T = 5 units and Radius, center M = 10 units Sum of radii = 15 units Circles touch . $TM = \text{Sum of radii}$	✓CA Sum of radii ✓CA Justification	(2)
			[22]

QUESTION 5

5.1			
5.1.1	$\cos 218^\circ$ $= -\cos 38^\circ$ $= -\frac{\sqrt{1-p^2}}{1}$	✓A Calculation of $\sqrt{1-p^2}$ ✓A Reduction ✓CA Answer	(3)

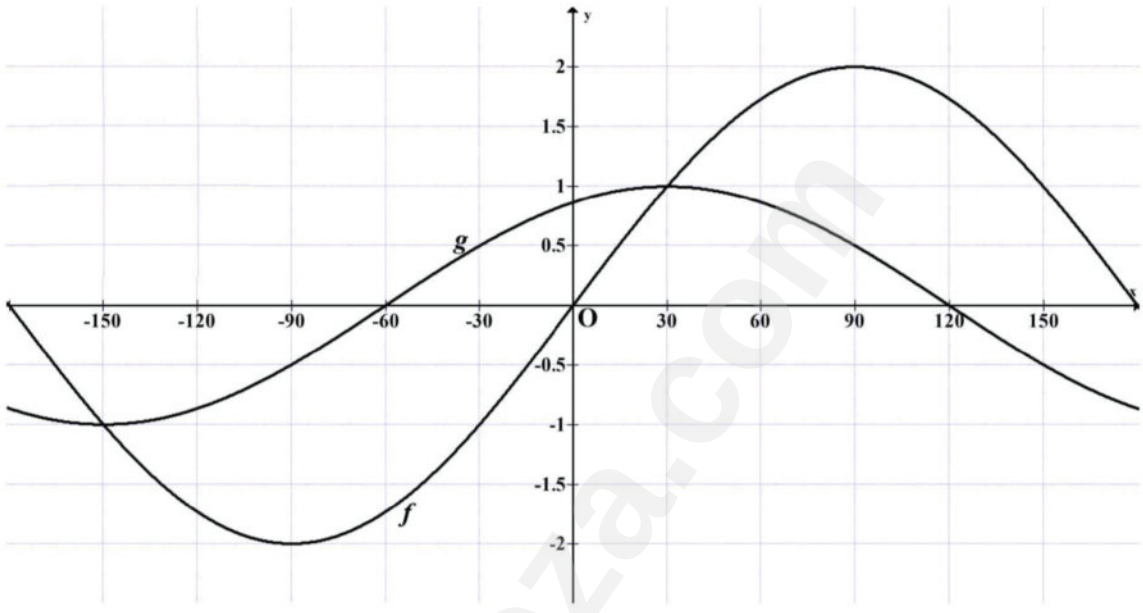
5.1.2	$\cos 14^\circ$ $= \cos(52^\circ - 38^\circ)$ $= \cos 52^\circ \cos 38^\circ + \sin 52^\circ \sin 38^\circ$ $= \left(\frac{p}{1}\right)\left(\frac{\sqrt{1-p^2}}{1}\right) + \left(\frac{\sqrt{1-p^2}}{1}\right)\left(\frac{p}{1}\right)$ $= 2p\sqrt{1-p^2}$ OR $\cos 24^\circ = \sin 76^\circ$ $= \sin 2(38^\circ)$ $= 2 \sin 38^\circ \cos 38^\circ$ $= 2p\sqrt{1-p^2}$	$\checkmark$ A Writing as difference $\checkmark$ A Expansion $\checkmark$ CA Answer $\checkmark$ $\sin 76^\circ$ $\checkmark$ double angle $\checkmark$ answer	(3)
5.1.3	$\sin 26^\circ \cos 26^\circ$ $= \frac{1}{2} \sin 52^\circ$ $= \frac{1}{2} \sqrt{1-p^2}$	$\checkmark$ A Double angle $\checkmark$ CA Answer	(2)
5.2	$\frac{2 \sin 165^\circ \cos 195^\circ}{\cos 45^\circ \sin 15^\circ - \cos 15^\circ \sin 45^\circ}$ $= \frac{2 \sin 15^\circ \cdot (-\cos 15^\circ)}{\cos 45^\circ \sin 15^\circ - \cos 15^\circ \sin 45^\circ}$ $= \frac{-2 \sin 30^\circ}{\sin(15^\circ - 45^\circ)}$ $= \frac{-2 \sin 30^\circ}{\sin(-30^\circ)}$ $= \frac{-2 \sin 30^\circ}{-\sin 30^\circ}$ $= 2$	$\checkmark$ A $-\cos 15^\circ$ $\checkmark$ A $-2 \sin 30^\circ$ $\checkmark$ A $\sin(15^\circ - 45^\circ)$ $\checkmark$ A $-\sin 30^\circ$ $\checkmark$ CA Answer	(5)
5.3.1	$K = \sqrt{3} \cos x + \sin x$ $K = 2 \left(\frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x \right)$ $K = 2(\sin 60^\circ \cos x + \cos 60^\circ \sin x)$ $K = 2 \sin(60^\circ + x)$	$\checkmark$ A $2 \left(\frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x \right)$ $\checkmark$ A $\sin 60^\circ$ and $\cos 60^\circ$ $\checkmark$ A $2 \sin(60^\circ + x)$	(3)

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5.3.2	$t = 2$ and $\theta = 60^\circ$	✓CA t – value and $\theta = 60^\circ$	(1)
5.3.3	2	✓CA Answer	(1)
5.4.1	$\text{LHS} = \frac{2 \tan \theta - \sin 2\theta}{2 \sin^2 \theta}$ $= \frac{2 \left(\frac{\sin \theta}{\cos \theta} \right) - 2 \sin \theta \cos \theta}{2 \sin^2 \theta} \times \frac{\cos \theta}{\cos \theta}$ $= \frac{2 \sin \theta - 2 \sin \theta \cos^2 \theta}{2 \sin^2 \theta \cos \theta}$ $= \frac{2 \sin \theta (1 - \cos^2 \theta)}{2 \sin^2 \theta \cos \theta}$ $= \frac{2 \sin \theta \cdot \sin^2 \theta}{2 \sin^2 \theta \cdot \cos \theta}$ $= \frac{\sin \theta}{\cos \theta}$ $= \tan \theta$ $= \text{LHS}$	✓A $\frac{\sin \theta}{\cos \theta}$ ✓A $2 \sin \theta \cos \theta$ ✓A simplification ✓A factorizing ✓A $1 - \cos^2 \theta = \sin^2 \theta$ ✓A simplified to $\frac{\sin \theta}{\cos \theta}$	(6)
5.4.2	$2 \sin^2 \theta = 0$ $\sin \theta = 0$ $\therefore \theta = 180^\circ \text{ and } 360^\circ$ $\theta = 270^\circ$	✓ 180° and 360° ✓ 270°	(2)
			[26]

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QUESTION 6

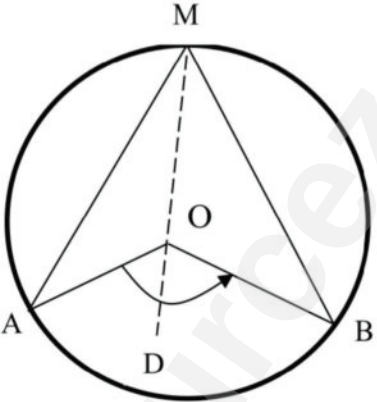
6.1			
	Graph of f: 1A mark for x – intercepts 1A marks for minimum and maximum points 1A mark for shape Graph of g: 1A mark for end points 1A mark for x – intercepts 1A mark for y – intercept (6)		
6.2.1	360°	✓A Answer	(1)
6.2.2	$x \in [-180^\circ; -150^\circ) \cup (30^\circ; 180^\circ]$	$[-180^\circ; -150^\circ)$ ✓A : Notation ✓CA : values $(30^\circ; 180^\circ]$ ✓A : Notation ✓CA : values	(4)
6.2.3	$f(x) = 1.5 + g(x)$ $f(x) - g(x) = 1.5$ $x = 90^\circ$ or $x = 150^\circ$	✓A $f(x) - g(x) = 1.5$ ✓A Answer	(2)
			[13]

QUESTION 7

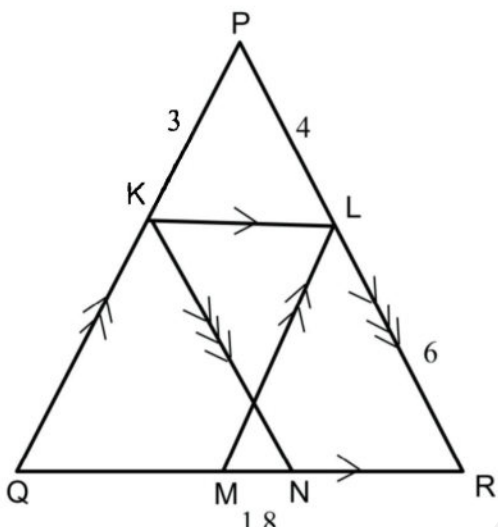
7.1	$\frac{PQ}{QR} = \tan \theta$ $PQ = QR \tan \theta$	✓ A Answer	(1)
7.2	$\frac{QR}{\sin \hat{S}} = \frac{QS}{\sin \hat{QSR}}$ $\frac{QR}{\sin \beta} = \frac{x}{\sin \alpha}$ $QR = \frac{x \sin \beta}{\sin \alpha}$ $PQ = \frac{x \sin \beta \tan \theta}{\sin \alpha}$	✓ A Sine rule formula ✓ A Subs. Sine rule ✓ A Making QR a subj. of the formula ✓ A Subst. of QR	(4)
7.3	$\text{Area of } \triangle QSR = \frac{1}{2}(x)(8 - 2x) \sin 60^\circ$ $= \frac{1}{2}(x)(8 - 2x) \left(\frac{\sqrt{3}}{2}\right)$ $= \frac{\sqrt{3}}{4}(x)(8 - 2x)$ $= \sqrt{3}(x) \left(2 - \frac{1}{2}x\right)$ $= 2\sqrt{3}x - \frac{\sqrt{3}}{2}x^2$	✓ A Subst. into Area rule ✓ A $\frac{\sqrt{3}}{2}$ ✓ A Simplifying	(3)

7.4	For Max Area: $x = -\frac{b}{2a}$ $x = -\frac{(2\sqrt{3})}{2\left(-\frac{\sqrt{3}}{2}\right)}$ $x = 2$	✓ A Formula ✓ A Substitution into formula ✓ CA Answer	(3)
			[11]

QUESTION 8

8.1	 <u>Constr:</u> Join MO and produce to D. $\widehat{AOD} = \widehat{OAM} + \widehat{AMO} \dots$ (Ext. $\angle$ of Δ) $\widehat{BOD} = \widehat{OBM} + \widehat{BMO} \dots$ (Ext. $\angle$ of Δ) But $\widehat{OAM} = \widehat{AMO}$ and $\widehat{OBM} = \widehat{BMO} \dots$ (Radii =) $\therefore \widehat{AOD} + \widehat{BOD} = 2\widehat{AMO} + 2\widehat{BMO}$ $\widehat{AOB} = 2(\widehat{AMO} + \widehat{BMO})$ $\widehat{AOB} = 2\widehat{M}$ NOTE No construction : No marks	✓ A Construction ✓ A S/R ✓ A S ✓ A S ✓ A S	(5)
8.2.1	$\widehat{OVA} = \widehat{ODA} = 90^\circ \dots$ (Radius $\perp$ Tangent) VODA is a cyclic quad. (Converse of opposite angles of quad. Supplementary)	✓ A S/R ✓ A R	(2)
8.2.2	$\widehat{O}_1 = 40^\circ \dots$ (Exterior angle of cyclic quad = int. opp. Angle)	✓ S ✓ R	(2)

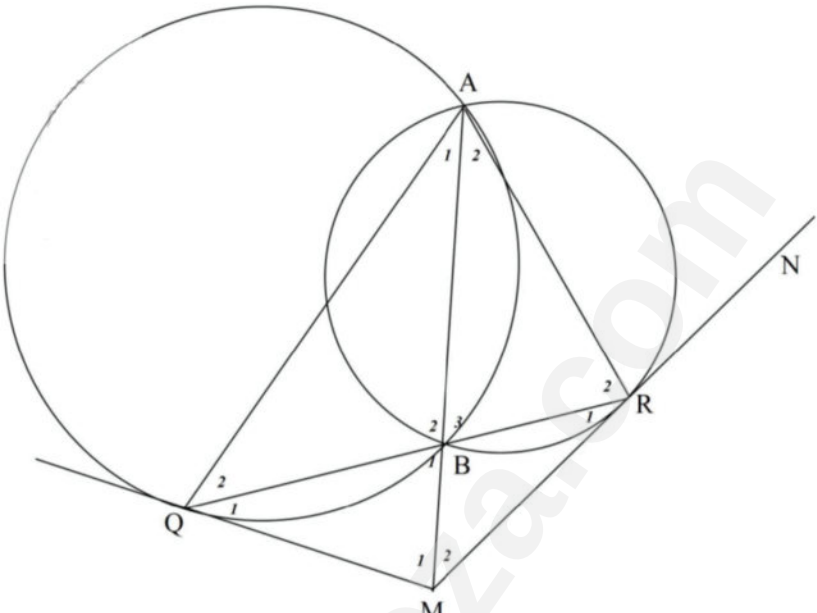
8.2.3	$\widehat{V}_1 = \frac{180^\circ - 40^\circ}{2} = 70^\circ$(sum of $\angle s$ of Δ ; radii) $\widehat{ADM} = \frac{180^\circ - 40^\circ}{2} = 70^\circ$(sum of $\angle s$ of Δ ; Tangents drawn from a common point A) $\widehat{ADM} = \widehat{O}_2 = 70^\circ$... (Angles subtended by common chord AV) $BV \parallel OA$(Converse of Alt $\angle s$ or Alt $\angle s$ are =) OR In ΔOVA and ΔODA $OV=OD$(radii) $OA=OA$(common) $AV=AD$(tangents from the same point) $\Delta OVA \equiv \Delta ODA$(SSS) $A_1=A_2$($\equiv \Delta s$) $=20^\circ$ $A_2=V_2$($\angle s$ in the same segment) $=20^\circ$ $V_1=90^\circ - 20^\circ=70^\circ$ $O_2=180^\circ - [OVA + A_1]$ $=180^\circ - [90^\circ + 20^\circ]$ $=70^\circ$ $\therefore \widehat{V}_1 = \widehat{O}_2$(both $=70^\circ$) $\therefore BV \parallel OA$ (alt $\angle s$ are = or conv.alt $\angle s$)	$\checkmark$ S/R $\checkmark$ S/R $\checkmark S$ $\checkmark R$ $\checkmark$ R $\checkmark A$ $\Delta OVA \equiv \Delta ODA$(SSS) $\checkmark A$ S/R $\checkmark A$ $V_1=70^\circ$ $\checkmark A$ $O_2=70^\circ$ $\checkmark A$ R	(5)
			[14]

9.1	Divides the other two sides, proportionally.	✓S ✓S	<u>divides</u> the other two sides proportionally	(2)
9.2				

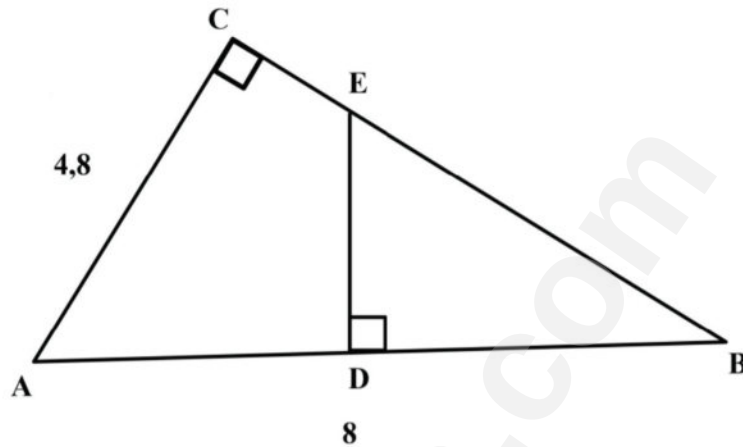
9.2.1	$\frac{KQ}{3} = \frac{6}{4} \dots\dots\dots(\text{Prop. Thm; } KL \parallel QR)$ $KQ = 4,5 \text{ units}$	✓ S/R ✓ Answer	(2)
9.2.2	$KL = QM \dots(\text{Opposite sides of } \parallel^m \text{ QKLM are equal})$ $KL = NR \dots (\text{Opposite sides of } \parallel^m \text{ NKLR are equal})$ Therefore $QM = NR$ (both $\dots KL$) OR $\frac{QN}{QR} = \frac{QP}{PR} \quad \text{Prop Thm, } KN \parallel PR$ $\frac{4,5}{3} = \frac{3}{2}$ $\frac{MR}{QM} = \frac{RL}{PL} \quad \text{Prop Thm, } ML \parallel QP$ $= \frac{6}{4} = \frac{3}{2}$ $\therefore \frac{QN}{NR} = \frac{MR}{QM}$ $\frac{QM+1,8}{NR} = \frac{NR+1,8}{QM}$ $\therefore QM(QM+1,8) = MR(NR+1,8)$ $\therefore QM = NR$	✓ S/R ✓ S/R $\checkmark \frac{QN}{QR} = \frac{QP}{PR} \quad \text{Prop Thm, } KN \parallel PR$ OR $\frac{MR}{QM} = \frac{RL}{PL} \quad \text{Prop Thm, } ML \parallel QP$ $\checkmark \frac{QM+1,8}{NR} = \frac{NR+1,8}{QM}$	(2)

[16]

QUESTION 10

			
10.1	In Δ's MQA and MBQ $\hat{M}_1 = \hat{M}_1$(Common) $\hat{A}_1 = \hat{Q}_1$(Tan-Chord Theorem) $\hat{AQM} = \hat{B}_1$(Remaining $\angle$s of Δ's) $\Delta MQA \parallel \Delta MBQ$(AAA)	✓ S/R ✓ S/R ✓ R	(3)
10.2	In Δ's MAR and MRB $\hat{M}_2 = \hat{M}_2$(Common) $\hat{A}_2 = \hat{R}_1$(Tan-Chord Thm) $\hat{ARM} = \hat{RBM}$(Remaining $\angle$s of Δ's) $\Delta MAR \parallel \Delta MRB$(AAA) $\frac{MA}{MR} = \frac{MR}{MB}$(Δ's similar) $MR^2 = AM \cdot MB$	✓ S ✓ S/R ✓ R ✓ S A✓R	(5)
10.3	$\frac{MQ}{MB} = \frac{MA}{MQ}$(from 10.1) $MQ^2 = MB \cdot MA = \dots \dots (1)$ Also $MR^2 = AM \cdot MB$ Now $\frac{MQ^2}{MR^2} = \frac{MB \cdot MA}{AM \cdot MB}$ $\therefore \frac{MQ^2}{MR^2} = 1$	✓ $\frac{MQ}{MB} = \frac{MA}{MQ}$ ✓ $MQ^2 = MB \cdot MA$ ✓✓ $\frac{MQ^2}{MR^2} = \frac{MB \cdot MA}{AM \cdot MB}$	(4)
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QUESTION 11



11.1	$BC^2 = 8^2 - 4,8^2 \dots\dots (TOP)$ $BC = 6,4 \text{ cm}$	✓ S ✓ S	(2)
11.2	$\triangle BED$	✓ S	(1)
11.3	$\frac{BA}{BE} = \frac{AC}{ED} = \frac{BC}{BD}$ $\frac{8}{BE} = \frac{4,8}{ED} = \frac{6,4}{4}$ $ED = \frac{4,8 \times 4}{6,4} = 3 \text{ cm}$ Area of $\triangle EDB$ $= \frac{1}{2} (4)(3) = 6 \text{ cm}^2$ Area of $\triangle ABC$ $= \frac{1}{2} (4,8)(6,4) = 15,6 \text{ cm}^2$ Therefore, Area of ADEC $= 15,6 - 6 = 9,6 \text{ cm}^2$	$\triangle BAC \parallel \triangle BED$ ✓A S ✓CA Value of ED ✓CA Area of $\triangle EDB$ ✓CA Area of $\triangle ABC$ ✓CA Area of ADEC	(5)
			[8]

GRAND TOTAL: 150

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

MATHEMATICS PAPER 2

SEPTEMBER 2023

MARKING GUIDELINES

MARKS: 150

NOTE:

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Please turn over/Blaai aseblief om

- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking guidelines.
- Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:

- As 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.
- As 'n kandidaat 'n antwoord deurgehaal en nie oorgedoen het nie, sien die deurgehaalde antwoord na.
- Volgehoue akkuraatheid is op ALLE aspekte van die nasienriglyn van toepassing.
- Dit is onaanvaarbaar om waardes/antwoorde te veronderstel om 'n probleem op te los.

QUESTION/VRAAG 1

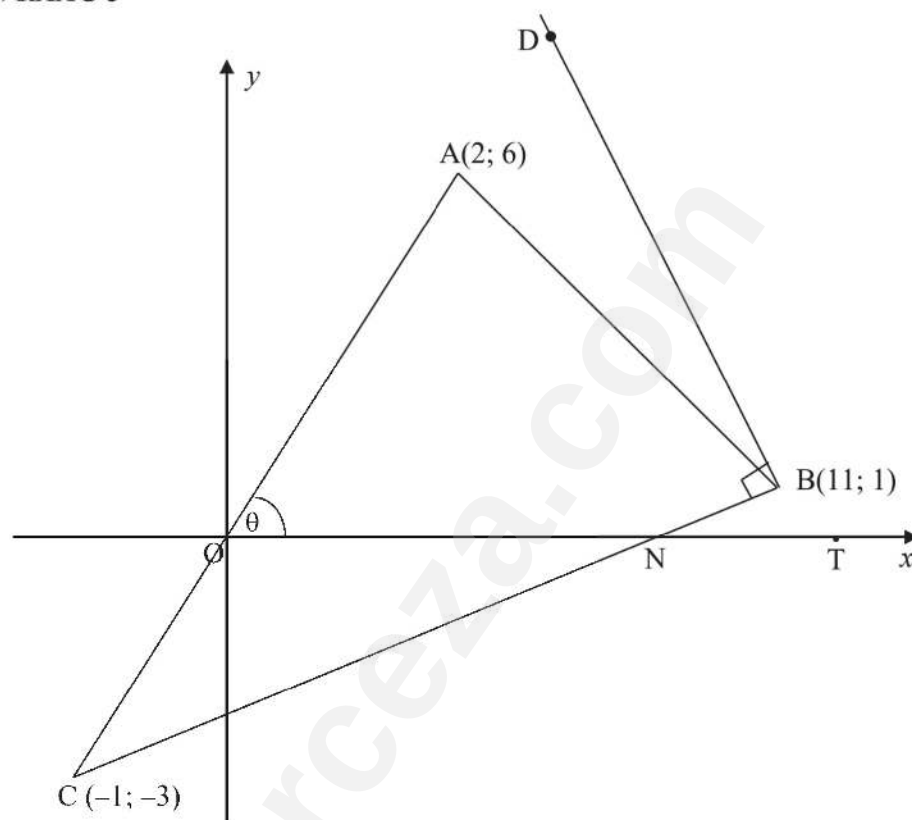
71	83	88	91	92	92	95	97	104	108	109	110	111	115	129
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1.1	$\frac{1495}{15} = 99,67$	✓✓ 99,67 (2)
1.2	$\sigma = 14,06$	✓SD (1)
1.3	$99,67 - 14,06 = 85,61$ $\frac{2}{15} \times 100 = 13,33\%$	✓boundary ✓ $\frac{2}{15}$ ✓answer (3)
1.4.1	96,67	✓answer (1)
1.4.2	14,06	✓answer (1)
		[8]

QUESTION/VRAAG 2

2.1.1	True	✓ answer (1)
2.1.2	Positively skewed / Skewed to the right	✓ answer (1)
2.1.3	Range for company A = $800 - 200 = 600$ Range for company B = $600 - 100 = 500$ Biggest range : Company A	✓ both ranges ✓ Company A (2)
2.1.4	$20 \times 75\% = 15$ workers	✓ 75% ✓ 15 workers (2)
2.2.1	$y = 5965,51 - 2,93x$	✓ 5329,84 ✓ -2,61 ✓ equation (3)
2.2.2	$r = -0,49$	✓ (1)
2.2.3	$y = 5965,51 - 2,93(2018)$ $= 52,77$ The correlation coefficient is moderate thus the value is fairly reliably predicted.	✓ prediction ✓ answer + reason (2)
		[12]

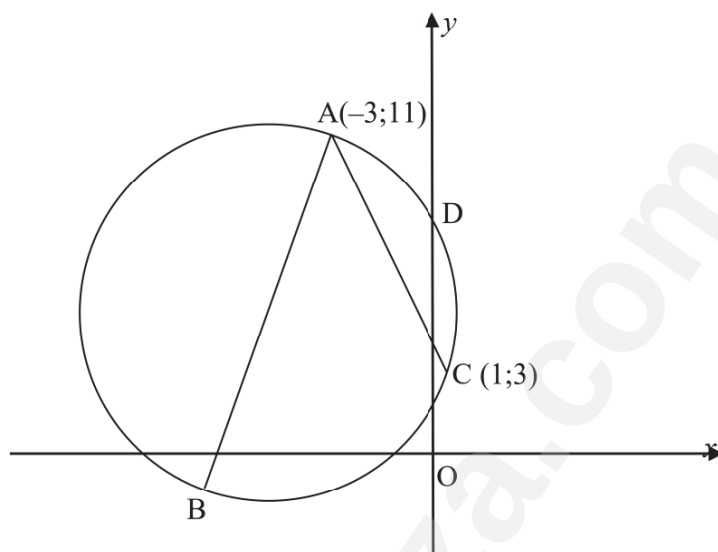
QUESTION/VRAAG 3



3.1.1	$N : (x; 0)$ $m_{NC} = \frac{0 - (-3)}{x - (-1)} = \frac{1}{3}$ $x + 1 = 9$ $x = 8$ OR $y - (-3) = \frac{1}{3}(x - (-1))$ $y = \frac{1}{3}x - \frac{8}{3}$ For $y = 0$: $x = 8$	✓ subst into gradient formula ✓ answer (2) ✓ subst into equation ✓ answer
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3.1.2	$\tan \theta = 3$ $\theta = 71,565...^\circ$ $\tan \hat{BNT} = \frac{1}{3}$ $\hat{BNT} = 18,434...^\circ$ $\hat{C} = 71,565...^\circ - 18,434...^\circ$ $= 53,13^\circ$ Accept $\hat{C} = 53,14^\circ$	$\checkmark$ substitution $\checkmark \theta$ $\checkmark$ substitution $\checkmark \hat{BNT}$ $\checkmark$ answer (5)
3.2	$y - 6 = 3(x - 2)$ $y = 3x$	$\checkmark$ substitution $\checkmark$ equation (2)
3.3	$m_\perp = -3$ $y - 1 = -3(x - 11)$ $y = -3x + 34$ $3x = -3x + 34$ $6x = 34$ $x = \frac{17}{3} = 5,67$ $y = 3\left(\frac{17}{3}\right) = 17$	$\checkmark m_\perp = -3$ $\checkmark$ equation $\checkmark$ setting equations equal $\checkmark$ value of x $\checkmark$ value of y (5)
		[14]

QUESTION/VRAAG 4

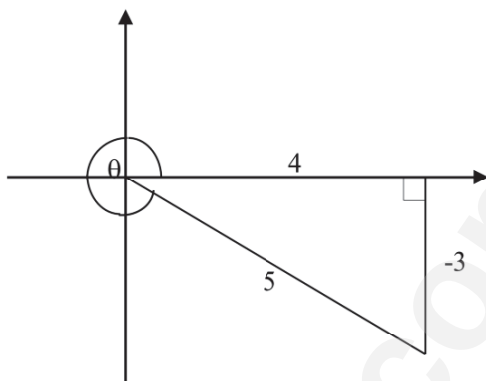


4.1	$m_{AC} = \frac{11 - 3}{-3 - 1} = \frac{8}{-4} = -2$ $mdpt_{AC} = (-1; 7)$ $y - 7 = \frac{1}{2}(x - (-1))$ $y = \frac{1}{2}x + \frac{15}{2}$	✓ gradient of AC. ✓ Midpt AC ✓ $\perp$ gradient and subst ✓ equation (4)
4.2	$\frac{1}{2}x + \frac{15}{2} = 3x + 20$ $-2,5x = 12,5$ $x = -5$ $y = 3(-5) + 20 = 5$ $\therefore \text{centre}(-5; 5)$	✓ set equations equal ✓ solve x ✓ solve y (3)
4.3	$\text{radius} = \sqrt{(-5 - (-3))^2 + (5 - 11)^2}$ $= \sqrt{40} = 2\sqrt{10}$ $\text{diameter} = 4\sqrt{10} \text{ units}$ $= 12,65$	✓ substitution in formula ✓ radius ✓ diameter (3)
4.4	$(x + 5)^2 + (y - 5)^2 = 40$	✓ $(x + 5), (y - 5)$ ✓ 40 (2)

4.5	$m_{rad} = \frac{11-5}{-3-(-5)} = 3$ $m_{tan} = -\frac{1}{3}$ $y - 11 = -\frac{1}{3}(x - (-3))$ $y = -\frac{1}{3}x + 10$ $(0; p): p = -\frac{1}{3}(0) + 10$ $p = 10$	✓ gradient ✓ subst in form ✓ equation ✓ value of p (4)
4.6	$(x + 2)^2 + (y - 7)^2 = (\sqrt{10})^2$ $(x + 2)^2 + (y - 7)^2 = 10$	✓ $(x + 2)$ ✓ $(y - 7)$ ✓ 10 (3)
4.7	Centres: $(2; 3)$ and $(-5; 5)$ Distance between centres = $\sqrt{(2 - (-5))^2 + (3 - 5)^2}$ $= \sqrt{53} = 7,28$ Sum of radii = $2 + 2\sqrt{10} = 8,32$ Distance between centres < Sum of radii $\therefore$ They will intersect.	✓ subst of centres ✓ distance ✓ Sum of radii ✓ conclusion (4)
		[23]

QUESTION 5

5.1

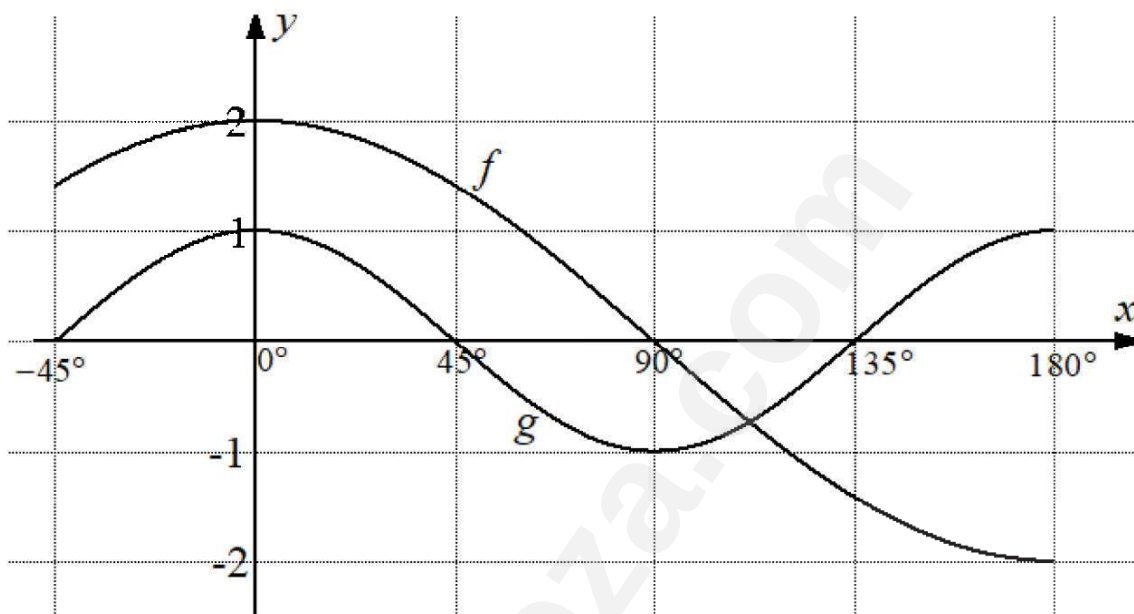


5.1.1	$(-3)^2 + 4^2 = r^2$ $r^2 = 25$ $r = 5$ $\sin \theta = \frac{-3}{5}$	✓ value of r ✓ answer (2)
5.1.2	$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ $= \left(\frac{4}{5}\right)^2 - \left(\frac{-3}{5}\right)^2$ $= \frac{16}{25} - \frac{9}{25}$ $= \frac{7}{25}$	✓ expansion ✓ substitution ✓ answer (3)
5.1.3	$\cos(\theta + 30^\circ) = \cos \theta \cos 30^\circ - \sin \theta \sin 30^\circ$ $= \left(\frac{4}{5}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{-3}{5}\right)\left(\frac{1}{2}\right)$ $= \frac{4\sqrt{3} + 3}{10}$	✓ expansion ✓ substitution ✓ answer (3)
5.2	$(4 \sin \alpha)^2 + (4 \cos \alpha)^2$ $= 16 \sin^2 \alpha + 16 \cos^2 \alpha$ $= 16(\sin^2 \alpha + \cos^2 \alpha)$ $= 16(1) = 16$	✓ simplification ✓ answer (2)

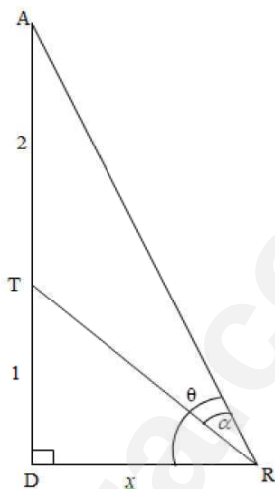
5.3	$\begin{aligned} & \sin(90^\circ - x) \cdot \cos(-x) - \sin(x - 180^\circ) \cdot \sin(90^\circ + x) \\ &= \sin(180^\circ - x) \cdot \cos(360^\circ - x) - \sin(180^\circ + x) \cdot \sin(90^\circ + x) \\ &= \sin x \cos x - (-\sin x) \cos x \\ &= \sin x \cos x + \sin x \cos x \\ &= 2 \sin x \cos x \\ &= \sin 2x \end{aligned}$	$\checkmark \sin x$ $\checkmark \cos x$ $\checkmark -\sin x$ $\checkmark \cos x$ $\checkmark \text{simplification}$ $\checkmark \text{double angle identity}$	(6)
5.4.1	$\frac{\sin 7x + \sin x}{2 \cos 3x} = \sin 4x$ $\begin{aligned} \text{LHS} &= \frac{\sin(4x + 3x) + \sin(4x - 3x)}{2 \cos 3x} \\ &= \frac{\sin 4x \cos 3x + \cos 4x \sin 3x + \sin 4x \cos 3x - \cos 4x \sin 3x}{2 \cos 3x} \\ &= \frac{\sin 4x \cos 3x + \sin 4x \cos 3x}{2 \cos 3x} \\ &= \frac{2 \sin 4x \cos 3x}{2 \cos 3x} \\ &= \sin 4x = \text{RHS} \end{aligned}$	$\checkmark \text{both brackets}$ $\checkmark \sin(4x + 3x)$ expansion $\checkmark \sin(4x - 3x)$ expansion $\checkmark \text{simplification}$	(4)
5.4.2	$\begin{aligned} 2 \cos 3x &= 0 \\ \cos 3x &= 0 \\ 3x &= \begin{cases} 90^\circ + 360^\circ k \\ -90^\circ + 360^\circ k \end{cases} \text{ for } k \in \mathbb{Z} \\ x &= \begin{cases} 30^\circ + 120^\circ k \\ -30^\circ + 120^\circ k \end{cases} \end{aligned}$ OR $\begin{aligned} 2 \cos 3x &= 0 \\ \cos 3x &= 0 \\ 3x &= \begin{cases} 90^\circ + 360^\circ k \\ 270^\circ + 360^\circ k \end{cases} \text{ for } k \in \mathbb{Z} \\ x &= \begin{cases} 30^\circ + 120^\circ k \\ 90^\circ + 120^\circ k \end{cases} \end{aligned}$	$\checkmark 2 \cos 3x = 0$ $\checkmark 3x = \begin{cases} 90^\circ + 360^\circ k \\ -90^\circ + 360^\circ k \end{cases} \text{ for } k \in \mathbb{Z}$ $\checkmark x = \begin{cases} 30^\circ + 120^\circ k \\ -30^\circ + 120^\circ k \end{cases}$ OR $\checkmark 2 \cos 3x = 0$ $\checkmark 3x = \begin{cases} 90^\circ + 360^\circ k \\ 270^\circ + 360^\circ k \end{cases} \text{ for } k \in \mathbb{Z}$ $\checkmark x = \begin{cases} 30^\circ + 120^\circ k \\ 90^\circ + 120^\circ k \end{cases}$	(3)

5.5	$\sin(3x + 20^\circ) = \cos x$ $\sin(3x + 20^\circ) = \sin(90^\circ - x)$ $3x + 20^\circ = 90^\circ - x + 360^\circ k \quad k \in \mathbb{Z}$ $4x = 70^\circ + 360^\circ k$ $x = 17,5^\circ + 90^\circ k$ OR $3x + 20^\circ = 180^\circ - (90^\circ - x) + 360^\circ k \quad k \in \mathbb{Z}$ $3x + 20^\circ = 180^\circ - 90^\circ + x + 360^\circ k$ $2x = 70^\circ + 360^\circ k$ $x = 35^\circ + 180^\circ k$ OR $\sin(3x + 20^\circ) = \cos x$ $\cos[90^\circ - (3x + 20^\circ)] = \cos x$ $-3x + 70^\circ = x + 360^\circ k \quad k \in \mathbb{Z}$ $-4x = -70^\circ + 360^\circ k$ $x = 17,5^\circ + 90^\circ k$ OR $-3x + 70^\circ = -x + 360^\circ k \quad k \in \mathbb{Z}$ $-2x = -70^\circ + 360^\circ k$ $x = 35^\circ + 180^\circ k$	$\checkmark \sin(90 - x)$ $\checkmark 3x + 20^\circ = 90^\circ - x + 360^\circ k \quad k \in \mathbb{Z}$ $\checkmark x = 17,5^\circ + 90^\circ k$ $\checkmark 3x + 20^\circ = 180^\circ - (90^\circ - x) + 360^\circ k$ $\checkmark 2x = 70^\circ + 360^\circ k$ $\checkmark x = 35^\circ + 180^\circ k$ (6) $\checkmark \cos[90^\circ - (3x + 20^\circ)]$ $\checkmark -4x = -70^\circ + 360^\circ k$ $\checkmark x = 17,5^\circ + 90^\circ k$ $\checkmark -3x + 70^\circ = -x + 360^\circ k \quad k \in \mathbb{Z}$ $\checkmark -2x = -70^\circ + 360^\circ k$ $\checkmark x = 35^\circ + 180^\circ k$
		[29]

QUESTION 6



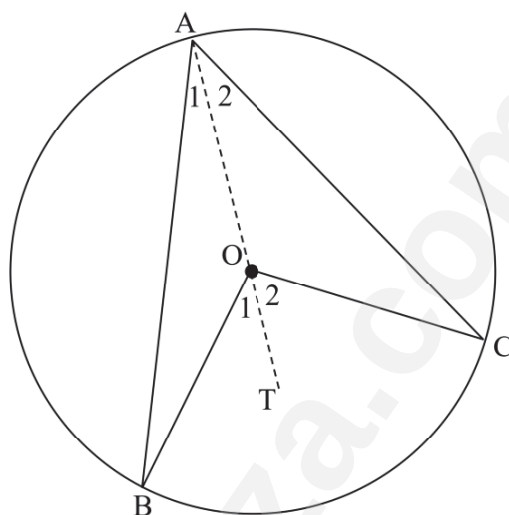
6.1	180°	✓ answer (1)
6.2		✓ critical values ✓ notation (2)
6.3	Range of g : $y \in [-1; 1]$ Range of $3g$: $y \in [-3; 3]$ Range of $y = 3g(x) - 1$: $-4 \leq y \leq 2$ or $y \in [-4; 2]$	✓ critical values ✓ notation (2)
6.4	$2\cos x = \frac{1}{2}$ $\cos x = \frac{1}{4}$ $x = 75,5^\circ$ $-45^\circ \leq x \leq 75,5^\circ$	✓ equation ✓ $x = 75,5^\circ$ ✓ critical values ✓ notation (4)
6.5	$\frac{1}{2}\cos^2 x - \frac{1}{4}$ $= \frac{1}{4}(2\cos^2 x - 1)$ $= \frac{1}{4}(\cos 2x)$ $= \frac{1}{4}(-1)$ $= -\frac{1}{4}$	✓ factor ✓ double angle ✓ min of $\cos 2x$ ✓ answer (4)



7.1	$\hat{TRD} = \theta - \alpha$ $\cos(\theta - \alpha) = \frac{x}{TR}$ $TR = \frac{x}{\cos(\theta - \alpha)}$	$\checkmark \hat{TRD}$ $\checkmark \cos \text{ definition}$	(2)
7.2	$\hat{A} = 90^\circ - \theta$ $\frac{TR}{\sin(90^\circ - \theta)} = \frac{AT}{\sin \alpha}$ $\frac{TR}{\cos \theta} = \frac{2}{\sin \alpha}$ $TR = \frac{2 \cos \theta}{\sin \alpha}$ $\frac{x}{\cos(\theta - \alpha)} = \frac{2 \cos \theta}{\sin \alpha}$ $x = \frac{2 \cos \theta \cos(\theta - \alpha)}{\sin \alpha}$	$\checkmark x$ $\checkmark TR$ $\checkmark \text{Subst in area-rule}$ $\checkmark \text{answer}$	(4)

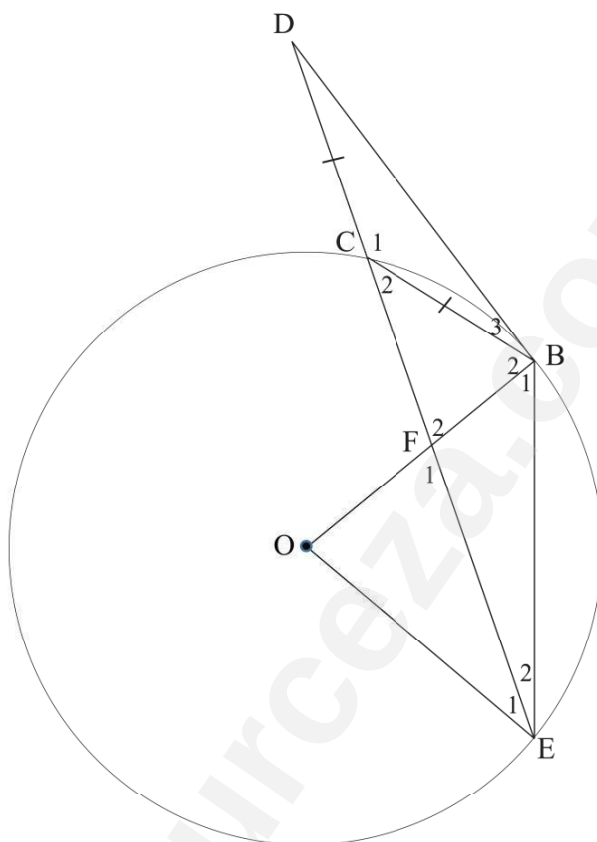
7.3	$x = \frac{2 \cos 68,33 \cos(68,33 - 28^\circ)}{\sin 28^\circ}$ $= 1,1992...$ $TR = \frac{1,1992}{\cos(\theta - \alpha)}$ $= 1,5730...$ $\hat{A} = 21,67^\circ$ $\hat{ATR} = 130,33^\circ$ $Area \Delta ATR = \frac{1}{2}(2)(1,5730) \sin 130,33^\circ$ $= 1,1992...$ $= 1,20 units^2$	✓ x ✓ TR ✓ subst in area formula ✓ answer (4)
		[10]

QUESTION 8



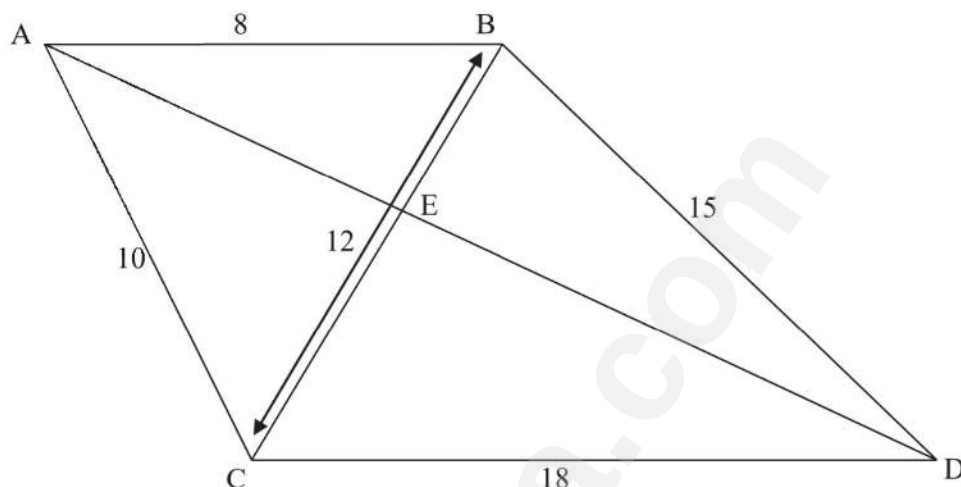
8.1	Draw line from A through O to T let $\hat{A}_1 = x$ and $\hat{A}_2 = y$ $\hat{A}_1 = \hat{B} = x$ ($\angle$'s opp = sides) $\hat{O}_1 = 2x$ (<i>ext</i> $\angle$ of Δ) similarly $\hat{O}_2 = 2y$ $\hat{BOC} = 2x + 2y = 2(x + y)$ $\therefore \hat{BOC} = 2 \times \hat{BAC}$	✓ construction ✓ S/R ✓ S/R ✓ S ✓ deduction (5)
-----	--	--

8.2



8.2.1	$\hat{D} = x$ ($\angle$ opp = sides) $\hat{E}_2 = x$ (tan-chord theorem)	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R (4)
8.2.2	$\hat{C}_2 = 2x$ (ext $\angle$ of Δ) $2\hat{C}_2 = \hat{EOB} = 4x$ ($\angle$ at centre = $2 \times \angle$ at circumf)	$\checkmark$ S/R $\checkmark$ S $\checkmark$ R (3)
8.2.3	$\hat{OBD} = 90^\circ$ (tan $\perp$ rad) $\hat{B}_2 = 90^\circ - x$	$\checkmark$ S $\checkmark$ R (2)
8.2.4	$\hat{OBD} = 90^\circ$ (tan $\perp$ rad) $\hat{F}_2 = 180^\circ - x - \hat{OBD}$ (sum $\angle$ s Δ) $\hat{F}_2 = 90^\circ - x$ $\therefore BC = FC$ (opp $\angle$'s =) $\therefore DC = BC = FC$	$\checkmark$ S $\checkmark$ S $\checkmark$ S/R (3)
		[17]

QUESTION 9



9.1	In $\triangle BCA$ and $\triangle CDB$ $\frac{BC}{CD} = \frac{12}{18} = \frac{2}{3}$ $\frac{AB}{BC} = \frac{8}{12} = \frac{2}{3}$ $\frac{AC}{DB} = \frac{10}{15} = \frac{2}{3}$ $\triangle BCA \parallel \triangle CDB$ (sides in proportion)	✓ S ✓ S ✓ S ✓ S/R (4)
9.2	$\angle \hat{B}C = \angle \hat{C}D$ ($\triangle BCA \parallel \triangle CDB$) $\therefore AB \parallel CD$ (alt $\angle$s=)	✓ S ✓ R (2)
9.3	In $\triangle ABE$ and $\triangle DCE$: $\angle \hat{B}C = \angle \hat{C}D$ (proven) $\angle \hat{E}B = \angle \hat{C}D$ (vertically opp $\angle$) $\triangle ABE \parallel \triangle DCE$ ($\angle \angle \angle$) $\frac{AB}{DC} = \frac{BE}{CE} \quad (\parallel \Delta's)$ $\frac{8}{18} = \frac{x}{12-x}$ $8(12-x) = 18x$ $96 - 8x = 18x$ $96 = 26x$ $x = 3,69$ $CE = 8,31$	✓ $\triangle ABE \parallel \triangle DCE$ ✓ $\frac{AB}{DC} = \frac{BE}{CE}$ ✓ $\frac{8}{18} = \frac{x}{12-x}$ ✓ $CE = 8,31$ (4)

10.1	$\hat{P}\hat{S}\hat{T} = 90^\circ$ ($\angle$ in semi-circle)	✓ S ✓ R (2)
10.2	$\hat{T}_1 = \hat{T}_2$ (ST is a bisector) $\hat{T}_2 = \hat{V}$ ($\angle$ opp=sides) $\hat{T}_1 = \hat{V}$ $\hat{R}_2 = \hat{P}$ (ext $\angle$ of cyclic quad) $\hat{S}_4 = \hat{P}\hat{S}\hat{T} = 90^\circ$ (sum $\angle$ of Δ)	✓ S ✓ S ✓ S ✓ R ✓ S + R (5)
10.3	$\Delta TSO \parallel \Delta TVS$ $\hat{T}_1 = \hat{T}_2$ (ST is a bisector) $\hat{S}_2 = \hat{V} = \left(both = \hat{T}_1 \right)$ $\hat{O}_1 = T\hat{S}\hat{V}$ (3rd $\angle$ Δ) $\Delta TSO \parallel \Delta TVS$ ($\angle\angle\angle$)	✓ S ✓ S ✓ S / R (3)

10.4	$\frac{TS}{TV} = \frac{OS}{VS} \quad \Delta TSO \parallel \Delta TVS \quad (\angle \angle \angle)$ $TS.VS = OS.TV$ <i>but</i> $TS = VS$ <i>given</i> <i>and</i> $OS = \frac{1}{2}PT$ <i>both radii</i> $\therefore VS.VS = \frac{1}{2}PT.TV$ $\therefore 2VS^2 = PT.TV$	✓ S ✓ S ✓ S+R ✓ substitution (4)
		[14]

TOTAL: 150



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Education

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REPUBLIC OF SOUTH AFRICA

**NATIONAL SENIOR CERTIFICATE/
NASIONALE SENIOR SERTIFIKAAT**

GRADE/GRAAD 12

MATHEMATICS P2/WISKUNDE V2

SEPTEMBER 2023

MARKING GUIDELINES/NASIENRIGLYNE

MARKS/PUNTE: 150

**These marking guidelines consist of 20 pages and a 3 page cognitive grid/
Hierdie nasienriglyne bestaan uit 20 bladsye en 'n 3 bladsye kognitiewe tabel**

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- As 'n kandidaat 'n vraag TWEE KEER beantwoord, merk slegs die EERSTE poging.
- As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, merk die doodgetrekte poging.
- Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.
- Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.

QUESTION/VRAAG 1

63	79	50	74	75	66	150	86	72	74	60
----	----	----	----	----	----	-----	----	----	----	----

1.1.1	$\bar{x} = \frac{849}{11}$ $= 77,18$	Answer only: full marks	✓ 849 (addition of results/ <i>optel van uitslae</i>) ✓ answer/ <i>antwoord</i> (CA if/as ÷ 11) (2)
1.1.2	$\sigma = 24,86$	No penalty for rounding:	✓ answer/ <i>antwoord</i> (A) (1)
1.1.3	$(\bar{x} - \sigma ; \bar{x} + \sigma)$ $= (52,32 ; 102,04)$ $\therefore$ 2 results/ <i>uitslae</i>	Answer only: full marks provided 1.1.1 & 1.1.2 both correct	✓ 52,32 ✓ 102,04 ✓ answer/ <i>antwoord</i> (3)
1.2	150		✓ answer/ <i>antwoord</i> (1)
1.3	D		✓✓ answer/ <i>antwoord</i> (2)
1.4	By doubling any of the six lowest results, the new result will be more than 74 and lie to the right hand side of 74/ <i>Deur om enige uitslag van die ses laagste uitslae te verdubbel sal die nuwe uitslag meer as 74 wees en regs van 74 lê.</i> Because 74 is also the mode, the new median will still be 74/ <i>Omdat 74 die modus is sal die nuwe mediaan 74 bly.</i> The median therefore has not changed/ <i>Die mediaan het dus nie verander nie.</i>		✓74 mode/ <i>modus</i> ✓no change/ <i>geen verandering</i> (2)
			[11]

QUESTION/VRAAG 2

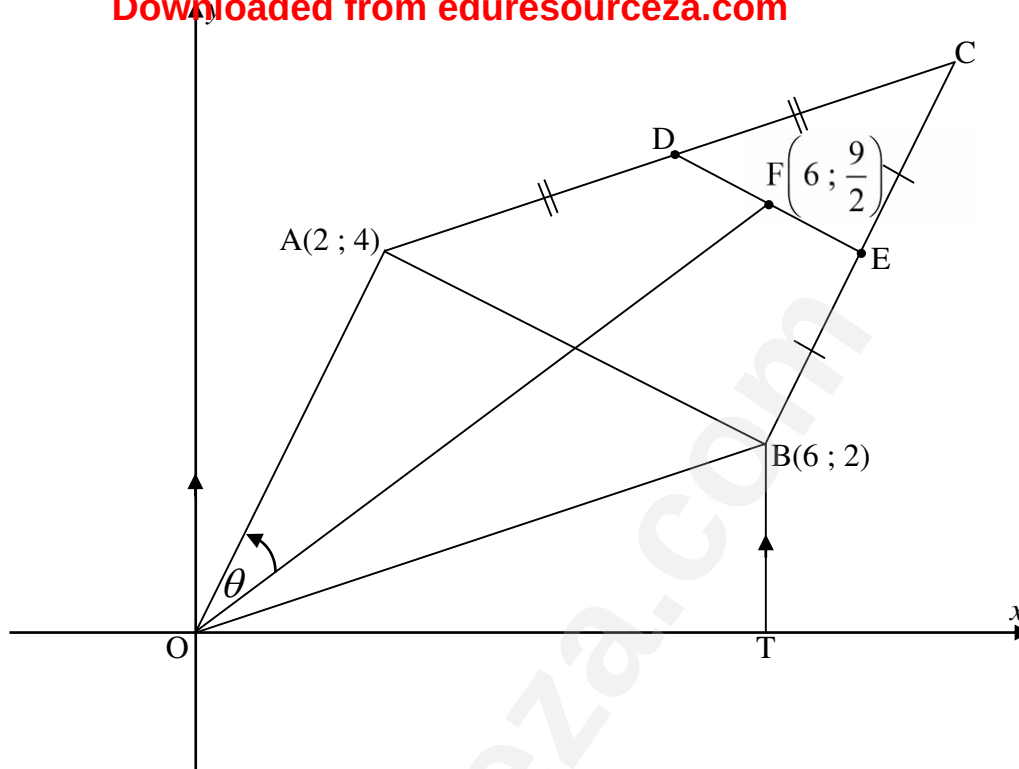
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Temperature at midday (in °C)/ Middaguur-temperatuur (in °C)	16	20	23	29	33	38	40	38	35	30
Number of ice creams / Aantal roomyse	12	17	19	44	64	70	74	66	60	40

2.1	Strong/Sterk The majority of the points lie close to the regression line/ <i>Die meerderheid punte lê naby die regressielyn</i> OR/OF Strong/Sterk $r = 0,98$	✓ strong/sterk ✓ reason/rede (2) ✓ strong/sterk ✓ $r = 0,98$ (2)
2.2	$a = -38,4828...$ $b = 2,8173...$ $\hat{y} = -38,48 + 2,82x$ Answer only: full marks, but if a and b are swapped only 1/3 marks/ <i>maar as a en b omgeruil is, slegs 1/3 punte.</i>	✓ a ✓ b ✓ equation/ vergelyking (3)
2.3	$\hat{y} = -38,48 + 2,82(26)$ $= 34,84$ $\therefore 34$ ice creams / roomyse Answer only: full marks Can also accept 35 OR/OF $26\hat{y} = 34,77$ (calculator / sakrekenaar) $\therefore 34$ ice cream / roomyse	✓ substitute 26 into eq. / vervang 26 in vgl. ✓ 34 (2) ✓✓ 34 (2)
2.4	Regression line will be pulled slightly upwards/regressielyn sal effe opwaarts getrek word The prediction will be that more ice cream will be sold/ <i>Die voorspelling is dat meer roomyse verkoop sal word.</i>	✓ explanation/ verduideliking ✓ more ice cream/ meer roomyse (2)
		[9]

QUESTION/VRAAG 3

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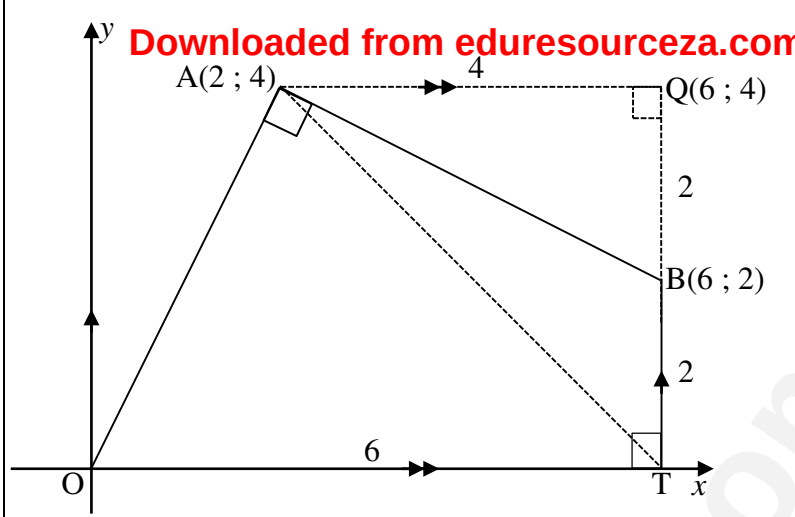


3.1.1	$AB = \sqrt{(6-2)^2 + (2-4)^2}$ $= 2\sqrt{5}$	✓ subst./vervang ✓ answer/antwoord (2)
3.1.2	$m_{AB} = \frac{2-4}{6-2}$ $= -\frac{1}{2}$ Answer only: full marks	✓ subst./vervang ✓ answer/antwoord (2)
3.2	$m_{OA} = 2$ But / maar : $m_{OA} \times m_{AB}$ $= 2\left(-\frac{1}{2}\right)$ $= -1$ $\therefore OA \perp AB$	✓ $m_{OA} = 2$ ✓ $2\left(-\frac{1}{2}\right) = -1$ (2)

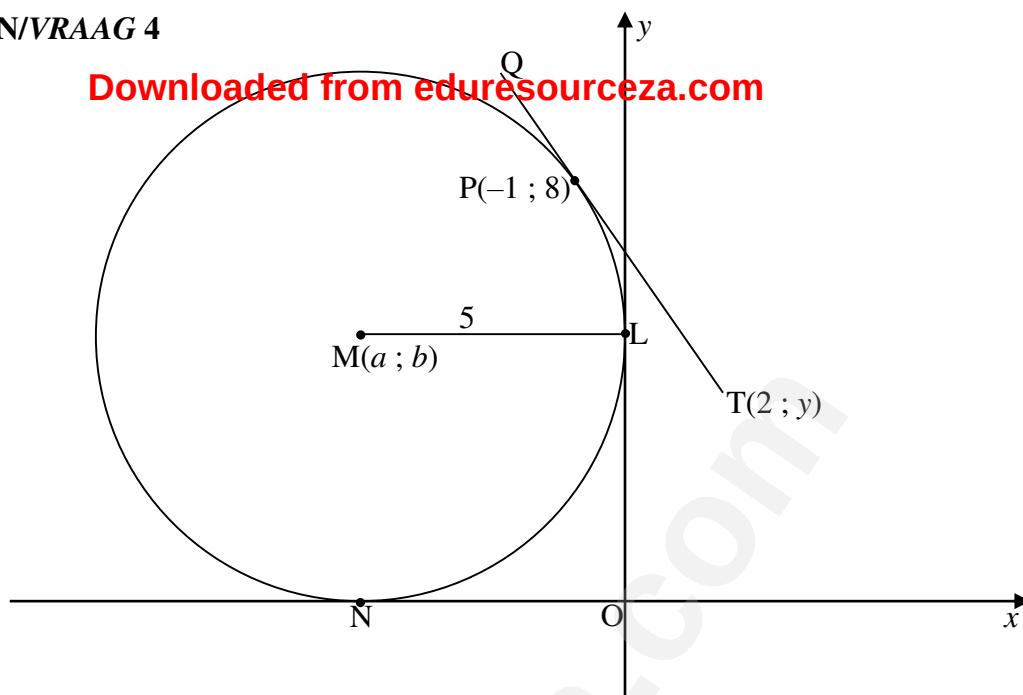
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3.3	DE $\parallel$ AB [midpt.theorem / midpt. – stelling] $\therefore m_{DE} = -\frac{1}{2}$ through / deur $\left(6; \frac{9}{2}\right)$ $y - \frac{9}{2} = -\frac{1}{2}(x - 6)$ $\therefore y = -\frac{1}{2}x + \frac{15}{2}$	✓ S OR/OF R ✓ m_{DE} ✓ subst./vervang $\left(6; \frac{9}{2}\right)$ ✓ equation/ vergelyking (4)
3.4	If / As AOBC is parm: OA $\parallel$ BC and / en OB $\parallel$ AC $x_O \rightarrow x_B = x_O + 6$ (translation / translasie) $\therefore x_A \rightarrow x_C = 2 + 6$ $\therefore x_C = 8$ in the same way / op dieselfde wyse: $y_A \rightarrow y_C = 4 + 2$ $\therefore y_C = 6$ $\therefore C(8; 6)$ OR/OF midpt. AB = $(4; 3)$ $\therefore$ midpt. OA = $(4; 3)$ [diag./hoekln. parm.] $\frac{0 + x_C}{2} = 4$ and / en $\frac{0 + y_C}{2} = 3$ $\therefore x_C = 8$ $\therefore y_C = 6$ $\therefore C(8; 6)$ OR/OF eq./vgl. BC: $y = 2x - 10$ eq./vgl. AC: $y = \frac{1}{3}x + \frac{10}{3}$ $2x - 10 = \frac{1}{3}x + \frac{10}{3}$ $5x = 40$ $x = 8$ and / en: $y = 2(8) - 10 = 6$ $\therefore C(8; 6)$	Answer only: full marks ✓ method/metode ✓ $x_C = 8$ ✓ $y_C = 6$ (3) ✓ method/metode ✓ $x_C = 8$ ✓ $y_C = 6$ (3) ✓ method/metode ✓ $x_C = 8$ ✓ $y_C = 6$ (3)

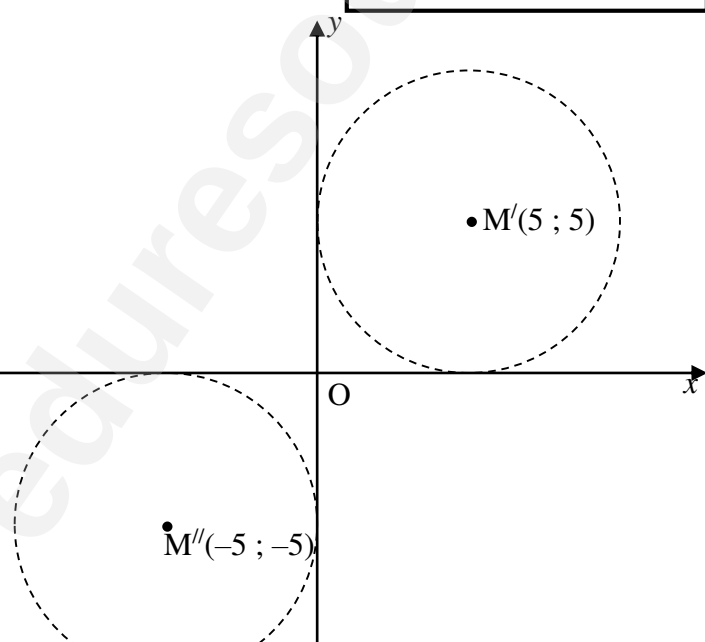
3.5.1	$\tan \hat{AOT} = 2$ $\therefore \hat{AOT} = 63,43^\circ$ $\tan \hat{FOT} = \frac{3}{4}$ $\therefore \hat{FOT} = 36,87^\circ$ $\theta = 63,43^\circ - 36,87^\circ$ $\therefore \theta = 26,56^\circ$	$\checkmark \tan \hat{AOT} = m_{OA}$ $\checkmark \text{answer/antwoord}$ $\checkmark \tan \hat{FOT} = \frac{3}{4}$ $\checkmark \text{answer/antwoord}$ $\checkmark \text{answer/antwoord}$ (5)
3.5.2	BT = 2 units/eenhede $\perp_h = 4$ area $\Delta ABT = \frac{1}{2}(2)(4)$ $= 4 \text{ units}^2 / \text{eenhede}^2$ OR/OF BT = 2 units/eenhede $\hat{ABT} = 116,57^\circ$ [$\angle$s of quad./ $\angle$e van vierhk.] area $\Delta ABT = \frac{1}{2}(2\sqrt{5})(2)\sin 116,57^\circ$ $= 3,9998\dots$ $\approx 4 \text{ units}^2 / \text{eenhede}^2$ OR/OF	$\checkmark BT = 2$ $\checkmark \perp_h = 4$ $\checkmark \text{subst./vervang}$ $\checkmark \text{answer/antwoord}$ (4) $\checkmark BT = 2$ $\checkmark \hat{ABT}$ $\checkmark \text{subst./vervang}$ $\checkmark \text{answer/antwoord}$ (4)

	 Area $\triangle ABT$ = Area AOTQ – Area $\triangle AOT$ – Area $\triangle ABQ$ $= \frac{1}{2}(4 + 6) \times 4 - \frac{1}{2}(6)(4) - \frac{1}{2}(2)(4)$ $= 4 \text{ units}^2 / \text{eenhede}^2$	✓ method/metode ✓ $\frac{1}{2}(4 + 6) \times 4$ ✓ $\frac{1}{2}(6)(4) \& \frac{1}{2}(2)(4)$ ✓ answer/antwoord (4) [22]
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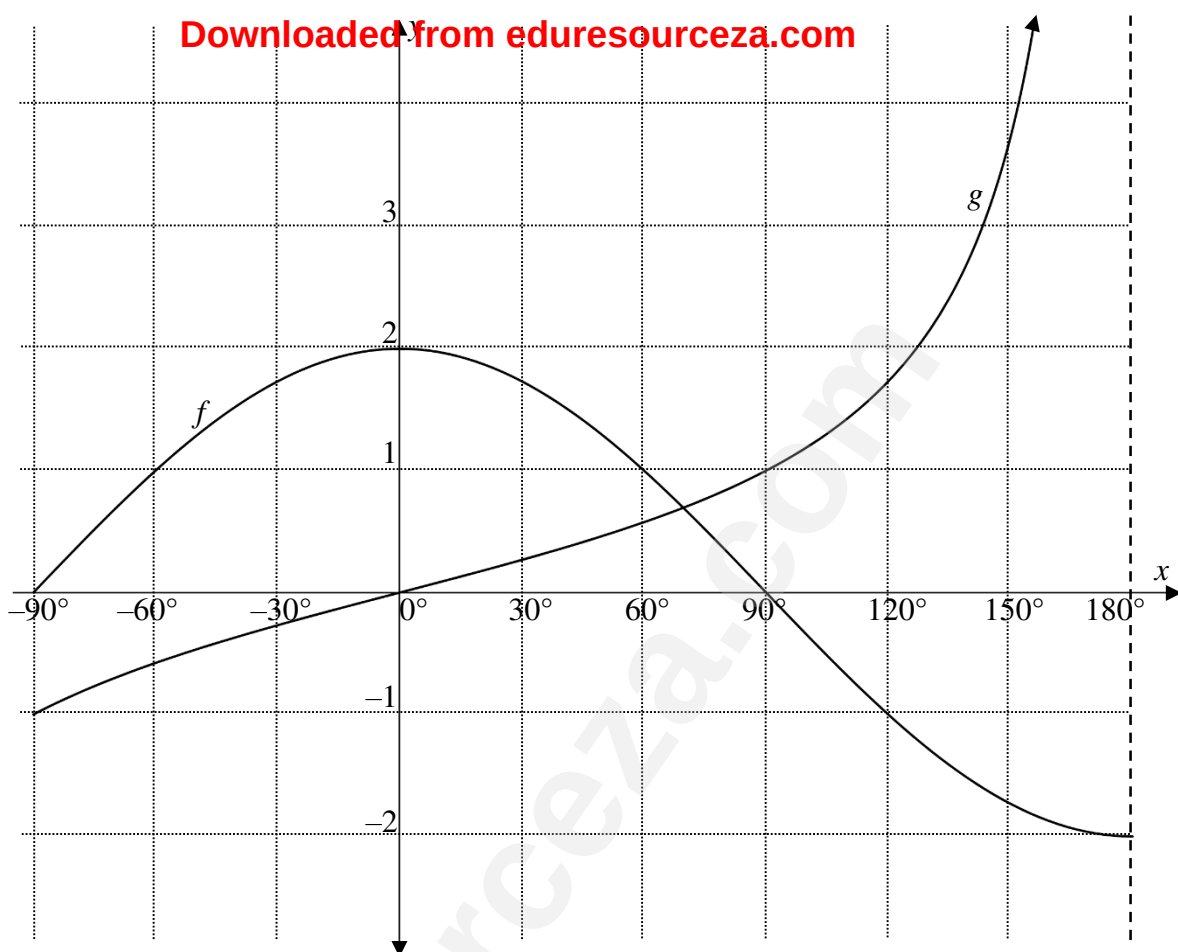
QUESTION/VRAAG 4



4.1	radius $\perp$ tangent / <i>radius $\perp$ raaklyn</i>	✓rede/reason (1)
4.2.1	M(-5 ; 5)	✓ $x = -5$ ✓ $y = 5$ (2)
4.2.2	$(x + 5)^2 + (y - 5)^2 = 25$	✓ $(x + 5)^2 + (y - 5)^2$ ✓ $r^2 = 25$ (2)
4.2.3	$m_{MP} = \frac{8-5}{-1-(-5)}$ $= \frac{3}{4}$ $\therefore m_{QT} = -\frac{4}{3} \text{ [rad } \perp \text{ tangent / rad } \perp \text{ raaklyn]}$ $8 = -\frac{4}{3}(-1) + c \quad \text{OR / OF} \quad y - 8 = -\frac{4}{3}(x - (-1))$ $c = \frac{20}{3}$ $y = -\frac{4}{3}x + \frac{20}{3}$	✓subst./vervanging ✓ $m_{MP} = \frac{3}{4}$ ✓ $m_{QT} = -\frac{4}{3}$ ✓subst./vervang m & $(-1 ; 8)$ ✓equation / vergelyking (5)

4.3	$y = -\frac{4}{3}(2) + \frac{20}{3}$ $y = 4$ $\therefore T(2;4)$ $MT = \sqrt{(2 - (-5))^2 + (4 - 5)^2}$ $= 5\sqrt{2}$ radius circle T / radius sirkel T = $5\sqrt{2} - 5$ $(x - 2)^2 + (y - 4)^2 = 75 - 50\sqrt{2}$ $\approx 4,29$	✓ subst./vervanging ✓ $y_T = 4$ ✓ subst./vervanging ✓ answer/antwoord ✓ MT – 5 ✓ equation / vergelyking (6)
4.4	If the x-axis and y-axis have to remain simultaneously as tangents to the circle M, then the circle is reflected across the axes/As die x-as en y-as gelyktydig as raaklyne aan die sirkel M bly, dan word die sirkel oor die asse gereflekteer. The only quadrants where $xy \geq 0$, are quadrants 1 and 3/ Die enigste kwadrante waar $xy \geq 0$, is kwadrante 1 en 3 $M'(5 ; 5)$ & $M''(-5 ; -5)$ Accuracy /Akkuraatheid But CA from 4.2.1 / Maar CA van 4.2.1 	✓✓ $M'(5 ; 5)$ ✓✓ $M''(-5 ; -5)$ (4)

QUESTION/VRAAG 5



5.1	$b = \frac{1}{2}$	✓ answer/antwoord (1)
5.2	$y \geq -1$ OR/OF $y \in [-1 ; \infty)$	✓ critical values/kritiese waardes ✓ notation/notasie (2)
5.3	360°	✓ 360° (1)
5.4	$x = 85^\circ$	✓ 85° (1)
5.5	$x = 0^\circ$ or/of $x = 180^\circ$	✓ 0° ✓ 180° (2)
5.6	$p = 180^\circ$	✓✓ 180° (2)
		[9]

QUESTION/VRAAG 6

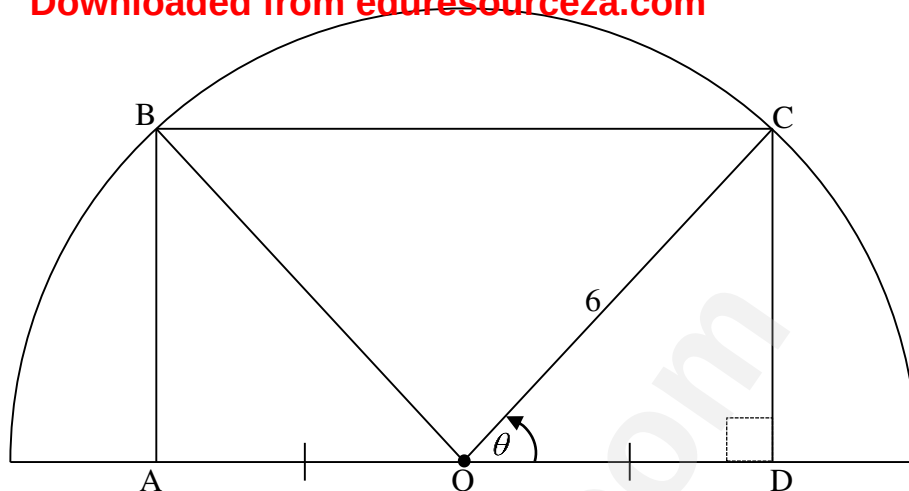
6.1.1	$\sin 335^\circ$ $= -\sin 25^\circ$	$\checkmark -\sin 25^\circ$ (1)
6.1.2	$\cos 50^\circ$ $= \cos 2(25^\circ)$ $= 1 - 2\sin^2 25$	Answer only: full marks $\checkmark \cos 2(25^\circ)$ $\checkmark 1 - 2\sin^2 25$ (2)
6.2	$\frac{\sin(-2x) \cdot (1 - \sin^2 x)}{\sin(90^\circ + x) \cdot \tan x}$ $= \frac{(-\sin 2x)(\cos^2 x)}{(\cos x)\left(\frac{\sin x}{\cos x}\right)}$ $= \frac{-2\sin x \cdot \cos x \cdot \cos^2 x}{\sin x}$ $= -2\cos^3 x$	$\checkmark -\sin 2x$ $\checkmark \cos^2 x$ $\checkmark \cos x$ $\checkmark \frac{\sin x}{\cos x}$ $\checkmark -2\sin x \cdot \cos x$ $\checkmark -2\cos^3 x$ (6)
6.3	$(p \tan 30^\circ + q \sin 60^\circ)^2$ $= \left(p \cdot \frac{1}{\sqrt{3}} + q \cdot \frac{\sqrt{3}}{2}\right)^2$ $= \left(\frac{p}{\sqrt{3}} + \frac{\sqrt{3}q}{2}\right)^2$ $= \frac{p^2}{3} + pq + \frac{3q^2}{4}$ $= \frac{4p^2 + 12pq + 9q^2}{12}$	$\checkmark$ substitution/vervangings $\checkmark$ expansion/uitbreiding $\checkmark$ answer/antwoord (3)

6.4.1	$\sin(A - B)$ $= \cos[90^\circ - (A - B)]$ $= \cos[(90^\circ + B) - A]$ $= \cos(90^\circ + B) \cdot \cos A + \sin(90^\circ + B) \cdot \sin A$ $= (-\sin B) \cdot \cos A + \cos B \cdot \sin A$ $= \sin A \cdot \cos B - \sin B \cdot \cos A$ OR/OF $\sin(A - B)$ $= \cos[90^\circ - (A - B)]$ $= \cos[(90^\circ - A) - (-B)]$ $= \cos(90^\circ - A) \cdot \cos(-B) + \sin(90^\circ - A) \cdot \sin(-B)$ $= \sin A \cdot \cos B + \cos A \cdot (-\sin B)$ $= \sin A \cdot \cos B - \sin B \cdot \cos A$	✓ co-ratio/ko-verhouding ✓ writing as difference of A and B / skryf as die verskil van A en B ✓ expansion/ uitbreiding ✓ all reductions/alle reduksies (4)
6.4.2	$LHS/LK = \sin 9A + \sin A$ $= \sin(5A + 4A) + \sin(5A - 4A)$ $= \sin 5A \cdot \cos 4A + \sin 4A \cdot \cos 5A + \sin 5A \cdot \cos 4A - \sin 4A \cdot \cos 5A$ $= 2\sin 5A \cdot \cos 4A$	✓ $\sin(5A + 4A)$ ✓ $\sin(5A - 4A)$ ✓ expansion/ uitbreiding (3)
6.4.3	Max value of/Maks waarde van: $2 \sin 5A \cdot \cos 4A = 2$ $\therefore 3^2 = 9$ $\therefore \text{max value / maks. waarde} : 3^{2\sin 5A \cdot \cos 4A} = 9$	✓ max value/maks. waarde $2 \sin 5A \cdot \cos 4A = 2$ ✓ 9 (2)

6.5	$\cos 2x - 5 \cos x - 2 = 0$ $2 \cos^2 x - 5 \cos x - 2 = 0$ $2 \cos^2 x - 5 \cos x - 3 = 0$ $(2 \cos x + 1)(\cos x - 3) = 0$ $\cos x = -\frac{1}{2}$ or / of $\cos x = 3$ $\text{ref. } \angle = 60^\circ \quad \cos x \neq 3$ $x = 120^\circ + k.360^\circ ; \text{ or / of } x = 240^\circ + k.360^\circ ; k \in \mathbb{Z}$	$\checkmark 2 \cos^2 x - 1$ $\checkmark \text{factors/faktore}$ $\checkmark \text{both equations/beide vergls.}$ $\checkmark \cos x \neq 3$ $\checkmark x = 120^\circ \ \& \ x = 240^\circ$ $\checkmark + k.360^\circ, k \in \mathbb{Z}$ (6)
6.6	$\tan x = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots}}}}$ $\tan^2 x = \sin x + \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots}}}}$ $\tan^2 x = \sin x + \tan x$ $\frac{\sin^2 x}{\cos^2 x} = \sin x + \frac{\sin x}{\cos x}$ $\sin^2 x = \sin x \cdot \cos^2 x + \sin x \cdot \cos x$ $2 \sin^2 x = 2 \sin x \cdot \cos^2 x + 2 \sin x \cdot \cos x$ $2 \sin^2 x = \sin 2x \cdot \cos x + \sin 2x$ $2 \sin^2 x = \sin 2x (\cos x + 1)$	$\checkmark$ $\sin x + \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots}}}$ $\checkmark \tan^2 x = \sin x + \tan x$ $\checkmark \frac{\sin^2 x}{\cos^2 x} = \sin x + \frac{\sin x}{\cos x}$ $\checkmark \text{multiply by/vermenigvuldig}$ $\text{met } 2 \cos^2 x$ $\checkmark \text{double angle identity/}$ $\text{dubbelhoekidentiteit}$ (5)
		[32]

QUESTION/VRAAG 7

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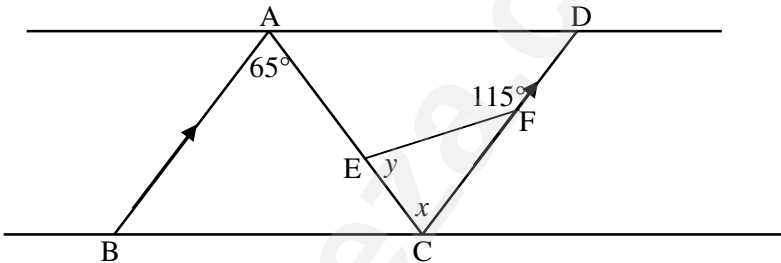
7.1	$\hat{B}OA = \theta$ $\therefore \hat{B}OC = 180^\circ - 2\theta$	$\checkmark \hat{B}OA = \theta$ $\checkmark$ answer/antwoord (2)
7.2	$\hat{B}OC = 94^\circ$ $BC^2 = OB^2 + OC^2 - 2OB \cdot OC \cdot \cos \hat{B}OC$ $BC^2 = 6^2 + 6^2 - 2(6)(6)\cos 94^\circ$ $BC^2 = 77,0224...$ $\therefore BC = 8,78$ units / eenhede	$\checkmark \hat{B}OC$ $\checkmark$ Subst. in cosine rule correctly/vervang korrek in cos-reël $\checkmark$ answer/antwoord (3)
7.3	For ABCD to be a square/Vir ABCD om vierkant te wees: $AD = DC$ $\therefore DC = 2OD$ In $\triangle ODC$ $\tan \theta = \frac{DC}{OD}$ $= \frac{2OD}{OD}$ $= 2$ $\therefore \theta = \tan^{-1}(2)$ $\theta = 63,43^\circ$	$\checkmark DC = 2OD$ $\checkmark \tan \theta = \frac{DC}{OD}$ $\checkmark \tan \theta = 2$ $\checkmark \theta = 63,43^\circ$ (4)
		[9]

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GEOMETRY/MEETKUNDE

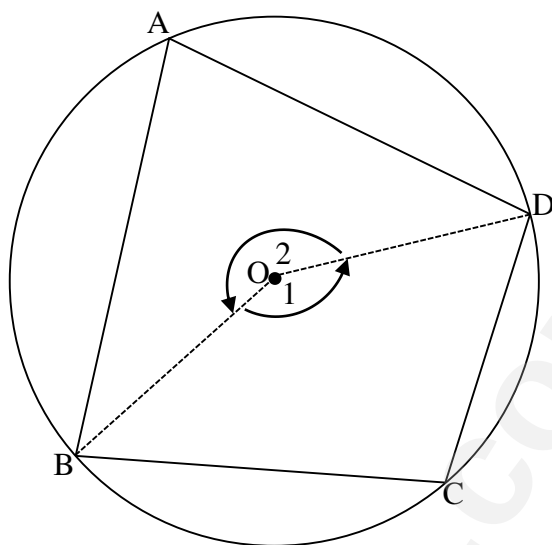
Please read carefully through the following table before marking **QUESTION 8–10/**

Lees asseblief sorgvuldig deur die volgende tabel alvorens **VRAAG 8–10** nagesien word.

	The order in which the candidate answers a geometry question must follow logically/Die volgorde waarin 'n kandidaat 'n meetkundevraag beantwoord moet logies volg. Example/Voorbeeld Given/Gegee $AB \parallel CD$ and/en $\hat{EFD} = 115^\circ$  The candidate first needs to calculate x BEFORE he/she can calculate y/Die kandidaat moet eerste vir x bereken VOORDAT hy/sy vir y kan bereken.
S	A mark for a correct statement (A statement mark is independent of a reason) 'n Punt vir 'n korrekte bewering ('n Punt vir 'n bewering is onafhanklik van die rede)
R	A mark for the correct reason (A reason mark may only be awarded if the statement is correct) 'n Punt vir 'n korrekte rede ('n Punt word slegs vir die rede toegeken as die bewering korrek is)
S/R	Award a mark if the statement AND reason are both correct (Both MUST be correct to get one mark) Ken 'n punt toe as die bewering EN rede beide korrek is (Beide MOET korrek wees om een punt te kry)

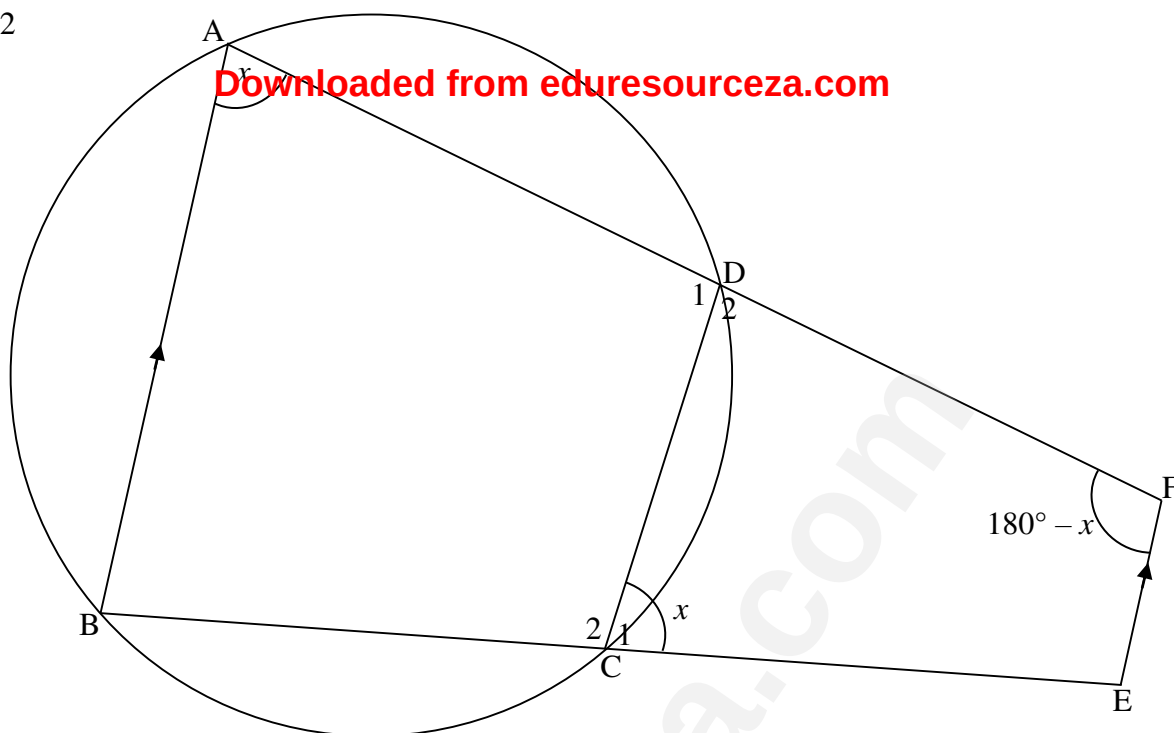
QUESTION/VRAAG 8

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8.1	Construction/<i>Konstr.</i>: Draw BO and OD/<i>Trek BO en OD.</i> $\hat{O}_1 = 2\hat{A}$ [$\angle$ at centre = $2 \times \angle$ at circ./ <i>midpts</i> $\angle = 2 \times$ <i>omtreks</i> $\angle$] $\hat{O}_2 = 2\hat{C}$ [$\angle$ at centre = $2 \times \angle$ at circ./ <i>midpts</i> $\angle = 2 \times$ <i>omtreks</i> $\angle$] $\hat{O}_1 + \hat{O}_2 = 360^\circ$ [$\angle$s around a point/$\angle$ e om 'n punt] $\therefore 2\hat{A} + 2\hat{C} = 360^\circ$ $\therefore 2(\hat{A} + \hat{C}) = 360^\circ$ $\therefore \hat{A} + \hat{C} = 180^\circ$ The last statement mark / <i>Die laaste punt vir bewering:</i> $2\hat{A} + 2\hat{C} = 360^\circ$ OR / OF $2(\hat{A} + \hat{C}) = 360^\circ$	✓ constr./ <i>konstr.</i> ✓ S/R ✓ S ✓ S/R ✓ S (5)
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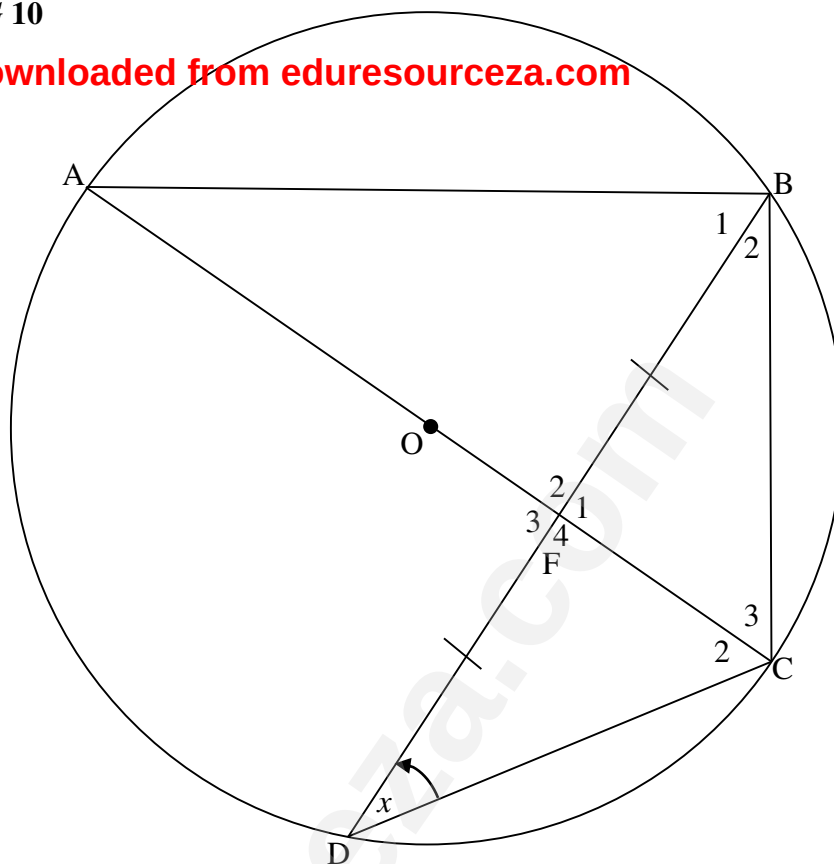
8.2

Let/Laat: $\hat{A} = x$

8.2	$\hat{C}_1 = \hat{A} = x$ [ext. $\angle$ cyclic quad./buite $\angle$ kvh.] $\hat{F} = 180^\circ - \hat{A} = 180^\circ - x$ [co-interior $\angle$s/ko – binne $\angle$e; $AB \parallel FE$] $\hat{C}_1 + \hat{F} = x + 180^\circ - x = 180^\circ$ $\therefore$ CEFD is a cyclic quad/is 'n kvh. [converse opp. $\angle$s cyclic quad/omgekeerde teenoorst. $\angle$e kvh] / [ext. $\angle$ of quad = opp. interior $\angle$/buite $\angle$ van vierhoek = teenoorst. binne $\angle$] OR / OF $\hat{C}_2 = 180^\circ - \hat{A} = 180^\circ - x$ [opp. $\angle$s cyclic quad/teenoorst. $\angle$e kvh] $\hat{F} = 180^\circ - \hat{A} = 180^\circ - x$ [co-interior $\angle$s/ko – binne $\angle$e; $AB \parallel FE$] $\hat{C}_2 = \hat{F} = 180^\circ - x$ $\therefore$ CEFD is a cyclic quad/is 'n kvh. [converse ext. $\angle$ cyclic quad/omgekeerde buite. $\angle$ kvh] / [opp. $\angle$s of quad supplementary/teenoorst. $\angle$e van vierhoek supplementêr]	$\checkmark$S$\checkmark$R $\checkmark$S/R $\checkmark$S $\checkmark$R (5) $\checkmark$S$\checkmark$R $\checkmark$S/R $\checkmark$S $\checkmark$R (5)
		[10]

QUESTION/VRAAG 10

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10.1.1	$\hat{A} = x$ [$\angle$ s in same segm./ $\angle$ e in dies. segm.]	✓S ✓R (2)
10.1.2	$\hat{A}BC = 90^\circ$ [$\angle$ in semi circle./ $\angle$ in halfsirkel] $\hat{C}_3 = 90^\circ - x$ [sum of $\angle$ s of Δ /som van $\angle$ e van Δ]	✓S ✓R ✓S/R (3)
10.2.1	$OF \perp DB$ [line from centre to midpt.chord/midpt.sirkel, midpt.koord] $\therefore \hat{F}_2 = \hat{F}_3 = 90^\circ$	✓S ✓R (2)
10.2.2	In ΔCFB and / en ΔCBA $\hat{C}_3 = \hat{C}_3$ [common/gemeenskaplik] $\hat{F}_1 = \hat{B} = 90^\circ$ [from 10.1.2 & 10.2.1] $\therefore \Delta CFB \parallel \Delta CBA$ [$\angle \angle \angle$] OR/OF In ΔCFB and / en ΔCBA $\hat{C}_3 = \hat{C}_3$ [common/gemeenskaplik] $\hat{F}_1 = \hat{B} = 90^\circ$ [from 10.1.2 & 10.2.1] $\hat{B}_2 = \hat{A}$ [sum of $\angle$ s of Δ /som van $\angle$ e van Δ] $\therefore \Delta CFB \parallel \Delta CBA$	✓S/R ✓S ✓R (3) ✓S/R ✓S ✓S/R (3)

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10.2.3	$\frac{CF}{CB} = \frac{CB}{CA}$ [Two sides $\parallel$] $\therefore CB^2 = AC \cdot FC$ but / maar: $\triangle DFC \equiv \triangle FBC$ [S $\angle$ S] $\therefore DC = BC$ $\therefore DC^2 = AC \cdot FC$	✓S/R ✓S ✓S/R ✓S (4)
10.2.4	$AC^2 = AB^2 + BC^2$ [Pythagoras] $AC^2 = AB^2 + FC \cdot AC$ [$BC^2 = DC^2$] $1 = \frac{AB^2}{AC^2} + \frac{FC}{AC}$ $\frac{FC}{AC} = 1 - \frac{AB^2}{AC^2}$ $\frac{FC}{AC} = \left(1 - \frac{AB}{AC}\right) \left(1 + \frac{AB}{AC}\right)$ but / maar $AC = AO + OC$ $\therefore \frac{FC}{AC} = \left(1 - \frac{AB}{AO + OC}\right) \left(1 + \frac{AB}{AO + OC}\right)$	✓S ✓ $BC^2 = FC \cdot AC$ ✓ $\div AC^2$ ✓ $\frac{FC}{AC} = 1 - \frac{AB^2}{AC^2}$ ✓factorise/faktoriseer (5)
		[19]

TOTAL/TOTAAL: 150

GRID-Analysis (According to BLOOMS TAXONOMY)													
		KNOWLEDGE			ROUTINE PROCEDURES			COMPLEXED PROCEDURES			PROBLEM SOLVING		
Question	Description	Low	Mod	High	Low	Mod	High	Low	Mod	High	Low	Mod	High
	Statistics(Gr.11) [11]												
1.1.1	Mean	2											
1.1.2	Standard deviation (calculator)				1								
1.1.3	Outside one SD					3							
1.2	Outlier	1											
1.3	Complex range							2					
1.4	Critical thinking median										2		
2	Regression(Gr.12) [9]												
2.1	Interpret scatter plot + regr. line			2									
2.2	Equation of regression line				3								
2.3	Estimation by using regr. line	2											
2.4	Critical thinking SD										2		
3	Analytical (Gr.11) [22]												
3.1.1	Distance		2										
3.1.2	Gradient		2										
3.2	Prove lines perpendicular				2								
3.3	Equation str.line + midpt.th							4					
3.4	4 th coordinate of parm						3						
3.5.1	Inclination							5					
3.5.2	Integrating trig to find area Δ								4				
4	Analytical(Gr.12) [20]												
4.1	Geometry theorem			1									
4.2.1	Coordinates centre			1	1								
4.2.2	Eq. of circle			2									
4.2.3	Equation of tangent					5							
4.3	Eq. of new circle with specific conditions									6			
4.4	Critical thinking new equation											4	

		KNOWLEDGE			ROUTINE PROCEDURES			COMPLEX PROCEDURES			PROBLEM SOLVING		
Question	Description	Low	Mod	High	Low	Mod	High	Low	Mod	High	Low	Mod	High
5	Trig graphs [9]												
5.1	Parameters			1									
5.2	Range				2								
5.3	Period	1											
5.4	Translation					1							
5.5	Undefined							2					
5.6	Critical thinking + integrating concepts										2		
6	Trigonometry [32]												
6.1.1	Reduction	1											
6.1.2	Double angle			2									
6.2.	Simplification + combination of identities		2					4					
6.3	Special angles + combining algebra			2			1						
6.4.1	Derive formula $\sin(A - B)$				4								
6.4.2	Identity						3						
6.4.3	Max value										2		
6.5	General solution						6						
6.6	Critical thinking + evaluation											5	
7	2D/3D Trig [9]												
7.1	Angle in terms of θ			2									
7.2.	Cosine rule					3							
7.3	Transforming a quadrilateral									4			
8	Geometry [10]												
8.1	Theorem				5								
8.2	Prove quad. to be cyclic quad.							5					

NSC/NSS – Cognitive grid/*Kognitiewe tabel*[illegible]



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DEPARTMENT OF EDUCATION
DEPARTEMENT VAN ONDERWYS
LEFAPHA LA THUTO
ISEBE LEZEMFUNDO

**PROVINCIAL PREPARATORY EXAMINATION/
PROVINSIALE VOORBEREIDINGSEKSAMEN**

GRADE/GRAAD 12

**MATHEMATICS P2/WIKUNDE V2
MARKING GUIDELINES/NASIENRIGLYNE
SEPTEMBER 2023**

MARKS/PUNTE: 150

**These marking guidelines consist of 22 pages/
*Hierdie nasienriglyne bestaan uit 22 bladsye.***

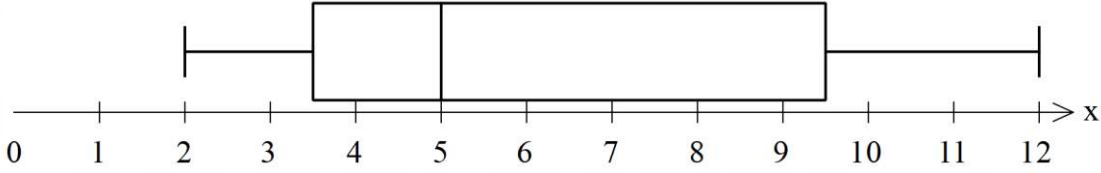
NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values to solve a problem is NOT acceptable.

GEOMETRY	
S	A mark for a correct statement (A statement mark is independent of a reason)
R	A mark for the correct reason (A reason mark may only be awarded if the statement is correct)
S/R	Award a mark if statement AND reason are both correct

QUESTION 1

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1.1		✓ Median ✓ Min + Max ✓ Q2, Q3 (3)
1.2	Skew to the right.	✓ answer (1)
1.3	6,44	✓✓ answer (2)
1.4	$\sigma = 3,24$ $[6,44 - 3,24 ; 6,44 + 3,24]$ $[3,2 ; 9,68]$ 4 values lie outside / 4 waardes lê buite Round or square is o.k. Interested in the interval AO 1 mark	✓ 3,24 ✓ interval ✓ answer (3)
		[9]

QUESTION 2

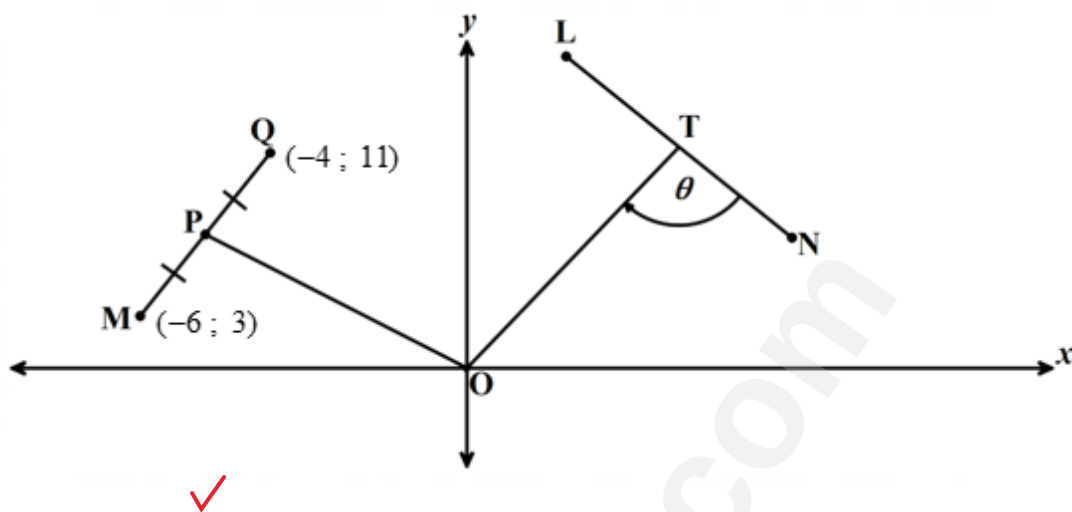
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NUMBER OF HOURS WITHOUT SLEEP (x)	10	11	12	13	14	15	16	17	18	19	21	24	25
NUMBER OF MISTAKES MADE (y)	4	4	5	8	8	10	11	11	13	15	21	23	25

2.1	$25 - 10$ $= 15$	✓ Max – min ✓ answer (2)
2.2	$a = -12,05$ $b = 1,46$ $\hat{y} = -12,05 + 1,46x$	✓ $a = -12,05$ ✓ $b = 1,46$ ✓ equation (3)
2.3	$r = 0,99$	✓ answer (1)
2.4	$y = -12,05 + 1,46(23)$ $y = 21,53$ $y \approx 22$	✓ substitution ✓ answer (2)
2.5	Yes. the correlation coefficient is 0,99, <u>strong positive correlation</u> .	✓ Yes ✓ strong, positive, linear correlation (2)
		[10]

QUESTION 3

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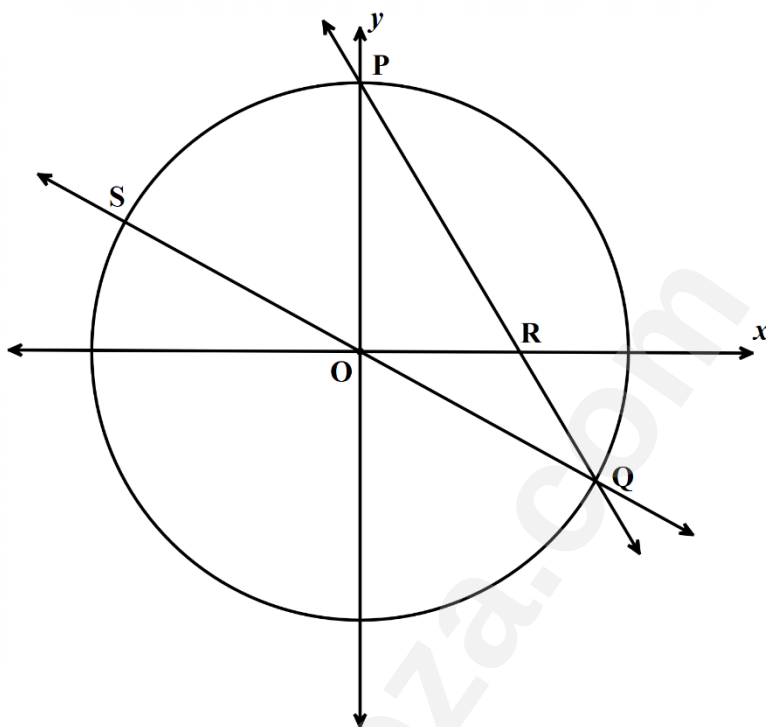
3.1.1	$\left(\frac{-6-4}{2} ; \frac{3+11}{2} \right)$ $P(-5 ; 7) \quad \checkmark \quad \checkmark$	$\checkmark x = -5$ $\checkmark y = 7$ (2)
3.1.2	$\frac{3}{2}x = 15 - x \quad \checkmark$ $3x = 30 - 2x$ $5x = 30$ $x = 6 \quad \checkmark \quad y = \frac{3}{2}(6) = 9 \quad \checkmark$ $T(6 ; 9)$ OR $x + y = 15$ $x + \frac{3}{2}x = 15$ $x = 6$ $y = \frac{3}{2}(6) = 9$ $T(6 ; 9)$	$\checkmark$ equating $\checkmark$ x-value $\checkmark$ y-value (3) OR $\checkmark$ substitution $\checkmark$ x-value $\checkmark$ y-value (3)

	OR $x = 15 - y$ $y = \frac{3}{2}(15 - y)$ $y = \frac{45}{2} - \frac{3}{2}y$ $\frac{5}{2}y = \frac{45}{2}$ $y = 9$ $x + 9 = 15$ $x = 6$ T(6 ; 9)	OR ✓ substitution ✓ y – value ✓ x – value (3)
3.1.3	$y = -x + 15$ $\tan \alpha = -1$ $\alpha = 135^\circ$ ✓ $\tan \beta = \frac{3}{2}$ $\beta = 56,31^\circ$ ✓ $\theta = 180^\circ - 56,31^\circ - 45^\circ$ ✓ $\theta = 78,69^\circ$ ✓ OR $y = -x + 15$ $\tan \alpha = -1$ $\alpha = 135^\circ$ $\tan \beta = \frac{3}{2}$ $\beta = 56,31^\circ$ $\theta = 135^\circ - 56,31^\circ = 78,69^\circ$	✓ $\alpha = 135^\circ$ ✓ $56,31^\circ$ ✓ method ✓ answer (4) OR ✓ $\alpha = 135^\circ$ ✓ method ✓ $56,31^\circ$ ✓ answer (4)
3.2	$\sqrt{(-2-0)^2 + (k-1-0)^2} = 2k$ ✓ $(-2)^2 + (k-1)^2 = 4k^2$ $4 + k^2 - 2k + 1 = 4k^2$ $3k^2 + 2k - 5 = 0$ ✓ $(3k+5)(k-1) = 0$ $k \neq -\frac{5}{3}$ or $k=1$ ✓	✓ substitution in distance formula ✓ standard form ✓ factors ✓ answer with selection. (4)

3.3.1	$m_{SY} = \frac{3-5a-3}{2+4a-2} = \frac{-5a}{4a} = \frac{-5}{4}$ $m_{SU} = \frac{3-5b-3}{2+4b-2} = \frac{-5b}{4b} = \frac{-5}{4}$ $m_{YU} = \frac{3-5b-(3-5a)}{2+4b-(2+4a)} = \frac{-5b+5a}{4b-4a} = \frac{-5(b-a)}{4(b-a)} = \frac{-5}{4}$ S, Y and U collinear (gradients are the same)	✓✓ any 2 gradients ✓ Reason (3)
3.3.2	$y = \frac{-5}{4}(x) + c$ $3 = \frac{-5}{4}(2) + c$ $c = \frac{11}{2}$ $y = \frac{-5}{4}x + \frac{11}{2}$ OR $y - 3 = -\frac{5}{4}(x - 2)$ $y = -\frac{5}{4}x + \frac{10}{4} + 3$ $y = -\frac{5}{4}x + \frac{11}{2}$	✓ $m = -\frac{5}{4}$ ✓ substitute (2 ; 3) ✓ $c = \frac{11}{2}$ (3) OR ✓ $m = -\frac{5}{4}$ ✓ substitute (2 ; 3) ✓ $c = \frac{11}{2}$ (3)
		[19]

QUESTION 4

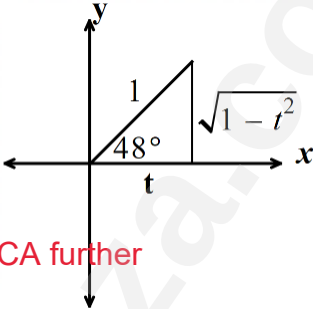
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4.1	P (0 ; 4)	✓ answer (A) (1)
4.2	$y = -2x + 4$ and $x^2 + y^2 = 16$ $x^2 + (-2x + 4)^2 = 16$ $x^2 + 4x^2 - 16x + 16 - 16 = 0$ $5x^2 - 16x = 0$ $x(5x - 16) = 0$ $x \neq 0$ or $x = \frac{16}{5}$ $y = -2\left(\frac{16}{5}\right) + 4$ $y = \frac{-12}{5}$	✓ substitution ✓ standard form ✓ answer with selection ✓ substitution of x-value (4)
4.3	$y = -2x + 4$ $0 = -2x + 4$ $2x = 4$ $x = 2$ $R(2 ; 0)$ or $R(2 ; 0)$ Radius = 2 $(x - 2)^2 + y^2 = 4$	✓ substitute $y = 0$ ✓ $x = 2$ ✓ $(x - 2)^2 + y^2$ ✓ 4 (4)

4.4	$m_{OQ} = \frac{0 + \frac{12}{5} - \frac{3}{4}}{0 - \frac{16}{5}} = \frac{\frac{12}{5} - \frac{3}{4}}{-\frac{16}{5}}$ ref angle = $36,87^\circ$ Inclination SQ = $180^\circ - 36,87^\circ = 143,13^\circ$ Inclination PR: $m = -2$, ref angle = $63,43^\circ$ $180^\circ - 63,43^\circ = 116,57^\circ$ Adjustment: $143,13^\circ - 116,57^\circ = 26,56^\circ$	✓ gradient $-\frac{3}{4}$ ✓ $143,13^\circ$ ✓ $116,57^\circ$ ✓ $26,56^\circ$ (4)
4.5.1	$x^2 + 2x + 1 + y^2 - 6y + 9 = 6 + 1 + 9$ $(x+1)^2 + (y-3)^2 = 16$	✓ completing the Square ✓ $(x+1)^2 + (y-3)^2$ ✓ 16 (3)
4.5.2	centre: $(-1 ; 3)$	✓ centre: $(-1 ; 3)$ (1)
4.6	Distance between 2 centres of circles: $(0 ; 0)$ and $(-1 ; 3)$ $\sqrt{(0+1)^2 + (0-3)^2}$ $= \sqrt{10} = 3,16 \text{ units}$ Radius + Radius = $4 + 4 = 8$ units Distance between centres < Radius₁ + Radius₂ Circles intersect.	✓ substitution in distance formula ✓ $\sqrt{10}$ ✓ method ✓ Conclusion (4)
		[21]

QUESTION 5

5.1.1	$\cos 228^\circ$ $= -\cos 48^\circ$ $= -t$	$\checkmark = -\cos 48^\circ$ $\checkmark$ answer (2)
5.1.2	$\sin 48^\circ = \sqrt{1 - \cos^2 48^\circ}$ $= \sqrt{1 - t^2}$ OR $\sin 48^\circ = \sqrt{1 - t^2}$  Penalise wrong sketch, then CA further	$\checkmark$ identity $\checkmark$ answer (2) OR $\checkmark$ sketch, Pythagoras $\checkmark$ answer (2)
5.1.3	$\cos 2(48^\circ)$ $= 2\cos^2 48^\circ - 1$ $= 2t^2 - 1$ OR $\cos 2(48^\circ)$ $= 1 - 2\sin^2 48^\circ$ $= 1 - 2(1 - t^2)$ $= 2t^2 - 1$ OR $\cos 2(48^\circ)$ $= \cos^2 48^\circ - \sin^2 48^\circ$ $= t^2 - (\sqrt{1 - t^2})^2$ $= 2t^2 - 1$	$\checkmark$ double angle $\checkmark$ expansion $\checkmark$ answer (3) OR $\checkmark$ double angle $\checkmark$ expansion $\checkmark$ answer (3) OR $\checkmark$ double angle $\checkmark$ expansion $\checkmark$ answer (3)

5.1.4	$\sin(45^\circ + 48^\circ)$ ✓ $= \sin 45^\circ \cos 48^\circ + \cos 45^\circ \sin 48^\circ$ ✓ $= \frac{\sqrt{2}}{2} \cdot t + \frac{\sqrt{2}}{2} \cdot \sqrt{1-t^2}$ ✓	✓ $\sin(45^\circ + 48^\circ)$ ✓ expansion ✓ substitution ✓ special angle value (4)
5.2	$\frac{\sin 36^\circ \cdot \sin \theta \cdot \cos(90^\circ - \theta)}{\sin 756^\circ} + \frac{\sin 2\theta \cdot \cos \theta}{2 \sin \theta}$ $\frac{\sin 36^\circ \cdot \sin \theta \cdot \sin \theta}{\sin 36^\circ} + \frac{2 \sin \theta \cdot \cos \theta \cdot \cos \theta}{2 \sin \theta}$ $= \sin^2 \theta + \cos^2 \theta$ ✓ $= 1$ ✓ if they leave out the angle - ZERO If they change another letter no penalty	✓ $\sin \theta$ ✓ $\sin 36^\circ$ ✓ $2 \sin \theta \cos \theta$ ✓ simplify to identity. ✓ answer (5)
5.3.1	$\tan^2 x \left(\frac{1}{\tan^2 x} - 1 \right) = \frac{\cos 2x}{\cos^2 x}$ $LHS = 1 - \tan^2 x$ ✓ $= 1 - \frac{\sin^2 x}{\cos^2 x}$ ✓ $= \frac{\cos^2 x - \sin^2 x}{\cos^2 x}$ ✓ ✓ $= \frac{\cos 2x}{\cos^2 x} = RHS$	✓ simplification ✓ identity ✓ simplification ✓ double angle identity (4)
5.3.2	$x = 90^\circ$ ✓ $x = 0^\circ$ ✓ $x = 270^\circ$ ✓ $x = 180^\circ$; $x = 360^\circ$	✓ answer ✓ answer (2)

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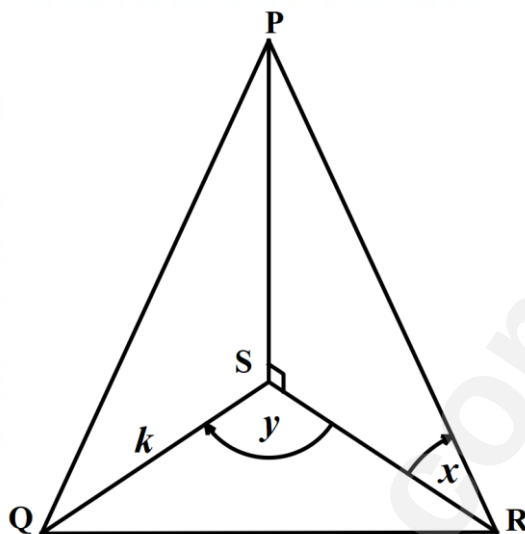
5.3.3	$1 - \tan^2 x = 0$ $1 = \tan^2$ Downloaded from eduresourceza.com $\tan x = \pm 1$ ✓ ref angle: 45° ✓ $x = 45^\circ + 180^\circ k, k \in \mathbb{Z}$ $x = 135^\circ + 180^\circ k, k \in \mathbb{Z}$ ✓ OR $\frac{\cos 2x}{\cos^2 x} = 0$ $\therefore \cos 2x = 0$ ✓ Ref angle: 90° $2x = 90^\circ + 360^\circ k$ $x = 45^\circ + 180^\circ k; k \in \mathbb{Z}$ ✓ $2x = 270^\circ + 360^\circ k$ $x = 135^\circ + 180^\circ k; k \in \mathbb{Z}$ ✓ If only +1 Then 3 out of 4 $x = 180k + 45$ $x = 180k - 45$	$\checkmark \tan x = \pm 1$ $\checkmark 45^\circ$ $\checkmark 135^\circ$ $\checkmark +180^\circ k; k \in \mathbb{Z}$ (4) OR $\checkmark \cos 2x = 0$ ~~$\checkmark 90^\circ$~~ $\checkmark 45^\circ$ $\checkmark 135^\circ$ $\checkmark +180^\circ k; k \in \mathbb{Z}$ (4)
5.4	$g(x) - f(x)$ $= 1 - 2\sin^2 x - \sin x + 1$ $= -2\sin^2 x - \sin x + 2$ ✓ Maximum at $\sin x = \frac{-b}{2a} = \frac{-(-1)}{2(-2)} = -\frac{1}{4}$ ✓ $\text{ref } \angle : 14,48^\circ$ ✓ $x = 345,52^\circ$ ✓ If x = instead of sinx = then 2 out of 5	$\checkmark -2\sin^2 x - \sin x + 2$ $\checkmark$ substitution $\checkmark \sin x = -\frac{1}{4}$ $\checkmark$ ref angle $\checkmark$ answer (5)
		[31]

QUESTION 6

6.1	Downloaded from eduresourceza.com ✓ x-intercepts ✓ y-intercept ✓ Turning points $(-45^\circ; -2)$; $(135^\circ; -2)$ dont have to indicate coordinates (3)
6.2	1 ✓ ✓ answer (1)
6.3	$(210^\circ; -1)$ ✓ ✓ ✓ x – value ✓ y – value (2)
6.4	$\sin 2x \cdot \cos x - \cos x = 0$ $\cos x (\sin 2x - 1) = 0$ ✓ x- intercepts: $g(x): 225^\circ$ ✓ $f(x): 90^\circ; 270^\circ$ ✓ ✓ ✓ factorization ✓ 225° ✓ 90° ✓ 270° (4) [10]

QUESTION 7

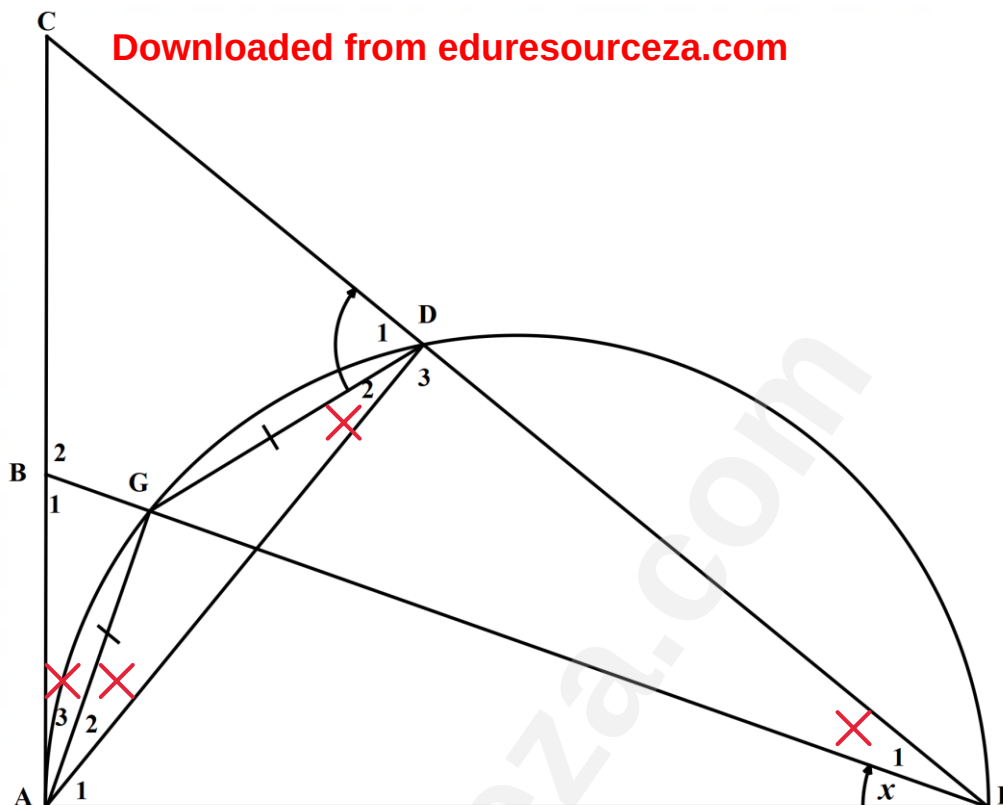
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7.1	$\tan x = \frac{PS}{SR}$ ✓ $SR = \frac{PS}{\tan x}$ ✓	✓ trig ratio ✓ answer (2)
7.2	$\text{Area } \triangle QRS = \frac{1}{2} \cdot k \cdot SR \cdot \sin y$ ✓ $A = \frac{1}{2} \cdot k \cdot \left(\frac{PS}{\tan x} \right) \cdot \sin y$ ✓ $A = \frac{k \cdot PS \cdot \sin y}{2 \tan x}$ ✓ $PS = \frac{2A \tan x}{k \sin y}$ ✓	✓ Use of area rule ✓ substitute $SR = \frac{PS}{\tan x}$ ✓ simplification ✓ Change of subject (4)
7.3	$\frac{2(480,9) \cdot \tan 46,5^\circ}{87,36 \cdot \sin y} = 76,8$ ✓ $\frac{11,6017...}{\sin y} = 76,8$ $\frac{11,6017...}{76,8} = \sin y$ $0,151... = \sin y$ ✓ $y = 8,69^\circ$ ✓	✓ substitution ✓ simplification ✓ answer (3)
		[9]

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QUESTION 9



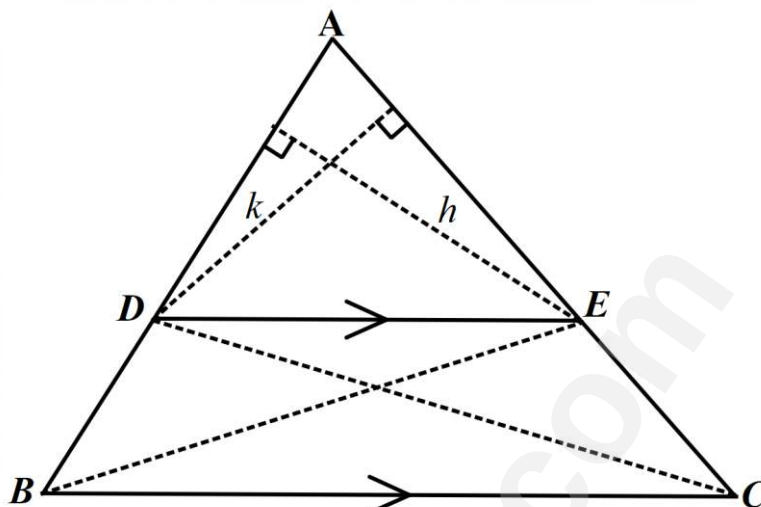
9.1	ANY THREE OF THE FOUR ANGLES WITH REASONS IN CORRECT ORDER $\hat{A}_3 = x$ (tan-chord th / raaklyn-koord st) ✓ s/r $\hat{D}_2 = x$ (∠'s in the same segment/ hoeke in dieselfde segment) ✓ OR (tan-chord th / raaklyn-koord st) $\hat{A}_2 = x$ (angles opp = sides / hoeke teenoor gelyke sye) ✓ OR $\hat{A}_3 = x$ (tan-chord th / raaklyn-koord st) ✓ s/r $\hat{D}_2 = x$ (∠'s in the same segment/ hoeke in dieselfde segment) ✓ OR (tan-chord th / raaklyn-koord st) $\hat{E}_1 = x$ (∠'s subtended by = chords / Hoeke onderspan deur gelyke koorde) ✓ OR $\hat{A}_3 = x$ (tan-chord th / raaklyn-koord st) ✓ s/r $\hat{E}_1 = x$ (∠'s subtended by = chords / Hoeke onderspan deur gelyke koorde) ✓ $\hat{A}_2 = x$ (angles opp = sides / hoeke teenoor gelyke sye) ✓	✓ S/R ✓ S ✓ R ✓ S ✓ R (5) OR ✓ S/R ✓ S ✓ R ✓ S ✓ R (5) OR ✓ S/R ✓ S ✓ R ✓ S ✓ R (5)
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You can ALSO get three angles WITHOUT angle A3. Then award 5 marks.

9.2	$\hat{D}_3 = 90^\circ$ ($\angle$'s in a semi-circle / Hoeke in 'n halwe sirkel) ✓ s/r $\hat{D}_1 = 180^\circ - 90^\circ - x$ ✓ s $= 90^\circ - x$ (Angles on a straight line/ Hoeke op 'n reguit lyn) $\hat{A} = 90^\circ$ (Rad $\perp$ Tangent / Rad $\perp$ Raaklyn) ✓ s/r $\hat{B}_2 = 90^\circ + x$ (Ext angle of Δ / Buite hoek v Δ) $\therefore \hat{B}_2 + \hat{D}_1 = 180^\circ$ BCDG a cyclic quad (Opp angles of <u>quad</u> supp) ✓ BCDG 'n Koordevierhoek (Teenoorst hoeke van vierhoek supp) OR CONVERSE opp angles cyclic quad OR $\hat{D}_3 = 90^\circ$ ($\angle$'s in a semi-circle / Hoeke in 'n halwe sirkel) ✓ s/r $\hat{D}_2 + \hat{D}_3 = 90^\circ + x$ ✓ $\hat{A} = 90^\circ$ (Rad $\perp$ Tangent / Rad $\perp$ Raaklyn) ✓ s/r $\hat{B}_2 = 90^\circ + x$ (Ext angle of Δ / Buite hoek v Δ) $\hat{D}_2 + \hat{D}_3 = 90^\circ + x = \hat{B}_2$ BCDG a cyclic quad (Ext angle of quad = opp int angle) ✓ BCDG 'n Koordevierhoek (Buite hoek van vierhoek = teenoorstaande binnehoek)	✓S/R ✓S $90^\circ - x$ ✓S/R ✓S $90^\circ + x$ ✓R OR ✓S/R ✓S $\hat{D}_2 + \hat{D}_3 = 90^\circ + x$ ✓SR ✓S $\hat{B}_2 = 90^\circ + x$ ✓R (5) (5)
	or (converse Ext angle cyclic quad)	[10]

QUESTION 10

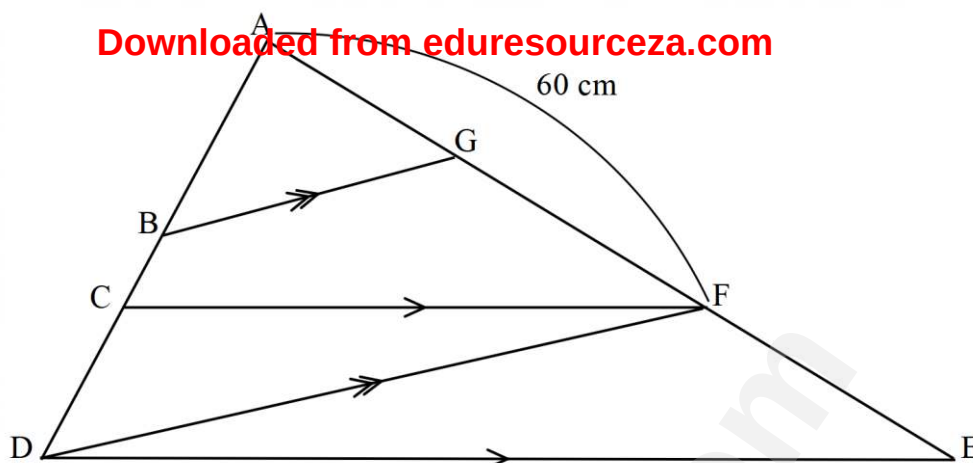
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Construction need only be on sketch or in words. No construction - no further marking

10.1	Constr: Draw heigh h with base AD and height k with base AE ✓ Draw DC and BE PROOF: $\frac{\text{Area } \triangle ADE}{\text{Area } \triangle BDE} = \frac{\frac{1}{2} \cdot AD \cdot h}{\frac{1}{2} \cdot BD \cdot h} = \frac{AD}{BD}$ $\frac{\text{Area } \triangle ADE}{\text{Area } \triangle CDE} = \frac{\frac{1}{2} \cdot AE \cdot k}{\frac{1}{2} \cdot EC \cdot k} = \frac{AE}{EC}$ s/r Area $\triangle BDE$ = Area $\triangle CDE$ (Same base, same height/ Dieselfde basis en hoogte) $\frac{\text{Area } \triangle ADE}{\text{Area } \triangle BDE} = \frac{\text{Area } \triangle ADE}{\text{Area } \triangle CDE} \quad \checkmark$ $\frac{AD}{BD} = \frac{AE}{EC}$	✓ construction ✓ S ✓ S ✓ S /R ✓ S (5)
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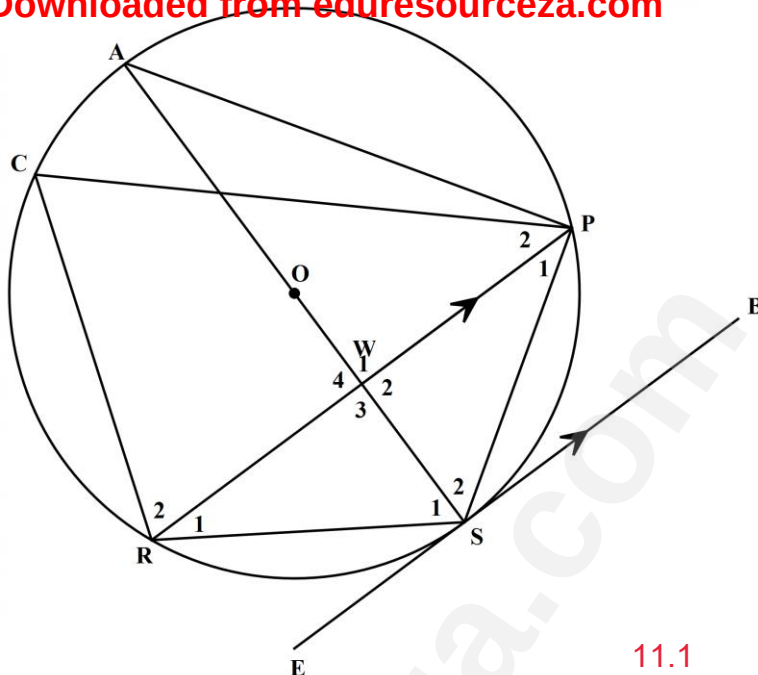


10.2.1	$60 \div 3 \times 2 = 40\text{cm}$ ✓ ✓ OR $\frac{FE}{60} = \frac{2}{3}$ $\therefore FE = 40\text{cm}$ ✓ ✓ ANSWER ONLY – FULL MARKS	✓ method ✓ answer (2) OR ✓ S ✓ answer (2)
10.2.2	$\frac{AF}{FE} = \frac{AC}{CD} = \frac{3}{2}$ (Line parallel one side of Δ / Lyn parallel een sy van Δ) ✓ s/r $\frac{AG}{GF} = \frac{AB}{BD} = \frac{7}{8}$ (Line parallel one side of Δ / Lyn parallel een sy van Δ) ✓ s/r $\begin{array}{ccccccc} & 7x & & 8x & & & \\ & \bullet & \text{---} & \bullet & \text{---} & \bullet & \\ A & & B & C & & D & \\ & 3y & & 2y & & & \end{array}$ From ratios: $AB = 7x$ $BD = 8x$ $AC = 3y$ $CD = 2y$ $5y = 15x$ $y = 3x$ ✓ method MUST HAVE $\frac{BC}{CD} = \frac{8x - 2y}{2y}$ $\frac{BC}{CD} = \frac{8x - 2(3x)}{2(3x)} = \frac{2x}{6x} = \frac{1}{3}$ ✓	✓ S/R ✓ S/R ✓ method ✓ Answer (4)

	Downloaded from eduresourceza.com OR $DC = \frac{2}{5} AD$ (prop th, CF//DE / eweredigheids st, CF//DE) ✓ s/r $BD = \frac{8}{15} AD$ (prop th, BG//DF / eweredigheidst, BG//DF) ✓ s/r $BC = BD - DC$ $BC = \frac{8}{15} AD - \frac{2}{5} AD = \frac{2}{15} AD$ ✓ method $\frac{BC}{CD} = \frac{\frac{2}{15} AD}{\frac{2}{5} AD} = \frac{1}{3}$ ✓	OR ✓ SR ✓ SR ✓ method ✓ Answer (4)
		[11]

QUESTION 11

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11.1

Penalise for // left out

11.1	$\hat{O}SB = 90^\circ$ (Rad $\perp$ tangent / Rad $\perp$ raaklyn) ✓ s $\hat{W}_1 = 90^\circ$ (Corr $\angle$'s PR//ED / ooreenk $\angle^e$ PR//ED) ✓ s/r $WR = WP$ (Line from centre $\perp$ to chord / lyn uit mdpt $\perp$ op koord) ✓ r	✓ S ✓ R ✓ S/R ✓ R (4)
11.2	In $\triangle WRS$ and $\triangle PAS$: $\hat{A}PS = 90^\circ$ ($\angle$ in a semi circle / Hoek in semi sirkel) ✓ s/r $\hat{W}_3 = 90^\circ$ (Vertically opp $\angle$'s / Regoorst hoeke) ✓ s/r $\hat{W}_3 = \hat{A}PS$ $\hat{A} = \hat{R}$ ($\angle$'s in the same segment / $\angle^e$ in dieselfde segment) ✓ s/r $\triangle PAS \equiv \triangle WRS$ (AAA) ✓ r	✓ S/R ✓ s/r ✓ S/R ANY 2 ANGL ES ✓ R (4)
11.3	$\triangle WRS$ and $\triangle WPS$: $\hat{W}_3 = \hat{W}_2 = 90^\circ$ ($\angle$'s on a straight line / Hoeke op 'n reguit lyn) ✓ s/r $WR = WP$ (Proven / Bewys) ✓ s WS is common/gemeenskaplik ✓ s $\triangle WRS \equiv \triangle WPS$ (SAS) ✓ order and reason $\therefore \triangle PAS \equiv \triangle WPS$ $\frac{AS}{PS} = \frac{PS}{WS}$ (From $\equiv \triangle$) ✓ r $PS^2 = AS \cdot WS$	✓ S/R ✓ S ✓ S ✓ Order and Reason ✓ S ✓ R (6)

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	OR/OF	OR/OF
	$\frac{WS}{RS} = \frac{PS}{AS}$ (corresponding sides in proportion, ooreenkomstige sye in verhouding) ✓ s/r $RS \cdot PS = WS \cdot AS$ ✓ s $\triangle WRS$ and $\triangle WPS$ ✓ s $\hat{W}_3 = \hat{W}_2 = 90^\circ$ (∠'s on a straight line / Hoeke op 'n reguit lyn) ✓ s $WR = WP$ (Proven / Bewys) ✓ s WS is common/gemeenskaplik $\triangle WRS \equiv \triangle WPS$ (SAS) ✓ order and reason $\Rightarrow RS = PS$ ✓ s $PS^2 = AS \cdot WS$	✓ S/R ✓ S ✓ S ✓ S ✓ Order and Reason ✓ S (6)
		[14]

TOTAL: 150

ALTERNATIVE:
TRIANGLE APS /// TRIANGLE PWS



PREPARATORY EXAMINATION *VOORBEREIDENDE EKSAMEN*

2023

MARKING GUIDELINES/*NASIENRIGLYNE*

MATHEMATICS/<i>WISKUNDE</i> (PAPER/<i>VRAESTEL 2</i>) (10612)
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19 pages/*bladsye*

NOTE:

- If a candidate does not redo a question in the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking guidelines. Stop marking at the second calculation error.
- Assuming answers/values to solve a problem is NOT acceptable.

LET WEL:

- As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.
- As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.
- Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.
- Aannames van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.

GEOMETRY/MEETKUNDE	
S	A mark for a correct statement (A statement mark is independent of a reason.)
	<i>'n Punt vir 'n korrekte bewering</i> (<i>'n Punt vir 'n bewering is onafhanklik van die rede.</i>)
R	A mark for a correct reason (A reason mark may only be awarded if the statement is correct.)
	<i>'n Punt vir 'n korrekte rede</i> (<i>'n Punt word slegs vir die rede toegeken as die bewering korrek is.</i>)
S/R	Award a mark if the statement AND reason are both correct.
	(<i>Ken 'n punt toe as beide die bewering EN rede korrek is.</i>)

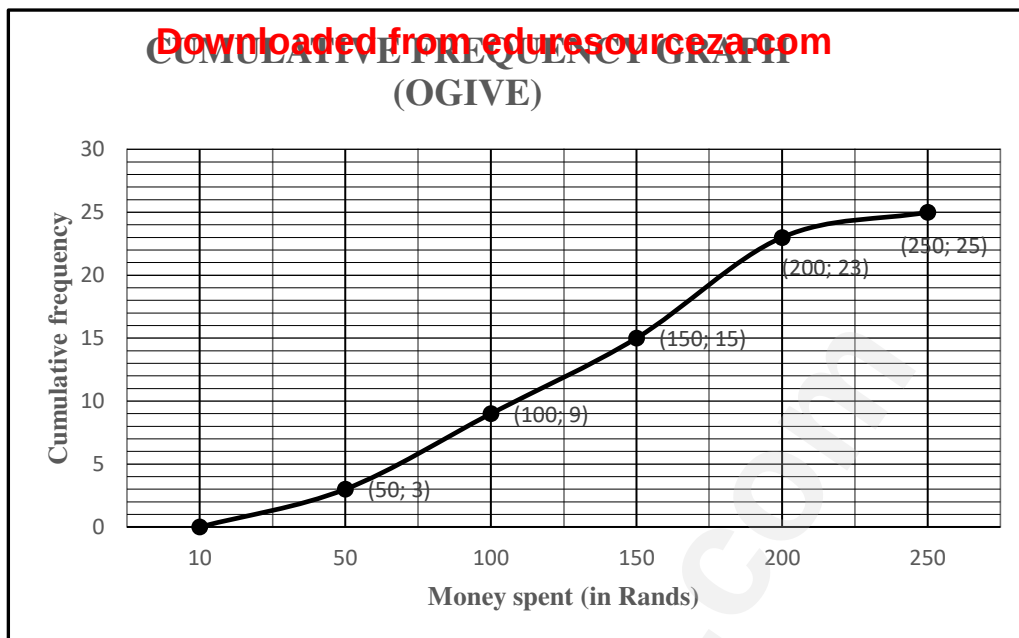
TIME USED ON <i>INSTAGRAM</i> (in minutes)	30	45	58	63	75	90
TIME USED ON <i>TIKTOK</i> (in minutes)	40	55	70	60	90	100



Answer only: Full marks
Slegs antwoord: Volpunte

1.5	$73,4 = \frac{\sum x}{10}$ OR/OF $73,4 = \frac{\sum x}{20}$ Total/Totaal = 734 minutes/minute Total for Instagram and TikTok = 1 468 minutes Totaal vir Instagram en TikTok = 1 468 minute $\frac{1\ 468}{60} = 24,47$ $\therefore$ Yes I agree. More than a day was spent./ $\therefore$ Ja, ek stem saam. Meer as 'n dag is spandeer.	✓ 1 468 minutes/minute ✓ $\frac{1468}{60} = 24,47$ hours/ure ✓ conclusion/gevolgtrekking (3)
		[10]

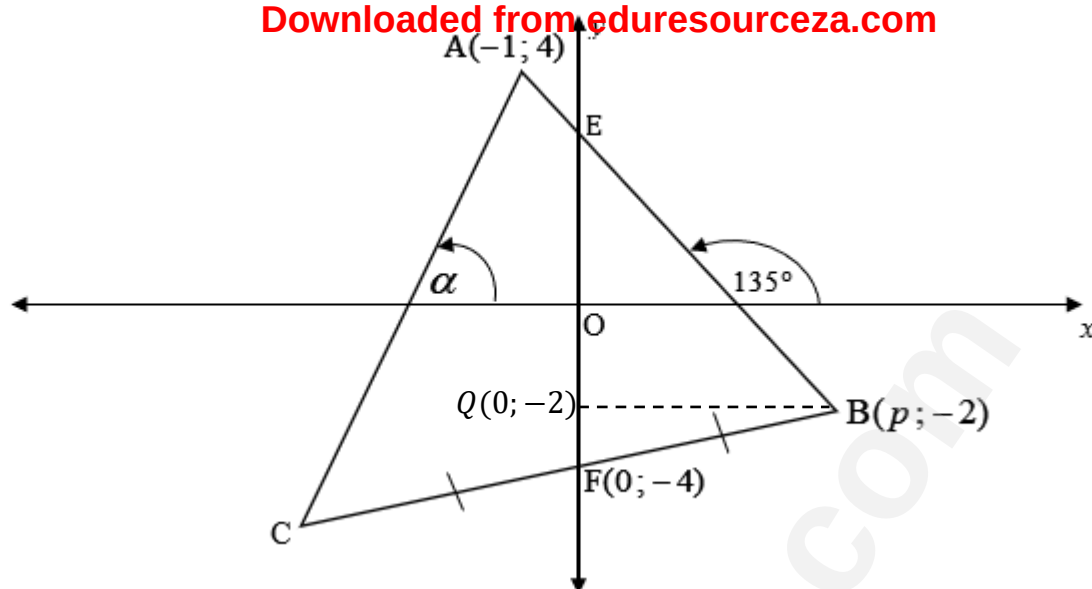
QUESTION/VRAAG 2



2.1.1	25 learners/ <i>leerders</i>	✓ answer/ <i>Antwoord</i> (1)
2.1.2	$a = 3$ $b = 6$	✓ $a = 3$ ✓ $b = 6$ (2)
2.1.3	$25 - 19 = 6$ learners/ <i>leerders</i> $\frac{6}{25} \times 100 = 24\%$ $\therefore 24\%$ of the learners spent more than R175/ <i>van die leerders het meer as R175 gespandeer</i>	✓ 6 ✓ 24% (2)
2.2	$\bar{x} - 2\sigma = 4,8$ $13,20 - 2\sigma = 4,8$ $-2\sigma = -8,4$ $\sigma = 4,2$ OR $\bar{x} + 2\sigma = 9,2$ $13,20 + 2\sigma = 9,2$ $2\sigma = -4$ OR The interval is incorrect/meaningless because $13,20 < 9,2$ which is mean < upperlimit, therefore no further calculations can be done. / <i>Die gegewe interval is verkeerd / betekenisloos, want $13,20 < 9,2$, dit is die gemiddeld < boonste limiet. Dus kan geen verdere berekenings gemaak word nie.</i>	✓ ✓ equation/ <i>vergelyking</i> ✓ ✓ $\sigma = 4,2$ OR ✓ ✓ equation/ <i>vergelyking</i> ✓ ✓ $2\sigma = -4$ OR ✓ ✓ ✓ ✓ explanation/ <i>verduideliking</i> (4)
		[9]

QUESTION/VRAAG 3

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3.1	$m_{AB} = \tan 135^\circ = -1$	✓ $m_{AB} = \tan 135^\circ$ ✓ answer (2)
3.2	$m_{AB} = \frac{-2-4}{p-(-1)}$ $-1 = \frac{-6}{p+1}$ $p+1=6$ $p=5$	✓ correct substitution of A and B into gradient formula/korrekte substitusie van A en B in die gradiënt formule ✓ equate to/gelykstel aan -1 (2)
3.3	$\frac{5+x_c}{2} = 0 \quad \text{and/en} \quad \frac{-2+y_c}{2} = -4$ $x_c = -5 \quad \text{and/en} \quad y_c = -8+2$ $C(-5;-6) \quad y_c = -6$ NOTE: Does not have to be in coordinate form <i>Let op: Dit is nie nodig om in koördinaatvorm te skryf nie.</i>	✓ $x_c = -5$ ✓ $y_c = -6$ (2)

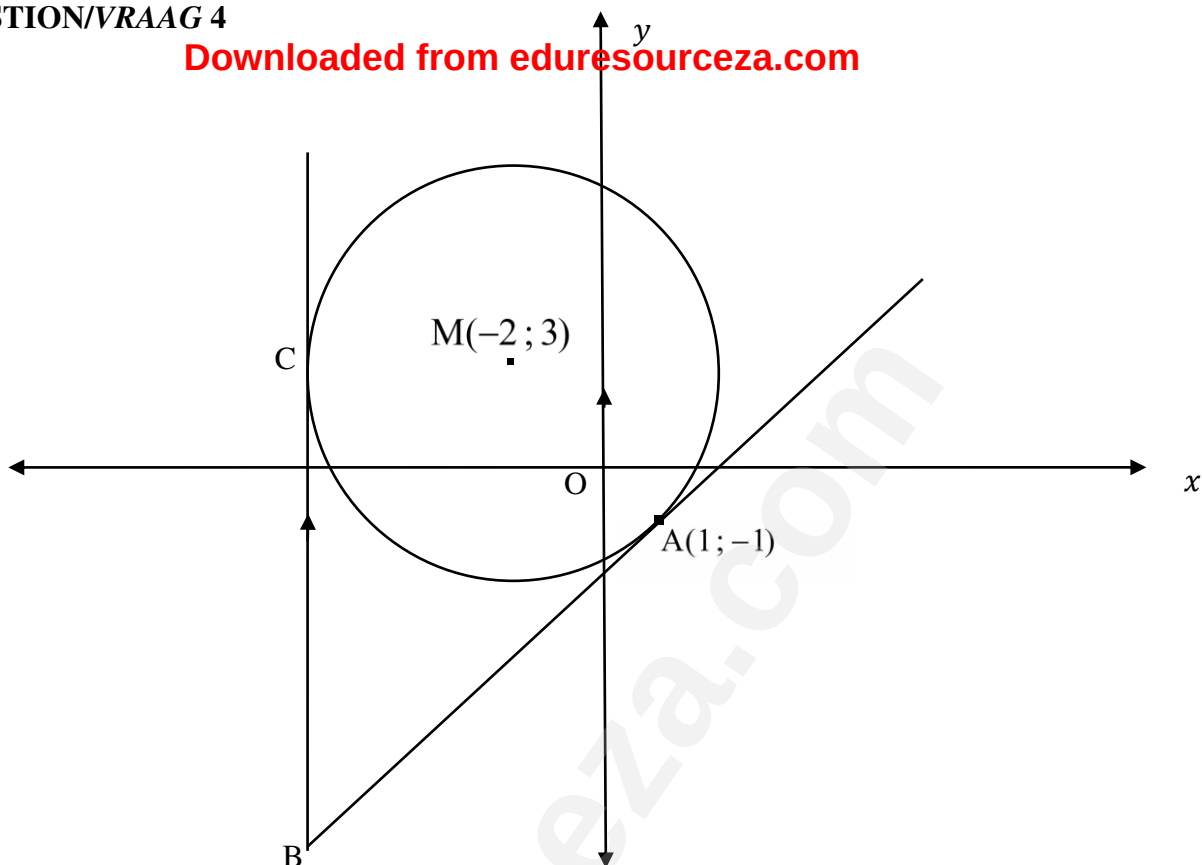
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3.4	$m_{AC} = \frac{4 - (-6)}{-1 - (-9)} = \frac{10}{-10} = -1$ Substitute/Vervang A(-1 ; 4) $y - 4 = \frac{5}{2}(x - (-1)) \quad \text{OR/OF} \quad 4 = \frac{5}{2}(-1) + c$ $y = \frac{5}{2}x + \frac{5}{2} + 4 \quad 4 = \frac{-5}{2} + c$ $y = \frac{5}{2}x + \frac{13}{2} \quad c = 6\frac{1}{2}$ $y = \frac{5}{2}x + 6,5 \quad y = \frac{5}{2}x + 6\frac{1}{2}$	✓ correct substitution of A and C into gradient formula/korrekte substitusie van A en C in die gradiënt formule ✓ $m_{AC} = \frac{5}{2}$ ✓ subs of m and point A or C/vervang m en punt A of C ✓ equation/vergelyking (4)
3.5	$\tan \alpha = \frac{5}{2}$ $\alpha = 68,20^\circ$ $\hat{CAB} = 135^\circ - 68,20^\circ \quad (\text{Ext } \angle \text{ of } \Delta)$ $= 66,8^\circ$	✓ $\tan \alpha = \frac{5}{2}$ ✓ $\alpha = 68,20^\circ$ ✓ answer/antwoord (3)
3.6	Subst./Vervang A (-1 ; 4) OF/OF Subst/Vervang B(5; -2) $E : 4 = -1(-1) + c \quad \text{OR/OF} \quad y + 2 = -1(x - 5)$ $3 = c \quad y + 2 = -x + 5$ $y = -x + 3$ E(0 ; 3) F(0 ; -4) $\text{Area/Opp } \triangle BEF = \frac{1}{2} \times EF \times \perp h$ $= \frac{1}{2} \times 7 \times 5$ $= 17,5 \text{ square units/vierkante eenhede}$ OR/OF $E : 4 = -1(-1) + c$ $3 = c$ E(0 ; 3) $\hat{FEB} = 45^\circ$ $BE^2 = (5 - 0)^2 + (-2 - 3)^2$ $BE = \sqrt{50}$ $\text{Area/Opp } \triangle BEF = \frac{1}{2} \times EF \times BE \times \sin \hat{FEB}$ $= \frac{1}{2} \times 7 \times \sqrt{50} \times \sin 45^\circ$ $= 17,5 \text{ square units / vierkante eenhede}$	✓ value of c/waarde van c ✓ $\perp$ height/hoogte = 5 ✓ answer/antwoord (3) OR/OF ✓ value of c/waarde van c ✓ $BE = \sqrt{50}$ ✓ answer/antwoord (3)

	OR/OF $E: 4 = -1(-1) + c$ $3 = c$ Downloaded from eduresourceza.com $E(0; 3)$ $Area / Opp \Delta FQB = \frac{1}{2} \times 2 \times 5 = 5$ $Area / Opp \Delta EQB = \frac{1}{2} \times 5 \times 5 = 12,5$ $Area / Opp \Delta EFB = 5 + 12,5$ $= 17,5 \text{ square units / vierkante eenhede}$	OR/OF ✓ value of c/waarde van c ✓ $\perp$ height/hoogte = 5 ✓ answer/antwoord (3)
3.7	$AK = 5\sqrt{5}$ $\sqrt{(t+1)^2 + (t-4)^2} = (5\sqrt{5})$ $t^2 + 2t + 1 + t^2 - 8t + 16 = 125$ $t^2 - 3t - 54 = 0$ $(t-9)(t+6) = 0$ $t = 9 \text{ (n.a.) or/of } t = -6$ $K(-6; -6)$ NOTE: Must be in coordinate form. <i>Let op: Moet in koördinaatvorm wees.</i>	✓ substitute A and K into distance formula/ vervang A en K in die afstandformule ✓ equating/gelykstelling ✓ standard form/ standaardvorm ✓ factors/faktore ✓ $K(-6; -6)$ (5)
		[21]

QUESTION/VRAAG 4

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4.1	$(x+2)^2 + (y-3)^2 = r^2$ $(1+2)^2 + (-1-3)^2 = r^2$ $9+16 = r^2$ $\therefore (x+2)^2 + (y-3)^2 = 25$ OR/OF $AM = \sqrt{(-2-1)^2 + (3+1)^2}$ $AM = \sqrt{9+16}$ $r = 5$ $\therefore r^2 = 25$ $\therefore (x+2)^2 + (y-3)^2 = 25$	✓ subs of M and A into the distance formula/ vervang M en A in die afstandformule ✓ $r^2 = 25$ ✓ equation/vergelyking OR/OF ✓ subs of M and A into the distance formula/ vervang M en A in die afstandformule ✓ $r^2 = 25$ ✓ equation/vergelyking (3)
4.2	C(-2-5; 3) (by symmetry/deur simmetrie) C(-7; 3)	✓ $x_C = -7$ ✓ $y_C = 3$ (2)

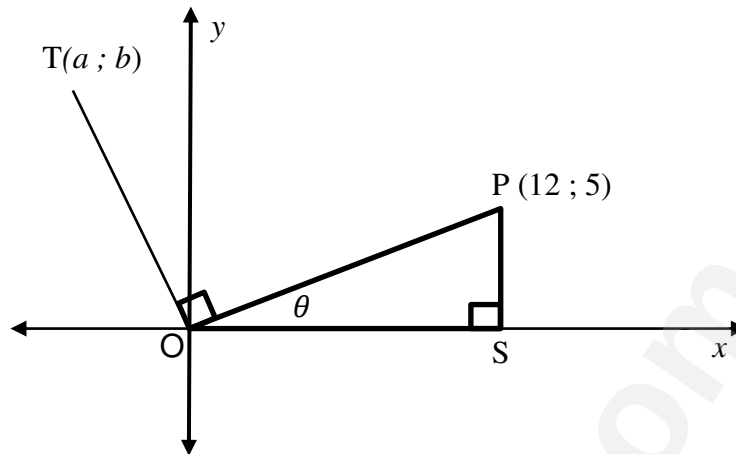
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4.3	$m_{AM} = \frac{3 - (-1)}{-2 - 1} = -\frac{4}{3}$ $m_{AB} = \frac{3}{4} \quad (\text{radius} \perp \text{tangent/raaklyn})$ $y - (-1) = \frac{3}{4}(x - 1) \quad (\text{subst./vervang A}(1; -1))$ $y = \frac{3}{4}x - \frac{3}{4} - 1$ $y = \frac{3}{4}x - \frac{7}{4}$	✓ subs A and M into gradient formula/ vervang A en M in die gradiënt formule ✓ $m_{\text{radius}} = -\frac{4}{3}$ ✓ $m_{\text{tangent}} = \frac{3}{4}$ ✓ substitute m and A/ vervang m en A ✓ equation/vergelyking (5)
4.4	$B(-7; y_B)$ subs./vervang $B(-7; y_B)$ into eqn of AB/in die vergelyking van AB $y_B = \frac{3}{4}(-7) - \frac{7}{4}$ $y_B = -7$ $B(-7; -7)$ $BC = 10 \text{ units/eenhede}$	✓ subs./vervang $x = -7$ ✓ $y_B = -7$ ✓ $BC = 10$ (3)
4.5	$(x-1)^2 + (y+1)^2 = 1$	✓ LHS/LK ✓ RHS/RK (2)
4.6	$r_2 - r_1 < MN < r_2 + r_1$ $1 < p + 2 < 9 \text{ or/of } 1 < -2 - p < 9$ $-1 < p < 7 \text{ or/of } 3 < -p < 11$ $-1 < p < 7 \text{ or/of } -11 < p < -3$ OR/OF Two circles will touch when:/ Twee sirkels sal sny indien: Distance between centre/Afstand tussen middelpunte = $r_1 + r_2$ or /of distance between centres/ Afstand tussen middelpunte = $r_2 - r_1$ $\sqrt{(p+2)^2 + (3-3)^2} = 4+5 \text{ or /of } \sqrt{(p+2)^2 + (3-3)^2} = 5-4$ $(p+2)^2 = 9^2 \text{ or /of } (p+2)^2 = 1$ $p+2 = \pm 9 \text{ or /of } p+2 = \pm 1$ $p = \pm 9 - 2 \text{ or /of } p = -2 \pm 1$ $p = -11 \text{ or /of } p = 7 \text{ or /of } p = -3 \text{ or /of } p = -1$ Two circles intersect in two different points if / twee sirkels sny in twee verskillende punte as $-11 < p < -3 \text{ or } -1 < p < 7$	✓ value of 1 and 9 (Sum and difference of radii)/Waarde van 1 en 9 (som en verskil van radiusse) ✓✓ $-1 < p < 7$ ✓✓ $-11 < p < -3$ (5) OR/OF ✓ value of 1 and 9 (Sum and difference of radii)/Waarde van 1 en 9 (som en verskil van radiusse) ✓✓ $-1 < p < 7$ ✓✓ $-11 < p < -3$ (5)
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QUESTION/VRAAG 5

5.1

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5.1.1	$\tan \theta = \frac{5}{12}$	✓ answer/antwoord (1)
5.1.2	$r^2 = 5^2 + 12^2$ $r = 13$ $\sin \theta = \frac{5}{13}$	✓ value of/waarde van r ✓ answer/antwoord Answer only: Full marks Slegs antwoord: Volpunte (2)
5.1.3	$\cos(90^\circ + \theta) = \frac{a}{19,5}$ $-\sin \theta = \frac{a}{19,5}$ $-\frac{5}{13} = \frac{a}{19,5}$ $a = \frac{-5 \times 19,5}{13}$ $a = -7,5$	✓ ratio/verhouding ✓ $-\sin \theta$ ✓ substitute/vervang $\sin \theta$ ✓ value of/waarde van a (4)
5.2	$\frac{\sin(360^\circ - 2x) \cdot \sin(-x)}{\sin(90^\circ + x)} + 2 \cos^2(180^\circ + x)$ $= \frac{(-\sin 2x) \cdot (-\sin x)}{\cos x} + 2 \cos^2 x$ $= \frac{(-2 \sin x \cdot \cos x) \cdot (-\sin x)}{\cos x} + 2 \cos^2 x$ $= 2 \sin^2 x + 2 \cos^2 x$ $= 2(\sin^2 x + \cos^2 x)$ $= 2$	✓ $-\sin 2x$ ✓ $-\sin x$ ✓ $\cos x$ ✓ $2 \cos^2 x$ ✓ $\sin 2x = 2 \sin x \cdot \cos x$ ✓ answer/antwoord (6)

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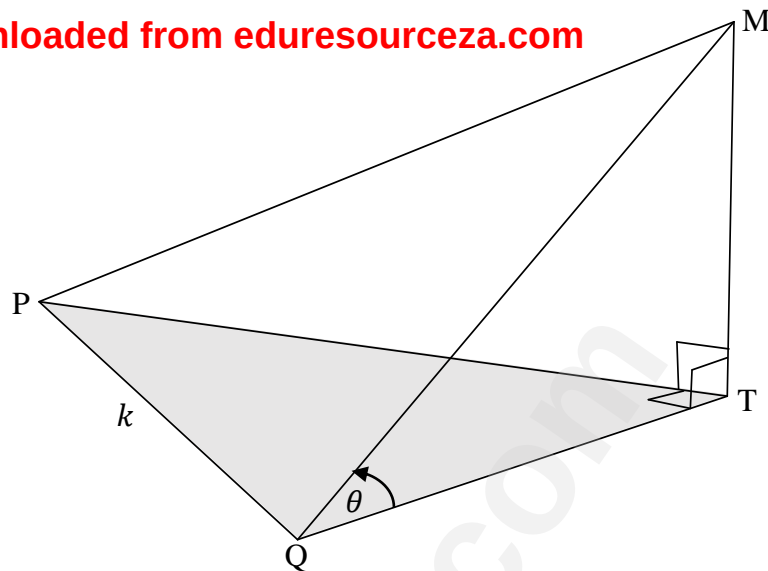
5.3	$\cos 42^\circ = 2 \cos^2 21^\circ - 1$ $\sqrt{k} = 2 \sin^2 69^\circ - 1$ $\frac{\sqrt{k} + 1}{2} = \sin^2 69^\circ$	✓ double angle expansion/ <i>dubbelhoek uitbreiding</i> ✓ co-function/ <i>ko-funksie</i> ✓ answer/ <i>antwoord</i> (3)
5.4.1	$\begin{aligned} \text{LHS/LK} &= \frac{\sin 5x \cdot \cos 3x - \cos 5x \cdot \sin 3x}{\tan 2x} - 1 \\ &= \frac{\sin(5x - 3x)}{\frac{\sin 2x}{\cos 2x}} - 1 \\ &= \cos 2x - 1 \\ &= (1 - 2 \sin^2 x) - 1 \\ &= -2 \sin^2 x \\ &= \text{RHS} \end{aligned}$	✓ $\sin(5x - 3x)$ ✓ $\tan 2x = \frac{\sin 2x}{\cos 2x}$ ✓ $\cos 2x - 1$ ✓ double angle identity/ <i>dubbelhoek identiteit</i> (4)
5.4.2	Undefined if/ <i>Ongedefinieërd as</i> $\tan 2x = 0$ $x = 0^\circ$ or/ <i>of</i> $\tan 2x$ is undefined if/ <i>ongedefinieërd as</i> $2x = 90^\circ$ $x = 45^\circ$	✓ $x = 0^\circ$ ✓ $x = 45^\circ$ (2)
5.5.1	$\begin{aligned} f(x) &= 2 \cos x - \sin^2 x \\ &= 2 \cos x - (1 - \cos^2 x) \\ &= 2 \cos x - 1 + \cos^2 x \\ &= \cos^2 x + 2 \cos x + 1 - 1 - 1 \\ &= (\cos x + 1)^2 - 2 \end{aligned}$	✓ identity/ <i>identiteit</i> ✓ completing the square/ <i>voltooi die vierkant</i> (2)
5.5.2	Maximum of/ <i>Maksimum van</i> $\cos x = 1$ Max. of/ <i>Maks. van</i> $(\cos x + 1)^2$ is $(1 + 1)^2 = 4$ Max. of/ <i>Maks. van</i> $(\cos x + 1)^2 - 2$ is 2	✓ Max. of/ <i>Maks. van</i> $\cos x = 1$ ✓ answer/ <i>antwoord</i> Answer only: Full marks Slegs antwoord: Volpunte (2)
		[26]

QUESTION/VRAAG 6

6.1	$\cos(x - 30^\circ) + 2 \sin x = 0$ $\cos x \cos 30^\circ + \sin x \sin 30^\circ + 2 \sin x = 0$ $\frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x + 2 \sin x = 0$ $\sqrt{3} \cos x + \sin x + 4 \sin x = 0$ $5 \sin x = -\sqrt{3} \cos x$ $\tan x = -\frac{\sqrt{3}}{5}$	✓ expansion of compound angle/ <i>uitbreiding van saamgestelde hoek</i> ✓ special angles/ <i>spesiale hoeke</i> ✓ simplification/ <i>vereenvoudiging</i> ✓ $5 \sin x = -\sqrt{3} \cos x$
6.2	$\tan x = \frac{-\sqrt{3}}{5}$ ref. angle/ <i>verwys. hoek</i> = $19,106^\circ$ $x = 160,89^\circ + k \cdot 180^\circ; k \in \mathbb{Z}$ $x = -19,11^\circ$ or/of $x = 160,89^\circ$ OR/OF $\tan x = \frac{-\sqrt{3}}{5}$ $x = -19,11^\circ + k \cdot 180^\circ; k \in \mathbb{Z}$ $x = -19,11^\circ$ or $x = 160,89^\circ$	✓ ref angle/ <i>verwys. hoek</i> ✓ $x = -19,11^\circ$ ✓ $x = 160,89^\circ$ OR/OF ✓ $x = -19,11^\circ + k \cdot 180^\circ; k \in \mathbb{Z}$ ✓ $x = -19,11^\circ$ ✓ $x = 160,89^\circ$
6.3.1	Amplitude = 2	✓ answer/ <i>antwoord</i>
6.3.2		
	✓ both <i>x</i> -intercepts/ <i>albei x-afsnitte</i> ✓ turning points/ <i>draaipunte</i> $(-150^\circ; -1)$, $(30^\circ; 1)$, $(210^\circ; -1)$ ✓ shape/ <i>vorm</i>	(3)
6.3.3	(a) $x \in (-19,11^\circ; 160,89^\circ)$ or $-19,11^\circ < x < 160,89^\circ$	✓ Correct intervals/ <i>korrekte intervale</i> ✓ Correct notations/ <i>korrekte notasies</i>
	(b) $x = 180^\circ$	✓ answer
		[14]

QUESTION/VRAAG 7

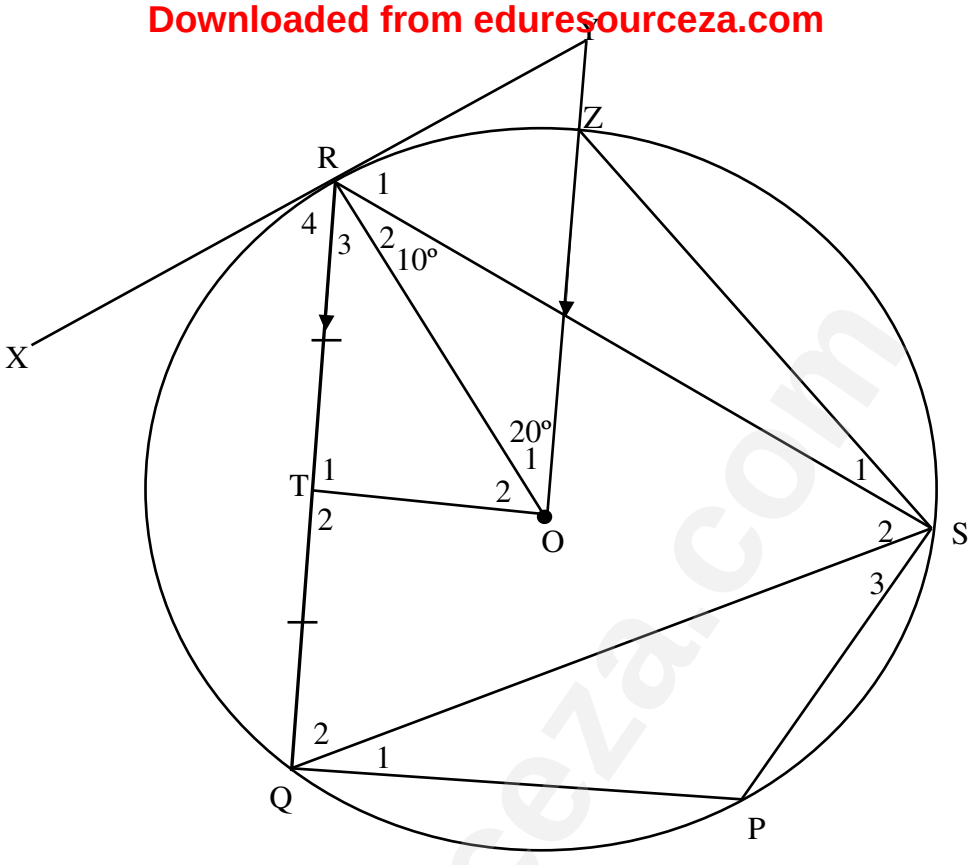
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7.1	$\text{Area / Opp} = \frac{1}{2} \times k \times 2k \times \sin \hat{MPQ}$ $2k^2 \cdot \sin \theta \cos \theta = \frac{1}{2} \times k \times 2k \times \sin \hat{MPQ}$ $k^2 \cdot \sin 2\theta = k^2 \times \sin \hat{MPQ}$ $\sin 2\theta = \sin \hat{MPQ}$ $\hat{MPQ} = 2\theta$	✓ subs in area rule/vervang in opp. reël ✓ area of/van $\Delta MPQ = k^2 \times \sin \hat{MPQ}$ ✓ $2k^2 \cdot \sin \theta \cdot \cos \theta = k^2 \sin 2\theta$ (3)
7.2	$MQ^2 = k^2 + (2k)^2 - 2(k)(2k) \cos 2\theta$ $MQ^2 = k^2 + 4k^2 - 4k^2 \cos 2\theta$ $= 5k^2 - 4k^2 \cos 2\theta$ $= 5k^2 - 4k^2 (1 - 2 \sin^2 \theta)$ $= 5k^2 - 4k^2 + 8k^2 \sin^2 \theta$ $= k^2 + 8k^2 \sin^2 \theta$ $= k^2 (1 + 8 \sin^2 \theta)$ $MQ = k \sqrt{1 + 8 \sin^2 \theta}$	✓ correct subst into cosine rule/korrekte substitusie in die cos reël ✓ $5k^2 - 4k^2 \cos 2\theta$ ✓ $\cos 2\theta = 1 - 2 \sin^2 \theta$ ✓ $k^2 (1 + 8 \sin^2 \theta)$ (4)
7.3	$MQ = 139,5 \sqrt{1 + 8 \sin^2 42^\circ}$ $MQ = 298,6045..$ $\frac{MT}{MQ} = \sin \theta$ $MT = 298,6045.. \times \sin 42^\circ$ $MT = 199,805..$ $MT \approx 200 \text{ m}$	✓ value of/waarde van MQ ✓ ratio/verhouding ✓ $MT = 200 \text{ m}$ (3) 1 mark penalty for incorrect rounding/ Penaliseer met 1 punt vir verkeerde afronding
[10]		

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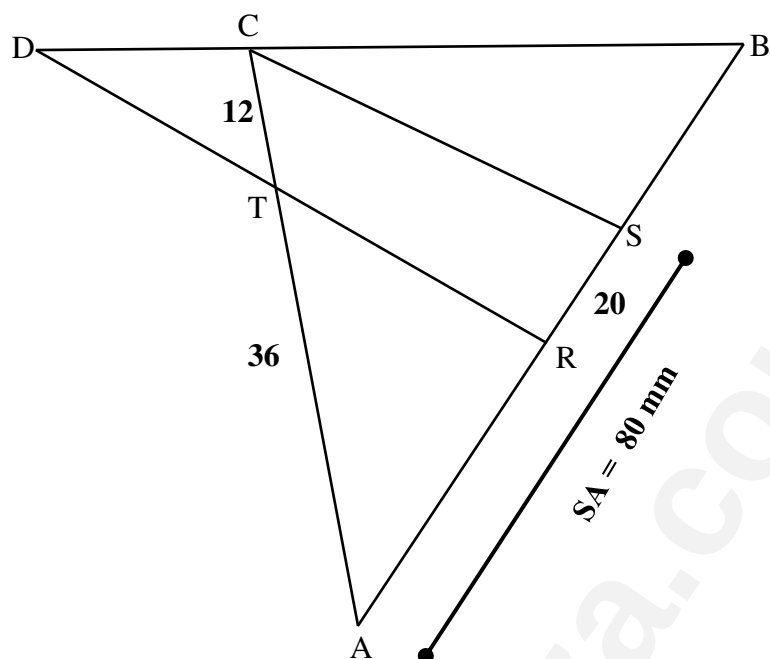
QUESTION/VRAAG 8

8.1	Downloaded from eduresourceza.com 	
8.1.1	$\hat{S}_1 = 10^\circ$ $\angle \text{centre} = 2 \times \angle \text{at circumference/midpt}$ $\angle = 2 \times \text{omtrek } \angle$	$\checkmark S$ $\checkmark R$ (2)
8.1.2	$\hat{R}_3 = 20^\circ$ Alt $\angle$'s/Verwis. $\angle^e$, RQ//YO	$\checkmark S/R$ (1)
8.1.3	$\hat{P} = 150^\circ$ opposite $\angle^s$ of cyclic quad/teenoorst. $\angle^e$ van koordevierhoek	$\checkmark S$ $\checkmark R$ (2)
8.1.4	$\hat{R}_3 + \hat{R}_4 = 90^\circ$ $\hat{R}_4 = 90^\circ - 20^\circ = 70^\circ$ $\hat{S}_2 = \hat{R}_4 = 70^\circ$ OR/OF $\hat{R}_1 + \hat{R}_2 = 90^\circ$ $\hat{R}_1 = 80^\circ$ $\hat{Q}_2 = 80^\circ$ $\hat{S}_2 = 70^\circ$	radius $\perp$ tangent/raaklyn tan chord theorem/raaklyn koord stelling OR/OF radius $\perp$ tangent/raaklyn tan chord theorem/raaklyn koord stelling sum of $\angle^s$ in a triangle/binne $\angle^e$ van 'n driehoek $\checkmark R$ $\checkmark S$ $\checkmark S \checkmark R$ OR/OF $\checkmark R$ $\checkmark S$ $\checkmark S / R$ $\checkmark S$ (4)

8.2	$\hat{T}_1 = 90^\circ$ <i>line from centre to midpoint of chord/lyn vanaf middelpunt tot middelpunt van koord</i> $\hat{O}_2 = 70^\circ$ <i>sum of $\angle^s$ in a triangle/binne $\angle^e$ van 'n driehoek</i> $\hat{R}_4 = \hat{O}_2 = 70^\circ$ $\therefore$ XRY is a tangent/ 'n raaklyn <i>converse tan chord theorem/omgekeerde raaklyn koord stelling</i> OR/OF $\hat{R}_1 + \hat{R}_2 = 90^\circ$ <i>radius $\perp$ tangent/raaklyn</i> $\hat{T}_1 = 90^\circ$ <i>line from centre to midpoint of chord/lyn vanaf middelpunt tot middelpunt koord</i> $OR \hat{Y} = \hat{T}_1$ <i>converse tan chord theorem/omgekeerde raaklyn koord stelling</i> $\therefore$ XRY is a tangent/ 'n raaklyne	✓S/R ✓S ✓R (3) ✓S ✓S/R ✓R (3)
		[12]

QUESTION/VRAAG 9

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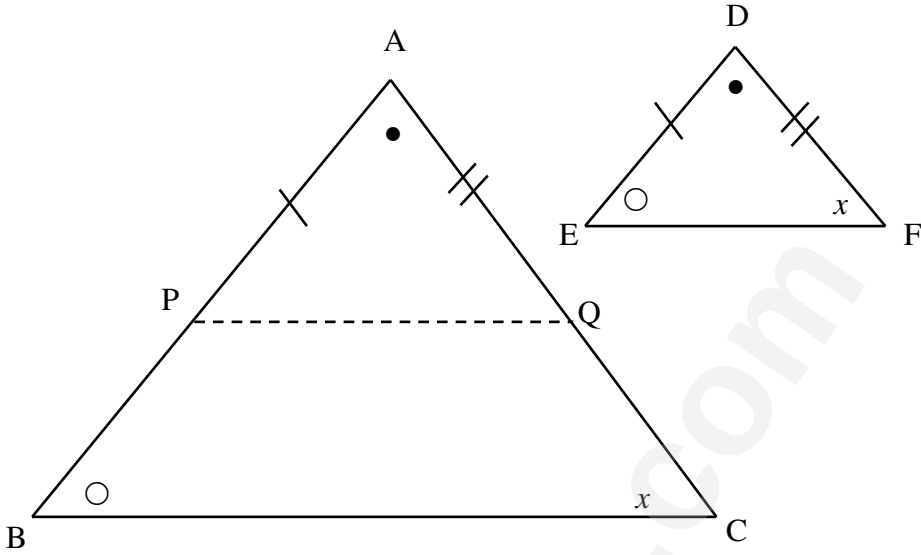
9.1	$\frac{CT}{TA} = \frac{12}{36} = \frac{1}{3}$ $\frac{SR}{RA} = \frac{20}{60} = \frac{1}{3}$ $\therefore \frac{CT}{TA} = \frac{SR}{RA}$ $\therefore CS \parallel TR$ line divides the sides of Δ in proportion/lyn verdeel die sye van die Δ in verhouding OR/OF $\frac{SR}{SA} = \frac{20}{80} = \frac{1}{4}$ $\frac{CT}{CA} = \frac{12}{48} = \frac{1}{4}$ $\therefore \frac{CT}{CA} = \frac{SR}{SA}$ $\therefore CS \parallel TR$ line divides the sides of Δ in proportion/lyn verdeel die sye van die Δ in verhouding	✓S ✓S ✓R (3) ✓S ✓S ✓R (3)
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9.2	$\frac{AR}{RB} = \frac{2}{3}$ $\frac{60}{RB} = \frac{2}{3}$ $RB = 90 \text{ mm}$ $\therefore SB = 70 \text{ mm}$ $\frac{BS}{SR} = \frac{BC}{CD}$ proportion theorem/<i>eweredigheidstelling</i> CS TR $\frac{70}{20} = \frac{2x}{\frac{1}{2}x + 1}$ $40x = 35x + 70$ $x = 14 \text{ mm}$ OR/OF $2k = 60$ $\therefore k = 30$ $\therefore 3k = 90$ $\frac{SR}{RB} = \frac{DC}{DB}$ prop. theorem/<i>eweredigheidstelling</i> CS TR $\frac{20}{90} = \frac{\frac{1}{2}x + 1}{\frac{5}{2}x + 1}$ $5x + 2 = \frac{9}{2}x + 9$ $\frac{1}{2}x = 7$ $\therefore x = 14 \text{ mm}$	✓S ✓value of/<i>waarde van</i> RB ✓S ✓R ✓substitution/<i>substitusie</i> ✓answer/<i>antwoord</i> (6) ✓value of/<i>waarde van</i> k ✓value of/<i>waarde van</i> 3k ✓S ✓R ✓substitution/<i>substitusie</i> ✓answer/<i>antwoord</i> (6) [9]
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QUESTION/VRAAG 10

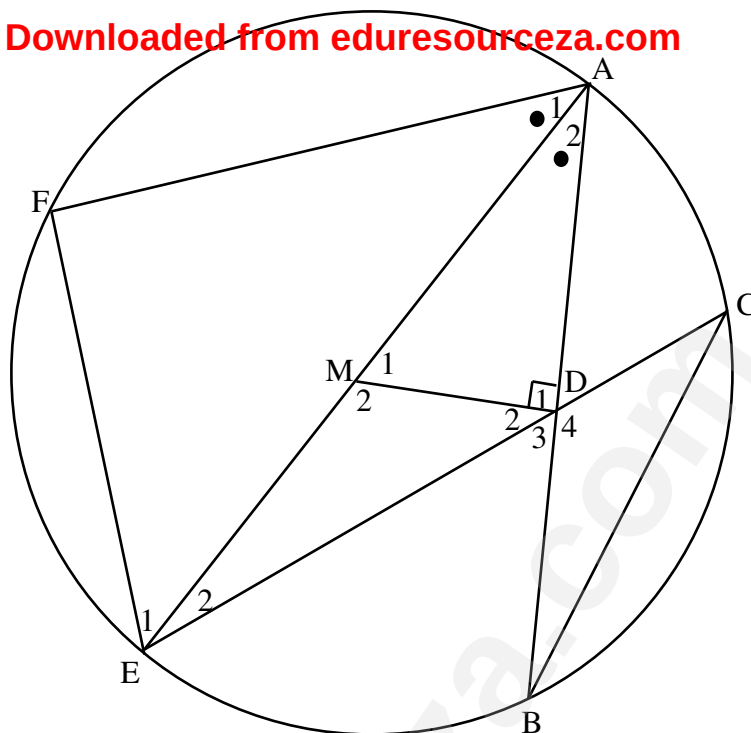
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10.1	 NB: No construction/Geen konstruksie nie 0/6	
	On AB, mark off AP = DE and on AC, mark off AQ = DF. / Op AB, merk AP = DE af en op AC, merk AQ = DF af. Draw/Trek PQ. Proof/Bewys: In $\triangle APQ$ and/en $\triangle DEF$ $\hat{A} = \hat{D}$ given/gegee $AQ = DF$ construction/konstruksie $AP = DE$ construction/konstruksie $\therefore \triangle APQ \equiv \triangle DEF$ S.S $\hat{A}PQ = \hat{E}$ $\hat{A}PQ = \hat{B}$ [$\hat{E} = \hat{B}$] $\therefore PQ \parallel BC$ corresponding angles are equal/ooreenk. hoeke is gelyk $\frac{AB}{AP} = \frac{AC}{AQ}$ line $\parallel$ to one side of $\triangle$/lyn $\parallel$ aan een sy van $\triangle$ $AP = DE$ and/en $AQ = DF$ $\therefore \frac{AB}{DE} = \frac{AC}{DF}$	✓ construction/ konstruksie ✓ S ✓ R ✓ S ✓ R ✓ S/R (6)

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10.2

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10.2.1	$\hat{F} = 90^\circ$ $\angle$ in semi - circle/ <i>halwe sirkel</i> In $\triangle AEF$ and/en $\triangle AMD$ $\hat{A}_1 = \hat{A}_2$ given/ <i>gegee</i> $\hat{F} = \hat{D}_1 = 90^\circ$ proved/ <i>bewys</i> $\therefore \triangle AEF \parallel \triangle AMD$ $\angle \angle \angle$	$\checkmark$ S/R $\checkmark$ S $\checkmark$ S $\checkmark$ R (4)
	OR/OF $\hat{F} = 90^\circ$ $\angle$ in semi - circle/ <i>halwe sirkel</i> In $\triangle AEF$ and/en $\triangle AMD$ $\hat{A}_1 = \hat{A}_2$ given/ <i>gegee</i> $\hat{F} = \hat{D}_1 = 90^\circ$ proved/ <i>bewys</i> $\hat{E}_1 = \hat{M}_1$ sum of $\angle$'s in $\triangle$ / <i>binne $\angle$ e van $\triangle$</i> $\therefore \triangle AEF \parallel \triangle AMD$	OR/OF $\checkmark$ S/R $\checkmark$ S $\checkmark$ S $\checkmark$ R (4)

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10.2.2	$\frac{AF}{AD} = \frac{AE}{AM}$ $\Delta AEF \parallel \Delta AMD$ but/maar $AE = 2AM$ $\frac{AF}{AD} = \frac{2AM}{AM} = 2$	✓ S/R ✓ S ✓ answer/antwoord (3)
10.2.3	In ΔCDB and/en ΔADE $\hat{C} = \hat{A}_2$ $\angle s$ in same segment/ $\angle e$ in dieselfde segment $\hat{D}_4 = \hat{E}DA$ vertically opposite $\angle$'s/ regoorstaande $\angle e$ $\Delta CDB \parallel \Delta ADE$ $\angle \angle \angle$ $\therefore \frac{CD}{AD} = \frac{DB}{DE}$ but/maar $AD = DB$ line from centre $\perp$ to chord/lyn vanaf midpt. $\perp$ op koord $\frac{CD}{AD} = \frac{AD}{DE}$ $\therefore AD^2 = CD \times DE$ OR/OF In ΔCDB and/en ΔADE $\hat{C} = \hat{A}_2$ $\angle s$ in same segment/ $\angle e$ in dieselfde segment $\hat{D}_4 = \hat{E}DA$ vertically opposite $\angle$'s/ regoorstaande $\angle e$ $\hat{B} = \hat{E}_2$ $\angle s$ in same segment/ $\angle e$ in dieselfde segment $\Delta CDB \parallel \Delta ADE$ $\therefore \frac{CD}{AD} = \frac{DB}{DE}$ but/maar $AD = DB$ line from centre $\perp$ to chord/lyn vanaf midpt. $\perp$ op koord $\frac{CD}{AD} = \frac{AD}{DE}$ $\therefore AD^2 = CD \times DE$	✓ identifying Δs / identifiseer Δe ✓ S/R ✓ S ✓ R ✓ S ✓ S/R (6) OR/OF ✓ identifying Δs / identifiseer Δe ✓ S/R ✓ S ✓ R ✓ S ✓ S/R [19]

TOTAL/TOTAAL: 150



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**NATIONAL
SENIOR CERTIFICATE
*NASIONALE
SENIORSERTIFIKAAT***

GRADE/*GRAAD* 12

SEPTEMBER 2023

**MATHEMATICS P2/*WISKUNDE V2*
MARKING GUIDELINE/*NASIENRIGLYN***

MARKS/*PUNTE*: 150

This marking guideline consists of 14 pages.
Hierdie nasienriglyn bestaan uit 14 bladsye.

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QUESTION 1/VRAAG 1

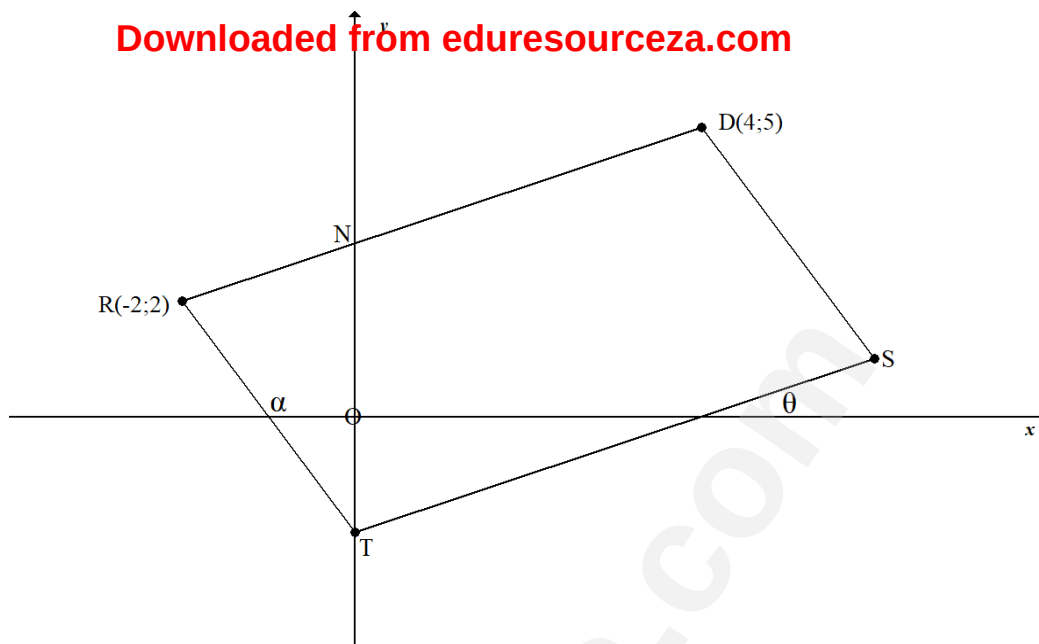
Data Set / Datastel	$\frac{29+27+24+31+22+19+30}{7} = 26$	✓✓ mean / gemiddelde	(2)
1.1.1(a)	Mean / Gemiddelde = $\frac{182}{7} = 26$	✓✓ SD / standaardafwyking	(1)
1.1.1(b)	Standard deviation / standaardafwyking = 4,14	✓✓ calculations / berekeninge ✓ answer / antwoord	(3)
1.1.2	{26 – 4,14; 26 + 4,14} = {21, 86; 30, 14} 5 players / spelers	✓✓ for any two valid comments using SD and the mean <i>Vir enige twee geldige opmerkings in gebruik van standaardafwyking en die gemiddelde</i>	(2)
1.1.3	Rugby team has the same average number of push-ups. The rugby team results were clustered around the mean because of the smaller standard deviation. <i>Rugbyspan het dieselfde gemiddelde aantal opstote. Die rugbyspan se uitslae was rondom die gemiddelde gegroepeer agv die kleiner standaardafwyking.</i>	✓ answer / antwoord ✓ reason / rede	(2)
1.2.1	50%	✓ answer / antwoord	(1)
1.2.2	Mean / Gemiddelde Distribution skewed to the right. (positively skewed) <i>Verspreiding is skeef na regs (positief skeef)</i>	✓ answer / antwoord ✓ reason / rede	(2)
			[11]

QUESTION 2/VRAAG 2

Third Term % Derde Kw. %	71	80	59	38	41	98	80	88	91	94	64	94	70	42	64
Final Term % Finale Kw. %	74	77	58	41	42	98	78	92	85	92	68	96	73	52	71

2.1	a = 9, 035 b = 0, 895 $\hat{y} = 9,035 + 0,895x$	✓ for a / vir a ✓ for b / vir b ✓ for equation / vir vergelyking	(3)
2.2	r = 0, 98	✓ answer / antwoord	(1)
2.3.1	$y = 9,035 + 0,895(48)$ $y \approx 52$	✓ substitution / vervanging ✓ answer / antwoord	(2)
2.3.2	correlation is very strong OR 48 is within domain of regression line. <i>korrelasie is baie sterk OF 48 is binne die gebied van die regressie-lyn.</i>	✓ answer / antwoord	(1)
2.4.1	50% is outside the domain of the line (data set) OR (50 ; 80) is an outlier. <i>50% is buite die gebied van die lyn (datastel) OF (50 ; 80) is 'n uitskieter</i>	✓ answer / antwoord	(1)
2.4.2	Increase the gradient / Vermeerder die gradiënt	✓ answer / antwoord	(1)
			[9]

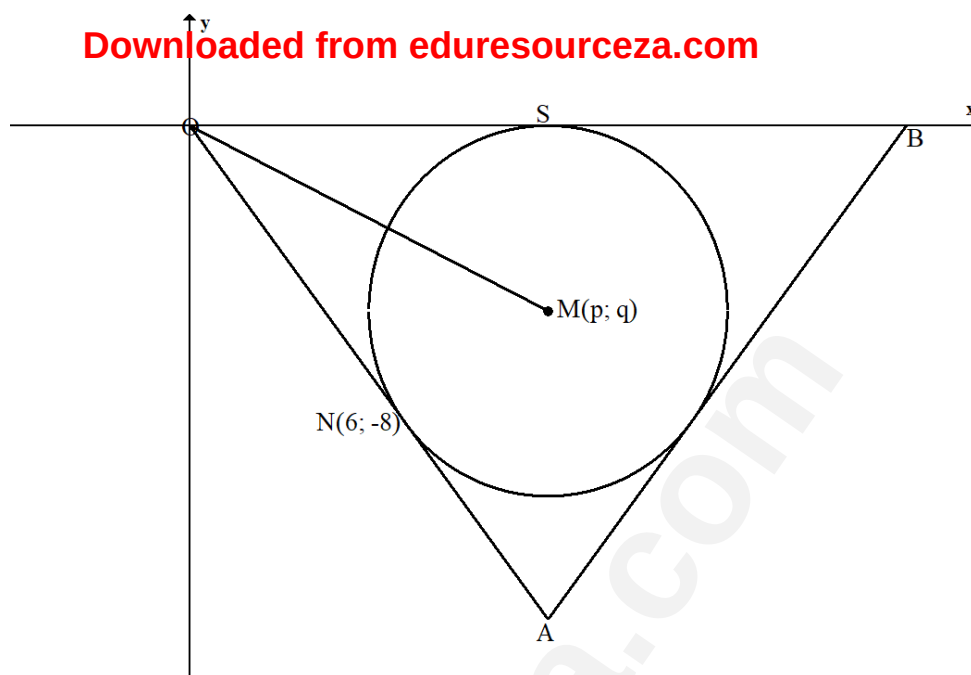
QUESTION 3 / VRAAG 3



3.1	$T(0; -2)$	✓ answer / antwoord	(1)
3.2.1	$m_{RT} = \frac{2 - (-2)}{-2 - 0} = -\frac{4}{2} = -2$	✓ substitution / vervanging ✓ answer / antwoord	(2)
3.2.2	$\tan \theta = \frac{1}{2}$ $\therefore \theta = 26,57^\circ$ $\tan \alpha = -2$ $\therefore \alpha = 116,57^\circ$ $\therefore \hat{R\hat{T}S} = 116,57^\circ - 26,57^\circ = 90^\circ$ OR / OF $m_{TS} = \frac{1}{2}$ $m_{RT} = -2$ $m_{TS} \times m_{RT} = -1$ $\hat{R\hat{T}S} = 90^\circ$	✓ for / vir $\tan \theta = \frac{1}{2}$ ✓ for / vir $\theta = 26,57^\circ$ ✓ for / vir $\tan \alpha = -2$ ✓ for / vir $\alpha = 116,57^\circ$ ✓ $\hat{R\hat{T}S} = 90^\circ$ OR / OF ✓ $m_{TS} = \frac{1}{2}$ ✓ $m_{RT} = -2$ ✓✓ product of gradients <i>produk van gradiënte</i> ✓ $\hat{R\hat{T}S} = 90^\circ$	(5)
3.3	$y = \frac{1}{2}x + c$ $2 = \frac{1}{2}(-2) + c$ $c = 3$ $\therefore y = \frac{1}{2}x + 3$	✓ gradient / gradiënt ✓ substitution of point R or D <i>vervang van punt R of D</i> ✓ answer / antwoord	(3)

3.4	RDST is a parallelogram (opposite sides parallel) RDST is 'n parallelogram (teenoorst. sye ewewydig) Midpoint of ~~RD~~ <i>DT</i> $\left(\frac{4+0}{2}; \frac{5-2}{2}\right) = \left(2; \frac{3}{2}\right)$ Midpoint of RS is the same as midpoint of DT (diagonals bisect each other.) <i>Middelpunt van RS is dieselfde as die middelpunt van DT (hoeklyne halveer mekaar)</i> Midpoint of RS / <i>Middelpunt van RS</i>: $\left(2; \frac{3}{2}\right)$ OR / OF $S(6; 1)$ Midpoint of RS / <i>Middelpunt van RS</i>: $\left(\frac{6-2}{2}; \frac{1+2}{2}\right) = \left(2; \frac{3}{2}\right)$	✓ substitution in the MP formula <i>vervang in die MP formule</i> ✓ answer / <i>antwoord</i> OR / OF ✓✓ $S(6; 1)$ ✓ answer / <i>antwoord</i>	(3)
3.5	$N(0; 3)$ $RN = \sqrt{2^2 + 1^2} = \sqrt{5}$ $RT = \sqrt{2^2 + 4^2} = \sqrt{20}$ $\text{Area / Oppervlakte} = \frac{1}{2} \times \sqrt{20} \times \sqrt{5}$ $= 5 \text{ square units / eenhede}^2$ OR / OF $TN = 5 \text{ units / eenhede}$ Height / <i>Hoogte</i> = 2 units / <i>eenhede</i> $\text{Area / Oppervlakte} = \frac{1}{2} \times 5 \times 2$ Area / <i>Oppervlakte</i> = 5 square units / <i>eenhede</i>²	✓ coordinates of N <i>koördinate van N</i> ✓ for / <i>vir</i> RN ✓ for / <i>vir</i> RT ✓ for the answer / <i>vir die antwoord</i> OR / OF ✓ $TN = 5 \text{ units / eenhede}$ ✓ Height/<i>hoogte</i> = 2 units / <i>eenhede</i> ✓ sub. into formula <i>vervang in formule</i> ✓ answer / <i>antwoord</i>	(4)
			[18]

QUESTION 4 / VRAAG 4

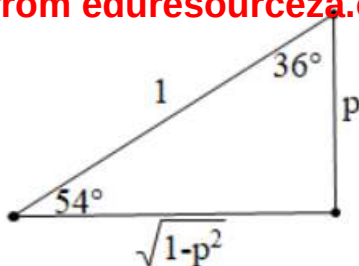


4.1.1	$ON = \sqrt{(6-0)^2 + (-8)^2}$ $= \sqrt{100} = 10 \text{ units / eenhede}$	✓ substitution in correct formula <i>vervanging in korrekte formule</i> ✓ answer / antwoord	(2)
4.1.2	$ON = OS$ (tangents from the same point) <i>(raaklyne vanaf dieselfde punt)</i> $\therefore p = 10 \text{ units / eenhede}$	✓ S and/en R ✓ answer / antwoord	(2)
4.1.3	$ON \perp NM$ (tan – radius) / <i>(raaklyn – radius)</i> $m_{ON} = \frac{-8}{6} = \frac{-4}{3}$ $m_{NM} = \frac{3}{4}$	✓ S and/en R ✓ gradient of ON / <i>gradiënt van ON</i> ✓ gradient of NM <i>gradiënt van NM</i>	(3)
4.1.4	$m_{NM} = \frac{q+8}{10-6} = \frac{3}{4}$ $\frac{q+8}{4} = \frac{3}{4}$ $q = -5$	✓ for subs and equating <i>vir vervanging en gelyk stel</i> ✓ answer / antwoord	(2)
4.2	$MS = r = 5 \text{ units / eenhede}$ $(x-10)^2 + (y+5)^2 = 25$	✓ radius / radius ✓ centre sub/ vervang middelpunt ✓ answer / antwoord	(3)
4.3	$k = 5$ OR/OF $k = 15$	✓ $k = 5$ OR/OF ✓ $k = 15$	(2)

4.4	Coordinates of the point directly opposite N is C. <i>Koördinate van die punt regoor N is C.</i> $C\left(\frac{x+6}{2} = 10; \frac{y-8}{2} = -5\right)$ $C(14; -2)$ Equation of the tangent at C: <i>Vergelyking van die raaklyn by C:</i> $y + 2 = -\frac{4}{3}(x - 14)$ $y = -\frac{4}{3}x + \frac{50}{3}$ $\therefore 0 < t < \frac{50}{3}$	✓ formula and sub / <i>formule en vervanging</i> ✓ for x-coordinate <i>vir x-koördinaat</i> ✓ for y-coordinate <i>vir y-koördinaat</i> ✓ substitution / vervanging ✓ for the answer / <i>vir die antwoord</i> ✓ for the value of t. <i>vir die waarde van t</i>	(6)
4.5	They will not touch. The new circle is the old circle shifted up by 11. <i>Hulle sal nie raak nie.</i> <i>Die nuwe sirkel is die ou sirkel 11 eenhede opwaarts geskuif.</i>	✓ answer / <i>antwoord</i> ✓ any valid reason / <i>enige geldige antwoord</i>	(2)
			[22]

QUESTION 5/VRAAG 5

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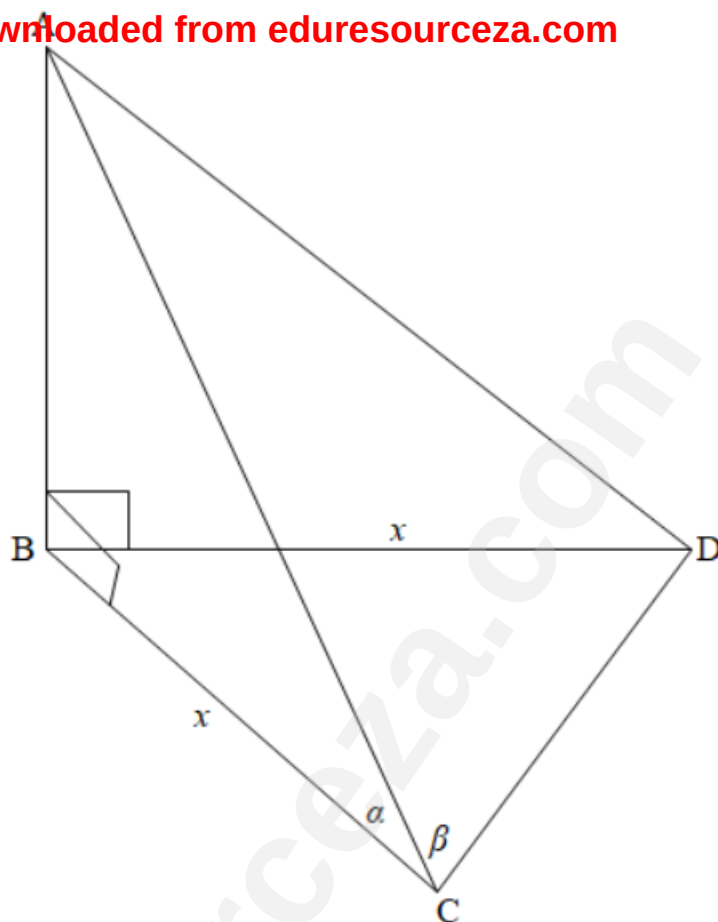
5.1.1	$\sin 594^\circ$ $= \sin 234^\circ$ $= -\sin 54^\circ$ $= -p$	$\checkmark -\sin 54^\circ$ $\checkmark -p$	(2)
5.1.2	$\cos 36^\circ$ $= p$	$\checkmark \checkmark$ answer / antwoord	(2)
5.1.3	$\cos 18^\circ$ $= \sin 72^\circ$ $= \sin 2(36^\circ)$ $= 2 \sin 36^\circ \cos 36^\circ$ $= 2 \left(\frac{\sqrt{1-p^2}}{1} \right) \left(\frac{p}{1} \right)$ $= 2p(\sqrt{1-p^2})$	$\checkmark$ for / vir $\sin 72^\circ$ $\checkmark$ for / vir $\sin 2(36^\circ)$ $\checkmark$ for / vir $2 \sin 36^\circ \cos 36^\circ$ $\checkmark$ for / vir $2p(\sqrt{1-p^2})$	(4)
5.2	$\frac{\cos 140^\circ - \sin(90^\circ - \theta)}{\sin 410^\circ + \cos(-\theta)}$ $= \frac{\cos(90^\circ + 50^\circ) - \cos \theta}{\sin 50^\circ + \cos \theta}$ $= \frac{-\sin 50^\circ - \cos \theta}{\sin 50^\circ + \cos \theta}$ $= \frac{-(\sin 50^\circ + \cos \theta)}{(\sin 50^\circ + \cos \theta)}$ $= -1$	$\checkmark -\sin 50^\circ \quad \checkmark \cos \theta$ $\checkmark \sin 50^\circ \quad \checkmark \cos \theta$ $\checkmark$ for the common factor vir die gemene faktor $\checkmark$ for the answer vir die antwoord	(6)

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5.3	$\cos(x + 65^\circ) \cdot \cos(x + 20^\circ) - \sin(x + 245^\circ) \cdot \sin(x + 20^\circ)$ $= \cos(x + 65^\circ) \cdot \cos(x + 20^\circ) + \sin(x + 65^\circ) \cdot \sin(x + 20^\circ)$ $= \cos[(x + 65^\circ) - (x + 20^\circ)]$ $= \cos 45^\circ$ $= \frac{1}{\sqrt{2}}$	✓ reduction / <i>reduksie</i> ✓ compound angle <i>saamgestelde hoek</i> ✓ $\cos 45^\circ$ ✓ answer / <i>antwoord</i>	(4)
5.4	$\cos^2 x - \sin^2 x = \frac{1}{2}$ $\cos 2x = \frac{1}{2}$ $2x = 60^\circ + 360^\circ \cdot k$ or/of $2x = 300^\circ + 360^\circ \cdot k$ $x = 30^\circ + 180^\circ \cdot k$ or/of $x = 150^\circ + 180^\circ \cdot k \quad k \in \mathbb{Z}$ OR / OF $2 \cos^2 x - 2 \sin^2 x = 1$ $2 \cos^2 x - 2 \sin^2 x = \sin^2 x + \cos^2 x$ $3 \sin^2 x = \cos^2 x$ $\tan^2 x = \frac{1}{3}$ $\tan x = \frac{1}{\sqrt{3}}$ $x = 30^\circ$ (reference angle / <i>verwysingshoek</i>) $x = 30^\circ + 180^\circ \cdot k, k \in \mathbb{Z}$	$\cos 2x = \frac{1}{2}$ ✓ for / <i>vir</i> $2x$ in both quadrants / <i>in beide</i> <i>kwadrante</i> ✓ $x = 30^\circ + 180^\circ \cdot k$ ✓ $x = 150^\circ + 180^\circ \cdot k$ OR / OF ✓ multiplying by 2 and using identity / <i>vermenigvuldig met</i> <i>2 en gebruik van identiteit</i> ✓ $3 \sin^2 x = \cos^2 x$ ✓ $\tan x = \frac{1}{\sqrt{3}}$ ✓ answer / <i>antwoord</i>	(4)
5.5.1	$\text{LHS} = \frac{\sin 2\theta \cdot \tan \theta}{\cos 2\theta + 1}$ $= \frac{2 \sin \theta \cos \theta \cdot \frac{\sin \theta}{\cos \theta}}{2 \cos^2 \theta - 1 + 1}$ $= \frac{2 \sin^2 \theta}{2 \cos^2 \theta}$ $= \tan^2 \theta$	✓ $2 \sin \theta \cos \theta$ ✓ $\frac{\sin \theta}{\cos \theta}$ ✓ $2 \cos^2 \theta - 1$ ✓ $\frac{2 \sin^2 \theta}{2 \cos^2 \theta}$	(4)
5.5.2	$\cos 2\theta + 1 = 0$ $\cos 2\theta = -1$ $2\theta = 180^\circ$ $\theta = 90^\circ$	✓ $\cos 2\theta + 1 = 0$ ✓ $\cos 2\theta = -1$ ✓ $2\theta = 180^\circ$ ✓ $\theta = 90^\circ$	(4)
			[30]

QUESTION 6 / VRAAG 6

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6.1	$\triangle ABC \equiv \triangle ABD$ (SAS) / (SHS)	✓ S and/en R	(1)
6.2	$\cos \alpha = \frac{x}{AC}$ $AC = \frac{x}{\cos \alpha}$	✓ S ✓ answer / antwoord	(2)
6.3	$CD^2 = \left(\frac{x}{\cos \alpha}\right)^2 + \left(\frac{x}{\cos \alpha}\right)^2 - 2\left(\frac{x}{\cos \alpha}\right)\left(\frac{x}{\cos \alpha}\right)\cos(180^\circ - 2\beta)$ $CD^2 = \frac{x^2}{\cos^2 \alpha} + \frac{x^2}{\cos^2 \alpha} + 2\left(\frac{x^2}{\cos^2 \alpha}\right)\cos 2\beta$ $CD^2 = \frac{2x^2}{\cos^2 \alpha}(1 + \cos 2\beta)$ $CD^2 = \frac{2x^2}{\cos^2 \alpha}(1 + (1 - 2\sin^2 \beta))$ $CD^2 = \frac{2x^2}{\cos^2 \alpha} \times 2\cos^2 \beta$ $CD^2 = \frac{4x^2 \cos^2 \beta}{\cos^2 \alpha}$ $\therefore CD = \frac{2x \cos \beta}{\cos \alpha}$ OR / OF	✓ cos rule / cos-reël ✓ simplification vereenvoudiging ✓ double angles expansion dubbelhoeke uitbreiding ✓ simplification vereenvoudiging OR / OF	

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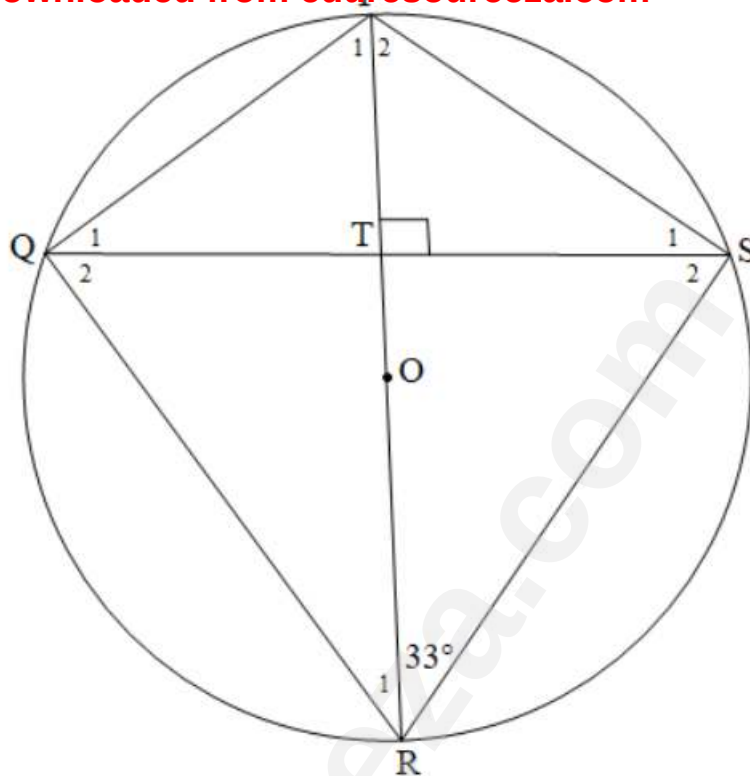
	$\widehat{ADB} = \beta$ $\widehat{CAD} = 180^\circ - 2\beta$ $\frac{CD}{\sin(180^\circ - 2\beta)} = \frac{AC}{\sin \beta}$ $CD = \frac{AC \sin 2\beta}{\sin \beta}$ $CD = \frac{x \cdot 2 \sin \beta \cos \beta}{\cos \alpha \sin \beta}$ $CD = \frac{2x \cos \beta}{\cos \alpha}$	✓ for /vir $\widehat{ADB} = \beta$ and/en $\widehat{CAD} = 180^\circ - 2\beta$ ✓ use of sin rule <i>gebruik van sinus-reël</i> ✓ substitution of AC <i>vervangings van AC</i> ✓ simplification / <i>vereenvoudiging</i>	(4)
6.4	$CD = \frac{2x \cos \beta}{\cos \alpha}$ $CD = \frac{2(25) \cos 65,62^\circ}{\cos 30^\circ}$ $CD = 23,83 \text{ cm}$	✓ substitution / <i>vereenvoudiging</i> ✓ answer / <i>antwoord</i>	(2)
[9]			

QUESTION 7 / VRAAG 7

7.1	$f(180^\circ) = -0,71$ $\therefore$ Range: <i>Terrein: Waardeversameling:</i> $-0,71 \leq y \leq 1$ OR / OF $[-0,71; 1]$	✓ $f(180^\circ)$ ✓ answer / <i>antwoord</i>	(2)
7.2	✓ intercepts / <i>afsnitte</i> ✓ shape / <i>vorm</i> ✓ turning points / <i>draaipunte</i>		(3)
7.3	Period / <i>Periode</i> = 180°	✓ answer / <i>antwoord</i>	(1)
7.4	$-45^\circ < x < 45^\circ$	✓✓ answer / <i>antwoord</i>	(2)
7.5	$x = -45^\circ$ or / <i>of</i> $x = 135^\circ$	✓ $x = -45^\circ$ ✓ $x = 135^\circ$	(2)
7.6	$g(x) = \cos(x + 15^\circ)$	✓ answer / <i>antwoord</i>	(1)
[11]			

QUESTION 8 / VRAAG 8

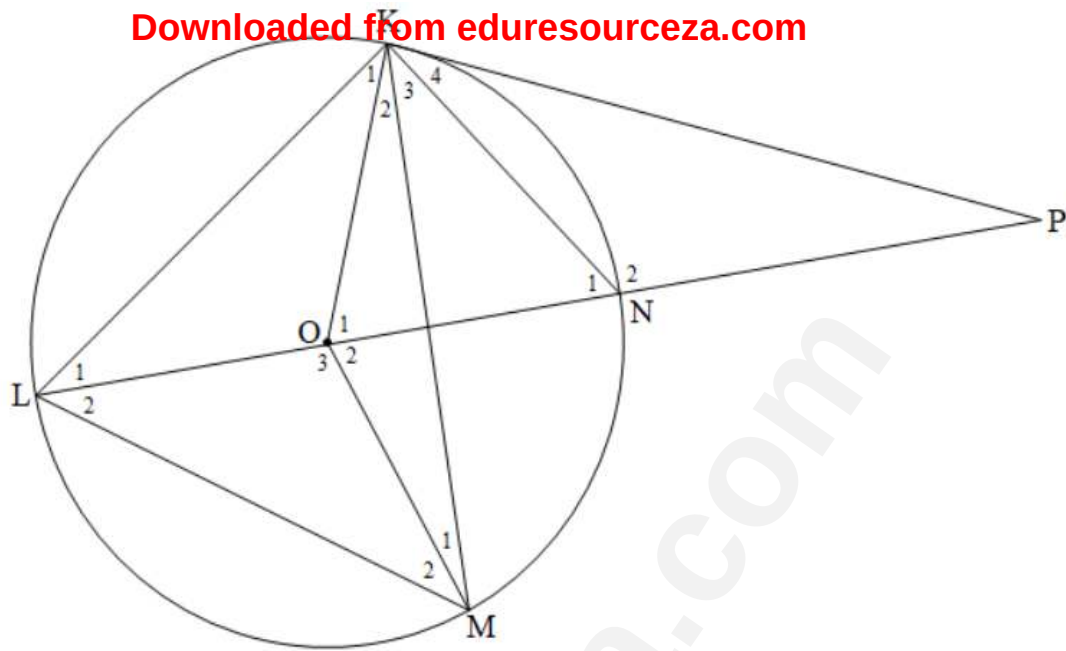
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8.1.1	$\hat{Q}_1 = 33^\circ$ ($\angle$ s in the same segment) ($\angle$ e in dieselfde segment)	✓ S ✓ R	
	$\hat{P}_1 = 57^\circ$ ($\angle$ s of a triangle) / ($\angle$ e van 'n driehoek)	✓ S and/en R	(3)
8.1.2	$\hat{Q} = 90^\circ$ ($\angle$ subtended by the diameter) ($\angle$ onderspan deur middellyn)	✓ S and/en R	
	$\hat{Q}_2 = 57^\circ$ (complementary $\angle$ s / $\angle$ s of a triangle) (komplementêre $\angle$ e / $\angle$ e van 'n driehoek)	✓ S and/en R	(2)
8.2	$QT = TS = 8 \text{ cm}$ (line from centre perp to chord) (lyn vanaf middelpunt loodreg op koord)	✓ S and/en R	
	$OQ = OS = 10 \text{ cm}$ (radii) / (radiusse)	✓ S and/en R	
	$OQ^2 = TO^2 + QT^2$ (Pythagoras) / (Pythagoras)	✓ S and/en R	
	$10^2 = OT^2 + 8^2$		
	$TO = \sqrt{100 - 64} = 6 \text{ cm}$	✓ answer / antwoord	(4)
			[9]

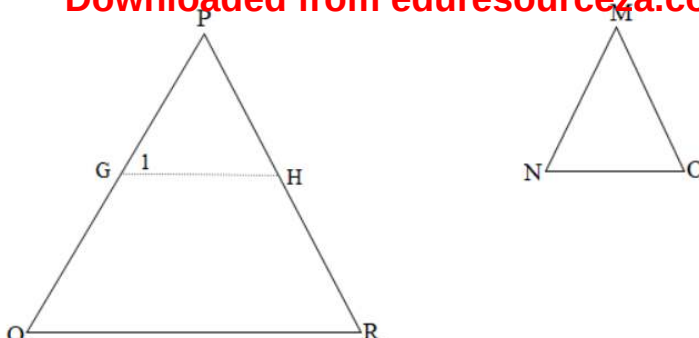
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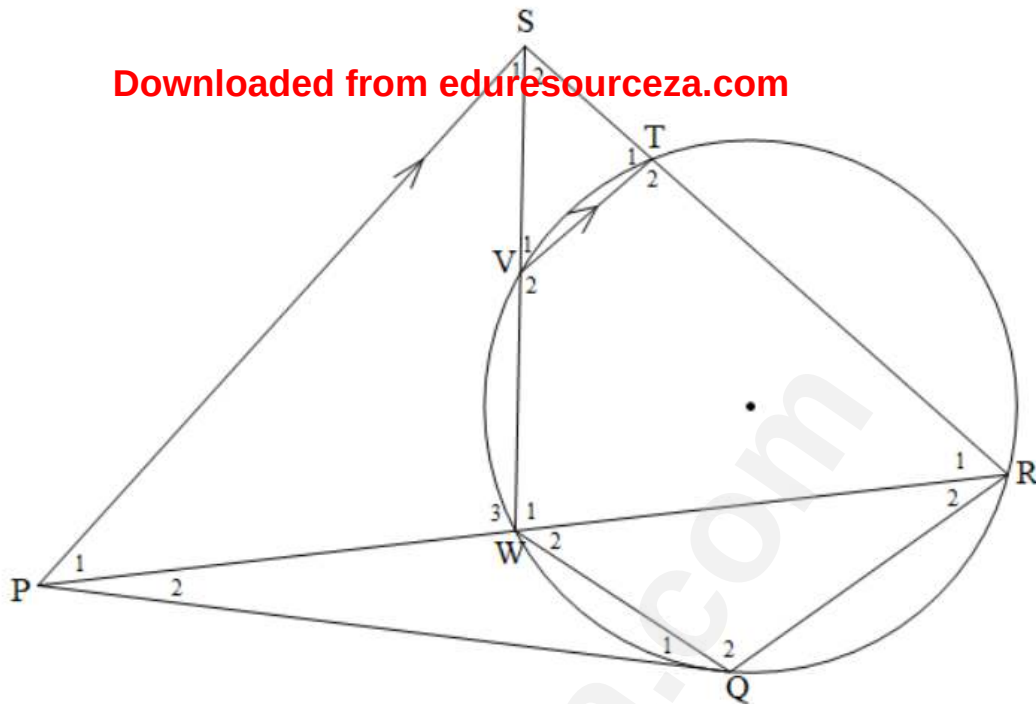
QUESTION 9 / VRAAG 9



9.1	$\hat{L}\hat{K}N = 90^\circ$ $\hat{O}\hat{K}P = 90^\circ$	✓ answer / antwoord ✓ answer / antwoord	(2)
9.2.1	$\hat{L}_1 = x$ (tan – chord theorem) / (raaklyn-koord stelling)	✓ S ✓ R	(2)
9.2.2	$\hat{K}_1 = x$ ($\angle$ s opp. = sides) / ($\angle$ e teenoor = sye)	✓ S ✓ R	(2)
9.2.3	In ΔLKP ; $\hat{L}\hat{K}P = 90^\circ + x$ and/en $\hat{L}_1 = x$ $\therefore \hat{P} = 90^\circ - 2x$ ($\angle$ s of a triangle) ($\angle$ e van 'n driehoek) OR / OF $\hat{N}_2 = 90^\circ + x$ (ext $\angle$ of Δ) / (buite $\angle$ van Δ) $\therefore \hat{P} = 90^\circ - 2x$ ($\angle$ s of a triangle) ($\angle$ e van 'n driehoek)	✓ S ✓ R OR / OF ✓ S/R ✓ S/R	(2)
9.3	$\hat{O}\hat{M}P = 90^\circ$ (tan $\perp$ rad) / (raaklyn $\perp$ radius) $\hat{O}\hat{K}P = 90^\circ$ (proven) / (bewys) $\therefore KOMP$ is a c.q. (opp. $\angle$ s suppl.) $KOMP$ is 'n kv (teenoorst. $\angle$ e supplementêr)	✓ S & R ✓ S ✓ R	(3)
			[11]

QUESTION 10 / VRAAG 10

10.1	Downloaded from eduresourceza.com  On PQ, mark PG = MN and on PR, mark off PH = MO Join GH In $\triangle PGH$ and $\triangle MNO$ (1) PG = MN (construction) (2) $\hat{P} = \hat{M}$ (given) (3) PH = MO (construction) $\therefore \triangle PGH \equiv \triangle MNO$ (SAS) $\therefore \hat{G}_1 = \hat{N}$ (Congruency) But $\hat{Q} = \hat{N}$ (given) $\therefore \hat{G}_1 = \hat{Q}$ $\therefore GH \parallel QR$ (corresponding angles formed =) $\therefore \frac{PG}{PQ} = \frac{PH}{PR}$ (prop.int; $GH \parallel QR$) But PG = MN and PH = MO $\therefore \frac{MN}{PQ} = \frac{MO}{PR}$	✓ construction ✓ congruency proof ✓ S ✓ R ✓ S and/en R ✓ S	
	Afrikaans Op PQ, merk af PG = MN en op PR, merk af PH = MO Verbind GH In $\triangle PGH$ en $\triangle MNO$ (1) PG = MN (konstruksie) (2) $\hat{P} = \hat{M}$ (gegee) (3) PH = MO (konstruksie) $\therefore \triangle PGH \equiv \triangle MNO$ (SHS) $\therefore \hat{G}_1 = \hat{N}$ (Kongruensie) Maar, $\hat{Q} = \hat{N}$ (gegee) $\therefore \hat{G}_1 = \hat{Q}$ $\therefore GH \parallel QR$ (ooreenkomstige hoeke gevorm =) $\therefore \frac{PG}{PQ} = \frac{PH}{PR}$ (eweredigheid; $GH \parallel QR$) Maar, PG = MN en PH = MO $\therefore \frac{MN}{PQ} = \frac{MO}{PR}$	✓ konstruksie ✓ kongruensie bewys ✓ S ✓ R ✓ S and/en R ✓ S	(6)



10.2.1	$\hat{S}_1 = \hat{V}_1$ (alt $\angle$ s; $PS \parallel VT$) / (verw. $\angle e$; $PS \parallel VT$) $\hat{V}_1 = \hat{R}_1$ (ext. $\angle$ of $\triangle RTVW$) / (buite $\angle$ van $\triangle RTVW$) $\therefore \hat{S}_1 = \hat{R}_1$	$\checkmark$ S and/en R $\checkmark$ S $\checkmark$ R	(3)
10.2.2	In $\triangle PWS$ and/en $\triangle PSR$ (1) $\hat{P}_1$ is common / is gemeen (2) $\hat{S}_1 = \hat{R}_1$ (proven) / (bewys) (3) 3 rd angle of a triangle / 3 ^{de} hoek van driehoek $\triangle PWS \parallel \triangle PSR$ (AAA) / (HHH) / ($\angle \angle \angle$)	$\checkmark$ S $\checkmark$ R $\checkmark$ S and/en R	(3)
10.2.3	In $\triangle PQW$ and/en $\triangle PRQ$ (1) $\hat{Q}_1 = \hat{R}_2$ (tan – chord theorem) / (raaklyn – koord stelling) (2) $\hat{P}_2$ is common / is gemeen (3) 3 rd angle of a triangle / 3 ^{de} hoek van driehoek $\triangle PQW \parallel \triangle PRQ$ (AAA) $\therefore \frac{PQ}{PR} = \frac{PW}{PQ}$ (similarity) / (gelykvormig) $\therefore PQ^2 = PW \cdot PR$	$\checkmark$ S and/en R $\checkmark$ S $\checkmark$ S and/en R $\checkmark$ S $\checkmark$ S	(5)
10.2.4	From / Vanaf 10.2.1 $\frac{PW}{PS} = \frac{PS}{SR}$ ($\triangle PWS \parallel \triangle PSR$) $\therefore PS^2 = PW \cdot PR$ From / Vanaf 10.2.2 $PQ^2 = PW \cdot PR$ $\therefore PQ = PS$	$\checkmark$ S $\checkmark$ S $\checkmark$	(3)
			[20]

TOTAL / TOTAAL: 150