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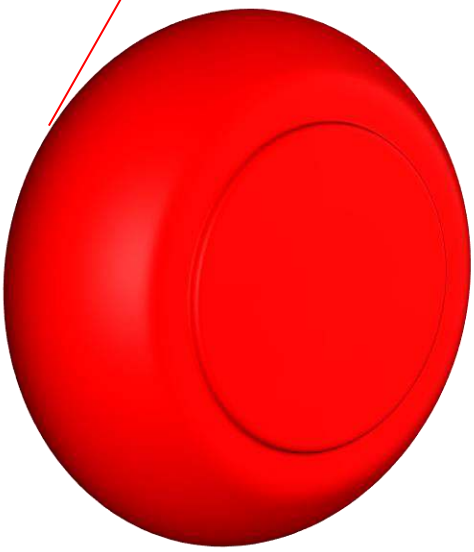
By Naeem Mohamed





Mathematics Grade 12 Paper 2
Section by Section
Past Paper Revision 2020

For Free Distribution



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MODULE 0: IMPORTANT INFORMATION

A. CONTENTS

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B. GENERAL ADVICE FOR 2020 MATRICULANTS

2020 – The year you write your final school examinations! Those who excel during this vital year will open up your learning and earning opportunities. Use your final year of study well. Attend all your classes in school and any extra tuition offered to you, both privately and by your school. Eat healthy, play sports and avoid any activity that undermines your studying and preparation. At the same time, remain calm as stressing too much can work against you.

Keep up to date with the latest matric news in newspapers and on the internet. Work through previous year's examination papers, exemplar papers as well as departmental papers and any sample papers prepared for you by your tutors and teachers. Ask your tutors / teachers to work through the questions, answers and examination technique with you.

Do everything you possibly can to convert twelve years of school into a National Senior Certificate with excellent marks, while making the most of probably the most enjoyable year of your schooling life.

Good luck!



_____
Naeem Mohamed

C. GUIDE STRUCTURE

This booklet is focused on preparing you for the trial and final examinations. Each module is based on a specific question in the paper and starts off with important aspects in the subject guideline. Theory and questions from national, IEB and exemplar papers are then worked through during the course.

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D. MATRIC HELP ON THE INTERNET

Visit the following websites regularly to download past papers, media publications, workbooks and other valuable mathematics resources:

- www.education.gov.za
- www.mathsexcellence.co.za
- www.thutong.org.za
- www.learn.co.za
- www.maths911.com
- www.youngeinsteins.co.za

E. MARK DISTRIBUTION FOR MATHEMATICS CAPS GRADE 12

Paper 1:

Description	% of Paper 1
Patterns & Sequences	± 17%
Annuities & Finance	± 10%
Functions & Graphs	± 23%
Algebra and Equations (and inequalities)	± 17%
Calculus	± 23%
Probability	± 10%
	100%

- **Bookwork** (Theory and proofs) of a maximum 6 marks could be included in the paper.
- Questions will not necessarily be compartmentalised in sections as this table indicates. Various topics can be **integrated** in the same question.

Paper 2:

Description	% of Paper 2
Analytical Geometry	± 27%
Trigonometry	± 27%
Statistics	± 13%
Euclidean Geometry and Measurement	± 33%
	100%

- **Bookwork** (Theorems and trigonometric proofs) of a maximum 12 marks could be included in the paper.
- Questions will not necessarily be compartmentalised in sections as this table indicates. Various topics can be **integrated** in the same question.

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INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \; ; \quad r \neq 1 \qquad S_\infty = \frac{a}{1-r} \; ; \; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$In\ \Delta ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A \qquad area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

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MODULE 1
DATA HANDLING: REGRESSION AND CORRELATION

Unit 1: Summary Statistics

MEASURES OF CENTRAL TENDENCY

- **Mean (Average)** $= \frac{\text{Total}}{\text{Number of Values}}$
- **Median** – is the middle value in the ordered list.
- **Mode** – is the value that occurs most.

MEASURES OF DISPERSION OR SPREAD

- **Range** = Highest Value – Lowest Value
- **Quartiles** – divide ordered data into quarters.
- **Deciles** – divide ordered data in 10 equal parts.
- **Percentiles**– divide ordered data into 100 equal parts.
- **Interquartile Range** = Upper Quartile (Q_3) – Lower Quartile (Q_1)
- **Five number summary** – uses only the minimum, Q_1, Median, Q_3 and the maximum value.

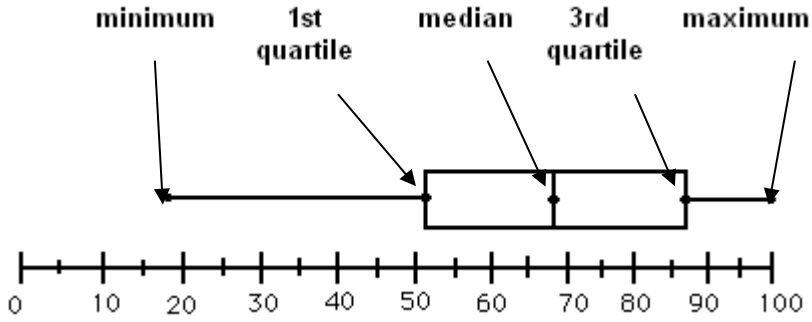
DIAGRAMS AND CONCEPTS

- **Frequency** – the number of occurrences of data / groups of data in a set.
- **Cumulative Frequency**- is a total count for all data values less than a specific value.
- **Histogram** – is a graphical display of tabulated frequencies, shown as bars. It shows what proportion of cases fall into each of several categories. The bars must be adjacent. The intervals must be of the same size.
- **Frequency polygon** – this is similar to a histogram as illustrated in the diagram above by the line joining the midpoints of each bar.
- **Bar Graph** - is a chart with rectangular bars with lengths proportional to the values that they represent. The bars can be horizontal or vertical. It is a visual display used to compare the amount or frequency of occurrence of different characteristics of data and it is used to compare groups of data. Bars normally have spaces between them.
- **Ogive curve** (pronounced as O-jive) – is a graph of the endpoint plotted against cumulative frequency. The curve is usually 'S' shaped.

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- **Box-and-Whisker diagram** – graphical version of the 5 number summary.



VARIANCE AND STANDARD DEVIATION

- The variance is a measure of the dispersion of a set of data points around their mean value.

$$\textbf{Variance} = \frac{\sum (x-\bar{x})^2}{n}$$

- The **Standard Deviation** is a measure of the dispersion of a set of data from its mean. The more spread apart the data, the higher the deviation. If data are close together, this results in a lower standard deviation. Standard deviation is calculated as the square root of variance.

$$\textbf{Standard Deviation} = \sqrt{\text{variance}}$$

- **USE OF A SCIENTIFIC CALCULATOR FOR DATA HANDLING**



Calculating the mean and standard deviation

  - Mode 2: STAT.
  - 1 – VAR.
  - Enter each number pressing '=' to get to the next line.
  - Press 'AC'.
  - Now press 'SHIFT' and '1'.
  - VAR.
  - Choose 'x' for the mean or 'xσn' for the standard deviation.

Turning the 'Frequency' Function on / off

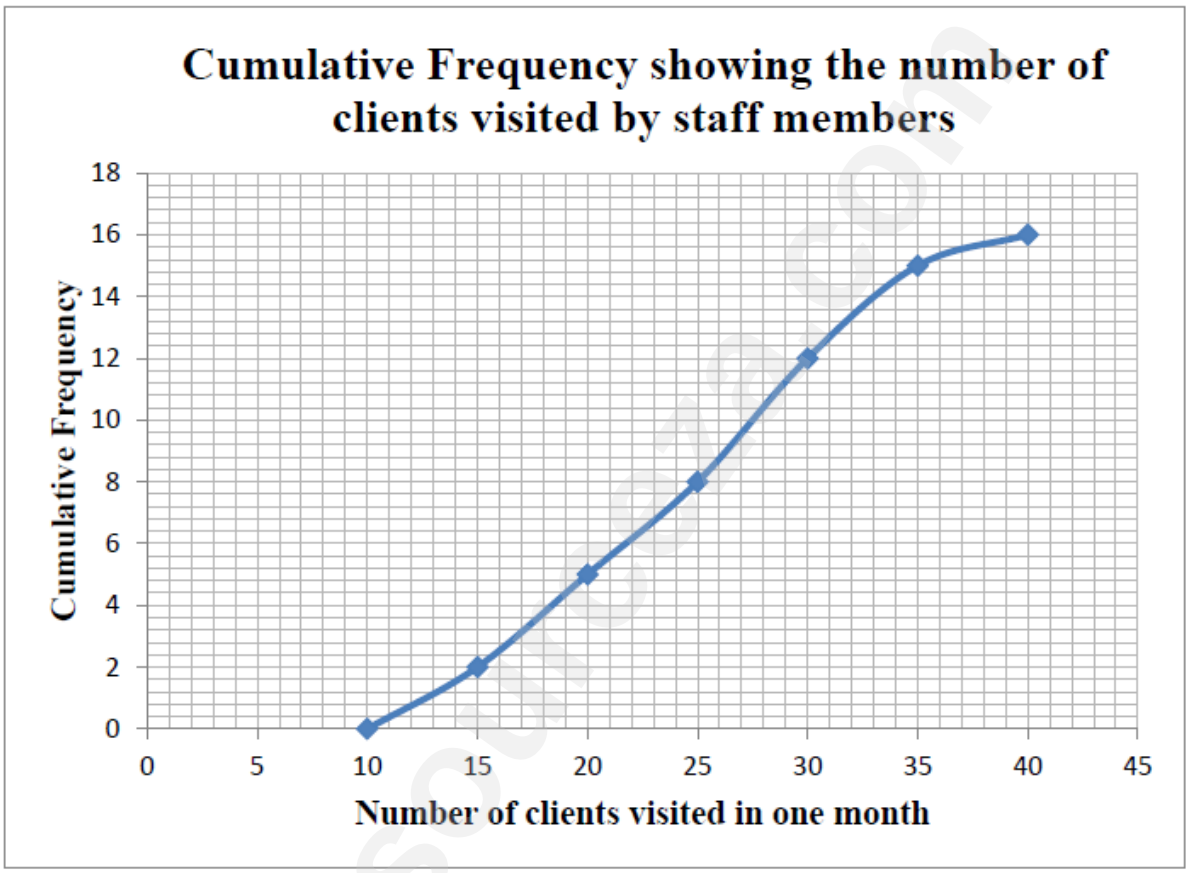
  - SHIFT 'SETUP'.
  - Press the arrow key 'down' to get to another menu.
  - 3: STAT
  - Choose 1: ON or 2: OFF

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QUESTION 1

1.1 Ros is the sales manager for a print and design company. His staff members must report about the number of clients they visit every month. Below is the ogive representing the collected data in one month.



1.1 How many staff members were there? (1)

1.2 Determine the semi-interquartile range. (3)

1.3 Draw a box whisker diagram for the data. (3)

1.4 Comment on the distribution of data using your result in QUESTION 1.3. (1)
[8]

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QUESTION 2

In an experiment, a group of 23 girls were presented with a page containing 30 coloured rectangles. They were asked to name the colours of the rectangles correctly as quickly as possible. The time, in seconds, taken by each of the girls is given in the table below.

12	13	13	14	14	16	17	18	18	18	19	20
21	21	22	22	23	24	25	27	29	30	36	

2.1 Calculate:

2.1.1 The mean of the data (2)

2.1.2 The interquartile range of the data (3)

2.2 The standard deviation of the times taken by the girls is 5,94. How many girls took longer than ONE standard deviation from the mean to name the colours? (2)

2.3 Draw a box and whisker diagram to represent the data on the number line provided in the ANSWER BOOK. (3)

2.4 The five-number summary of the times taken by a group of 23 boys in naming the colours of the rectangles correctly is (15 ; 21 ; 23,5 ; 26 ; 38).

2.4.1 Which of the two groups, girls or boys, had the lower median time to correctly name the colours of the rectangles? (1)

2.4.2 The first three learners who named the colours of all 30 rectangles correctly in the shortest time will receive a prize. How many boys will be among these three prizewinners? Motivate your answer. (2)

[13]

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QUESTION 1

The following table shows the test marks (in %) of Grade 11 learners in Frances Baard High School.

INTERVAL OF TEST MARKS	NUMBER OF LEARNERS
$0 \leq x < 20$	4
$20 \leq x < 40$	5
$40 \leq x < 60$	9
$60 \leq x < 80$	13
$80 \leq x < 100$	10
Totals	**41**

1.1 Write down the modal class. (1)

1.2 Calculate the estimated mean. (3)

1.3 Complete the cumulative frequency table provided in the ANSWER BOOK. (2)

1.4 Draw a cumulative frequency curve (ogive) to represent the data on the grid provided in the ANSWER BOOK. (3)

1.5 Use the cumulative frequency curve (ogive) to determine the interquartile range for the data. (3)

[12]

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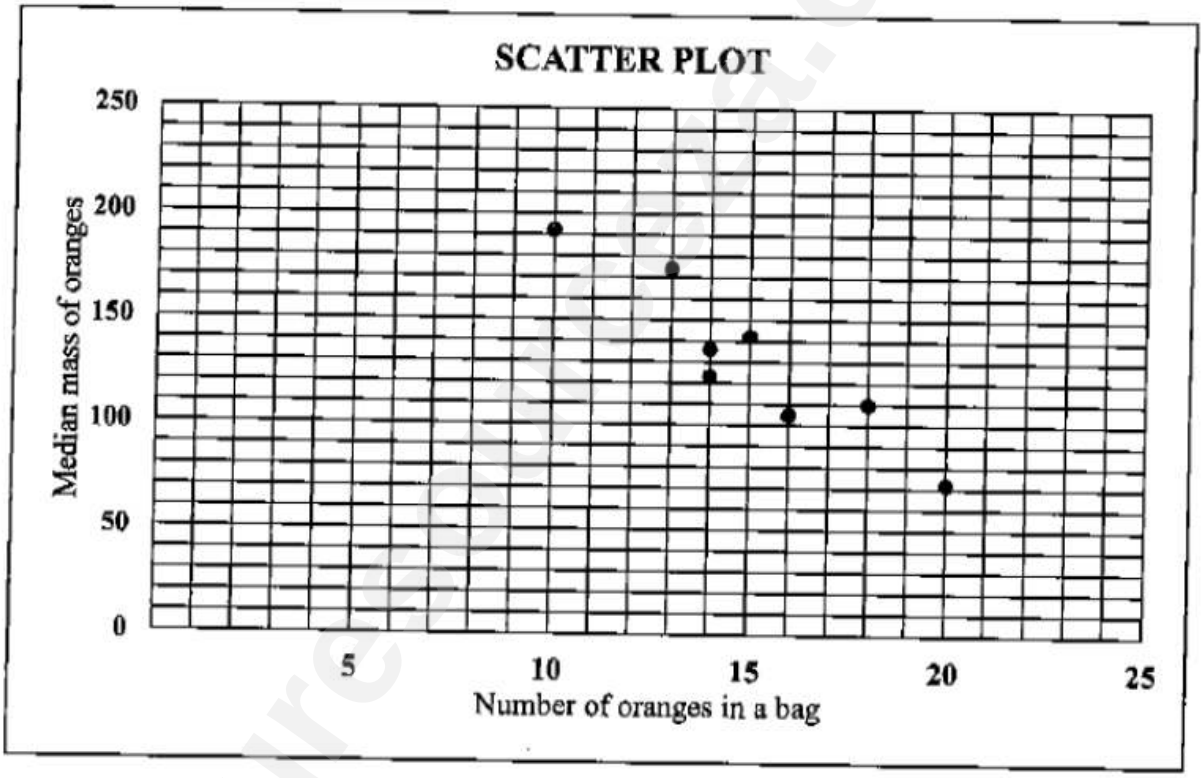
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QUESTION 1

A student is investigating the number of oranges in a bag in relation to the median mass of the oranges filled in the same bag. The findings are recorded in the table below.

Number of oranges in a bag	18	16	20	15	14	13	14	10
Median mass of oranges in the same bag (to the nearest gram)	110	105	72	142	123	174	136	192



1.1 Determine the equation of the least squares regression line for the data. (3)

1.2 Write down the correlation coefficient of the data. (1)

1.3 Draw the least squares regression line on the scatter plot given in your ANSWER BOOK. (2)

1.4 Comment on the strength of the relationship between the number of oranges in the bag and the median mass of the oranges. (1)

1.5 Determine the possible median mass of oranges in a bag, if there are 12 oranges in that bag. (2)
[9]

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QUESTION 1

1. The following information represents the amount of maize exported to other countries over 11 years in 1000 tons.

39	42	48	54	62	68	78	78	82	91	93

1.1 Calculate the mean amount of maize exported over the 11 years. (2)

1.2 Calculate the standard deviation of the data. (2)

1.3 Calculate the number of years that are within one standard deviation of the mean. (2)

1.4 Draw a box and whisker diagram to represent the data. (4)

1.5 Comment on the skewness of the data. (1)

1.6 There was an error in the data. The mean amount of maize exported over the 11 years should increase by 1,25 thousands of tons. What impact will this error have on the:

1.6.1 yearly data provided in the above table? (1)

1.6.2 on the interquartile range of the given data above? (1)

[13]

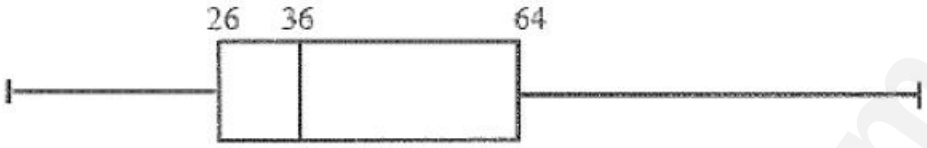
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QUESTION 1

Some of the test results of 21 learners are given below. There was only one result of 26 marks and only one result of 64 marks.



1.1 What information is omitted on the diagram above? (1)

1.2 The results were read to the learners in ascending order. If the 5th learner's result was 26, which learner obtained a result of 64? (2)

1.3 One of the learners was arguing that the distribution of the data wasn't symmetrical. Is the learner correct? Give a reason for the learner's remark. (3)

1.4 The class calculated the following, using the test results:

$\bar{x} = 45{,}5$ and $\delta\bar{x} = 19{,}2$.

Use the above information and determine the number of learners whose results fall outside ONE standard deviation of the mean. (3)

1.5 If the marks of each learner would increase by 5 marks, what effect would it have on the mean and standard deviation? (2)

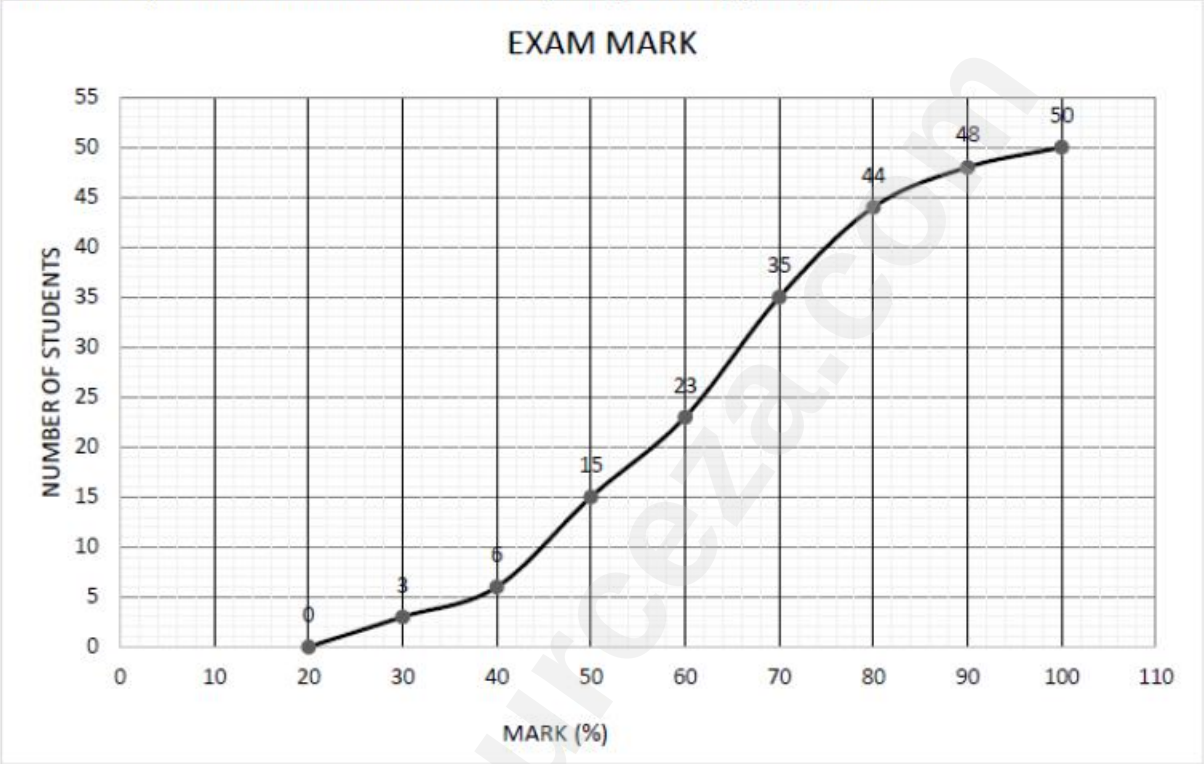
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QUESTION 1

1.1 A number of learners' marks for acertain exam were requested in a survey. Marks from the exam are represented in the cumulative frequency curve (ogive) below:



1.1.1 How many learners participated in this survey? (1)

1.1.2 Complete the frequency column in the table given in the ANSWER BOOK.

Interval	Frequency	Cumulative frequency
$20 \leq x < 30$		3
$30 \leq x < 40$		6
$40 \leq x < 50$		15
$50 \leq x < 60$		23
$60 \leq x < 70$		35
$70 \leq x < 80$		44
$80 \leq x < 90$		48
$90 \leq x < 100$		50

(2)

1.1.3 Use the graph to determine the median mark. (2)

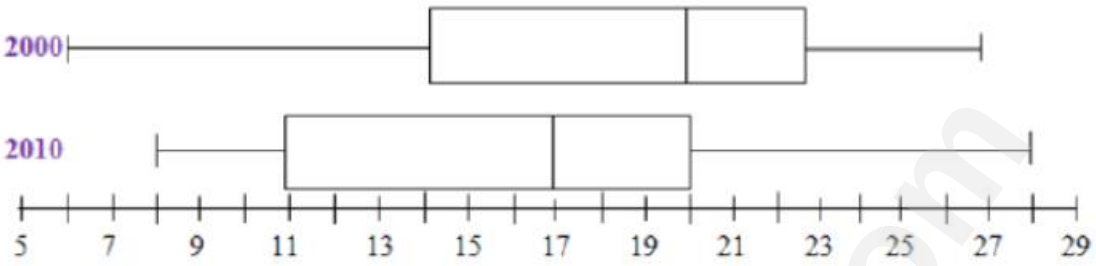
1.1.4 Write down the modal class. (1)

1.1.5 If a learner needs 70% to qualify for a reward, how many learners qualified? (2)

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1.2 The results from a test for students for the year 2000 and the year 2010 are illustrated in the box and whisker plot. The total mark for the test was 30.



1.2.1 Determine the interquartile range for 2010. (2)

1.2.2 In which year did students perform better in the test? Motivate your answer. (2)

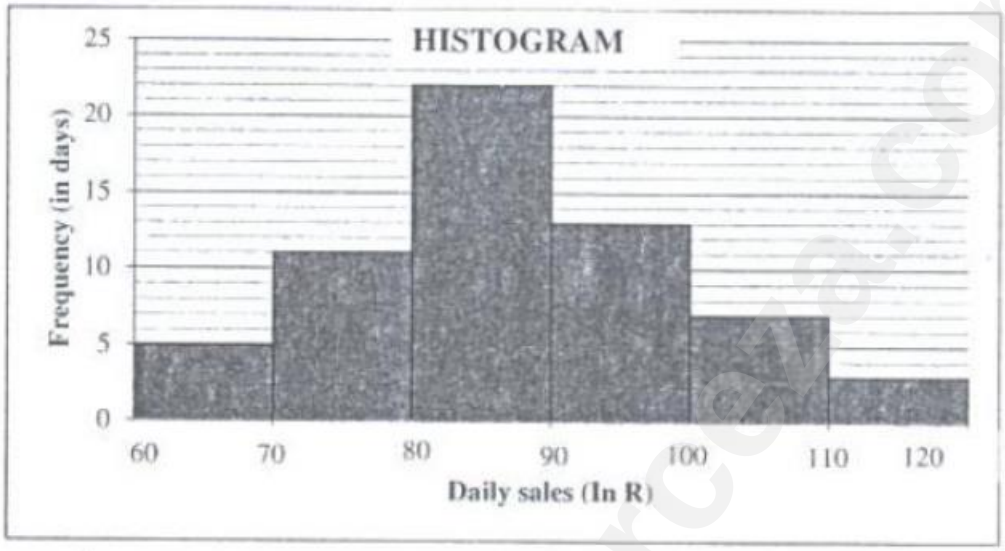
[12]

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QUESTION 1

A street vendor has kept record of his daily sales for November and December 2017. The daily sales(in Rand) is shown in the histogram below.



1.1 A table summarising the daily sales for November and December 2017 is drawn in the answer book. Complete the cumulative frequency column in the table. (3)

1.2 Draw an ogive (cumulative frequency graph) of the daily sales for November and December 2017 on the grid provided IN THE ANSWER BOOK. (4)

1.3 On how many days did the daily sales exceed 100? (2)

1.4 Use the ogive IN THE ANSWER BOOK to estimate the median value of the daily sales. (2)

[11]

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QUESTION 1

The time (in seconds) between the consecutive landings of aeroplanes at an airport on day 1 was recorded. The data is given in the Cumulative Frequency table below.

Time in seconds	Number of aeroplanes (Frequency)	Cumulative Frequency
$60 < t \leq 90$	2	2
$90 < t \leq 120$	16	18
$120 < t \leq 150$	28	46
$150 < t \leq 180$	17	63
$180 < t \leq 210$	k	p
$210 < t \leq 240$	7	80

1.1 Show that $k = 10$. (1)

1.2 Write down the value of p. (1)

1.3 Calculate the estimated mean time between the landings of two consecutive aeroplanes. (3)

1.4 It is given that $(q\ ;\ 186{,}89)$ is the interval of the landing time between aeroplanes within ONE standard deviation from the estimated mean.

1.4.1 Write down the estimated standard deviation of the time between the consecutive landings of the aeroplanes. (2)

1.4.2 Calculate the value of q. (1)

1.5 On day 2, the same number of aeroplanes that landed on day 1, land at the airport. The elapsed time between all the consecutive landings of all the aeroplanes is m seconds shorter than the time that is given in the table above.

If an ogive is to be drawn of the data of day 2, the following will be true:

- The ogive will be grounded at $(57\ ;\ 0)$
- The maximum value of the ogive will be at $(237\ ;\ 80)$

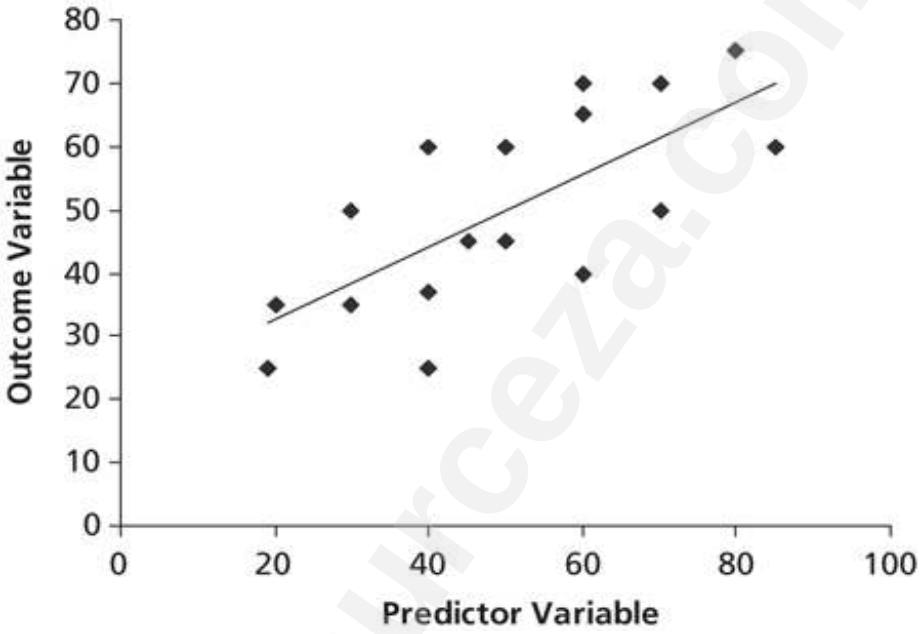
Determine the average time between the landing of two aeroplanes on DAY 2, if it is given that the frequency distribution of the two days are the same. (2)
[10]

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Unit 2: Bivariate Data and Regression Modelling

- When you think of **regression** think prediction. A regression uses the historical relationship between an independent and a dependent variable to predict the future values of the dependent variable. Businesses use regression to predict such things as stock prices.
- **Scatter Plot –** scattered points arising from two different data sets, forming linear, quadratic, exponential and other trends.
-



- A **line of best fit (trend / regression / least squares line)** is a line on a scatter plot which can be drawn near the points to more clearly show the trend between two sets of data. Data points that are very far from the line of best fit are called **outliers**.
- **Interpolation** – when a value within the domain or range of data values is used to make the prediction.
- **Extrapolation** – when a value outside the domain or range of the data set is used to make the predication.

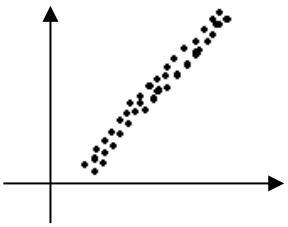
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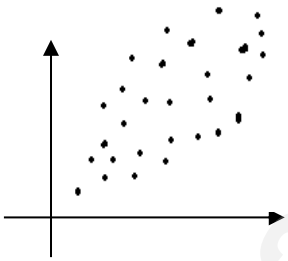
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- **Using the calculator to fit lines:**
  - Press **Mode 2** (STAT), Choose option **2: a + bx** (LINEAR REGRESSION)
  - Enter **bivariate data** (x and y values) into the table – when complete press **AC**
  - Now press **SHIFT: 1 (STAT)** and choose **REG** – to calculate a, b and r.

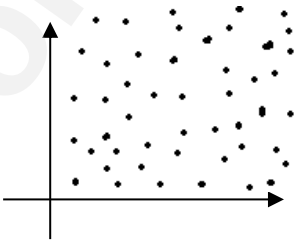
  'a' represents the intercept and 'b' represents the slope.
- **Correlation** is a measure of how strongly 2 variables are related. Correlation only occurs when the relationship is linear. Types of correlation:



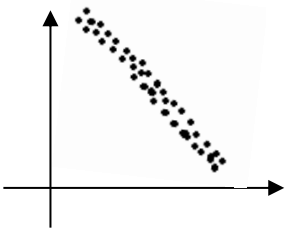
Positive and strong



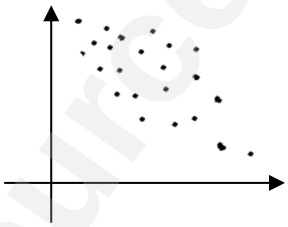
Positive and weak



No correlation



Negative and strong



Negative and weak

- The **correlation coefficient (r)** lies in the interval: $-1 \leq r \leq 1$
  'r' has the same sign as the slope (b) of the regression line.
  If r = 0, there is **no correlation**.
  If r is close to 0, there is **weak correlation**.
  If r is close to 1 or -1, there is **strong correlation**.

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QUESTION 2

The data below shows the marks obtained by ten Grade 12 learners from two different Mathematics classes sitting in the first row of each class.

Class A	16	36	20	38	40	30	35	22	40	24
Class B	45	70	44	56	60	48	75	60	63	38

2.1 Make use of the grid provided on the DIAGRAM SHEET 1 to draw a scatter plot for the data. (2)

2.2 Calculate the equation of the least squares regression line for this data. (2)

2.3 Draw the least squares regression line for the data on the scatter plot diagram drawn in QUESTION 2.1 (DIAGRAM SHEET 1). (1)

2.4 Calculate the correlation coefficient for the above data. (2)

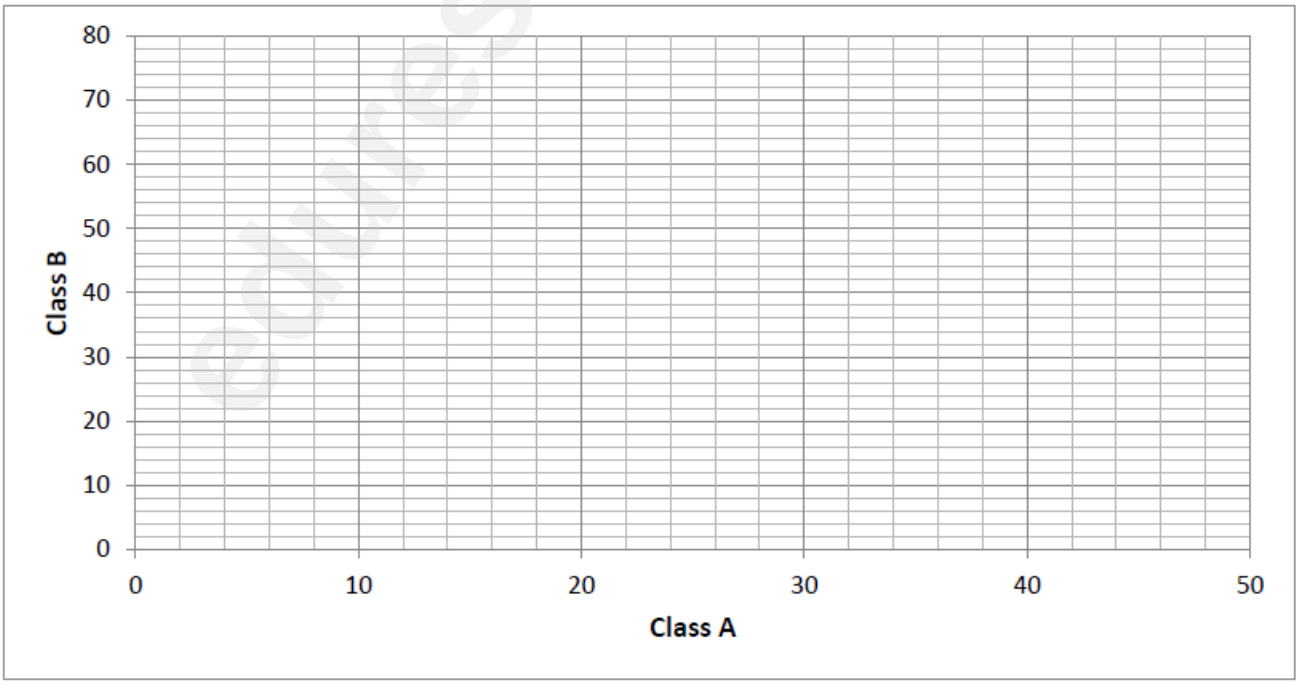
2.5 Calculate the mean and the standard deviation for Class B. (4)

2.6 How many scores fall within one standard deviation of the mean? (2)

[13]

DIAGRAM SHEET 1

QUESTIONS 2.1 AND 2.3



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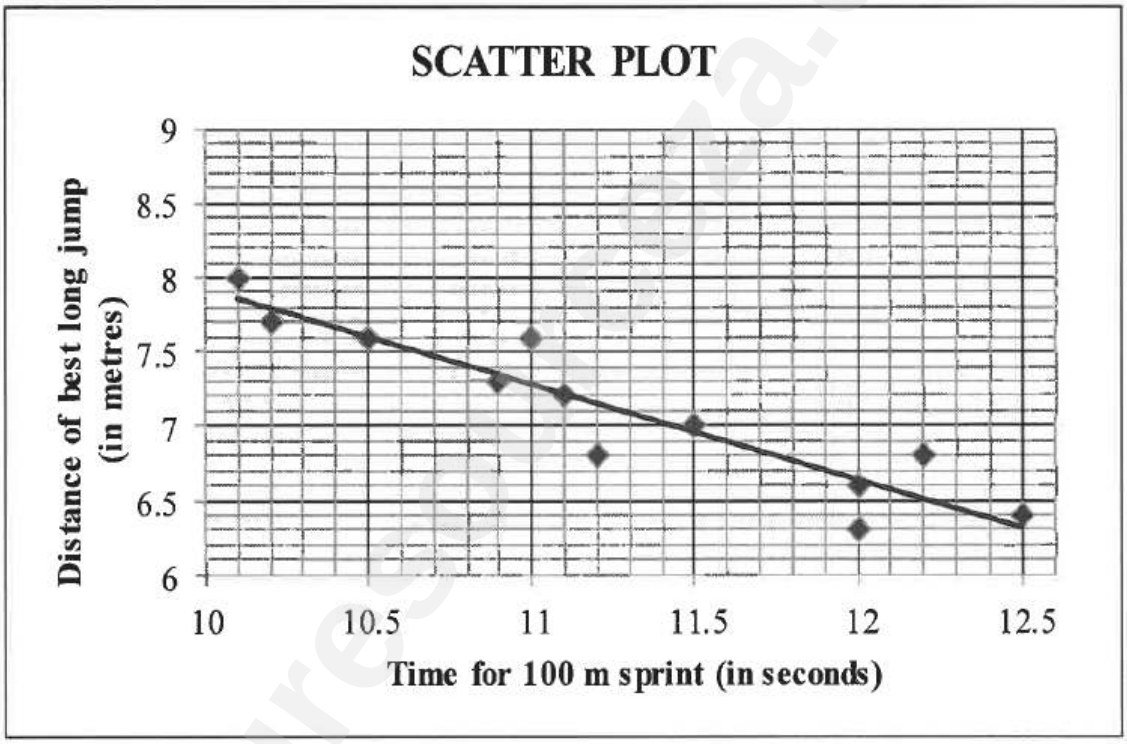
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QUESTION 1

The table below shows the time (in seconds, rounded to ONE decimal place) taken by 12 athletes to run the 100 metre sprint and the distance (in metres, rounded to ONE decimal place) of their best long jump.

Time for 100 m sprint (in seconds)	10,1	10,2	10,5	10,9	11	11,1	11,2	11,5	12	12	12,2	12,5
Distance of best long jump (in metres)	8	7,7	7,6	7,3	7,6	7,2	6,8	7	6,6	6,3	6,8	6,4

The scatter plot representing the data above is given below.



The equation of the least squares regression line is $\hat{y} = a + bx$.

1.1 Determine the values of a and b. (3)

1.2 An athlete runs the 100 metre sprint in 11,7 seconds. Use $\hat{y} = a + bx$ to predict the distance of the best long jump of this athlete. (2)

1.3 Another athlete completes the 100 metre sprint in 12,3 seconds and the distance of his best long jump is 7,6 metres. If this is included in the data, will the gradient of the least squares regression line increase or decrease? Motivate your answer without any further calculations. (2)
[7]

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QUESTION 2

Research is done to determine if the number of hours reading over a certain period of time has an effect on the results of a candidate's mark in a general knowledge test (out of 120).

Number of hours	15	20	22	25	32	40	44	50	55	58
Mark out of 120	40	30	55	80	70	75	100	105	98	79

2.1 Determine the equation of the least squares regression line. (3)

2.2 Estimate the mark that a person that reads 36 hours in that same period will obtain in the test. (2)

2.3 What is the correlation between the number of hours reading and the mark a person scores for the test? Motivate your answer. (3)
[8]

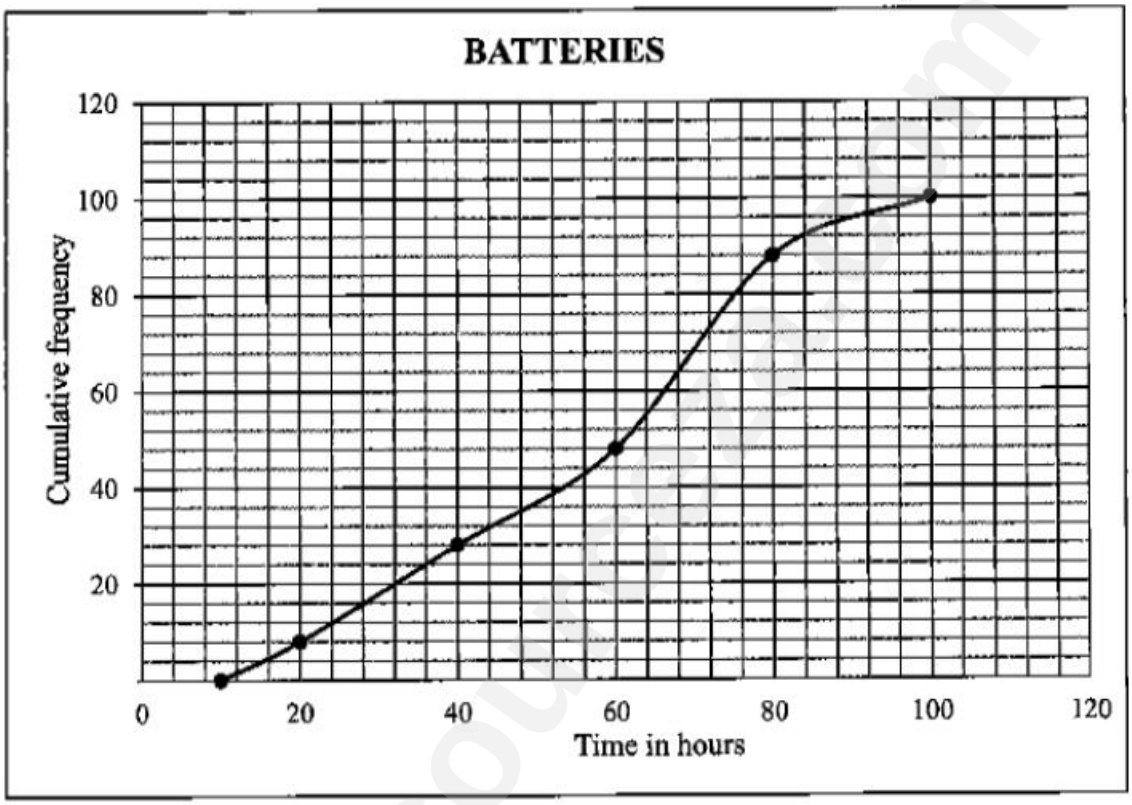
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QUESTION 2

2.1 Batteries are used in everyday life. The Grade 12 Physical Sciences learners investigated the life span of batteries under constant test conditions.

The ogive (cumulative frequency graph) below shows the lifespan (in hours) of the batteries.



2.1.1 How many batteries were tested for this investigation? (1)

2.1.2 Use the graph to estimate the median time for the life span (in hours) of the batteries. (2)

2.1.3 The minimum lifespan of batteries is 10 hours and the maximum lifespan is 100 hours. Use the cumulative frequency graph to draw a box and whisker diagram in your ANSWER BOOK. (3)

2.1.4 Comment on the skewness of the distribution of the lifespan of the batteries. (1)

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2.2 The table below represents values in a data set written in increasing order. None of the values in the data set are repeated.

5	a	19	b	c	d	35

Determine the values of a, b, c, and d if:

- The median is 20.
- The semi interquartile range is 8.
- The upper quartile is twice the lower quartile.
- The mean is 22.

(4)
[11]

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QUESTION 2

The following information (in %) represent contributions made by the Agricultural and Mining industries in order to evaluate the GDP(GROSS DOMESTIC PRODUCT) of a certain country.

YEAR	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Agriculture (x)	4,2	3,4	3,1	2,7	2,9	3,0	2,9	3,0	2,6	2,5	2,6
Mining (y)	19,2	19,4	19,2	18,5	17,5	17,0	16,8	15,2	14,2	12,8	12,4

2.1 Determine the equation of the least squares regression line. (3)

2.2 Estimate the percentage that the mining industry will contribute if the agriculture industry dropped to 1,2 %. (2)

2.3 Comment on the strength of the correlation between the contributions made by these two industries. Motivate your answer. (2)

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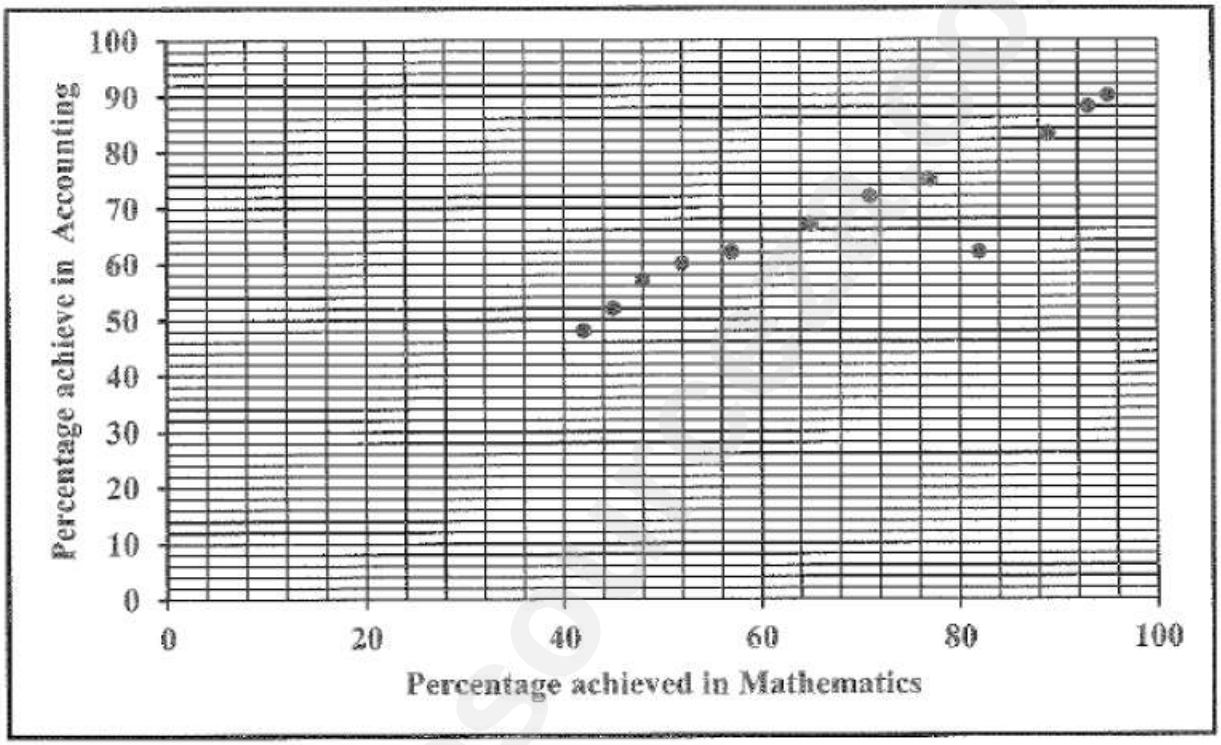
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QUESTION 2

At a certain High School only 12 candidates take Mathematics and Accounting. The marks as a percentage scored by these candidates in the Trial Examinations for these two subjects, are shown in the table and scatter plot below.

Mathematics	52	82	93	95	71	65	77	42	89	48	45	57
Accounting	60	62	88	90	72	67	75	48	83	57	52	62

SCATTER PLOT



2.1 Use the scatter plot to comment on the relationship between the learner's performance in Mathematics and Accounting. Provide a reason for your answer. (2)

2.2 Calculate the value of the correlation coefficient. (1)

2.3 Calculate the equation for the least squares regression line for the data. (3)

2.4 If a candidate from this group scored 60% in the Mathematics examination, but was absent for the Accounting examination, predict the percentage he would have scored in the Accounting examination. (2)

2.5 Use the scatter plot and identify any outlier(s) in the data. (1)

[9]

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QUESTION 2

A group of 12 learners have been asked to measure their resting heart rate (beats per minute) and the time (in minutes) that they exercise in a week. The data below was gathered.

Minutes of exercise per week	Resting heart rate (BPM)
30	82
40	77
60	75
90	70
140	68
180	67
270	60
350	58
360	52
420	50
440	48
500	45

2.1 Represent the data as a scatter plot on the grid provided (3)

2.2 Calculate the correlation coefficient for the given data. (2)

2.3 Use the data to calculate the equation of the least squares regression line. (3)

2.4 If a learner has a resting heart rate of 65 beats per minute, how many hours would you expect him to exercise per week? (3)

[11]

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QUESTION 2

2.1 A learner conducted an experiment to investigate the relationship between the age and resting heart rate (in beats per minute). He sought the assistance of the local clinic. The information for 12 people is shown in the table below:

Age	59	32	42	50	22	39	21	20	27	40	29	47
Resting heart rate (beats per minute)	88	74	74	93	85	71	78	82	70	75	95	75

2.1.1 Determine the equation of the least squares regression line. (3)

2.1.2 Predict the resting heart rate of a person aged 57 years. (2)

2.1.3 Write down the correlation coefficient for the data. (1)

2.1.4 Use the correlation coefficient to comment on the prediction done in Question 2.1.2. (1)

2.2 A group of students wrote a statistics test. The following information was obtained from the marks obtained in this test:

$$\sum_{n=1}^{22} x_n = 1320$$

2.2.1 How many students wrote the test? (1)

2.2.2 Calculate the mean mark for the test. (2)

[10]

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QUESTION 2

The marks, in percentage, obtained in an Accounting and Mathematics test by a group of ten Grade 12 learners is shown in the table below.

Accounting Test	76	65	88	68	70	79	51	66	59	74
Mathematics Test	80	69	93	19	76	85	57	79	62	78

2.1 Identify an outlier of the above data. (1)

2.2 Determine the equation for the least squares regression line after ignoring the outlier in the above data. (3)

2.3 Another learner in the same class obtained 83% in the Accounting test, but due to illness could not write the Mathematics test. Use the equation established in 2.2 to predict the learner's mark for the Mathematics test. (2)

2.4 The teacher decided to award the learner who was absent the predicted mark obtained in 2.3 for the Mathematics test. Other learners in the class felt that it was unfair.

Motivate to these learners why the predicted mark is a good indication of what the learner may have scored in the Mathematics test. (2)

2.5 After the Mathematics subject advisor has moderated the answer books of the Mathematics tests, she decides to lower every test mark by p%. Explain, **without any calculations,** what influence the lowering in the marks of the Mathematics test has on the slope of the least squares regression line of the above data when the outlier is ignored. (2)

[10]

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MODULE 2: ANALYTICAL GEOMETRY

1. EXAMINATION GUIDELINE / EXAM TECHNIQUE

- **Mark allocation**: ± 40 marks. **Suggested Time allocation**: 48 minutes.
- Know how to calculate the **area** of figures such as triangles and rectangles.
- The following Euclidean Geometry theorems are useful when dealing with questions on inclination:
  - The angles on a **straight line** add up to 180°.
  - The angles of a **triangle** add up to 180°.
  - The **exterior angle** of a triangle equals the sum of the interior opposite angles.
  - **Vertically opposite angles** are equal.
- The following definitions are useful:
  - The **median** of a triangle is the drawn from the vertex to the midpoint of the opposite side.
  - The **altitude** of a triangle is the perpendicular height drawn from any vertex.
  - A **perpendicular bisector** bisects a line and is also perpendicular to that line.

2. THEORY AND QUESTIONS

PROPERTIES OF QUADRILATERALS

Use these properties to prove the following quadrilaterals analytically:

- A **trapezium** is a quadrilateral with exactly one pair of opposite sides parallel.
- A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel or a quadrilateral with both pairs of opposite sides equal.
- A **rectangle** is a parallelogram with 1 interior < equal to 90° (∴ all interior angles are equal to 90°).
- A **rhombus** is a parallelogram with one pair of adjacent sides equal in length or a parallelogram with diagonals bisecting each other (∴ all sides are equal).
- A **square** is a rectangle with one pair of adjacent sides equal in length or a rhombus with one interior angle equal to 90° (∴ all interior angles are equal to 90°).
- A **kite** is a quadrilateral with 2 pairs of adjacent sides equal in length.

REVISION OF FORMULAE

Distance formula: $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint formula: $x_m = \frac{x_1 + x_2}{2}$; $y_m = \frac{y_1 + y_2}{2}$

Gradient Formula $m = \frac{y_2 - y_1}{x_2 - x_1}$

Inclination: $\tan\theta = m$

Parallel Lines: The gradients are equal

Perpendicular Lines: $m_1 . m_2 = -1$
(A quick way of doing this is to invert the gradient and change the sign)

A, B & C are collinear if $m_{AB} = m_{BC} = m_{AC}$

To find the equation of a straight line

Given 2 pts: $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

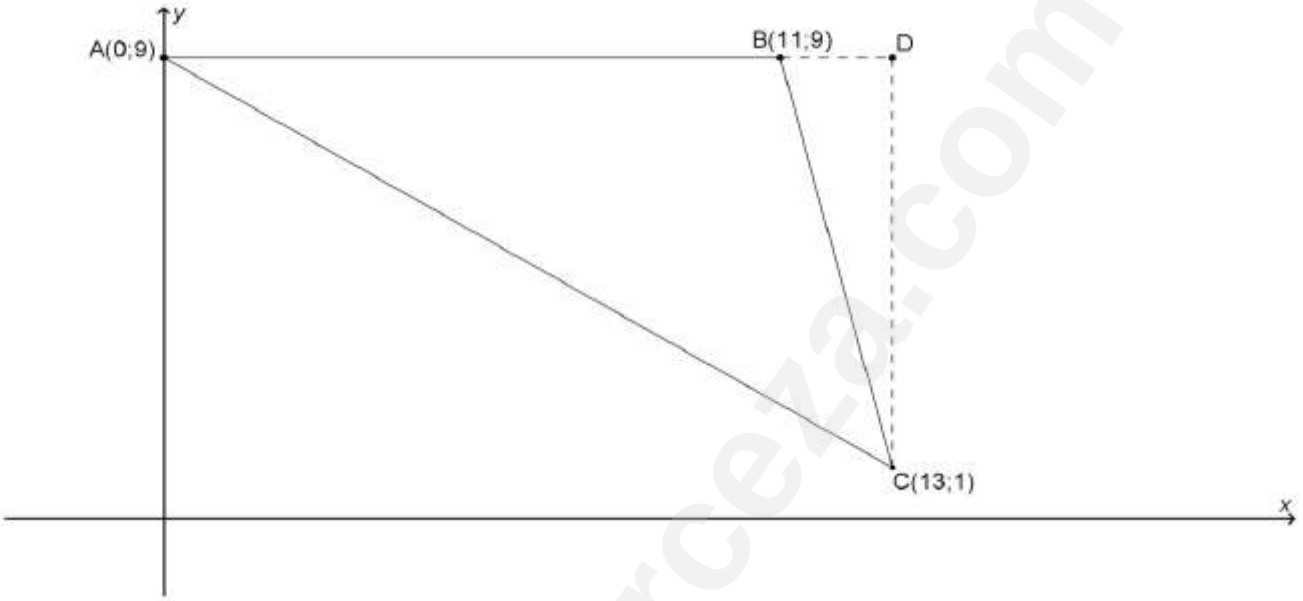
Given gradient and 1 pt: $y - y_1 = m(x - x_1)$

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QUESTION 3

In the figure, A(0 ; 9), B(11 ; 9) and C(13 ; 1) are the vertices of ΔABC with altitude CD.



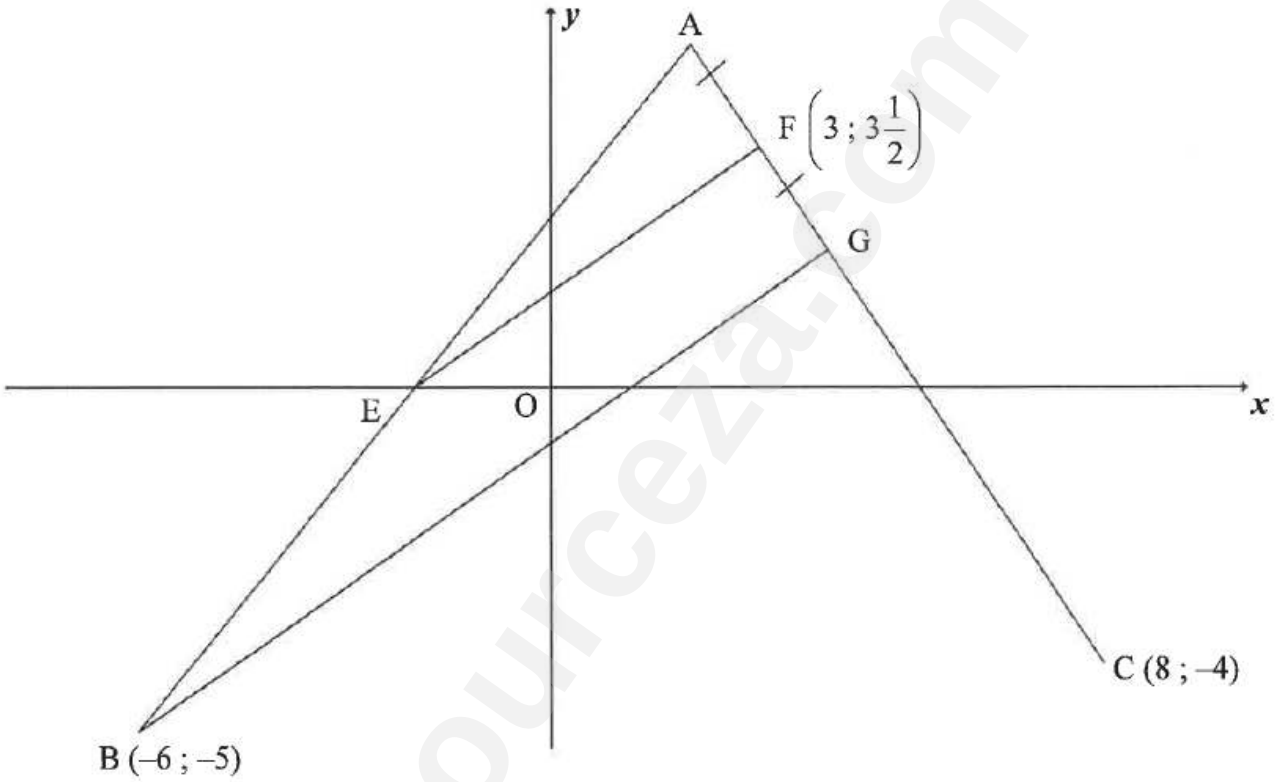
3.1	Write down the equation of the line AB.	(1)
3.2	Write down the length of AB.	(1)
3.3	Determine the co-ordinates of D.	(1)
3.4	Calculate the area of ΔABC.	(3)
3.5	Determine the co-ordinates of M, the midpoint of AC.	(2)
3.6	Hence, determine the equation of the perpendicular bisector of AC.	(4)
3.7	Does the perpendicular bisector in QUESTION 3.6 pass through point B or not? Justify your answer using relevant calculations.	(2)
3.8	Determine ∠ABC correct to the nearest degree.	(3)
3.9	Determine the equation of a line parallel to AC and passing through D.	(2)
		[19]

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QUESTION 3

In the diagram, A, B(–6 ; –5) and C(8 ; –4) are points in the Cartesian plane. $F\left(3;3\frac{1}{2}\right)$ and G are points on line AC such that AF = FG. E is the x-intercept of AB.



3.1 Calculate:

3.1.1 The equation of AC in the form $y = mx + c$ (4)

3.1.2 The coordinates of G if the equation of BG is $7x - 10y = 8$ (3)

3.2 Show by calculation that the coordinates of A is (2 ; 5). (2)

3.3 Prove that EF || BG. (4)

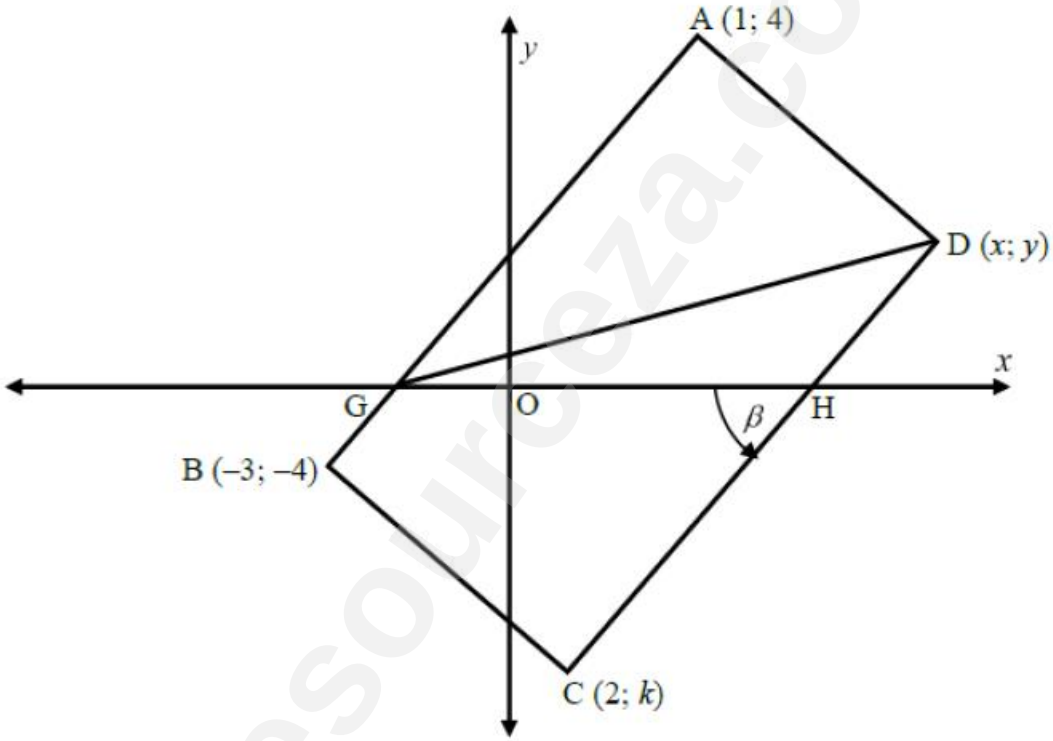
3.4 ABCD is a parallelogram with D in the first quadrant. Calculate the coordinates of D. (4)
[17]

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QUESTION 3

In the diagram below, A (1 ; 4), B (–3 ; – 4) , C (2 ; k) and D (x ; y) are the vertices of a rectangle. AB and DC cuts the $x-$axis at G and H respectively. GD is drawn. $\hat{GHC} = \beta$.



3.1 Calculate the gradient of BG. (2)

3.2 Determine the equation of AB in the form $y = mx + c$. (2)

3.3 Calculate the:

3.3.1 Value of k (y-coordinate of C). (4)

3.3.2 Coordinates of D. (3)

3.3.3 Size of β. (3)

3.3.4 Area of ΔDHG. (7)

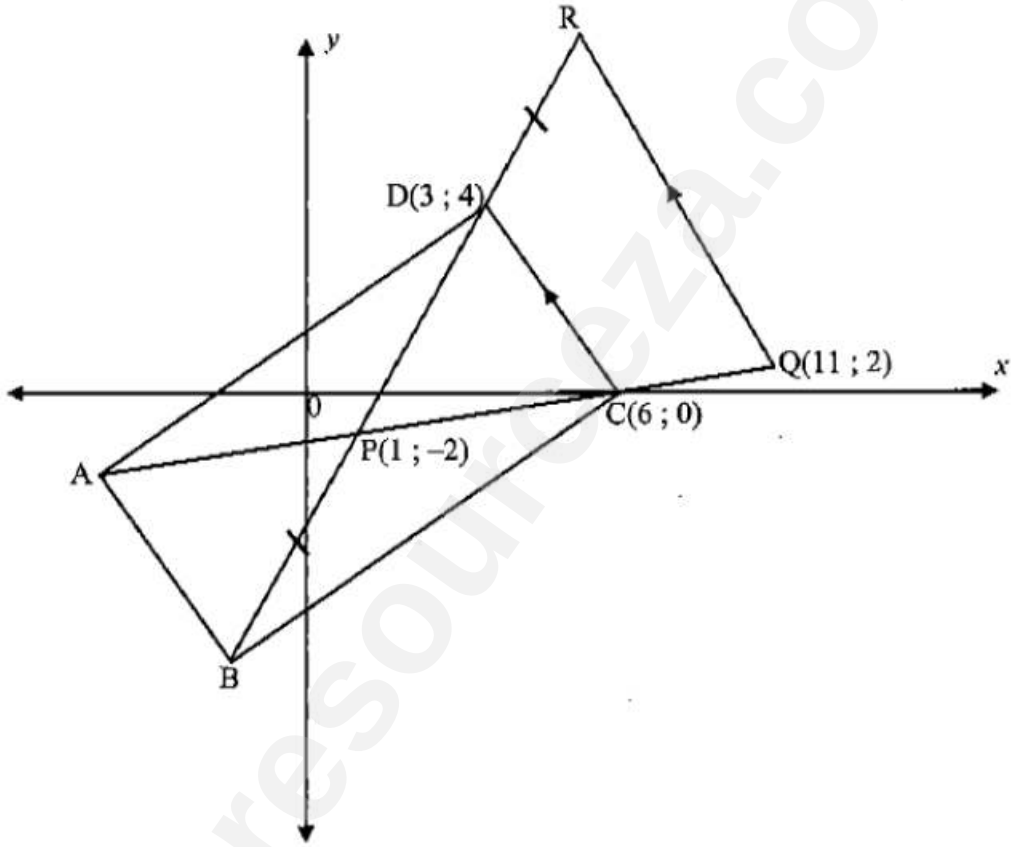
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QUESTION 3

3.1 In the diagram below, A, B, C (6 ; 0) and D (3 ; 4) are the vertices of rectangle ABCD. Diagonals AC and BD bisect each other at P(1 ; −2). AC is produced to Q(11 ; 2) and BD is produced to R such that BP = DR and CD ∥ QR.



3.1.1 Calculate the coordinates of B. (3)

3.1.2 Determine the gradient of CD. (2)

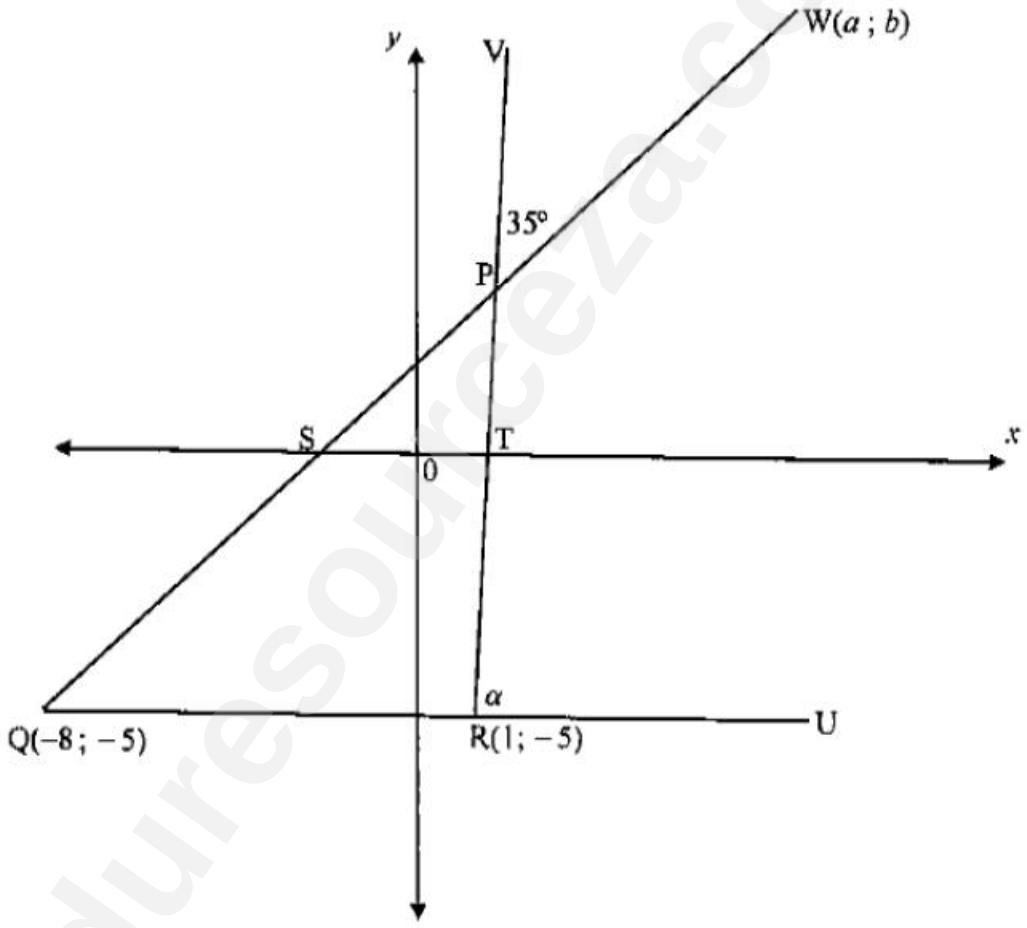
3.1.3 Show that the equation of QR is $y = -\frac{4}{3}x + \frac{50}{3}$. (2)

3.1.4 If K(4 ; y) is a point in the 4th quadrant such that PK = RQ, calculate the value of y. (6)

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3.2 In the diagram below, P, Q(−8 ; −5) and R(1 ; −5) are the vertices of ΔPQR. RP is produced to V and QP is produced to W(a ; b) such that $V\hat{P}W = 35°$. The equation of QW is $y = x + \frac{2}{3}$. QR is produced to U and $U\hat{R}V = \alpha$. QW and RV intersect the x-axis at S and T respectively.



3.2.1 Calculate the size of α. (5)

3.2.2 It is further given that QU ⊥ WU and R is the midpoint of QU. Calculate the area of ΔQWU. (6)

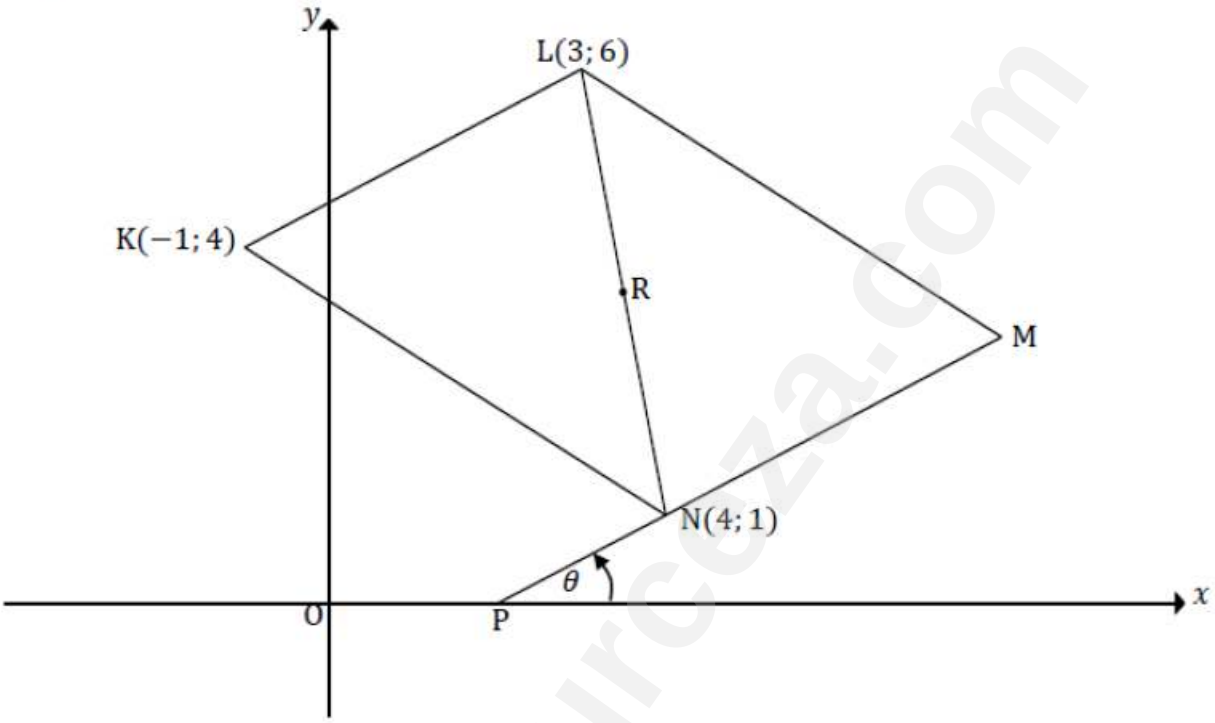
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QUESTION 3

In the diagram, K (–1 ; 4) ; L(3 ; 6); M and N(4 ; 1) are vertices of a parallelogram. R is the midpoint of LN. P is the x-intercept of the line MN produced.



3.1 Calculate the:

3.1.1 gradient of KL. (2)

3.1.2 coordinates of R. (3)

3.1.3 coordinates of M. (4)

3.2 Determine the equation of NM in the form $y = mx + c$. (3)

3.3 Calculate the:

3.3.1 coordinates of P. (2)

3.3.2 size of θ, the inclination of PM. (2)

3.3.3 size of $\hat{KPN}$. (4)

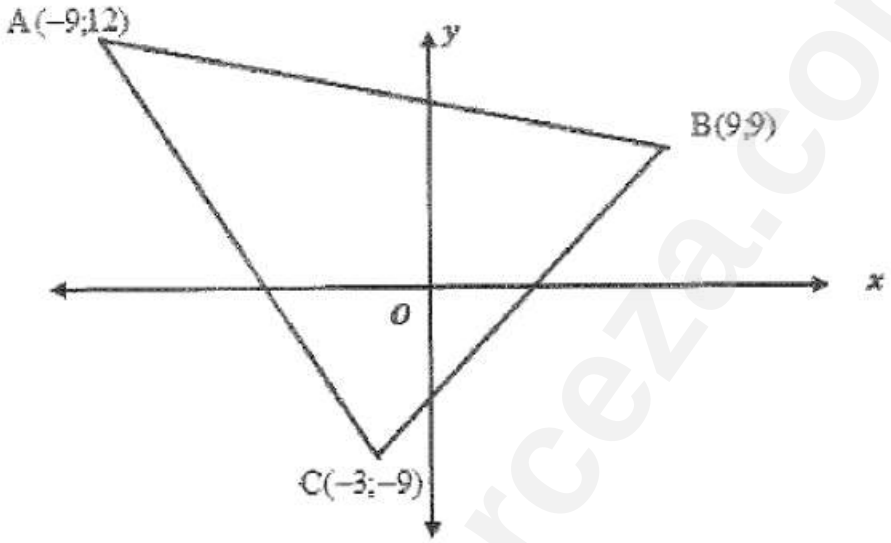
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QUESTION 3

In the diagram A(−9;12), B(9;9) and C(−3;−9) are the vertices of Δ ABC. K $(m;n)$ is a point in the second quadrant.



3.1 Calculate the gradient of AB. (2)

3.2 Calculate the size of $\hat{B}$, rounded off to two decimal digits. (5)

3.3 Determine the coordinates of M, the midpoint of BC. (2)

3.4 Determine the equation of the median AM. (3)

3.5 Determine the coordinates of K, if A, K and M are collinear and BK = $5\sqrt{5}$ units. (8)

[20]

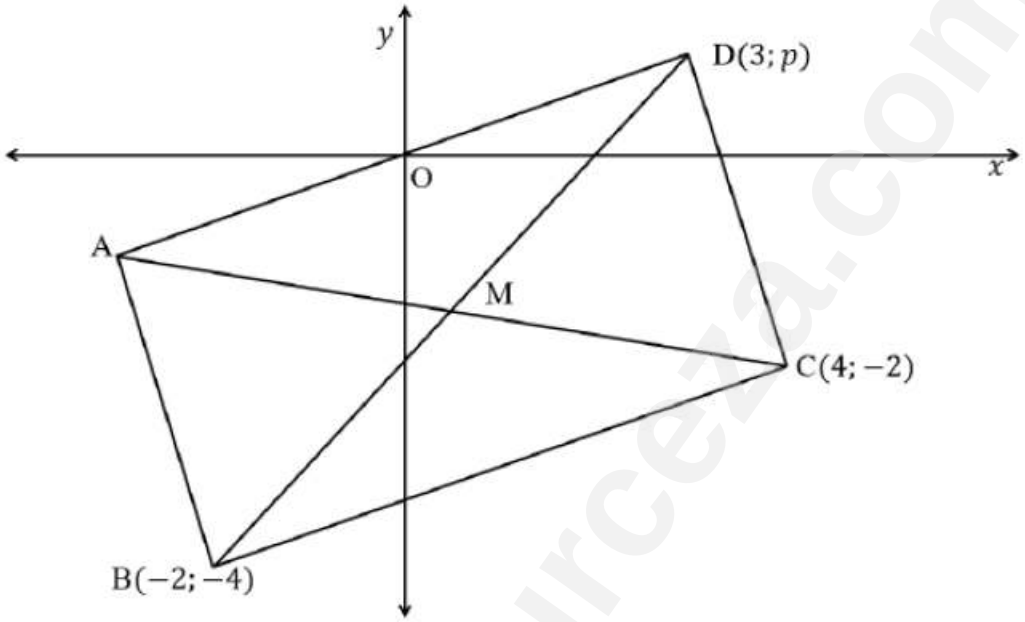
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QUESTION 3

In the diagram A, B$(-2;-4)$, C$(4;-2)$ and D$(3;\ p)$ are the vertices of a rectangle. The diagonals AC and BD intersect at M.



3.1 Given that the length of AC is $\sqrt{50}$ units, show that $p = 1$. (4)

3.2 Determine the coordinates of M. (3)

3.3 Calculate the gradient of DC (2)

3.4 Determine the equation of line AB in the form $y = mx + c$. (2)

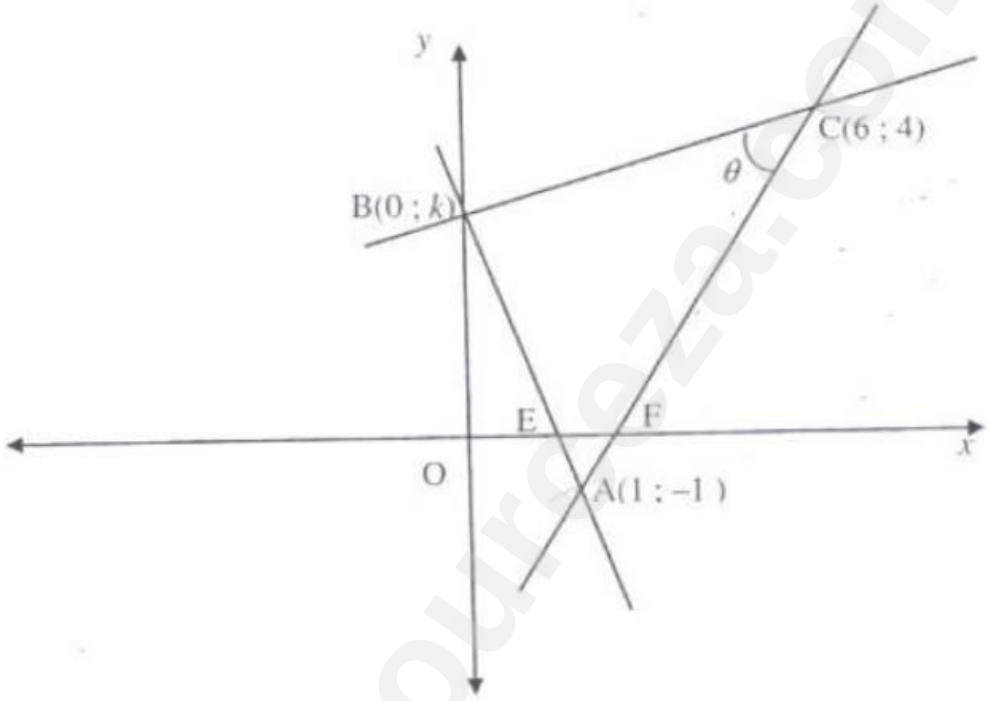
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QUESTION 3

In the diagram, A(1 ; −1), B(0 ; k) and C(6 ; 4) are the vertices of ΔABC. The equations of the sides AB and AC are $y+3x-2=0$ and $y=x-2$ respectively. AB cuts the x-axis at E and AC cuts the x-axis at F.



3.1 Write down the value of k. (2)

3.2 Calculate the length of AC and leave your answer in simplest surd form. (2)

3.3 Prove that $A\hat{B}C = 90°$. (3)

3.4 Calculate the size of θ. (5)

3.5 Determine the equation of the circle passing through A, B and C in the form $(x-a)^2+(y-b)^2=r^2$. (4)

3.6 If D is a point in the first quadrant, calculate the coordinates of D such that ABCD in that order, forms a parallelogram. (4)

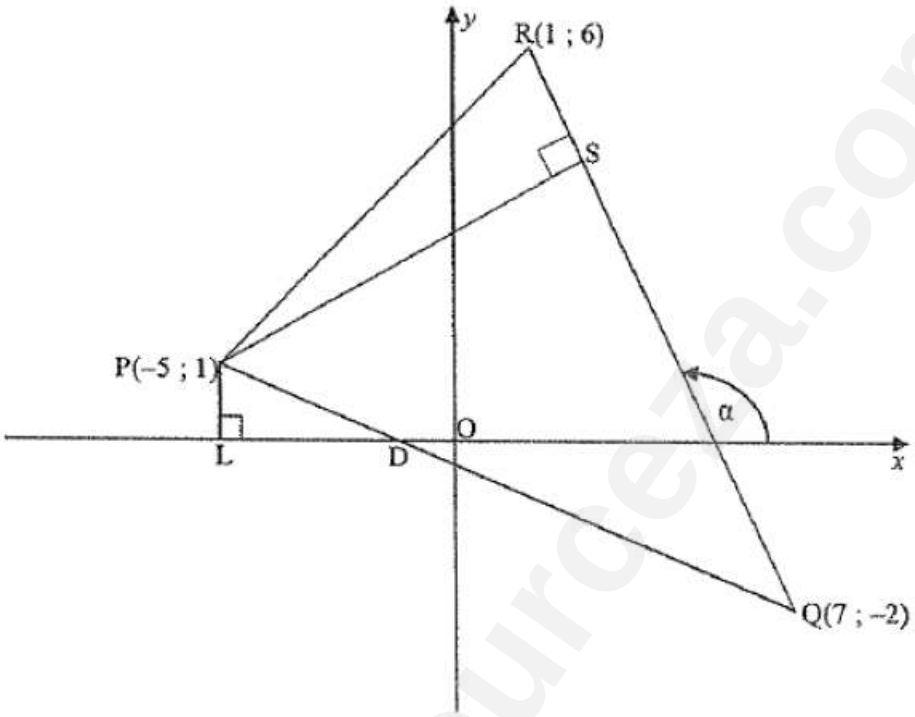
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QUESTION 3

In the diagram below, P (–5 ; 1), Q(7 ; –2) and R(1 ; 6) are the vertices of ΔPQR.
PQ intersects the x-axis at D. The angle of elevation of QR is α.
PS $\perp$ RQ and L lies on the x-axis such that PL $\perp$ $x-$axis.



3.1 Write down the equation of the line PL. (1)

3.2 Calculate the gradient of QR. (2)

3.3 Determine the equation of the line PS. (4)

3.4 Calculate the size of the angle of inclination of PQ. (3)

3.5 Calculate the size of $\hat{PQS}$. (4)

3.6 It is given that the areas of $\Delta PRS = 4x^2$ and $\Delta PQS = 16x^2$.
Calculate the length of SQ, WITHOUT calculating the coordinates of S. (5)
[19]

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CIRCLES

Equation of circle with center origin (0; 0): $x^2 + y^2 = r^2$

- To find the equation of a circle, find r^2 by substituting 1 point on the circle.

Circle with centre at any point: $ax^2 + bx + cy^2 + dy + e = 0$

- Complete the square on both sides to find centre / radius: $(x-a)^2 + (y-b)^2 = r^2$

TANGENTS

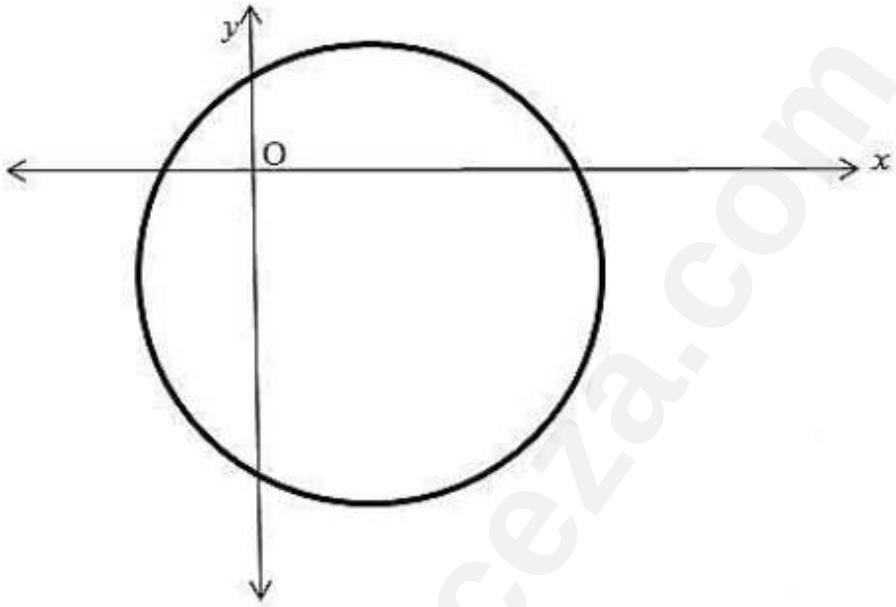
- **Finding the equation of a tangent to the circle**
  - Join the centre of the circle to the point of tangency to form the radius.
  - Find the **gradient of the radius**.
  - Find the **gradient of the tangent** by inverting and changing the sign of the gradient of the radius (tangent is always perpendicular to radius).
  - Substitute the gradient and the point of tangency (x; y) into **$y - y_1 = m(x - x_1)$.**

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QUESTION 4

The length of the radius of the circle with equation: $x^2 - 2x + y^2 + 4y = a$ is 5 units.



4.1 Show by means of calculations that $a = 20$ units. (4)

4.2 Write down the coordinates of the centre M of the circle. (1)

4.3 A$(x\,;y)$, with $y > 0$, is one of the points of intersection of the circle and the straight line $x = 4$. Determine the values of x and y. (4)

4.4 Determine the equation of the tangent to the circle at the point A. (4)

4.5 Determine whether the point T$(-1\,;-2)$ lies inside or outside the circle. (3)

4.6 If the circle is translated 3 units to the left and 1 unit up, determine the equation of the new circle. (3)

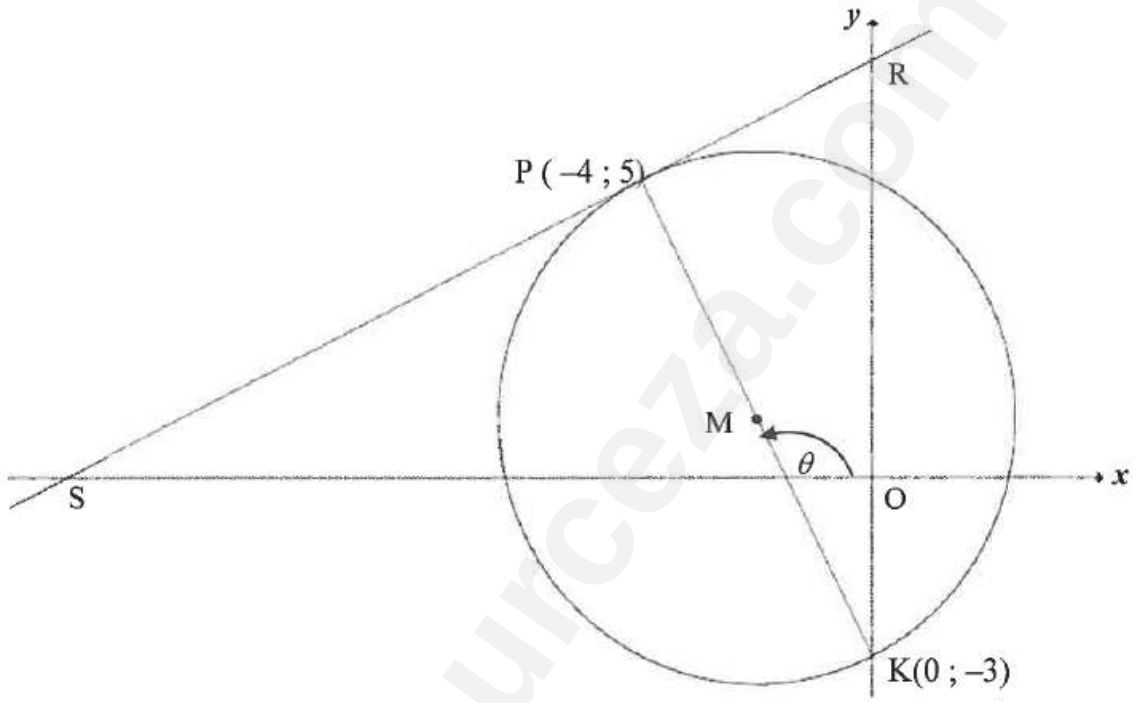
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QUESTION 4

In the diagram, P(–4 ; 5) and K(0 ; –3) are the end points of the diameter of a circle with centre M. S and R are respectively the x- and y-intercept of the tangent to the circle at P. θ is the inclination of PK with the positive x-axis.



4.1 Determine:

4.1.1 The gradient of SR (4)

4.1.2 The equation of SR in the form $y = mx + c$ (2)

4.1.3 The equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$ (4)

4.1.4 The size of $P\hat{K}R$ (3)

4.1.5 The equation of the tangent to the circle at K in the form $y = mx + c$ (2)

4.2 Determine the values of t such that the line $y = \frac{1}{2}x + t$ cuts the circle at two different points. (3)

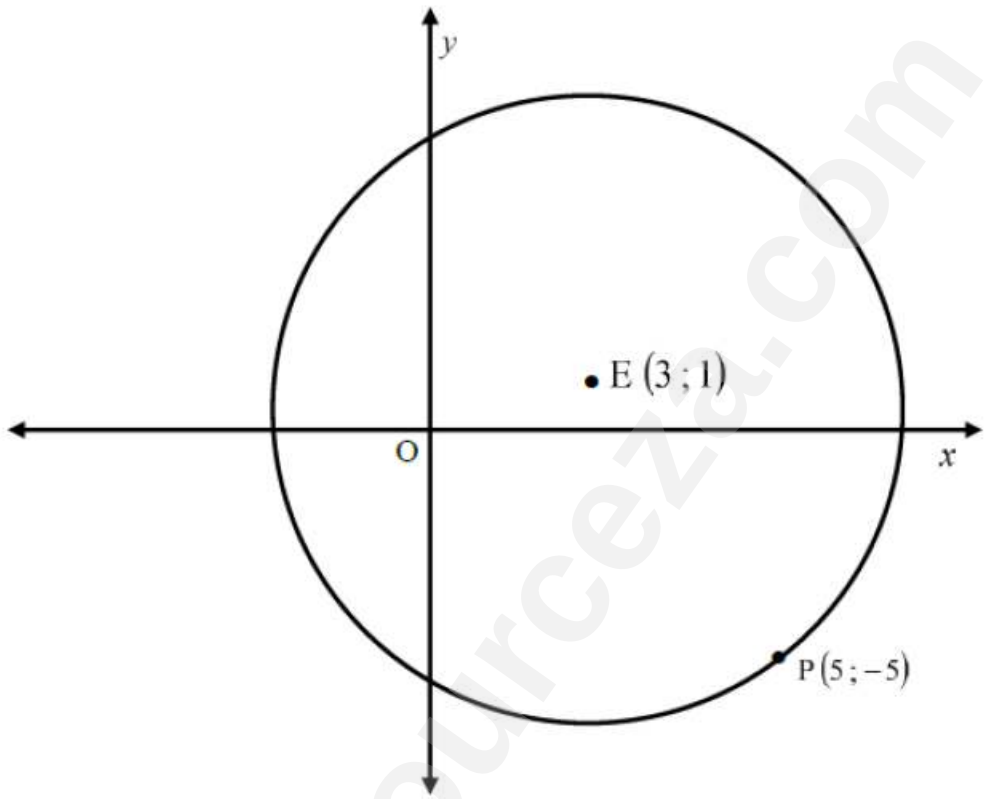
4.3 Calculate the area of ΔSMK. (5)
[23]

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QUESTION 4

In the diagram below, the circle centred at $E(3;1)$ passes through point $P(5;-5)$.



4.1 Determine the equation of:

4.1.1 The circle in the form $x^2 + y^2 + Ax + By + C = 0$. (4)

4.1.2 The tangent to the circle at $P(5;-5)$ in the form $y = mx + c$. (5)

4.2 A smaller circle is drawn inside the circle. Line EP is a diameter of the small circle. Determine the:

4.2.1 Coordinates of the centre of the smaller circle. (3)

4.2.2 Length of the radius. (3)

4.3 Hence, or otherwise, determine whether point $C(9;3)$ lies inside or outside the circle centre at E. (3)

[18]

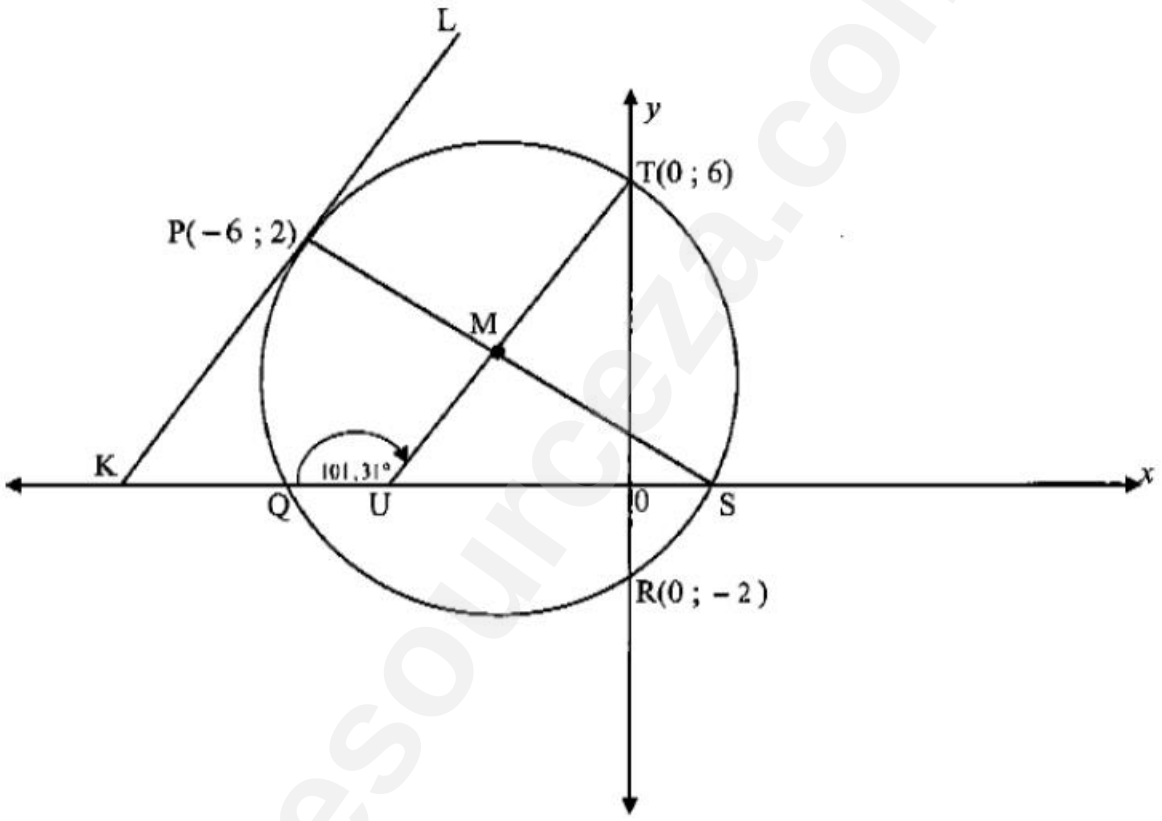
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QUESTION 4

In the diagram below, a circle with centre M, cuts the x-axis at Q and S and the y-axis at T(0 ; 6) and R(0 ; −2). The equation of diameter SMP is $y=\frac{-1}{5}x+\frac{4}{5}$.

KPL is a tangent to the circle at P(−6 ; 2). TM produced cuts the x-axis at U. $\hat{QUT}=101{,}31°$.



4.1 Determine the equation of TU. (3)

4.2 Calculate the coordinates of M. (3)

4.3 If the coordinates of M are (−1 ; 1), determine the equation of the circle in the form $(x-a)^2+(y-b)^2=r^2$. (3)

4.4 Prove that KL is parallel to TU. (3)

4.5 Is the point $V\left(-\frac{1}{2};7\right)$ inside the circle? Support your answer with calculations. (3)

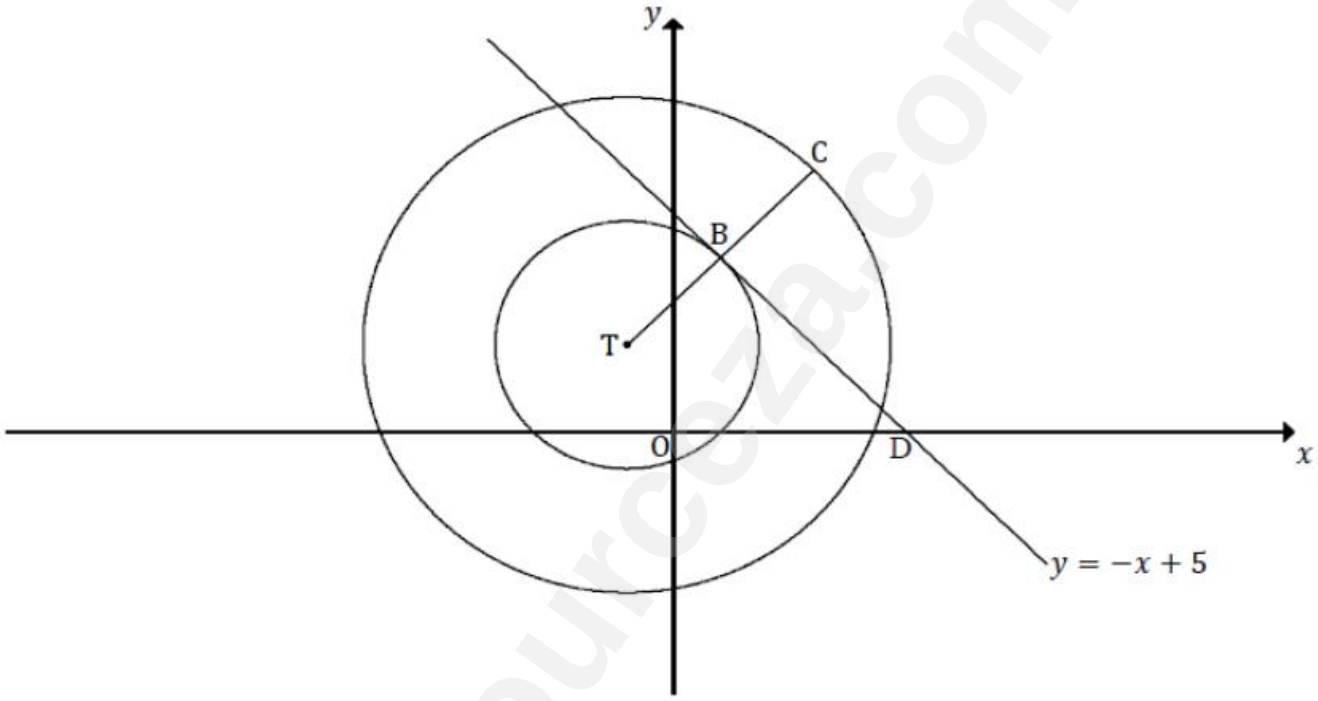
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QUESTION 4

In the diagram, T is the centre of two concentric circles. The larger circle has equation $x^2 + y^2 - 4y + 2x - 27 = 0$. The smaller circle touches the straight line $y = -x + 5$ at point B. BD is a tangent to smaller circle T. D is the x-intercept of the straight line. C is a point on the larger circle such that TBC is a straight line.



4.1 Calculate the coordinates of T. (4)

4.2 Show that equation of TB is given by $y = x + 3$. (3)

4.3 Calculate the coordinates of B (3)

4.4 Determine the equation of the smaller circle in the form $(x - a)^2 + (y - b)^2 = r^2$. (3)

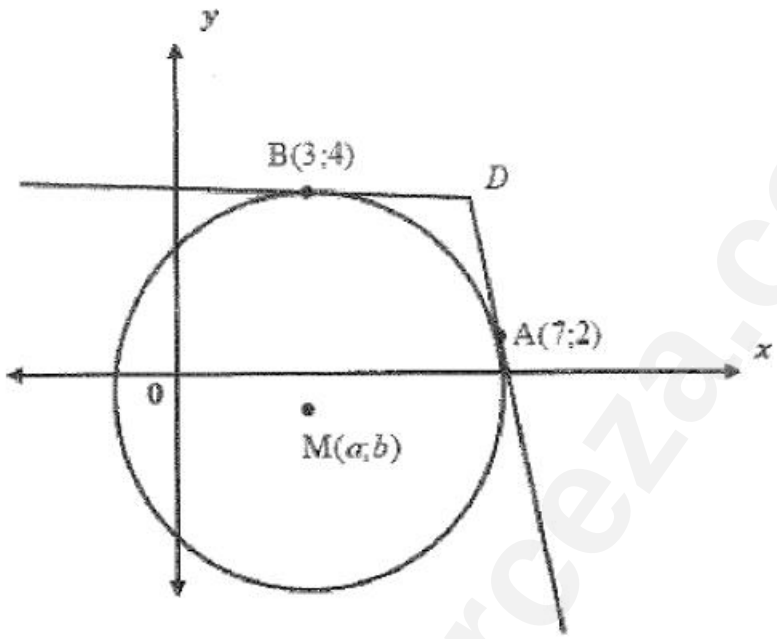
4.5 Calculate the area of quadrilateral OTBD. (7)
[20]

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QUESTION 4

In the diagram a circle $x^2 - 6x + y^2 + 2y = 15$ passes through the points A(7;2) and B(3;4). AD and BD are tangents to the circle, with M the centre of the circle



4.1 Determine the:

4.1.1 Coordinates of the centre M, and the radius of the circle. (5)

4.1.2 Equation of tangent AD. (4)

4.1.3 Coordinates of D, if BD and AD meet at D. (4)

4.2 Identify the type of quadrilateral ADBM is, and provide a valid reason. (2)

4.3 Determine the area of ADBM. (4)

[19]

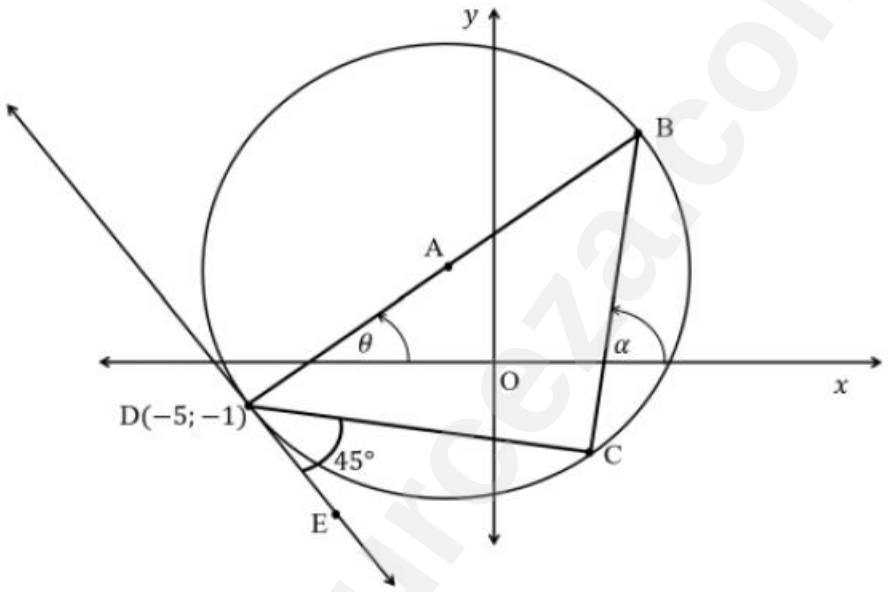
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QUESTION 4

In the diagram is the circle with equation $(x+1)^2+(y-2)^2=25$.
DB is the diameter of the circle and A the centre of the circle. DE is a tangent to the circle at D$(-5;-1)$. The angle $E\widehat{D}C=45°$. The inclination angles of AD and BC is θ and α respectively.
B and C are points on the circumference of the circle.



4.1 Determine:

4.1.1 The coordinates of A, the centre of the circle. (2)

4.1.2 The coordinates of B (3)

4.1.3 The gradient of AD. (2)

4.1.4 The value of θ, the inclination angle of AD. (2)

4.1.5 The equation of the tangent DE. (3)

4.2 Calculate the gradient of BC. (4)

4.3 Another circle with equation $x^2+y^2-6x+2y=8$ is given.

Show that:

4.3.1 The coordinates of the centre of the circle is M$(3;-1)$. (4)

4.3.2 The two circles will intersect each other. Show all calculations. (4)
[24]

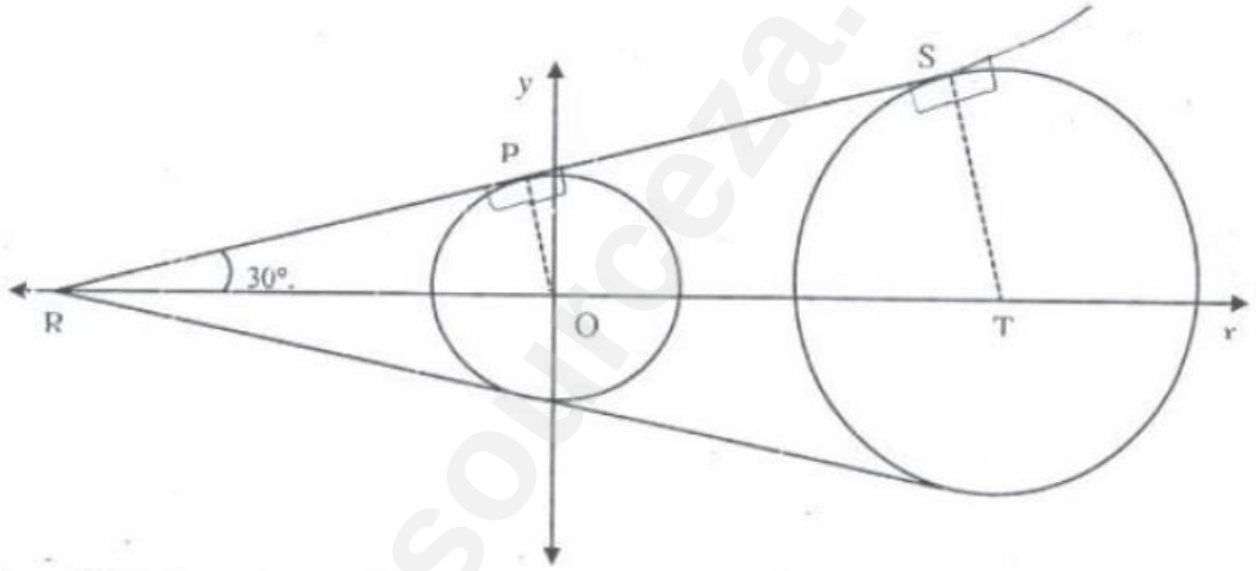
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QUESTION 4

The diagram below shows a representation of the chain of a bicycle attached to two circular cogs as represented in the Cartesian plane. The equations of the circles are given by $x^2 + y^2 = 1$ and $(x-6)^2 + y^2 = 9$.

RPS is a common tangent to the smaller and larger circles at P and S respectively, with R a point on the negative x-axis. T is the centre of the larger circle. The angle of inclination between the tangent RPS and the horizontal axis is 30°.



4.1 Write down the coordinates of T. (1)

4.2 Write down the length of the radius of the larger circle. (1)

4.3 If R$(-3\ ;0)$, determine the equation of tangent PRS in the form $y = mx + c$. Leave answer in surd form. (4)

4.4 Determine the equation of the radius ST in the form $y = mx + c$. Leave answer in surd form. (3)

4.5 Determine the coordinates of S. (5)

4.6 Determine the distance between the two circular cogs. (4)

[18]

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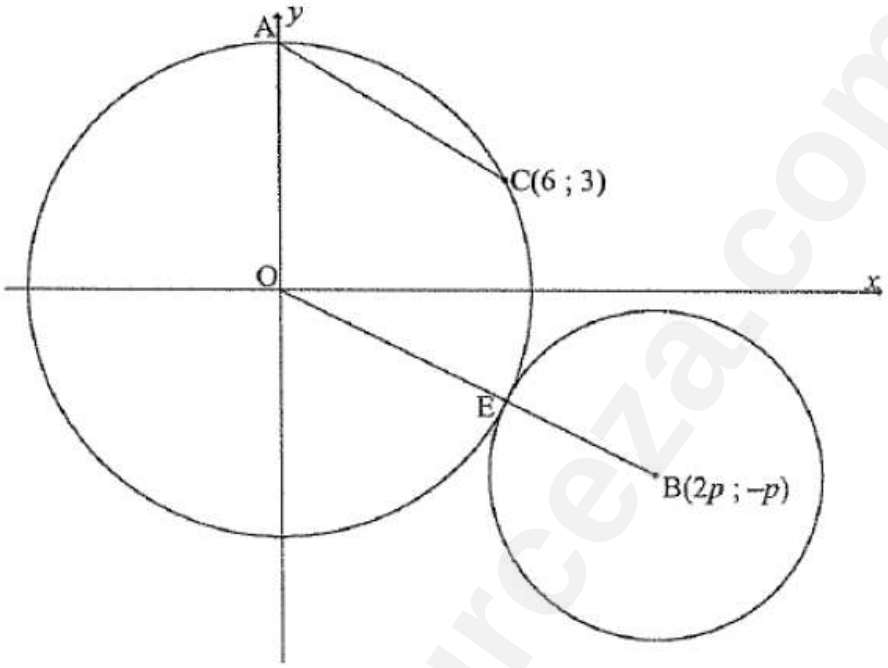
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QUESTION 4

In the diagram below, two circles are given. Circle O, having the origin as centre, intersects the y-axis at A and passes through the point C(6 ; 3). The circle having centre B($2p$; $-p$) touches circle O externally in point E. The centres of the two circles are joined by the line OB.



4.1 Determine the equation of the circle having centre O. (2)

4.2 Determine the coordinates of A. (2)

4.3 Determine the equation of AC. (3)

4.4 Calculate the value(s) of k for which the line $y = \frac{1-\sqrt{5}}{2}x + k$ will intersect the circle having centre O at two points, one of which has a positive x-value and the other a negative x-value. (2)

4.5 It is given that the length of EB $= \sqrt{20}$.

4.5.1 Write down, in terms of p, the equation of circle B in the form $(x-a)^2 + (y-b)^2 = r^2$. (2)

4.5.2 Determine the value of p if $p > 0$. (5)

4.6 Suppose a third circle with the following equation is given:
$x^2 + y^2 + 4x\cos\theta + 8y\sin\theta + 3 = 0$

Determine the maximum length that the radius of this circle can be for any value of θ. (6)

[22]

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MODULE 3: TRIGONOMETRY

1. EXAMINATION GUIDELINE / EXAM TECHNIQUE

- **Mark allocation**: ± 40 marks. **Suggested Time allocation**: 48 minutes.
- Use the values of the trigonometric functions (in surd form where applicable) of **30º, 45º and 60º**.
- Proofs of trigonometric identities are **NOT** required for examination purposes.
- Make sure that your calculator is set on "**DEGREE**" mode for all trigonometric calculations.
- Draw the **CAST** diagram before starting the trigonometry section.

2. THEORY AND QUESTIONS

TRIGONOMETRIC THEORY

Grade 11 Identities: **Quotient Identity:** $\tan\theta = \frac{\sin\theta}{\cos\theta}$ **Square Identity:** $\sin^2\theta + \cos^2\theta = 1$

Compound Angle Identities

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$
$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$
$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

Double Angle Identities

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$

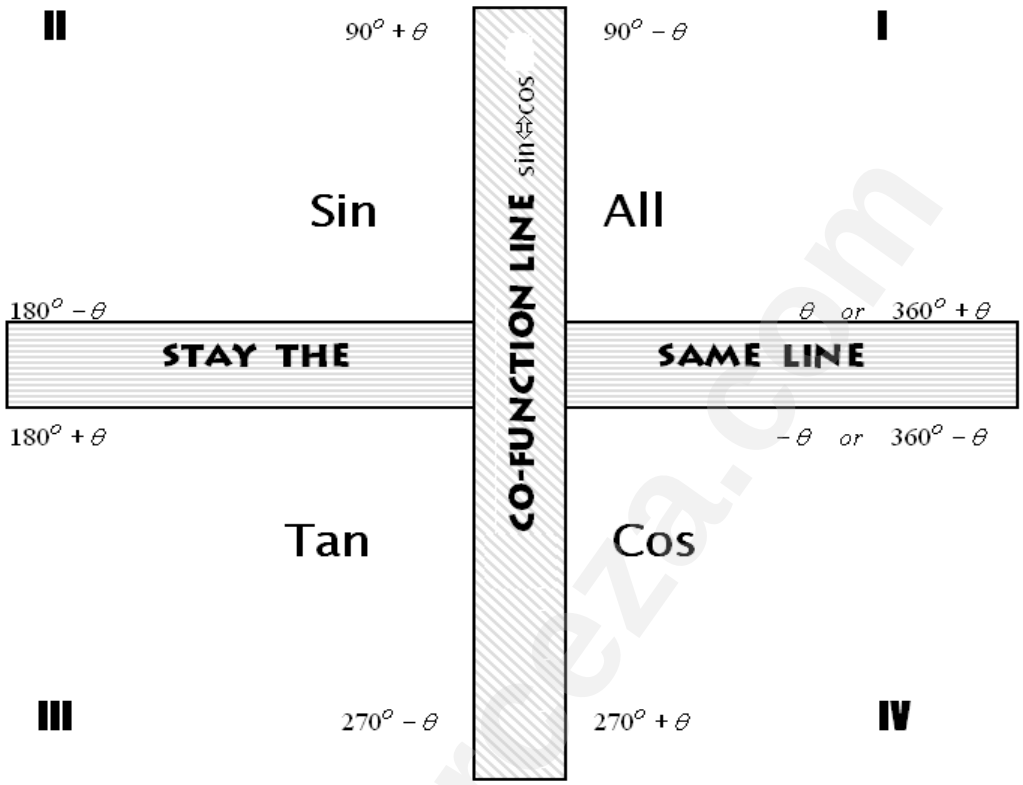
Special Cases for General Solution

$\sin\theta = 0: \quad \theta = 180k; k\varepsilon Z$ $\cos\theta = 0: \quad \theta = 90^\circ + 180k; k\varepsilon Z$ $\tan\theta = 0: \quad \theta = 180k; k\varepsilon Z$

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CAST Diagram:



REDUCTION

1. Identify the **quadrant** in which the angle lies.
2. For **very large positive or negative angles**, add or subtract 360^0 until you get an acute angle, e.g. sin 750^0 = sin ($750^0 - 360^0 - 360^0$) = sin30^0.
3. **Determine the sign** of the given ratio in that quadrant.
4. Write down the answer in acute form, e.g. sin (180^O+x) = -sin x.
5. For the **90/270 quadrants**, change to the co-function, e.g. sin (90^0–x) = cos x.

THE GENERAL SOLUTION OF TRIGONOMETRIC EQUATIONS

1. **Isolate the trigonometric ratio (s)** on the LHS and **factorise** if necessary.
2. Decide whether the **RHS is Positive or Negative**.
3. Choose the **appropriate quadrant** in relation to the ratio.
4. Find the **inverse function** (2^{nd} Function of 'Shift' Calculator), using the positive equivalent of the RHS and apply this to the appropriate quadrants.
5. **Insert 360k for sin/cos** or **180k for tan** on the RHS & solve to get the general solution. We most commonly use 2 quadrants for sin / cos and 1 quadrant for tan.
6. If required, **substitute integers for k** to solve for a specified domain, e.g. [-180^0; 180^0].
7. Interpretation and drawing of **trigonometric graphs** were covered in paper 1 – graphs may be combined with a general solution in paper 2.

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QUESTION 5

5.1 If $\cos 62° = m$, determine the value of each of the following in terms of m.

5.1.1 $\sin 28°$ (2)

5.1.2 $\cos 362°$ (4)

5.2 Simplify to a single ratio:

$$\frac{\tan(360° - x).sin\ (90^0 + x)}{\sin(-x)}$$ (5)

5.3 If $4\,sin^2\theta - 3 = 0,\ 90° \leq \theta \leq 180°$, calculate the value of:

$$\cos\frac{1}{4}\theta.\sin\frac{1}{2}\ \theta - \tan(3\theta - 45°)$$ (6)

[17]

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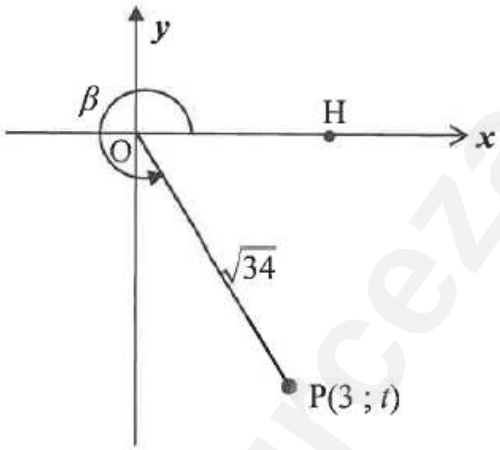
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QUESTION 5

5.1 Given: $\dfrac{\sin(A-360°).\cos(90°+A)}{\cos(90°-A).\tan(-A)}$

Simplify the expression to a single trigonometric ratio. (6)

5.2 In the diagram, P(3 ; t) is a point in the Cartesian plane. OP $=\sqrt{34}$ and $\hat{HOP}=\beta$ is a reflex angle.



Without using a calculator, determine the value of:

5.2.1 t (2)

5.2.2 $\tan\beta$ (1)

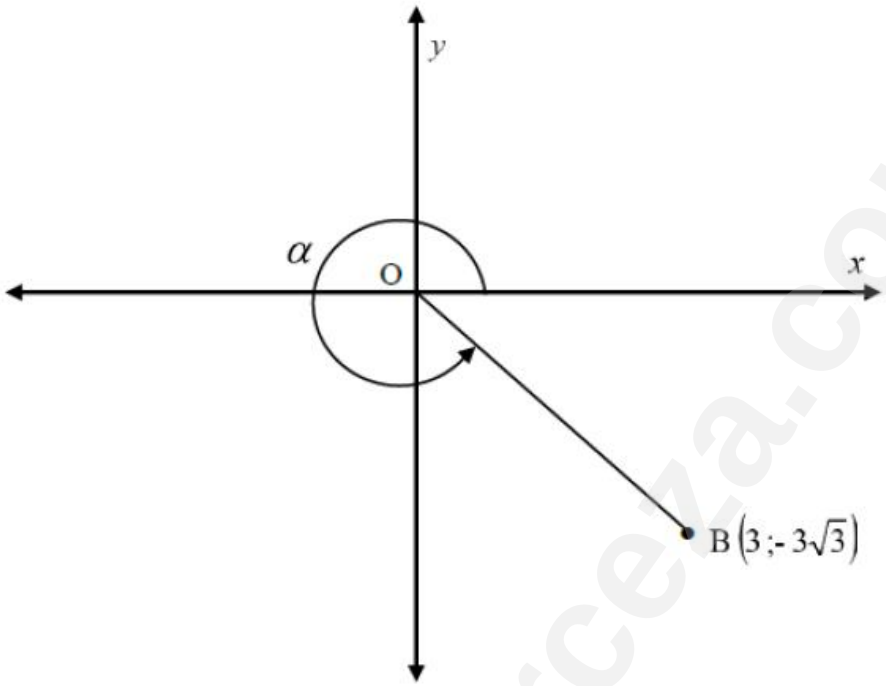
5.2.3 $\cos 2\beta$ (4)

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QUESTION 5

5.1 In the Cartesian plane below, the point $B(3;-3\sqrt{3})$ and the reflex angle, α, are shown.



Determine **(without using a calculator)** the value of:

5.1.1 OB (2)

5.1.2 $\cos(\alpha + 30°)$ (4)

5.2 Simplify:

$$\frac{\sin^2(90° - x)\tan(360° - x)}{\sin(-x)}$$ (4)

5.3 Prove that:

$$\cos(60° + \theta) - \cos(60° - \theta) = -\sqrt{3}\sin\theta$$ (3)

5.4 Consider the identity: $\frac{1 - \sin 2A}{\sin A - \cos A} = \sin A - \cos A$

5.4.1 Prove the identity. (4)

5.4.2 For which values of A in the interval $0° < A < 180°$ will the identity be undefined? (2)

[19]

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QUESTION 5

5.1 If $\sin 16° = \frac{1}{\sqrt{1+k^2}}$, express the following in terms of k, **without the use of a calculator.**

5.1.1 $\tan 16°$ (2)

5.1.2 $\cos 32°$ (3)

5.2 Simplify the following expression.

$$\frac{\cos(90°+x)\sin(x-180°)-\cos^2(180°-x)}{\cos(-2x)}$$ (6)

5.3 Calculate the value of the following, **without the use of a calculator.**

$\cos 75°.\cos 45° - \cos 15°.\cos 45°$ (4)

5.4 Given: $\tan\theta\left(\sin 2\theta + \frac{3\cos^2\theta}{\sin\theta}\right) = -2\cos^2\theta + 3\cos\theta + 2$

5.4.1 Prove the identity. (3)

5.4.2 Determine the general solution of:

$$\tan\theta\left(\sin 2\theta + \frac{3\cos^2\theta}{\sin\theta}\right) = 0$$ (4)

5.5 Solve for a and b:

$\cos(a+b) = -\frac{\sqrt{2}}{2}$ if $a+b \in [0° ; 180°]$

$\cos(a-2b) = \frac{1}{2}$ if $a-2b \in [0° ; 180°]$ (4)

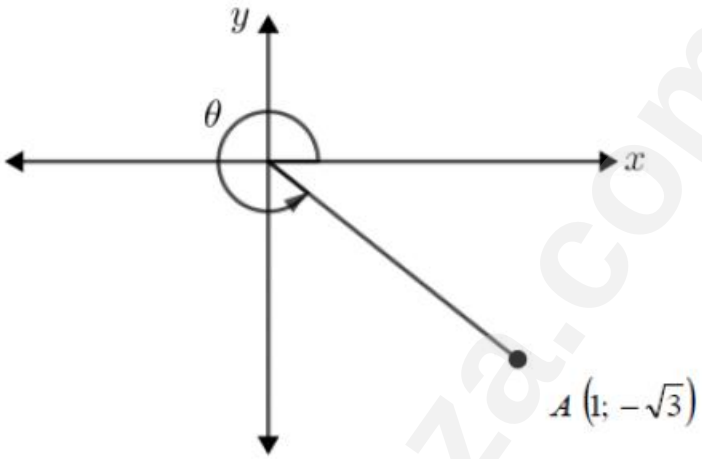
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QUESTION 5

5.1 Use the diagram below to calculate, **without the use of a calculator,** the following



5.1.1 $\tan\theta$ (1)

5.1.2 $\sin(-\theta)$ (3)

5.1.3 $\sin(\theta - 60°)$ (4)

5.2 Determine the value of the following trigonometric expression:

$$\frac{\tan(180° - \theta)\sin(90° + \theta)}{\cos 300° \sin(\theta - 360°)}$$ (6)

5.3 Consider: $\frac{\cos 2x - 1}{\sin 2x} = -\tan x$

5.3.1 Prove the identity (3)

5.3.2 For which value(s) of x, $0° < x < 360°$, is this identity undefined? (3)

5.3.3 Hence or otherwise, find the general solution of $\frac{\sin 4x}{\cos 4x - 1} = 4$. (4)

[24]

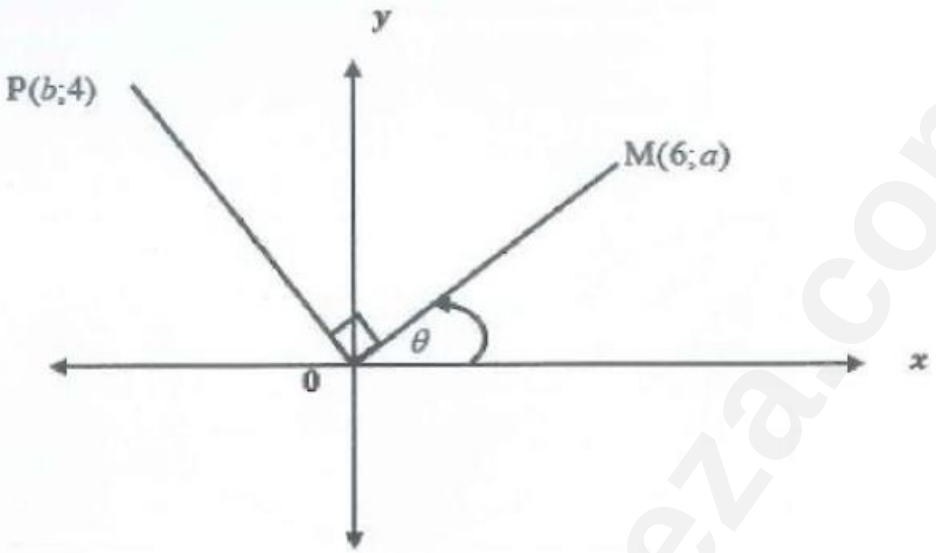
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QUESTION 5

In the diagram alongside $P\hat{O}M = 90°$, $M\hat{O}X = \theta$, $M(6;a)$, $P(b;4)$ and $\sqrt{5}\cos\theta - 2 = 0$.



Hint: $\tan(90° + \theta) = -\dfrac{1}{\tan\theta}$.

5.1 Determine, without the use of a calculator, the values of:

5.1.1 a (3)

5.1.2 b (4)

5.2 Simplify the following expression to a single trigonometric ratio of θ:

5.2.1 $\dfrac{\sin 163°}{\cos 73°} - \dfrac{\sin(90° + \theta).\tan^2(\theta - 360°)}{\cos(180° - \theta)}$ (7)

5.2.2 $\dfrac{\sin^2\theta\cos\theta}{\tan 2\theta(\sin^2\theta - \cos^2\theta)}$ (5)

5.3 Determine the general solution of $\cos(x - 60°) = 3\cos x$. (6)

[25]

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QUESTION 5

5.1 If $\sin 40°.\cos 22° + \cos 40°.\sin 22° = k$, determine without the use of a calculator, the value of the following in terms of k.

5.1.1 $\sin 62°$ (2)

5.1.2 $\tan 118°$ (4)

5.1.3 $\sin 14°.\cos 14°$ (3)

5.2 Prove the following identity:

$$\frac{1-\cos 2\theta}{\sin 2\theta \times \tan\theta} = 1 \quad (4)$$

5.3 For which value(s) of A will the following expression be real?

$$\sqrt{\sin(180° + A).\cos(90° + A) - \tan 45°} \quad (6)$$

[19]

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QUESTION 5

5.1 Simplify to one trigonometric ratio:

$$\frac{\sin(A-180°).\tan(180°-A).\cos A}{\cos(90°+A)}$$ (5)

5.2 If $\sin 28° = a$ and $\cos 32° = b$, determine the following in terms a and b.

5.2.1 $\cos 28°$

5.2.2 $\cos 64°$

5.2.3 $\sin 4°$

5.3 Prove the identity: $\frac{1}{1-\sin x} - \frac{1}{1+\sin x} = \frac{2\tan x}{\cos x}$ (5)

[18]

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QUESTION 5

5.1 Simplify each of the following **without the use of a calculator.**
Show ALL calculations.

5.1.1 $$\frac{\sin 110°.\tan 60°}{\cos 540°.\tan 250°.\sin 380°}$$ (7)

5.1.2 $$(1-\sqrt{2}\sin 22,5°)(\sqrt{2}\sin 22,5°+1)$$ (4)

5.2 Given the expression: $\frac{\cos 2x.\tan x}{\sin^2 x}$

5.2.1 For which value(s) of x, in the interval $x \in [0°;180°]$, will this expression be undefined? (3)

5.2.2 Prove that $\frac{\cos 2x.\tan x}{\sin^2 x} = \frac{\cos x}{\sin x} - \tan x$ (5)

[19]

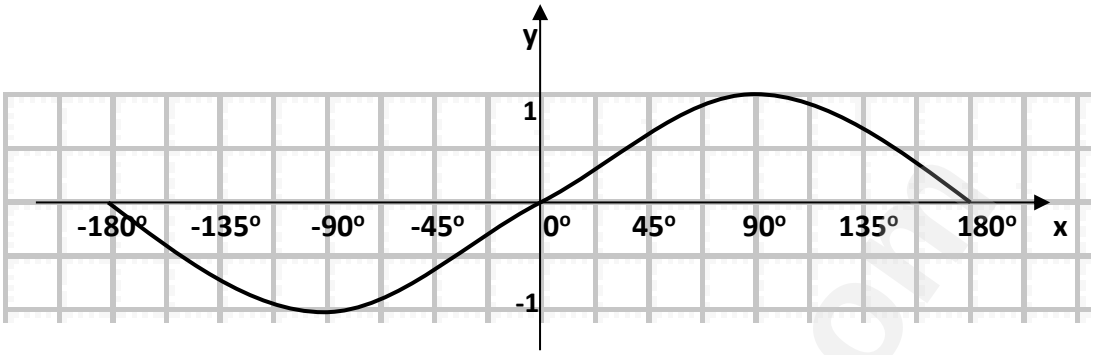
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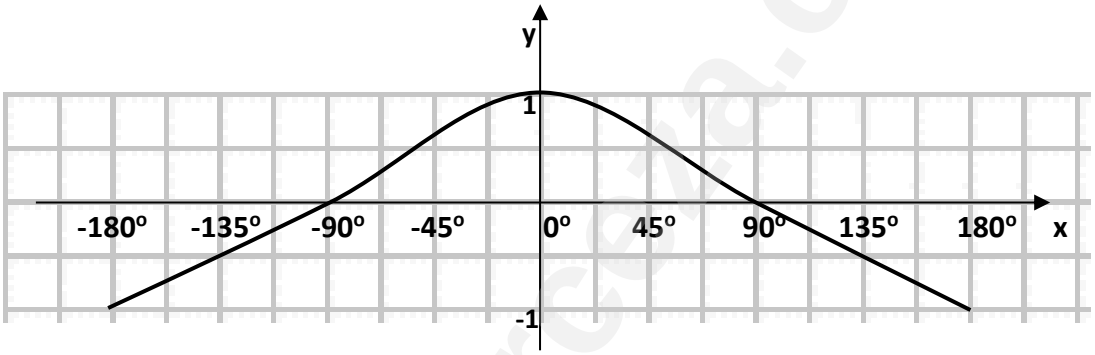
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TRIGONOMETRIC GRAPHS - $y = a\langle \sin/\cos/\tan\rangle(bx)$ **OR** $y = \langle \sin/\cos/\tan\rangle(x+c)+d$

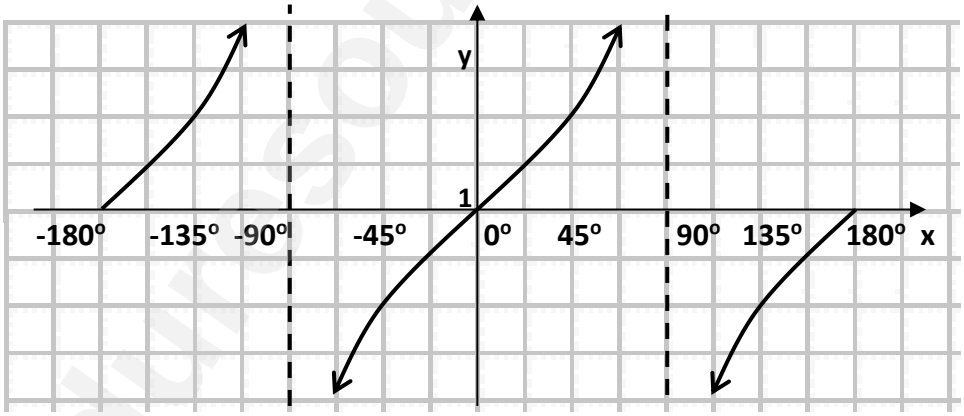
y= sinx



y=cosx



y = tanx



- The **amplitude** is represented by 'a'
- The **period** = $\frac{360^\circ}{b}$ for sin and cos graphs and $\frac{180^\circ}{b}$ for tan graphs.
- **Asymptotes** for tan graphs are at $\pm\frac{90^\circ}{b}$ and $\pm\frac{270^\circ}{b}$ and every $\pm\frac{180^\circ}{b}$ interval thereafter.
- 'c' represents a **horizontal shift** (+ → left, – → right)
- 'd' represents a **vertical shift** (+ → up, – → down)
- **Best interval** for sin and cos = $\frac{90^\circ}{b}$ and for tan = $\frac{45^\circ}{b}$ (used to mark scale of x-axis).

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QUESTION 8

Given $f(x) = 2\cos x$ and $g(x) = \sin(x + 30°)$

8.1 Write down the amplitude of f. (1)

8.2 Sketch the graph of f and g on the same set of axes on DIAGRAM SHEET 2. (5)

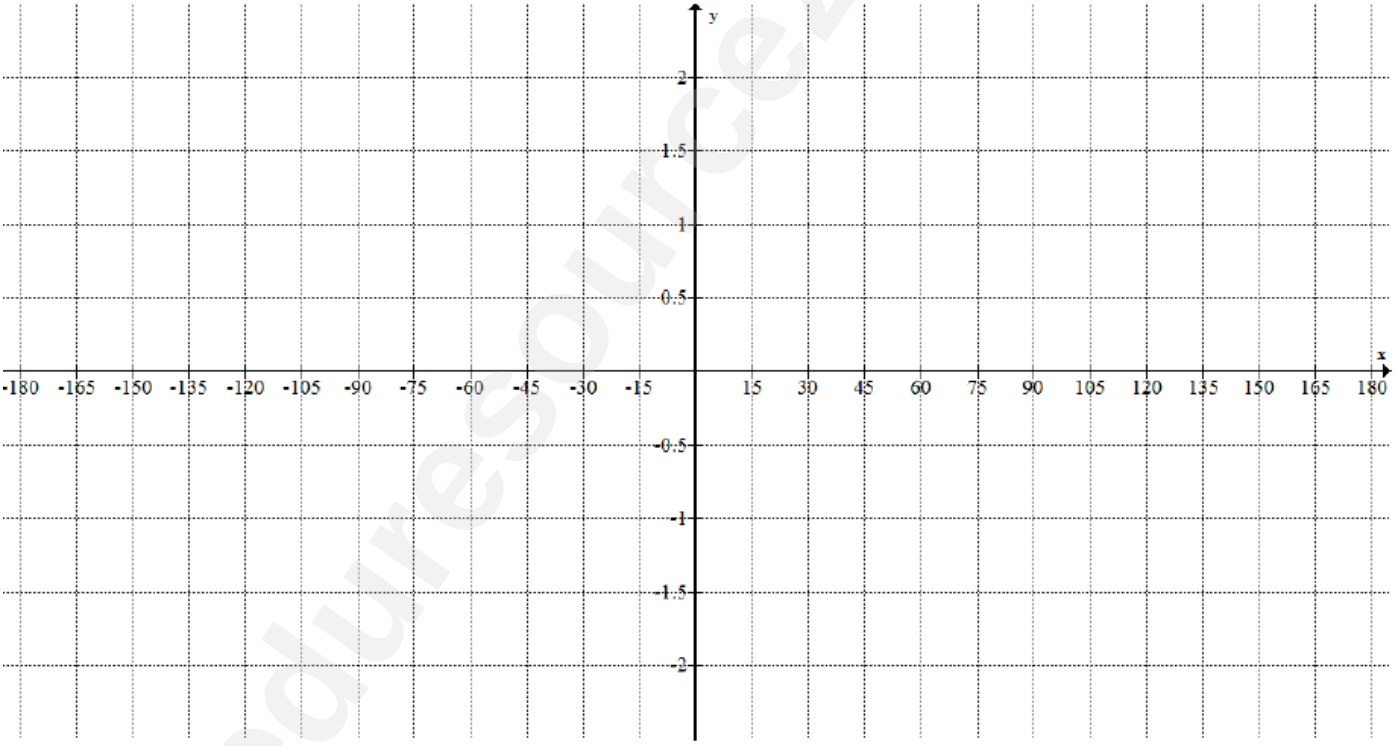
8.3 Determine the value(s) of x for which $f(x) - g(x) \geq 0$ for $x \in [-180°; 90°]$. (2)

8.4 Give TWO possible ways of transforming the graph of f such that its y-intercept is at the origin. (2)

[10]

DIAGRAM SHEET 2

QUESTION 8.2



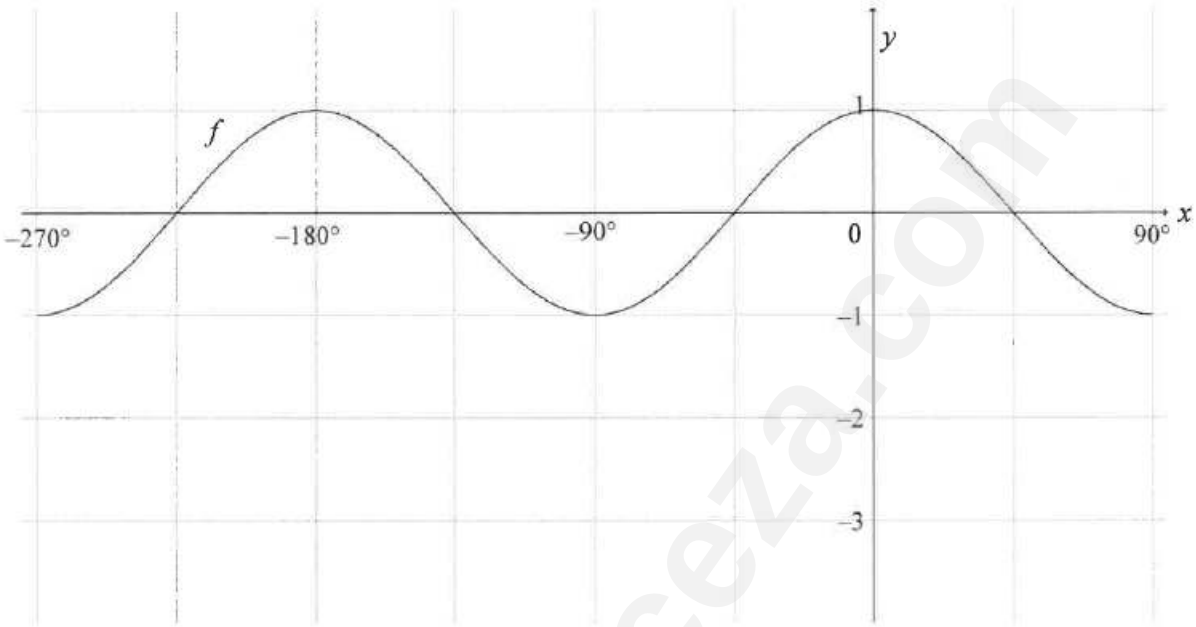
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QUESTION 6

In the diagram, the graph of $f(x) = \cos 2x$ is drawn for the interval $x \in [-270°; 90°]$.



6.1 Draw the graph of $g(x) = 2\sin x - 1$ for the interval $x \in [-270°; 90°]$ on the grid given in your ANSWER BOOK. Show ALL the intercepts with the axes, as well as the turning points. (4)

6.2 Let A be a point of intersection of the graphs of f and g. Show that the x-coordinate of A satisfies the equation $\sin x = \frac{-1+\sqrt{5}}{2}$. (4)

6.3 Hence, calculate the coordinates of the points of intersection of graphs of f and g for the interval $x \in [-270°; 90°]$. (4)

[12]

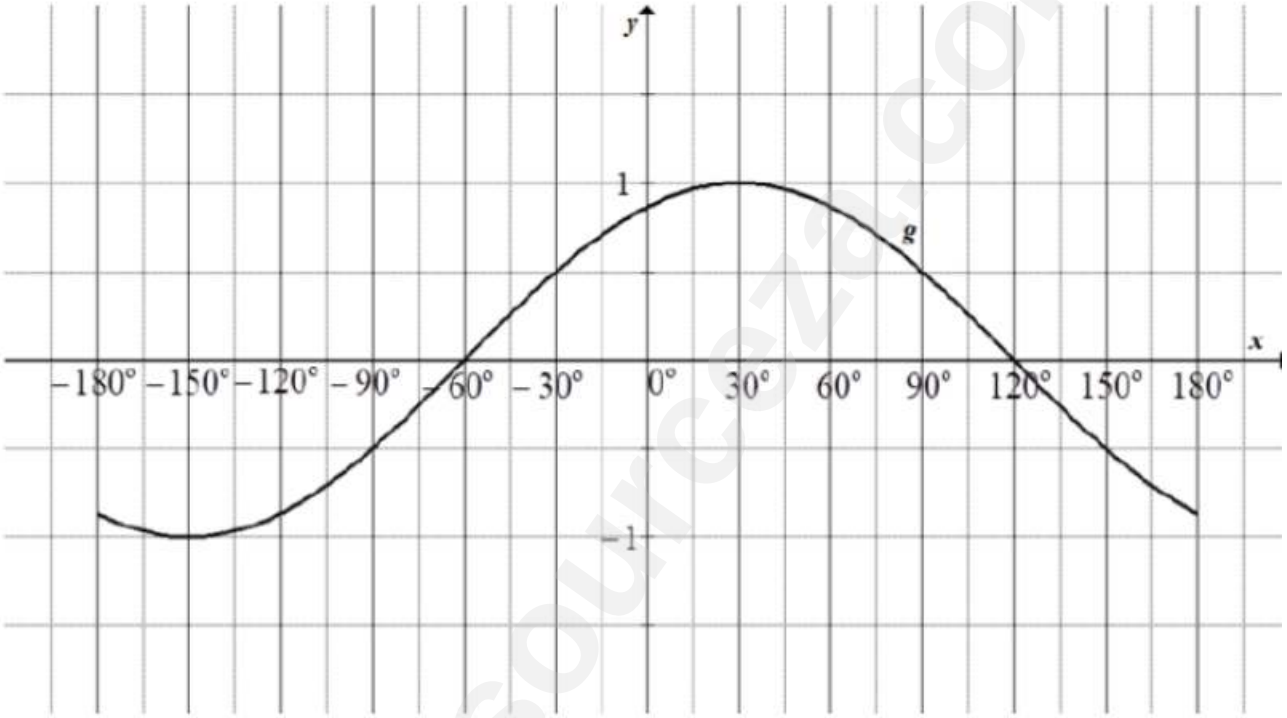
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QUESTION 6

6.1 Determine the general solution for $\sin 2x = \cos(x - 30°)$. (5)

6.2 The diagram below shows the graph of $g(x) = \cos(x - 30°)$ for the interval $x \in [-180°; 180°]$.



6.2.1 Write down the period of g. (1)

6.2.2 Determine the values of x for which the graph of g increasing. (2)

6.2.3 On the same system of axes draw the graph of $f(x) = \sin 2x$ for $x \in [-180°; 180°]$ in your ANSWER BOOK. (3)

6.2.4 Hence or otherwise, determine the values of x in the interval $-180° \leq x \leq 180°$ for which $f(x).g(x) < 0$. (3)

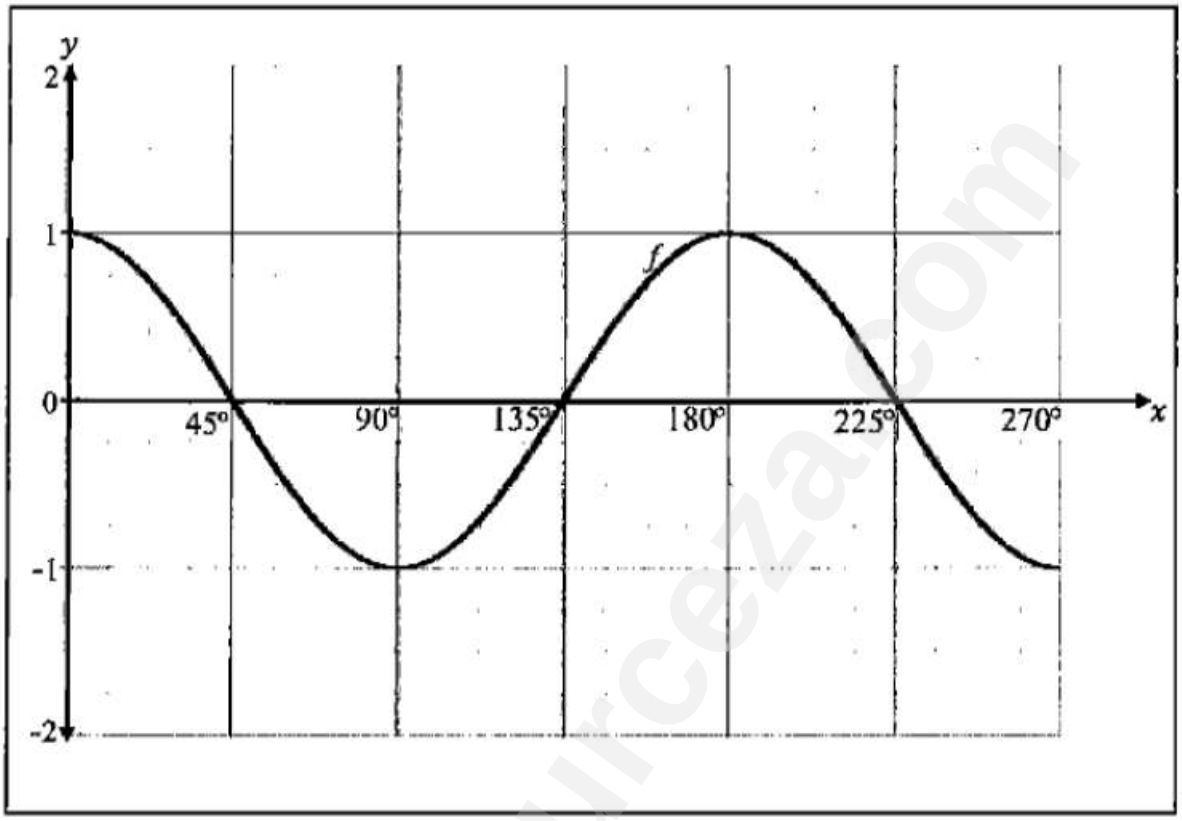
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QUESTION 6

In the diagram below, the graph of $f(x) = \cos 2x$ is drawn for the interval $x \in [0°; 270°]$.



6.1 Draw the graph of $g(x) = -\frac{1}{2}\tan x$ for the interval $x \in [0° ; 270°]$ on the grid provided in the ANSWER BOOK. Show all intercepts with the axes and asymptotes. (4)

6.2 Write down the range of $h(x) = 3 - f(x)$. (1)

6.3 Use the graph to determine the value(s) of x in the interval $x \in [135° ; 270°]$ for which $\frac{f(x)}{g(x)} \geq 0$. (2)

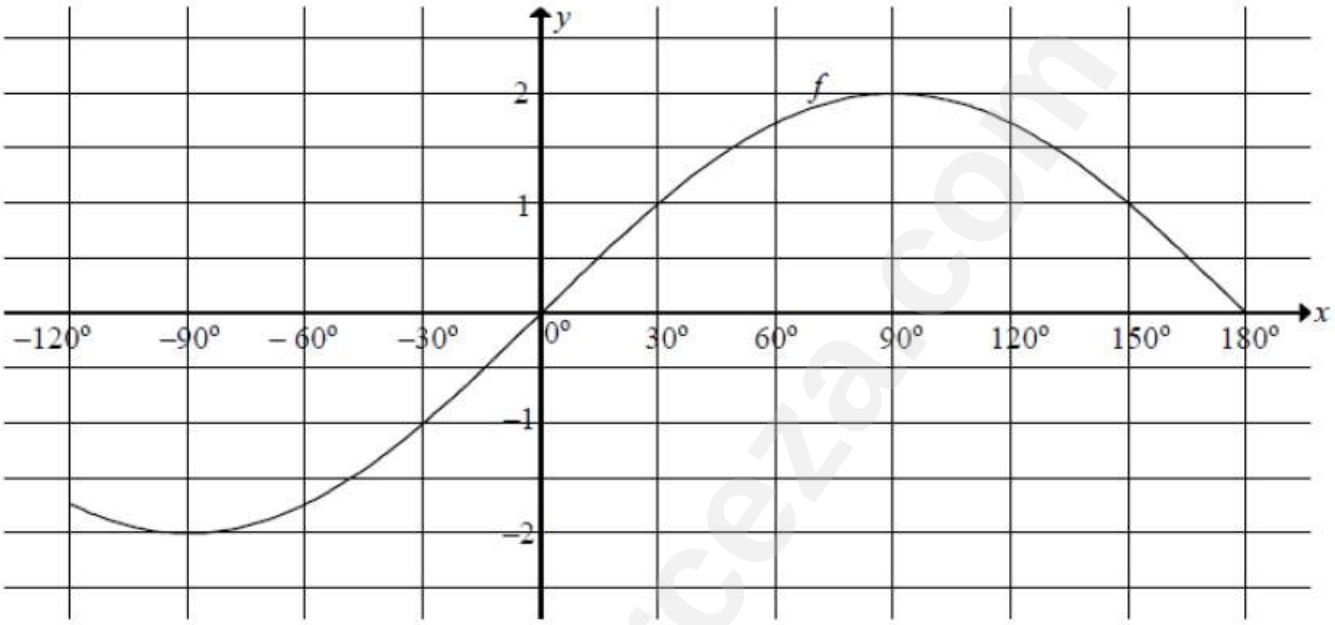
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QUESTION 6

In the diagram below, the graph of $f(x) = 2\sin x$ is drawn for the interval $x \in [-120° ; 180°]$.



6.1 Draw on the same system of axes the graph of $g(x) = \cos(x + 30°)$, for the interval $x \in [-120° ; 180°]$. Show all intercepts with the axes as well as the turning and end Points of the graph. (4)

6.2 Write down the period of f. (1)

6.3 For which values of x in the interval $x \in [-120° ; 180°]$ is:

6.3.1 The graph of g decreasing? (2)

6.3.2 $f(x).g(x) > 0$? (2)

6.4 If the graph of g is moved 60° to the left, determine the equation of the new graph which is formed, in its simplest form. (2)

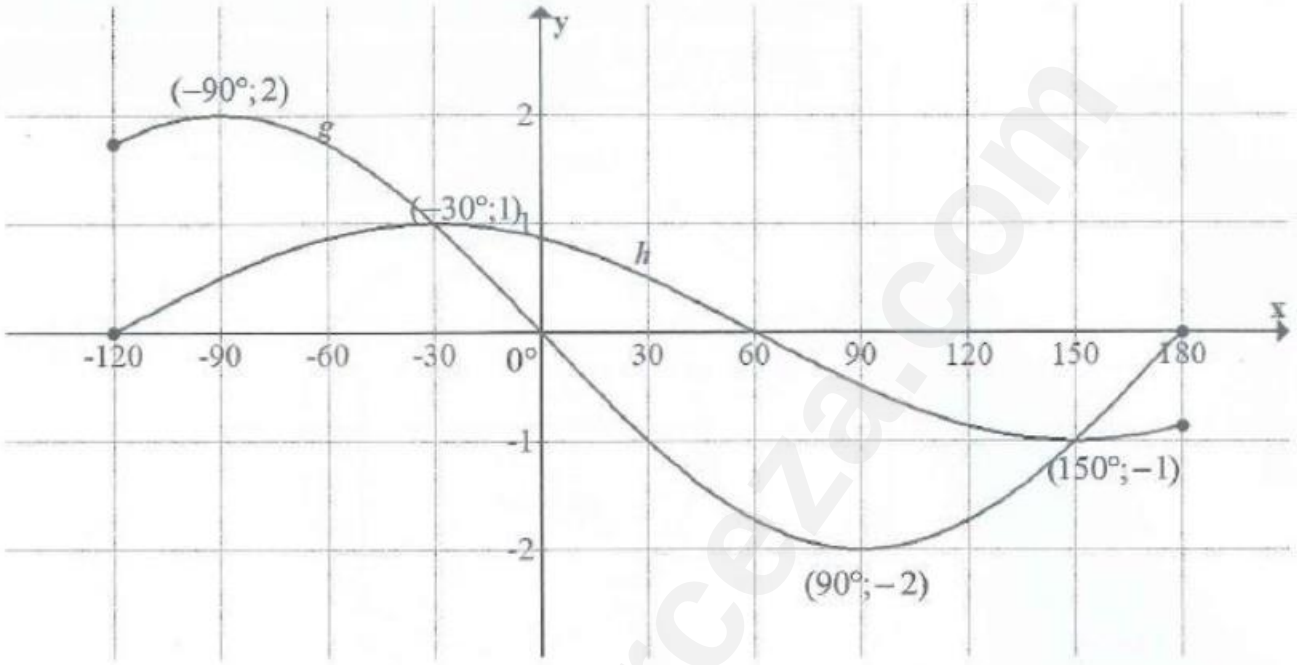
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QUESTION 6

The graphs of $h(x) = \cos(x + 30°)$ and $g(x) = -2\sin x, x \in [-120°; 180°]$ is drawn below.



Use the graphs above to determine for which values of x:

6.1 $2\sin x + \cos x \cos 30° \geq \sin x \sin 30°, \quad x \in [-120°; 180°]$ (4)

6.2 Will both $g(x)$ and $h(x)$ increase as x increases for $x \in [-120°; 180°]$ (4)

[8]

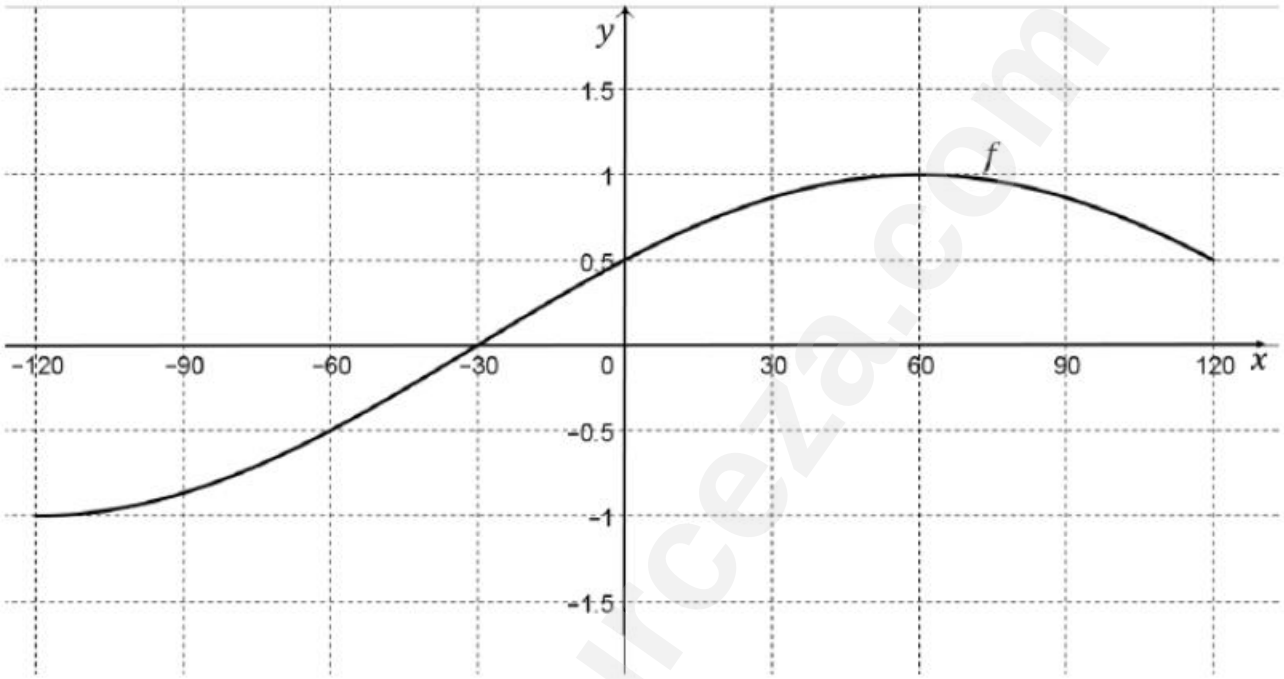
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QUESTION 6

In the diagram is the graph of $f(x) = \sin(x + a)$ for the interval [-120°;120°]



6.1 Determine the numerical value of a. (1)

6.2 On the grid provided in the ANSWER BOOK, draw the graph of $g(x) = \cos(3x)$ for the interval $x \in [-120°; 120°]$. Clearly show ALL intercepts with the axes, the turning point(s) and endpoint(s) of the graph. (4)

6.3 Determine the general solution for the following: $f(x) = g(x)$ (5)

6.4 Determine the values of x in the interval $x \in [0°; 120°]$, for which $f(x) > g(x)$. (2)

6.5 Describe the transformation from graph g to the graph of $k(x) = \cos(60° - 3x)$. (2)

[14]

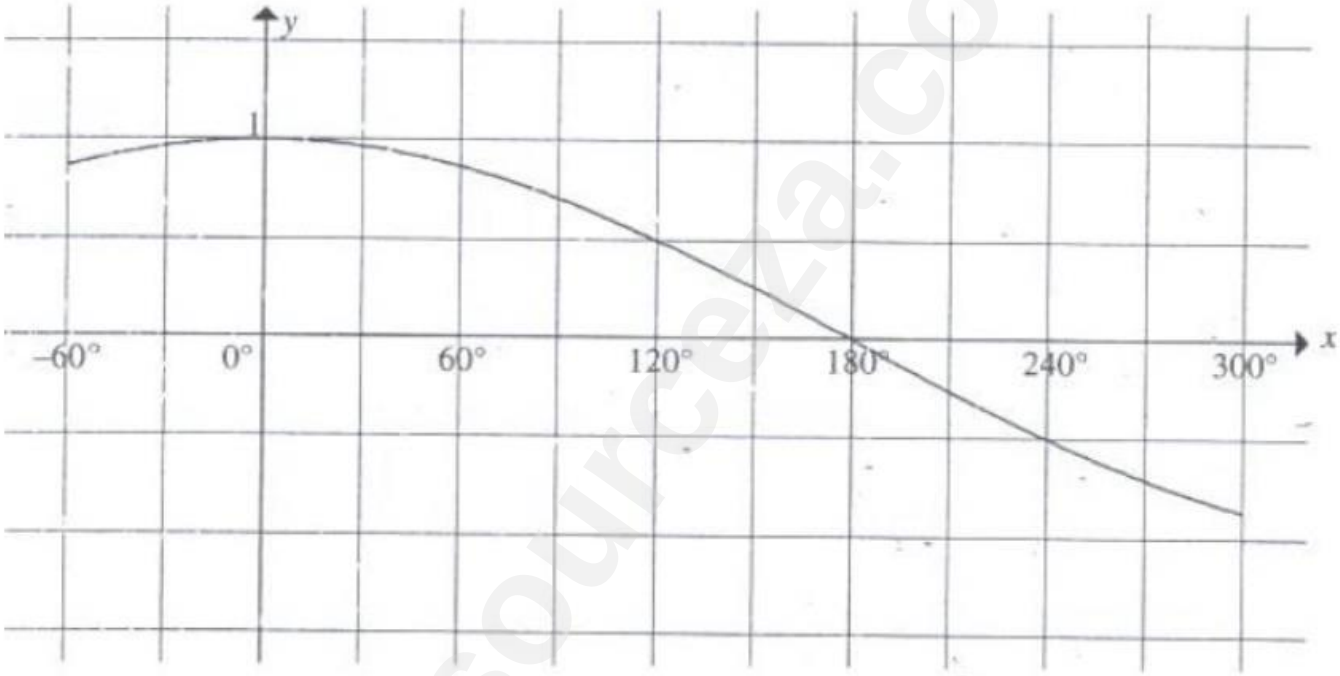
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QUESTION 6

6.1 Solve for x if $\sin(x+60^\circ)=\cos\frac{1}{2}x$ and $x\in[-60^\circ;300^\circ]$. (7)

6.2 In the diagram below, the graph of $f(x)=\cos\frac{1}{2}x$ is drawn for $x\in[-60^\circ;300^\circ]$.



6.2.1 On the grid provided IN THE ANSWER BOOK, draw the graph of $g(x)=\sin(x+60^\circ)$ for $x\in[-60^\circ;300^\circ]$. Clearly show all maximum and minimum points, the intercepts with the axes and the endpoints. (3)

6.2.2 Use the solutions obtained in 6.1 as well as the graphs drawn in 6.2.1 to determine the value(s) of $x\in[-60^\circ;300^\circ]$ for which:

6.3.1 $f(x)<g(x)$ (3)

6.3.2 $f(x).g(x)\leq 0$ (3)

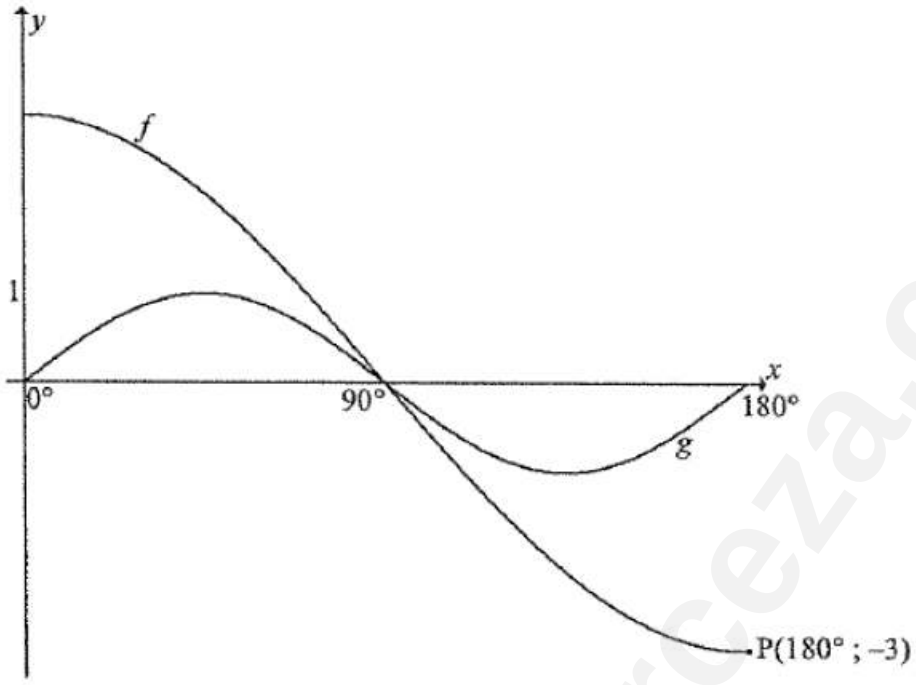
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QUESTION 6

In the diagram below, the graphs of $f(x) = a\cos x$ and $g(x) = \sin bx$ are drawn for the interval $x \in [0° ; 180°]$. The point P(180° ; –3) is on the graph of f.



6.1 Write down the values of a and b. (2)

6.2 Write down the period of f. (1)

6.3 Write down the range of $g(x) + 3$ (2)

6.4 For which values of x, in the given interval, is $f(x).g'(x)>0$ (3)

6.5 When the graph of g is shifted $q°$ to the left, it coincides with the function $y - \cos^2 x = -\sin^2 x$. Determine the value of q. (3)

[11]

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IDENTITIES

1. Choose the **more complex side** (never interfere with the equation). Sometimes, simplify both sides.
2. Convert **everything to sin and cos**.
3. Add and subtract fractions **using common denominators**.
4. Be on the lookout for **squares** and use identities.
5. **Factorise** where necessary.
6. When you have an equation of the form: $\frac{1 + \cos x}{\sin x}$
   then multiply using the difference of 2 squares as follows: $\frac{1 + \cos x}{\sin x} \times \frac{1 - \cos x}{1 - \cos x}$
   to obtain a square identity.
7. Don't use the cos double angle identity until you are absolutely sure of which one to use.
   For cos 2x, use:
   - **$\cos 2x = 1 - 2\sin^2 x$** to get rid of a '–1' or to form an expression with only sin.
   - **$\cos 2x = 2\cos^2 x - 1$** to get rid of a '1' or to form an expression with only cos.
   - **$\cos 2x = \cos^2 x - \sin^2 x$** to get rid of a $\sin^2 x$ or to form an expression with cos and sin.

   In using the above 3, try to get an expression that can be **factorised**, e.g. using trinomials.
8. For **triple or quadruple angles**, change as follows: **cos 3x = cos (2x + x), sin 4x = sin (2x + 2x)** then apply compound angle formulae.
9. **Work backwards** with the compound and double angle formulae as follows:
   a. **$\cos 100^0 \cos 40^0 + \sin 100^0 \sin 40^0 = \cos(100^0 - 40^0) = \cos 60^0$**
   b. **$\cos 2x = 1 - 2\sin^2 x$, $\therefore 1 - \cos 2x = \sin^2 x$, $\therefore = \frac{1 - \cos 2x}{2} = \sin^2 x$**

QUESTION 6

6.1 Prove that $\frac{\cos 2x}{1+\sin 2x} = \frac{\cos x - \sin x}{\sin x + \cos x}$ [4]

5.3 Prove:

5.3.1 $\sin(A+B) - \sin(A-B) = 2\cos A.\sin B$ (2)

5.3.2 **Without using a calculator,** that $\sin 77° - \sin 43° = \sin 17°$ (4)

[19]

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SINE RULE

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

USE WHEN GIVEN:

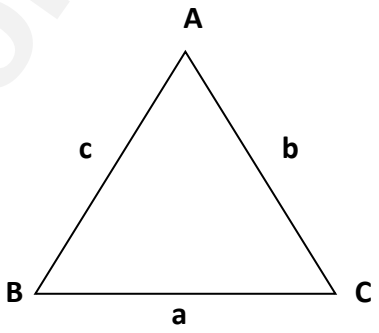
[<<S] or [SS<]

COSINE RULE

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

USE WHEN GIVEN:

[S<S]



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

[SSS]

AREA RULE

$$\text{Area } \Delta ABC = \frac{1}{2}ab\sin C$$
$$= \frac{1}{2}ac\sin B$$
$$= \frac{1}{2}bc\sin A$$

[S<S]

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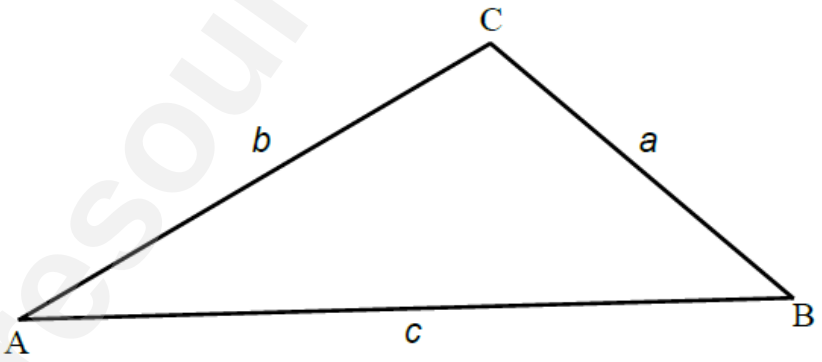
2-D AND 3-D PROBLEM SOLVING RULES

1. Mark all **right angles** and other given information on the figure. Note that the **vertical plane (e.g. walls, cliffs, etc.)** is perpendicular to the **horizontal plane (floors, ceiling, etc.).**
2. Always **indicate the triangle** you are working with.
3. Always start with the **triangle containing the most information**.
4. Use the **formulae** above to solve for sides and angles.
5. Link the triangle in Rule 3 to the triangle containing the unknown side **by finding the common side**.
6. Use the common side found to solve for the unknown side in the 2nd triangle.
7. Remember all **angles of elevation and depression** are taken from the horizontal plane.
8. The use of **reduction formula** and **identities** are important in simplifying the expression.

QUESTION 7

7.1 Use the diagram on DIAGRAM SHEET to prove that:

$a^2 = b^2 + c^2 - 2bc.cosA$

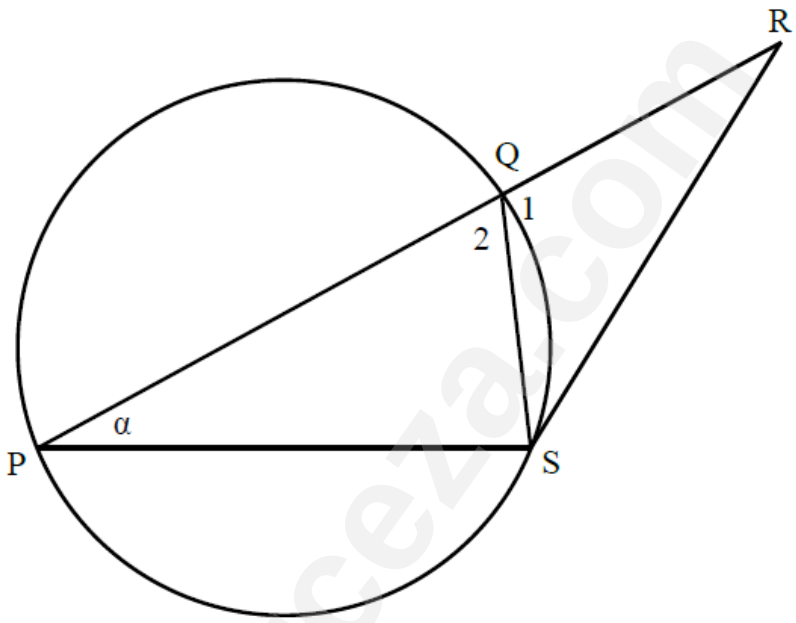


(4)

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7.2 In the figure, the diameter PQ of the circle is produced to R. S is a point on the circumference such that QR = QS = x.
P and S are joined and $Q\hat{P}S = \alpha$.



7.2.1 Prove that : SR = $x\sqrt{2(1 + sin \propto)}$ (4)

7.2.2 If SR = $5\sqrt{3}$ and $x = 5$, show that PQ = 10 (4)
[12]

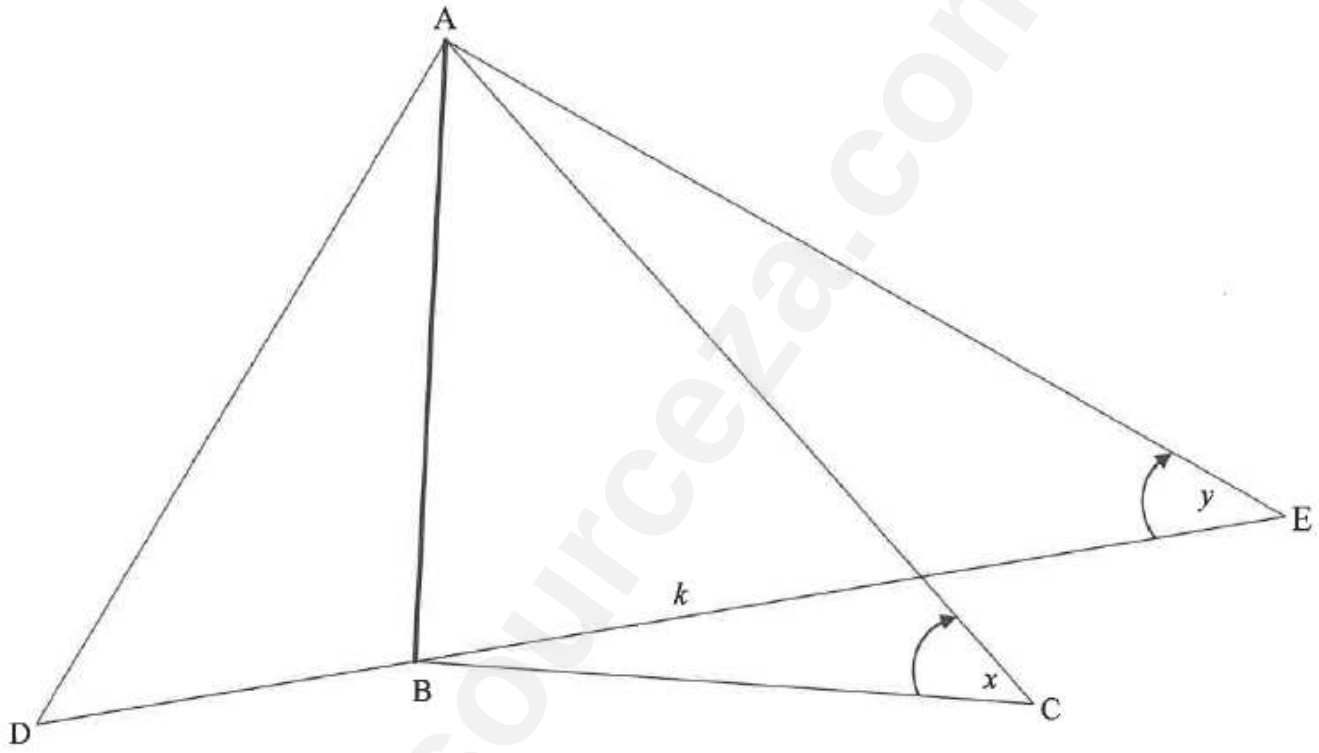
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QUESTION 7

AB represents a vertical netball pole. Two players are positioned on either side of the netball pole at points D and E such that D, B and E are on the same straight line. A third player is positioned at C. The points B, C, D and E are in the same horizontal plane. The angles of elevation from C to A and from E to A are x and y respectively. The distance from B to E is k.



7.1 Write down the size of $A\hat{B}C$. (1)

7.2 Show that $AC = \dfrac{k.\tan y}{\sin x}$ (4)

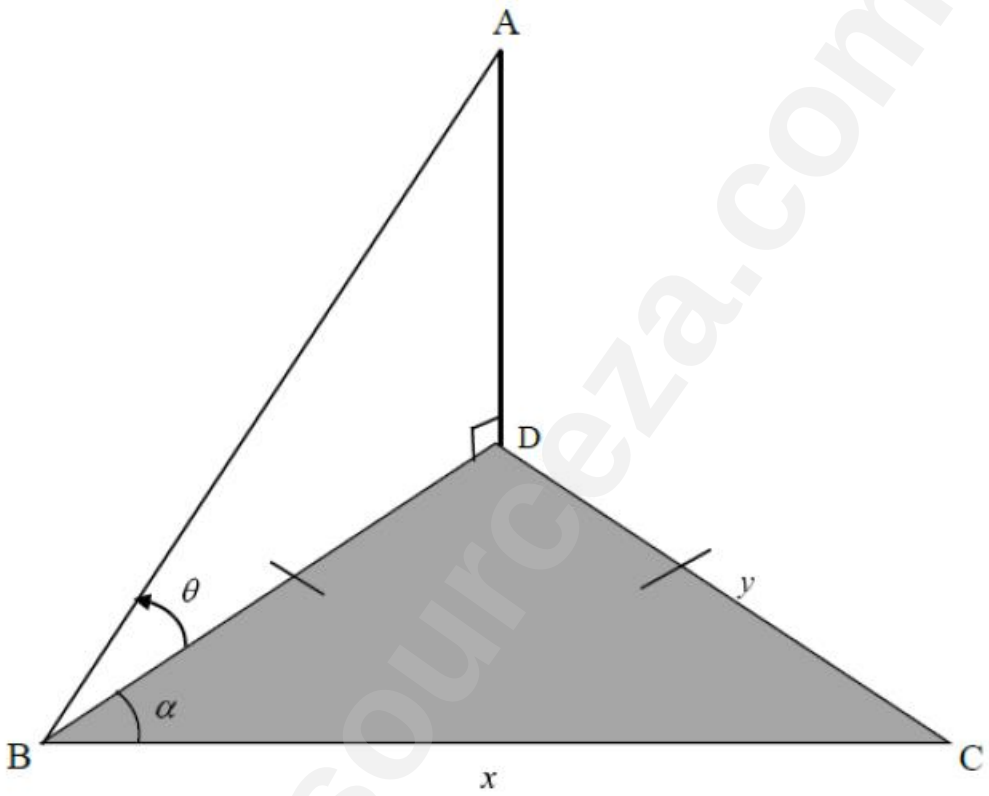
7.3 If it is further given that $D\hat{A}C = 2x$ and AD = AC, show that the distance DC between the players at D and C is $2k\tan y$. (5)
[10]

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QUESTION 7

In the diagram below, B, C and D are three points on the same horizontal plane such that BD = DC = y. $C\hat{B}D = \alpha$ and $A\hat{B}D = \theta$. Line BC = x.



Prove that $AB = \dfrac{x}{2\cos\alpha\cos\theta}$ [7]

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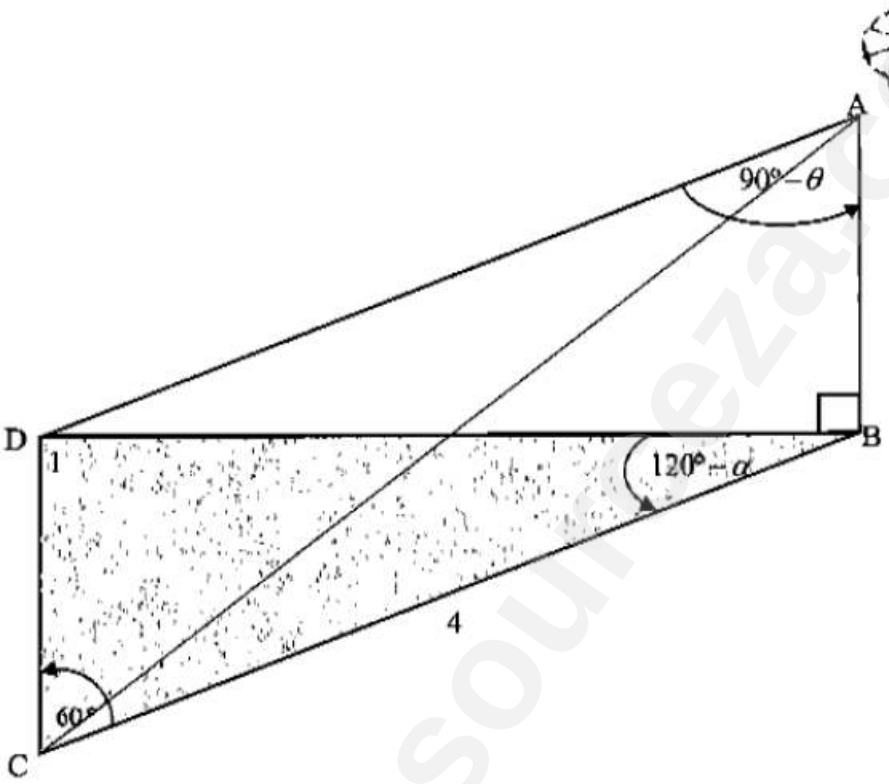
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QUESTION 7

In the diagram below, B, C and D are in the same horizontal plane. A drone positioned at A, captures the images of two objects at B and C. B is directly below the drone and C is 4 units away from B.

$D\hat{C}B = 60°$; $D\hat{B}C = 120° - \alpha$ and $D\hat{A}B = 90° - \theta$.



7.1 Determine $\hat{D}_1$ in terms of α. (2)

7.2 **Without the use of a calculator,** determine BD in terms of α. (3)

7.3 Show that $AB = \dfrac{2\sqrt{3}\tan\theta}{\sin\alpha}$. (3)

[8]

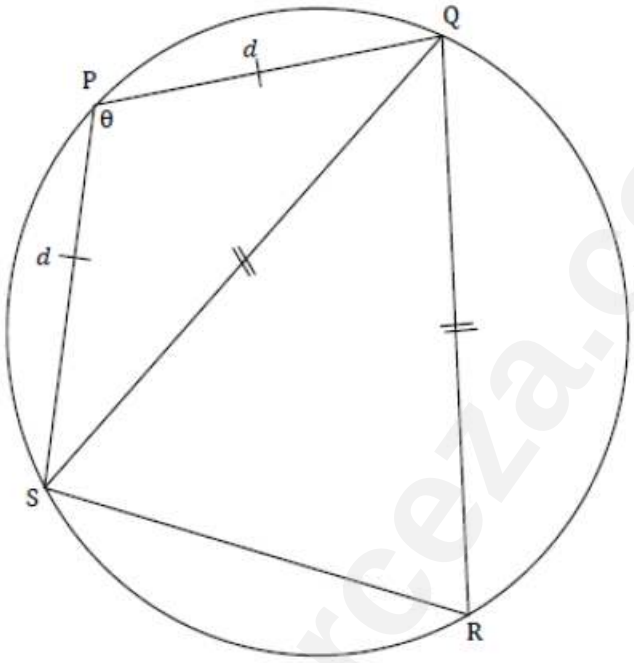
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QUESTION 7

In the diagram, PQRS is a cyclic quadrilateral with QS = QR and PQ = PS = d units. $\hat{QPS} = \theta$.



Use the diagram to prove that:

7.1. $QS = d\sqrt{2(1-\cos\theta)}$ (2)

7.2 The area of $\Delta QRS = -d^2 \sin 2\theta\,(1-\cos\theta)$ (3)

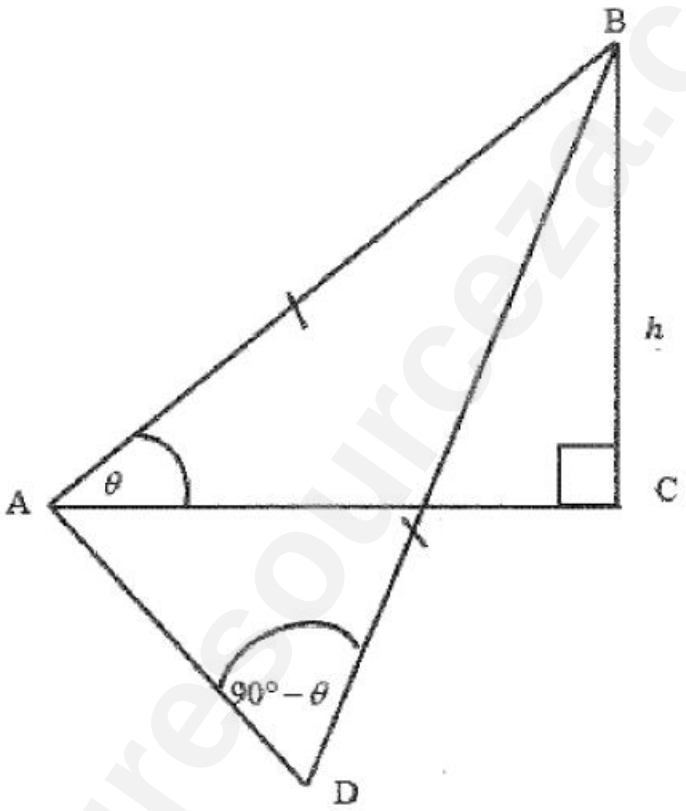
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QUESTION 7

In the diagram below, BC is a pole anchored by two cables at A and D. A, D and C are in the same horizontal plane. The height of the pole is h and the angle of elevation from A to the top of the pole B, is θ. BA = BD and $B\hat{D}A = 90° - \theta$.



7.1 Express AB in terms of h and a trigonometric ratio of θ. (1)

7.2 Determine the magnitude of $A\hat{B}D$ in terms of θ. (3)

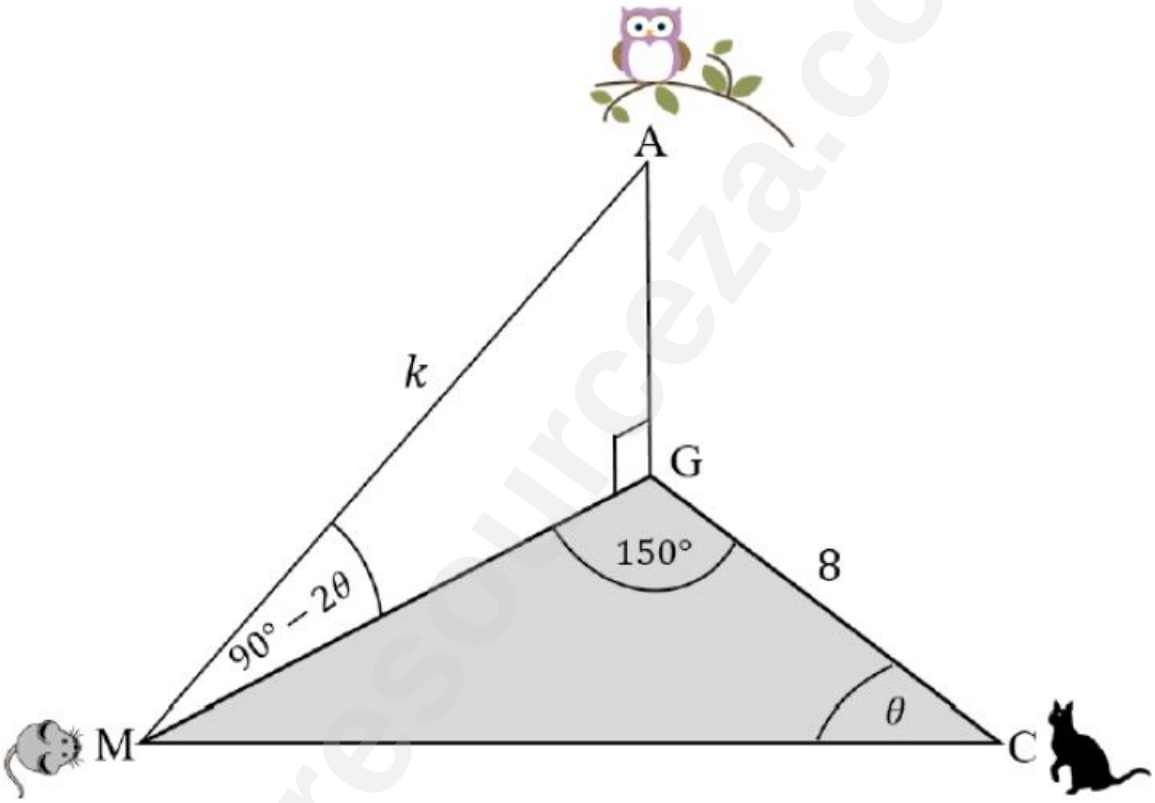
7.3 Determine the distance between the two anchors in terms of h. (3)
[7]

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QUESTION 7

A mouse on the ground is looking up to an owl in a tree and a cat to his right on the ground. The angle of elevation from the mouse to the owl is $(90° - 2\theta)$.
AM $= k$ *units*, GC$= 8$ *units* , M$\hat{G}$C$= 150°$ and M$\hat{C}$G $= \theta$



7.1 Give the size of M$\hat{A}$G in terms of θ. (1)

7.2 Show that MG$= k \sin 2\theta$ (2)

7.3 Show that MC$= k \cos \theta$ (4)

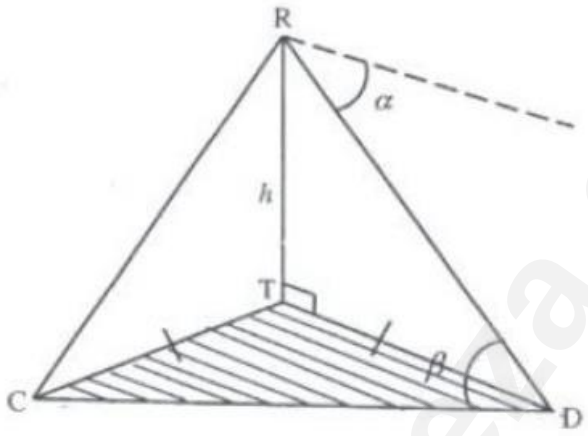
7.4 Show that the area of ΔMGC$= 2k \sin 2\theta$ (2)
[9]

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QUESTION 7

In the diagram, RT represents the height of a vertical tower. C and D represent two points equidistant from T and which lie on the same horizontal plane as T. The height of the tower is h. The angle of depression of D from R is α. $\hat{RDC} = \beta$.



7.1 Determine the size of $\hat{CRD}$ in terms of β. (2)

7.2 Prove that $CD = \dfrac{2h.\cos\beta}{\sin\alpha}$. (5)

7.3 Calculate the height of the tower, rounded off to nearest unit, if

$CD = 5{,}4$ units, $\alpha = 51°$ and $\beta = 65°$. (2)

[9]

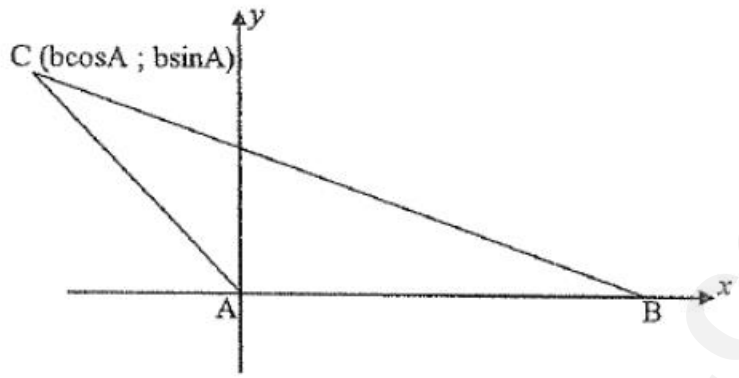
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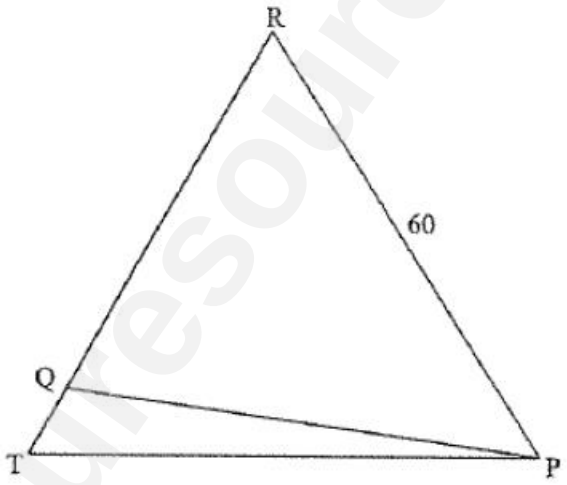
QUESTION 7

7.1 In the diagram below, ΔABC is drawn having A at the origin, B on the x-axis and the vertex C has the coordinates (bcosA ; bsinA).



Use the above diagram to prove that $a^2 = b^2 + c^2 - 2bc\cos A$ (4)

7.2 In the diagram below, ΔTPR is equilateral with PR = 60 units. Q is a point on RT such that RQ:QT = 5:1.



7.2.1 Show, by calculations, that PQ = 55,68 units. (4)

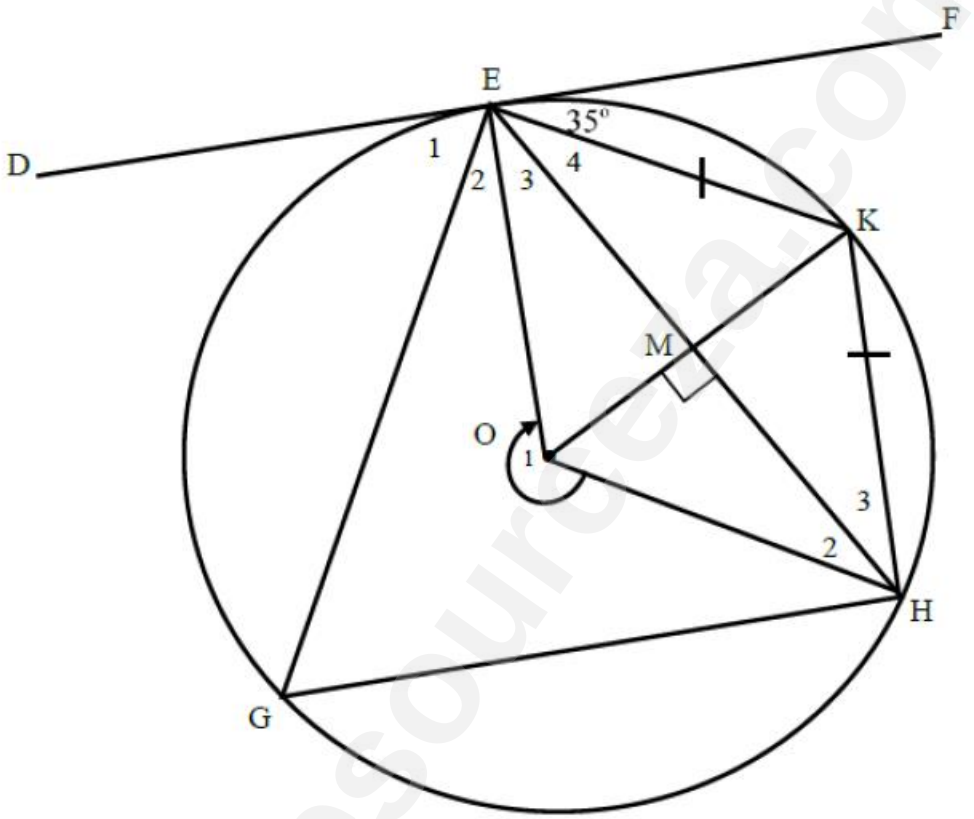
7.2.2 It is given that S is any point on the straight line PQ. Calculate the distance QS when S is the nearest to R. (4)
[12]

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QUESTION 8

DF is a tangent to the circle at E. EKHG is a cyclic quadrilateral. $K\hat{E}F = 35°$.
O is the centre of the circle. OK ⊥ EH and EK = HK.



8.1 Determine, with reasons, the size of each of the following:

8.1.1 $\hat{E}_4$ (3)

8.1.2 $E\hat{K}H$ (2)

8.1.3 $\hat{G}$ (2)

8.1.4 $\hat{O}_1$ (2)

8.2 It is further given that EH = 24 units. KM = 4 units and the radius of the circle EKHG is x. Determine the value of x. (4)

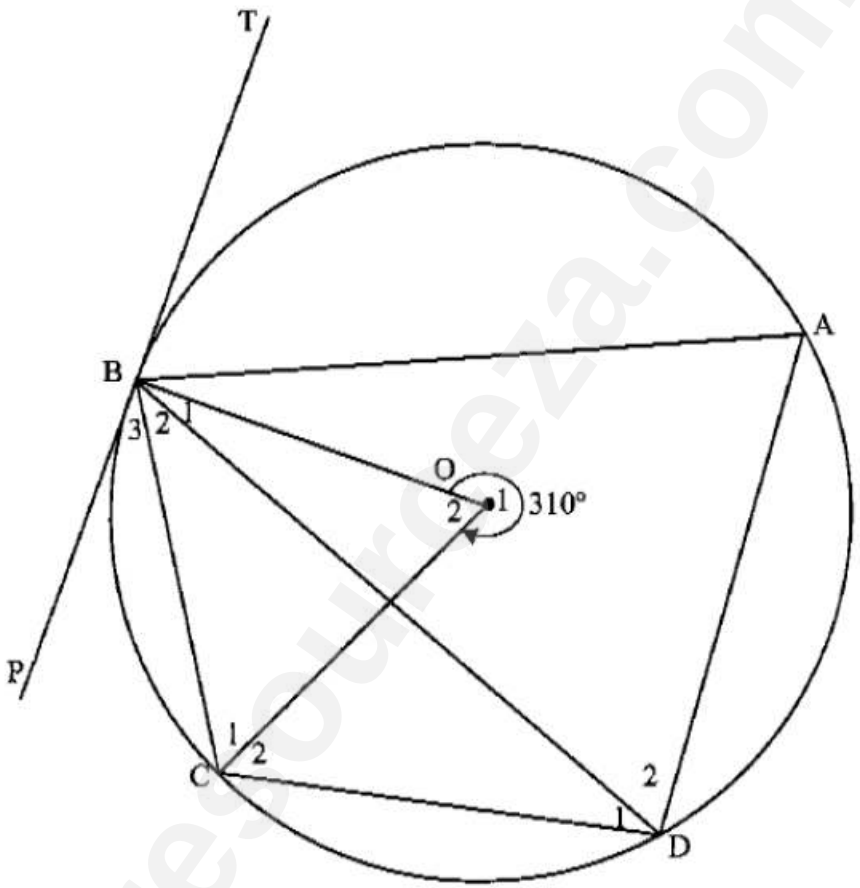
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QUESTION 8

In the diagram below, A, B, C and D are points on a circle having centre O.
PBT is a tangent to the circle at B.
Reflex $B\hat{O}C = \hat{O}_1 = 310°$ as shown in the diagram below.



Calculate, giving reasons, the size of:

8.1 $\hat{D}_1$ (3)

8.2 $\hat{B}_3$ (2)

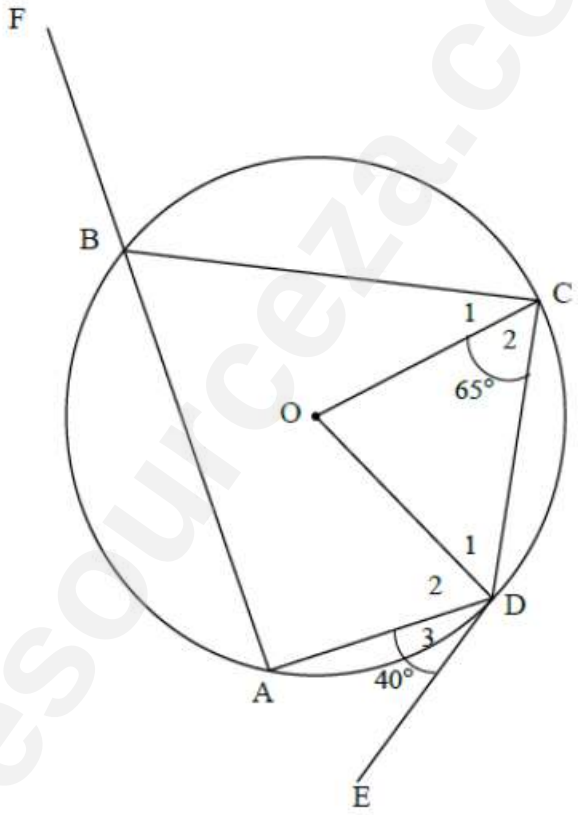
8.3 $\hat{B}_1$, if it is given that $\hat{A} = 60°$. (4)
[9]

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QUESTION 8

8.1 In the diagram, ABCD is a cyclic quadrilateral in the circle centered at O. ED is a tangent to the circle at D. Chord AB is produced to F. Radii OC and OD are drawn. $A\hat{D}E = 40°$ and $\hat{C}_2 = 65°$,



Determine, giving reasons, the size of each of the following angles:

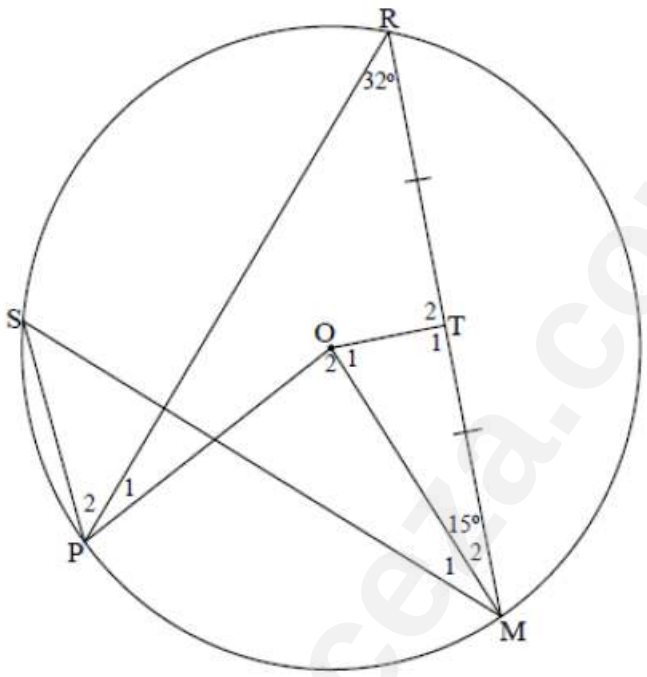
8.1.1 $\hat{D}_2$ (3)

8.1.2 $F\hat{B}C$ (4)

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8.2 In the diagram, O is the centre of the circle RMPS. OT bisects RM with T a point on RM. $\hat{PRM} = 32°$. SP, SM and radii OP and OM are drawn. $\hat{OMT} = 15°$.



Calculate, with reasons, the size of the angles:

8.2.1 $\hat{S}$ (2)

8.2.2 $\hat{O}_2$ (2)

8.2.3 $\hat{O}_1$ (3)

[14]

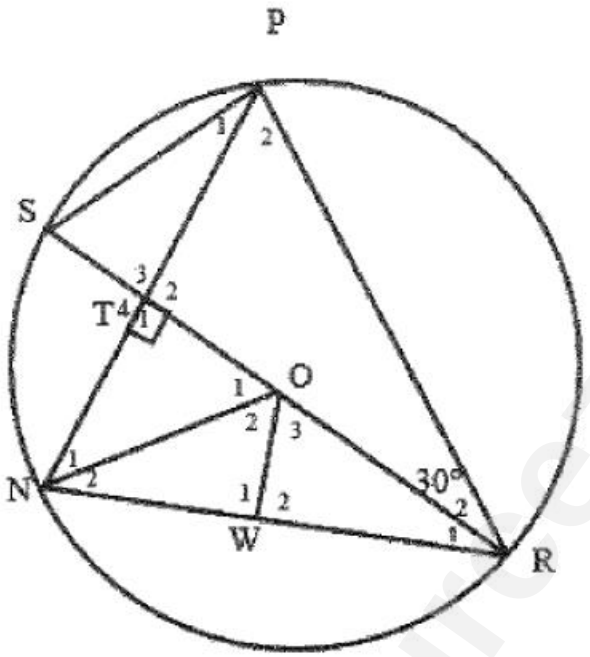
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QUESTION 8

In the diagram the vertices of Δ PNR lie on the circle with centre O. Diameter SR and chord NP intersect at T. OT ⊥ NP and $\hat{R}_2 = 30°$.



8.1 Determine, stating reasons, the size of:

8.1.1 $\hat{S}$ (3)

8.1.2 $\hat{R}_1$ (3)

8.1.3 $\hat{N}_1$ (2)

8.2 If it is further given that NW = WR, prove that TNWO is a cyclic quadrilateral. (4)

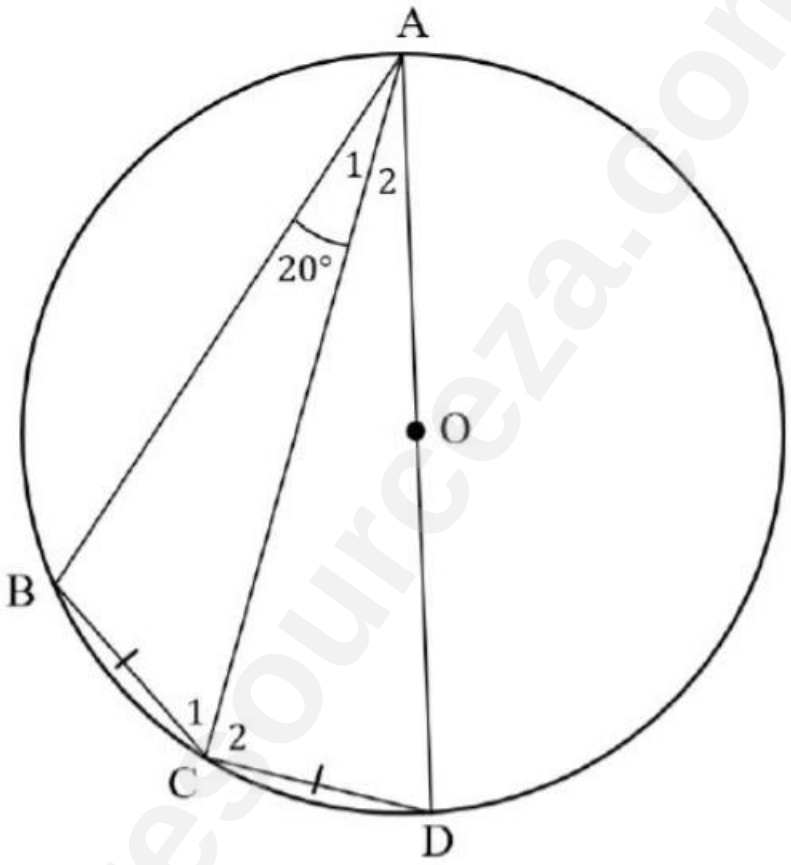
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QUESTION 8

8.1 In the diagram, AOD is a diameter of the circle centred at O.
BC = CD and $\hat{A}_1 = 20°$.



Determine, with reasons, the size of each of the following angles:

8.1.1 $\hat{A}_2$ (2)

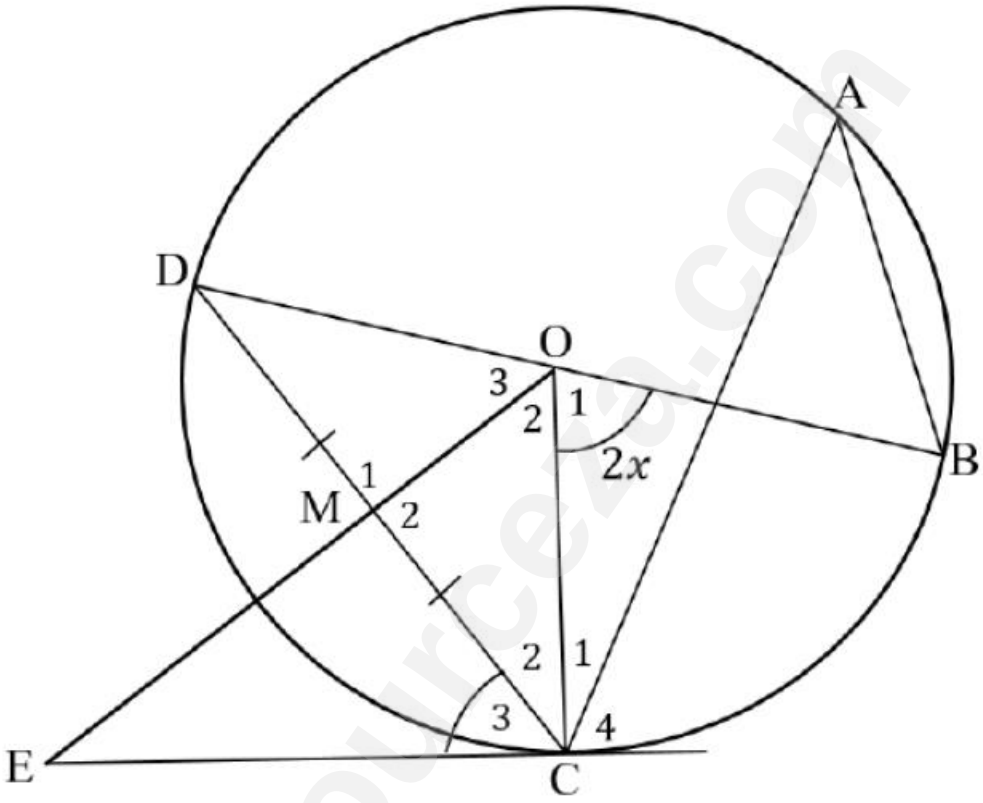
8.1.2 $\hat{C}_2$ (2)

8.1.3 $A\hat{B}C$ (3)

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8.2 In the diagram, O is the centre of the circle and EC is a tangent to the circle at C.
DM = MC and OME is a straight line.
Let $\hat{O}_1 = 2x$.



8.2.1 Give, with reasons, THREE angles equal to x. (6)

8.2.2 Prove that $\hat{O}_2 = 90° - x$ (3)

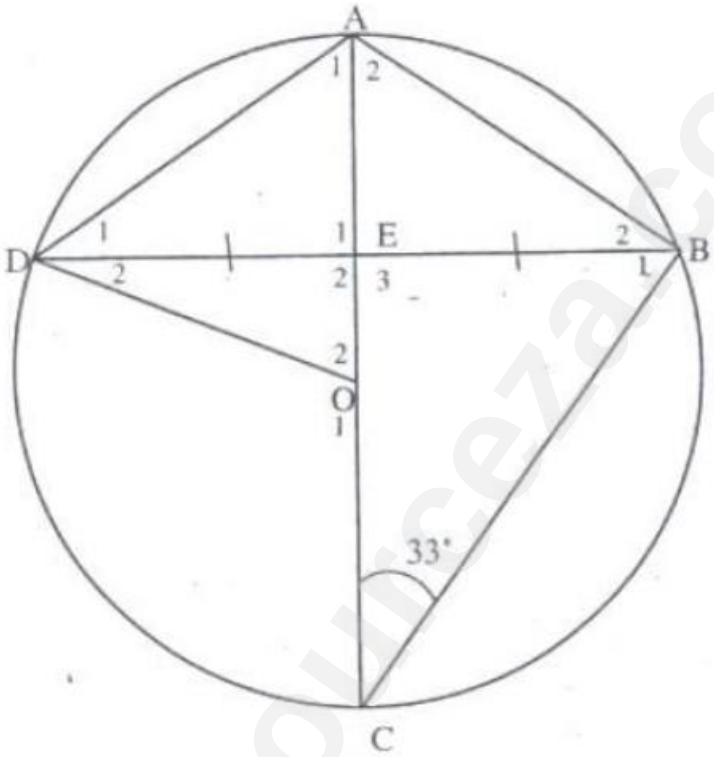
8.2.3 Prove that DOCE is a cyclic quadrilateral. (4)
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QUESTION 8

In the diagram, AC is the diameter of the circle with centre O. AC and chord DB intersect at E such that DE = EB. Chords AB, BC and AD and radius OD are drawn. $A\hat{C}B = 33°$.



Determine the size of:

8.1 $\hat{D}_1$ (2)

8.2 $\hat{A}_1$ (3)

8.3 $\hat{O}_1$ (2)

8.4 $\hat{D}_2$ (2)

8.5 $\hat{A}_2$ (3)

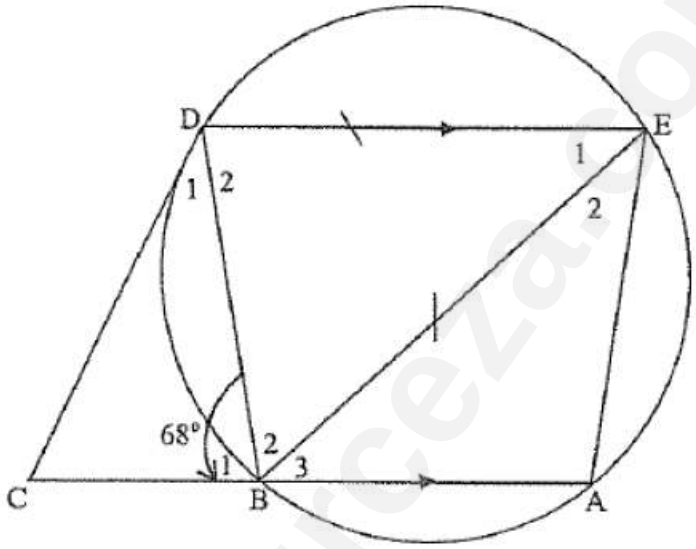
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QUESTION 8

In the diagram below, BAED is a cyclic quadrilateral with BA ∥ DE. BE = DE and $D\hat{B}C = 68°$. The tangent to the circle at D meets AB produced to C.



8.1 Calculate, with reasons, the size of:

8.1.1 $D\hat{E}A$ (2)

8.1.2 $\hat{A}$ (1)

8.1.3 $\hat{D}_2$ (2)

8.1.4 $\hat{B}_2$ (1)

8.1.5 $\hat{D}_1$ (3)

8.2 Prove that ΔBDC is isosceles. (2)

8.3 Prove that DE is a tangent to the circle that passes through the points C, B and D at D. (2)

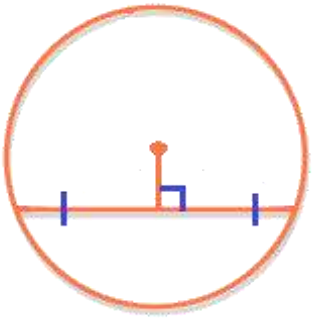
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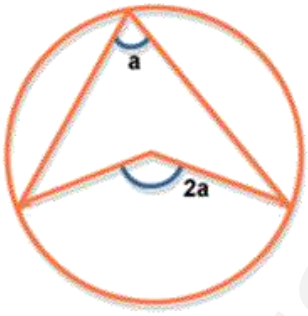
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MODULE 4 – EUCLIDEAN GEOMETRY: PROPORTIONAL DIVISION AND SIMILAR TRIANGLES

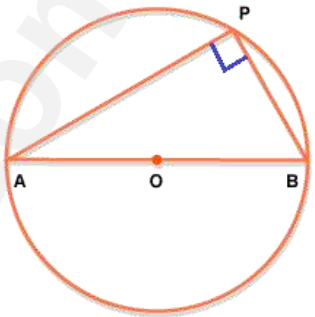
Unit 1: Revision of Grade 11 Circle Theorems



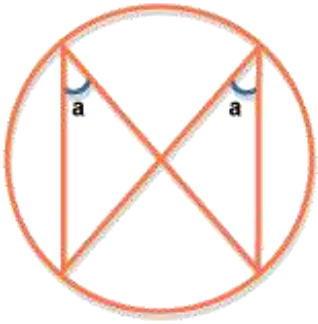
Theorem 1(a)
Midpt-chord
Theorem 1(b)
⊥ from centre to chord



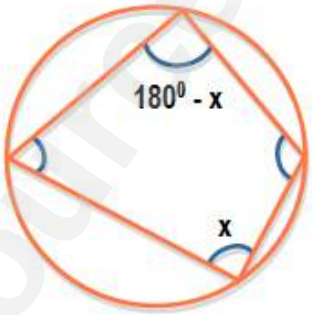
Theorem 2
< at centre = 2 x < @ circ.



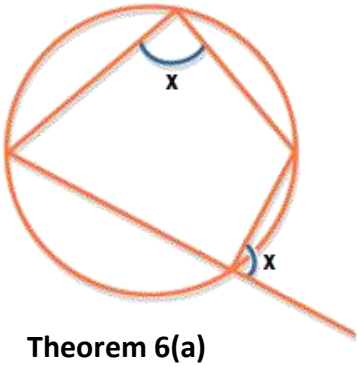
Theorem 3(a)
< in a semi-circle
Theorem 3(b)
Chord subtends < of 90°



Theorem 4(a)
<'s in the same segment
Theorem 4(b)
Line seg subtends equal <'s



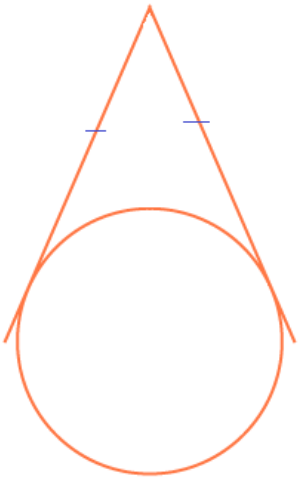
Theorem 5(a)
Opp <'s of cyclic quad
Theorem 5(b)
Opp <'s of quad suppl



Theorem 6(a)
Ext < of cyclic quad
Theorem 6(b)
Ext < of quad = int.



Theorem 7(a)
Tan ⊥ radius
Theorem 7(b)
Line ⊥ radius



Theorem 8
Tans from a comm. pt



Theorem 9(a)
Tan-chord
Theorem 9(b)
< betw. Line & chord = < in alt.

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Unit 2: General Problem-Solving Technique

Useful Guidelines for Solving Geometry Problems

1. Carefully read what is given while filling it in on your sketch. Ask yourself what the implication is of the given information.
2. Pay special attention to key words (underline them) and you must be able to recall all the theorems related to these key words (see table below).
3. Look additionally at the diagram for information that is not given in the question, e.g. look for cyclic quadrilaterals and angles in the same segment.
4. When a rider is preceded by a theorem which you had to prove, you can expect this theorem to play a significant role in the problem.
5. If you have to prove that 2 sides of a triangle are equal, prove that the opposite angles equal.
6. When trying to prove sides or angles equal, consider congruency as an option.
7. After completing a question, update the diagram, so that all additional information is available for the next question.

Useful Keywords

Key Word	Implication
1. Parallel Lines	Alt., corresp., co-int. <'s and proportionate sides.
2. Parallelograms	Look for the properties of them.
3. Centre of circle	Midpt theorem, ∠ at centre theorem and radii.
4. Diameter	∠ in a semi-circle.
5. Cyclic quadrilaterals	Opp ∠'s suppl, ext ∠ and ∠'s in the same segm.
6. Tangents	Tan-radius, tans from a comm pt, tan-chord.

Techniques for Proving Geometric Properties

To prove	Show
1. Parallel Lines	1. alternate ∠'s = 2. corresponding ∠'s = 3. co-interior ∠'s suppl. 4. Prove that the sides of a Δ are in proportion.
2. Congruency	1. SSS 2. S∠S 3. ∠∠S (or ∠S∠) 4. 90° HS
3. A quadrilateral is cyclic	1. Prove that the ∠'s in the same segm are =. 2. Prove that the opp ∠'s of the quad are suppl. 3. Prove that the ext ∠ of a quad = the int opp ∠
4. Tangents	1. Prove that the line is ⊥ to the radius. 2. Prove that the ∠ between chord and line is equal to the ∠ in the alternate segm.
5. Two Δ's similar	1. Prove Δ's equiangular. 2. Prove the sides of the Δ in proportion.

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4. ACCEPTABLE REASONS: EUCLIDEAN GEOMETRY

In order to have some kind of uniformity, the use of the following shortened versions of the theorem statements is encouraged.

4.1 ACCEPTABLE REASONS: EUCLIDEAN GEOMETRY (ENGLISH)

THEOREM STATEMENT	ACCEPTABLE REASON(S)
LINES	
The adjacent angles on a straight line are supplementary.	∠s on a str line
If the adjacent angles are supplementary, the outer arms of these angles form a straight line.	adj ∠s supp
The adjacent angles in a revolution add up to 360°.	∠s round a pt **OR** ∠s in a rev
Vertically opposite angles are equal.	vert opp ∠s =
If AB ∥ CD, then the alternate angles are equal.	alt ∠s; AB ∥ CD
If AB ∥ CD, then the corresponding angles are equal.	corresp ∠s; AB ∥ CD
If AB ∥ CD, then the co-interior angles are supplementary.	co-int ∠s; AB ∥ CD
If the alternate angles between two lines are equal, then the lines are parallel.	alt ∠s =
If the corresponding angles between two lines are equal, then the lines are parallel.	corresp ∠s =
If the cointerior angles between two lines are supplementary, then the lines are parallel.	coint ∠s supp
TRIANGLES	
The interior angles of a triangle are supplementary.	∠ sum in Δ **OR** sum of ∠s in Δ **OR** Int ∠s Δ
The exterior angle of a triangle is equal to the sum of the interior opposite angles.	ext ∠ of Δ
The angles opposite the equal sides in an isosceles triangle are equal.	∠s opp equal sides
The sides opposite the equal angles in an isosceles triangle are equal.	sides opp equal ∠s
In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.	Pythagoras **OR** Theorem of Pythagoras
If the square of the longest side in a triangle is equal to the sum of the squares of the other two sides then the triangle is right-angled.	Converse Pythagoras **OR** Converse Theorem of Pythagoras
If three sides of one triangle are respectively equal to three sides of another triangle, the triangles are congruent.	SSS
If two sides and an included angle of one triangle are respectively equal to two sides and an included angle of another triangle, the triangles are congruent.	SAS **OR** S∠S
If two angles and one side of one triangle are respectively equal to two angles and the corresponding side in another triangle, the triangles are congruent.	AAS **OR** ∠∠S
If in. two right angled triangles, the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of the other, the triangles are congruent	RHS **OR** 90°HS
The line segment joining the midpoints of two sides of a triangle is	Midpt Theorem

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Examination Guidelines

THEOREM STATEMENT	ACCEPTABLE REASON(S)
parallel to the third side and equal to half the length of the third side	
The line drawn from the midpoint of one side of a triangle, parallel to another side, bisects the third side.	line through midpt ∥ to 2nd side
A line drawn parallel to one side of a triangle divides the other two sides proportionally.	line ∥ one side of Δ **OR** prop theorem; name ∥ lines
If a line divides two sides of a triangle in the same proportion, then the line is parallel to the third side.	line divides two sides of Δ in prop
If two triangles are equiangular, then the corresponding sides are in proportion (and consequently the triangles are similar).	∥ Δs **OR** equiangular Δs
If the corresponding sides of two triangles are proportional, then the triangles are equiangular (and consequently the triangles are similar).	Sides of Δ in prop
If triangles (or parallelograms) are on the same base (or on bases of equal length) and between the same parallel lines, then the triangles (or parallelograms) have equal areas.	same base; same height **OR** equal bases; equal height
CIRCLES	
The tangent to a circle is perpendicular to the radius/diameter of the circle at the point of contact.	tan ⊥ radius tan ⊥ diameter
If a line is drawn perpendicular to a radius/diameter at the point where the radius/diameter meets the circle, then the line is a tangent to the circle.	line ⊥ radius **OR** converse tan ⊥ radius **OR** converse tan ⊥ diameter
The line drawn from the centre of a circle to the midpoint of a chord is perpendicular to the chord.	line from centre to midpt of chord
The line drawn from the centre of a circle perpendicular to a chord bisects the chord.	line from centre ⊥ to chord
The perpendicular bisector of a chord passes through the centre of the circle;	perp bisector of chord
The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre)	∠ at centre = 2 ×∠ at circumference
The angle subtended by the diameter at the circumference of the circle is 90°.	∠s in semi circle **OR** diameter subtends right angle **OR** $\angle\frac{1}{2}\odot$
If the angle subtended by a chord at the circumference of the circle is 90°, then the chord is a diameter.	chord subtends 90° **OR** converse ∠s in semi circle
Angles subtended by a chord of the circle, on the same side of the chord, are equal	∠s in the same seg
If a line segment joining two points subtends equal angles at two points on the same side of the line segment, then the four points are concyclic.	line subtends equal ∠s **OR** converse ∠s in the same seg
Equal chords subtend equal angles at the circumference of the circle.	equal chords; equal ∠s
Equal chords subtend equal angles at the centre of the circle.	equal chords; equal ∠s
Equal chords in equal circles subtend equal angles at the circumference of the circles.	equal circles; equal chords; equal ∠s

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Examination Guidelines

THEOREM STATEMENT	ACCEPTABLE REASON(S)
Equal chords in equal circles subtend equal angles at the centre of the circles.	equal circles; equal chords; equal ∠s
The opposite angles of a cyclic quadrilateral are supplementary	opp ∠s of cyclic quad
If the opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic.	opp ∠s quad sup **OR** converse opp ∠s of cyclic quad
The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.	ext ∠ of cyclic quad
If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic.	ext ∠ = int opp ∠ **OR** converse ext ∠ of cyclic quad
Two tangents drawn to a circle from the same point outside the circle are equal in length	Tans from common ptOR Tans from same pt
The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.	tan chord theorem
If a line is drawn through the end-point of a chord, making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle.	converse tan chord theorem **OR** ∠ between line and chord
QUADRILATERALS	
The interior angles of a quadrilateral add up to 360°.	sum of ∠s in quad
The opposite sides of a parallelogram are parallel.	opp sides of ‖m
If the opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram.	opp sides of quad are ‖
The opposite sides of a parallelogram are equal in length.	opp sides of ‖m
If the opposite sides of a quadrilateral are equal , then the quadrilateral is a parallelogram.	opp sides of quad are = **OR** converse opp sides of a parm
The opposite angles of a parallelogram are equal.	opp ∠s of ‖m
If the opposite angles of a quadrilateral are equal then the quadrilateral is a parallelogram.	opp ∠s of quad are = **OR** converse opp angles of a parm
The diagonals of a parallelogram bisect each other.	diag of ‖m
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.	diags of quad bisect each other **OR** converse diags of a parm
If one pair of opposite sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram.	pair of opp sides = and ‖
The diagonals of a parallelogram bisect its area.	diag bisect area of ‖m
The diagonals of a rhombus bisect at right angles.	diags of rhombus
The diagonals of a rhombus bisect the interior angles.	diags of rhombus
All four sides of a rhombus are equal in length.	sides of rhombus
All four sides of a square are equal in length.	sides of square
The diagonals of a rectangle are equal in length.	diags of rect
The diagonals of a kite intersect at right-angles.	diags of kite
A diagonal of a kite bisects the other diagonal.	diag of kite
A diagonal of a kite bisects the opposite angles	diag of kite

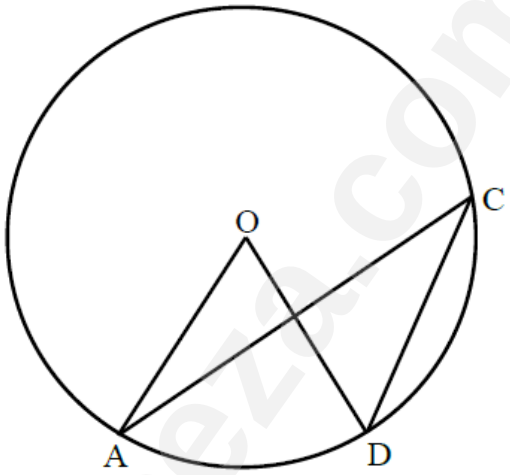
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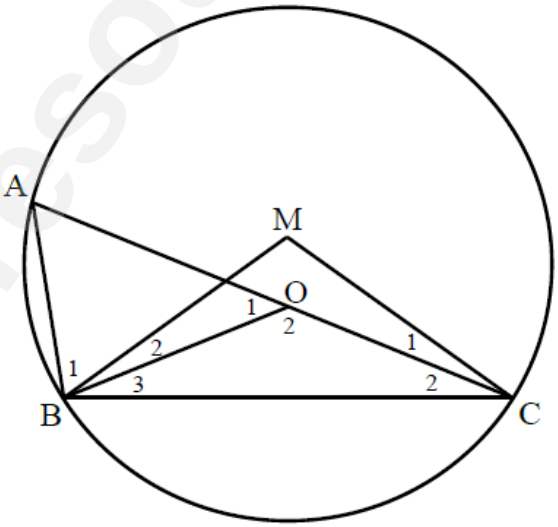
QUESTION 9

9.1 O is the centre of the circle and A, D and C are on the circumference of the circle. Use the diagram to prove, USING Euclidean geometry methods, the theorem which states that $\angle AOD = 2\angle ACD$.



(5)

9.2 In the figure, M is the centre of the circle through A, B, and C. OB and AC bisect $M\hat{B}C$ and M$\hat{C}$B respectively with O on AC. AB is joined.



Let $\hat{B}_2 = x$

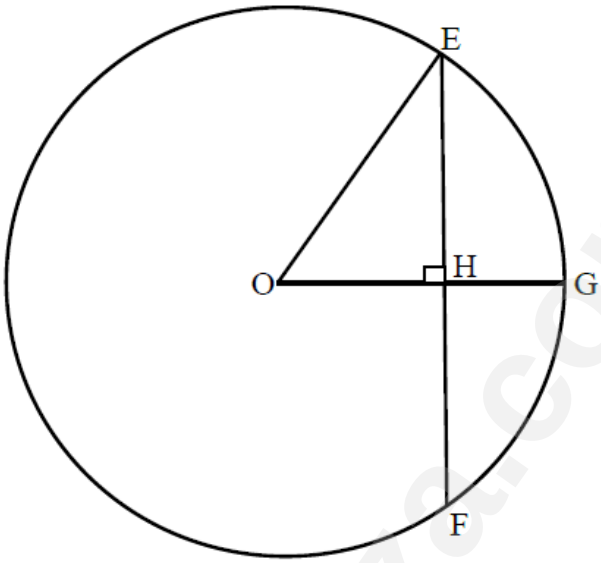
9.2.1 Determine the size of $\hat{O}_2$ in terms of x. (4)

9.2.2 Prove that AO is a diameter of circle ABO. (5)

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9.3 In the diagram below, EF is the chord of a circle with centre O. OG is perpendicular to EF.



If OE = x cm and HG = 2 cm, determine with reasons the size of EF (in its simplest form) in terms of x. (4)

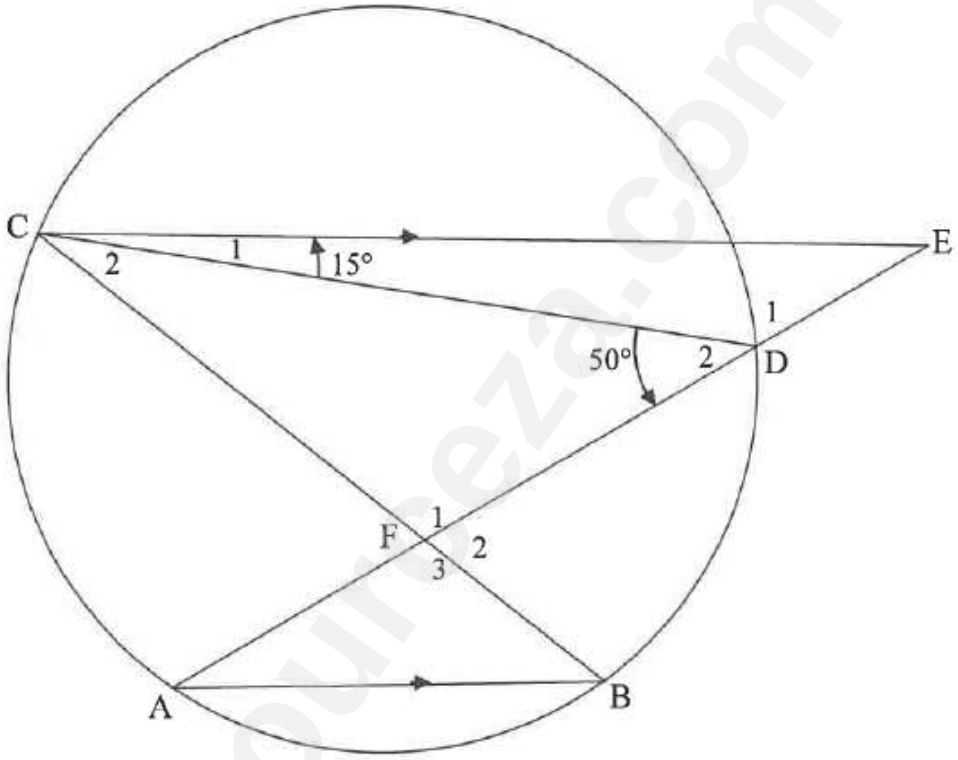
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QUESTION 8

In the diagram, points A, B, D and C lie on a circle. CE || AB with E on AD produced. Chords CB and AD intersect at F. $\hat{D}_2 = 50°$ and $\hat{C}_1 = 15°$.



8.1 Calculate, with reasons, the size of:

8.1.1 $\hat{A}$ (3)

8.1.2 $\hat{C}_2$ (2)

8.2 Prove, with a reason, that CF is a tangent to the circle passing through points C, D and E. (2)

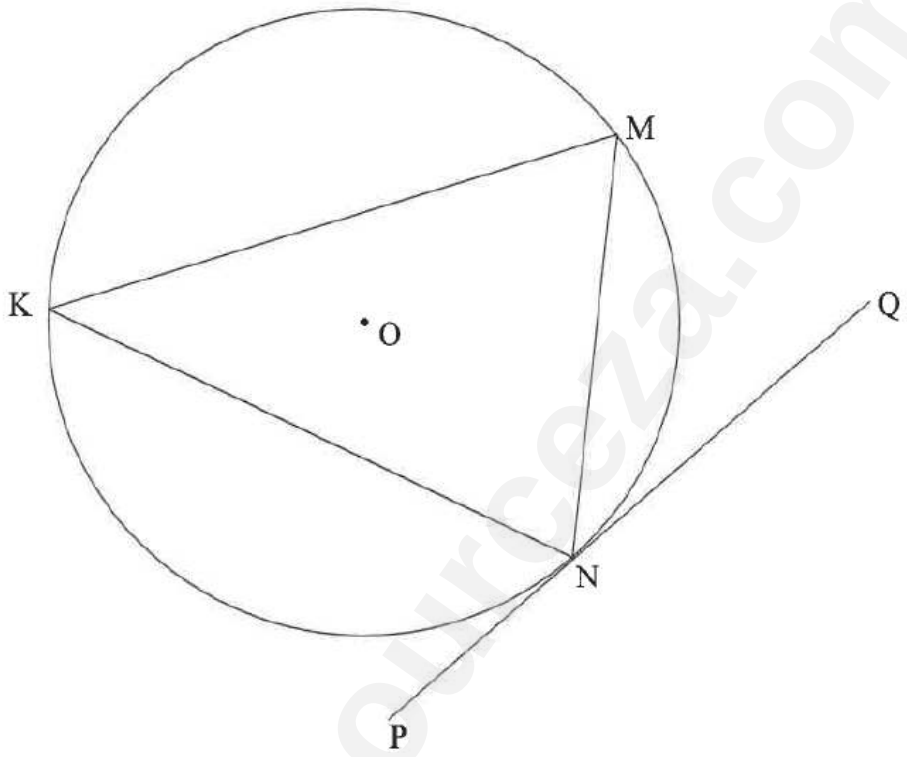
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QUESTION 11

11.1 In the diagram, chords KM, MN and KN are drawn in the circle with centre O. PNQ is the tangent to the circle at N.



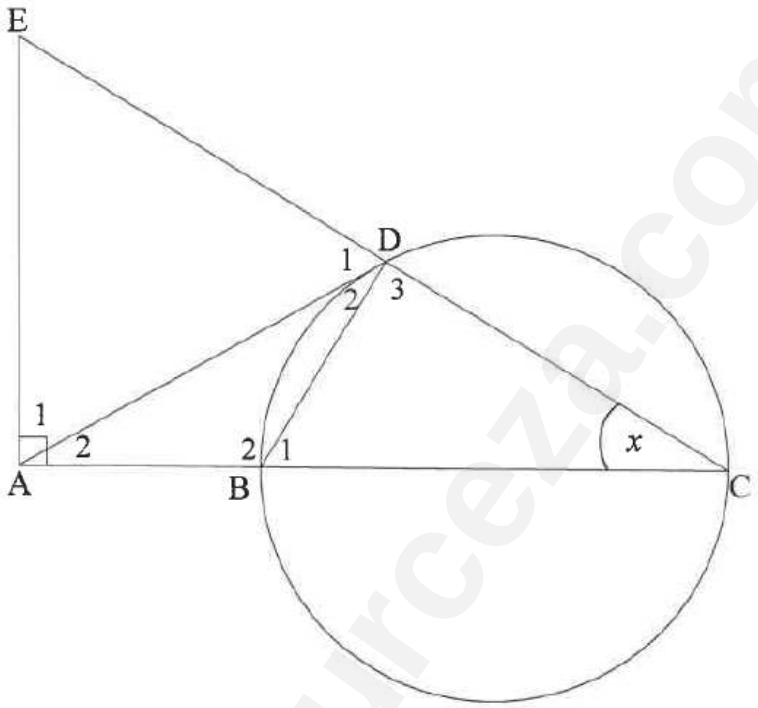
Prove the theorem which states that $M\hat{N}Q = \hat{K}$. (5)

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11.2 In the diagram, BC is a diameter of the circle. The tangent at point D on the circle meets CB produced at A. CD is produced to E such that EA ⊥ AC. BD is drawn.

Let $\hat{C} = x$.



11.2.1 Give a reason why:

(a) $\hat{D}_3 = 90°$ (1)

(b) ABDE is a cyclic quadrilateral (1)

(c) $\hat{D}_2 = x$ (1)

11.2.2 Prove that:

(a) AD = AE (3)

(b) $\Delta ADB \,|||\, \Delta ACD$ (3)

11.2.3 It is further given that BC = 2AB = 2r.

(a) Prove that $AD^2 = 3r^2$ (2)

(b) Hence, prove that ΔADE is equilateral. (4)

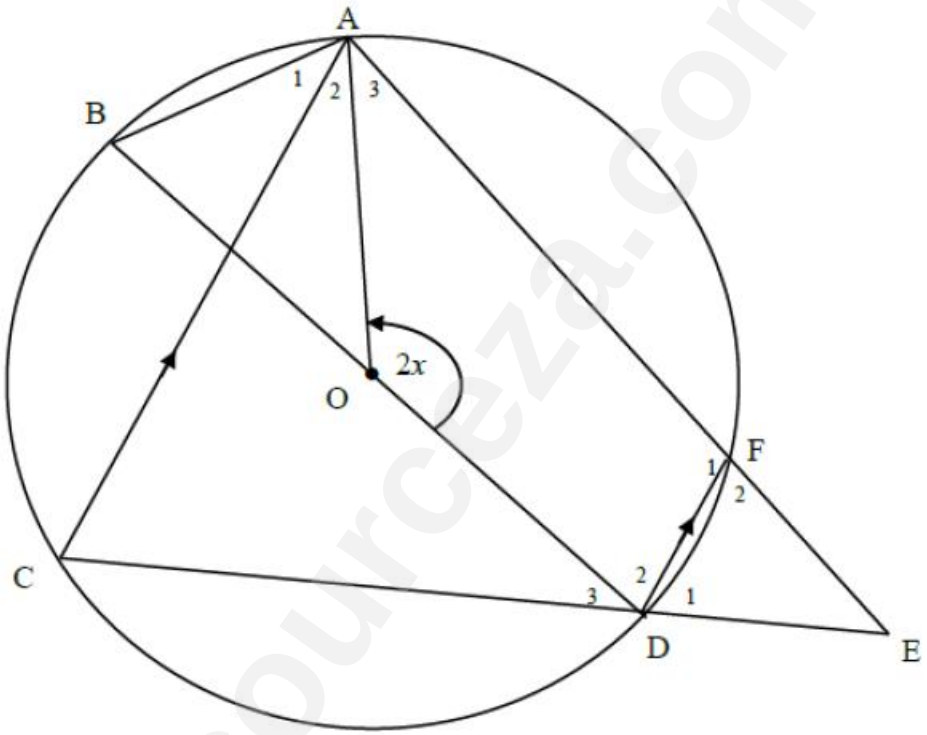
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QUESTION 9

9.1 A circle with centre O is given below. Lines CD and AF are produced to E. $A\hat{O}D = 2x$ and BD is the diameter. AC||FD.



9.1.1 Determine, with reasons, four other angles that are each equal to x. (6)

9.1.2 Express $\hat{E}$ in terms of x. (2)

9.1.3 Prove that AODE is a cyclic quadrilateral. (2)

9.2 It is further given that ED : DC = 8 : 12 and FE = 10. Calculate the length of AF. (3)

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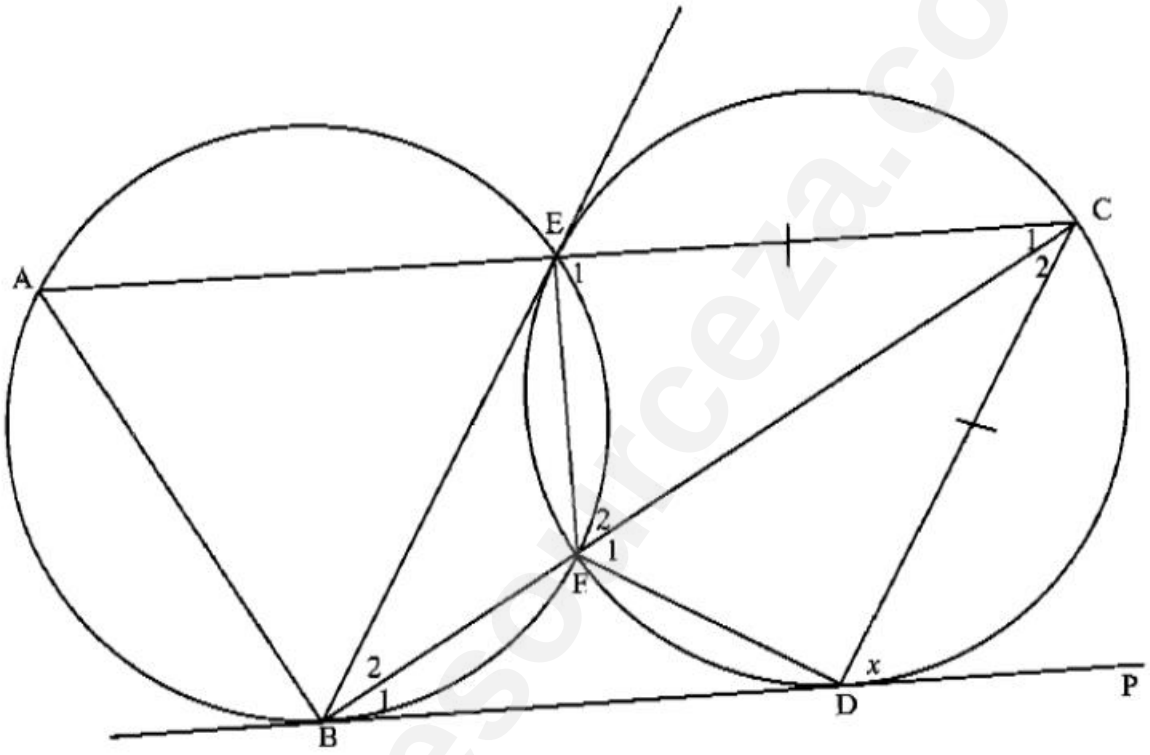
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QUESTION 9

9.1 Complete the statement so that it is TRUE.
Angles subtended by a chord of a circle, on the same side of a chord, are … (1)

9.2 In the diagram below, ABFE and EFDC are cyclic quadrilaterals in two equal circles that intersect at E and F. BFC and AEC are straight lines. BD is a common tangent to the circles at B and D respectively. EC = CD.
Let $C\hat{D}P = x$



Prove, giving reasons, that:

9.2.1 $\hat{F}_1 = \hat{F}_2$. (3)

9.2.2 ABDC is a cyclic quadrilateral. (3)

9.2.3 BE || CD. (2)

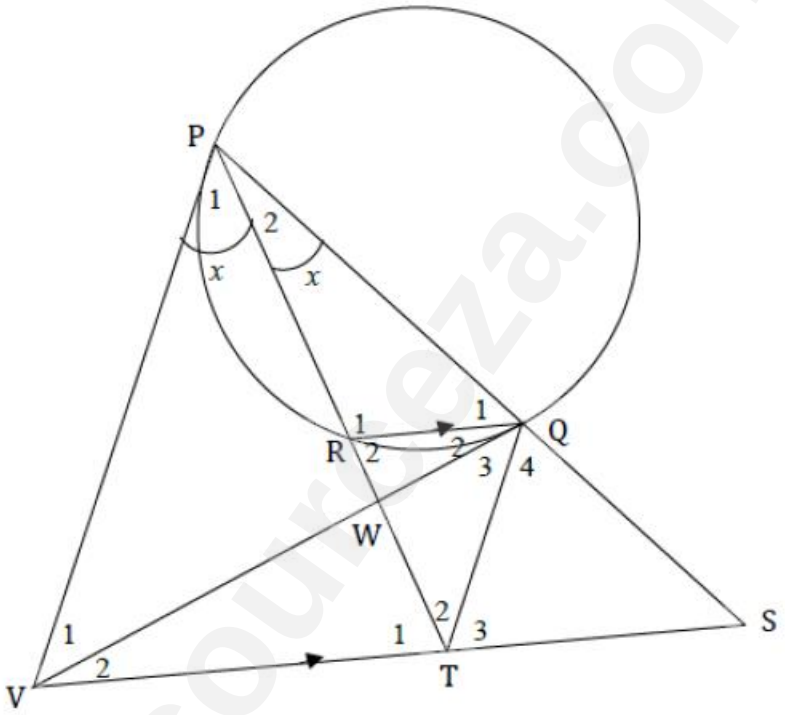
9.2.4 FC is a diameter of circle FDCE if it is given that EBDC is a rhombus. (5)
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QUESTION 9

In the diagram, PV and VQ are tangents to the circle at P and Q. PQ is produced to S and chord PR is produced to T such that VTS || RQ. VQ and RT intersect at W. $\hat{P}_1 = \hat{P}_2 = x$.



Prove that:

9.1 $\hat{S} = x$ (4)

9.2 PQTV is a cyclic quadrilateral (5)

9.3 TQ is a tangent to the circle passing through Q, W and P (3)

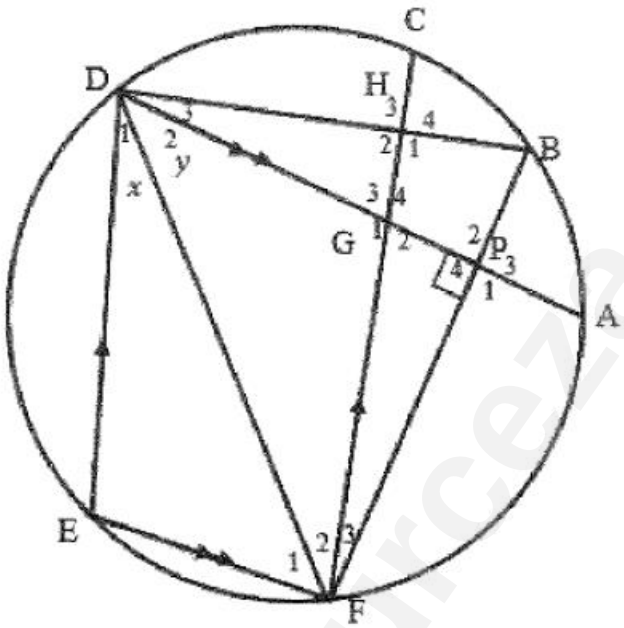
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QUESTION 9

9.1 A, B, C, D, E and F are points on the circumference of the circle. ED ∥ FC and AD ∥ FE. DA ⊥ FB. FC intersects DA at G and DB at H.



Let $E\hat{D}F = x$ and $A\hat{D}F = y$.

Prove that:

9.1.1 $\hat{B} = x + y$ (4)

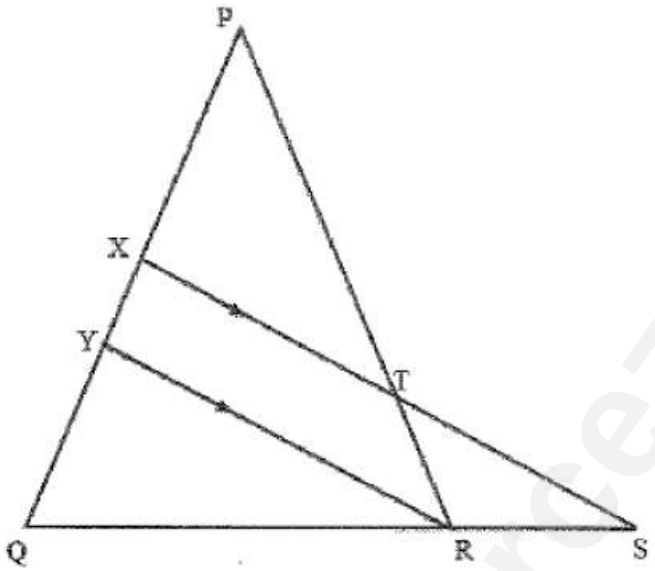
9.1.2 $\hat{H}_1 = 90°$ (6)

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9.2 In the diagram QR of Δ PQR is produced such that XS ∥ YR.

$\frac{PX}{XQ} = \frac{2}{5}$ and $\frac{PT}{TR} = \frac{3}{2}$.



Calculate, giving reasons, the value of $\frac{QR}{RS}$. (6)

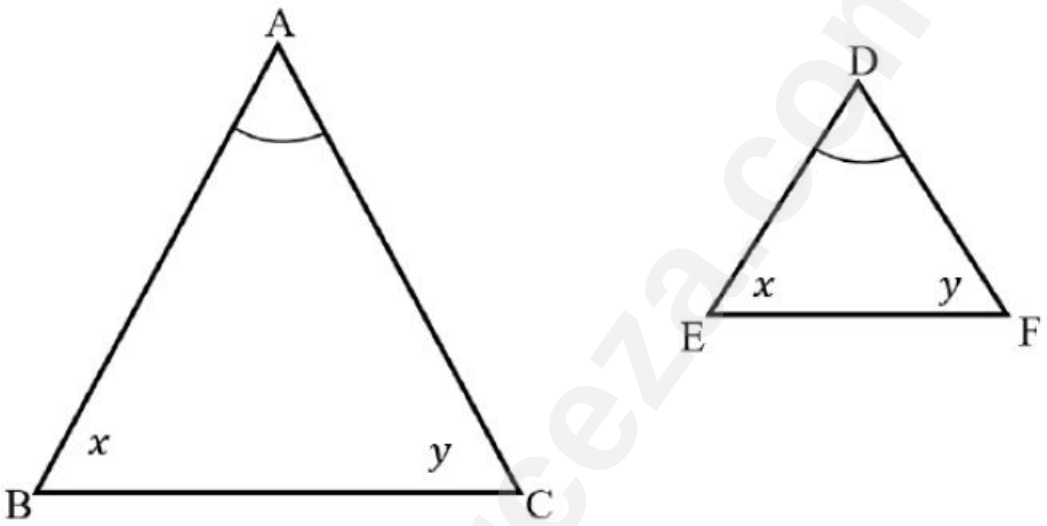
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QUESTION 9

9.1 In the diagram, ΔABC and ΔDEF are drawn such that $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.



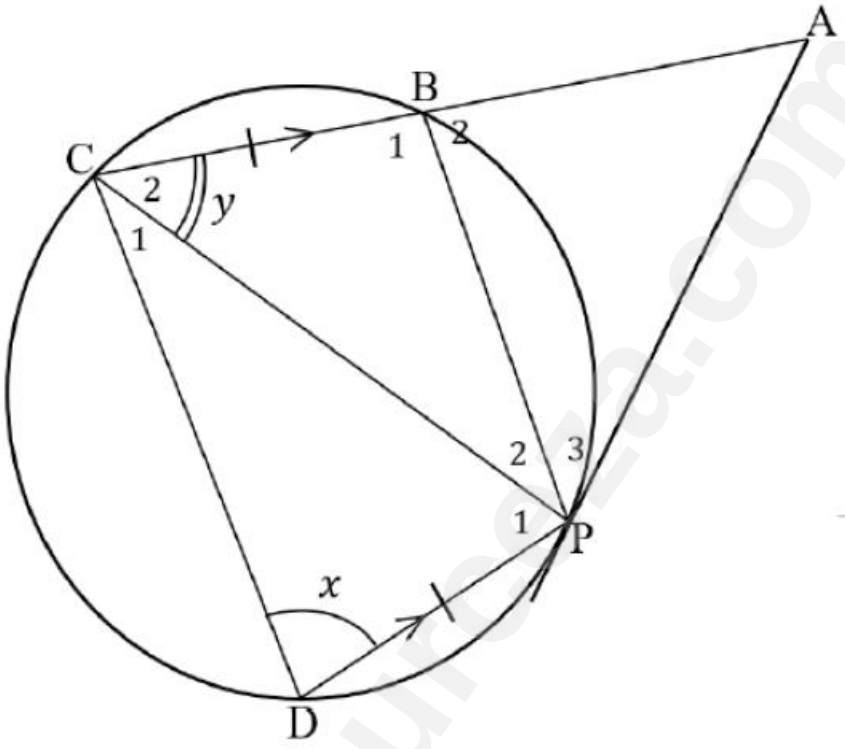
Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, that is $\frac{DE}{AB} = \frac{DF}{AC}$

(6)

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9.2 AP is a tangent to the circle at P. CB ∥ DP and CB=DP. CBA is a straight line.
Let $\hat{D} = x$ and $\hat{C}_2 = y$.



Prove, with reasons that:

9.2.1 ΔAPC ||| Δ ABP (4)

9.2.2 $AP^2 = AB \cdot AC$ (1)

9.2.3 ΔAPC ||| Δ CDP (4)

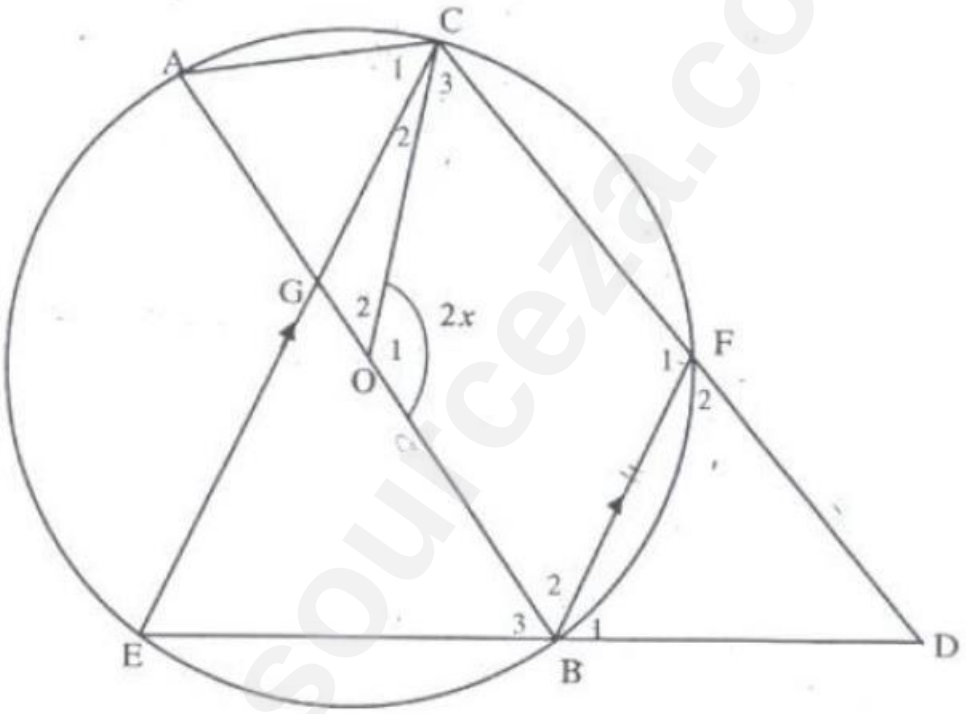
9.2.4 $AP^2 + PC^2 = AC^2$ (4)
[19]

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QUESTION 9

In the diagram below, AB passes through the centre O of the circle. Chords CF and EB are produced to meet at D. EC ∥ BF . Chord AC and radius OC are drawn. Let $\hat{O}_1 = 2x$.



9.1 Determine, in terms of x, the size of $\hat{F}_2$. (3)

9.2 Prove that DF = BD. (4)

9.3 Show that $\hat{C}_1 = \hat{C}_3$. (4)

9.4 If it is further given that CG = FB, FD = 1 and BG = 2, determine the value of $\frac{BD}{ED}$. (5)

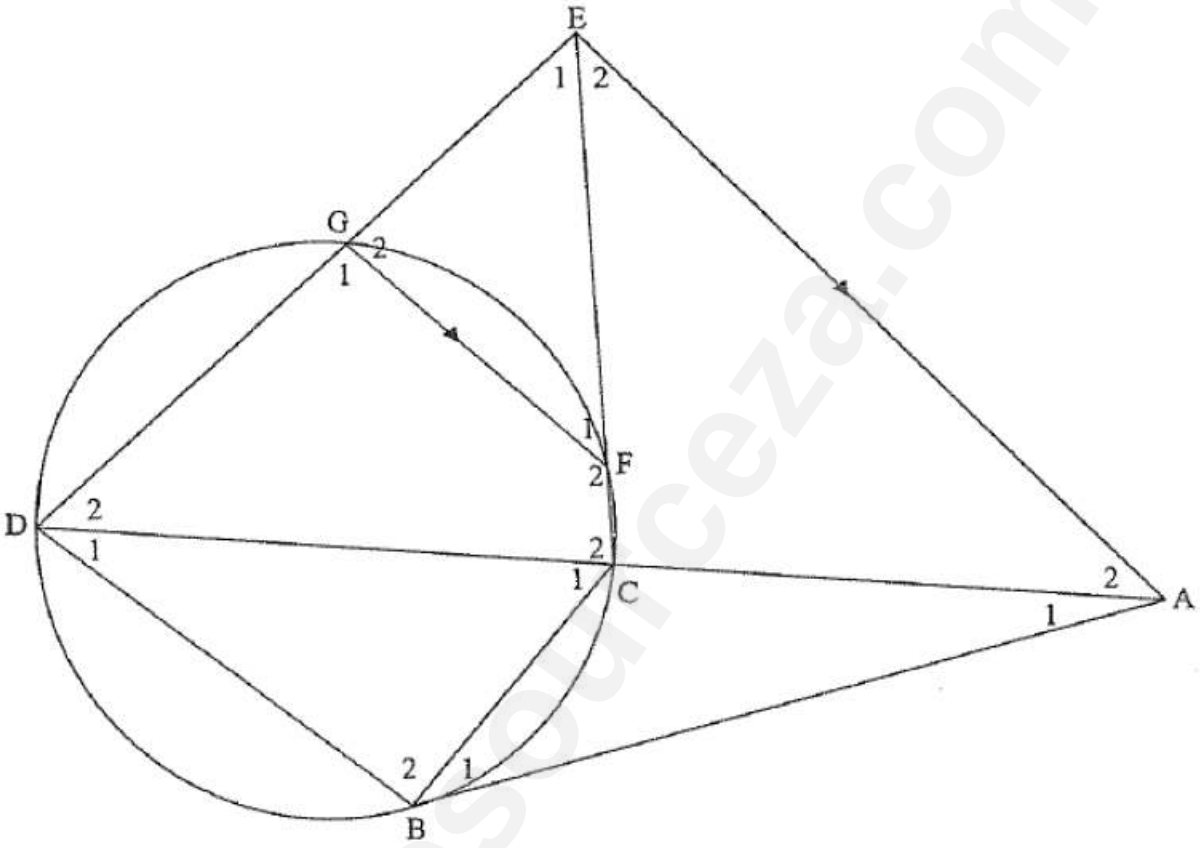
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QUESTION 9

In the diagram, DGFC is a cyclic quadrilateral and AB is a tangent to the circle at B. Chords DB and BC are drawn. DG produced and CF produced meet in E and DC is produced to A. EA || GF



9.1 Give a reason why $\hat{B}_1 = \hat{D}_1$. (1)

9.2 Prove $\Delta ABC ||| \Delta ADB$. (3)

9.3 Prove $\hat{E}_2 = \hat{D}_2$ (4)

9.4 Prove $AE = \sqrt{AD \times AC}$. (5)

9.5 Hence, show that AE = AB. (3)
[16]

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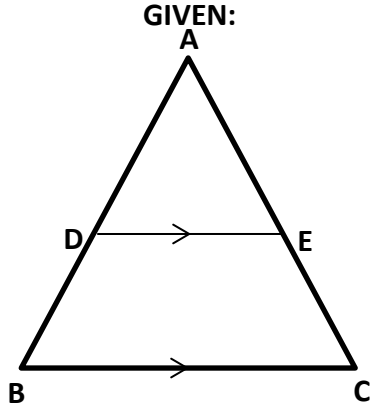
Unit 3: Proportional Division Theorem

Theorem 1 (a) ***

[proof required for examination purposes]

A line drawn parallel to one side of a triangle divides the other two sides proportionally.
[Line // 1 side of Δ]

GIVEN:



IMPLICATION:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Theorem 1 (b) ***

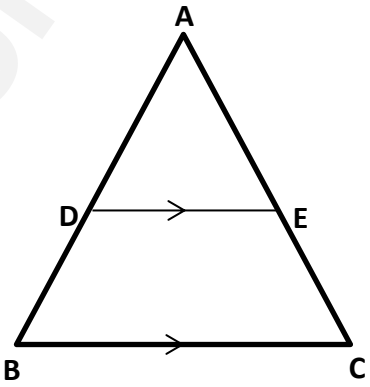
[proof required for examination purposes]

If a line divides 2 sides of a triangle in the same proportion, then the line is parallel to the 3rd side.
[Line divides 2 sides of Δ prop.]

GIVEN:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

IMPLICATION:



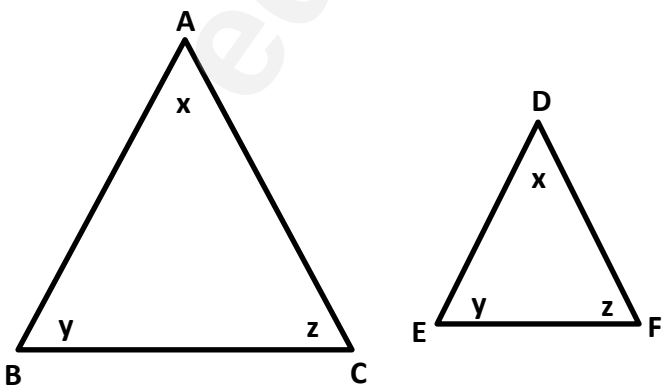
Special Case of the Proportional Division Theorem (Revision of Grade 10)

- A line drawn from the mid-point of one side of a triangle parallel to a 2nd side bisects the 3rd side.
- A line segment joining the mid-points of 2 sides of a triangle is parallel to the 3rd side and equal to one half of the 3rd side. **[Mid-pt Th]**

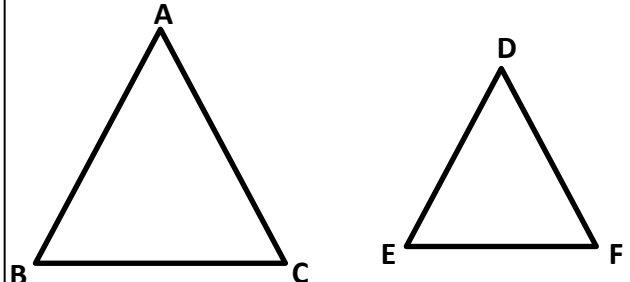
Unit 4: Similar Polygons and Triangles

Similar (|||) Triangles
2 triangles are similar if they are either equiangular OR if their corresponding sides are in proportion.
This is different to similar polygons where both conditions have to be met.

Theorem 2 (a) ***
[proof required for examination purposes]
If 2 triangles are equiangular, then the triangles are similar. **[Equiangular Δs or <<<]**



Theorem 2 (b) ***
[proof required for examination purposes]
If the corresponding sides of 2 triangles are proportional then the triangles are similar.
[sides of Δs prop.]



GIVEN:

If $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

IMPLICATION:

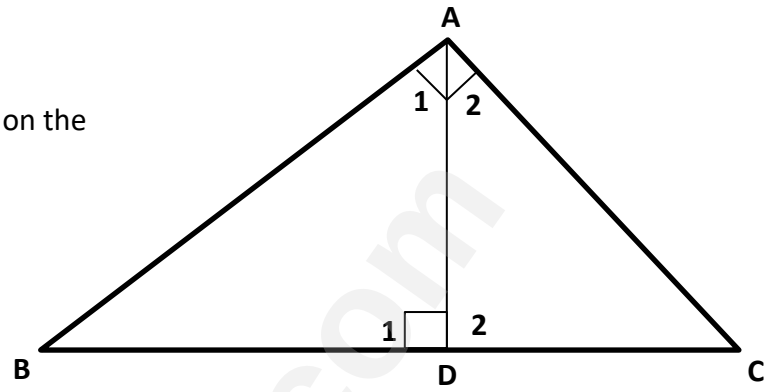
$\therefore$ Δ ABC ||| Δ DEF11

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Unit 5: Theorem of Pythagoras

Theorem 3 (a) ***
[proof required for examination purposes]
In any right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.
[Pythag.]
In the figure above, $\Delta ABD \,|||\, \Delta CAD \,|||\, \Delta CBA$**.**



$$\frac{AB}{BC} = \frac{BD}{AB} \quad \therefore AB^2 = BD \times BC \ldots (1)$$

$$\frac{AC}{BC} = \frac{CD}{AC} \quad \therefore AC^2 = BC \times CD \ldots (2)$$

$$\text{From (1) and (2): } AB^2 + AC^2 = BD \times BC + CD \times CB$$
$$= BC(BD + DC)$$
$$= BC \times BC$$
$$= BC^2$$

$$\therefore BC^2 = AB^2 + AC^2$$

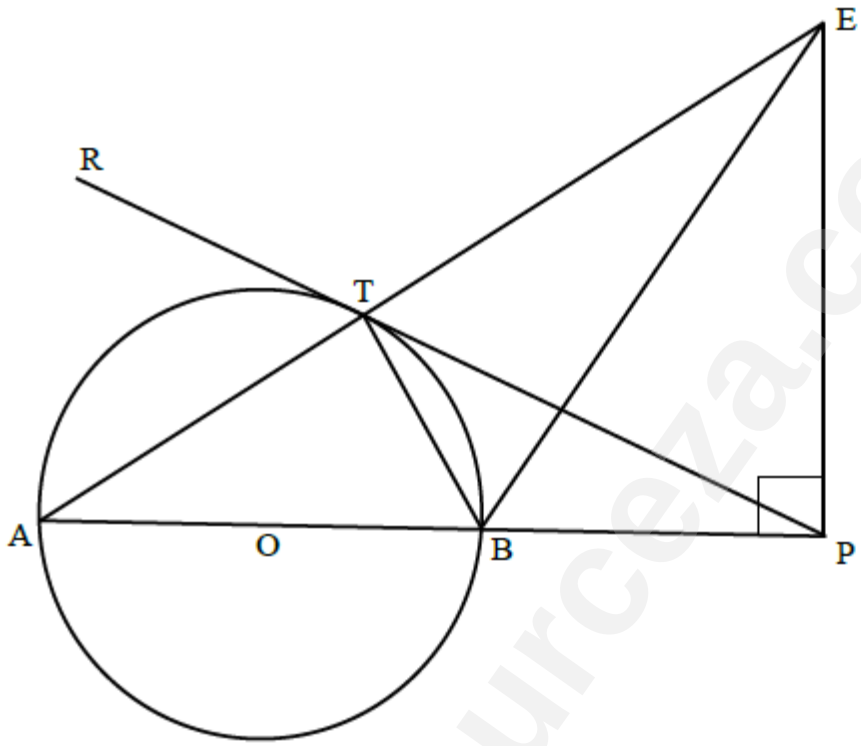
Theorem 3 (b)
[proof NOT required for examination purposes]
If the square on 1 side of a triangle is equal to the sum of the squares on the other 2 sides, then the angle contained by these 2 sides is a right angle.
[Pythag. converse]

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QUESTION 10

In the diagram below, O is the centre with A, B and T on the circumference, BP = OB = AO, PTR is a tangent and EP ⊥ AP.



Prove that:

10.1 TEPB is a cyclic quad (3)

10.2 ΔATB /// ΔAPE (3)

10.3 TP = PE (6)

10.4 ΔATB /// ΔEPB (5)

10.5 $2BP^2 = BE.TB$ (4)

[21]

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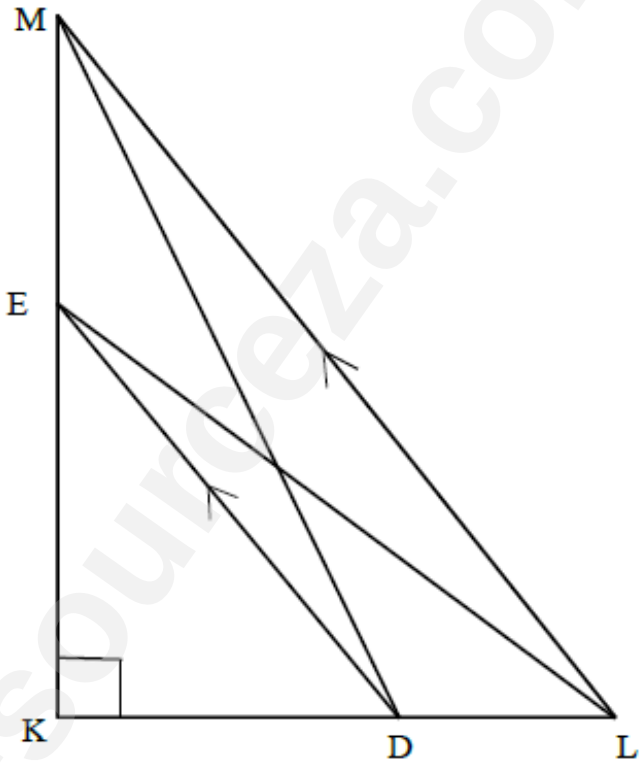
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QUESTION 11

11.1 Complete the following statement:

A line parallel to one side of a triangle divides … (1)

11.2 In Δ KLM, $\hat{K} = 90^{\circ}$ and D is a point on KL so that KD:DL = 2:1 and DE // LM with E on KM.



11.2.1 If DL = x and EM = y, express LM^2 in terms of x and y. (4)

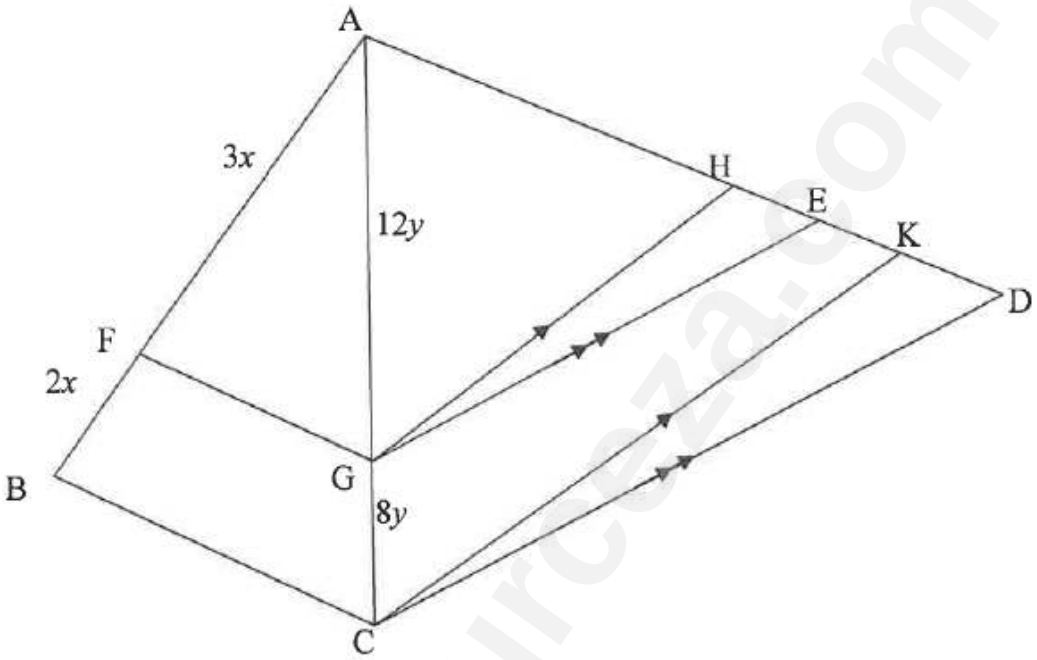
11.2.2 Show that $DM^2 + LE^2 = \frac{13}{9}LM^2$. (4)
[9]

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QUESTION 9

In the diagram, ΔABC and ΔACD are drawn. F and G are points on sides AB and AC respectively such that $AF = 3x$, $FB = 2x$, $AG = 12y$ and $GC = 8y$. H, E and K are points on side AD such that $GH \parallel CK$ and $GE \parallel CD$.



9.1 Prove that:

9.1.1 $FG \parallel BC$ (2)

9.1.2 $\frac{AH}{HK} = \frac{AE}{ED}$ (3)

9.2 If it is further given that $AH = 15$ and $ED = 12$, calculate the length of EK. (5)

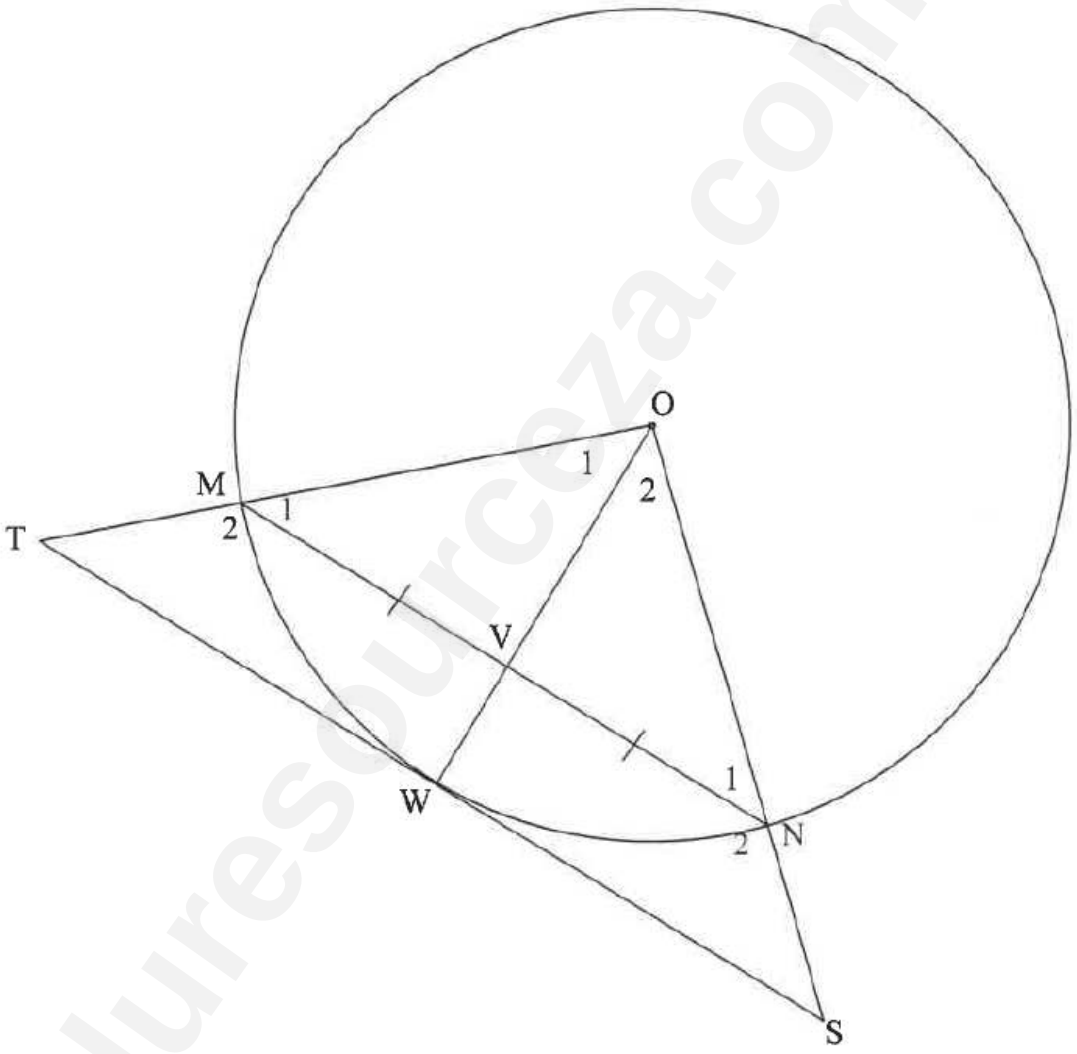
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QUESTION 10

In the diagram, W is a point on the circle with centre O. V is a point on OW. Chord MN is drawn such that MV = VN. The tangent at W meets OM produced at T and ON produced at S.



10.1 Give a reason why OV ⊥ MN. (1)

10.2 Prove that:

10.2.1 MN ∥ TS (2)

10.2.2 TMNS is a cyclic quadrilateral (4)

10.2.3 OS . MN = 2ON . WS (5)

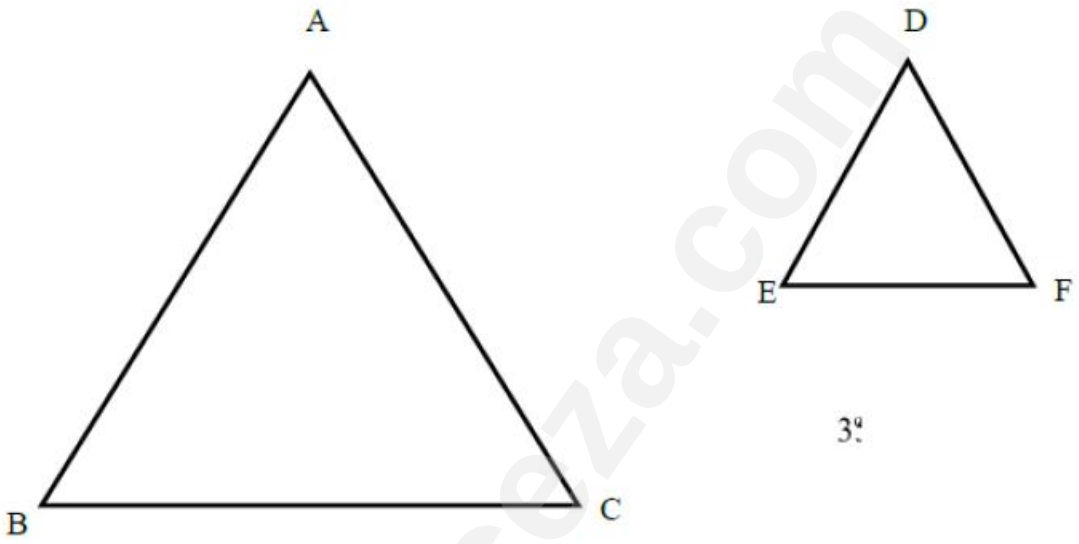
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QUESTION 10

10.1 In the diagram below, ΔABC and ΔDEF are drawn with $\hat{A} = \hat{D}; \hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.



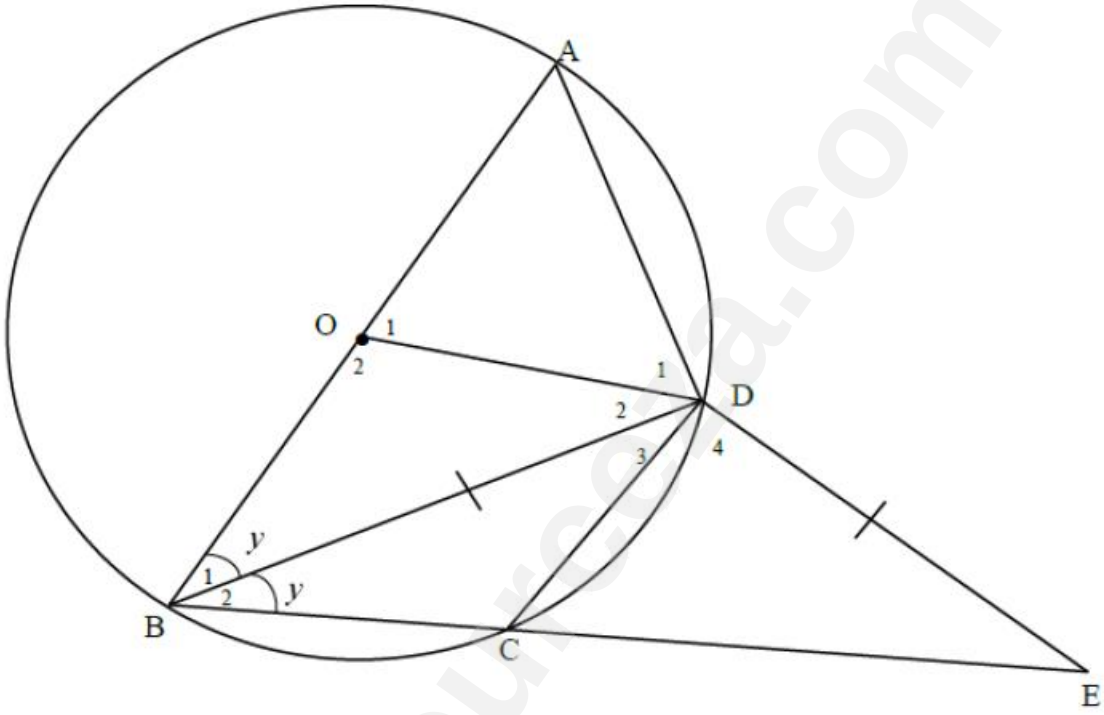
3?

Prove the theorem that states that if two triangles are similar, then the sides are proportional, i.e. $\frac{DE}{AB} = \frac{EF}{BC} = \frac{DF}{AC}$. (5)

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10.2 In the diagram below, AB is the diameter of a circle with centre O. BD and BC are chords. BD = DE. BCE is a line. $\hat{B}_1 = \hat{B}_2 = y$.



Prove that:

10.2.1 $\hat{D}_4 = 90°$ (5)

10.2.2 $\Delta BOD \;|||\; \Delta BDE$ (3)

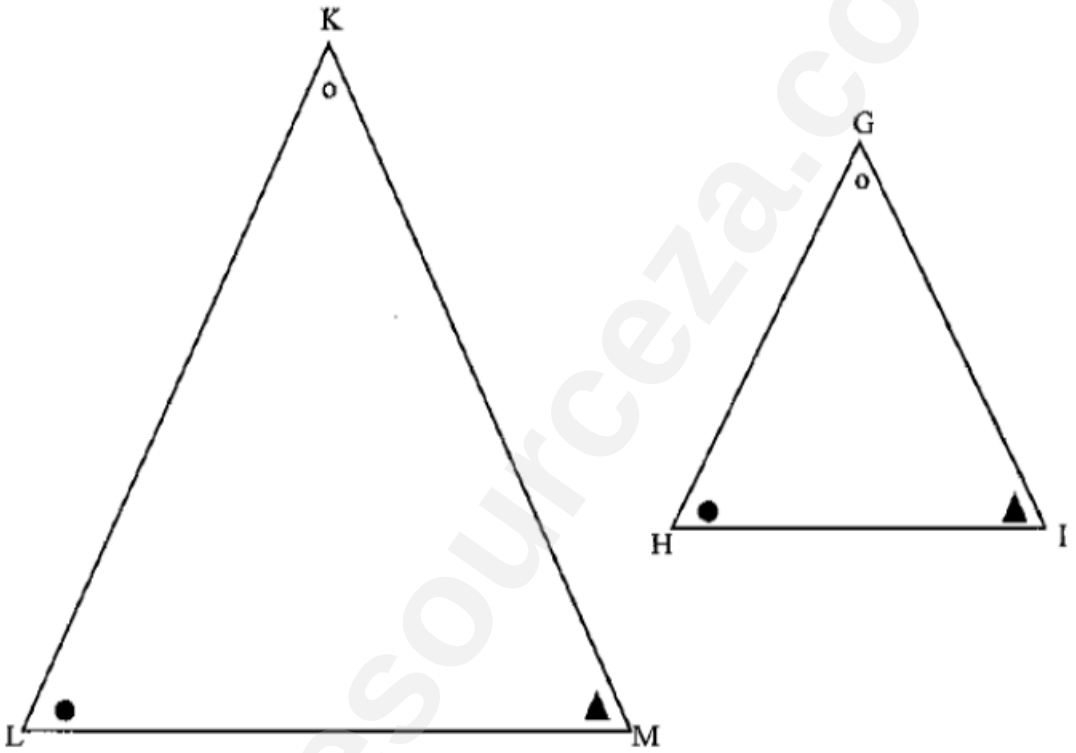
10.2.3 $DE^2 = BE.OD$ (4)
[17]

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QUESTION 10

10.1 In the diagram below, ΔKLM and ΔGHI are drawn such that $\hat{K} = \hat{G}$, $\hat{L} = \hat{H}$ and $\hat{M} = \hat{I}$. Prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, i.e. prove that $\frac{KL}{GH} = \frac{KM}{GI}$.

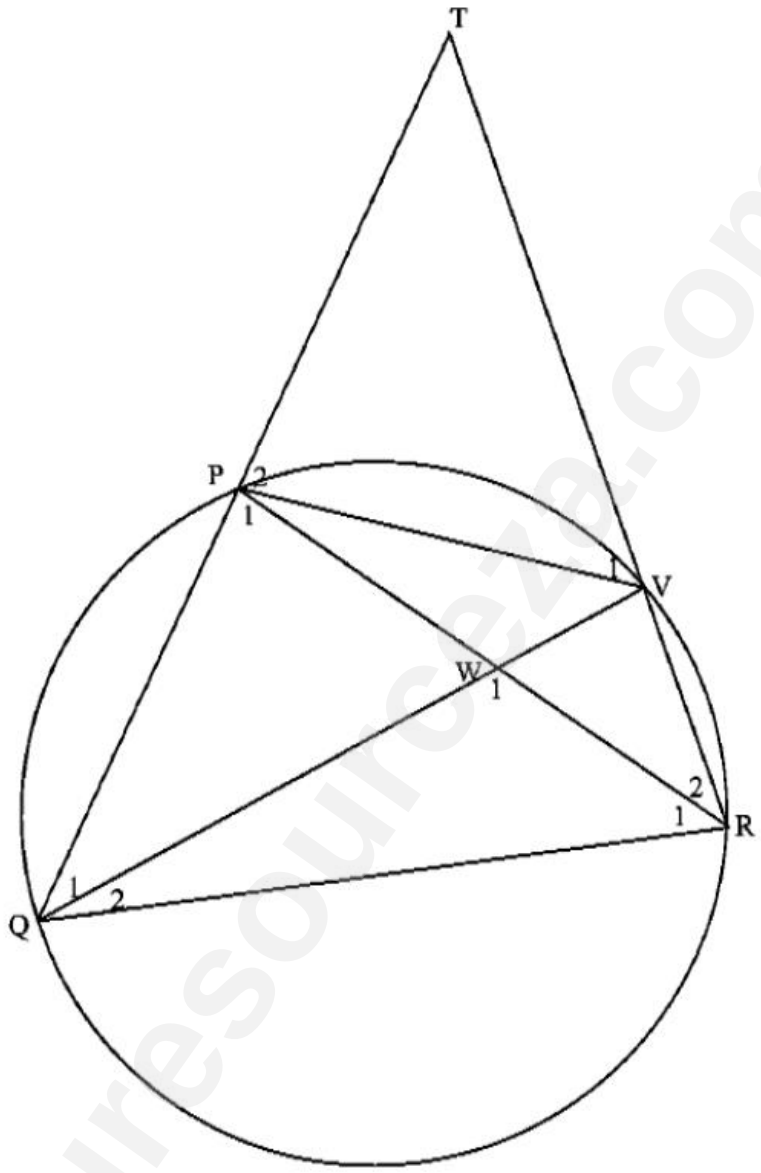


(5)

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10.2 In the diagram below, ΔPQR is an equilateral triangle inscribed in a circle. V is a point on the circle. QP produced meets RV produced at T. PR and QV intersect at W.



Prove, giving reasons, that:

10.2.1 $\hat{W}_1 = T\hat{R}Q$ (3)

10.2.2 $\Delta TQR \,|||\, \Delta QRW$ (3)

10.2.3 $\frac{PT}{QW} = \frac{PV}{WR}$ (6)

[17]

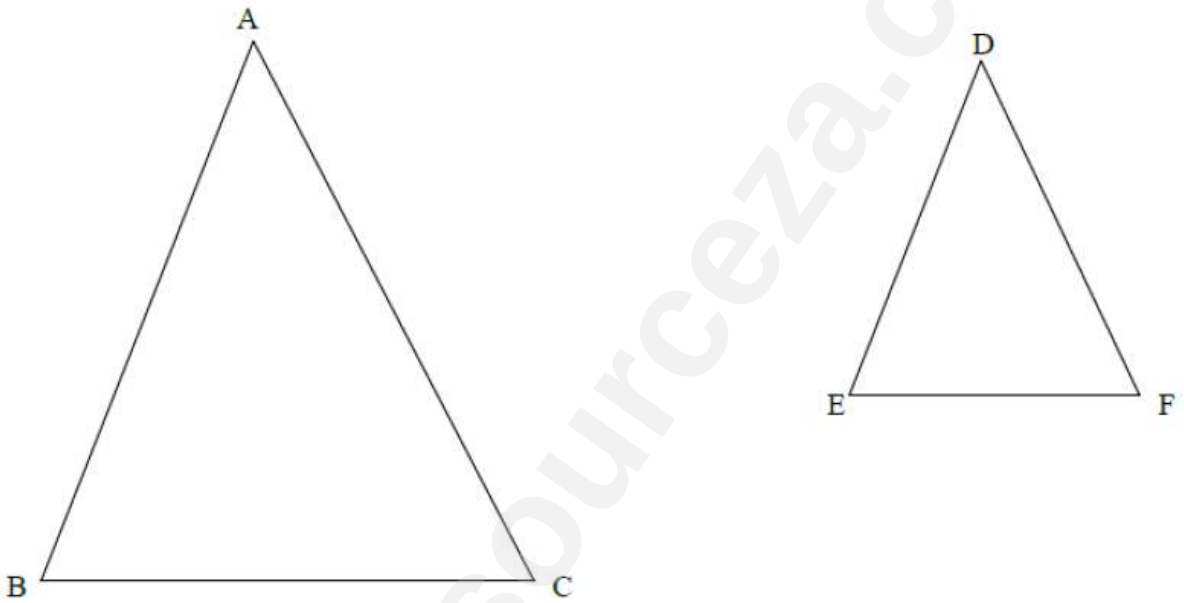
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QUESTION 10

10.1 In the diagram below, ΔABC and ΔDEF are drawn with $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.

Prove the theorem that states that equiangular triangles are similar and therefore $\frac{AB}{DE} = \frac{AC}{DF}$. (7)

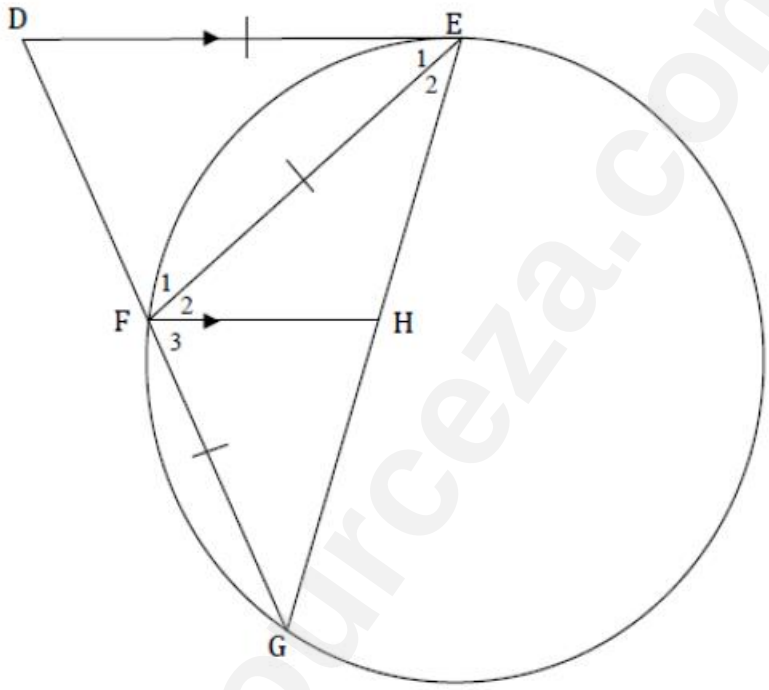


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10.2 In the diagram, DE is a tangent to the circle at E and DFG is a secant intersecting the circle at F and G. DE = EF = FG. H is a point on EG such that FH || DE.



10.2.1 Determine, giving reasons, 3 angles each equal to $\hat{DEF}$. (4)

10.2.2 Prove that:

a) $\Delta DEF \,|||\, \Delta DGE$ (3)

b) $\hat{D} = 72^0$. (5)

10.2.3 If it is further given that DF = k units and FG = 2 units, prove that $k^2 + 2k = 4$. (3)

10.2.4 Determine, giving reasons, the ratio of $\frac{GH}{GE}$ in terms of k. (2)

[24]

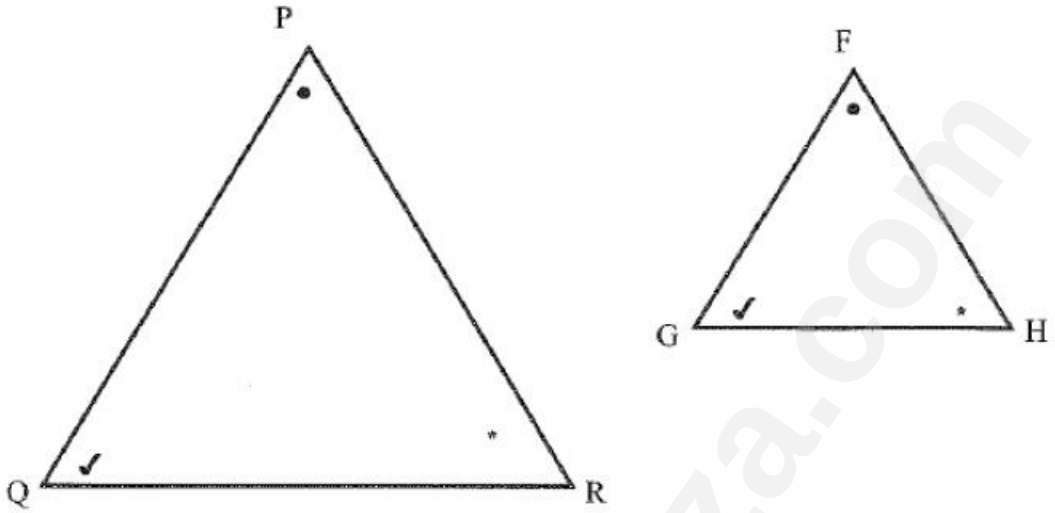
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QUESTION 10

10.1 ΔPQR and Δ FGH are given. $\hat{P} = \hat{F}, \hat{Q} = \hat{G}$ and $\hat{R} = \hat{H}$.



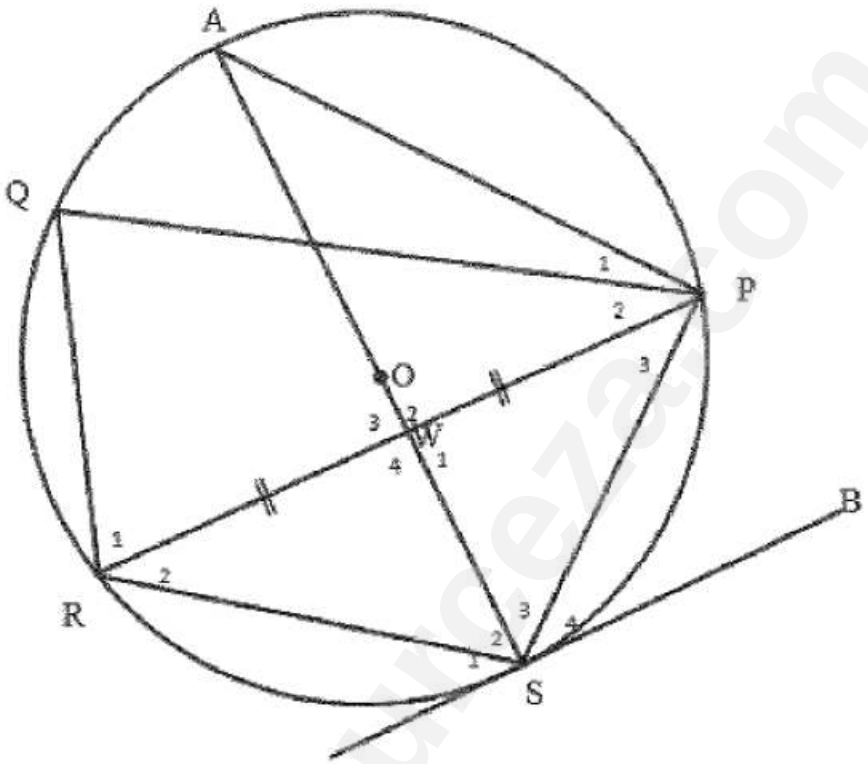
Prove the theorem which states that if $\Delta PQR \,|||\, \Delta$ FGH, then $\frac{PQ}{FG} = \frac{PR}{FH}$. (6)

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10.2 P, A, Q, R and S lie on the circle with centre O. SB touches the circle at S and RW = WP. AS and RP are straight lines.



Prove that:

10.2.1 $SB \parallel RP$ (5)

10.2.2 $\Delta APS \,|||\, \Delta RWS$ (4)

10.2.3 $RS^2 = WS.AS$ (4)

10.2.4 $AS = \frac{RW^2}{WS} + WS$ (4)

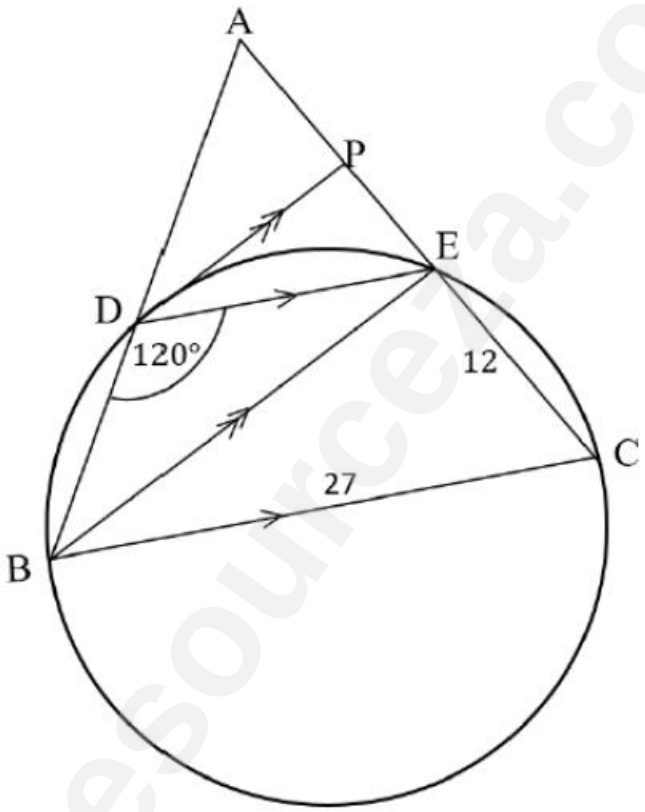
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QUESTION 10

In ΔABC in the diagram, D is a point on AB such that AD : DB = 5 : 4.
P and E are points on AC such that DE ∥ BC and DP ∥ BE.
BC is NOT a diameter of the circle.
Given: $\hat{BDE} = 120°$, EC = 12 units and BC = 27units.



10.1 Determine, with reasons:

10.1.1 The length of AE (3)

10.1.2 $\frac{\text{area of } \Delta AEB}{\text{area of } \Delta ECB}$ (2)

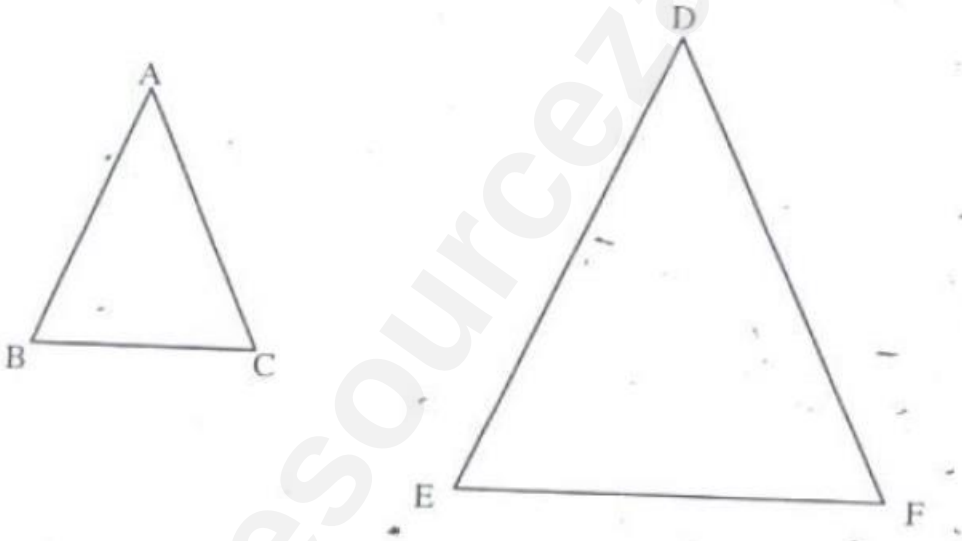
10.2 Hence, determine the length of DP if $\Delta ADP \text{ III } \Delta ABE$ (6)
[11]

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QUESTION 10

10.1 In the diagram below, ΔABC and ΔDEF are given with $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$. Proof the theorem that states that $\frac{AB}{DE} = \frac{AC}{DF}$. (6)



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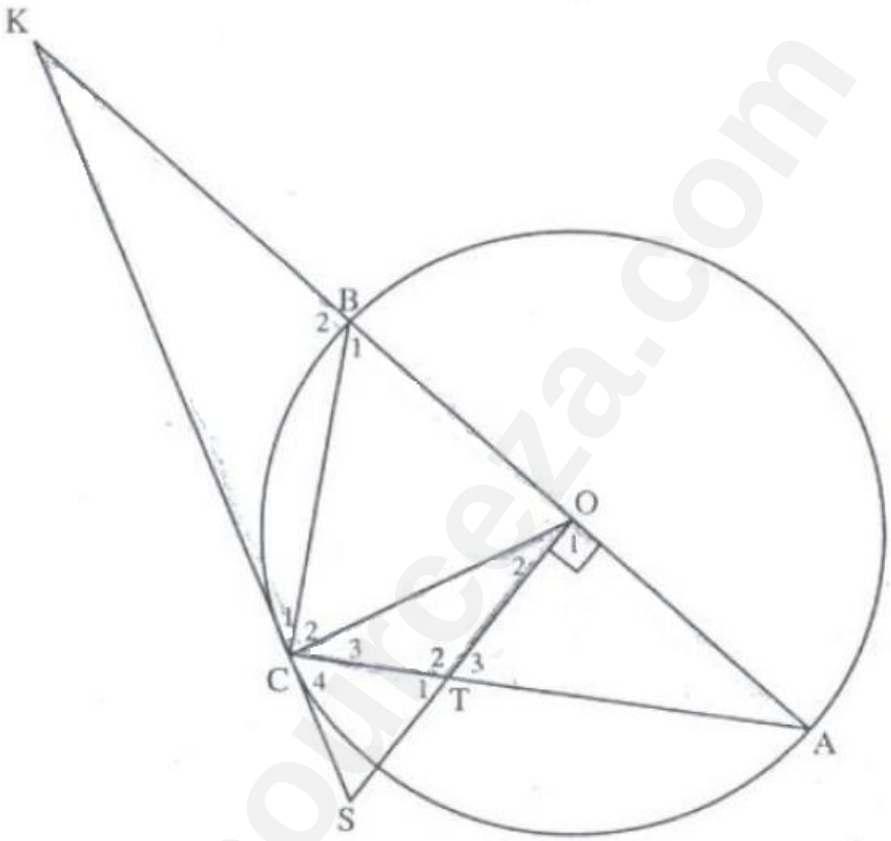
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10.2 In the diagram, AB is the diameter of the circle with centre O. AB is produced to K such that SK is a tangent to the circle at C. SO ⊥ AB. CA and SO intersect at T.



Prove that:

10.2.1 $\Delta CKB \;|||\; \Delta AKC$ (3)

10.2.2 $K\hat{C}T = \hat{T}_2$ (4)

10.2.3 $\Delta COT \;|||\; \Delta AKC$ (3)

10.2.4 $BK.AK = \dfrac{OT^2.CA^2}{CT^2}$ (4)

[20]

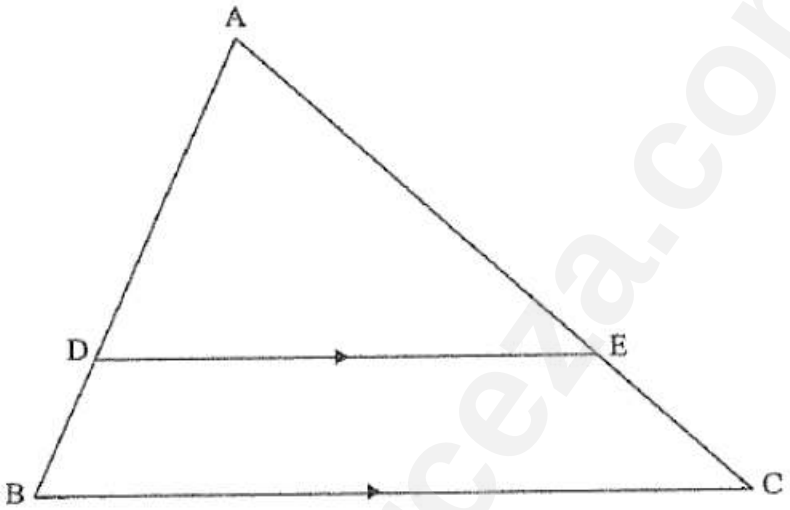
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QUESTION 10

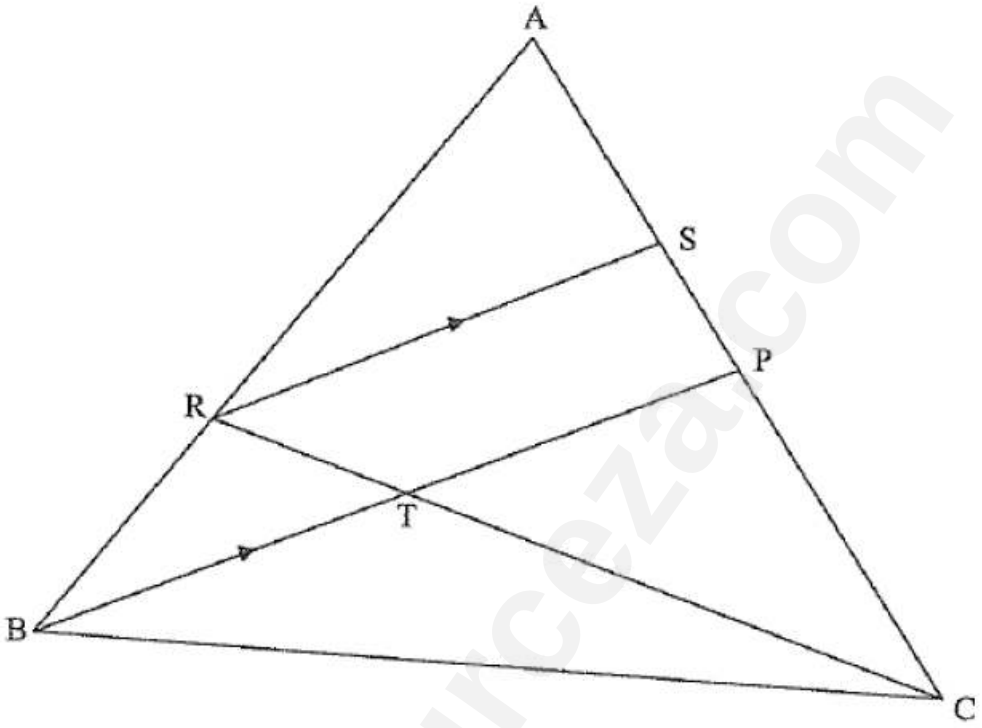
10.1 In ΔABC below, D and E are points on AB and AC respectively such that DE || BC. Prove the theorem which states that $\frac{AD}{DB} = \frac{AE}{EC}$. (6)



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10.2 In the diagram below, P is the midpoint of AC in ΔABC. R is a point on AB such that RS || BP and $\frac{AR}{AB} = \frac{3}{5}$. RC intersects BP in T.



Determine, with reasons, the following ratios:

10.2.1 $\frac{AS}{SC}$ (4)

10.2.2 $\frac{RT}{TC}$ (3)

10.2.3 $\frac{\text{Area of } \Delta RAS}{\text{Area of } \Delta RSC}$ (2)

10.2.4 $\frac{\text{Area of } \Delta TPC}{\text{Area of } \Delta RSC}$ (3)

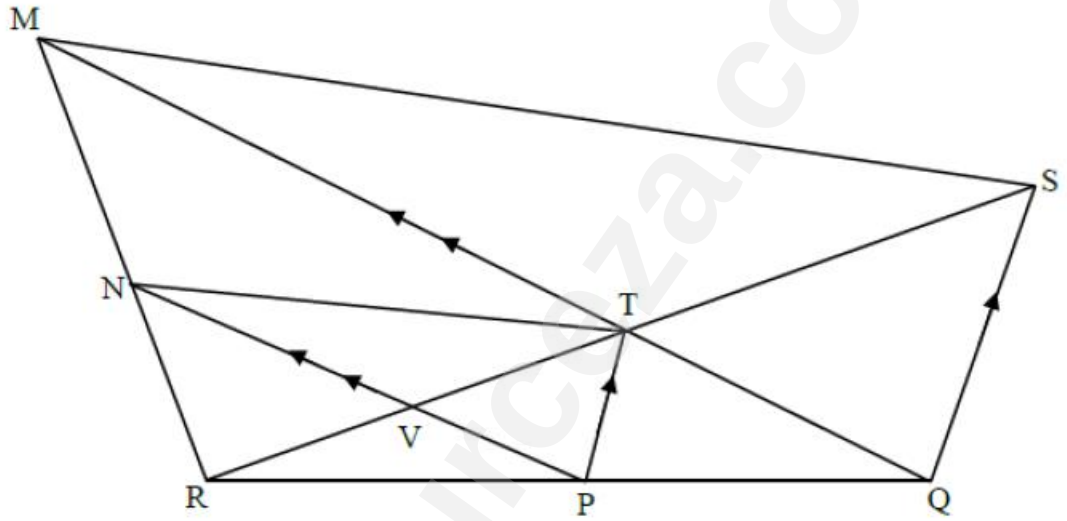
[18]

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QUESTION 11

In the diagram below, RQSM is a quadrilateral. N and P are points on MR and RQ respectively such that MQ || NP. The diagonals intersect at T. P is a point on RQ such that TP || SQ. TR and NP intersect at V.



11.1 Prove that NT || MS. (4)

11.2 If $RN = \frac{3}{5}NM$ and RS = 32, determine VT. (4)

[8]

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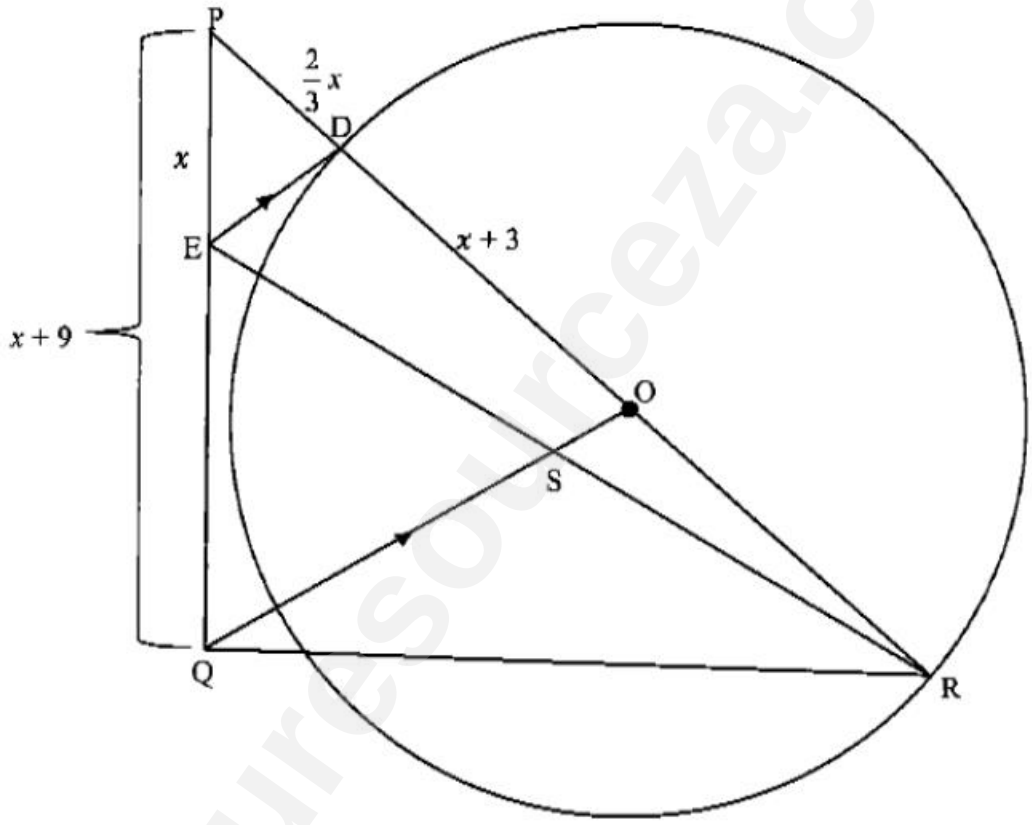
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QUESTION 11

In the diagram below, the circle with centre O is drawn. OQ is drawn parallel to a tangent to the circle at D. ER is drawn with S on OQ.
RD is produced to P and PQ is joined.

PE = x units, PQ = $x + 9$ units, PD = $\frac{2}{3}x$ units and DO = $x + 3$ units.



11.1 Calculate the length of RO. (4)

11.2 If OS = 1,4 units and S is the midpoint of ER, determine the length of DE. (2)

11.3 If the area of $\Delta PED = 2,7$ units^2, find the area of ΔPER. (4)
[10]

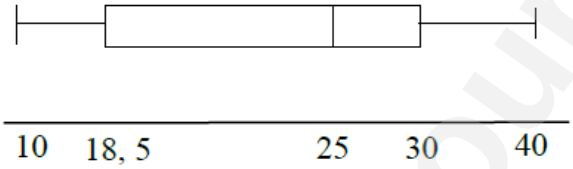
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MEMOS

MODULE 1: DATA HANDLING

GENERAL DATA HANDLING

QUESTION/*VRAAG* 1			
1.1	16	✓answer/*antwoord*	(1)
1.2	Semi-IQR $= \frac{30-18,5}{2}$ $= 5,75$	✓ correct values *korrekte waardes* ✓ substitution into correct formula / *vervanging in korrekte formule* ✓ answer/*antwoord*	(3)
1.3	 	✓ Min and/*en* Max/*Maks* ✓ Q_1 & Q_3 ✓ Q_2	(3)
1.4	The distribution is skewed to the left/*Die verspreiding is skeef na links.*	✓ answer/*antwoord*	(1)
			[8]

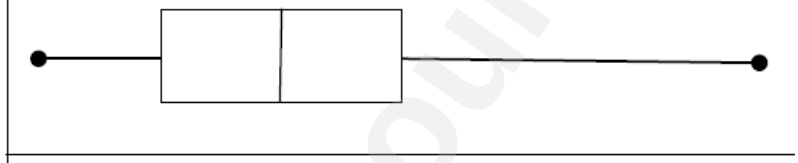
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QUESTION/*VRAAG* 2

12	13	13	14	14	16	17	18	18	18	19	20
21	21	22	22	23	24	25	27	29	30	36	

2.1.1	$\bar{x} = \frac{472}{23}$ $\bar{x} = 20{,}52$ seconds / *sekonde*	✓ $\frac{472}{23}$ ✓ answer (2)
2.1.2	$Q_1 = 16$ $Q_3 = 24$ IQR/*IKO* $= Q_3 - Q_1$ $= 24 - 16 = 8$	✓ Q_1 ✓ Q_3 ✓ answer (3)
2.2	$20{,}52 + 5{,}94 = 26{,}46$ $\therefore > 26{,}46$ $\therefore$ 4 girls/*dogters*	✓ 26,46 ✓ answer (2)
2.3	 **12** 14 **16** 18 **20** 22 **24** 26 28 30 **36**	✓ whiskers ending at 12 & 36 ✓ $Q_1 = 16$ & $Q_3 = 24$ (box) ✓ $Q_2 = 20$ (3)
2.4.1	Girls / *Meisies*	✓ answer (1)
2.4.2	Five-number summary of boys: (15 ; 21 ; 23,5 ; 26 ; 38) **None** of the boys / ***Nie een*** *van die seuns* ***nie*** 5 girls completed in less than 15 seconds which was the minimum time taken by the boys. *5 meisies voltooi in minder as 15 sekondes, wat die minimumtyd is wat die seuns geneem het.*	✓ answer ✓ reason/*rede* (2) **[13]**

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QUESTION/*VRAAG* 1

	SUGGESTED ANSWER/ *VOORGESTELDE ANTWOORD*	DESCRIPTORS/ *BESKRYWERS*
1.1	$60 \leq x < 80$	✓ answer/*antwoord* (1)
1.2		✓ All correct values/*Alle korrekte waardes*

INTERVAL OF TEST MARKS/ *INTERVAL VAN TOETSPUNTE*	NUMBER OF LEARNERS/ *AANTAL LEERDERS*	$X.f$
$0 \leq x < 20$	4	40
$20 \leq x < 40$	5	150
$40 \leq x < 60$	9	450
$60 \leq x < 80$	13	910
$80 \leq x < 100$	10	900
Totals/*Totale*	**41**	$\sum 2450$

✓ 2450

$\bar{x} = \frac{\sum Xf}{n} = \frac{2450}{41} = 59{,}76$

ANSWER ONLY: FULL MARKS/
SLEGS ANTWOORD: VOLPUNTE

✓ $\bar{x} = 59{,}76$ (3)

1.3			✓✓ ALL correct values in the table/ *ALLE korrekte waardes in die tabel*

INTERVAL OF TEST MARKS/ *INTERVAL VAN TOETSPUNTE*	NUMBER OF LEARNERS/ *AANTAL LEERDERS*	CUMULATIVE FREQUENCY/ *KUMULATIEWE-FREKWENSIE*
$0 \leq x < 20$	4	4
$20 \leq x < 40$	5	9
$40 \leq x < 60$	9	18
$60 \leq x < 80$	13	31
$80 \leq x < 100$	10	41

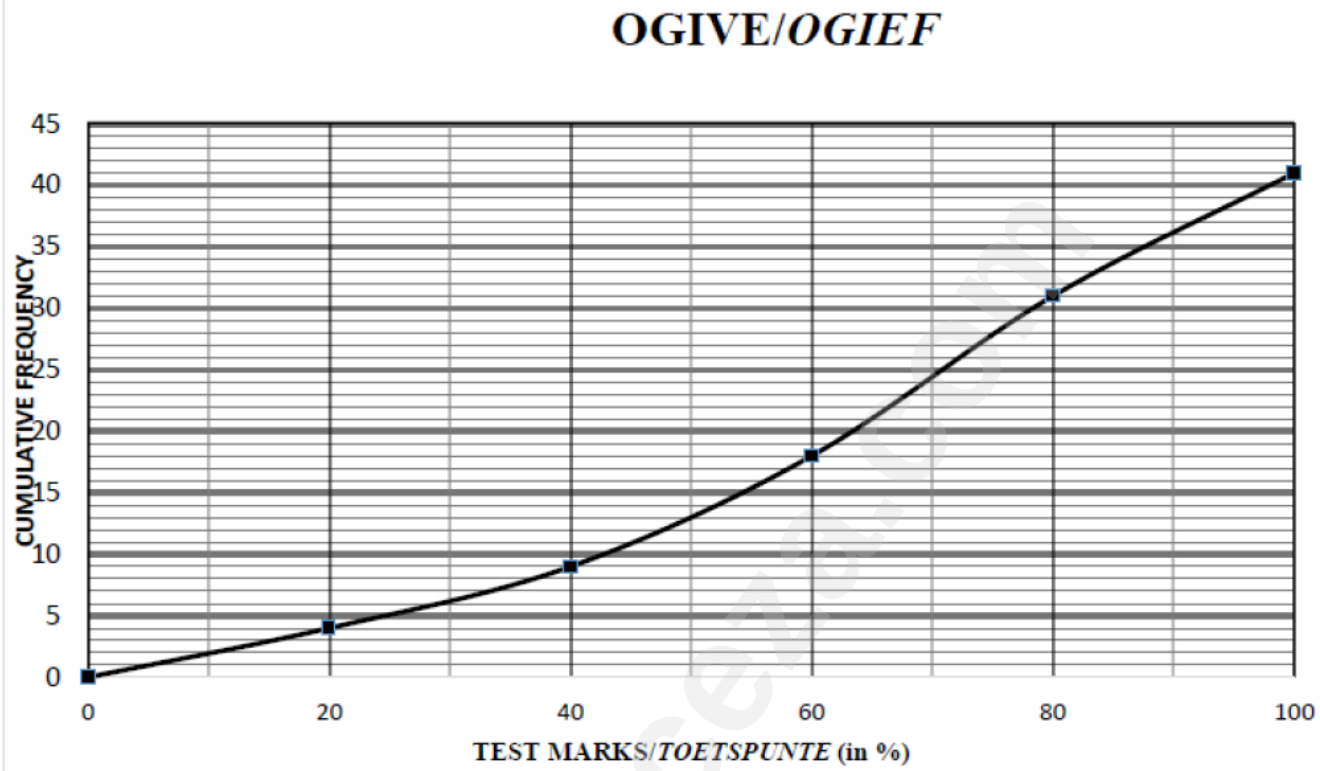
Max of 1/2 for 3 correct values in the table/
Maksimum van 1/2 vir 3 korrekte waardes in die tabel

(2)

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1.4



✓ using the cumulative frequency/*gebruik die kumulatiewefrekwensie*
✓ smooth curve/*gladde kurwe*
✓ using upper limits/*gebruik boonste limiete*

(3)

1.5	$IQR = Q_3 - Q1$ $= 68 - 33$ $= 35$	ACCEPT/*AANVAAR*: • 32 **OR/OF** 34 lower quartile/*laer kwartiel* • 67 **OR/OF** 69 upper quartile/*boonste kwartiel*	✓L Q ✓U Q ✓ *IQR* (3)

[12]

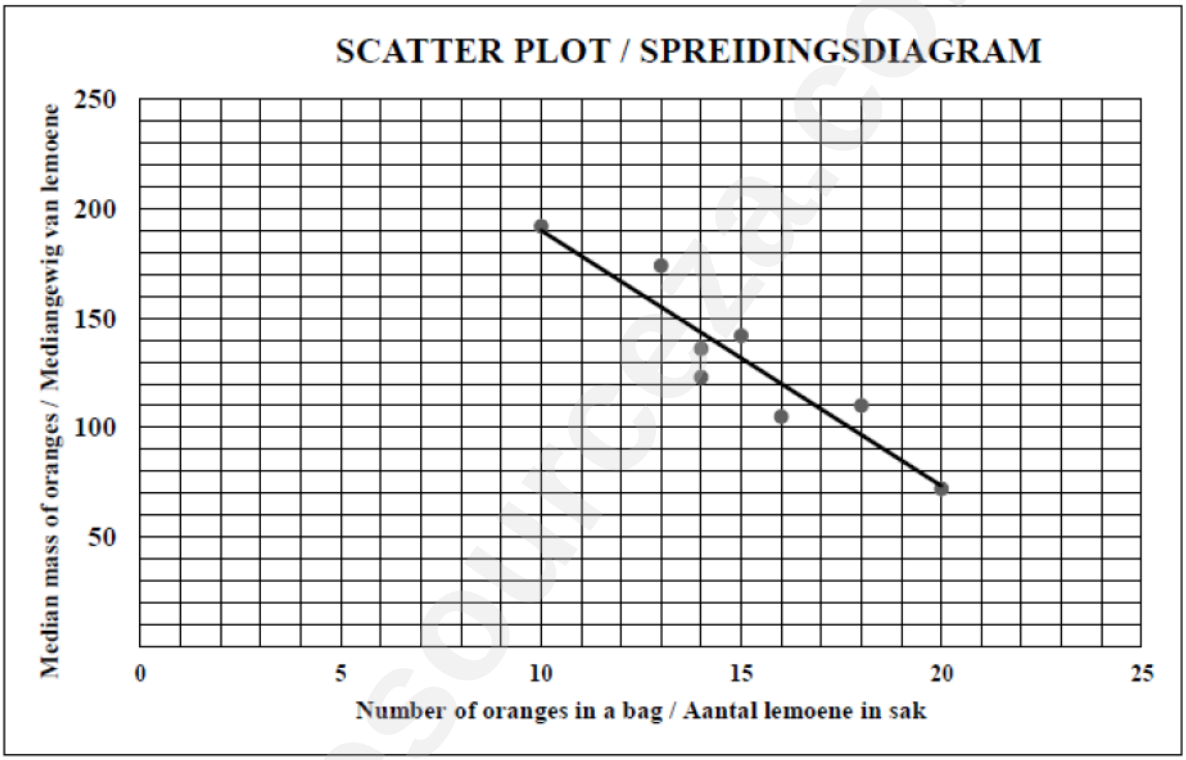
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QUESTION / *VRAAG* 1

Number of oranges in the bag *Aantal lemoene in sak*	18	16	20	15	14	13	14	10
Median mass of oranges in the same bag (to the nearest gram) / *Mediaangewig van die lemoene in dieselfde sak (tot die naaste gram)*	110	105	72	142	123	174	136	192



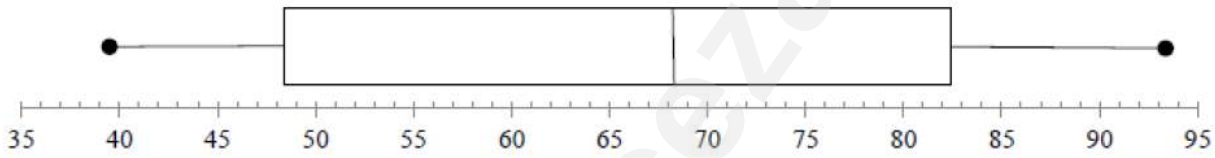
1.1	$a=307{,}20$ $b=-11{,}70$ $\hat{y}=307{,}20-11{,}7x$	✓ $a=307{,}20$ ✓ $b=-11{,}70$ ✓ $\hat{y}=307{,}20-11{,}7x$ (3)
1.2	$r=-0{,}93$	✓ $r=-0{,}93$ (1)
1.3	See scatter plot above/*sien spreidingsdiagram hierbo* (10 ; 190,2) (20 ; 73,2)	✓ (10 ; 190,2) ✓ (20 ; 73,2) (2)
1.4	Negative strong association / *Negatiewe sterk assosiasie*	✓answer / *antwoord* (1)
1.5	$\hat{y}=307{,}20-11{,}7(12)$ $=166{,}8$	✓substitution / *vervang* ✓answer / *antwoord* (2)
		[9]

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QUESTION 1

39	42	48	54	62	68	78	78	82	91	93

1.1	$\bar{x} = 66{,}82$ or 66,82 thousands	A ✓calculation CA ✓ answer	(2)
1.2	$\sigma = 18{,}3$	AA ✓✓answer If formula is used CA will apply.	(2)
1.3	$(\bar{x} - \sigma;\ \bar{x} + \sigma) = (48{,}52\ ;\ 85{,}12)$ $\therefore$ 6 countries	CA✓ substitution CA✓ answer	(2)
1.4	 A✓min & max values A✓quartile 1 value A✓median value & form A✓quartile 3 value		(4)
1.5	The data is skewed to the left/negatively skewed	CA✓answer	(1)
1.6.1	The sum of the data provided for the years must increase.	A✓answer	(1)
1.6.2	IQR will remain the same or change.	A✓answer (both same or change)	(1)
			[13]

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QUESTION/*VRAAG* 1

1.1	Minimum and maximum values/ *Minimum en maksimum waardes*	✓values both /albei waardes (1)
1.2	$0,75 \times 21 = 15,75$	✓ method/metode ✓answer (2)
1.3	Yes./*Ja* The difference between the median and lower quartile is much smaller than the difference between the median and upper quartile. / *Die verskil tussen die mediaan en onderst kwartiel is baie kleiner as die verkil tussen die mediaan en die boonste kwartiel.*	✓yes./*Ja* ✓✓reason/ *rede* (3)
1.4	The interval to consider: / Die interval: (26,3 ; 64,7) 10 learners lie outside 1 standard deviation. / 10 leerders lê buite een standard afwyking .	✓✓boundaries grense ✓10 (3)
1.5	The mean will increase by 5./ Die gemiddeld sal 5 meer wees, The standard deviation will remain the same./ die standaard afwyking sal dieselfde bly.	✓ ✓ (2)
		[11]

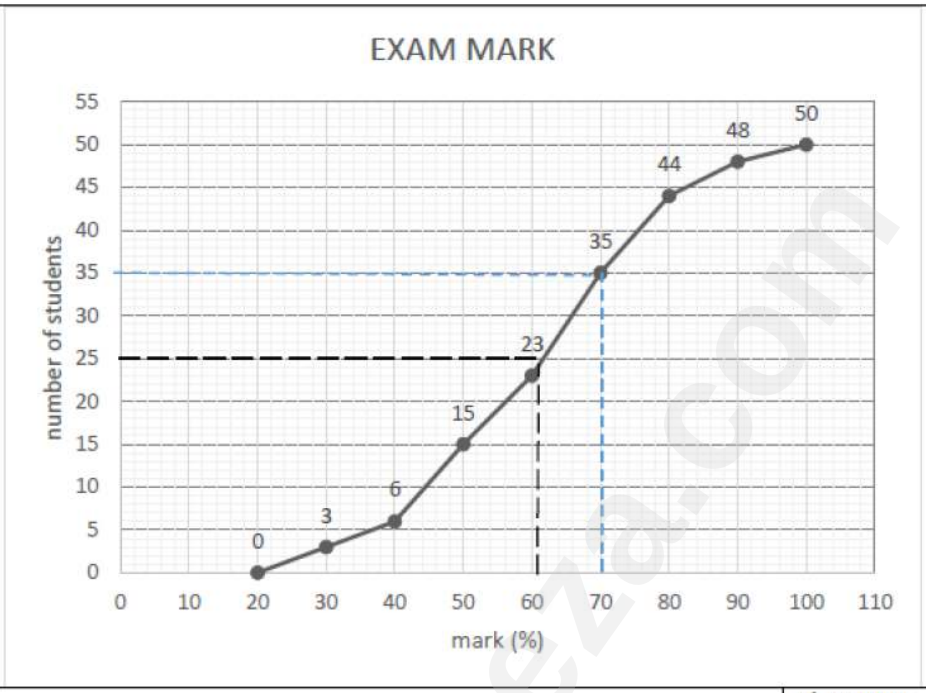
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QUESTION 1

1.1

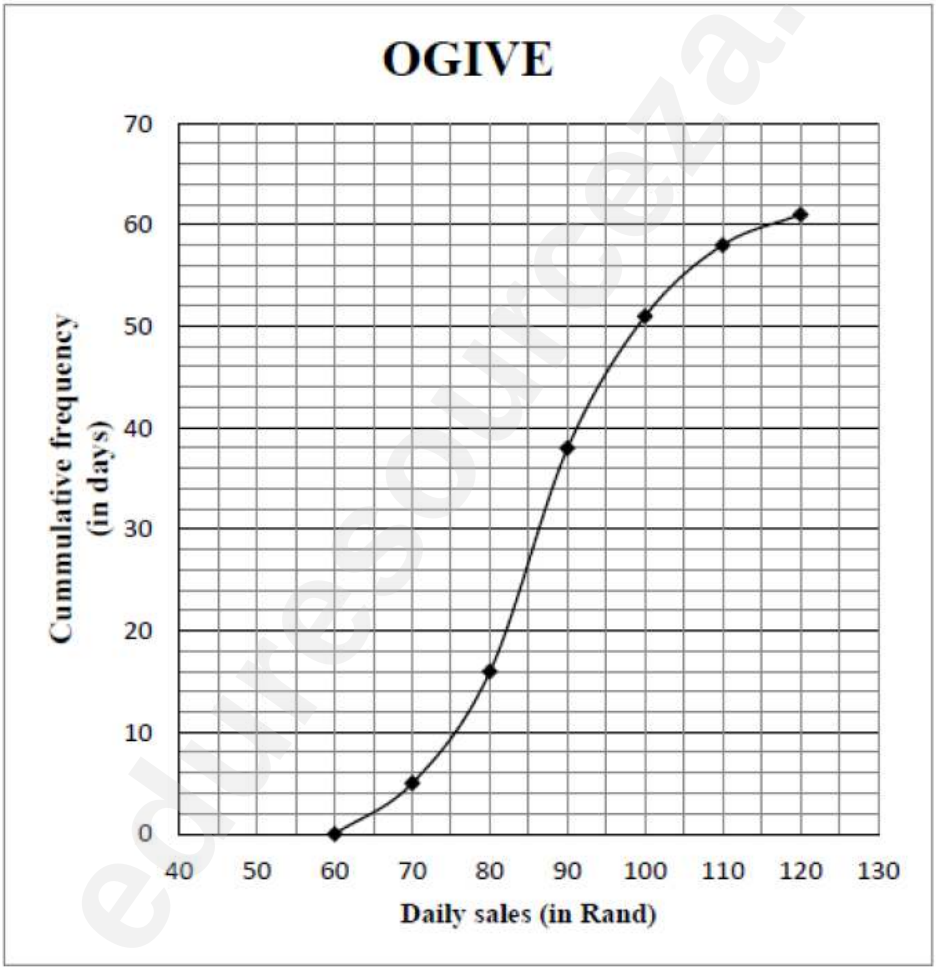


1.1.1	50	✓ *answer*	(1) K
1.1.2	<table><tr><th>Interval</th><th>Frequency</th><th>Cumulative frequency</th></tr><tr><td>$20 \leq x < 30$</td><td>3</td><td>3</td></tr><tr><td>$30 \leq x < 40$</td><td>3</td><td>6</td></tr><tr><td>$40 \leq x < 50$</td><td>9</td><td>15</td></tr><tr><td>$50 \leq x < 60$</td><td>8</td><td>23</td></tr><tr><td>$60 \leq x < 70$</td><td>12</td><td>35</td></tr><tr><td>$70 \leq x < 80$</td><td>9</td><td>44</td></tr><tr><td>$80 \leq x < 90$</td><td>4</td><td>48</td></tr><tr><td>$90 \leq x < 100$</td><td>2</td><td>50</td></tr></table>	✓ *first 4 values* ✓ *last 4 values*	(2) K
1.1.3	Median $= 62 \pm 1$	✓25 on graphs ✓ *answer* OR ✓✓Answer only	(2) R
1.1.4	$60 \leq x < 70$	✓ *answer*	(1) K
1.1.5	$50 - 35 = 15$ *learners*	✓ 35 on graph ✓ *answer* OR ✓✓Answer only	(2) R
1.2.1	$IQR = 20 - 11$ $= 9$	✓ 20-11 ✓ *answer* OR ✓✓Answer only	(2) K
1.2.2	2000 • the median score of 20 in 2000 is greater in value than the median score of 17 in 2010. • the upper half (50%)of the students in 2000 scored in the same score range as the upper one-fourth (25%) of the students in 2010. By considering the upper quarter, upper half, and upper three-quarters instead of just the lowest and highest scores, we would conclude that the students as a whole did much better in 2000 than in 2010.	✓ 2000 ✓ *reason*	(2) P
			[12]

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Question 1

1.1	<table><tr><th>Interval</th><th>Frequency</th><th>Cummulative frequency</th></tr><tr><td>$60 < x \leq 70$</td><td>5</td><td>5</td></tr><tr><td>$70 < x \leq 80$</td><td>11</td><td>16</td></tr><tr><td>$80 < x \leq 90$</td><td>22</td><td>38</td></tr><tr><td>$90 < x \leq 100$</td><td>13</td><td>51</td></tr><tr><td>$100 < x \leq 110$</td><td>7</td><td>58</td></tr><tr><td>$110 < x \leq 120$</td><td>3</td><td>61</td></tr></table>	✓ frequency ✓✓ accumulative frequency (3)
1.2		✓angle point ✓use of cumulative frequency ✓ upper limits ✓ free hand curve (4)
1.3	$61-51=10$	✓✓ answer (2)
1.4	Medium = 87 (between 85 and 88)	✓ ✓answer (2)
		[11]

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QUESTION/*VRAAG 1*

1.1	$k + 70 = 80$ $\therefore k = 10$	✓ $k + 70 = 80$ (1)
1.2	$80 - 7 = p$ **OR / *OF*** $63 + 10 = p$ $\therefore p = 73$ $\therefore p = 73$	✓ 73 (1)
1.3	$\bar{x} = \frac{75\times2 + 105\times16 + 135\times28 + 165\times17 + 195\times10 + 225\times7}{80}$ $= \frac{11940}{80}$ $\therefore \bar{x} = 149,25$ Answer only: Full Marks	✓midpoints/*middelwaardes* ✓ ÷ 80 ✓answer/*antwoord* (3)
1.4.1	$\bar{x} + \sigma = 186,89$ $\sigma = 37,64$ Answer only: Full Marks	✓method/*metode* ✓answer/*antwoord* (2)
1.4.2	$q = \bar{x} - \sigma_x$ $q = 149,25 - 37,64$ $q = 111,61$	✓answer/*antwoord* (1)
1.5	value of / *waarde van* $m = 3$ seconds / *sekondes* average time elapsed of day 2 will be 3 seconds less / *gemiddelde tydsverloop van dag 2 is 3 sekondes min der* $\bar{x}_{\text{day/dag 2}} = 146,25$	✓ $m = 3$ ✓ $\bar{x}_{\text{day/dag 2}} = 146,25$ (2)
		[10]

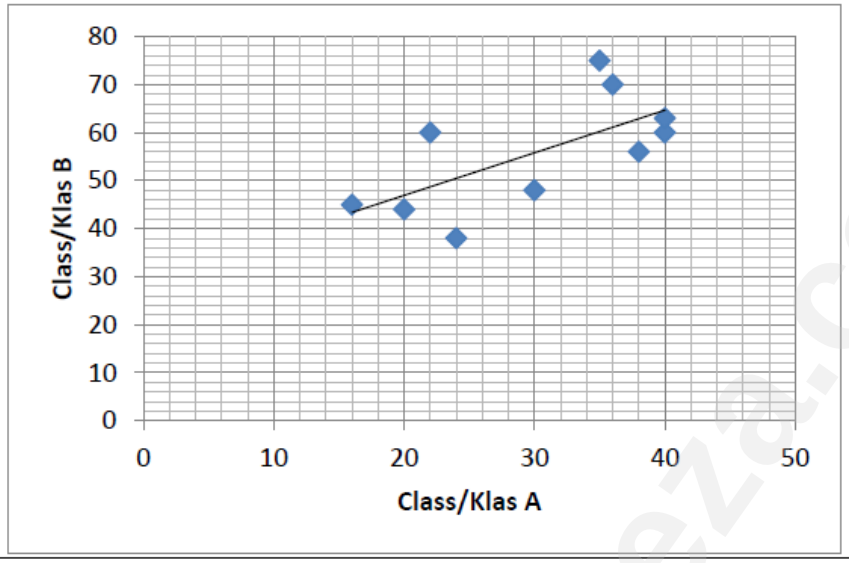
QUESTION/*VRAAG 2*

2.1	$y = A + Bx$ $y = 25,42 + 1,32x$	✓ 25,42 ✓ 1,32 ✓ $y = 25,42 + 1,32x$ (3)
2.2	$y = 25,42 + 1,32(36)$ $y = 72,94$	✓ correct substitution/*korrekte vervanging* ✓ answer/*antwoord* (2)
2.3	$r = 0,81$ Strong positive/*Sterk positiewe*	✓ $r = 0,81$ ✓ Strong/*Sterk* ✓ Positive/*Positiewe* (3)
		[8]

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REGRESSION MODELLING

QUESTION/*VRAAG* 2			
2.1		✓✓ plotting points/ *afsteek van punte* ✓ regression line/ *regressielyn* (2.3)	(2)
2.2	$a = 29{,}22$ (29,21542…..) $b = 0{,}89$ $\therefore y = 29{,}22 + 0{,}89x$	✓ calculating value of a and b *bereken waardes van a en b* ✓ equation for the line/*vergelyking vir die lyn*	(2)
2.3	On graph/*Op grafiek*		(1)

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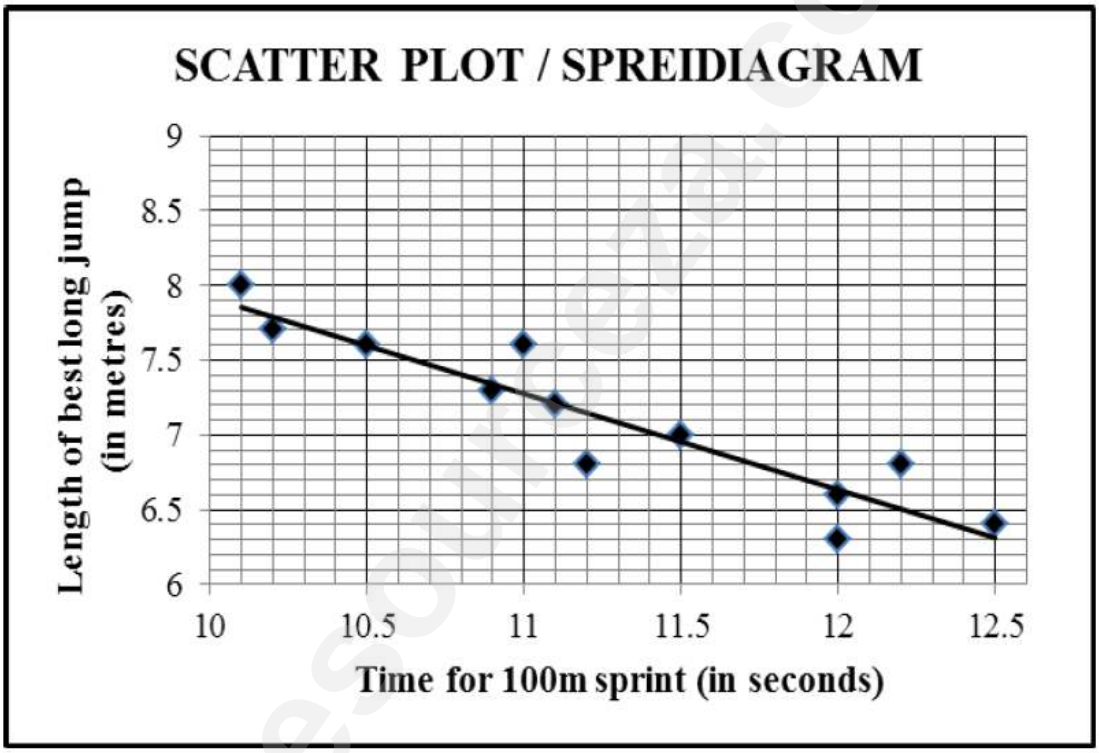
2.4	$r = 0{,}66$ OR/*OF* $s_y = \sqrt{\frac{\Sigma(y-\bar{y})^2}{n}}$ $= \sqrt{\frac{1290{,}9}{10}} = 11{,}36$ $s_x = \sqrt{\frac{\sum(x-\bar{x})^2}{n}}$ $= \sqrt{\frac{720{,}9}{10}} = 8{,}49$ $b = r\frac{s_y}{s_x}$ then/*dan* $0{,}89 = r\frac{11{,}36}{8{,}49}$; $r = 0{,}66$	✓✓answer/*antwoord* ✓ S_y, S_x ✓ answer/*antwoord*	(2)
2.5	Mean/*Gemiddelde*, $\bar{x} = \frac{559}{10} = 55{,}90$ (Answer only: Full marks) *(Slegs antwoord: Volpunte)* Standard deviation/*Standaardafwyking*, $\sigma = 11{,}36$	✓✓answer/*antwoord* ✓✓answer/*antwoord*	(4)
2.6	55,9 – 11,36 and 55,9 + 11,36 [44,54; 67,26] 6 scores/*tellings*	✓interval/*interval* ✓answer/*antwoord*	(2)
			[13]

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QUESTION/*VRAAG* 1

Time for 100 m sprint (in seconds) *Tyd vir 100 m-naelloop (in sekondes)*	10,1	10,2	10,5	10,9	11	11,1	11,2	11,5	12	12	12,2	12,5
Distance of best long jump (in metres) *Afstand van beste sprong in verspring (in meter)*	8	7,7	7,6	7,3	7,6	7,2	6,8	7	6,6	6,3	6,8	6,4

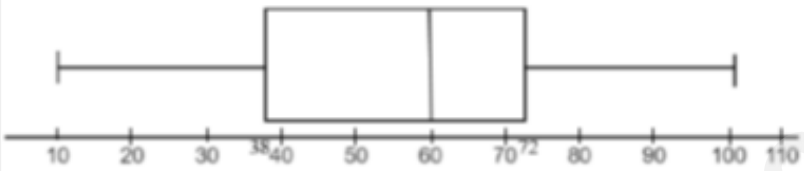


1.1	$a = 14{,}343\ldots = 14{,}34$ $b = -0{,}642\ldots = -0{,}64$	✓✓ value of a ✓ value of b (3)
1.2	$y = 14{,}34 - 0{,}64(11{,}7)$ $= 6{,}85$ **OR/OF** $y = 6{,}83$ (calculator / *sakrekenaar*)	✓ substitution correctly ✓ answer (2) ✓✓answer (2)
1.3	The gradient increases / *Die gradient neem toe* The point (12,3 ; 7,6) lies some distance above the current data. */Die punt (12,3 ; 7,6) lê bokant die huidige data.*	✓ increases/*neem toe* ✓ reasoning in words/ *redenasie in woorde* (2) **[7]**

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QUESTION / *VRAAG 2*

2.1.1	100	✓answer / *antwoord* (1)
2.1.2	Median / *Mediaan* = ±62	✓✓answer / *antwoord* (Accept / *Aanvaar* 61 / 62) (2)
2.1.3		✓ $Q_1 = 37/38$ ✓ $Q_3 = 72/73$ ✓ $Q_2 = 61/62$ & min & max / *min & maks* (10 & 100) (3)
2.1.4	Skewed to the left / *Skeef na links*	✓ left / *links* (1)
2.2	$b = 20$ $\frac{d-a}{2} = 8$ $2a = d$ *sub* $\frac{2a-a}{2} = 8$ $a = 16$ $d = 32$ $5+16+19+20+c+32+35 = 7\times 22$ $\therefore c = 27$	✓ $b = 20$ ✓ $a = 16$ ✓ $d = 32$ ✓ $c = 27$ (4)
		[11]

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QUESTION 2

YEAR	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Agriculture (x)	4.2	3,4	3,1	2,7	2,9	3,0	2,9	3,0	2,6	2,5	2,6
Mining (y)	19,2	19,4	19,2	18,5	17,5	17,0	16,8	15,2	14,2	12,8	12,4

2.1	a= 5,65 and b= 3,65 y= 5,65 +3,65x	A✓ a – value A✓ b – value CA✓ equation	(3)
2.2	y= 5,65+3,65x = 5,65+3,65(1,2) =10,03%	A✓ substitution of 1,2 CA✓ answer	(2)
2.3	strong positive correlation of the data r = 0,7	CA✓ strong positive A✓ value of correlation coefficient	(2)
			[7]

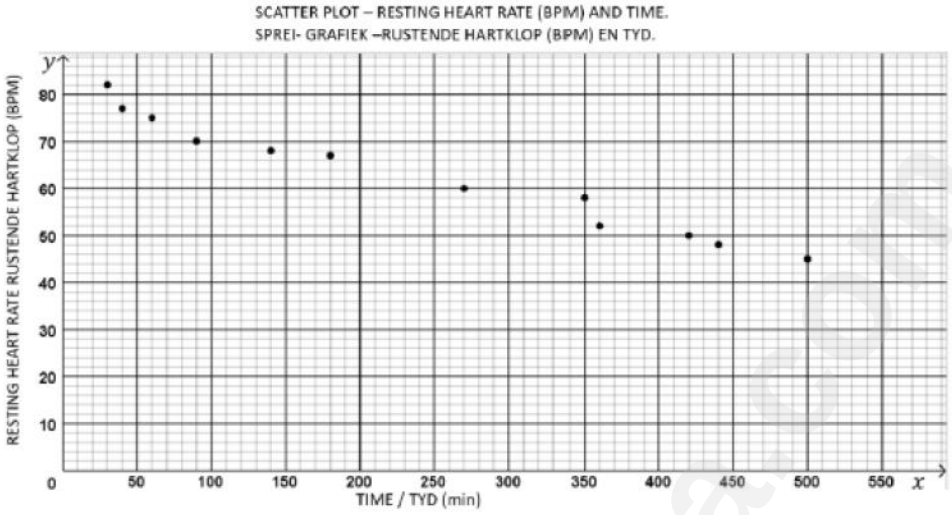
QUESTION/*VRAAG* 2

2.1	A very strong, positive relationship./ *'n baie sterk positiewe verwantskap* The points are close to one another./ *Die punte lê baie na aan mekaar*	✓statement/bewering ✓ explanation / verduideliking (2)
2.2	r = 0,93	✓Answer/*antwoord* (1)
2.3	a = 22,83 b = 0,66 $\hat{y}$ = 22,83+,66x	✓a ✓b ✓equation/ *vergelyking* (3)
2.4	$\hat{y}$ = 22,830,66(60) = 62,43 The candidate would have scored 62%	✓ substitution/*vervanging* ✓answer/*antwoord* (2)
2.5	(82;61)	✓answer /*antwoord* (1)
		[9]

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QUESTION 2

2.1		✓ 4 values ✓ 4 values ✓ 4 values (3) K	
2.2	$r = -0,98$	✓✓ answer -1 rouding	(2) R
2.3	$a = 79,71$ $b = -0,07$ $\hat{y} = 79,71 - 0,07x$	✓ value of a ✓ value of b ✓ equation	(3) R
2.4	$65 = 79,71 - 0,07x$ $-14,71 = -0,07x$ $210,14 \ldots min. = x$ 207.14 $\therefore$ 3,5 *hours per week* 3.45 h or 3h 27 min	✓ *subst* **CA from 2.3** ✓ 210,14 *min* ✓ 3,5 *hours*	(3) R
			[11]

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QUESTION 2

2.1.1	By using a calculator : $a = 76{,}60$ (76,59564211) $b = 0{,}1$(0.09544928654) ∴ equation of line of least squares is $y = 76{,}60 + 0{,}10x$	✓ a ✓ b ✓ equation (3)
2.1.2	$y = 76{,}60 + 0{,}10(57)$ ≈ 82	✓substitution ✓answer (2)
2.1.3	$r = 0{,}1408374211$ $r \approx 0{,}14$	✓✓ r $\approx 0{,}14$ (1)
2.1.4	r value is low which means that the data is spread around the regression line. Therefore prediction using the regression line will be **invalid**.	✓invalid (1)
2.2.1	22	✓ 22 (1)
2.2.2	$\frac{1320}{22} = 60$	✓ $\frac{1320}{22}$ ✓ 60 (2)
		[10]

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QUESTION/*VRAAG 2*

2.1	(68 ; 19)	✓ (68 ; 19) (1)
2.2	$\hat{y} = a + bx$ $a = 7{,}35$ and / *en* $b = 0{,}98$ $\therefore \hat{y} = 7{,}35 + 0{,}98x$	✓ $a = 7{,}35$ ✓ $b = 0{,}98$ ✓ equation/ *vergelyking* (3)
2.3	$\hat{y} = 7{,}35 + 0{,}98(83)$ $\hat{y} = 88{,}69\%$ **OR / *OF*** $\hat{y} = 88{,}35\%$ (calculator / *sakrekenaar*)	✓ substitute 83 into eq. / *vervang 83 in vgl.* ✓ 88,69 (2) ✓✓ 88,35% (2)
2.4	$r = 0.96$ ∴ Very strong positive correlation / *Baie Sterk positiewe korrelasie*	✓ $r = 0.96$ ✓Very strong / *Baie sterk* (2)
2.5	If the data points were to be plotted on a scatter plot diagram, the new plotted points will be shifted downwards with the same constant value. Hence the line of best fit will also shift downwards with the new plotted points, but the slope of the line will remain unchanged. Therefore it has NO influence on the gradient/ *As die datapunte op 'n spreidiagram geplot moet word, sal die nuwe datapunte afwaarts geskuif word met dieselfde konstante waarde. Vervolgens sal die paslyn ook saam met die nuwe datapunte afwaarts skuif, maar die helling van die lyn sal onveranderd bly. Dus sal dit GEEN invloed op die helling hê nie.*	✓New data points and line of best fit shifted downwards/*Nuwe datapunte en paslyn word afwaarts geskuif* ✓No influence on gradient/*Geen invloed op helling* (2) **[10]**

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MODULE 2: ANALYTICAL GEOMETRY

GENERAL ANALYTICAL GEOMETRY

QUESTION/*VRAAG* 3			
3.1	$y = 9$	✓ answer/*antwoord*	(1)
3.2	AB = 11 units/*eenhede* **OR/*OF*** $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $AB = \sqrt{(11 - 0)^2 + (9 - 9)^2}$ AB = 11 units/*eenhede*	✓ answer/*antwoord*	(1)
3.3	D(13; 9)	✓ answer/*antwoord*	(1)
3.4	Area of ΔABC = ½ AB.CD (*Oppervlakte van . . .*) = ½(11 × 8) = 44 square units/*vierkante eenhede*	✓ length of CD / *lengte van CD* ✓Substitute into correct formula / *Vervanging in korrekte formule* ✓answer/*antwoord*	(3)
3.5	$M = \left[\frac{x_2 + x_1}{2} ; \frac{y_2 + y_1}{2}\right]$ $= \left[\frac{13}{2} ; \frac{10}{2}\right]$ = [6,5 ; 5]	✓substitution into midpoint formula/ *vervanging in middelpunt formule* ✓answer/*antwoord*	(2)

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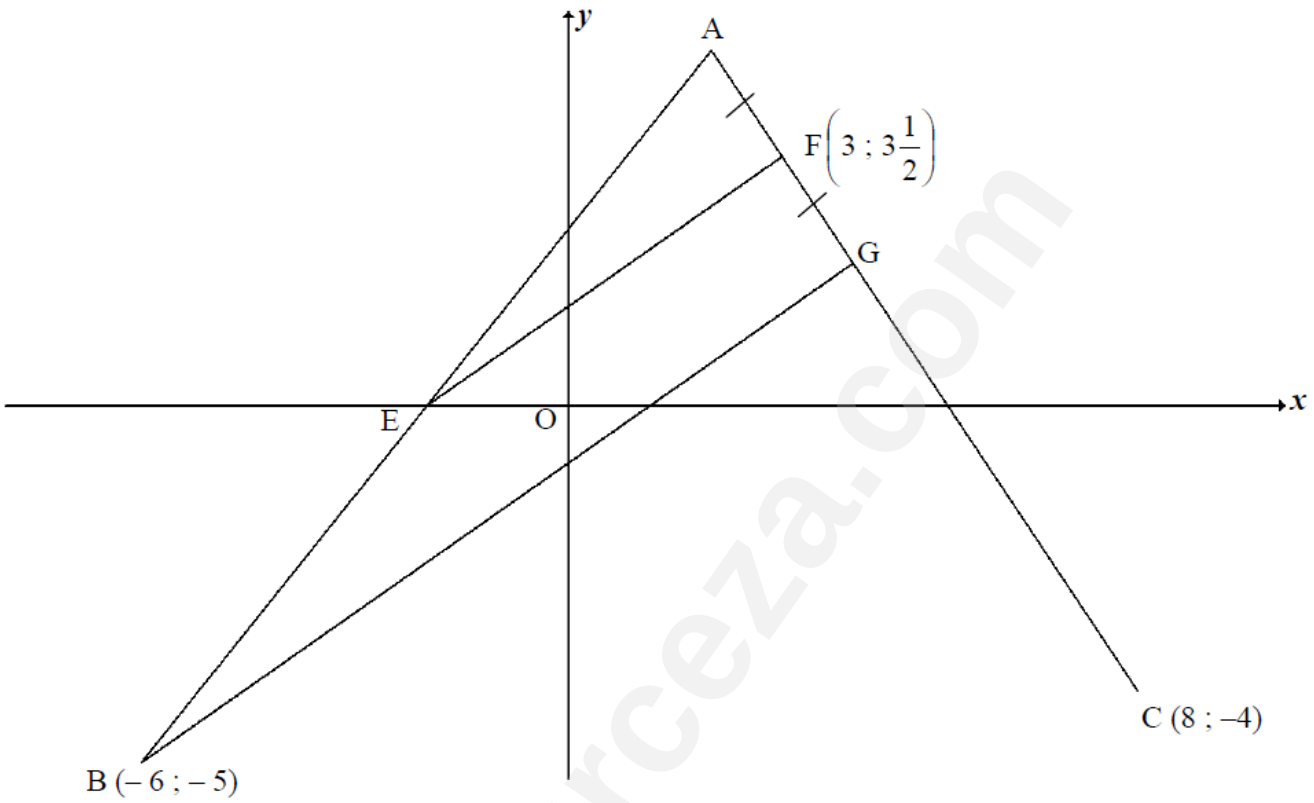
3.6	$m_{AC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{9-1}{0-13}$ $m_{AC} = -\frac{8}{13}$ $y - y_1 = m(x - x_1)$ $y - 5 = \frac{13}{8}(x - 6{,}5)$ $y = \frac{13}{8}x - 5\frac{9}{16}$	✓ m_{AC} ✓ correct gradient of perpendicular bisector/*korrekte gradient van middelloodlyn* ✓ Substitution into correct formula/ *Vervanging in korrekte formule* ✓answer/*antwoord*	(4)
3.7	$y = \frac{13}{8}(11) - \frac{89}{16}$ $y = \frac{143}{8} - \frac{89}{16}$ $y = \frac{197}{16}$ No, it does not pass through B/*Nee, dit gaan nie deur B nie.*	✓substitution into correct equation / *vervanging in korrekte vergelyking* ✓ answer/*antwoord* (justification/ *regverdiging*)	(2)

3.8	$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{9-1}{11-13}$ $= \frac{8}{-2} = -4$ $\tan\theta = -4$ $\theta = 104{,}04°$ $A\hat{B}C = 104{,}04°$	✓ m_{BC} ✓tan θ ✓ answer/*antwoord*	(3)
3.9	$y - y_1 = m(x - x_1)$ $y - 9 = -\frac{8}{13}(x - 13)$ $y = -\frac{8}{13}x + 8 + 9$ $y = -\frac{8}{13}x + 17$	✓ substitution/*vervanging* ✓ answer/*antwoord*	(2)
			[19]

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QUESTION/*VRAAG* 3



3.1.1	$m_{FC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{3\frac{1}{2} - (-4)}{3 - 8}$ $= -\frac{3}{2}$ $y = mx + c$ $\qquad$ $y - y_1 = m(x - x_1)$ $y = -\frac{3}{2}x + c$ $-4 = -\frac{3}{2}(8) + c$ **OR/*OF*** $(y - (-4)) = -\frac{3}{2}(x - 8)$ $c = 8$ $\qquad$ $y + 4 = -\frac{3}{2}x + 12$ $y = -\frac{3}{2}x + 8$ $\qquad$ $y = -\frac{3}{2}x + 8$ **OR/*OF***	✓ substitution of (8 ; –4) & $\left(3\,;3\frac{1}{2}\right)$ ✓ gradient ✓ substitution of m and (8 ; –4) ✓ equation of AC (4)

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	$m_{FC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-4) - \left(3\frac{1}{2}\right)}{8 - 3}$ $= -\frac{3}{2}$ $y = mx + c$ $3\frac{1}{2} = -\frac{3}{2}(3) + c$ $c = 8$ $y = -\frac{3}{2}x + 8$ **OR/*OF*** $y - y_1 = m(x - x_1)$ $\left(y - 3\frac{1}{2}\right) = -\frac{3}{2}(x - 3)$ $\left(y - 3\frac{1}{2}\right) = -\frac{3}{2}x + \frac{9}{2}$ $y = -\frac{3}{2}x + 8$	✓ substitution of $(8 ; -4)$ & $\left(3 ; 3\frac{1}{2}\right)$ ✓ gradient ✓ substitution of m and $\left(3 ; 3\frac{1}{2}\right)$ ✓ equation of AC (4)
3.1.2	AC: $3x + 2y = 16$ and BG: $7x - 10y = 8$ $15x + 10y = 80$ $\underline{7x - 10y = 8}$ $22x = 88$ $x = 4$ $3(4) + 2y = 16$ $y = 2$ $\therefore$ G(4 ; 2) **OR/*OF*** BG: $7x - 10y = 8 \quad \therefore y = \frac{7}{10}x - \frac{8}{10}$ $\therefore \frac{7}{10}x - \frac{8}{10} = -\frac{3}{2}x + 8$ [CA from 3.1.1] $\frac{11}{5}x = \frac{44}{5}$ $x = 4$ $3(4) + 2y = 16$ $y = 2$ $\therefore$ G(4 ; 2)	✓ method /*metode*: solving simultaneously / *los gelyktydig op* ✓ x coordinate $(x > 0)$ ✓ y coordinate (3) ✓ method: equating *metode: stel vgls gelyk* ✓ x coordinate $(x > 0)$ ✓ y coordinate (3)
3.2	$\frac{x_A + 4}{2} = 3$ and $\frac{y_A + 2}{2} = 3\frac{1}{2}$ $\therefore$ A(2 ; 5) **OR/*OF*** by translation/*deur translasie*: $x_A = 3 - (4 - 3) = 2$ $y_A = 3\frac{1}{2} + (3\frac{1}{2} - 2) = 5$ $\therefore$ A(2 ; 5)	✓ equation ito x ✓ equation ito y (2) ✓ equation ito x ✓ equation ito y (2)

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3.3	The coordinates of the midpt of AB / *Die koordinaat van midpt van AB is*: $\left(\frac{2+(-6)}{2};\frac{5+(-5)}{2}\right)=(-2\ ;\ 0)$ **But the *y*-coordinate of E is 0** **∴ E(–2 ; 0) is the midpoint of AB** ∴ EF ∥ BG [midpoint theorem/*middelpuntst* ***OR/OF*** line divides 2 sides of Δ in prop/*lyn verdeel 2 sye van Δ in dies verh*]	✓ subst A & B into midpt formula ✓ *y* coordinate = 0 ✓ E = midpt ✓ Reason (4)
	OR/*OF* The coordinates of the midpt of AB / *Die koordinaat van midpt van AB is*: $\left(\frac{2+(-6)}{2};\frac{5+(-5)}{2}\right)=(-2\ ;\ 0)$ $AE=\sqrt{(-2-2)^2+(0-5)^2}=\sqrt{41}$ $EB=\sqrt{(-2-(-6))^2+(0-(-5))^2}=\sqrt{41}$ ∴ In ΔABG: **AE = EB** and AF = FG ∴ EF ∥ BG [midpoint theorem/*middelpuntst*]	✓ subst A & B into midpt formula ✓ lengths of AE & EB ✓ AE = EB or E = midpt ✓ Reason (4)
	OR/*OF* Equation of AB: $y-(-5)=\left(\frac{5-(-5)}{2-(-6)}\right)(x-(-6))$ $y+5=\frac{10}{8}x+\frac{15}{2}\quad\therefore y=\frac{5}{4}x+\frac{5}{2}$ x-intercept of AB: $0=\frac{5}{4}x+\frac{5}{2}\quad\therefore x=-2$ ∴ E(–2 ; 0) $m_{EF}=\frac{3\frac{1}{2}-0}{3-(-2)}=\frac{7}{10}$ $m_{EF}=m_{BG}=\frac{7}{10}$ ∴ EF ∥ BG BG: $7x-10y=8$ ∴ $y=\frac{7}{10}x-\frac{8}{10}$ ∴ $m_{BG}=\frac{7}{10}$	✓ equation of AB ✓ coordinates of E ✓ gradient of EF ✓ gradient EF = gradient BG (4)

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3.4	Midpoint of AC $= \left(5 ; \frac{1}{2}\right)$ $\frac{x_D + (-6)}{2} = 5$ and $\frac{y_D + (-5)}{2} = \frac{1}{2}$ $\therefore$ D(16 ; 6)	✓✓ $\left(5 ; \frac{1}{2}\right)$ ✓ x value ✓ y value (4)
	OR/*OF* by translation/*dmv translasie*: D(16 ; 6)	✓ method finding x ✓ method finding y ✓ x value ✓ y value (4)
	OR/*OF* $m_{BC} = \frac{-5-(-4)}{-6-8} = \frac{1}{14}$ and $m_{AB} = \frac{5-(-5)}{2-(-6)} = \frac{5}{4}$ AD: $y - 5 = \frac{1}{14}(x-2) \Rightarrow y = \frac{1}{14}x + \frac{34}{7}$ CD: $y + 4 = \frac{5}{4}(x-8) \Rightarrow y = \frac{5}{4}x - 14$ $\frac{5}{4}x - 14 = \frac{1}{14}x + \frac{34}{7}$ $\therefore \quad x = 16$ $y = 6$	✓✓ equating ✓ x value ✓ y value (4) **[17]**

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QUESTION/*VRAAG* 3

3.1	$m_{BG} = \frac{-4-4}{-3-1}$ $= 2$	✓ substitution/*vervanging* ✓ $m_{BG} = 2$ (2)
3.2	$y = 2x + c$ $sub(-3;-4)$ $-4 = 2(-3) + c$ $\therefore c = 2$ $y = 2x + 2$ **OR/*OF*** $y = 2x + c$ $sub(1;4)$ $4 = 2(1) + c$ $\therefore c = 2$ $y = 2x + 2$ **OR/*OF*** $y - y_1 = m(x - x_1)$ $y + 4 = 2(x - (-3)$ $y = 2x + 2$ **OR/*OF*** $y - y_1 = m(x - x_1)$ $y - 4 = 2(x - 1)$ $y = 2x + 2$	✓ substitution/*vervanging* (–3; –4)/(1,4) ✓equation/*vergelyking* (2) **OR/*OF*** ✓ substitution/*vervanging* (–3; –4)/(1,4) ✓ equation/*vergelyking* (2)
3.3.1	$m_{BC} = -\frac{1}{2}$ $-\frac{1}{2} = \frac{k-(-4)}{2-(-3)}$ $-\frac{1}{2} = \frac{k+4}{5}$ $-5 = 2k + 8$ $2k = -13$ $k = -\frac{13}{2} = -6\frac{1}{2}$	✓ $m_{BC} = -\frac{1}{2}$ ✓ correct substitution/*korrekte vervanging* ✓simplification/*vereenvoudiging* ✓ k value/*k-waarde* (4)
3.3.2	By translation/*Deur translasie* $(x;y) \rightarrow \left(x+5; y-2\frac{1}{2}\right)$ $(1;4) \rightarrow \left(6; 1\frac{1}{2}\right)$ $D\left(6; \frac{3}{2}\right)$	✓ method/*metode* ✓ x value/*x-waarde* ✓ y value/*y-waarde* (3)

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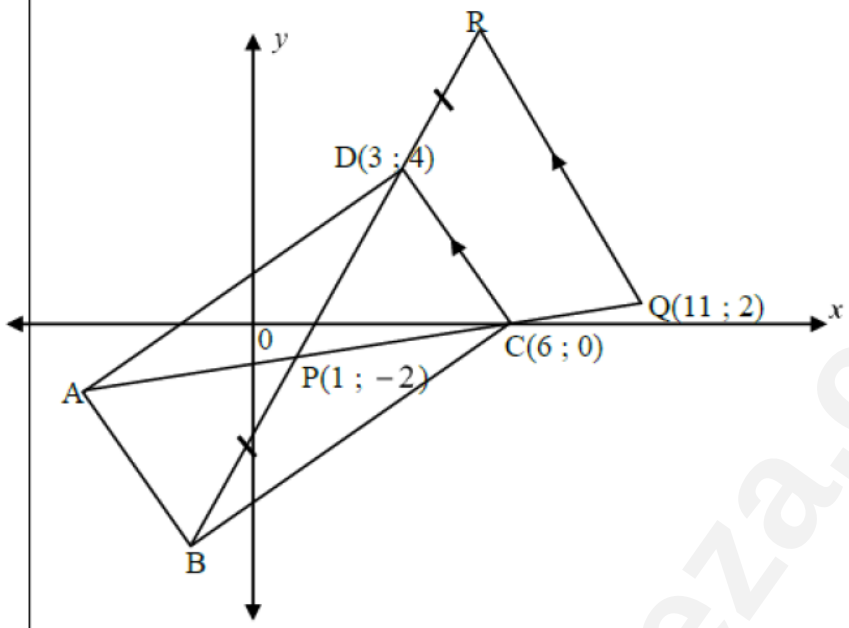
	OR/*OF*	**OR/*OF***
	$\frac{y-\left(-\frac{13}{2}\right)}{x-2}=\frac{2}{1}$	
	$y=2x-\frac{21}{2}$ .....(i)	✓ method/*metode*
	$\frac{y-4}{x-1}=-\frac{1}{2}$	
	$2y-8=-x+1$	
	$2\left(2x-\frac{21}{2}\right)-8=-x+1$	
	$4x-21-8=-x+1$	
	$\therefore x=6$	✓*x* value/*x-waarde*
	$y=2(6)-\frac{21}{2}$	
	$\therefore y=\frac{3}{2}$	✓*y* value/*y-waarde*
	$D\left(6;\frac{3}{2}\right)$	(3)
3.3.3	$m_{CD}=2$	✓ $m_{CD}=2$
	$\tan D\hat{H}X=2$	
	$D\hat{H}X=\tan^{-1}(2)$	✓ $D\hat{H}X$
	$D\hat{H}X=63{,}49°$	
	$\beta=63{,}49°$ [vertically opposite angles/*vertikaal teenoorgestelde hoeke*]	✓ $\beta=63{,}49°$ (3)
3.3.4	$y=2x+c$	
	$-\frac{13}{2}=2(2)+c$	✓ substitution/ *vervanging* $\left(2;-\frac{13}{2}\right)$
	$-\frac{21}{2}=c$	
	$y=2x-\frac{21}{2}$	
	$0=2x-\frac{21}{2}$	✓ $y=0$
	$\therefore x=5{,}25$	
	$GH=6{,}25$ **OR/*OF***	✓ $GH=6{,}25$
	$Area/Oppvl=\frac{1}{2}GH.y_D$	✓ method/*metode*
		✓ y_D
	$=\frac{1}{2}\times 6{,}25\times 1{,}5$	✓substitution/*vervanging*
	= 4,69 square units/*vierkanteeenhede*	✓ answer/*antwoord* (7)
		[21]

$HD=\sqrt{(6-5{,}25)^2+\left(0-\frac{3}{2}\right)^2}$
$=\frac{3\sqrt{5}}{4}$
$G\hat{H}D=180°-63{,}49°$ [∠'s in straight line/∠° *in reguitlyn*]
$=116{,}51°$
Area of/*Oppvl van* $GHD=\frac{1}{2}(GH)(HD)\sin G\hat{H}D$
$=\frac{1}{2}(6{,}25)\left(\frac{3\sqrt{5}}{4}\right)\sin 116{,}51°$
= 4,69 square units/*vierkanteeenhede*

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QUESTION / *VRAAG* 3

3.1		
3.1.1	$1=\frac{3+x}{2}$ $\quad -2=\frac{4+y}{2}$ $2=3+x$ $\quad -4=4+y$ $x=-1$ $\quad y=-8$ $B(-1;-8)$	✓$1=\frac{3+x}{2}$ ✓$-2=\frac{4+y}{2}$ ✓ B(−1;−8) (3)
3.1.2	$m_{CD}=\frac{0-4}{6-3}$ $=-\frac{4}{3}$	✓substitution into gradient formula / *vervang in gradient formule* ✓ $m_{CD}=-\frac{4}{3}$ (2)
3.1.3	$y-2=\frac{-4}{3}(x-11)$ $y=\frac{-4}{3}x+\frac{50}{3}$ **OR / *OF*** $y=\frac{-4}{3}x+c$ $2=\frac{-4}{3}(11)+c$ $c=\frac{50}{3}$ $y=\frac{-4}{3}x+\frac{50}{3}$	✓substitute m /*vervang* m ✓substitute / *vervang* Q(11;2) (2)

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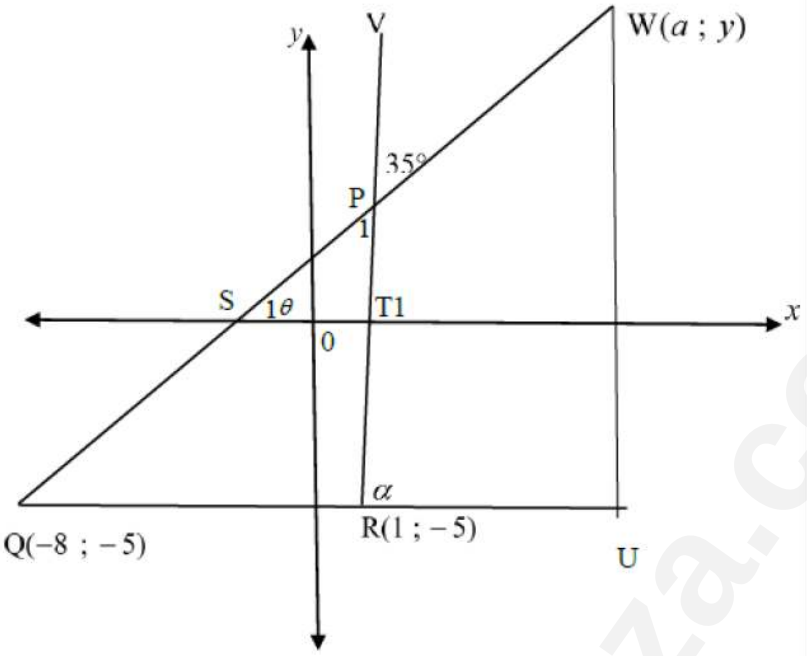
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3.1.4	$CD = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$ **OR / *OF*** $= \sqrt{(0-4)^2 + (6-3)^2}$ $= \sqrt{25}$ $CD = 5$ R(5;10) midpoint / *middelpunt* $RQ = \sqrt{(2-10)^2 + (11-5)^2}$ $RQ = 10$ D is the midpoint of PR / *D is die middelpunt van PR* C is the midpoint of PQ (line from midpoint of 1 side ‖ to 2nd side)/ *C is die middelpunt van PQ (lyn van middelpunt van 1 sy ‖ aan 2de sy)* $RQ = 2CD = 10$ (midpoint theorem/ *middelpuntstelling*) $PK = RQ$ $\sqrt{(y+2)^2 + (4-1)^2} = 10$ $(y+2)^2 + (4-1)^2 = 10^2$ $(y+2)^2 = 91$ **or / *of*** $y+2 = \pm\sqrt{91}$ $y = \pm\sqrt{91} - 2$ $y = -11{,}54$ **or / *of*** $y \neq 7{,}54$ $\sqrt{(y+2)^2 + (4-1)^2} = 10$ $y^2 + 4y + 4 + 9 = 100$ $y^2 + 4y - 87 = 0$ $y = \frac{-4 \pm \sqrt{4^2 - 4(1)(-87)}}{2(1)}$ $y = \frac{-4 \pm \sqrt{364}}{2}$ $y = -11{,}54$ **or / *of*** $y \neq 7{,}54$	✓CD = 5 **OR / *OF*** R(5 ; 10) ✓ statement / *bewering* ✓RQ = 10 ✓correct substitution into distance formula / *korrekte vervanging in die afstandsformule* ✓simplification / *vereenvoudiging* ✓ $y = -11{,}54$ (6)

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3.2		
3.2.1	$m_{PQ} = \tan\theta$ $\tan\theta = 1$ $\theta = 45°$ $\hat{P}_1 = 35°$ vertical opp ∠s / *regoorst ∠e* QR ‖ to the *x*-axis / *aan die x-as* $\hat{T}_1 = 35° + 45°$ ext ∠of Δ / *buite ∠ v Δ* $\hat{T}_1 = 80°$ $\alpha = \hat{T}_1 = 80°$ corr ∠s ST‖QR / *ooreenkomstige ∠e ST‖QR* **OR / *OF*** $m_{PQ} = \tan\theta$ $\tan\theta = 1$ $\theta = 45°$ QR ‖ to the *x*-axis / *aan die x-as* $\hat{S}_1 = \hat{Q} = 45°$ corr ∠'s ST‖QR / *ooreenkomstige ∠'e ST‖QR* $\hat{P}_1 = 35°$ vertical opp ∠s / *regoorst ∠e* $\alpha = 35° + 45°$ $\alpha = 80°$ ext ∠of Δ / *buite ∠ v Δ*	✓ $m_{PQ} = \tan\theta$ ✓ $\theta = 45°$ ✓ $\hat{P}_1 = 35°$ ✓ $\hat{T}_1 = 80°$ ✓ $\alpha = \hat{T}_1 = 80°$ **OR / *OF*** ✓ $m_{PQ} = \tan\theta$ ✓ $\theta = 45°$ ✓ $\hat{S}_1 = \hat{Q} = 45°$ ✓ $\hat{P}_1 = 35°$ ✓ $\alpha = \hat{T}_1 = 80°$ (5)

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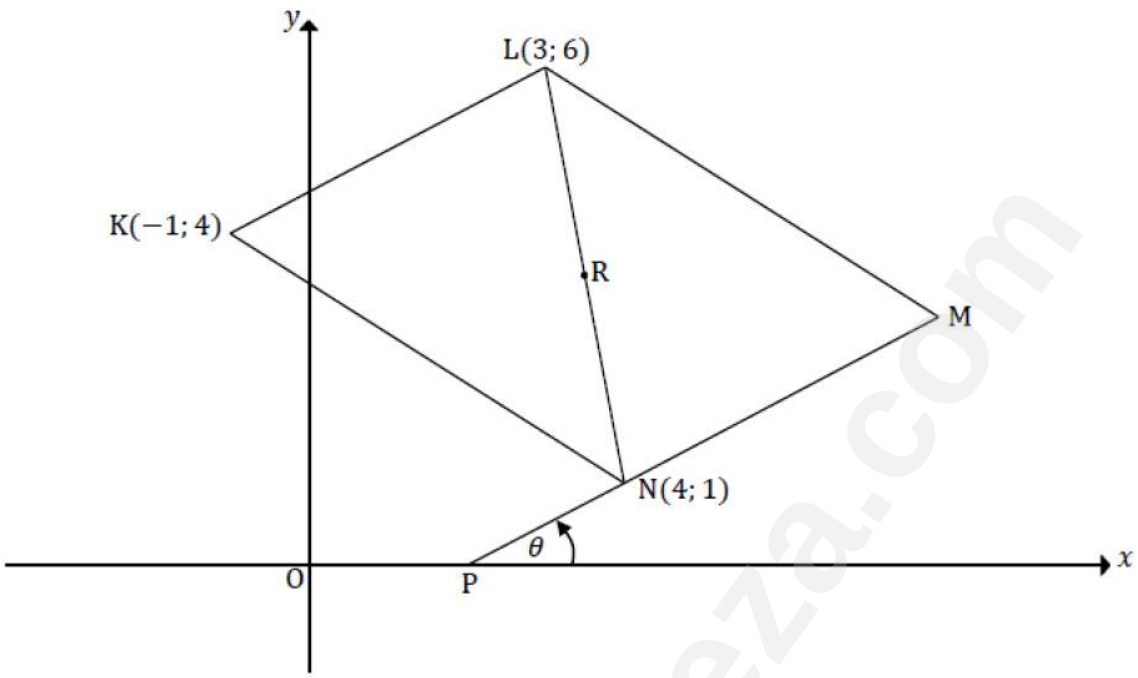
3.2.2	x at/*by* U: $\frac{U_2+(-8)}{2}=1$ $\therefore U_x = 10$ units / *eenhede* QU = 18 units / *eenhede* x at/*by* W = x at / *by* U = 10 y at/*by* W: $y = 10+\frac{2}{3}$ $=\frac{32}{3}$ $WU = \frac{32}{3}+5=\frac{47}{3}$ $\therefore$ Area $\Delta QWU = \frac{1}{2}(18)\left(\frac{47}{3}\right)$ $=141$ square units / *eenhede kwadraat*	✓ $U_x = 10$ units / *eenhede* ✓ QU = 18 units / *eenhede* ✓ U = 10 ✓ $WU = \frac{47}{3}$ ✓ correct substitution in area formula / *korrekte vervanging in oppv. formule* ✓ 141 square units / *eenhede kwadraat* (6)
		[24]

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QUESTION 3



3.1

3.1.1	$m_{KL} = \frac{6-4}{3-(-1)}$ $= \frac{2}{4}$ or $\frac{1}{2}$ answer only: full marks	A✓ substitution in the correct formula CA✓ answer	(2)
3.1.2	midpoint of R : $\left(\frac{x_N + x_L}{2}; \frac{Y_N + Y_L}{2}\right)$: $\left(\frac{3+4}{2}; \frac{6+1}{2}\right)$ R $\left(\frac{7}{2}; \frac{7}{2}\right)$ answer only: full marks	A✓ substitution in the correct formula CA✓ x CA✓ y	(3)
3.1.3	co-ordinates of M: $\frac{x-1}{2} = \frac{7}{2}$ $x - 1 = 7 \quad \therefore x = 8$ $\frac{y+4}{2} = \frac{7}{2}$ answer only: full marks $y = 7 - 4 \quad \therefore y = 3$ $\therefore$ M (8; 3)	CA✓ $\frac{x-1}{2} = \frac{7}{2}$ CA✓ $\frac{y+4}{2} = \frac{7}{2}$ CA✓ x CA✓ y	(4)
3.2	The equation of NM $m_{NM} = \frac{1}{2}$ [NM ∥ KL] $y - y_1 = m(x - x_1)$ $y - 1 = \frac{1}{2}(x - 4)$ $y = \frac{1}{2}x - 2 + 1 \therefore y = \frac{1}{2}x - 1$	CA✓ gradient CA✓ correct subst. of m and N(4 ; 1) CA✓ equation	(3)

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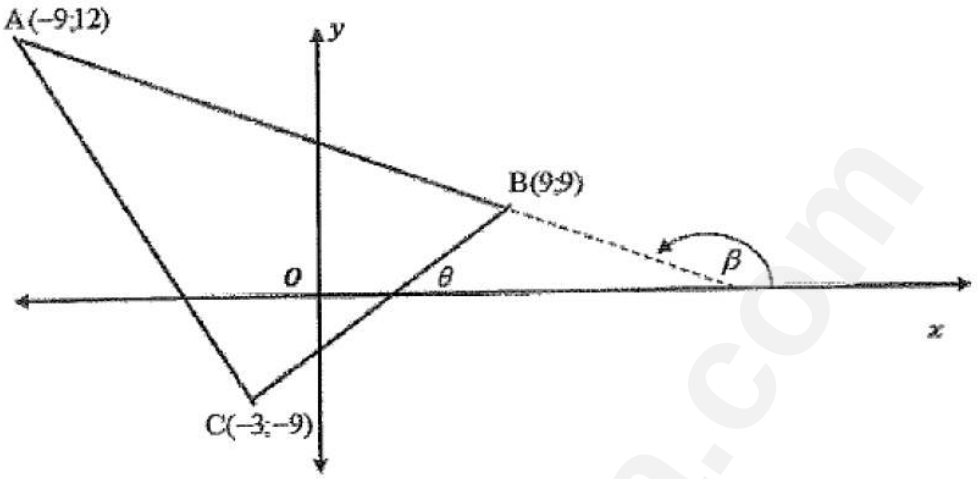
3.3

3.3.1	$0 = \frac{1}{2}x - 1$ $x = 2$ $\therefore$ P $(2; 0)$ answer only: full marks	A✓ y = 0 CA✓ x	(2)
3.3.2	$\tan\theta = \frac{1}{2}$ $\theta = 26{,}57°$ answer only: full marks	CA✓ $\tan\theta = m$ CA✓ answer	(2)
3.3.3	$m_{KP} = \frac{0-4}{2-(-1)} = -\frac{4}{3}$ $\tan^{-1}\left(\frac{-4}{3}\right) = 180° - 53{,}13° = 126{,}87°$ $K\hat{P}N = 126{,}87° - 26{,}57°$ $= 100{,}3°$	CA✓ m_{KP} CA✓ inclination of KP A✓ method CA✓ answer	(4)
			[20]

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QUESTION /VRAAG 3



3.1	$m_{AB} = \frac{9-12}{9+9}$ $= \frac{-1}{6}$	✓subst of A and B/*vervang A en B* ✓answer /*antwoord* (2)
3.2	Extend AB and construct inclination $\angle$'s β and θ. *Verleng AB en konstrueer* $\angle$'e β and θ $\tan\beta = -\frac{1}{6}$ $\beta = 170{,}54°$ $m_{BC} = \frac{9+9}{9+3} = \frac{3}{2}$ $\tan\theta = \frac{3}{2}$ $\theta = 56{,}31°$ $A\hat{B}C = 56{,}31° + 9{,}46°$ $= 65{,}77°$	✓ $\beta = 170{,}54°$ ✓ $\tan\theta = \frac{3}{2}$ ✓ $\theta = 56{,}31°$ ✓Method/metode ✓ $A\hat{B}C = 65{,}77°$ (5)
3.3	$M(\frac{-3+9}{2}; \frac{-9+9}{2})$ $M(3;0)$	✓x-value/waarde ✓y-value/waarde (2)
3.4	$m_{AM} = \frac{12-0}{-9-3} = -1$ $y-0 = -1(x-3)$ $y = -x+3$	✓ $m_{AM} = \frac{12-0}{-9-3}$ ✓Correct substitution/ korrekte vervanging

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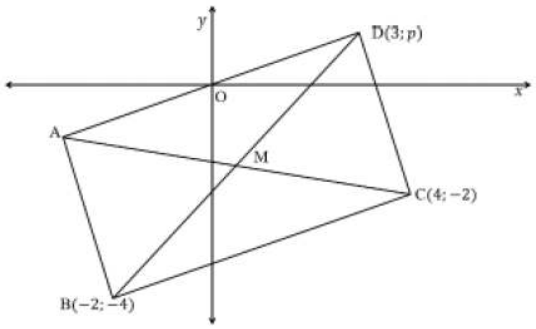
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		✓ equation/vergelyking (3)
3.5	$m_{KM} = \frac{n-0}{m-3} = -1$ $\therefore n = -m+3$ $(m-9)^2 + (n-9)^2 = (5\sqrt{5})^2$ Subst: $(m-9)^2 + (-m+3-9)^2 = (5\sqrt{5})^2$ $(m-9)^2 + (-m-6)^2 = (5\sqrt{5})^2$ $m^2 - 18m + 81 + m^2 + 12m + 36 = 125$ $2m^2 - 6m - 8 = 0$ $m^2 - 3m - 4 = 0$ $(m-4)(m+1) = 0$ $m = 4$ *or* $m = -1$ $n = -(-1) + 3 = 4$ $\therefore K(-1;4)$	✓ $m_{KM} = \frac{n-0}{m-3} = -1$ ✓ $n = -m+3$ ✓Substitution in dist formula/ *vervang in afstandsformule* ✓standard form / *standaard vorm* ✓factors/*faktore* ✓m-values/*waardes* ✓only $m = -1$ ✓ $K(-1;4)$ (8)
		[20]

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QUESTION 3

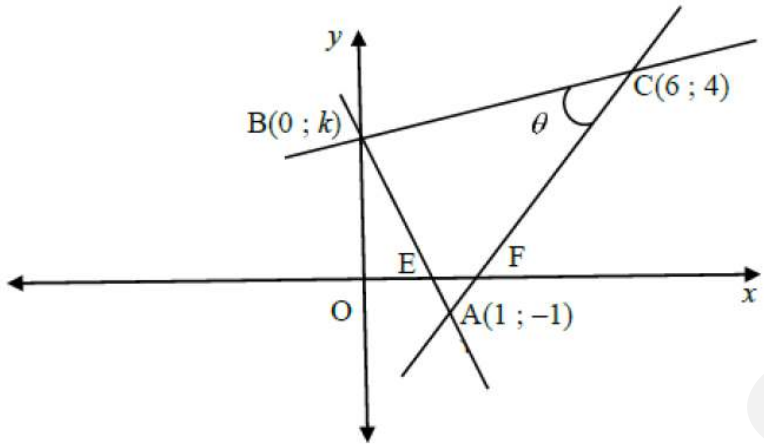


3.1	AC=BD $\sqrt{50} = \sqrt{(3-(-2))^2 + (p-(-4))^2}$ $50 = 25 + p^2 + 8p + 16$ $p^2 + 8p - 9 = 0$ $(p+9)(p-1) = 0$ $p \neq -9\,; p = 1$	✓ *subst into correct formula* ✓ $p^2 + 8p - 9$ ✓ *factors* ✓ *correct p value*	(4) C
3.2	$M\left(\frac{3-2}{2}; \frac{1-4}{2}\right)$ $M\left(\frac{1}{2}; -\frac{3}{2}\right)$	✓ *subst into correct formula* ✓ $\frac{1}{2}$ ✓ $\frac{-3}{2}$ OR ✓✓ Answer only	(3) R
3.3	$m_{DC} = \frac{1-(-2)}{3-4}$ $m_{DC} = \frac{3}{-1}$ $m_{DC} = -3$	✓ *subst into correct formula* ✓ -3 OR ✓✓ Answer only	(2) K
3.4	$y = -3x + c$ $-4 = -3(-2) + c$ $-10 = c$ $y = -3x - 10$	✓ $m = -3$ **CA** from 3.3 ✓ $c = -10$	(2) R
			[11]

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QUESTION 3



3.1	$AB: y+3x-2=0$ $\therefore y=-3x+2$ $\therefore k=2$	✓ $y=-3x+2$ ✓ $k=2$ (2)
3.2	$AC=\sqrt{(1-6)^2+(-1-4)^2}$ $=\sqrt{50}$ $=5\sqrt{2}$	✓ substitution ✓ answer (2)
3.3	$m_{AB}=\frac{-1-2}{1-0}=-3$ $m_{BC}=\frac{4-2}{6-0}=\frac{1}{3}$ $m_{AB}\times m_{BC}=-3\times\frac{1}{3}=-1$ $\therefore A\hat{B}C=90°.$	✓ $m_{AB}=-3$ ✓ $m_{BC}=\frac{1}{3}$ ✓ $m_{AB}\times m_{BC}=-1$ (3)
3.4	$\tan\beta=m_{AC}=1$ $\beta=45°$ $\tan\phi=m_{BC}=\frac{1}{3}$ $\phi=18{,}43°$ $\theta=45°-18{,}43°$ [ext ∠of Δ] $=26{,}57°$ **OR**	✓ $\tan\beta=m_{AC}=1$ ✓ β ✓ ϕ ✓method ✓answer (5)

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	$AC = \sqrt{(1-6)^2 + (-1-4)^2}$ $= \sqrt{50}$ $= 5\sqrt{2}$	✓ $AC = 5\sqrt{2}$
	$AB = \sqrt{(0-1)^2 + (2+1)^2}$ $= \sqrt{10}$	✓ $AB = \sqrt{10}$
	$\sin\theta = \frac{AB}{AC}$ $= \frac{\sqrt{10}}{5\sqrt{2}}$ $= \frac{\sqrt{5}}{2}$	✓ $\sin\theta = \frac{AB}{AC}$ ✓ $\sin\theta = \frac{\sqrt{10}}{5\sqrt{2}}$
	$\therefore \theta = 26.57°$	✓ answer (5)
	OR	
	$\frac{\sin\theta}{AB} = \frac{\sin B}{AC}$ $\sin\theta = \frac{\sqrt{10}\sin 90°}{5\sqrt{2}}$ $= \frac{\sqrt{5}}{5}$ $\therefore \theta = 26{,}57°$	✓ $\frac{\sin\theta}{AB} = \frac{\sin B}{AC}$ ✓ $AC = 5\sqrt{2}$ ✓ $AB = \sqrt{10}$ ✓ $\sin\theta = \frac{\sqrt{10}\sin 90°}{5\sqrt{2}}$ ✓ answer (5)

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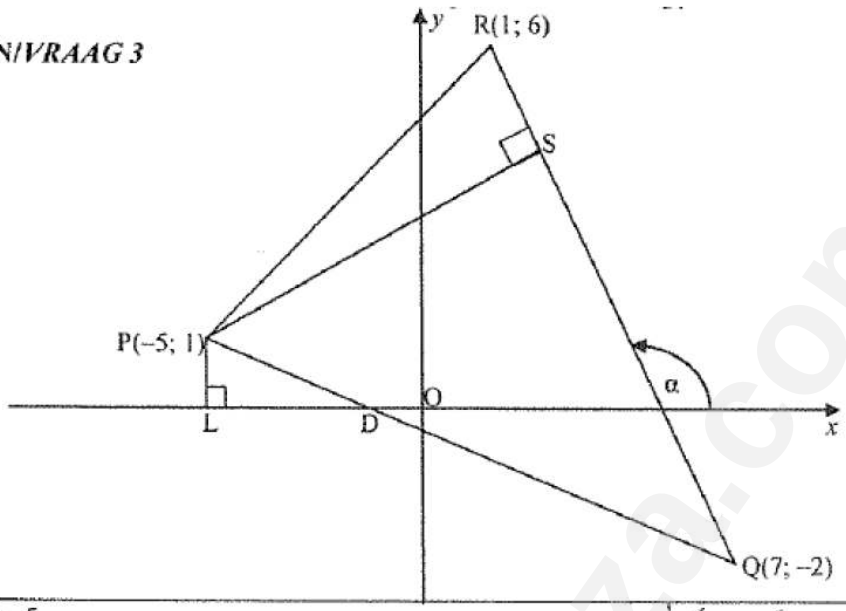
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3.5	$\hat{BDC} = 90^\circ$ altitude $\therefore$ BC is the diameter (line subtends $90^\circ\angle$)	✓ BC is diameter
	midpoint of BC : $\left(\frac{0+6}{2};\frac{2+4}{2}\right)$ $= (3;3)$	✓ $(3;3)$
	$(x-3)^2+(y-3)^2 = r^2$ subst B$(0;2)$ $(0-3)^2+(2-3)^2 = r^2$ $\therefore r^2 = 10$ $(x-3)^2+(y-3)^2 = 10$	✓ Substitution of (0 ; 2) ✓ equation (4)
	OR	
	$\hat{BDC} = 90^\circ$ altitude $\therefore$ BC is the diameter (line subtends $90^\circ\angle$)	✓ BC is diameter
	midpoint of BC : $\left(\frac{0+6}{2};\frac{2+4}{2}\right)$ $= (3;3)$	✓ $(3;3)$
	$(x-3)^2+(y-3)^2 = r^2$ subst B$(0;2)$ $(0-3)^2+(2-3)^2 = r^2$ $\therefore r^2 = 10$ $(x-3)^2+(y-3)^2 = 10$	✓ Substitution of (0 ; 2) ✓ equation (4)
3.6	$M_{AC} = \left(\frac{1+6}{2};\frac{2+1}{2}\right)$ $= \left(\frac{7}{2};\frac{3}{2}\right)$	✓ $\frac{7}{2}$ ✓ $\frac{3}{2}$
	$\frac{7}{2} = \frac{x+0}{2}$ $\qquad$ $\frac{3}{2} = \frac{2+y}{2}$ $x = 7$ $\qquad$ $y = 1$ D(7 ; 1)	✓ $x = 7$ ✓ $y = 1$ (4)
		[20]

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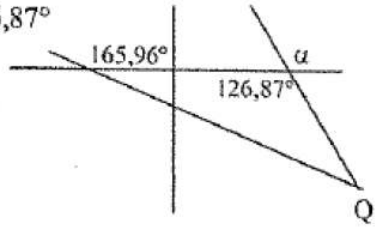
QUESTION/*VRAAG 3*



3.1	$x = -5$	✓ $x = -5$ (1)
3.2	$m_{QR} = \frac{-2-6}{7-1}$ $= \frac{-8}{6}$ $= -\frac{4}{3}$	✓Method/*Metode* ✓Answer/*Antwoord* (2)
3.3	PS ⊥ QR $\therefore m_{PS} \times m_{QR} = -1$ $\therefore m_{PS} = \frac{3}{4}$ Through the point / *Deur die punt* $(-5;1)$ $y-1 = \frac{3}{4}(x+5)$ $y = \frac{3}{4}x + \frac{15}{4} + 1$ $\therefore y = \frac{3}{4}x + \frac{19}{4}$	✓ $m_{PS} \times m_{QR} = -1$ [PS ⊥ QR] ✓ $m_{PS} = \frac{3}{4}$ ✓Subst./*Vervang* $(-5;1)$ ✓ $y = \frac{3}{4}x + \frac{19}{4}$ (4)
3.4	$m_{PQ} = \frac{1-(-2)}{-5-7}$ $= \frac{3}{-12}$ $= -\frac{1}{4}$ $\tan P\hat{D}O = \frac{-1}{4}$ $\therefore P\hat{D}O = -14{,}04° + 180° = 165{,}96°$	✓ $m_{PQ} = -\frac{1}{4}$ ✓ $\tan P\hat{D}O = \frac{-1}{4}$ ✓165,96° (3)

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3.5	$\tan\alpha = \frac{-4}{3}$ $\therefore \alpha = -53{,}13° + 180° = 126{,}87°$  $165{,}96° = 126{,}87° + P\hat{Q}R$ $\therefore P\hat{Q}R = 39.09°$	✓ $\tan\alpha = \frac{-4}{3}$ ✓ $\alpha = 126{,}87°$ ✓ $165{,}96° = 126{,}87° + P\hat{Q}R$ ✓39.09° (4)
3.6	$\frac{\text{area }\Delta PRS}{\text{area }\Delta PQS} = \frac{RS}{QS} = \frac{4}{16} = \frac{1}{4}$ [same height ; same base/*dieselfde hoogte ; dieselfde basis*] $\therefore RS = a$ and / *en* $QS = 4a$ Length of QR / *Lengte van QR* : $d_{QR} = \sqrt{(1-7)^2 + (6-(-2))^2}$ $d_{QR} = \sqrt{100} = 10$ units / *eenhede* $a + 4a = 10$ $\therefore 5a = 10$ $\therefore a = 2$ Length of QS / *Lengte van QS* $= 4(2) = 8$ units / *eenhede* **OR/*OF*** $QR^2 = (1-7)^2 + (6-(-2))^2$ $QR = 10$ $\frac{\text{area }\Delta PQS}{\text{area }\Delta PQR} = \frac{\frac{1}{2}PQ.QS.\sin Q}{\frac{1}{2}PQ.QR.\sin Q}$ $\frac{16x^2}{20x^2} = \frac{QS}{10}$ $\frac{4}{5} = \frac{QS}{10}$ $SQ = \frac{4}{5}(10)$ $= 8$ units / *eenhede*	✓✓ $\frac{\text{area }\Delta PRS}{\text{area }\Delta PQS} = \frac{1}{4}$ ✓Method/*Metode* ✓ $d_{QR} = 10$ ✓ QS = 8 (5) ✓Method/*Metode* ✓QR = 10 ✓✓ $\frac{\text{area }\Delta PQS}{\text{area }\Delta PQR} = \frac{4}{5}$ ✓Answer/*Antwoord* (5) [19]

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CIRCLE ANALYTICAL GEOMETRY

QUESTION/*VRAAG* 4			
4.1	$x^2 - 2x + y^2 + 4y = a$ $(x-1)^2 + (y+2)^2 = a + 1 + 4$ $(x-1)^2 + (y+2)^2 = a + 5$ $\therefore a + 5 = r^2$ $a + 5 = 25$ $a = 20$	✓✓completing the square/*voltooiing van vierkant* ✓equating r^2/*gelyk stel aan* r^2 ✓$r^2 = 25$	(4)
4.2	M (1; -2)	✓ answer in coordinate form/ *antwoord in koördinaat vorm*	(1)
4.3	$x^2 - 2x + y^2 + 4y = 20$ A(x; y) $\therefore (4)^2 - 2(4) + y^2 + 4y = 20$ $16 - 8 + y^2 + 4y = 20$ $y^2 + 4y - 12 = 0$ $(y + 6)(y - 2) = 0$ $\therefore$ y = -6 or/of y = 2 $y = 2$ $A(4; 2)$ $y > 0$	✓ substituting into correct formula/*vervanging in korrekte formule* ✓standard form/*standaardvorm* ✓ factors/*faktore* ✓correct answer/*korrekte antwoord*	(4)
4.4	$m_{MA} = \frac{2+2}{4-1} = \frac{4}{3}$ $m_{tangent} = -\frac{3}{4}$ $y - y_1 = m(x - x_1)$ $\therefore y - 2 = -\frac{3}{4}(x - 4)$ $y = -\frac{3}{4}x + 3 + 2$ $\therefore y = -\frac{3}{4}x + 5$	✓$m_{MA} = \frac{4}{3}$ ✓$m_{tan.} = -\frac{3}{4}$ ✓substituting into correct formula/*vervanging in korrekte formule* ✓answer/*antwoord*	(4)

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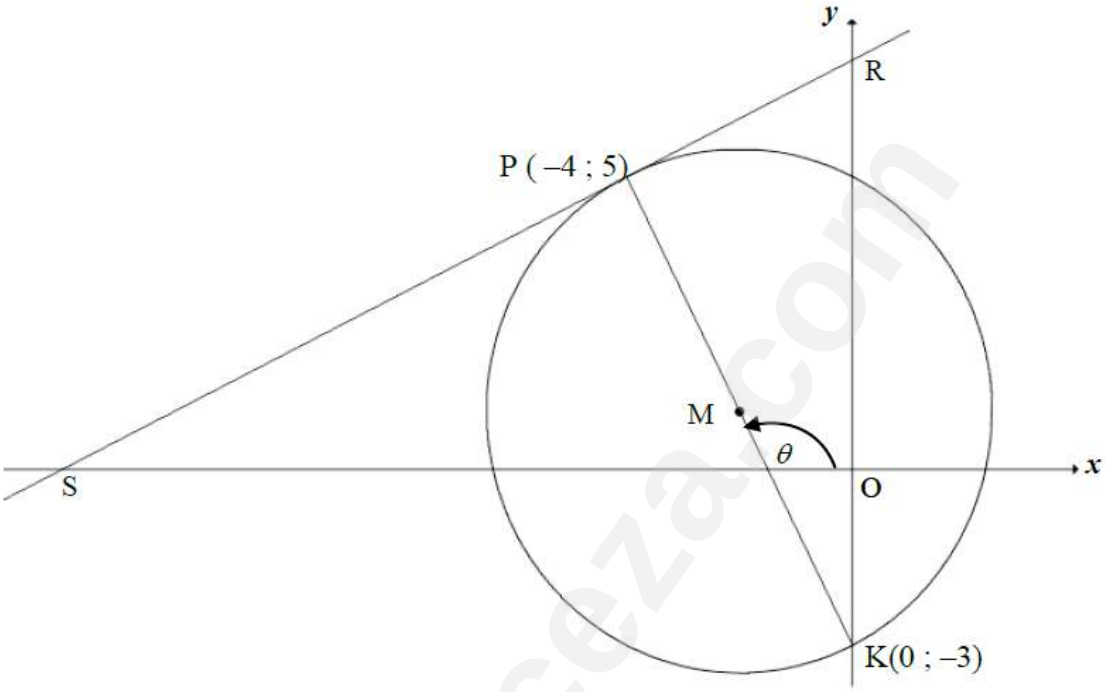
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4.5	M(1; -2) , T(-1;-2) MT $=\sqrt{(-1-1)^2+(-2+2)^2}$ $=2$ $2<\sqrt{20}$ **OR/*OF*** $x^2-2x+y^2+4y=20$ T (-1; -2) LHS $=(-1)^2-2(-1)+(-2)^2+4(-2)$ LHS $= 1+2+4-8$ LHS = -1 LHS $\neq$ RHS -1 < 20 $\therefore$ T lies inside the circle/*T lê binne die sirkel*	✓ substitution/*vervanging* ✓simplification/*vereenvoudiging* ✓ answer/*antwoord*	(3)
4.6	$(x-1)^2+(y+2)^2=20$ $(x-1+3)^2+(y+2-1)^2=20$ $(x+2)^2+(y+1)^2=20$	✓ substitution into correct formula/*vervanging in korrekte formule* ✓ ✓ simplification and answer $(x+2)$ and $(y+1)$ / *vereenvoudiging en antwoord* $(x+2)$ *en* $(y+1)$	(3)
			[19]

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QUESTION/*VRAAG* 4



4.1.1	$m_{PK} = \frac{5-(-3)}{-4-0}$ $= -2$ PK ⊥ SR [radius ⊥ tangent/*raaklyn*] $\therefore\ m_{PK} \times m_{RS} = -1$ $\therefore m_{RS} = \frac{1}{2}$	✓ substitution P & K into gradient formula ✓ gradient of PK ✓ PK ⊥ SR **OR** r ⊥ tangent ✓ answer (4)
4.1.2	$y = \frac{1}{2}x + c$ $5 = \frac{1}{2}(-4) + c$ **OR/*OF*** $(y-5) = \frac{1}{2}(x-(-4))$ $c = 7$ $(y-5) = \frac{1}{2}x + 2$ $y = \frac{1}{2}x + 7$ $y = \frac{1}{2}x + 7$	✓ substitution of m and P ✓ equation (2)

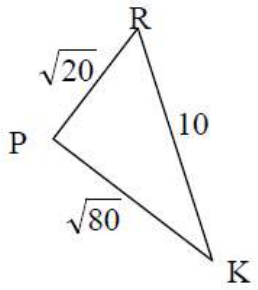
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4.1.3	$M\left(\frac{-4+0}{2};\frac{5+(-3)}{2}\right)$ $\therefore M(-2;1)$ $r^2=(x_2-x_1)^2+(y_2-y_1)^2$ $r^2=(-2+4)^2+(1-5)^2$ $\therefore r^2=20$ $\therefore (x+2)^2+(y-1)^2=20 \text{ or } (\sqrt{20})^2$ **OR/*OF*** $M\left(\frac{-4+0}{2};\frac{5+(-3)}{2}\right) \therefore M(-2;1)$ $(x+2)^2+(y-1)^2=r^2$ $(-4+2)^2+(5-1)^2=r^2$ $\therefore r^2=20$ $\therefore (x+2)^2+(y-1)^2=20 \text{ or } (\sqrt{20})^2$ **OR/*OF*** $M\left(\frac{-4+0}{2};\frac{5+(-3)}{2}\right) \therefore M(-2;1)$ $PK=\sqrt{(-4-0)^2+(5-(-3))^2}=\sqrt{80}$ $r=\frac{\sqrt{80}}{2}=\sqrt{20}$ $\therefore (x+2)^2+(y-1)^2=20 \text{ or } (\sqrt{20})^2$	✓ x value of M ✓ y value of M ✓ $r^2=20$ ✓ equation (4) ✓✓ M (– 2 ; 1) $r^2=20$ ✓ equation (4) ✓✓ M (– 2 ; 1) $r^2=20$ ✓ equation (4)

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4.1.4	$\tan\theta = m_{PK} = -2$ $\therefore\ \theta = 180° - 63{,}43°$ $= 116{,}57°$ $P\hat{K}R = 116{,}57° - 90°$ [ext ∠of ΔMOK] $= 26{,}57°$	✓ $\tan\theta = -2$ ✓ size of θ ✓ answer (3)
	OR/*OF*	
		
	In ΔRPK: $PK = \sqrt{(0-(-4))^2 + (-3-5)^2} = \sqrt{80}$ $PR = \sqrt{(-4-0)^2 + (5-7)^2} = \sqrt{20}$ $RK = 10$	✓ lengths of PK, PR & RK
	$\cos P\hat{K}R = \frac{PK^2 + KR^2 - PR^2}{2.PK.KR} = \frac{(\sqrt{80})^2 + (10)^2 - (\sqrt{20})^2}{2(\sqrt{80})(10)}$ $= \frac{2\sqrt{5}}{5}$ $P\hat{K}R = 26{,}57°$	✓ correct values into cos rule ✓ answer (3)
	OR/*OF*	
	$\sin P\hat{K}R = \frac{\sqrt{20}}{10}$ **OR/*OF*** $\cos P\hat{K}R = \frac{\sqrt{80}}{10}$ $P\hat{K}R = 26{,}57°$ $P\hat{K}R = 26{,}57°$	✓ lengths of sides ✓ ratio ✓ answer (3)
	OR/*OF*	
	$\tan P\hat{K}R = \frac{\sqrt{20}}{\sqrt{80}}$ $P\hat{K}R = 26{,}57°$	✓ lengths of sides ✓ ratio ✓ answer (3)

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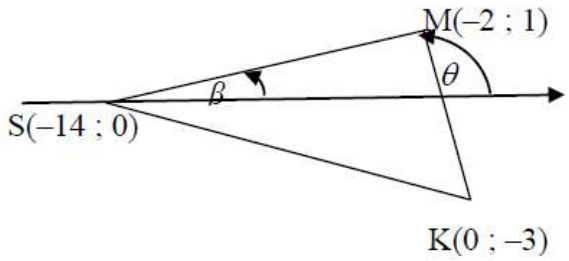
4.1.5	RS $\parallel$ tangent at K(0 ; -3) $\therefore m_{PS} = m_{tang} = \frac{1}{2}$ $\therefore y = \frac{1}{2}x - 3$ **OR/*OF*** $m_{PK} = \frac{1-5}{-2+4} = -2$ $m_{PK} \times m_{tang} = -1$ [radius $\perp$ tangent/*raaklyn*] $\therefore m_{tang} = \frac{1}{2}$ $\therefore y = \frac{1}{2}x - 3$	✓ gradient ✓ equation (2) ✓ gradient ✓ equation (2)
4.2	$t \in (-3 ; 7)$ **OR/*OF*** $-3 < t < 7$	✓ -3 (A) ✓ 7 (CA from 4.1.2) ✓ correct inequality (3) ✓ -3 (A) ✓ 7 (CA from 4.1.2) ✓ correct inequality (3)
4.3	RS: $y = \frac{1}{2}x + 7$ $\therefore$ S(-14 ; 0) $SP = \sqrt{(-14-(-4))^2 + (0-5)^2} = \sqrt{100+25} = \sqrt{125}$ Area $\Delta SMK = \frac{1}{2}$. MK . SP $= \frac{1}{2}(\sqrt{20})(\sqrt{125})$ = 25 square units	✓ coordinates of S ✓ length of SP ✓ correct base & height into Area rule ✓ correct substitution ✓ answer (5)

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OR/OF



Let β = inclination of SM/ *inklinasie van SM*

RS: $y = \frac{1}{2}x + 7 \quad \therefore$ S(–14 ; 0) ✓ coordinates of S

$SM = \sqrt{(-14-(-2))^2 + (0-1)^2} = \sqrt{145}$ ✓ length of SM

$\tan\beta = \frac{1-0}{-2-(-14)} = \frac{1}{12} \quad \therefore \beta = 4{,}76°$

$\therefore S\hat{M}K = 116{,}57° - 4{,}76° \quad$ [ext $\angle$ of Δ]
$= 111{,}81°$ ✓ size of/*grootte v* $S\hat{M}K$

$\text{Area } \Delta SMK = \frac{1}{2}(SM)(MK).\sin S\hat{M}K$

$= \frac{1}{2}(\sqrt{145})(\sqrt{20}).\sin 111{,}81°$ ✓ correct substitution into area rule

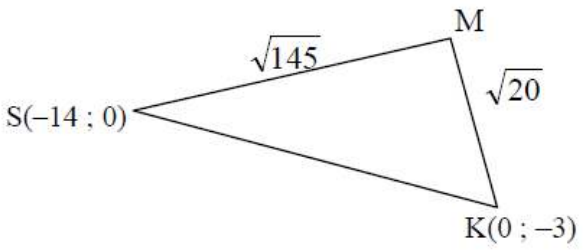
$= 24{,}9985 = 25$ square units ✓ answer

(5)

OR/*OF*

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RS: $y = \frac{1}{2}x + 7 \quad \therefore S(-14\,;\,0)$

$SK = \sqrt{(-14-0)^2 + (0+3)^2} = \sqrt{205}$

$\cos S\hat{M}K = \frac{(\sqrt{145})^2 + (\sqrt{20})^2 - (\sqrt{205})^2}{2(\sqrt{145})(\sqrt{20})} = -\frac{2\sqrt{29}}{29}$

$S\hat{M}K = 111{,}80°$

Area $\Delta SMK = \frac{1}{2}(SM)(MK).\sin S\hat{M}K$

$= \frac{1}{2}(\sqrt{145})(\sqrt{20}).\sin 111{,}81°$

$= 24{,}9985 = 25$ square units

✓ coordinates of S
✓ length of SK
✓ size of /*grootte v* $S\hat{M}K$
✓ correct substitution into area rule
✓ answer

(5)

OR/*OF*

Produce KS to T

RS: $y = \frac{1}{2}x + 7 \quad \therefore S(-14\,;\,0)$

$SK = \sqrt{(-14-0)^2 + (0+3)^2} = \sqrt{205}$

$SM = \sqrt{(-14-(-2))^2 + (0-1)^2} = \sqrt{145}$

$m_{SK} = -\frac{3}{14} \Rightarrow T\hat{S}O = 167{,}91°$

$m_{SM} = \frac{1}{12} \Rightarrow M\hat{S}O = 4{,}76°$

$M\hat{S}K = 180° - 167{,}91° + 4{,}76° = 16{,}85°$

Area $\Delta SMK = \frac{1}{2}(SM)(SK).\sin M\hat{S}K$

$= \frac{1}{2}(\sqrt{145})(\sqrt{205}).\sin 16{,}85°$

$= 24{,}9985 = 25$ square units

✓ coordinates of S
✓ length of SK & SM
✓ size of /*grootte v* $M\hat{S}K$
✓ correct substitution into area rule
✓ answer

(5)

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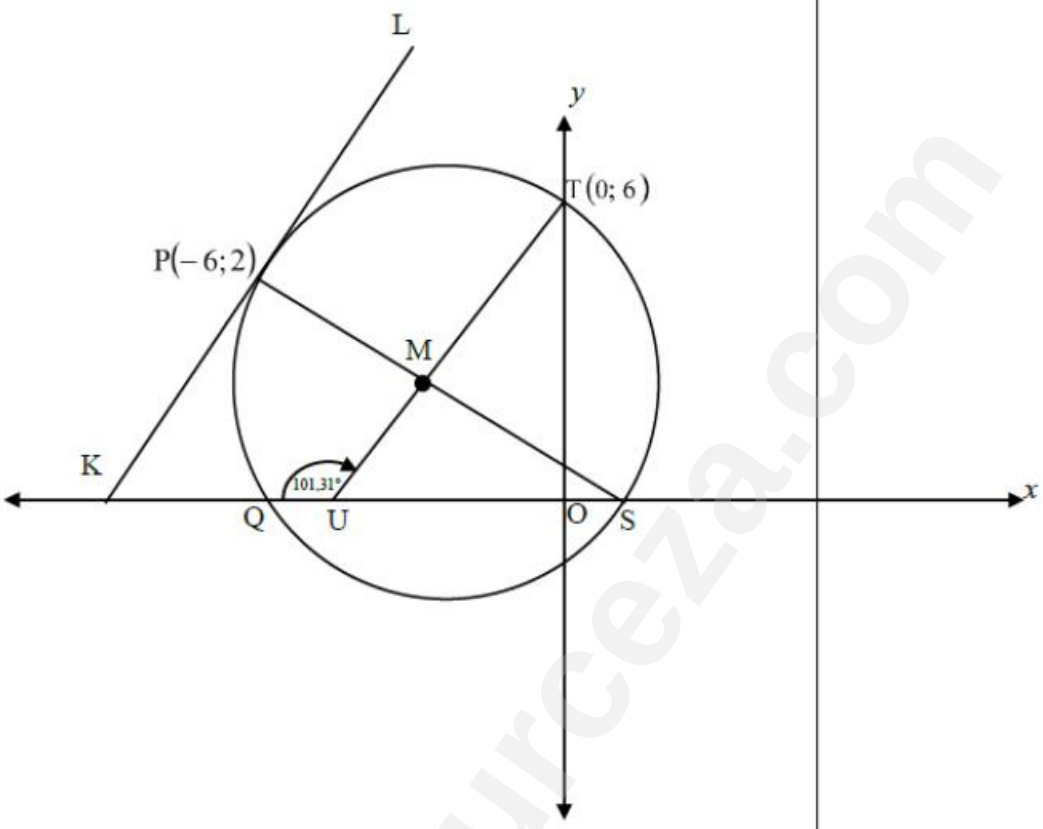
QUESTION/*VRAAG* 4

4.1.1	centre/*middelpunt* E(3;1) $(x-3)^2+(y-1)^2=r^2$ $(5-3)^2+(-5-1)^2=r^2$ $40=r^2$ $(x-3)^2+(y-1)^2=40$ $x^2-6x+9+y^2-2y+1=40$ $x^2-6x+y^2-2y=30$	✓ substitution/*vervanging* E(3; 1) ✓ substitution/*vervanging* P(5; –5) ✓ $40=r^2$ ✓standard form/*standaardvorm* (4)
4.1.2	$m_{rad}=\frac{1-(-5)}{3-5}$ $=\frac{6}{-2}\quad=-3$ $m_{\tan}=\frac{1}{3}\quad$ [radius ⊥ tan] $y=\frac{1}{3}x+c$ $-5=\frac{1}{3}(5)+c$ $c=\frac{-20}{3}$ $y=\frac{1}{3}x-\frac{20}{3}$	✓ correct substitution/*korrekte vervanging* ✓ $m_{rad}=-3$ ✓ $m_{rad}=\frac{1}{3}$ ✓ substitution/*vervanging* (5; –5) ✓ equation/*vergelyking* (5)
4.2.1	centre/*middelpunt* $=\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}$ $=\frac{3+5}{2};\frac{1-5}{2}$ centre/*middelpunt*: (4; -2)	✓ method/*metode* ✓ x value/*x-waarde* ✓ y value/*y-waarde* (3)
4.2.2	$(x-4)^2+(y+2)^2=r^2$ $(3-4)^2+(1+2)^2=r^2$ $10=r^2$ $r=\sqrt{10}$	✓substitution of centre/*vervanging van middelpunt* ✓ substitution/*vervanging* (3;1) ✓ $=\sqrt{10}$ (3)
4.3	$r=\sqrt{40}$ $EC=\sqrt{(9-3)^2+(3-1)^2}$ $=2\sqrt{10}$ C is on the circumference/C *is op die omtrek*	✓ $r=\sqrt{40}$ ✓distance/*afstand* EC ✓motivation/*motivering* (3)
		[18]

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QUESTION / *VRAAG* 4



4.1	$\hat{TUS} = 180° - 101{,}31° = 78{,}69°$ adj supp ∠s / *aangrensende suppl* ∠e $m_{TU} = \tan 78{,}69° = 5$ $c = 6$ $y = 5x + 6$	✓ $\hat{TUS} = 78{,}69°$ ✓ $m_{TU} = 5$ ✓ $y = 5x + 6$ (3)
4.2	x – int / *afsnit* $y = 0$ $\frac{-1}{5}x + \frac{4}{5} = 0$ $-x + 4 = 0$ $x = 4$ $\therefore S(4;0)$ $M = \left(\frac{-6+4}{2}; \frac{2+0}{2}\right)$ $\therefore M(-1;1)$ **OR / *OF***	✓ S(4;0) ✓ substitute correctly / *korrekte vervanging* ✓ M(−1;1)

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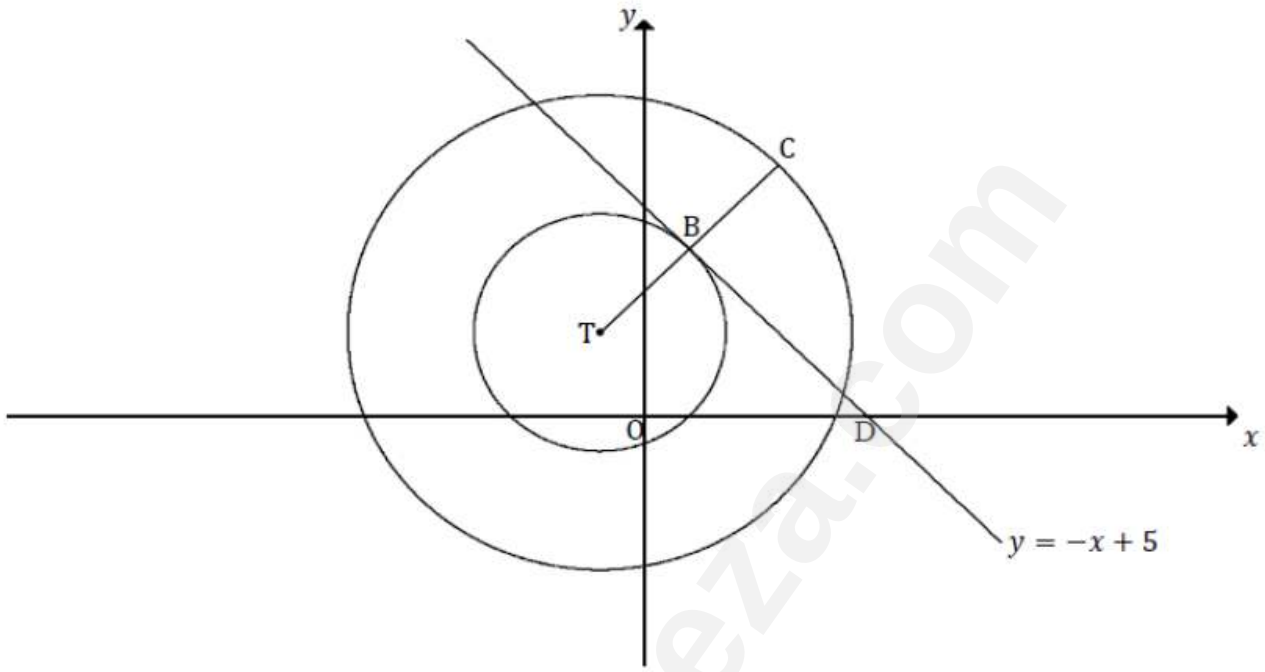
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	$5x+6=-\frac{1}{3}x+\frac{4}{5}$ $\frac{26}{5}x=-\frac{26}{5}$ At M: $x=-1$ $y=-1$ $\therefore$ M(−1;1) **CANDIDATE MUST SHOW CALCULATIONS TO GET MARKS IN THIS QUESTION / *KANDIDAAT MOET BEREKENINGS TOON OM PUNTE IN HIERDIE VRAAG TE VERDIEN***	✓ $5x+6=-\frac{1}{3}x+\frac{4}{5}$ ✓ $x=-1$ ✓ $y=1$ (3)
4.3	$(x+1)^2+(y-1)^2=r^2$ $(-6+1)^2+(2-1)^2=r^2$ $r^2=26$ $(x+1)^2+(y-1)^2=26$ **OR / *OF*** $(x+1)^2+(y-1)^2=r^2$ $(4+1)^2+(0-1)^2=r^2$ $r^2=26$ $(x+1)^2+(y-1)^2=26$	✓ substitute / *vervang* (−6;2) ✓ $r^2=26$ ✓ $(x+1)^2+(y-1)^2=26$ **OR / *OF*** ✓ substitute / *vervang* (4;0) ✓ $r^2=26$ ✓ $(x+1)^2+(y-1)^2=26$ (3)
4.4	$m_{MP}=-\frac{1}{5}$ $m_{MP}\times m_{KL}=-1$ $m_{KL}=5$ radius ⊥ tan / *radius ⊥ raaklyn* $m_{TU}=5$ proven / *reeds bewys* $\therefore m_{TU}=m_{KL}=5$ KL ∥ TU	✓ $m_{MP}=-\frac{1}{5}$ ✓ $m_{KL}=5$ ✓ $m_{TU}=5$ (3)
4.5	$VM=\sqrt{\left(-1+\frac{1}{2}\right)^2+(1-7)^2}$ $=6,02$ radius $=\sqrt{26}=5,1$ 6,02>5,1 $\therefore V\left(-\frac{1}{2};7\right)$ does not lie within the circle./ *lê nie binnedie sirkel nie.*	✓ VM = 6,02 ✓ 6,02>5,1 ✓ conclusion / *gevolgtrekking* (3)
		[15]

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QUESTION 4



4.1.	$x^2 + 2x + y^2 - 4y = 27$ $x^2 + 2x + 1 + y^2 - 4y + 4 = 27 + 1 + 4$ $(x+1)^2 + (y-2)^2 = 32$ $\therefore$ T $(-1; 2)$ **OR** $x = \frac{2}{-2} = -1$ $y = \frac{-4}{-2} = 2$	A✓ completing square LHS A✓ LHS CA✓ x CA✓ y AA✓✓ x AA✓✓ y	(4) (4)
4.2	$m_{TB} \times m_{BD} = -1$ [tangent $\perp$ radius] $m_{TB} = 1$ $y = mx + c$ $2 = 1 . (-1) + c$ $\therefore c = 3$ $\therefore$ Equation of TB is $y = x + 3$	A✓ $m_{TB} = 1$ A✓ subst of m andT A✓ value of c	(3)
4.3	equating TB and BD: $x + 3 = -x + 5$ $2x = 2$ $x = 1$ $y = 1 + 3$ or $y = -1 + 5$ $\therefore$B(1 ; 4)	A✓equating CA✓ $x = 1$ CA✓ $y = 4$ CA provided first quadrant.	(3)

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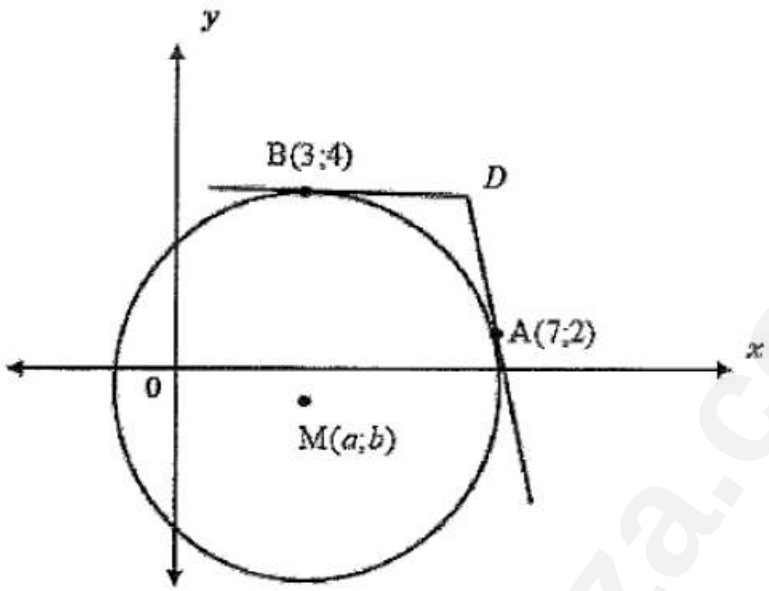
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4.4	$(x+1)^2+(y-2)^2=r^2$ $(1+1)^2+(4-2)^2=r^2$ $8=r^2$ $(x+1)^2+(y-2)^2=8$	CA✓ subst T in circle eq CA ✓ subst B CA✓equation	(3)
4.5	Draw a horizontal line TE with E on BD: Point D(5 ; 0) and Point E: $2=-x+5$ $\therefore$ E(3 ; 2) Area of trap TEDO $=\frac{1}{2}(TE+OD)\times\perp h$ $=\frac{1}{2}(4+5)\times 2$ $=9$ units Area of $\Delta TBE=\frac{1}{2}(TB\times BE)$ $=\frac{1}{2}(\sqrt{8})(\sqrt{8})$ $=4$ units Area of OTBD $=9+4$ $=13$ units^2	A✓ coordinates of D CA✓coordinates of E CA✓ length of TE CA✓ subst in area of trap CA✓subst in correct area of Δ formula A✓ method CA✓ answer	(7)
			[20]

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QUESTION/VRAAG 4



4.1.1	$x^2-6x+9+y^2+2y+1=15+9+1$ $(x-3)^2+(y+1)^2=25$ $M(3;-1)$ Radius = 5	✓LHS /*LK* ✓RHS/*RK* $x^2-6x+9+y^2+2y+1=15+9+1$

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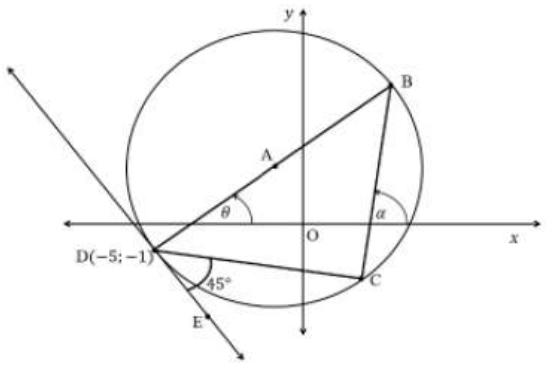
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		✓ $(x-3)^2+(y+1)^2=25$ ✓Midpoint/ *middelpunt* ✓radius (5)
4.1.2	$m_{AM}=\frac{2+1}{7-3}=\frac{3}{4}$ $m_{AD}=-\frac{4}{3}$ $y-2=-\frac{4}{3}(x-7)$ $y=-\frac{4}{3}x+\frac{34}{3}$	✓ $m_{AM}=\frac{2+1}{7-3}=\frac{3}{4}$ ✓m of tangent/ *m van raaklyn* ✓Substitution/ *vervanging* ✓equation /*vergelyking* (4)
4.1.3	Equation of BD: $y=4$ $-\frac{4}{3}x+\frac{34}{3}=4$ $-4x+34=12$ $-4x=-22$ $x=\frac{11}{2}$ $D(\frac{11}{2};4)$	✓equation of BD/ *vergelyking van BD* ✓equating the 2 tangent/ *Stel 2 vergelykings gelyk* ✓x-value/*waarde* ✓y-value/ *waarde* (4)
4.2	ADBM is a kite. Two pair of adjacent sides are equal.	✓statement/*bewering* ✓reason /*rede* (2)
4.3	$MD=\sqrt{(3-\frac{11}{2})^2+(-1-4)^2}=\frac{5\sqrt{5}}{2}$ $AB=\sqrt{(7-3)^2+(2-4)^2}=2\sqrt{5}$ Area ADBM $=\frac{1}{2}\times AB\times MD$ $=\frac{1}{2}\times 2\sqrt{5}\times\frac{5\sqrt{5}}{2}$ $=\frac{25}{2}$ squared units.	✓Length of MD/*lengte van MD* ✓Length of AB/*lengte van AB* ✓formula for area of a kite and correct substitution / *korrekte formule en vervanging* ✓answer /*antwoord* (4)
		[19]

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QUESTION 4



4.1.1	$A(-1;2)$	✓ -1 ✓ 2 A	(2) K
4.1.2	$\frac{x+(-5)}{2} = -1$ $x-5 = -2$ $x = 3$ $\frac{y+(-1)}{2} = 2$ $y-1 = 4$ $y = 5$ $B(3;5)$	✓ *both subst into mpt* ✓ $x = 3$ ✓ $y = 5$ **CA from 4.1.1** OR ✓✓Answer only	(3) C
4.1.3	$m_{AD} = \frac{2-(-1)}{-1-(-5)}$ $= \frac{3}{4}$	✓ *subst.* **CA from 4.1.1** ✓ *gradient*	(2) K
4.1.4	$\tan\theta = \frac{3}{4}$ $\theta = 36{,}87°$	✓ $\tan\theta = \frac{3}{4}$ **CA from 4.1.3** ✓ *answer* **CA**	(2) R
4.1.5	$m_{radius} = \frac{3}{4}$ $m_{tangent} = \frac{-4}{3}$ *radius* ⊥ *tangent* $y = -\frac{4}{3}x + c$ $-1 = -\frac{4}{3}(-5) + c$ $-1 = \frac{20}{3} + c$ $c = -\frac{23}{3}$ $\therefore y = -\frac{4}{3}x - \frac{23}{3}$	✓ $\frac{-4}{3}$ **CA from 4.1.3** **penalise if not neg but CA further.** ✓ *subst* $(-5;-1)$ ✓ $c = -\frac{23}{3}$	(3) R

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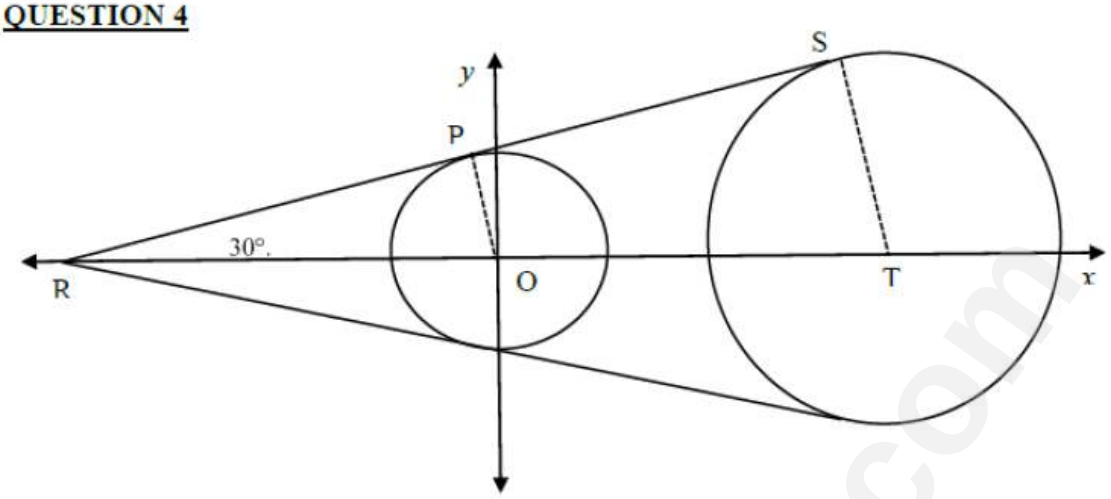
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4.2	$\hat{B} = 45°$ [tan;chord theorem / raaklyn;koord stelling] $\alpha = 45° + 36{,}87°$ [$ext\angle of\ \Delta / buite\angle van\Delta$] $\alpha = 81{,}87°$ $m_{BC} = \tan 81{,}87°$ $m_{BC} = 7$	✓ $\hat{B} = 45°$ ✓ $\alpha = 81{,}87°$ **CA from 4.1.4** ✓ $m_{BC} = \tan 81{,}87°$ ✓ $m_{BC} = 7$	(4) P
4.3.1	$x^2 - 6x + 9 + y^2 + 2y + 1 = 8 + 9 + 1$ $(x-3)^2 + (y+1)^2 = 18$ $\therefore$ M $(3; -1)$	✓ +9 both sides ✓ +1 both sides ✓ $(x-3)^2$ ✓ $(y+1)^2$ *answer given*	(4) R
4.3.2	M$(3; -1)$ and A $(-1; 2)$ $MA = \sqrt{(3-(-1))^2 + (-1-2)^2}$ $MA = \sqrt{16+9}$ $MA = \sqrt{25}$ $MA = 5$ $r_M + r_A = \sqrt{18} + 5$ or $= 9{,}24$ MA$< r_M + r_A$ $\therefore$ *circles intersect/sirkels sny*	✓ *subst* ✓5 ✓ $\sqrt{18} + 5$ ✓*reason for conclusion*	(4) P
			[24]

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QUESTION 4



4.1	T(6; 0)	✓ (6; 0) (1)
4.2	radius = 3	✓ radius = 3 (1)
4.3	$\tan 30° = \frac{\sqrt{3}}{3}$ $0 = \frac{\sqrt{3}}{3}(-3) + c$ $c = \sqrt{3}$ $y = \frac{\sqrt{3}}{3}x + \sqrt{3}$	✓ $\tan 30° = \frac{\sqrt{3}}{3}$ ✓ substitution ✓ $c = \sqrt{3}$ ✓ equation (4)
4.4	$y = -\frac{3}{\sqrt{3}}x + c$ radius ⊥ tangent $0 = -\frac{3}{\sqrt{3}}(6) + c$ $c = \frac{18}{\sqrt{3}}$ $y = -\frac{3}{\sqrt{3}}x + \frac{18}{\sqrt{3}}$	✓ gradient ✓ $c = \frac{18}{\sqrt{3}}$ ✓ equation (3)

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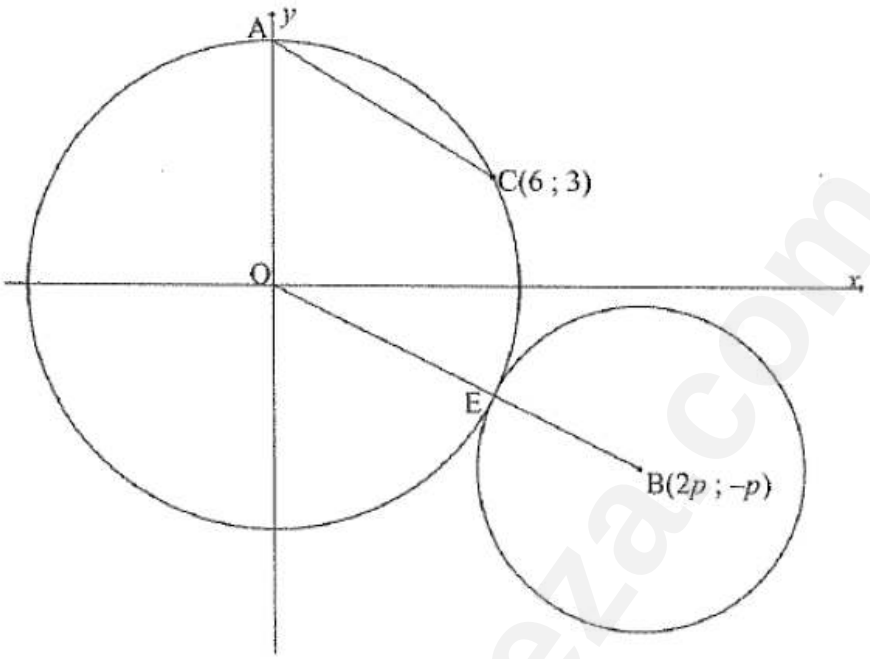
4.5	$\frac{\sqrt{3}}{3}x+\sqrt{3}=-\frac{3}{\sqrt{3}}x+\frac{18}{\sqrt{3}}$ $\frac{9x+54}{3\sqrt{3}}=+\frac{3x+9}{3\sqrt{3}}$ $12x=45$ $x=\frac{15}{4}=3\frac{3}{4}$ $y=\frac{\sqrt{3}}{3}\left(\frac{15}{4}\right)+\sqrt{3}$ $y=\left(\frac{9\sqrt{3}}{4}\right)$ $S\left(\frac{15}{4};\frac{9\sqrt{3}}{4}\right)$	✓equating ✓simplification ✓x-value ✓substitution ✓y-value (5)
4.6	**The distance between centres of circles: OT = 6** $r+R=1+3$ $=4$ The distance between the two circles = OT $-(r+R)$ $=6-4$ $=2$ units	✓ **OT = 6** ✓ $r+R=1+3$ ✓4 ✓2 (4)
		[18]

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QUESTION/*VRAAG 4*



4.1	$x^2+y^2=r^2$ $(6)^2+(3)^2=r^2$ $r^2=45$ $\therefore x^2+y^2=45$	✓$r^2=45$ ✓ answer/*antwoord* (2)
4.2	$A\left(0;\sqrt{45}\right)$	✓0 ✓$\sqrt{45}$ (2)
4.3	$m_{AC}=\frac{3-\sqrt{45}}{6-0}$ $=\frac{3-3\sqrt{5}}{6}$ $=\frac{1-\sqrt{5}}{2}$ Through the point / *Deur die punt* $\left(0;\sqrt{45}\right)$ $y=\frac{1-\sqrt{5}}{2}x+\sqrt{45}$	✓Method/*Metode* ✓ $m_{AC}=\frac{1-\sqrt{5}}{2}$ ✓Equation/*Vergelyking* (3)
4.4	$-\sqrt{45}<k<\sqrt{45}$ or / *of* $-3\sqrt{5}<k<3\sqrt{5}$ alternative notation / *alternatiewe notasie* $k\in(-\sqrt{45};\sqrt{45})$ or / *of* $k\in(-3\sqrt{5};3\sqrt{5})$	✓$-\sqrt{45}$ and / *en* $\sqrt{45}$ ✓Correct notation/ *Korrekte notasie* (2)
4.5.1	$(x-2p)^2+(y+p)^2=20$	✓$(x-2p)^2+(y+p)^2$ ✓ 20 (2)

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4.5.2	$d_{OB} = \sqrt{45} + \sqrt{20}$ $= 5\sqrt{5}$ or / *of* $\sqrt{125}$ $OB^2 = (2p)^2 + (p)^2$ [Pythagoras] $\left(5\sqrt{5}\right)^2 = 4p^2 + p^2$ $125 = 5p^2$ $25 = p^2$ $\therefore p = \pm 5$ But according / *Maar volgens diagram* $p > 0$ $\therefore p = 5$	✓ $d_{OB} = \sqrt{45} + \sqrt{20}$ ✓ $d_{OB} = 5\sqrt{5}$ ✓ $OB^2 = (2p)^2 + (p)^2$ ✓ $p^2 = 25$ ✓ $p = 5$ (5)
4.6	$x^2 + 4x\cos\theta + y^2 + 8y\sin\theta = -3$ $x^2 + 4x\cos\theta + (2\cos\theta)^2 + y^2 + 8y\sin\theta + (4\sin\theta)^2 =$ $-3 + (2\cos\theta)^2 + (4\sin\theta)^2$ $\therefore r^2 = -3 + 4\cos^2\theta + 16\sin^2\theta$ $r^2 = -3 + 4\cos^2\theta + 16(1 - \cos^2\theta)$ $r^2 = -3 + 4\cos^2\theta + 16 - 16\cos^2\theta$ $r^2 = 13 - 12\cos^2\theta$ For / *vir* $13 - 12\cos^2\theta$ to be a maximum / *om 'n maksimum te wees* : $\cos^2\theta$ needs to be a minimum / *moet 'n minimum wees* minimum of / *van* $\cos^2\theta = 0$ $\therefore r^2 = 13 - 12(0) = 13$ $\therefore r = \sqrt{13}$ **OR / *OF*** $r^2 = -3 + 4(1 - \sin^2\theta) + 16\sin^2\theta$ $r^2 = -3 + 4 - 4\sin^2\theta + 16\sin^2\theta$ $r^2 = 1 + 12\sin^2\theta$ For / *vir* $1 + 12\sin^2\theta$ to be a maximum / *om 'n maksimum te wees* : maximum of / *maksimum van* $\sin^2\theta = 1$ $\therefore r^2 = 1 + 12(1) = 13$ $\therefore r = \sqrt{13}$	✓ Complete the square/*Voltooi die vierkant* ✓ $r^2 = -3 + 4\cos^2\theta + 16\sin^2\theta$ ✓ $16(1 - \cos^2\theta)$ ✓ $r^2 = 13 - 12\cos^2\theta$ ✓ $\cos^2 = 0$ ✓ $r = \sqrt{13}$ ✓ $4(1 - \sin^2\theta)$ ✓ $r^2 = -3 + 4 - 4\sin^2\theta + 16\sin^2\theta$ ✓ $r^2 = 1 + 12\sin^2\theta$ ✓ $\sin^2\theta = 1$ ✓ $r = \sqrt{13}$ (6) [22]

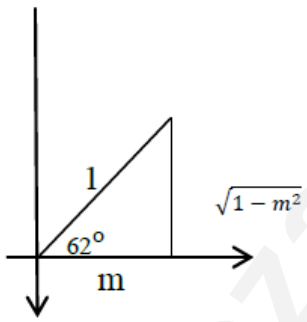
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MODULE 3: TRIGONOMETRY

GENERAL TRIGONOMETRY

QUESTION/*VRAAG* 5

	5.1.1	$\sin 28° = \sin(90° - 62°)$ $= \cos 62°$ $= m$  	✓ equation/*vergelyking* ✓ answer/*antwoord*	(2)
	5.1.2	$\cos 362° = \cos(2°)$ $= \cos(62° - 60°)$ $= \cos 62° \cos 60° + \sin 62° \sin 60°$ $= \frac{1}{2}m + \frac{\sqrt{3}}{2}(\sqrt{(1-m^2)})$	✓ $\cos 2°$ ✓ $62° - 60°$ ✓ expansion/*uitbreiding* ✓ substitution/*vervanging*	(4)
	5.2.1	$\frac{\tan(360° - x).\sin(90° + x)}{\sin(-x)}$ $= \frac{-\tan x.\cos x}{-\sin x}$ $= \frac{\frac{\sin x}{\cos x} \times \cos x}{\sin x}$ $= 1$	✓ $-\tan x$ ✓ $\cos x$ ✓ $-\sin x$ ✓ $\frac{\sin x}{\cos x}$ ✓ answer/*antwoord*	(5)

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5.3	$4\sin^2\theta = 3$ $\sin^2\theta = \frac{3}{4}$ $\therefore \sin\theta = \pm\frac{\sqrt{3}}{2}$ $\therefore \theta = 120°;\ 90° < \theta < 180°$ $= \cos\frac{1}{4}\theta.\sin\frac{1}{2}\theta - \tan(3\theta - 45°)$ $= \cos 30°.\sin 60° - \tan(360° - 45°)$ $= \cos 30°.\sin 60° + \tan 45°$ $= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + 1$ $= 1\frac{3}{4}$	✓ simplification/*vereenvoudiging* ✓ value of/*waarde van* θ ✓substitution of/*vervanging van* θ ✓ Cos $30° = \frac{\sqrt{3}}{2}$ ✓ Sin $60° = \frac{\sqrt{3}}{2}$ ✓ $\tan 45° = 1$	(6) **[17]**

QUESTION/*VRAAG* 5

5.1	$\frac{\sin(A - 360°).\cos(90° + A)}{\cos(90° - A).\tan(-A)}$ $= \frac{\sin A(-\sin A)}{\sin A(-\tan A)}$ $= \frac{\sin A}{\left(\frac{\sin A}{\cos A}\right)}$ $= \cos A$	✓ sin A ✓–sin A ✓ sin A ✓ –tan A ✓ $\tan A = \frac{\sin A}{\cos A}$ ✓ answer (6)
5.2.1	$t^2 = (\sqrt{34})^2 - (3)^2$ $\therefore t = -5$	✓ substitution ✓ answer (2)
5.2.2	$\tan\beta = \frac{-5}{3}$	✓ correct ratio (1)

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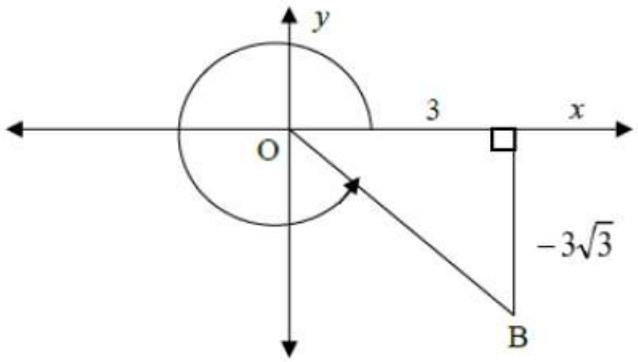
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5.2.3	$\cos 2\beta = 2\cos^2\beta - 1$ $= 2\left(\frac{3}{\sqrt{34}}\right)^2 - 1$ $= 2\left(\frac{9}{34}\right) - 1$ $= -\frac{16}{34}$ OR $-\frac{8}{17}$ **OR/*OF*** $\cos 2\beta = 1 - 2\sin^2\beta$ $= 1 - 2\left(-\frac{5}{\sqrt{34}}\right)^2$ $= 1 - 2\left(\frac{25}{34}\right)$ $= -\frac{16}{34}$ OR $-\frac{8}{17}$ **OR/*OF*** $\cos 2\beta = \cos^2\beta - \sin^2\beta$ $= \left(\frac{3}{\sqrt{34}}\right)^2 - \left(-\frac{5}{\sqrt{34}}\right)^2$ $= \frac{9}{34} - \frac{25}{34}$ $= -\frac{16}{34}$ OR $-\frac{8}{17}$	✓ compound formula ✓ substitution ✓ simplification ✓ answer (4) ✓ compound formula ✓ substitution ✓ simplification ✓ answer (4) ✓ compound formula ✓ substitution ✓ simplification ✓ answer (4)

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QUESTION/VRAAG 5

5.1.1	 $(3)^2+\left(-3\sqrt{3}\right)^2=OB^2$ $OB=6$	✓Pythagoras ✓OB (2)
5.1.2	$\cos(\alpha+30°)$ $=\cos\alpha\cos30°-\sin\alpha\sin30°$ $=\left(\frac{3}{6}\right)\left(\frac{\sqrt{3}}{2}\right)-\left(\frac{-3\sqrt{3}}{6}\right)\left(\frac{1}{2}\right)$ $=\left(\frac{\sqrt{3}}{2}\right)$	✓expansion/*uitbreiding* ✓✓ratios/*verhoudings* ✓answer/*antwoord* (4)
5.2	$\frac{\sin^2(90°-x)\tan(360°-x)}{\sin(-x)}$ $=\frac{\cos^2 x\,(-\tan x)}{(-\sin x)}$ $=\frac{\cos^2 x\left(-\frac{\sin x}{\cos x}\right)}{-\sin x}$ $=\cos x$	✓$\cos^2 x$ ✓$-\tan x$ ✓$-\sin x$ ✓$\frac{\sin x}{\cos x}$ (4)

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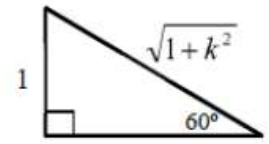
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5.3	$\cos(60°+\theta)-\cos(60°-\theta)$ $=[\cos 60°\cos\theta-\sin 60°\sin\theta]-[\cos 60°\cos\theta+\sin 60°\sin\theta]$ $=-2\sin 60°\sin\theta$ $=-2.\frac{\sqrt{3}}{2}.\sin\theta$ $=-\sqrt{3}\sin\theta$ **OR/*OF*** $\cos(60°+\theta)-\cos(60°-\theta)$ $=[\cos 60°\cos\theta-\sin 60°\sin\theta]-[\cos 60°\cos\theta+\sin 60°\sin\theta]$ $=\frac{1}{2}\cos\theta-\frac{\sqrt{3}}{2}\sin\theta-\frac{1}{2}\cos\theta-\frac{\sqrt{3}}{2}\sin\theta$ $=-2\left(\frac{\sqrt{3}}{2}\right)\sin\theta$ $=-\sqrt{3}\sin\theta$	✓ method/*metode* ✓ ratios/*verhoudings* ✓ answer/*antwoord* (3) **OR/*OF*** ✓ method/*metode* ✓ ratios/verhoudings ✓ answer/*antwoord* (3)
5.4.1	$LHS=\frac{1-\sin 2A}{\sin A-\cos A}$ $=\frac{1-2\sin A\cos A}{\sin A-\cos A}$ $=\frac{\sin^2 A+\cos^2 A-2\sin A\cos A}{\sin A-\cos A}$ $=\frac{(\sin A-\cos A)(\sin A-\cos A)}{\sin A-\cos A}$ $=\sin A-\cos A$ **OR/*OF*** $RHS=\sin A-\cos A\times\frac{\sin A-\cos A}{\sin A-\cos A}$ $=\frac{\sin^2 A-2\sin A.\cos A+\cos^2 A}{\sin A-\cos A}$ $=\frac{1-2\sin A.\cos A}{\sin A-\cos A}$	✓ $2\sin A\cos A$ ✓ $\sin^2 A+\cos^2 A$ ✓ factors/*faktore* ✓ answer/*antwoord* (4) **OR/*OF*** ✓ $\frac{\sin A-\cos A}{\sin A-\cos A}$ ✓ identities/*identiteite* ✓ simplification/ *vereenvoudiging* ✓ answer/*antwoord* (4)
5.4.2	$\sin A-\cos A=0$ $\sin A=\cos A$ $\tan A=1$ $A=45°$	✓ $\sin A-\cos A=0$ ✓ answer/*antwoord* (2)
		[19]

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QUESTION / *VRAAG* 5

5.1.1	$x^2=\left(\sqrt{1+k^2}\right)^2-(1)^2$ (Pythagoras) $x^2=k^2$ $x=k$ $\tan 16°=\frac{1}{k}$  	✓ $x=k$ ✓ $\tan 16°=\frac{1}{k}$ (2)
5.1.2	$\cos 32°$ $=\cos 2(16°)$ $=2\cos^2 16°-1$ $=2\left(\frac{k}{\sqrt{1+k^2}}\right)^2-1$ **OR / *OF*** $\cos 32°$ $=\cos 2(16°)$ $=\cos^2 16°-\sin^2 16°$ $=\left(\frac{k}{\sqrt{1+k^2}}\right)^2-\left(\frac{1}{\sqrt{1+k^2}}\right)^2$ **OR / *OF*** $\cos 32°$ $=\cos 2(16°)$ $=1-2\sin^2 16°$ $=1-2\left(\frac{1}{\sqrt{1+k^2}}\right)^2$	✓ $\cos 2(16°)$ ✓ $2\cos^2 16°-1$ ✓ correct substitution / *korrekte vervanging* **OR / *OF*** ✓ $\cos 2(16°)$ ✓ $\cos^2 16°-\sin^2 16°$ ✓ correct substitution / *korrekte vervanging* **OR / *OF*** ✓ $\cos 2(16°)$ ✓ $1-2\sin^2 16°$ ✓ correct substitution / *korrekte vervanging* (3)
5.2	$\frac{\cos(90°+x)\sin(x-180°)-\cos^2(180°-x)}{\cos(-2x)}$ $=\frac{(-\sin x)(-\sin x)-\cos^2 x}{\cos 2x}$ $=\frac{\sin^2 x-\cos^2 x}{\cos^2 x-\sin^2 x}$ **OR / *OF*** $\frac{-\cos 2x}{\cos 2x}$ $=\frac{-(\cos^2 x-\sin^2 x)}{\cos^2 x-\sin^2 x}$ $=-1$ $=-1$	✓ $-\sin x$ ✓ $-\sin x$ ✓ $-\cos^2 x$ ✓ $\cos 2x$ ✓ $-(\cos^2 x-\sin^2 x)$ **OR / *OF*** $-\cos 2x$ ✓ -1 (6)

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5.3	$\cos 75°.\cos 45° - \cos 15°.\cos 45°$ $= \cos 75°.\cos 45° - \sin 75°.\sin 45°$ $= \cos(75° + 45°)$ $= \cos 120°$ $= -\cos 60°$ $= -\frac{1}{2}$ **OR / *OF*** $\cos 75°.\cos 45° - \cos 15°.\cos 45°$ $= \sin 15°.\cos 45° - \cos 15°.\sin 45°$ $= \sin(15° - 45°)$ $= \sin(-30°)$ $= -\sin 30°$ $= -\frac{1}{2}$	✓ $\cos 75°.\cos 45° - \sin 75°.\sin 45°$ ✓ $\cos(75° + 45°)$ ✓ $-\cos 60°$ ✓ $-\frac{1}{2}$ **OR / *OF*** ✓ $\sin 15°.\cos 45° - \cos 15°.\sin 45°$ ✓ $\sin(15° - 45°)$ ✓ $-\sin 30°$ ✓ $-\frac{1}{2}$ (4)
5.4.1	$\tan\theta\left(\sin 2\theta + \frac{3\cos^2\theta}{\sin\theta}\right)$ $= \frac{\sin\theta}{\cos\theta}\left(2\sin\theta\cos\theta + \frac{3\cos^2\theta}{\sin\theta}\right)$ $= 2\sin^2\theta + 3\cos\theta$ $= 2(1 - \cos^2\theta) + 3\cos\theta$ $= -2\cos^2\theta + 3\cos\theta + 2$	✓ $2\sin\theta\cos\theta$ and / *en* $\frac{\sin\theta}{\cos\theta}$ ✓ simplification / *vereenvoudiging* ✓ $1 - \cos^2\theta$ (3)
5.4.2	$-2\cos^2\theta + 3\cos\theta + 2 = 0$ $2\cos^2\theta - 3\cos\theta - 2 = 0$ $(2\cos\theta + 1)(\cos\theta - 2) = 0$ $\cos\theta = -\frac{1}{2}$ **or / *of*** $\cos\theta = 2$ no solution / *geen oplossing* ref / *verwy* $\angle = 60°$ $\theta = \pm 120° + k360°; k \in Z$ **OR / *OF*** $\theta = 120° + k360°; k \in Z$ $\theta = 240° + k360°; k \in Z$	✓ factors / *faktore* $(2\cos\theta + 1)(\cos\theta - 2)$ ✓ both equations / *beide vergelykings* $\cos\theta = -\frac{1}{2}$ **or / *of*** $\cos\theta = 2$ ✓ no solution / *geen oplossing* ✓ $\theta = \pm 120° + k360°\ \ k \in Z$ **OR / *OF*** $\theta = 120° + k360°; k \in Z$ $\theta = 240° + k360°; k \in Z$ (4)

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5.5	$\cos(a+b) = -\frac{\sqrt{2}}{2}$ ref $\angle$ / verw $\angle = 45°$ $a+b = 180° - 45°$ $a+b = 135°$ ..... (1)	✓ $a+b = 135°$
	$\cos(a-2b) = \frac{1}{2}$ ref $\angle$ / verw $\angle = 60°$ $a-2b = 60°$ ..... (2)	✓ $a-2b = 60°$
	$3b = 75°$ (1) − (2) $b = 25°$ $a = 110°$	✓ $b = 25°$ ✓ $a = 110°$ (4)
		[26]

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QUESTION 5

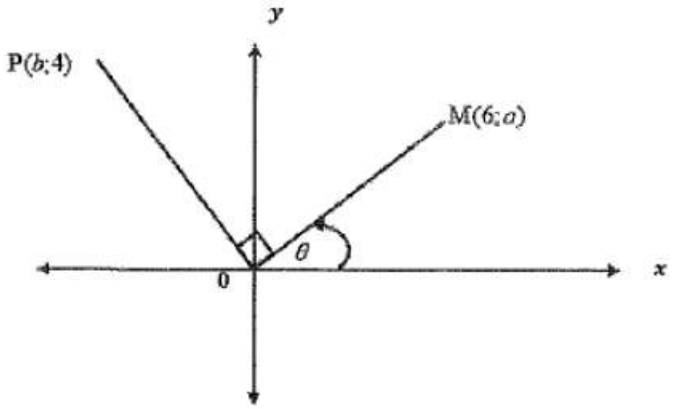
5.1.1	$\tan\theta = \frac{-\sqrt{3}}{1} = -\sqrt{3}$	A✓ answer	(1)
5.1.2	$r^2 = (1)^2 + (-\sqrt{3})^2$ Pyth $r^2 = 4$ $r = 2$ $\sin(-\theta) = -\sin\theta$ $= -\left(\frac{-\sqrt{3}}{2}\right)$ *or* $\frac{\sqrt{3}}{2}$	A✓ $r = 2$ A✓ reduction CA✓ answer	(3)
5.1.3	$\sin(\theta - 60°)$ $= \sin\theta\cos 60° - \cos\theta\sin 60°$ $= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{-\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = \frac{-2\sqrt{3}}{4} = \frac{-\sqrt{3}}{2}$	A✓ expansion CA✓ both special ratios CA✓ value of cos θ CA✓ answer	(4)
5.2	$\frac{\tan(180° - \theta)\sin(90° + \theta)}{\cos 300°\sin(\theta - 360°)}$ $= \frac{-\tan\theta\,.\cos\theta}{\cos 60°\,.\sin\theta}$ $= \frac{-\frac{\sin\theta}{\cos\theta}.\cos\theta}{\frac{1}{2}.\sin\theta}$ $= -2$	A✓ $-\tan\theta$ A✓ $\cos\theta$ A✓ $\cos 60°$ A✓ $\sin\theta$ A✓ identity CA✓ answer	(6)
5.3.1	$LHS = \frac{\cos 2x - 1}{\sin 2x}$ $= \frac{1 - 2\sin^2 x - 1}{2\sin x\cos x}$ $= -\frac{\sin x}{\cos x}$ $= -\tan x$ $= RHS$	A✓ $1 - 2\sin^2 x$ A✓ $2\sin x\cos x$ A✓ simplification	(3)
5.3.2	$x = 90°$; $180°$; $270°$	A✓ 90° A✓ 180° A✓ 270°	(3)

5.3.3	$-\tan 2x = \frac{1}{4}$ $\tan 2x = -\frac{1}{4}$ $2x = 165{,}96° + k.180°$ $x = 82{,}98° + k.90°$; $k \in Z$	A✓ $\tan 2x = -\frac{1}{4}$ A✓ 165,96° CA✓ 82,98° A✓ $k \in Z$	(4)
			[24]

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QUESTION/*VRAAG* 5



5.1.1	$\cos\theta = \frac{2}{\sqrt{5}}$ $y^2 = (\sqrt{5})^2 - 2^2 = 1$ $y = 1$ $\tan\theta = \frac{a}{6}$ $\frac{a}{6} = \frac{1}{2}$ $a = 3$	$\cos\theta = \frac{2}{\sqrt{5}}$ ✓ $y = 1$ ✓ $\frac{a}{6} = \frac{1}{2}$ ✓ $a = 3$ (3)
5.1.2	$\tan P\hat{O}X = \frac{4}{b}$ $\tan(90° + \theta) = \frac{4}{b}$ $-\frac{1}{\tan\theta} = \frac{4}{b}$ $\tan\theta = -\frac{b}{4} = \frac{1}{2}$ $b = -2$	✓ $\tan(90° + \theta) = \frac{4}{b}$ ✓ $-\frac{1}{\tan\theta} = \frac{4}{b}$ ✓ $\tan\theta = -\frac{b}{4} = \frac{1}{2}$ ✓answer/*antwoord* (4)
5.2.1	$\frac{\sin 163°}{\cos 73°} - \frac{\sin(90° + \theta).\tan^2(\theta - 360°)}{\cos(180° - \theta)}$ $= \frac{\sin(90° + 73°)}{\cos 73°} - \frac{\cos\theta.\tan^2\theta}{-\cos\theta}$ $= \frac{\cos 73°}{\cos 73°} + \tan^2\theta$	✓ $\cos 73°$ ✓ $\sin(90° + \theta) = \cos\theta$ ✓ $\tan^2\theta$ ✓ $-\cos\theta$

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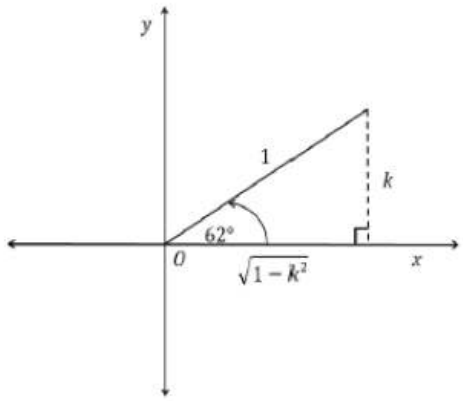
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	$= 1+\frac{\sin^2\theta}{\cos^2\theta}$ $=\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}$ $=\frac{1}{\cos^2\theta}$	✓$1+\frac{\sin^2\theta}{\cos^2\theta}$ ✓$\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}$ ✓answer/*antwoord* (7)
5.2.2	$\frac{\sin^2\theta\cos\theta}{\tan 2\theta(\sin^2\theta-\cos^2\theta)}$ $=\frac{\sin^2\theta\cos\theta}{\frac{\sin 2\theta}{\cos 2\theta}(\sin^2\theta-\cos^2\theta)}$ $=\frac{\sin^2\theta\cos\theta}{\frac{\sin 2\theta}{\cos 2\theta}(-\cos 2\theta)}$ $=\frac{\sin^2\theta\cos\theta}{-2\sin\theta\cos\theta}$ $=\frac{\sin\theta}{2}$	✓$\frac{\sin 2\theta}{\cos 2\theta}$ ✓$2\sin\theta\cos\theta$ ✓$-\cos 2\theta$ ✓Simplification/ *vereenvoudiging* ✓answer /*antwoord* (5)
5.3.	$\cos(x-60°)=3\cos x.$ $\cos x\cos 60°+\sin x\sin 60°=3\cos x$ $\frac{1}{2}\cos x+\frac{\sqrt{3}}{2}\sin x=3\cos x$ $\frac{\sqrt{3}}{2}\sin x=\frac{5}{2}\cos x$ $\sqrt{3}\sin x=5\cos x$ $\frac{\sin x}{\cos x}=\frac{5}{\sqrt{3}}$ $\tan x=\frac{5}{\sqrt{3}}$ $x=70{,}89°+k.180°, k\in Z$	✓expansion/*uitbreiding* ✓special angles/ *bekende hoeke* ✓simplification/ *vereenvoudig* ✓$\frac{\sin x}{\cos x}=\frac{5}{\sqrt{3}}$ ✓$\tan x=\frac{5}{\sqrt{3}}$ ✓$x=70{,}89°+k.180°, k\in Z$ (6)
		[25]

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QUESTION 5

5.1	If $\sin40°.\cos22° + \cos40°.\sin22° = k$, then without the use of a calculator, determine the value of the following in terms of k.		
5.1.1	$\sin40°.\cos22° + \cos40°.\sin22° = k$ $\sin(40° + 22°)$ $\sin62° = k$	✓ $\sin(40° + 22°)$ ✓ *answer* A	(2) K
5.1.2	 $tan(118°)$ $= tan(180° - 62°)$ $= -tan62°$ $= -\frac{k}{\sqrt{1-k^2}}$	✓ $y = k\,;r = 1$ ✓ $x = \sqrt{1-k^2}$ ✓ $-tan62°$A ✓ *answer* CA	(4) R
5.1.3	$\frac{sin14.\ cos14°}{2sin14°cos14°}{2}$ $= \frac{1}{2}sin28°$ $= \frac{1}{2}cos62°$ $= \frac{\sqrt{1-k^2}}{2}$	✓$\frac{1}{2}sin28°$ ✓$\frac{1}{2}cos62°$ (do not have to show give 2 marks for answer) ✓ *answer* CA	(3) C

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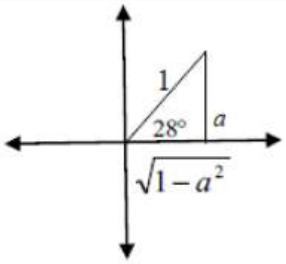
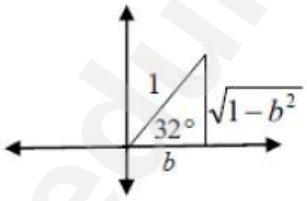
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5.2	$LHS = \frac{1-cos2\theta}{sin2\theta \times tan\theta}$ $LHS = \frac{1-(1-2sin^2\theta)}{2sin\theta cos\theta \times \frac{sin\theta}{cos\theta}}$ $LHS = \frac{2sin^2\theta}{2sin\theta \times \frac{sin\theta}{1}}$ $LHS = 1$ $LHS = RHS$	✓ $1-2sin^2\theta$ ✓ $2sin\theta cos\theta$ ✓ $\frac{sin\theta}{cos\theta}$ ✓ $\frac{sin^2\theta}{sin^2\theta}$	(4) C
5.3	$\sqrt{\sin(180° + A).\cos(90° + A) - tan45}$ $= \sqrt{-\sin A . -sinA - 1}$ $= \sqrt{\sin A . sinA - 1}$ $= \sqrt{-(1-\sin^2 A)}$ $= \sqrt{-cos^2 A}$ $\therefore cos^2 A = 0$ $A = 90° \pm 180°n\,; n \in Z$ **OR** $A = 90° + 360n \quad or\ A = 270° + 360°n$	✓ $-\sin A$ ✓ $-\sin A$ ✓ -1 ✓ $-cos^2 A$ ✓ $cos^2 A = 0$ or $sin = 1$ or $cosA = 0$ ✓ *answer*	(6) C
			[19]

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QUESTION 5

5.1	$\frac{\sin(A-180°).\tan(180°-A).\cos A}{\cos(90°+A)}$ $=\frac{\sin(-(180°-A)).(-\tan A).\cos A}{-\sin A}$ $=\frac{-\sin A.(-\tan A).\cos A}{-\sin A}$ $=-\frac{\sin A}{\cos A}.\cos A$ $=-\sin A$	✓ $-\tan A$ ✓ $-\sin A$ ✓ $-\sin A$ ✓ identity ✓ answer (5)
5.2.1	 $\cos 28° = \sqrt{1-a^2}$ **OR** $\cos^2 28° + \sin^2 28° = 1$ $\cos^2 28° + a^2 = 1$ $\cos^2 28° = 1-a^2$ $\cos 28° = \sqrt{1-a^2}$	✓ sketch ✓ answer (2) ✓ identity ✓ answer (2)
5.2.2	 $\cos 64°$ $= \cos(2\times 32°)$ $= 2\cos^2 32° - 1$ $= 2b^2 - 1$ **OR**	✓ double angle ✓ answer (2)

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	$\cos 64^\circ$ $= \cos(2\times 32^\circ)$ $= 1-2\sin^2 32$ $= 1-2\left(\sqrt{1-b^2}\right)^2$ $= 1-2+2b^2$ $= 2b^2-1$ **OR** $\cos(2\times 32^\circ)$ $= \cos^2 32^\circ - \sin^2 32^\circ$ $= b^2-\left(\sqrt{1-b^2}\right)^2$ $= b^2-1+b^2$ $= 2b^2-1$	✓double angle ✓answer (2) ✓double angle ✓answer (2)
5.2.3	$\sin 4^\circ$ $= \sin(32^\circ-28^\circ)$ $= \sin 32^\circ.\cos 28^\circ - \cos 32^\circ.\sin 28^\circ$ $= \left(\sqrt{1-b^2}\right)\left(\sqrt{1-a^2}\right)-b.a$ $= \sqrt{(1-b^2)(1-a^2)}-ab$	✓compound ∠ ✓ expansion ✓ $\sqrt{1-b^2}$ ✓ $\sqrt{1-a^2}$ (4)
5.3	$\frac{1}{1-\sin x}-\frac{1}{1+\sin x}=\frac{2\tan x}{\cos x}$ LHS: $\frac{1}{1-\sin x}-\frac{1}{1+\sin x}=\frac{1+\sin x-(1-\sin x)}{(1-\sin x)(1+\sin x)}$ $=\frac{1+\sin x-1+\sin x}{1-\sin^2 x}$ $=\frac{2\sin x}{\cos^2 x}$ RHS: $\frac{2\tan x}{\cos x}=\frac{2\frac{\sin x}{\cos x}}{\cos x}$ $=\frac{2\sin x}{\cos^2 x}$ $\therefore$ LHS = RHS	✓LCD ✓numerator ✓ $\frac{2\sin x}{\cos^2 x}$ ✓ $\frac{\sin x}{\cos x}$ ✓ $\frac{2\sin x}{\cos^2 x}$ (5)

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	OR $\frac{1}{1-\sin x}-\frac{1}{1+\sin x}=\frac{2\tan x}{\cos x}$ LHS : $\frac{1}{1-\sin x}-\frac{1}{1+\sin x}=\frac{1+\sin x-(1-\sin x)}{(1-\sin x)(1+\sin x)}$ $=\frac{1+\sin x-1+\sin x}{1-\sin^2 x}$ $=\frac{2\sin x}{\cos^2 x}$ $=\frac{2\sin x}{\cos x.\cos x}$ $=\frac{2\tan x}{\cos x}$ $\therefore$ LHS = RHS	✓LCD ✓numerator ✓simplification ✓ $\frac{2\sin x}{\cos^2 x}$ ✓ $\frac{\sin x}{\cos x}=\tan x$ (5)
		[18]

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QUESTION/*VRAAG 5*

5.1.1	$\frac{\sin(180°-70°).\tan 60°}{\cos 180°.\tan(180°+70°).\sin 20°}$ $\frac{\sin 70°.\tan 60°}{(-1)\tan 70°.\sin 20°}$ $\frac{\sin 70°.\sqrt{3}}{(-1).\frac{\sin 70°}{\cos 70°}.\cos 70°}$ $=-\sqrt{3}$	✓ $\sin 70°$ ✓ $\cos 180°$ ✓ $\tan 70°$ ✓ $\sin 20°$ ✓ $\frac{\sin 70°}{\cos 70°}$ ✓ $\sin 20° = \cos 70°$ ✓ $-\sqrt{3}$ (7)
5.1.2	$1-2\sin^2 22,5°$ $\cos 2(22,5°)$ $\cos 45°$ $=\frac{1}{\sqrt{2}}$ $=\frac{\sqrt{2}}{2}$	✓ $1-2\sin^2 22,5°$ ✓ $\cos 2(22,5°)$ ✓ $\cos 45°$ ✓ $\frac{1}{\sqrt{2}}$ (4)
5.2.1	$\sin^2 x=0$ and where / *en waar* $\tan x$ is undefined / *ongedefinieerd* $\therefore \sin x=0$ $\therefore x=0°$ or / *of* $x=180°$ or / *of* $x=90°$	✓ $x=0°$ ✓ $x=180°$ ✓ $x=90°$ (3)
5.2.2	LHS / *LK* $\frac{\cos 2x.\tan x}{\sin^2 x}$ $=\frac{(\cos^2 x-\sin^2 x)\left(\frac{\sin x}{\cos x}\right)}{\sin^2 x}$ $=(\cos^2 x-\sin^2 x)\left(\frac{\sin x}{\cos x}\right)\left(\frac{1}{\sin^2 x}\right)$ $=\frac{\cos^2 x-\sin^2 x}{\sin x\cos x}$ $=\frac{\cos^2 x}{\sin x\cos x}-\frac{\sin^2 x}{\sin x\cos x}$ $=\frac{\cos x}{\sin x}-\frac{\sin x}{\cos x}$ $=\frac{\cos x}{\sin x}-\tan x=$ RHS / *RK*	✓ $\cos 2x=\cos^2 x-\sin^2 x$ ✓ $\tan x=\frac{\sin x}{\cos x}$ ✓ $\frac{\cos^2 x-\sin^2 x}{\sin x\cos x}$ ✓ $\frac{\cos^2 x}{\sin x\cos x}-\frac{\sin^2 x}{\sin x\cos x}$ ✓ $\frac{\cos x}{\sin x}-\frac{\sin x}{\cos x}$ (5)

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	OR/*OF*	
	LHS / *LK* $\frac{\cos 2x.\tan x}{\sin^2 x}$ $=\frac{\left(\cos^2 x-\sin^2 x\right)\left(\frac{\sin x}{\cos x}\right)}{\sin^2 x}$ $=\frac{\cos x.\sin x}{\sin^2 x}-\frac{\frac{\sin^3 x}{\cos x}}{\sin^2 x}$ $=\frac{\cos x}{\sin x}-\frac{\sin x}{\cos x}$ $=\frac{\cos x}{\sin x}-\tan x=$ RHS / *RK*	✓ $\cos 2x=\cos^2 x-\sin^2 x$ ✓ $\tan x=\frac{\sin x}{\cos x}$ ✓ $\frac{\cos x.\sin x}{\sin^2 x}$ $\frac{\sin^3 x}{\cos x}$ ✓ $\frac{\cos x}{\sin^2 x}$ ✓ $\frac{\cos x}{\sin x}-\frac{\sin x}{\cos x}$ (5) [19]

TRIGONOMETRIC IDENTITIES

QUESTION/*VRAAG 6*		
	LHS $=\frac{\cos 2x}{1+\sin 2x}$ $=\frac{\cos^2 x-\sin^2 x}{1+2\sin x\cos x}$ $=\frac{\cos^2 x-\sin^2 x}{\sin^2 x+2\sin x\cos x+\cos^2 x}$ $=\frac{(\cos x+\sin x)(\cos x-\sin x)}{(\sin x+\cos x)(\sin x+\cos x)}$ $=\frac{\cos x-\sin x}{\sin x+\cos x}$	✓ expansion in numerator/ *uitbreiding van teller* ✓ expansion in denominator/ *uitbreiding van noemer* ✓ $\sin^2 x+\cos^2 x$ ✓ factorisation/*faktorisering* [4]

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5.3.1	LHS = sin(A + B) – sin(A – B) = sin A.cos B + cos A.sin B – (sin A. cos B – cos A. sin B) = sin A.cos B + cos A.sin B – sin A. cos B + cos A. sin B = 2cos A.sin B = RHS	✓ compound formula ✓ compound formula (2)
5.3.2	$\sin 77° - \sin 43° = \sin(60° + 17°) - \sin(60° - 17°)$ $= 2\cos 60°.\sin 17°$ $= 2 \times \frac{1}{2} \times \sin 17°$ $= \sin 17°$ **OR/*OF*** $\sin 77° - \sin 43° = \sin(60° + 17°) - \sin(60° - 17°)$ $= (\sin 60° \cos 17° + \cos 60° \sin 17°) -$ $(\sin 60° \cos 17° - \cos 60° \sin 17°)$ $= \frac{\sqrt{3}}{2}\cos 17° + \frac{1}{2}\sin 17° - \frac{\sqrt{3}}{2}\cos 17° + \frac{1}{2}\sin 17°$ $= \sin 17°$	✓ 60° + 17° ✓ 60° – 17° ✓ simplify ✓ $\frac{1}{2}$ (4) ✓ 60° + 17° ✓ 60° – 17° ✓ expansion ✓ $\frac{1}{2}$ (4) **[19]**

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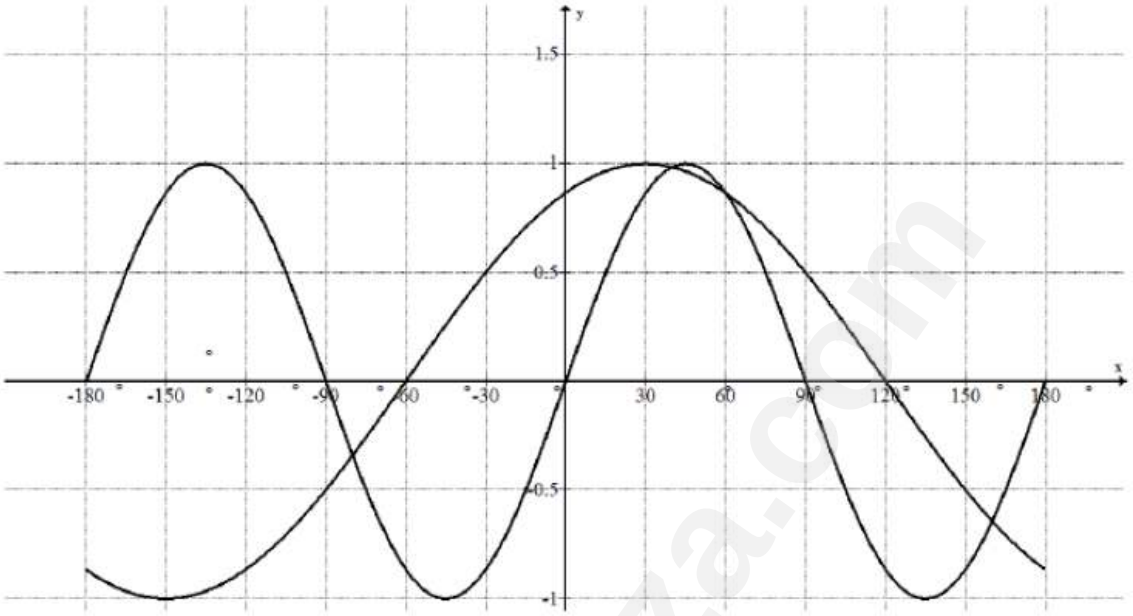
QUESTION/*VRAAG 6*

6.1	$\sin 2x = \sin(90° - (x - 30°)) + 360.k$ $2x = 90° - x + 30° + 360.k$ $2x = 120° - x + 360.k$ $x + 2x = 120° + 360.k$ $3x = 120° + 360.k$ $x = 40° + 120°k, k \in z$ **OR/*OF*** $2x = 180° - (120° - x)$ $2x = 180° - 120° + x$ $2x - x = 60°$ $x = 60° + 360°k$ **ALTERNATIVE/*ALTERNATIEF*:** $\cos(90° - 2x) = \cos(x - 30°) + 360.k$ $90° - 2x = x - 30° + 360.k$ $-3x = -120° + 360.k$ $x = 40° - 120°k, k \in z$ **OR/ *OF*** $90° - 2x = 360° - (x - 30°)$ $-x = 300° + 360°.k$ $x = -300° + 360°.k$ $x = 60° + 360°k$	✓ method/*metode* ✓ $2x = 90° - x + 30° + 360.k$ ✓ $x = 40° + 120°k, k \in z$ ✓ $2x = 180° - (120° - x)$ ✓ $x = 60° + 360°k$ (5) **ALTERNATIVE/*ALTERNATIEF*:** ✓ method/*metode* ✓ $90° - 2x = x - 30° + 360.k$ ✓ $x = 40° - 120°k, k \in z$ ✓ $90° - 2x = 360° - (x - 30°)$ ✓ $x = 60° + 360°k$ (5)
6.2.1	Period $= 360°$	✓ answer/*antwoord* (1)
6.2.2	$-150° < x < 30°$	✓values/*waardes* ✓inequalities/*ongelykhede* (2)

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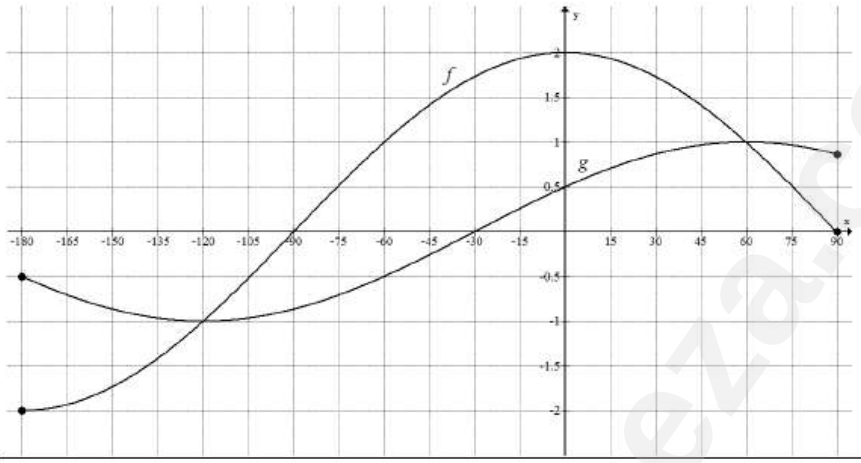
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6.2.3	 ✓ x intercepts/*x-afsnitte* ✓turning points/*draaipunte* ✓shape/*vorm* (3)	
6.2.4	$-180° \le x \le -90°$; $-60° \le x \le 0°$; $90° \le x \le 120°$	✓ $-180° \le x \le -90°$ ✓ $-60° \le x \le 0°$ ✓ $90° \le x \le 120°$ (3)
		[14]

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TRIGONOMETRIC GRAPHS

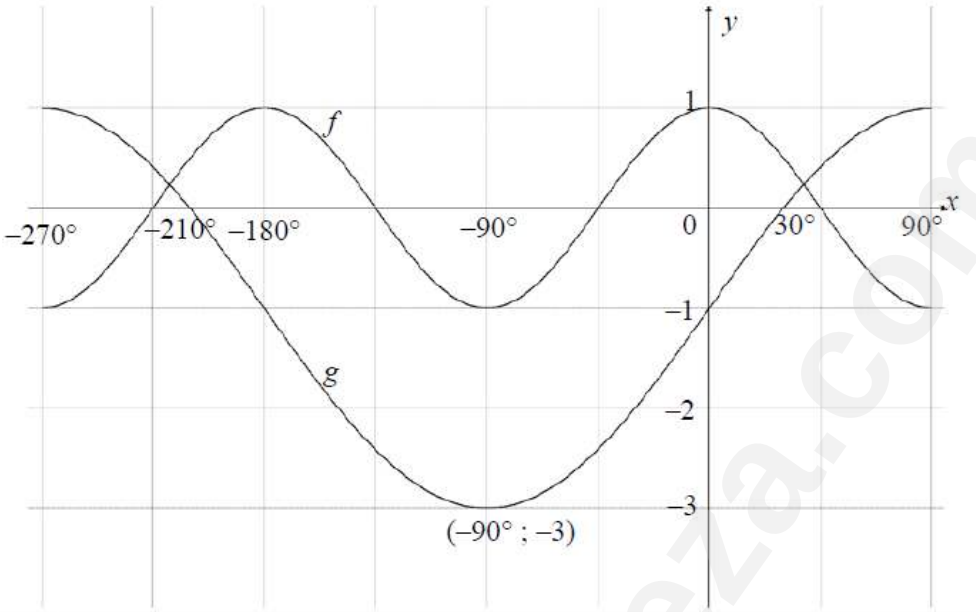
QUESTION/*VRAAG* 8			
8.1	2	✓ answer / *antwoord*	(1)
8.2		*f*: ✓ *x*- int/*x-afsnit* ✓ shape/*vorm* *g*: ✓ *x*-ints/*x-afsnitte* ✓ *y*-int/*y-afsnit* ✓ shape/*vorm*	(5)
8.3	$-120° \leq x \leq 60°$	✓ critical values / *kritiese waardes* ✓ notation/*notasie*	(2)
8.4	Translate *f* 30° to the right/Skuif *f* 30° na regs **OR/OF** Translate *f* down by ½ / *Skuif f 'n ½ eenheid af*	✓ correct translation/*korrekte translasie* ✓ correct translation/*korrekte translasie*	(2)
			[10]

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QUESTION/*VRAAG* 6

6.1



✓ (–90° ; –3)
✓ (0 ; –1)
✓ x – intercepts: –210° & 30°
✓ shape

(4)

6.2

$\cos 2x = 2\sin x - 1$

$1 - 2\sin^2 x = 2\sin x - 1$

$2\sin^2 x + 2\sin x - 2 = 0$

$\sin^2 x + \sin x - 1 = 0$

$\sin x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)}$

$\sin x = \dfrac{-1+\sqrt{5}}{2}$, since $\sin x = \dfrac{-1-\sqrt{5}}{2} < -1$ has no solution

✓ $\cos 2x = 1 - 2\sin^2 x$
✓ standard form
✓ using quadratic formula
✓ substitution into quadratic formula

(4)

6.3

$\sin x = \dfrac{-1+\sqrt{5}}{2} = 0{,}618...$

Reference $\angle = 38{,}17°$

$\therefore x = 38{,}17° + k.360°$ or $x = 141{,}83° + k.360°$; $k \in Z$

$\therefore x = 38{,}17°$ or $-218{,}17°$

$y = 0{,}24$

$\therefore$ Points of intersection/*snypunte*:
(38,17° ; 0,24) and (–218,17° ; 0,24)

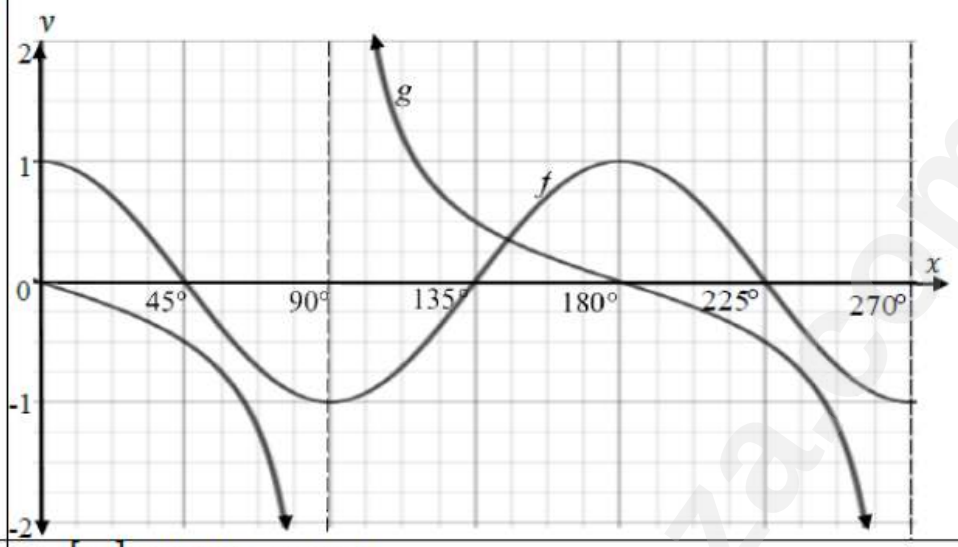
✓ 38,17°
✓ 141,83°
✓ –218,17°
✓ 0,24

(4)
[12]

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QUESTION / *VRAAG 6*

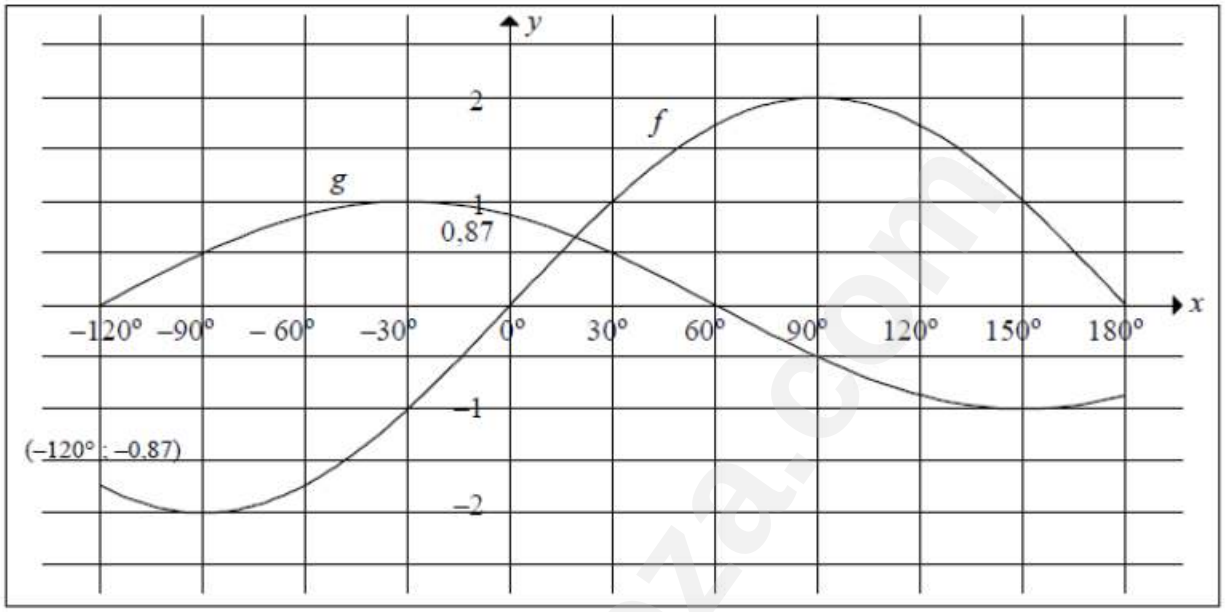
6.1		✓ $\left(45°;-\frac{1}{2}\right)$ ✓ *x*-intercepts / *x-afsnitte* ✓ shape / *vorm* ✓ asymptotes / *asimptote* (4)
6.2	$y \in [2;4]$ **OR / *OF*** $2 \le y \le 4$	✓ $y \in [2;4]$ **OR / *OF*** $2 \le y \le 4$ (1)
6.3	$x \in [135°;180°]$ **OR / *OF*** $135° \le x \le 180°$ $x \in [225°;270°)$ **OR / *OF*** $225° \le x < 270°$	✓ $x \in [135°;180°]$ **OR / *OF*** $135° \le x \le 180°$ ✓ $x \in [225°;270°)$ **OR / *OF*** $225° \le x < 270°$ (2)
		[7]

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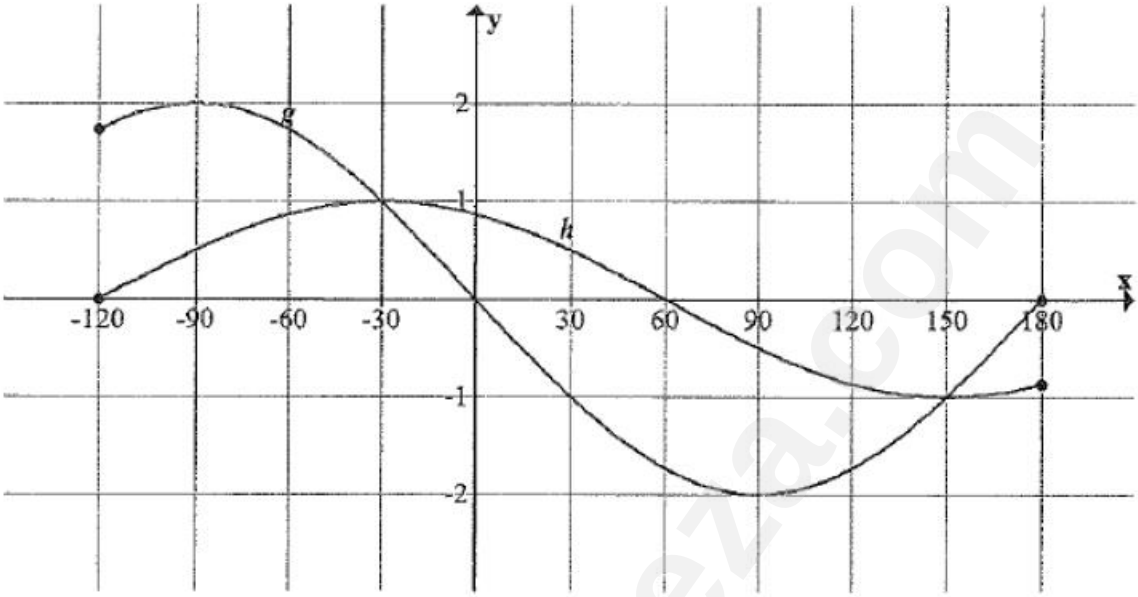
QUESTION 6

6.1	 A✓ x-intercepts A✓ shape A✓ $(-30° ; 1)$ & $(150° ; -1)$ A✓ $(-120° ; 0)$ & $(180° ; -0,87)$	(4)
6.2	Period $= 360°$	A✓ answer (1)
6.3.1	$-30° < x < 150°$	CA✓ endpoints A✓ correct interval (2)
6.3.2	$x \in (0° ; 60°)$	CA✓ endpoints A✓ correct interval (2)
6.4	$y = \cos(x + 30° + 60°)$ $y = \cos(x + 90°)$ $y = -\sin x$	A✓ $\cos(x + 90°)$ CA✓ answer (2)
		[11]

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QUESTION/*VRAAG* 6



6.1	$\cos x\cos 30° - \sin x\sin 30° \geq -2\sin x$ $\cos(x+30°) \geq -2\sin x$ $x \in [-30°;150°]$	✓manipulation/*manipulasie* ✓$\cos(x+30°) \geq -2\sin x$ ✓Critical values/*kritieke punte* ✓notation /*notasie* (4)
6.2	$x \in [-120°;-90°) \cup (150°;180°]$	✓✓boundaries and notation/ *grense en notasie* ✓✓ boundaries and notation $\cup$ *grense en notasie* (4)
		[8]

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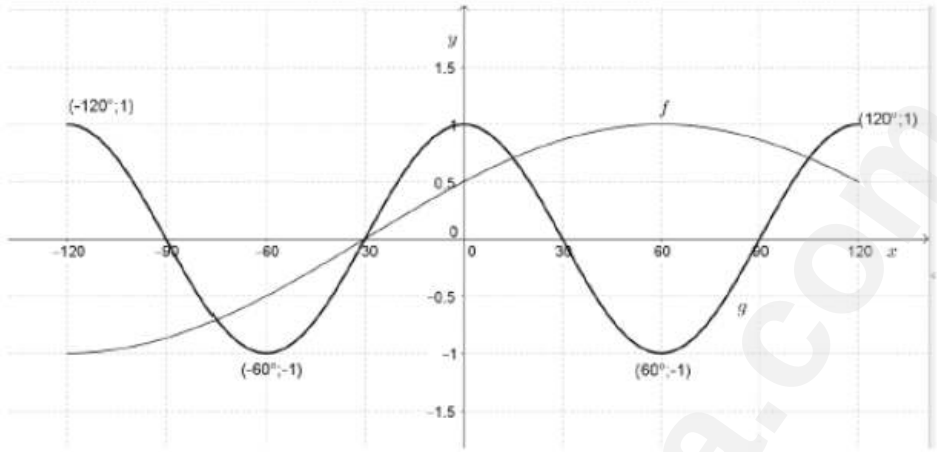
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QUESTION 6

6.1	$f(x) = \sin(x + a)$		
			
6.1	$a = 30°$	✓ *answer*	(1) K
6.2	Sketch	✓ x – *intercepts* ✓ y – *itercept* ✓ *Turning points* ✓ *end points*	(4) R
6.3	$f(x) = g(x)$ $\sin(x + 30°) = \cos(3x)$ $\sin(x + 30°) = \sin(90° - 3x)$ $x + 30° = 90° - 3x + 360°n \quad n \in Z$ $4x = 60° + 360°n$ $x = 15° + 90°n$ *or* $x + 30° = 180° - (90° - 3x) + 360°n$ $x + 30° = 90° + 3x + 360°n$ $-2x = 60° + 360°n$ $x = -30° - 180°n$	**CA from 6.1.1** ✓ $\sin(90° - 3x)$ ✓ *method in 1st quadrant* ✓ *answer* ✓ *method in 2nd quadrant* ✓ *answer*	(5) C
6.4	$15° < x < 105°$ or $x \in (15°; 105°)$	✓ *critical values* ✓ *notation* **CA from 6.3**	(2) C
6.1.5	$g(x) = \cos(3x)$ $k(x) = \cos(60° - 3x)$ $k(x) = \cos(3x - 60°)$ $k(x) = cos3(x - 20°)$ $\therefore$ *translated 20° to the right.* */ transleer 20° na regs*	✓✓ *answer* **A**	(2) P
			[14]

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QUESTION 6 **[16]**

6.1	$\sin(x+60°)=\cos\frac{1}{2}x$	✓co-function
	$\sin(x+60°)=\sin\left(90°-\frac{1}{2}x\right)$	
	*quadrant*1:	
	$x+60°=90°-\frac{1}{2}x+360.k, k\in Z$	✓1st quad equation
	$x+\frac{1}{2}x=30°+360.k$	
	$\frac{3}{2}x=30°+360.k$	
	$x=20°+240.k, k\in Z$	✓ $x=20°+240.k$
	*quadrant*2:	
	$x+60°=180°-(90°-\frac{1}{2}x)+360.k, k\in Z$	✓2nd quad equation
	$x+60°=180°-90°+\frac{1}{2}x+360.k$	
	$\frac{1}{2}x=30°+360.k$	
	$x=60°+720.k, k\in Z$	✓ $x=60°+720.k$
	$x\in\{20°;60°;260°\}$	✓✓ $x\in\{20°;60°;260°\}$
	OR	
		(7)

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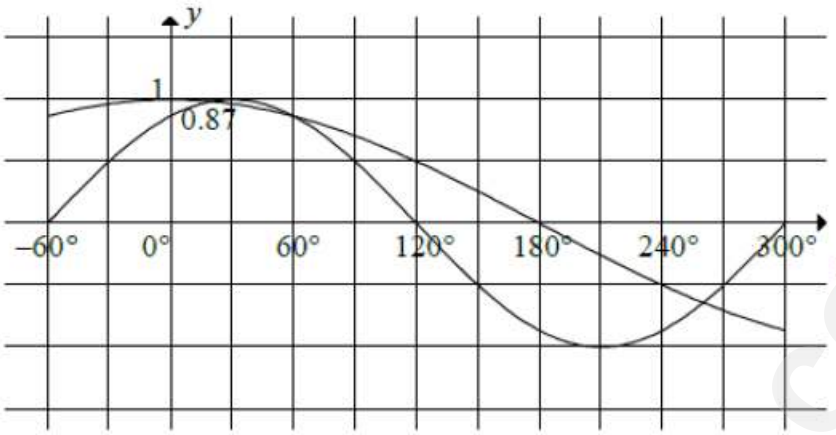
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	$\sin(x+60^\circ)=\cos\frac{1}{2}x$ $\cos[90-(x+60^\circ)]=\cos\frac{1}{2}x$ *quadrant* 1 $[90-(x+60^\circ)]=\frac{1}{2}x+360.k, k\in Z$ $90-x-60^\circ=\frac{1}{2}x+360.k$ $-\frac{3}{2}x=-30^\circ+360.k$ $x=20^\circ+240.k, k\in Z$ *quadrant* 4 $[90-(x+60^\circ)]=360^\circ-\frac{1}{2}x+360.k, k\in Z$ $90-x-60^\circ=360^\circ-\frac{1}{2}x+360k$ $30^\circ-x=360^\circ-\frac{1}{2}x+360.k$ $-\frac{1}{2}x=330^\circ+360.k$ $x=-660^\circ+720.k$ $x\in\{20^\circ;60^\circ;260^\circ\}$	✓ co-function ✓ 1st quad equation ✓ $x=20^\circ+240.k$ ✓ 4th quad equation ✓ $x=-660^\circ+720.k$ ✓✓ $x\in\{20^\circ;60^\circ;260^\circ\}$ (7)

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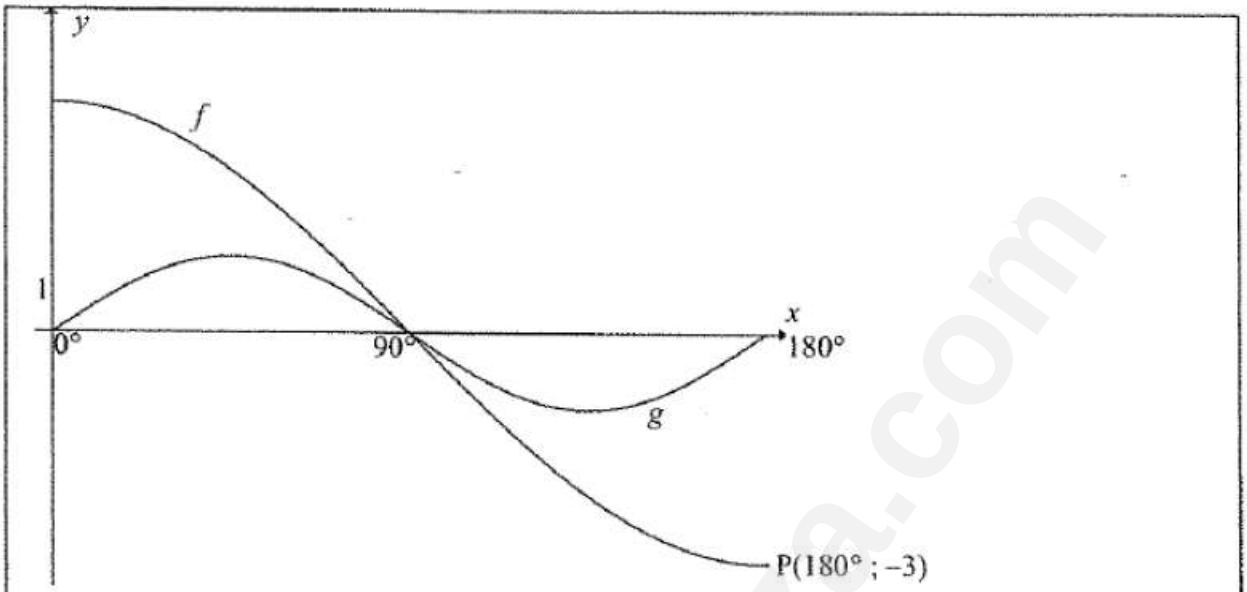
6.2.1		*g*: ✓ x – intercepts ✓ y – intercept ✓ turning points (3)
6.2.2	$20^\circ < x < 60^\circ$ or $260^\circ < x \leq 300^\circ$ **OR** $x \in (20^\circ;60^\circ) \cup x \in (600^\circ;300^\circ]$	✓ $20^\circ < x < 60^\circ$ ✓ $260^\circ < x \leq 300^\circ$ ✓ notation (3)
6.2.3	$120^\circ \leq x \leq 180^\circ$ $x = -60^\circ$ $x = 300^\circ$	✓✓ $120^\circ \leq x \leq 180^\circ$ ✓ $x = -60^\circ;300^\circ$ (3)
		[16]

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QUESTION/*VRAAG 6*



6.1	$a = 3$ and/*en* $b = 2$	✓ $a = 3$ ✓ $b = 2$ (2)
6.2	360°	✓ 360° (1)
6.3	$2 \leq y \leq 4$ or / *of* $y \in [2;4]$	✓ values/*waardes* 2 and/*en* 4 ✓ notation / *notasie*. (2)
6.4	$0° < x < 45°$ or / *of* $90° < x < 135°$ alternative notation / *alternatiewe notasie* $x \in (0°;45°) \cup (90°;135°)$	✓ 0° and/*en* 45° ✓ 90° and/*en* 135° ✓ notation / *notasie* (3)
6.5	$y = \cos^2 x - \sin^2 x$ $y = \cos 2x$ to coincide with g / *om saam te val met* g : $y = \sin(90° + 2x)$ $y = \sin 2(x + 45°)$ $\therefore q = 45°$	✓ $y = \cos 2x$ ✓ $\sin(90 + 2x)$ ✓ 45° (3) [11]

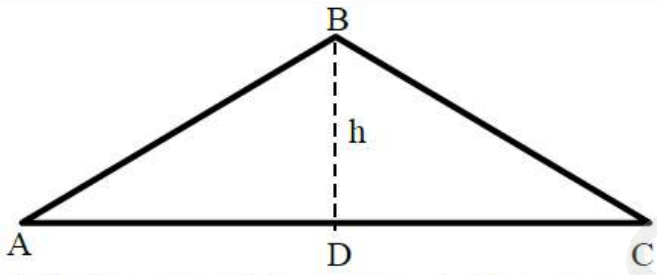
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SOLUTION OF TRIANGLES

QUESTION/*VRAAG* 7

7.1



Let/*Laat* BD = h (height of triangle/*hoogte van driehoek* ABC)
Let/*Laat* DC = x
$\therefore$ AD = $(b - x)$
Proof/*Bewys*:

$c^2 = h^2 + (b - x)^2$
$= h^2 + b^2 - 2bx + x^2$
$= b^2 + (h^2 + x^2) - 2bx$
$= b^2 + a^2 - 2bx$
$a^2 = h^2 + x^2$
$\frac{x}{a} = cos\ C$
$x = a \cos C$
$\therefore\ b^2 + a^2 - 2ab \cos C$

✓AD in terms of x / *AD in terme van x*

✓ c^2 (using/*gebruik van* Pythagoras)

✓ $(h^2 + x^2) = a^2$

✓substitution of x/ *vervanging van x*

(4)

7.2.1

PSQ = 90°
$Q_2 = 90° + \alpha$

In ΔQSR:
$SR^2 = SQ^2 + QR^2 - 2SQ.QR.\cos SQR$
$= x^2 + x^2 - 2x.x.\cos(90° + \alpha)$
$= 2x^2 - 2x^2(-\sin \alpha)$
$= 2x^2 + 2x^2(\sin \alpha)$
$= x^2(2 + 2\sin\alpha)$
$\therefore SR = x\sqrt{2(1 + \sin \alpha}$

✓ angle in semi-circle/*hoek in semi-sirkel*

✓ use of cos rule/ *gebruik van cos-reël*
✓ correct substitution/ *korrekte vervanging*

✓ x^2 $(2 + 2\sin\alpha)$

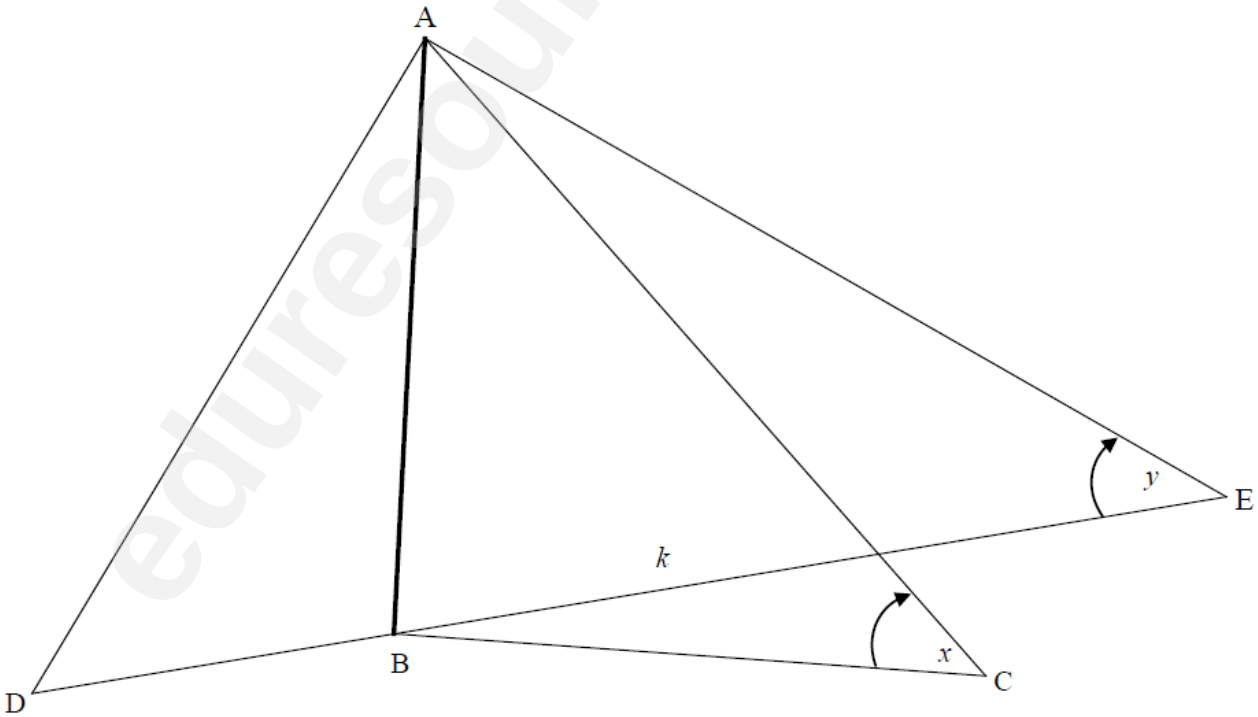
(4)

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7.2.2	$5\sqrt{3} = x\sqrt{2(1+sin\alpha)}$ $5\sqrt{3} = 5\sqrt{2(1+\sin\alpha)}$ $\sqrt{3} = \sqrt{2(1+\sin\alpha}$ $3 = 2(1+sin\alpha)$ $\frac{3}{2} = 1 + sin\alpha$ $\frac{1}{2} = sin\alpha$ $\alpha = 30°$ In ΔPSQ $\frac{QS}{PQ} = sin\,\alpha$ $PQ = \frac{QS}{\sin\alpha}$ $= \frac{5}{\sin 30°}$ = 10 units/*eenhede*	✓Substitution/ *Vervanging* ✓ $\sin\alpha = 0,5$ ✓ $\alpha = 30°$ ✓ answer/*antwoord*	(4)
			[12]

QUESTION/*VRAAG* 7



7.1	$A\hat{B}C = 90°$	✓ answer	(1)

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7.2	In Δ ABE: $\frac{AB}{BE} = \tan y$ $AB = k \tan y$ In Δ ABC: $\frac{AB}{AC} = \sin x$ $AC = \frac{AB}{\sin x}$ $= \frac{k \tan y}{\sin x}$	✓ correct ratio ✓ value AB ✓ correct ratio ✓ AC as subject and substitution (4)

7.3	$A\hat{D}C = A\hat{C}D = \frac{180° - 2x}{2} = 90° - x$ $\frac{DC}{\sin 2x} = \frac{AC}{\sin(90° - x)}$ $\frac{DC}{2 \sin x \cos x} = \frac{AC}{\cos x}$ $DC = \frac{AC(2 \sin x \cos x)}{\cos x}$ $= \frac{k \tan y}{\sin x} \cdot \frac{2 \sin x \cos x}{\cos x}$ $= 2k \tan y$ **OR/*OF*** $DC^2 = AD^2 + AC^2 - 2AD.AC \cos 2x$ $= AC^2 + AC^2 - 2AC^2 \cos 2x$ $= 2AC^2(1 - \cos 2x)$ $= 2AC^2(1 - 1 + \sin^2 x)$ $= 4AC^2 \sin^2 x$ $DC = 2AC.\sin x$ $= 2\left(\frac{k.\tan y}{\sin x}\right).\sin x$ $= 2k.\tan y$	✓ $90° - x$ ✓subst into sine rule ✓ $2 \sin x \cos x$ ✓ $\cos x$ ✓ substitution (5) ✓substitution into cos rule ✓ factorisation ✓ $1 - 2 \sin^2 x$ ✓ DC ito AC and $\sin x$ ✓ substitution (5)

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	OR/*OF* $DC^2 = AD^2 + AC^2 - 2AD.AC\cos 2x$ $= 2\left(\frac{k\tan y}{\sin x}\right)^2 - 2\left(\frac{k\tan y}{\sin x}\right)^2 \cos 2x$ $= \frac{2k^2\tan^2 y}{\sin^2 x} - \frac{2k^2\tan^2 y}{\sin^2 x}(1-2\sin^2 x)$ $= \frac{2k^2\tan^2 y}{\sin^2 x} - \frac{2k^2\tan^2 y}{\sin^2 x} + 4k^2\tan^2 y$ $DC = \sqrt{4k^2\tan^2 y}$ $= 2k\tan y$	✓correct cos rule ✓ substitution ✓ $1-2\sin^2 x$ ✓ squaring and multiplication ✓ $\sqrt{4k^2\tan^2 y}$ (5) **[10]**

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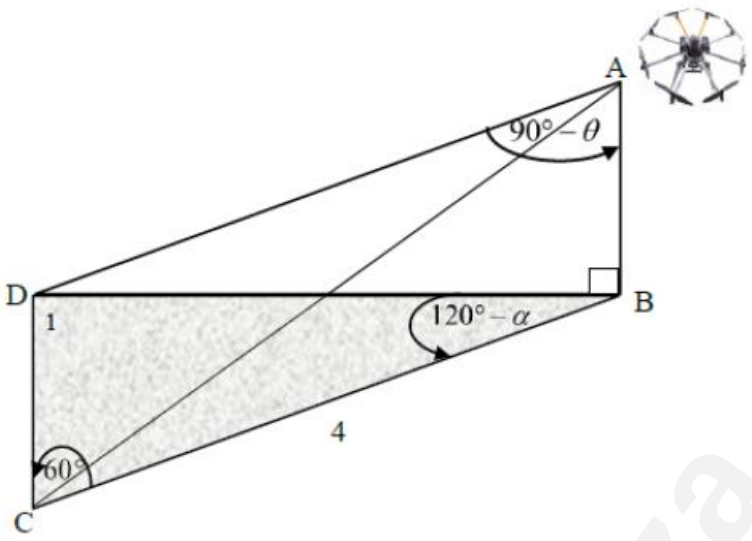
QUESTION/*VRAAG* 7

7.1	$\hat{BDC} = 180° - 2\alpha$ $\frac{y}{\sin\alpha} = \frac{x}{\sin 180° - 2\alpha}$ $y = \frac{x\sin\alpha}{\sin(180° - 2\alpha)}$ $y = \frac{x\sin\alpha}{2\sin\alpha\cos\alpha}$ $y = \frac{x}{2\cos\alpha}$ $\cos\theta = \frac{BD}{AB}$ $AB\cos\theta = BD$ $AB = \frac{BD}{\cos\theta}$ $AB = \frac{x}{2\cos\alpha} \div \cos\theta$ $AB = \frac{x}{2\cos\alpha} \times \frac{1}{\cos\theta}$ $AB = \frac{x}{2\cos\alpha\cos\theta}$	✓ $\hat{BDC} = 180° - 2\alpha$ ✓ method/*metode* ✓ substitution/*vervanging* ✓ $2\sin\alpha\cos\alpha$ ✓ $AB = \frac{BD}{\cos\theta}$ ✓ substitution/*vervanging* BD ✓ simplification/*vereenvoudiging* (7)
		[7]

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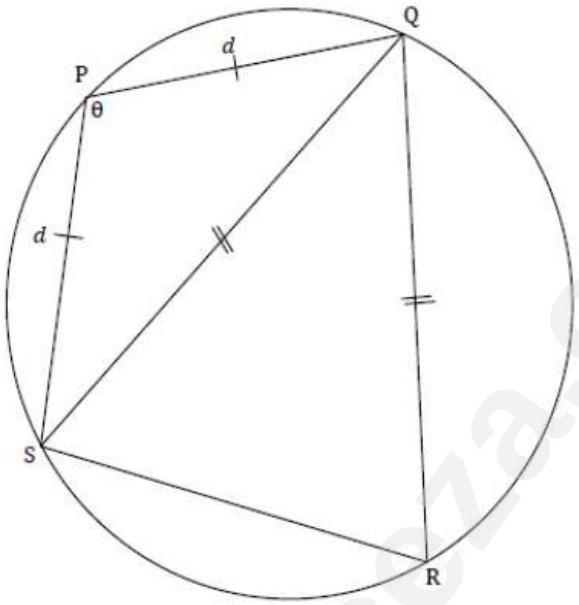
QUESTION / *VRAAG* 7

		
7.1	$\hat{D}_1 = 180° - 60° - (120° - \alpha)$ (sum of ∠s of Δ / *somvd ∠e v Δ*) $\hat{D}_1 = \alpha$	✓ $\hat{D}_1 = 180° - 60° - (120° - \alpha)$ ✓ $\hat{D}_1 = \alpha$ (2)
7.2	$\frac{BD}{\sin 60°} = \frac{4}{\sin\alpha}$ $BD \sin\alpha = 4 \sin 60°$ $BD = \frac{4\left(\frac{\sqrt{3}}{2}\right)}{\sin\alpha}$ $BD = \frac{2\sqrt{3}}{\sin\alpha}$	✓ substitution into correct sin rule / *vervang in korrekte sin reël* ✓ simplification / *vereenvoudiging* ✓ answer / *antwoord* (3)
7.3	In ΔADB : $A\hat{D}B = \theta$ (∠s of a Δ) $\frac{AB}{BD} = \tan\theta$ $AB = BD.\tan\theta$ $= \frac{2\sqrt{3}}{\sin\alpha}.\tan\theta$ $AB = \frac{2\sqrt{3}\tan\theta}{\sin\alpha}$	✓ $A\hat{D}B = \theta$ ✓ trig ratio / *trig verhouding* ✓ substitution of BD / *vervanging van BD* (3)
		[8]

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QUESTION 7

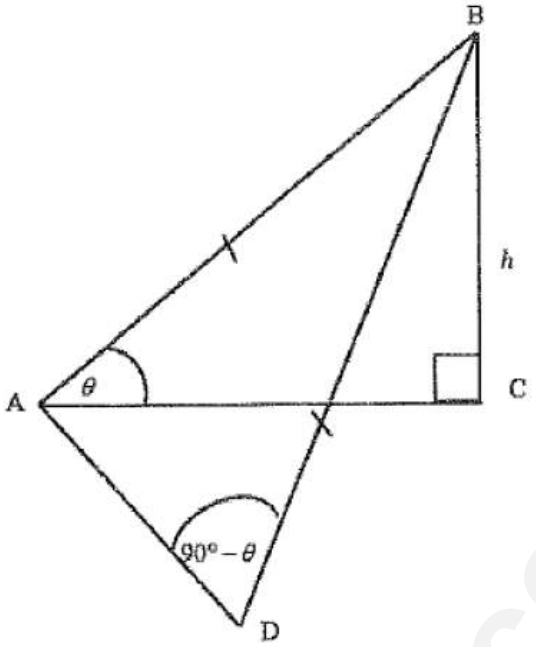


7.1	$QS^2 = d^2 + d^2 - 2d.d.\cos\theta$ $QS^2 = 2d^2 - 2d^2.\cos\theta$ $QS^2 = 2d^2(1-\cos\theta)$ $QS = d\sqrt{2(1-\cos\theta)}$	A✓ subst into cos-rule A✓ common factor	(2)
7.2	$\hat{R} = 180° - \theta$ opp. $\angle^s$ cyclic quad suppl $= Q\hat{S}R$ equal sides, equal angles $S\hat{Q}R = 2\theta - 180°$ sum $\angle^s\Delta$ $\Delta QRS = \frac{1}{2}.QS.QR\sin S\hat{Q}R$ $= \frac{1}{2}.d\sqrt{2(1-\cos\theta)}.d\sqrt{2(1-\cos\theta)}\sin(2\theta - 180°)$ $= \frac{1}{2}d^2.2(1-\cos\theta)(-\sin 2\theta)$ $= -d^2(1-\cos\theta).\sin 2\theta$	A✓ $S\hat{Q}R$ A✓ subst into area rule A✓ simplify & reduction	(3)
			[5]

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QUESTION/*VRAAG* 7



7.1	*In* ΔABC: $\sin\theta = \frac{h}{AB}$ $AB = \frac{h}{\sin\theta}$	✓ $AB = \frac{h}{\sin\theta}$ (1)
7.2	In Δ ABD: $B\hat{A}D = \hat{D} = 90° - \theta$ (sides opposite $= \angle's$) $90° - \theta + 90° - \theta + A\hat{B}D = 180°$ ($\angle's$ of Δ) $A\hat{B}D = 2\theta$	✓ $B\hat{A}D = \hat{D} = 90° - \theta$ ✓ $A\hat{B}D = 2\theta$ (2)
7.3	$\frac{AD}{\sin 2\theta} = \frac{AB}{\sin(90° - \theta)}$ $AD = \frac{\frac{h}{\sin\theta} \times \sin 2\theta}{\cos\theta}$ $AD = \frac{\frac{h}{\sin\theta} \times 2\sin\theta\cos\theta}{\cos\theta}$ $AD = 2h$	✓ $\frac{AD}{\sin 2\theta} = \frac{AB}{\sin(90° - \theta)}$ ✓ $\cos\theta$ ✓ $2\sin\theta\cos\theta$ ✓ answer /*antwoord* (4)
		[7]

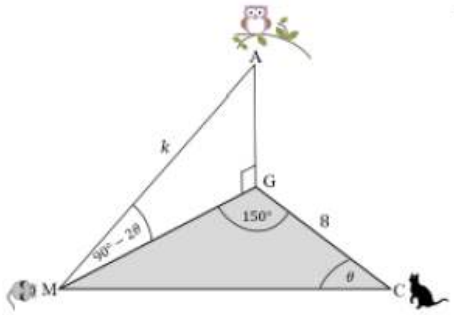
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QUESTION 7

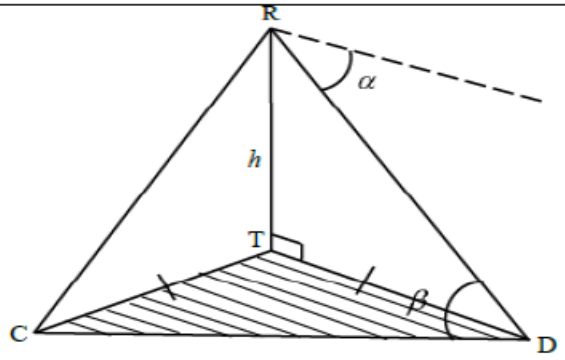


7.1	$M\hat{A}G = 2\theta$	✓ *answer*	(1) K
7.2	$\frac{MG}{sin2\theta} = \frac{k}{sin90°}$ $MG = \frac{ksin2\theta}{1}$ ... ... (1)	✓ $\frac{MG}{sin2\theta} = \frac{k}{sin90°}$ OR $\frac{MG}{k} = sin2\theta$ ✓ $MG = \frac{ksin2\theta}{1}$	(2) C
7.3	$\frac{MC}{sin150°} = \frac{MG}{sin\theta}$ $\frac{MC}{½} = \frac{MG}{sin\theta}$ $MC = \frac{½MG}{sin\theta}$ ... ... (1) Subst. (2) in (1) $MC = \frac{ksin2\theta}{2sin\theta}$ $MC = \frac{k2sin\theta.cos\theta}{2sin\theta}$ $MC = kcos\theta$	✓ $\frac{MC}{sin150°} = \frac{MG}{sin\theta}$ ✓ ½ ✓ $MC = \frac{½MG}{sin\theta}$ ✓ $2sin\theta.cos\theta$ *answer given*	(4) C
7.4	$\Delta MGC = \frac{1}{2}MG.CG\ sin150°$ $\Delta MGC = \frac{1}{2}ksin2\theta.8\ \frac{1}{2}$ $\Delta MGC = 2ksin2\theta$ **OR** $\Delta MGC = \frac{1}{2}MC.CG\ sin\theta$ $\Delta MGC = \frac{1}{2}kcos\theta.8\ sin\theta$ $\Delta MGC = 4kcos\theta sin\theta$ $\Delta MGC = 2k(2cos\theta sin\theta)$ $\Delta MGC = 2ksin2\theta$	✓ *subst.* ✓ method *answer given* **OR** ✓ *subst.* ✓ method *answer given*	(2) C
			[9]

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QUESTION 7

		
7.1	$CR = RD$ (congruency OR SAS) $C\hat{R}D = 180° - 2\beta$ ($\angle s$ in Δ)	✓ $CR = RD$ ✓answer (2)
7.2	$T\hat{R}D = 90° - \alpha$ $\cos(90° - \alpha) = \frac{h}{RD}$ $RD = \frac{h}{\sin\alpha}$ $CD^2 = \left(\frac{h}{\sin\alpha}\right)^2 + \left(\frac{h}{\sin\alpha}\right)^2 - 2\left(\frac{h}{\sin\alpha}\right)\left(\frac{h}{\sin\alpha}\right)\cos(180° - 2\beta)$ $= \frac{2h^2}{\sin^2\alpha} - 2\left(\frac{h^2}{\sin^2\alpha}\right)(-\cos 2\beta)$ $= \frac{2h^2}{\sin^2\alpha}(1 + \cos 2\beta)$ $= \frac{2h^2}{\sin^2\alpha}(1 + 2\cos^2\beta - 1)$ $= \frac{2h^2}{\sin^2\alpha}(2\cos^2\beta)$ $= \frac{4h^2}{\sin^2\alpha}(\cos^2\beta)$ $\therefore CD = \frac{2h.\cos\beta}{\sin\alpha}$	✓ratio ✓ $RD = \frac{h}{\sin\alpha}$ ✓substitution ✓ $\frac{2h^2}{\sin^2\alpha}(1 + 2\cos^2\beta - 1)$ ✓ $\frac{4h^2}{\sin^2\alpha}(\cos^2\beta)$ (5)
7.3	$CD = \frac{2h.\cos\beta}{\sin\alpha}$.	✓substitution

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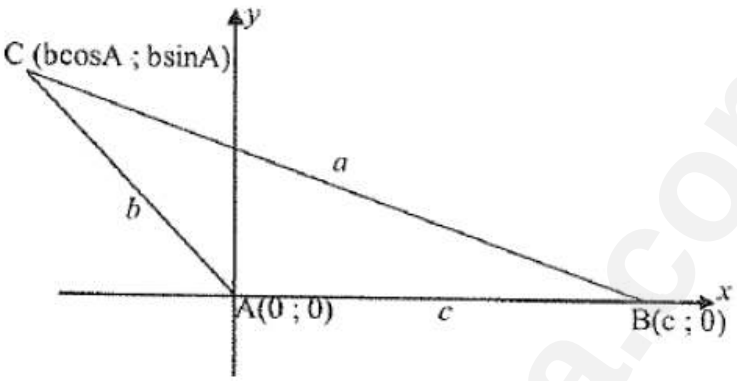
	$5,4 = \frac{2h.\cos 65°}{\sin 51°}$. $h = \frac{5,42.\sin 51°}{2\cos 65°}$. $h = 4,96$ $h \approx 5$	✓h to the nearest unit (2)
		[9]

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QUESTION/*VRAAG 7*

7.1



Coordinates of /*Koördinate van* B(c ; 0)

$BC = a = \sqrt{(b\cos A - c)^2 - (b\sin A - 0)^2}$

$\therefore a^2 = b^2\cos^2 A - 2bc\cos A + c^2 + b^2\sin^2 A$

$= b^2(\cos^2 A + \sin^2 A) - 2bc\cos A + c^2$

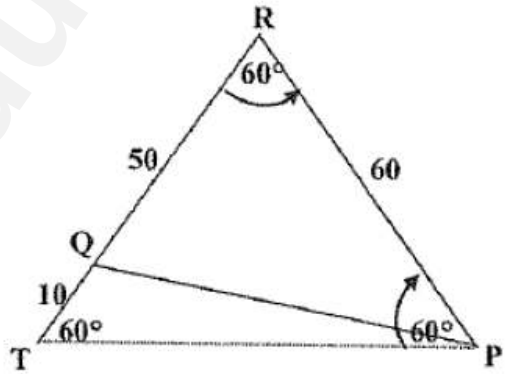
$= b^2(1) + c^2 - 2bc\cos A$

$= b^2 + c^2 - 2bc\cos A$

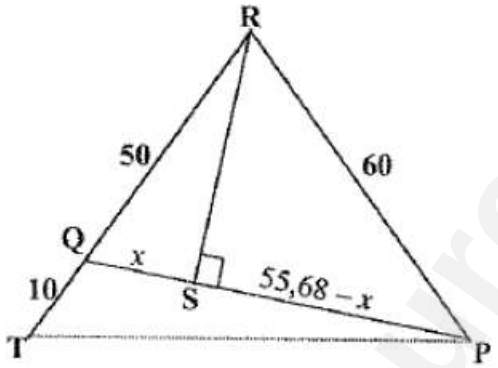
✓ coordinates of / *koördinate van B*
✓ correct subst. into distance formula / *korrekte vervanging in afstandsformule*
✓ $b^2\cos^2 A - 2bc\cos A + c^2 + b^2\sin^2 A$
✓ $b^2(\cos^2 A + \sin^2 A)$

(4)

7.2



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7.2.1	$\hat{R} = \hat{T} = \hat{P} = 60°$ $QT = 10$ and / *en* $QR = 50$ $PQ^2 = 60^2 + 50^2 - 2(50)(60)\cos 60°$ $= 3100$ **OR / *OF*** $PQ^2 = 10^2 + 60^2 - 2(10)60)\cos 60°$ $= 3100$ $\therefore PQ = 55,68$	✓ 60° ✓ 10 or/*of* 50 ✓ correct subst in cosine-rule/ *korrekte vervanging in cosreël* ✓ 3100 (4)
7.2.2	 S is the nearest to R where RS is the perpendicular heigt / *S is die naaste aan R waar RS die loodregte hoogte is.* Let / *Laat* $QS = x$ $\therefore PS = 55,68 - x$ $\left(50^2 - x^2\right) = \left(60^2 - [55,68 - x]^2\right) = RS^2$ $2500 - x^2 = 3600 - 3100,26 + 111,36x - x^2$ $2000,26 = 111,36x$ $\therefore x = 17,96$ units / *eenhede*	✓ Diagram ✓ Pythagoras ✓ Simplify / *vereenvoudig* ✓ answer / *antwoord* (4) **[12]**

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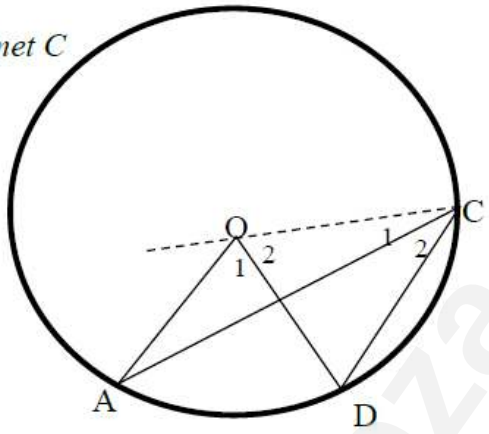
MODULE 4: EUCLIDEAN GEOMETRY

CIRCLE GEOMETRY

QUESTION/*VRAAG* 9

9.1

Const: Join O to C
Konstr: Verbind O met C



Construction/*Konstruksie*: Join CO produced/*Verbind CO verleng.*
Proof/*Bewys*:
Let/*Laat* $C_1 = x$ and/*en* $C_2 = y$
$\angle A = x$ [AO = OC= *r*]
$\angle D = x + y$ [OD = OC= *r*]
$\angle AOC = 180° - 2x$ [Sum of angles of/*Som v/d hoeke van* Δ]
$\angle DOC = 180° - (2x + 2y)$ [Sum of angles of Δ / "]
But/*Maar* $\angle AOD = \angle AOC - \angle DOC$
$\angle AOD = 2y$
$\angle AOD = 2\ \angle ACD$

✓ Construction/ *konstruksie*

✓ A and D (in terms of *x* and *y*) with reasons
A en D in terme van x en y met redes

✓ AOC (in terms of *x*) with a reason
AOC in terme van x met rede

✓ equation for AOD/ *gelykstel vir AOD*

✓ $\angle AOD = 2y$

(5)

9.2

9.2.1

MBC = MCB [MB = MC; radii]
$\angle B_3 = C_2 = x$ [BO bisects/*halveer* MBC and/*en* OC bisects/*halveer* MCB]
$\angle O_2 = 180° - 2x$ [angles of/*hoeke van* Δ]

✓reason/*rede*
✓✓B_3 and/*en* $C_2 = x$ and reason/*en rede*
✓ answer/*antwoord*

(4)

9.2.2

$\angle A = ½BMC$ [angle at centre = 2 × angle at circ /*middelpuntshoek = 2 × omtrekshoek*]
$= ½(180° - 4x)$ [angles of/*hoeke van* Δ]
$\angle A = 90° - 2x$

Therefore/*Daarom*:
$\angle ABO = O_2 - A$ [ext. angle of/*buitehoek van* ΔOAB]
$= (180° - 2x) - (90° - 2x)$
$= 90°$
AO subtends a right angle/*AO onderspan 'n reghoek*
∴ AO is a diameter/*AO is 'n middellyn*

✓ $\angle A = ½BMC$ (statement) and reason. *(stelling) en rede*
✓ substitution and answer/*vervanging en antwoord*
✓ $\angle ABO = O_2 - A$ (statement) and reason *(stelling) en rede*
✓substitution/ *vervanging*
ABO = 90°
✓ answer/*antwoord*

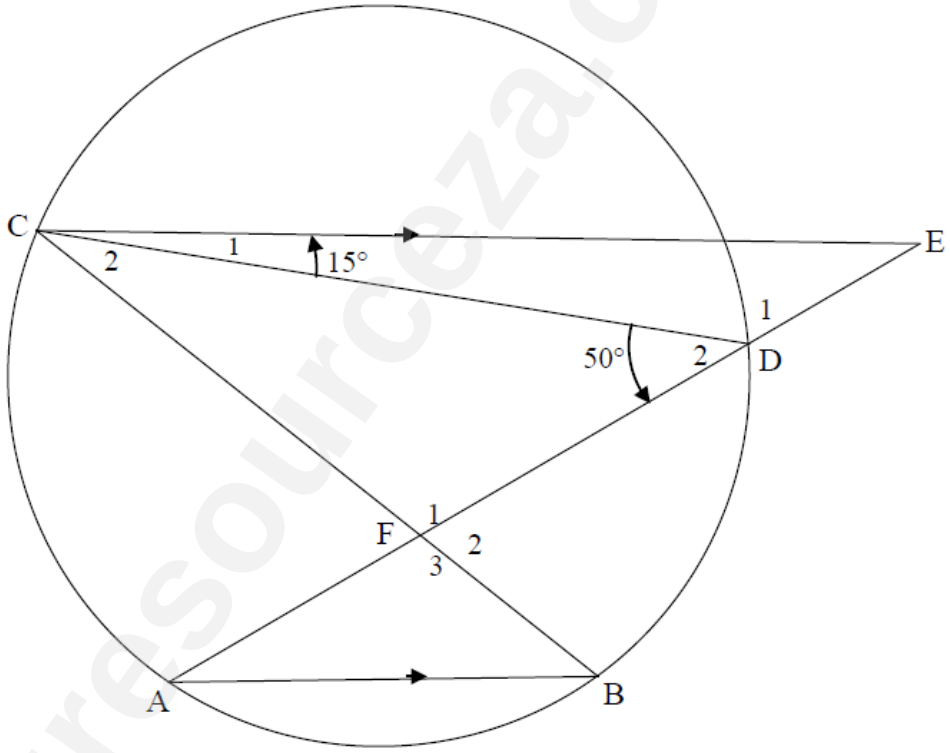
(5)

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9.3	In ΔEHO, $OE = x$ and $OH = x - 2$ $EH^2 = x^2 - (x-2)^2$ [Theorem of Pythagoras] $EH^2 = x^2 - x^2 + 4x - 4$ $= 4(x-1)$ $\therefore EH = 2\sqrt{x-1}$ $\therefore EF = 4\sqrt{x-1}$ line from centre of circle …/*lyn van middelpunt van sirkel* ...]	✓ $OH = x - 2$ ✓Theorem of/*Stelling van* Pythagoras ✓EH ✓EF [with reason/*met rede*]	(4)
			[19]

QUESTION/*VRAAG* 8



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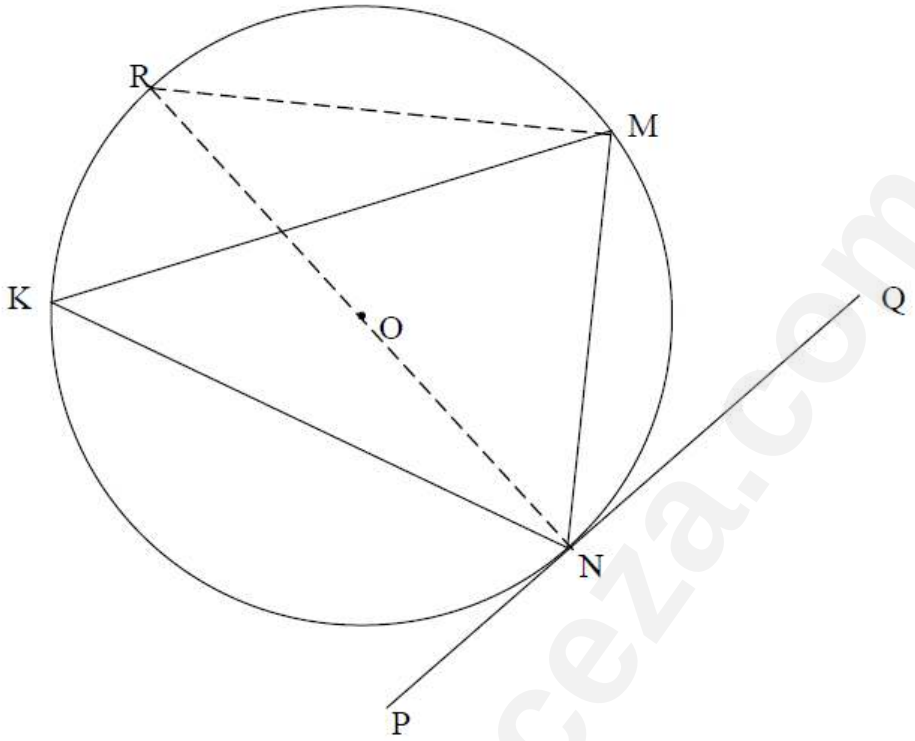
8.1.1	$\hat{E} = 50° - 15° = 35°$ [ext ∠ of Δ/*buite ∠ van Δ*] $\hat{A} = 35°$ [alt ∠s / *verwiss ∠e*; CE ∥ AB] **OR/*OF*** $\hat{E} = 180° - (130° + 15°) = 35°$ [str line; ∠s of Δ/*rt lyn; ∠e van Δ*] $\hat{A} = 35°$ [alt ∠s / *verwiss ∠e*; CE ∥ AB] **OR/*OF*** $\hat{B} = 50°$ [∠s in same segment/*∠e in dieselfde segment*] $\hat{C}_2 + 15° = 50°$ [alt ∠s / *verwiss ∠e*; CE ∥ AB] $\therefore \hat{C}_2 = 35°$ $\hat{A} = 35°$ [∠s in same segment/*∠e in dieselfde segment*]	✓ S ✓S ✓ R (3) ✓ S ✓S ✓ R (3) ✓ S ✓S ✓ R (3)
8.1.2	$\hat{C}_2 = 35°$ [∠s in same segment/*∠e in dieselfde segment*]	✓ S ✓ R (2)
8.2	$\hat{C}_2 = \hat{E}$ [from 8.1.1 and 8.1.2] ∴ CF is a tangent to the circle [converse tan chord theorem] ∴ *CF is 'n raaklyn aan die sirkel [omgekeerde raakl koordst]*	✓ S ✓ R (2) [7]

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QUESTION/*VRAAG* 11



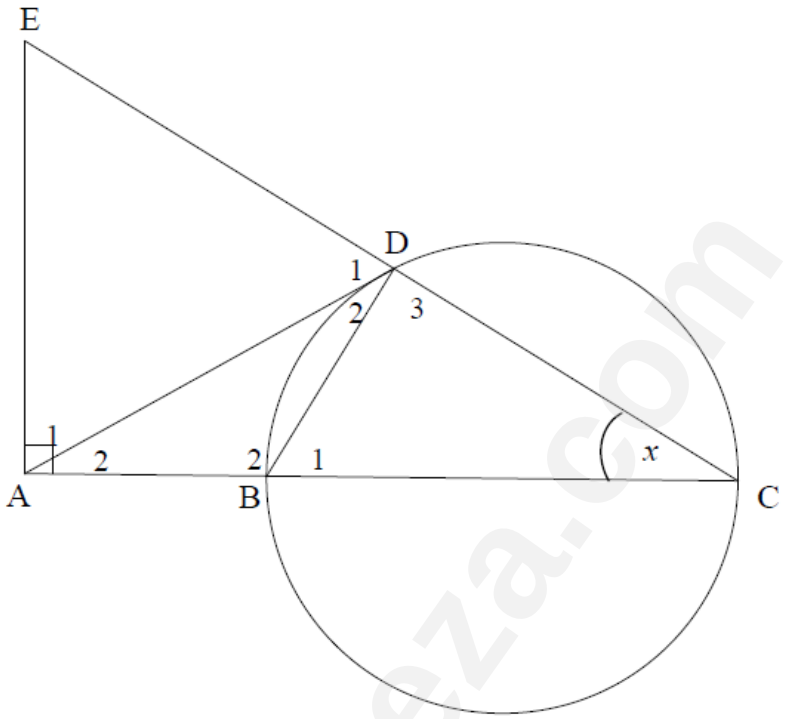
11.1	Construction: Draw diameter NR and draw RM *Konstruksie: Trek middellyn NR en verbind RM* $O\hat{N}M + M\hat{N}Q = 90°$ [radius ⊥ tangent/*raaklyn*] $N\hat{M}R = 90°$ [∠ in semi circle/*semi-sirkel*] $\therefore M\hat{R}N = 180° - (90° + 90° - M\hat{N}Q)$ [sum ∠s Δ] $= M\hat{N}Q$ but $M\hat{R}N = M\hat{K}N$ [∠s same segment/*∠e dieselfde segment*] $\therefore M\hat{N}Q = \hat{K}$	✓ construction ✓ S /R ✓ S/ R ✓ S ✓ S / R (5)

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11.2



11.2.1(a)	Angle in a semi circle/ *Hoek in halfsirkel*	✓ R (1)
11.2.1(b)	Exterior ∠of quad = opp interior ∠/ *Buite ∠van vierh = teenoorst binne ∠* **OR/*OF*** Opp ∠s of quad supplementary/ *Teenoorst ∠e van vierh is supplementêr*	✓ R (1)
11.2.1(c)	tangent chord theorem/ *raaklyn koord stelling*	✓ R (1)

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11.2.2(a)	In ΔAEC $\hat{E} = 180° - (90° + x)$ [sum $\angle$s Δ] $= 90° - x$ $\hat{D}_1 = 180° - (90° + x)$ [$\angle$s on a straight line] $= \hat{E} = 90° - x$ $\therefore AD = AE$ [sides opp = $\angle$s/ *sye teenoor* = $\angle$*e*]	✓ S ✓ S ✓ R (3)
11.2.2(b)	In ΔADB and ΔACD $\hat{A}_2 = \hat{A}_2$ [common] $\hat{D}_2 = \hat{C}$ [proven] $\hat{B}_2 = \hat{D}_2 + \hat{D}_3$ [sum $\angle^e \Delta$] $\therefore \Delta ADB \,\|\|\, \Delta ACD$ **OR/*OF*** In ΔADB and ΔACD $\hat{A}_2 = \hat{A}_2$ [common] $\hat{D}_2 = \hat{C}$ [proven] $\therefore \Delta ADB \,\|\|\, \Delta ACD$ [$\angle, \angle, \angle$]	✓ S ✓ S ✓ S (3) ✓ S ✓ S ✓ R (3)

11.2.3(a)	$\frac{AD}{AC} = \frac{AB}{AD}$ [$\|\|\|$ Δs] $AD^2 = AC \,.\, AB$ $= 3r \times r$ $= 3r^2$	✓ ratio ✓ substitution (2)
11.2.3(b)	$AD = AE = \sqrt{3}r$ [from 11.2.2(a) &11.2.3(a)] $AB = r$ and $BC = 2r$ $\therefore AC = 3r$ In ΔACE: $\tan \hat{E} = \frac{AC}{AE}$ $= \frac{3r}{\sqrt{3}r} = \sqrt{3}$ $\therefore \hat{E} = 60°$ $\therefore \hat{D}_1 = 60°$ [from 11.2.2(a)] $\therefore \hat{A}_1 = 60°$ [$\angle$s of $\Delta = 180°$] $\therefore \Delta ADE$ is equilateral/*is gelyksydig*	✓ AC ito r ✓ trig ratio ✓ simplification ✓ all 3 $\angle$s = 60° (4)

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OR/*OF*

$\frac{AD}{AC} = \frac{DB}{CD}$ [||| Δs]

$\frac{\sqrt{3}r}{3r} = \frac{DB}{CD}$ ✓ $\frac{\sqrt{3}r}{3r} = \frac{DB}{CD}$

$\tan x = \frac{1}{\sqrt{3}}$ ✓ $\frac{1}{\sqrt{3}} = \tan x$

$\therefore$ In ΔBDC: $x = 30°$ ✓ $x = 30°$

$\therefore \hat{E} = 60°$

$\therefore \hat{D}_1 = 60°$ [from 11.2.2(a)] ✓ all 3 ∠s = 60°

$\therefore \hat{A}_1 = 60°$ [∠s of Δ = 180°]

$\therefore$ ΔADE is equilateral/*is gelyksydig* (4)

OR/*OF*

$\frac{AD}{AC} = \frac{DB}{CD}$ [||| Δs]

$\frac{\sqrt{3}r}{3r} = \frac{DB}{CD}$ $\therefore BD = \frac{CD}{\sqrt{3}}$ ✓ $BD = \frac{CD}{\sqrt{3}}$

$DC^2 = BC^2 - DB^2$

$= 4r^2 - \frac{CD^2}{3}$

$3DC^2 = 12r^2 - CD^2$

$4CD^2 = 12r^2$

$DC = \sqrt{3}r$ ✓ $DC = \sqrt{3}r$

$EC^2 = EA^2 + AC^2$

$= 3r^2 + 9r^2$

$EC = 2\sqrt{3}r$ ✓ $EC = 2\sqrt{3}r$

$\therefore ED = EC - DC$

$= \sqrt{3}r$

$\therefore ED = EA = AD$ ✓ $ED = EA = AD$

$\therefore$ ΔADE is equilateral/*is gelyksydig* (4)

[20]

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QUESTION/*VRAAG* 8

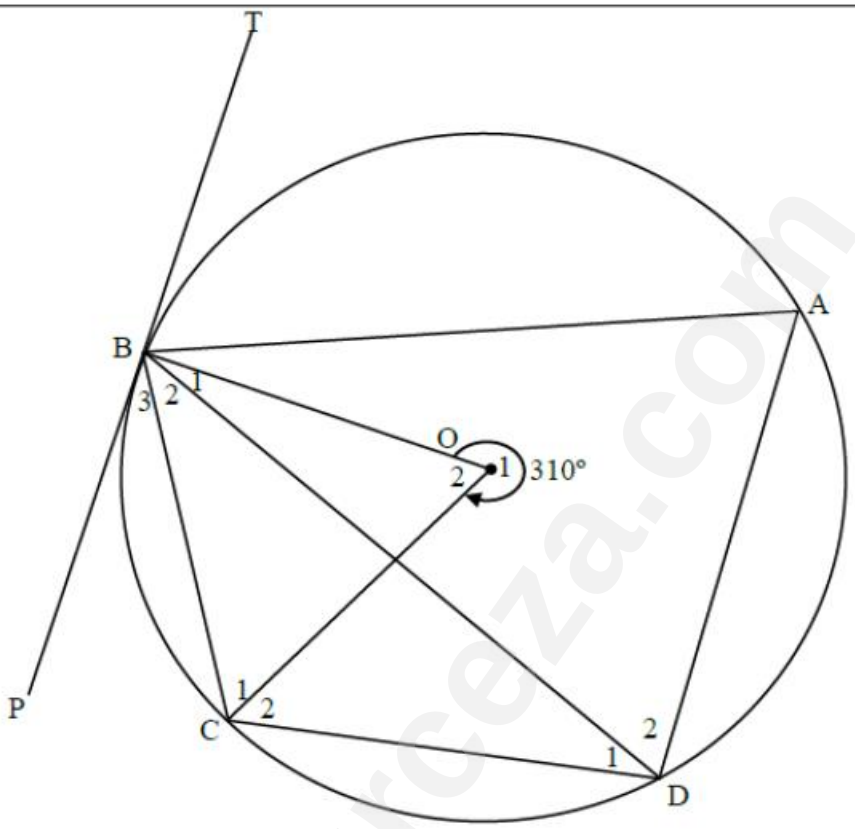
8.1.1	$\hat{H}_3 = 35°$ [tan chord theorem/*tan koordteorie*] $\hat{E}_4 = \hat{H}_3 = 35°$ [$\angle$ opp = sides/$\angle$*e teenoor = sye*]	✓S ✓R ✓S/R (3)
8.1.2	$E\hat{K}H = 180° - 2(35°)$ [sum of $\angle$'s in Δ/*som van* $\angle$*e* in Δ] $= 110°$	✓S ✓ answer/*antwoord* (2)
8.1.3	$\hat{G} = 180° - 110°$ [opp $\angle$ of cyclic quad/$\angle$*e teenoor van kdvh*] $= 70°$	✓S ✓R (2)
8.1.4	$\hat{O}_1 = 220°$ [$\angle$ at centre = 2 × $\angle$ at circumference/midpts $\angle^e = 2$ *omtreks* $\angle^e$]	✓S ✓R (2)
8.2	$(x-4)^2 + 12^2 = x^2$ [Pythagoras] $x^2 - 8x + 16 + 144 = x^2$ $-8x = -160$ $x = 20$	✓S/R ✓expansion/ *uitbreiding* ✓ simplification/ *vereenvoudiging* ✓answer/*antwoord* (4)
		[13]

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QUESTION / *VRAAG* 8



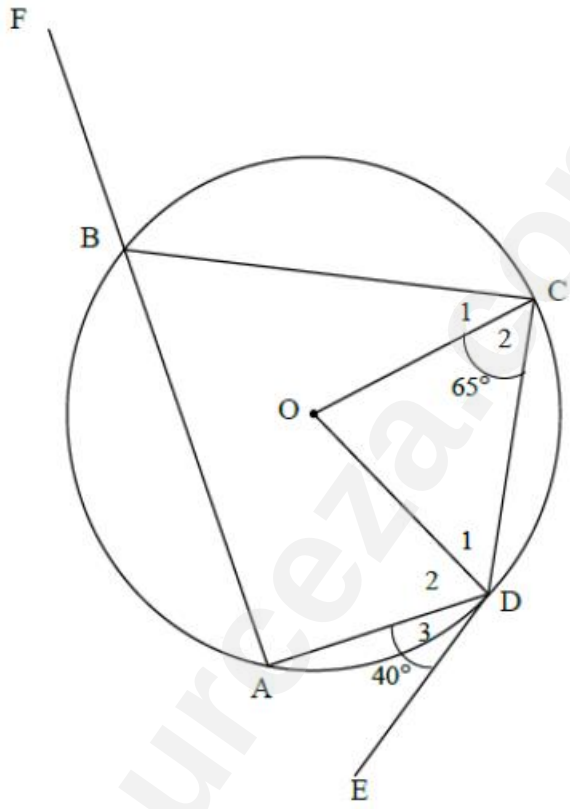
8.1	$\hat{O}_2 = 50°$ $\hat{D}_1 = 25°$	$\angle s$ around a point / *$\angle e$ om 'n punt* $\angle$ centre $= 2 \times \angle$ at circumference *midpts $\angle$ $= 2 \times$ omtreks $\angle$*	✓S ✓S ✓R (3)
8.2	$\hat{B}_3 = 25°$	tan chord theorem / *raaklyn koordstelling*	✓S ✓R (2)
8.3	$B\hat{C}D = 120°$ $\hat{B}_2 = 35°$ $O\hat{B}C = O\hat{C}B = 65°$ $\therefore \hat{B}_1 = 65° - 35°$ $\hat{B}_1 = 30°$	opp $\angle s$ of a cyclic quad /*teenoorst $\angle e$ v kvh* sum of $\angle s$ of a triangle / *som $\angle e$ v Δ* $\angle s$ opp. equal radii / *$\angle e$ teenoor gelyke radiuse*	✓S / R ✓S ✓S ✓answer / *antwoord*
	OR / *OF*	**OR / *OF***	
	$B\hat{C}D = 120°$ $\hat{B}_2 = 35°$ $\hat{B}_1 + \hat{B}_2 + \hat{B}_3 = 90°$ $\hat{B}_1 = 30°$	opp $\angle s$ of a cyclic quad/*teenoorst $\angle e$ vkvh* sum of $\angle s$ of a triangle / *som $\angle e$ v Δ* radius $\perp$ tangent / *radius $\perp$ raaklyn*	✓S / R ✓S ✓S ✓answer / *antwoord* (4)
			[9]

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QUESTION 8

8.1

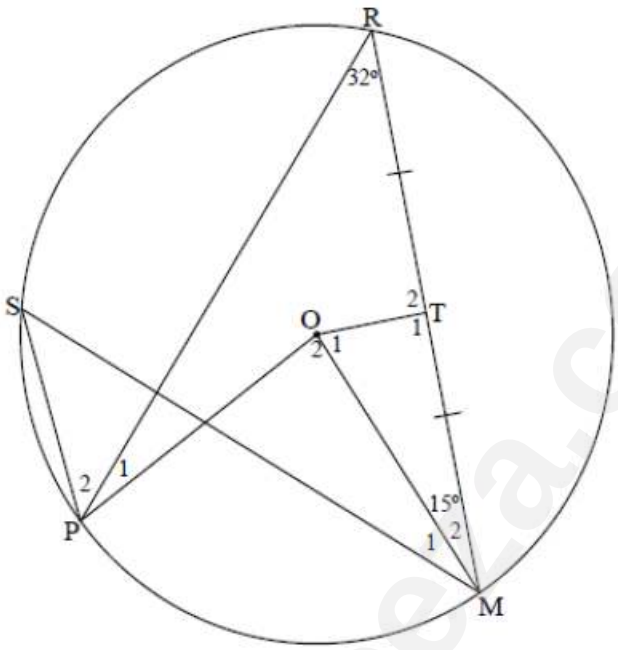


8.1.1	$\hat{D}_2 + \hat{D}_3 = 90^O$ (tan $\perp$ radius) $\therefore \hat{D}_2 = 90° - 40° = 50°$	A✓S A✓R A ✓answer	(3)
8.1.2	$\hat{D}_1 = \hat{C}_2 = 65°$ (angles opposite equal sides) $\hat{D}_1 + \hat{D}_2 = F\hat{B}C$ (ext angle of a cyclic quad) $65° + 50° = F\hat{B}C$ $\therefore F\hat{B}C = 115°$	A✓S/R A✓S A✓R CA✓answer	(4)

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8.2

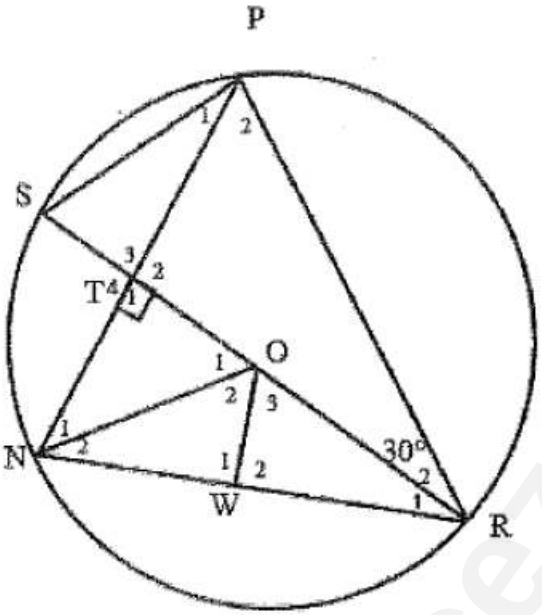


8.2.1	$\hat{S} = 32°$ (∠s in the same segment)	A✓S A✓R	(2)
8.2.2	$\hat{O}_2 = 64°$ (∠ at centre = 2 × ∠ at circumference)	A✓S A✓R	(2)
8.2.3	$\hat{T}_1 = 90°$ (line from centre to midpoint of chord) $\hat{O}_1 + \hat{T}_1 + \hat{M}_2 = 180°$ (sum of angles of Δ) $\hat{O}_1 + 90° + 15° = 180°$ $\hat{O}_1 = 75°$	A✓S/R A✓Method A✓answer	(3)
			[14]

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QUESTION/*VRAAG* 8

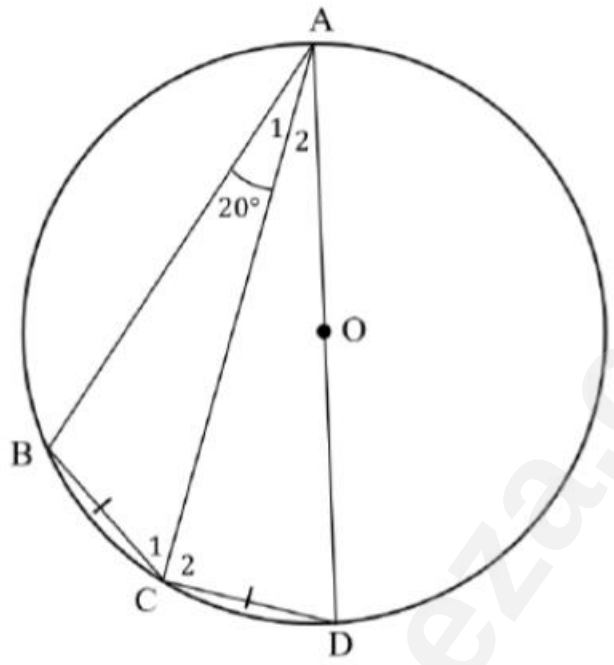


8.1.1	$\hat{P}_1 + \hat{P}_2 = 90°$ (∠ in semi-circle/ ∠ *in semi sirkel*) $\therefore \hat{S} = 60°$ (∠*'s of* Δ/ ∠*'e van* Δ)	✓S✓R ✓S (3)
8.1.2	$\hat{N}_1 + \hat{N}_2 = \hat{S} = 60°$ (∠'s in same segment/ ∠'e in dieselfde segment) $\therefore \hat{R}_1 = 30°$ (∠*'s of* ΔTNR/∠*'e van* ΔTNR)	✓S✓R ✓S (3)
8.1.3	$\hat{N}_2 = \hat{R}_1 = 30°$ (∠'s opp equal sides/ ∠*'e teenoor gelyke sye*) But $\hat{N}_1 + \hat{N}_2 = 60°$ $\therefore \hat{N}_1 = 30°$	✓S/R ✓S (2)
8.2	$\hat{W}_2 = 90°$ (line from centre to midpoint/ *lyn uit mdpt van sirkel na middelpunt van koord*) $\hat{T}_1 = \hat{W}_2$ (= 90°) $\therefore$ TNWO is a cyclic quadrilateral/*is 'n koordevierhoek* (CONVERSE of exterior ∠ of a cyclic quad)/ *OMGEKEERDE van buitehoek van 'n koordevierhoek*)	✓S✓R ✓S ✓R (4)
		[12]

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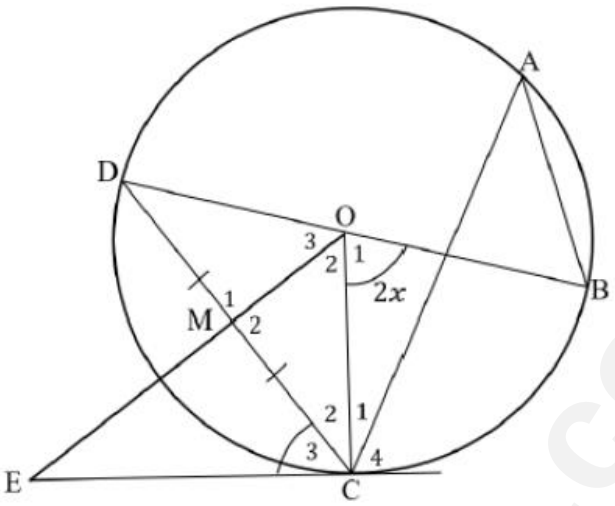
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QUESTION 8

8.1			
8.1.1	$\hat{A}_2 = 20°$ [equal chords; equal angles / *gelyke koorde; gelyke hoeke*]	✓ *S* ✓ *R*	(2) K
8.1.2	$\hat{C}_2 = 90°$ [∠in semi-circle /∠ in semi-sirkel]	✓ *S* ✓ *R*	(2) K
8.1.3	$\hat{D} = 70°$ [sum ∠s Δ/*som ∠e* Δ] $A\hat{B}C = 110°$ [opp ∠ cyclic quad / *teenoorst.∠ v kvh*]	✓ *S* & *R* ✓ *S* ✓ *R*	(3) R

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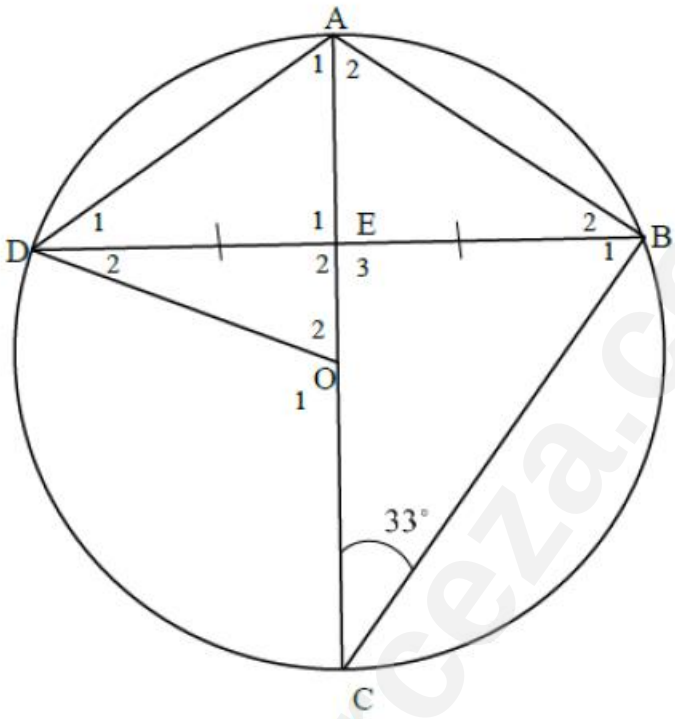
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8.2			
8.2.1	$\hat{A} = x$ [∠ at centre = 2 ∠ at circumf/*midpts∠ = 2 omtreks∠*] $\hat{D} = x$ [∠s in same segment/*∠e in dieselfde segment*] $\widehat{C_2} = x$ [∠s opp = radii/ *∠e teenoor = radiusse*]	✓ S ✓ R ✓ S ✓ R ✓ S ✓ R	(6) R
8.2.2	$\widehat{M_2} = 90°$ [Line from centre to midpoint of chord/ *lyn vanaf midpt na midpt van koord*] $\widehat{O_2} = 90° - x$ [sum ∠s Δ/*som ∠e* Δ]	✓ S ✓ R ✓ R	(3) C
8.2.3	$\widehat{O_2} = 90° - x$ [proven/ *reeds bewys*] $O\hat{C}E = 90°$ [radius ⊥ tangent/*raaklyn*] $\hat{E} = x$ [sum ∠s Δ/*som ∠e* Δ] $\therefore \hat{E} = \hat{D} = x$ ∴ DOCE is a cyclic quadrilateral. [converse theorem: angles in the same segment] ∴ *DOCE is 'n kvh* [*omgekeede stelling: hoeke in dies segment.*]	✓ S ✓ R ✓ S & R ✓ R	(4) C
			[20]

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QUESTION 8

8.1		
8.1	$\hat{D}_1 = 33°$ (∠'s in same circle segment)	✓S ✓R (2)
8.2	In Δ DAE : $\hat{E}_1 = 90°$ (line from centre to midpt of chord) $\hat{D}_1 = \hat{C} = 33°$ (∠s in same circle segment) $\therefore \hat{A}_1 = 57°$ (∠'s of Δ) **OR** In Δ EBC : $\hat{E}_3 = 90°$ (line from centre to midpt of chord) $\hat{C} = 33°$ (given) $\therefore \hat{B}_1 = 57°$ (∠s of Δ) $\therefore \hat{B}_1 = \hat{A}_1 = 57°$ (∠'s in same circle segment)	✓S ✓R ✓S (3) ✓S ✓R ✓S (3)
8.3	$\hat{O}_1 = 2\hat{A}_1 = 114°$ (∠at center = 2 × ∠at circumference)	✓S ✓R (2)

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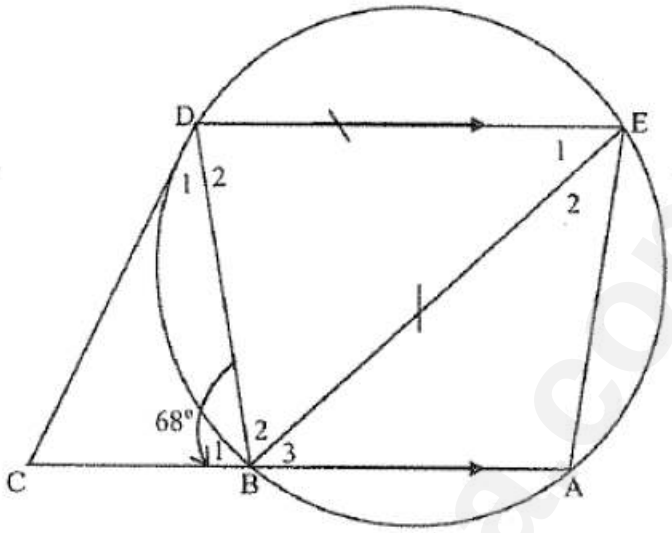
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8.4	$\hat{D}_1 + \hat{D}_2 = \hat{A}_1 = 57^\circ$ ($\angle$s opposite = sides) But $\hat{D}_1 = \hat{C} = 33^\circ$ ($\angle$s *in same circle segment*) $\therefore \hat{D}_2 = 24^\circ$ **OR** $D\hat{O}C = 2 \times \hat{B}_1$ ($\angle$@centre = 2 × $\angle$@circum) $D\hat{O}C = 2 \times 57^\circ$ $= 114^\circ$ $D\hat{O}C = \hat{E}_2 + \hat{D}_2$ (ext $\angle$ of Δ) $114^\circ = 90^\circ + \hat{D}_2$ $\therefore \hat{D}_2 = 24^\circ$	✓S/R ✓S (2) ✓S/R ✓S (2)
8.5	$\hat{B}_1 + \hat{B}_2 = 90^\circ$ ($\angle$ in semi−circle) $\hat{A}_2 = 57^\circ$ ($\angle$s ofΔ) **OR** $\Delta ADE \equiv \Delta ABE$ (S$\angle$S) $\therefore \hat{A}_1 = \hat{A}_2 = 57^\circ$	✓S ✓R ✓S (3) ✓S ✓R ✓S (3)
		[12]

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QUESTION/*VRAAG 8*



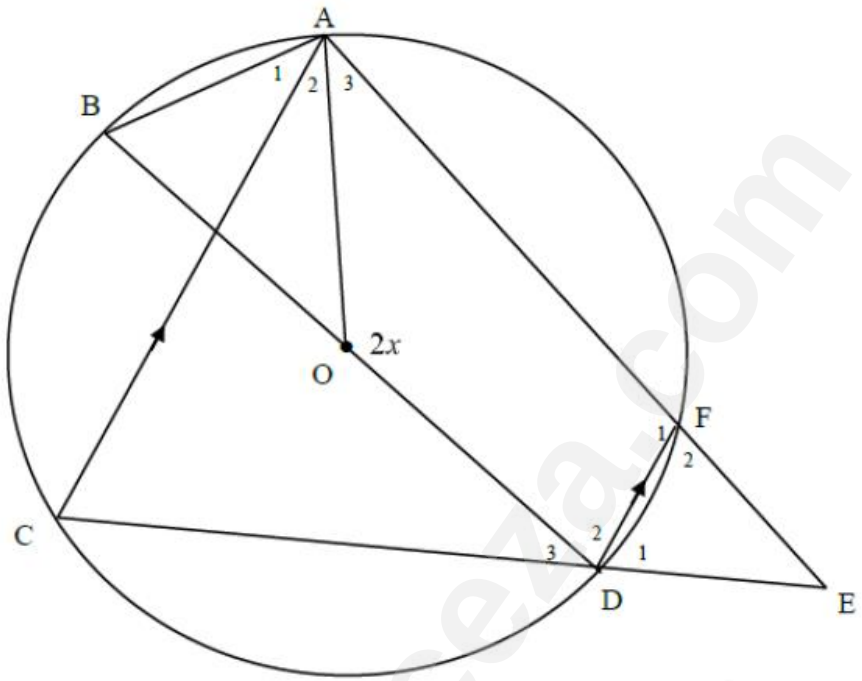
8.1.1	$D\hat{E}A = 68°$ [ext.∠ cyclic quad / *buite∠ van kvh*]	✓S✓R (2)
8.1.2	$\hat{A} = 112°$ [co-interior ∠s / *ko – binne ∠e; DE // CA*]	✓S/R (1)
8.1.3	$\hat{D}_2 = 68°$ [opp∠s of cyclic quad / *teenoorst.∠e van kvh*] **OR/*OF*** $\hat{D}_2 = 68°$ [alt ∠s / *verwiss ∠e; DE // CA*]	✓S✓R ✓S✓R (2)
8.1.4	$\hat{B}_2 = 68°$ [*∠s opp. = sides / ∠e teenoor = sye*]	✓S/R (1)
8.1.5	$\hat{E}_1 = 44°$ [sum of ∠s of Δ/*som van binne∠e van Δ*] $\hat{D}_1 = 44°$ [tangent chord theorem / *raaklyn – koordstelling*]	✓S/R ✓S✓R (3)
8.2	$\hat{C} = 68°$ [sum of ∠s of Δ/*som van binne∠e van Δ*] But / *maar* $\hat{B}_1 = 68°$ [given / *gegee*] DC = DB [sides opp. = ∠s / *sye teenoor = ∠e*] ∴Δ BDC is isosceles / *gelykbenig*	✓S/R ✓S/R (2)
8.3	$\hat{D}_2 = \hat{C} = 68°$ ∴ DE is a tangent / *'n raaklyn* [converse tangent chord theorem / *omgekeerde raaklyn – koordstelling*]	✓S ✓R (2) [13]

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QUESTION/*VRAAG* 9

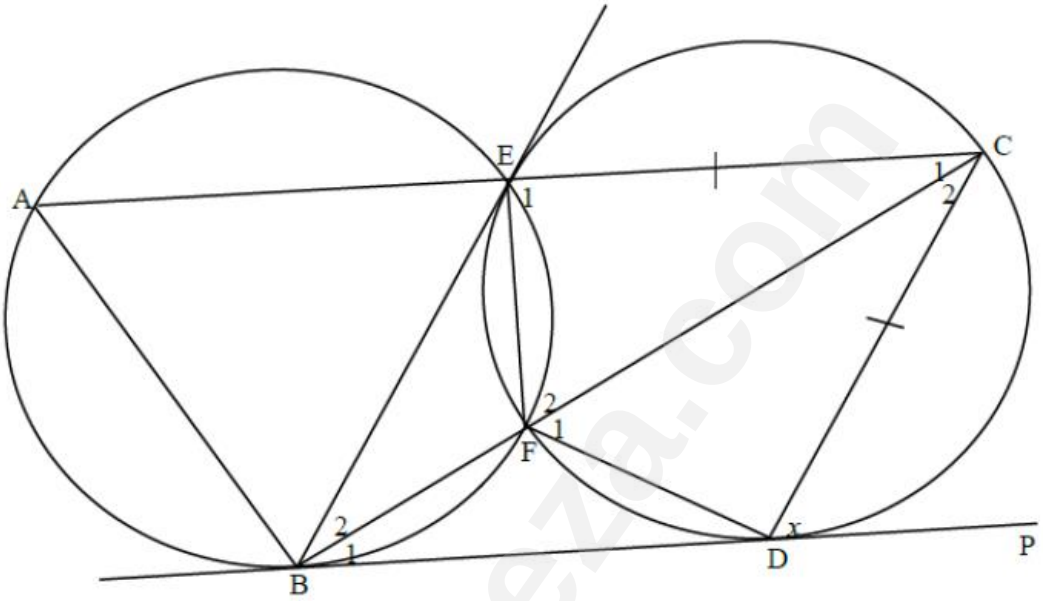


9.1.1	$\hat{C} = x$ [∠ at centre = 2 × ∠ at circumference/midpts ∠ = 2 × *omtreks* ∠] $\hat{B} = x$ [∠*s* in same segment/∠*e in dieselfdesegment*] $\hat{D}_1 = x$ [corresp ∠s /*ooreenkomstige* ∠e; AC ∥ DF] $B\hat{A}F = x$ [exterior ∠ of cyclic quad/*buite* ∠*kdvh*]	✓S/R ✓S/R ✓S✓R ✓S✓R (6)
9.1.2	$\hat{D}_1 = \hat{F}_2 = x$ ∴ DE = FE [∠*s* opp = sides/∠*e teenoor* = *sye*] $\hat{E} = 180° - 2x$ [sum of ∠*s* in Δ /*som van* ∠*e* in Δ]	✓ $\hat{D}_1 = \hat{F}_2$ ✓ $\hat{E} = 180° - 2x$ (2)
9.1.3	$\hat{E} = 180° - 2x$ [sum of ∠*s* in Δ /*som van* ∠*e* in Δ] $A\hat{O}D = 2x$ ∴ AODE is a cyclic quadrilateral/*is 'n kdvh* [converse of a cyclic quad/*omgek van kdvh*]	✓S ✓R (2)
9.2	$\frac{AF}{FE} = \frac{CD}{DE}$ [prop theorem/*eweredigh*] $\frac{12}{8} = \frac{AF}{10}$ AF = 15	✓S ✓R ✓ answer/*antwoord* (3)
		[13]

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QUESTION / *VRAAG 9*

9.1	Equal / *gelyk.*		✓answer / *antwoord* (1)
9.2			
9.2.1	$\hat{D}_1 = \hat{F}_1 = x$ $\hat{F}_1 = \hat{F}_2 = x$	tan chord theorem / *raaklyn koordstelling* = chords subtend = ∠*s* = *koorde onderspan* = ∠*e*	✓S / R ✓S ✓R (3)
9.2.2	$\hat{F}_2 = \hat{A} = x$ $\hat{D}_1 = \hat{A} = x$ ABDC is a cyclic quad / *ABDC is 'n kvh*	ext.∠ of cyclic quad / *buite ∠ v kvh* ext ∠= opp int ∠ **OR** converse of ext.∠ of cyclic quad / *buite ∠ = oorst binne ∠* ***OF*** *omgekeerde buite ∠ v kvh*	✓S ✓R ✓R (3)
9.2.3	$\hat{B}_1 + \hat{B}_2 = \hat{A}$ $\hat{A} = \hat{D}_1$ $\hat{B}_1 + \hat{B}_2 = \hat{D}_1$ BE ∥ CD	tan chord theorem / *raaklyn koordstelling* proved / *reeds bewys* correspond ∠*s* = / *ooreenkomst ∠e* =	✓S ✓R (2)

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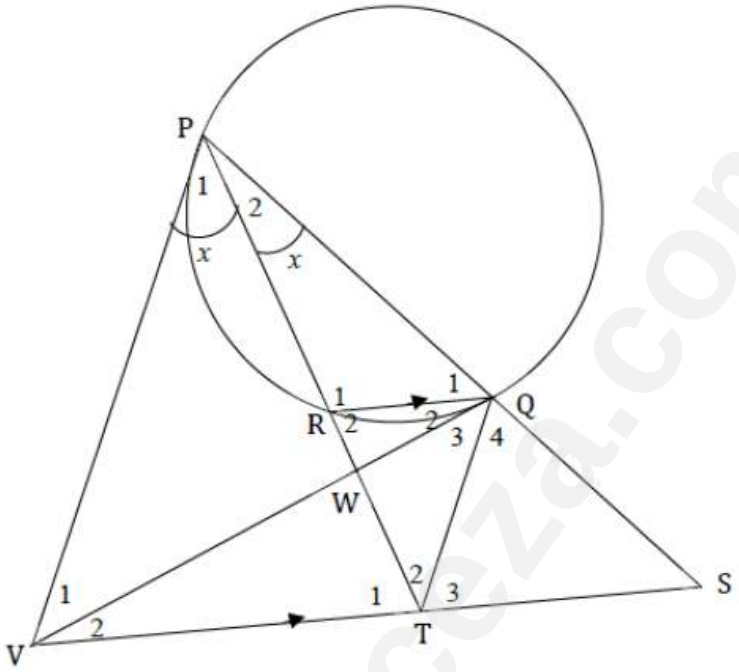
9.2.4	$\hat{C}_1+\hat{C}_2+\hat{F}_1+\hat{F}_2=180°$	opp $\angle s$ of a cyclic quad / *teenoorst $\angle e$ v kvh*	✓S ✓R
	$\hat{C}_1=\hat{C}_2$	diag rhombus bisect $\angle$ / *diag ruit halveer $\angle$*	✓S
	$\hat{F}_1=\hat{F}_2$	proved / *reeds bewys*	
	$2\hat{C}_1+2\hat{F}_2=180°$		
	$\hat{C}_1+\hat{F}_2=90°$		✓S
	$\hat{E}_1=90°$	sum of $\angle s$ of Δ / *som van $\angle e$ v Δ*	
	FC is a diameter of circle FDCE. *FC is 'n middellyn van sirkel FDCE.*	converse $\angle$ in a semi circle / *omgekeerde $\angle$ in half sirkel*	✓R
	OR / *OF*		**OR / *OF***
	Let $\hat{F}_1=\hat{F}_2=x$	proved / *reeds bewys*	✓S
	$\hat{C}=180°-2x$	opp $\angle s$ of a cyclic quad / *teenoorst $\angle e$ v kvh*	✓R ✓S
	$\hat{C}_1=\hat{C}_2=90°-x$	diag rhombus bisect $\angle$ / *diag ruit halveer $\angle$*	
	In ΔFDC **or** / *of* ΔEFC $\hat{D}=90°$ **or** / *of* $\hat{E}=90°$	sum of $\angle s$ of Δ / *som van $\angle e$ v Δ*	✓S
	FC is a diameter of circle FDCE. *FC is 'n middellyn van sirkel FDCE.*	converse $\angle$ in a semi circle / *omgekeerde $\angle$ in half sirkel*	✓R (5)
			[14]

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QUESTION 9



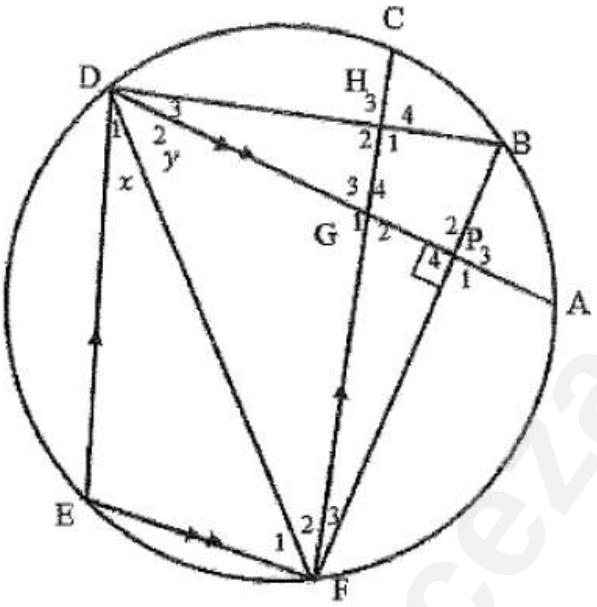
9.1	$\hat{P}_1 = \hat{Q}_1 = x$ (tan chord theorem) $\hat{Q}_1 = S = x$ (corresp ∠'s (PQ//VS) $\hat{S} = x$	A✓S A✓R A✓ S A✓R	(4)
9.2	$\hat{P}_2 = \hat{Q}_2$ (tan chord theorem) $\hat{Q}_2 = \hat{V}_2$ (alt ∠'s (PQ//VS) $\therefore \hat{P}_2 = \hat{V}_2$ $\therefore$ PQTV is a cyclic quadrilateral (converse of angles in the same seg.)	A✓ S A✓R A✓ S A✓R A✓ R	(5)
9.3	$\hat{Q}_3 = \hat{P}_1$ (∠'s in the same segment) $\therefore \hat{Q}_3 = \hat{P}_2$ $\therefore$ TQ is a tangent (converse: tan chord theorem)	A✓ S A✓R A✓ R	(3)
			[12]

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QUESTION/VRAAG 9

9.1



9.1.1	$\hat{E} = 180° - (x + y)$	(co-interior ∠'s, AD ∥ FE/ *ko-binne∠'e; AD ∥ FE)*	✓S ✓R
	$\therefore \hat{B} = x + y$	(opposite ∠'s of cyclic quadrilateral/ *teenoorstaande ∠'e van koordevierhoek)*	✓S ✓R (4)
9.1.2	DEFG a parm	(both pair of opposite sides ∥/ *albei pare teenoorstaande sye ∥)*	✓S/ R
	$\hat{G}_1 = 180° - (x + y)$	(opposite angles of parallelogram/ *teenoorst ∠ 'e van parm)*	✓S
	$\hat{G}_4 = \hat{G}_1 = 180° - (x + y)$	(vertically opp ∠ 's/*regoorst ∠'e)*	
	$\hat{G}_4 + \hat{B} = 180°$		✓S
	HPBG is a cyclic quad/ *HPBG is 'n koordevierhoek*		
	(CONVERSE: opposite ∠'s of cyclic quad/ *OMGEKEERDE: teenoorst ∠'e van 'n koordevierhoek)*		
	$\therefore \hat{H}_1 = \hat{P}_1$	(exterior ∠ of cyclic quad/ *buite ∠ van 'n koordevierhoek)*	✓S ✓R
			✓S (6)

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SIMILAR TRIANGLES AND RATIO AND PROPORTION

QUESTION/*VRAAG* 10			
10.1	$\angle ATB = 90°$ (subtended by diameter/*onderspan deur middellyn*) $\therefore \angle ATB = \angle EPB = 90°$ $\therefore$ TEPB is a cyclic quad/*is 'n koordevierhoek* (ext/*buite* $\angle$ = int opp/*teenoorst. binne* $\angle$)	✓ $\angle$ ATB = 90° ✓ $\angle$ ATB = $\angle$ EPB = 90° ✓conclusion with reason/ *gevolgtrekking met rede*	(3)
10.2	In ΔATB and/*en* ΔAPE $\angle A = \angle A$ [common/*gemeen*] $\angle ATB = \angle APE = 90°$ $\therefore \Delta ATB \,///\, \Delta APE$ $(\angle,\angle,\angle)$	✓statement with reason / *stelling met rede* ✓ $\angle ATB = \angle APE = 90°$ ✓ $\Delta ATB \,///\, \Delta APE$ with reason /*met rede*	(3)
10.3	$\angle RTA = \angle TBA$ (tangent chord/*raaklyn-koord*) $\angle RTA = \angle ETP$ (vertically opposite/*regoorst*) $\angle TBA = \angle TEP$ (ext/*buite* $\angle$ of cyclic quad/ *van koordevierhoek*) ΔTPE is Isosceles/*gelykbenig*. ($\angle ETP = \angle TEP$) $\therefore$ TP = PE	✓$\angle RTA = \angle TBA$ (statement) with reason. *(stelling) met rede* ✓$\angle RTA = \angle ETP$ (statement) with reason *(stelling) met rede* ✓statement with reason *(stelling) met rede* ✓ΔTPE is Isosceles/*gelykbenig* ✓ reason/*rede* ✓ conclusion/*slotsom*	(6)
10.4	$\angle ATB = \angle EPB = 90°$ $\angle PTB = \angle A$ [tan chord/*raaklyn-koord*] $\angle PTB = \angle PEB$ [subtended by BP/*onderspan deur BP*] $\therefore \angle A = \angle PEB$ Hence/*Vervolgens* $\Delta ATB \,///\Delta EPB$ [equiangular/ *gelykhoekig*]	✓ $\angle ATB = \angle EPB = 90°$ ✓ $\angle PTB = \angle A$ (statement) with reason *(stelling) met rede* ✓ $\angle PTB = \angle PEB$ (statement) with reason *(stelling) met rede* ✓ $\angle A = \angle PEB$ ✓ conclusion/*slotsom*	(5)
10.5	$\frac{BP}{TB} = \frac{BE}{AB}$ $(\Delta ATB \,///\, \Delta EPB)$ AB.BP = BE.TB 2BP.BP = BE.TB [AO=OB=BP] $2BP^2 = BE.TB$	✓statement with reason *stelling met rede* ✓ AB.BP = BE.TB ✓2BP.BP = BE.TB (statement) with reason *(stelling) met rede* ✓answer/*antwoord*	(4)
			[21]

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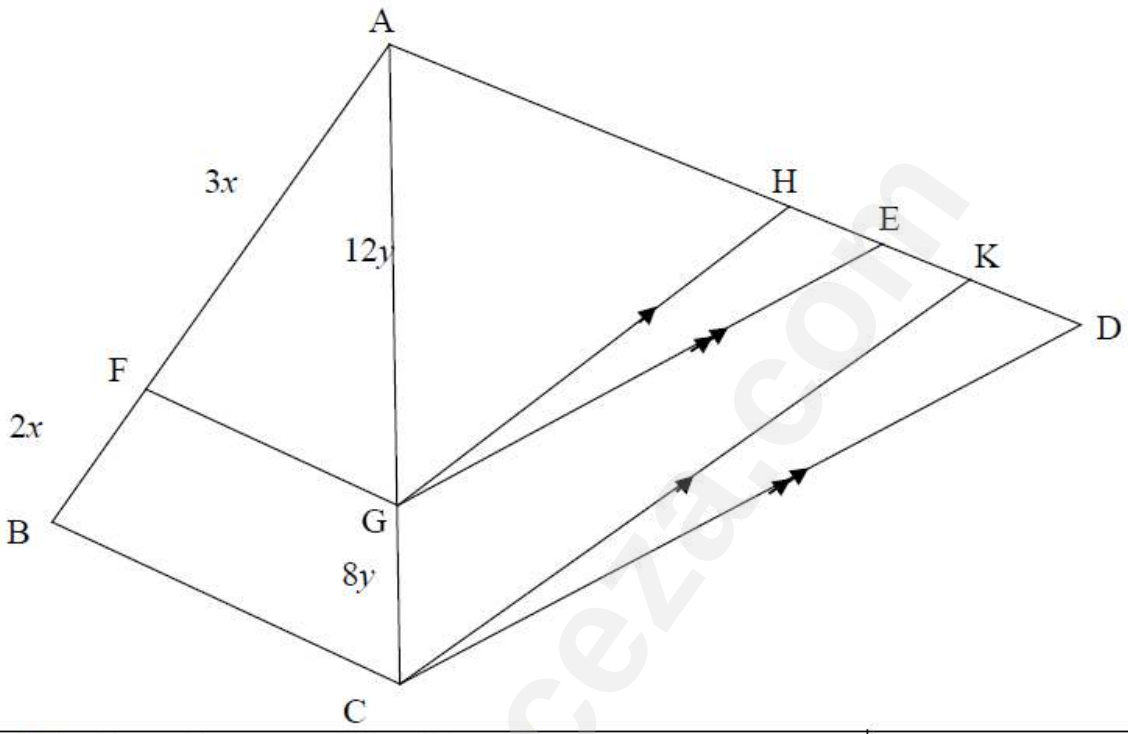
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QUESTION/VRAAG 11				
11.1	Divides the two other sides proportionally/*verdeel die twee ander sye eweredig.*		✓ answer/*antwoord*	(1)
11.2	11.2.1	$\frac{KD}{DL} = \frac{2x}{x}$ $\frac{KE}{EM} = \frac{2y}{y}$ [ED // LM] $LM^2 = KL^2 + KM^2$ [Pythagoras] $= (3x)^2 + (3y)^2$ $= 9x^2 + 9y^2$ $= 9\,DL^2 + 9\,EM^2$ 1	✓✓ proportions / *eweredighede* ✓ Pythagoras ✓ answer/*antwoord*	(4)
	11.2.2	$DM^2 + LE^2 = KM^2 + KD^2 + EK^2 + KL^2$ $= (3y)^2 + (2x)^2 + (2y)^2 + (3x)^2$ $= 9y^2 + 4x^2 + 4y^2 + 9x^2$ $= 13y^2 + 13x^2$ $= 13\,EM^2 + 13\,DL^2$ $= 13(EM^2 + DL^2)$ $= 13 \times \frac{LM^2}{9}$ $= \frac{13}{9} LM^2$	✓ statement/*stelling* ✓substitution and simplification/ *vervanging en vereenvoudiging* ✓use of 1/*gebruik van 1* ✓ conclusion/*slotsom*	(4)
				[9]

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QUESTION/*VRAAG* 9



9.1.1	$\frac{AF}{BF}=\frac{3x}{2x}=\frac{3}{2}$ & $\frac{AG}{CG}=\frac{12y}{8y}=\frac{3}{2}$ $\therefore \frac{AF}{BF}=\frac{AG}{CG}$ $\therefore$ FG ∥ BC [conv prop th/*omg eweredigh st.* **OR** line divides 2 sides of Δ in prop/*lyn verdeel 2 sye v Δ in dies verh*]	✓ $\frac{AF}{BF}=\frac{AG}{CG}$ ✓R (2)

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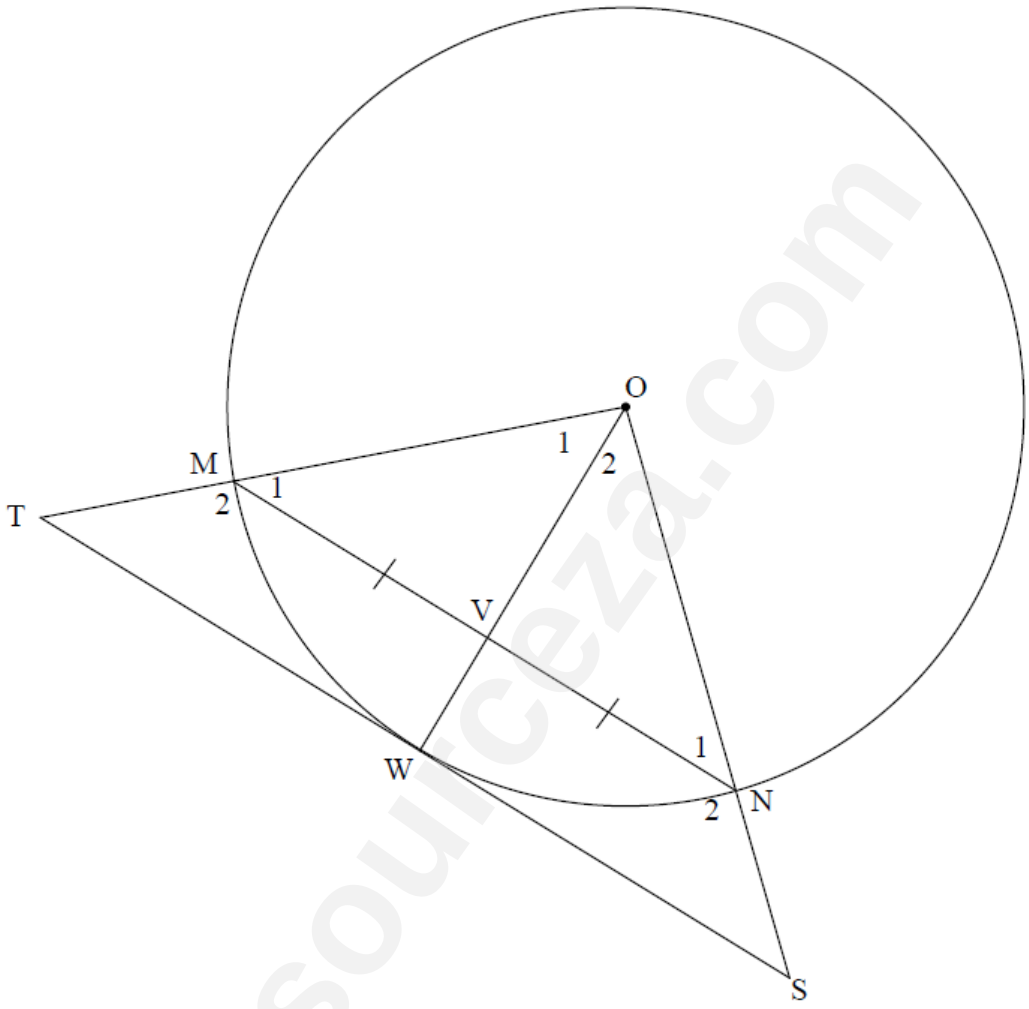
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9.1.2	$\frac{AG}{GC}=\frac{AH}{HK}$ [prop theorem/*eweredigh st*: GH ‖ CK **OR** line ‖ to 1 side of Δ/*lyn* ‖ *1 sy van Δ*] $\frac{AG}{GC}=\frac{AE}{ED}$ [prop theorem/*eweredigh st*: GE ‖ CD] $\therefore \frac{AH}{HK}=\frac{AE}{ED}$	✓S ✓R ✓ S (3)
9.2	$\frac{AE}{ED}=\frac{3}{2}$ and $\frac{AH}{HK}=\frac{3}{2}$ $\frac{AE}{12}=\frac{3}{2}$ and $\frac{15}{HK}=\frac{3}{2}$ $\therefore$ AE = 18 and HK = 10 $\therefore$ HE = AE – AH = 18 – 15 = 3 $\therefore$ EK = HK – HE = 10 – 3 = 7 **OR/*OF*** AD = 30 KD = AD – AH – HK = 30 – 15 – 10 = 5 EK = ED – KD = 12 – 5 = 7	✓ use of ratios ✓ AE = 18 ✓ HK = 10 ✓ HE = 3 **or** KD = 5 ✓ EK = 7 (5) **[10]**

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QUESTION/*VRAAG* 10



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10.1	Line from centre to midpoint of chord/ *lyn vanaf midpt na midpt van koord*	✓ R (1)
10.2.1	$O\hat{W}T = O\hat{W}S = 90°$ [radius ⊥ tangent/*raaklyn*] $\therefore MN \parallel TS$ [corresp ∠s =/*ooreenkomstige ∠e* = **OR** co-int ∠s 180°/*ko-binne∠e 180°* **OR** alternate ∠s/*verwiss ∠e*]	✓ R ✓ R (2)
10.2.2	$\hat{M}_1 = \hat{N}_1$ [∠s opp = sides/ *∠e teenoor = sye*] $\hat{M}_1 = \hat{T}$ [corresp ∠s/*ooreenk ∠e*; MN ∥ TS] $\therefore \hat{N}_1 = \hat{T}$ ∴ TMNS is a cyclic quadrilateral [conv: ext∠ cyclic quad] TMNS *is 'n koordevierhoek* [*omgek: buite∠ kdvh*] **OR/*OF*** $\hat{M}_1 = \hat{N}_1$ [∠s opp = sides/*∠e teenoor = sye*] $\hat{N}_1 = \hat{S}$ [corresp ∠s/*ooreenk ∠e*; MN ∥ TS] $\therefore \hat{S} = \hat{M}_1$ ∴ TMNS is a cyclic quadrilateral [conv: ext∠ cyclic quad] TMNS *is 'n koordevierhoek* [*omgek: buite∠ kdvh*]	✓ S ✓ S ✓ S ✓ R (4) ✓ S ✓ S ✓ S ✓ R (4)

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10.2.3	In ΔOVN and ΔOWS		
	$\hat{O}_2 = \hat{O}_2$	[common/*gemeenskaplik*]	✓ S; S; S **OR** S; S; R
	$O\hat{V}N = O\hat{W}S = 90°$	[from 10.1]	
	$O\hat{N}V = O\hat{S}W$	[sum ∠s Δ/*som ∠e Δ*]	
	$\therefore \Delta OVN \mid\mid\mid \Delta OWS$	[∠, ∠, ∠]	✓$\Delta OVN \mid\mid\mid \Delta OWS$
	$\therefore \frac{VN}{WS} = \frac{ON}{OS}$		✓ $\frac{VN}{WS} = \frac{ON}{OS}$
	But $VN = \frac{1}{2}MN$	[given]	✓ $VN = \frac{1}{2}MN$
	$\therefore \frac{\frac{1}{2}MN}{WS} = \frac{ON}{OS}$		✓ substitution
	$\therefore OS.MN = 2ON.\ WS$		(5)
	OR/*OF*		
	In ΔOVM and ΔOWS		✓ S; S; S **OR** S; S; R
	$O\hat{V}M = O\hat{W}S = 90°$	[from 10.1]	
	$O\hat{M}V = O\hat{S}W$	[sum ∠s Δ/*som ∠e Δ*]	
	$\therefore \Delta OVM \mid\mid\mid \Delta OWS$	[∠, ∠, ∠]	✓$\Delta OVM \mid\mid\mid \Delta OWS$
	$\therefore \frac{OM}{OS} = \frac{VM}{WS}$		✓ $\frac{OM}{OS} = \frac{VM}{WS}$
	But $VN = \frac{1}{2}MN$	[given]	✓ $VN = \frac{1}{2}MN$
	$\therefore \frac{\frac{1}{2}MN}{WS} = \frac{OM}{OS}$		✓ substitution
	$\therefore OS.MN = 2ON.\ WS$	[VM = VN]	(5)

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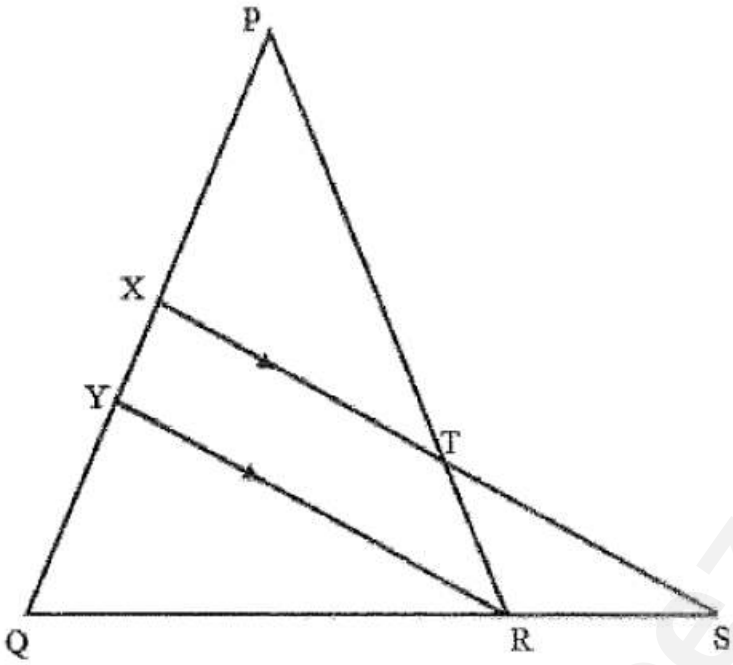
	OR/*OF*	
	If any other 2 Δs are used, first need to prove that TW = WS by proving ΔOWT ≡ ΔOWS	
	In ΔOVM and ΔOWT	✓✓ similarity
	$\hat{O}_1 = \hat{O}_1$ [common/*gemeenskaplik*]	✓✓ congruency
	$O\hat{V}M = O\hat{W}T = 90°$ [from 10.1]	
	$O\hat{M}V = O\hat{T}W$ [sum ∠s Δ/*som ∠e Δ*]	
	$\therefore \Delta OVM \,|||\, \Delta OWT$ [∠, ∠, ∠]	✓ VN = VM = $\frac{1}{2}$MN
	$\therefore \frac{VM}{WT} = \frac{OM}{OT}$	
	But VN = VM = $\frac{1}{2}$MN [given]	
	and WT = WS and OT = OS [ΔOWT ≡ ΔOWS]	
	$\therefore \frac{\frac{1}{2}MN}{WS} = \frac{ON}{OS}$	(5)
	$\therefore$ OS.MN = 2ON. WS	**[12]**

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9.2



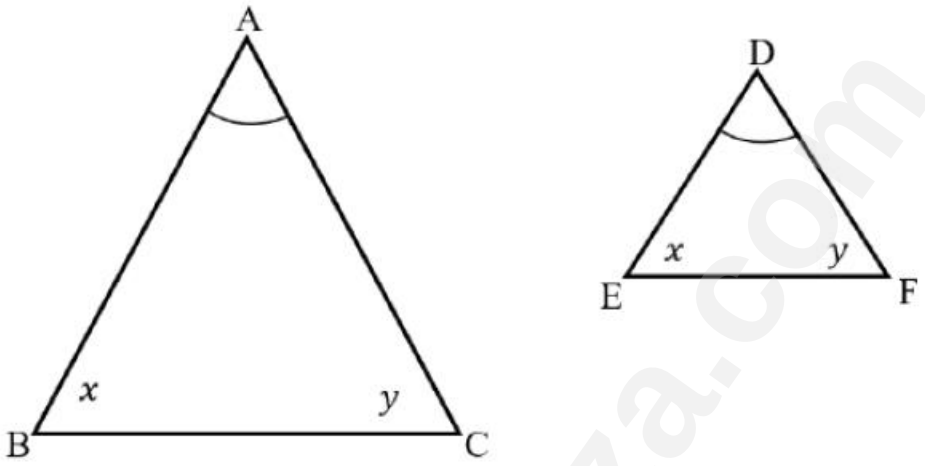
9.2	$\frac{PT}{TR}=\frac{PX}{XY}=\frac{3}{2}$ (proportionality theorem, XS ∥ YR Eweredigheidstelling, XS ∥ YR)	✓S✓R
	$\frac{PX}{XQ}=\frac{2}{5}$	✓S
	$\therefore \frac{PX}{XY}=\frac{3}{2}=\frac{6}{4}$ and/ en $\frac{PX}{XQ}=\frac{6}{15}$ $PX:XY:YQ=6:4:11$	✓manipulation to get equal numerators /manipulasie om tellers dieselfde te kry
	$\therefore \frac{QR}{RS}=\frac{QY}{YX}$ (proportionality theorem, XS ∥ YR Eweredigheidstelling, XS ∥ YR	✓S
	$\therefore \frac{QR}{RS}=\frac{11}{4}$	✓answer (6)
		[16]

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QUESTION 9

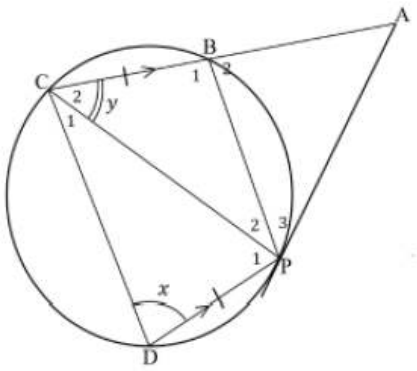
9.1



Constr: On sides AB and AC of ΔABC, mark points G and H respectively such that AG = DE and AH = DF. Draw GH/*Merk punt G en H op sy AB en AC van ΔABC onderskeidelik af sodanig dat AG = DE en AH = DF. Trek GH.*	✓ construction/ *konstruksie* B/D if no construction
Proof/*Bewys*:	
$\Delta AGH \equiv \Delta DEF$ [s, ∠, s]	✓ S/R
$\therefore A\hat{G}H = \hat{E}$	
$= \hat{B}$ [$\hat{B} = \hat{E}$, given/*gegee*]	✓ S
$\therefore GH \parallel BC$ [corresp/*ooreenk* ∠^{s} =]	✓ S /R
$\therefore \frac{AG}{AB} = \frac{AH}{AC}$ [line ∥ one side of Δ] or [prop theorem; GH ∥ BC] [*lyn ∥ een sy van* Δ] *of* [*eweredigheid*; GH ∥ BC]	✓ S ✓ R
$\therefore \frac{DE}{AB} = \frac{DF}{AC}$ [constr/*konstruksie*]	(6) K

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9.2			
9.2.1	*In* ΔAPC and / en Δ ABP: 1) $\hat{A} = \hat{A}$ [common ∠ ; gemene ∠) 2) $\widehat{P_3} = \widehat{C_2} = y$ [tangent chord theorem/ *raaklyn koord stelling*] 3) $A\hat{P}C = \widehat{B_2}$ [sum ∠s Δ/*som ∠e* Δ] ∴ ΔAPC lll Δ ABP **OR** *In* ΔAPC and / en Δ ABP: 1) $\hat{A} = \hat{A}$ [common ∠ ; gemene ∠) 2) $\widehat{P_3} = \widehat{C_2} = y$ [tangent chord theorem/ *raaklyn koord stelling*] ∴ ΔAPC lll Δ ABP (∠; ∠; ∠)	✓ S & R ✓ S ✓ R ✓ S & R ✓ S & R ✓ S ✓ R ✓ R (∠; ∠; ∠)	
	OR		
	In ΔAPC and / en Δ ABP 1) $\hat{A} = \hat{A}$ [common ∠ ; gemene ∠) 2) $\widehat{B_2} = \hat{D} = x$ [ext ∠ of cyc quad / buite ∠ koordevh.] $A\hat{P}C = \hat{D} = x$ [tangent chord theorem/ *raaklyn koord stelling*] ∴ $A\hat{P}C = \widehat{B_2}$ 3) $\widehat{P_3} = \widehat{C_2}$ [sum ∠s Δ/*som ∠e* Δ] ∴ ΔAPC lll Δ ABP **OR** *In* ΔAPC and / en Δ ABP 1) $\hat{A} = \hat{A}$ [common ∠ ; gemene ∠) 2) $\widehat{B_2} = \hat{D} = x$ [ext ∠ of cyc quad / buite ∠ koordevh.] $A\hat{P}C = \hat{D} = x$ [tangent chord theorem/ *raaklyn koord stelling*] ∴ $A\hat{P}C = \widehat{B_2}$ ∴ ΔAPC lll Δ ABP (∠; ∠; ∠)	✓ S & R ✓ S & R ✓ S & R ✓ S&R ✓ S & R ✓ S & R ✓ S & R ✓ R (∠; ∠; ∠)	(4) R

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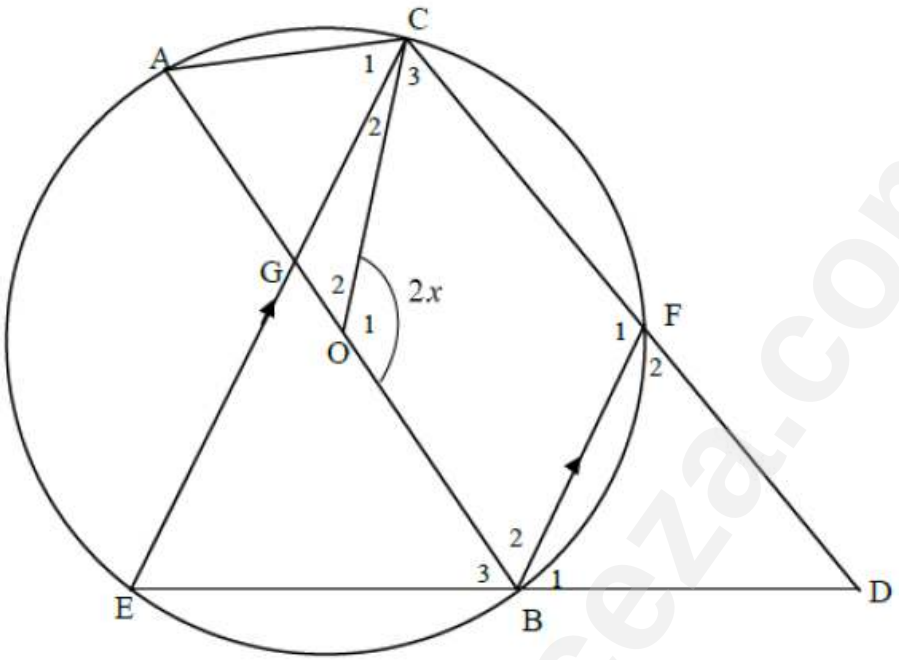
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9.2.2	$\frac{AP}{AB} = \frac{AC}{AP}$ $\Delta APC \text{ lll } \Delta ABP$ $\therefore AP^2 = AB.AC$	✓ S	(1) K
9.2.3	In $\Delta APC \text{ lll } \Delta CDP$ 1) $\hat{P}_1 = y$ [alt. ∠s/ verwis. ∠e; CB ‖ DP] 2) $\hat{D} = C\hat{P}A = x$ [tangent chord theorem/ *raaklyn koord stelling*] 3) $\hat{A} = \widehat{C_1}$ [sum ∠s Δ/*som* ∠*e* Δ] $\therefore \Delta APC \text{ lll } \Delta CDP$ **OR** In $\Delta APC \text{ lll } \Delta CDP$ 1) $\hat{P}_1 = y$ [alt. ∠s/ verwis. ∠e; CB ‖ DP] 2) $\hat{D} = C\hat{P}A = x$ [tangent chord theorem/ *raaklyn koord stelling*] $\therefore \Delta APC \text{ lll } \Delta CDP$ (∠; ∠; ∠)	✓ S & R ✓ S ✓ R ✓ S & R ✓ S & R ✓ S ✓ R ✓ R (∠; ∠; ∠)	(4) R
9.2.4	$\frac{AC}{CP} = \frac{PC}{DP}$ [$\Delta APC \text{ lll } \Delta CDP$] $\therefore PC^2 = DP.AC$ $AP^2 + PC^2 = AB.AC + AC.DP$ $= AC(AB + DP)$ $= AC(AB + BC)$ [$DP = BC$] $= AC.AC$ $= AC^2$	✓ S ✓ S ✓ *common factor* ✓ $= AC(AB + \boldsymbol{BC})$ *answer given*	(4) P
			[19]

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QUESTION 9



9.1	$\hat{E} = x$ ($\angle$at centre $= 2 \times \angle$at circum) $\hat{F}_1 = 180° - x$ (opposite $\angle$s of cyclic quad) $\hat{F}_2 = x$ ($\angle$s on straight line) **OR** $\hat{E} = x$ ($\angle$at centre $= 2 \times \angle$at circum) $\hat{F}_2 = x$ (ext $\angle$ of cyclicquad)	✓S ✓R ✓S (3) ✓S ✓R ✓S/R (3)
9.2	$\hat{F}_2 = x$ (ext $\angle$ of cyclic quad OR straight line) $\hat{B}_1 = x = \hat{E}$ (corresponding $\angle$s, EC $\parallel$ BF) $\therefore \hat{F}_2 = \hat{B}_1 = x$ $\therefore$ DF = DB (sides oppposite = $\angle$s)	✓S ✓R ✓S ✓R (4)

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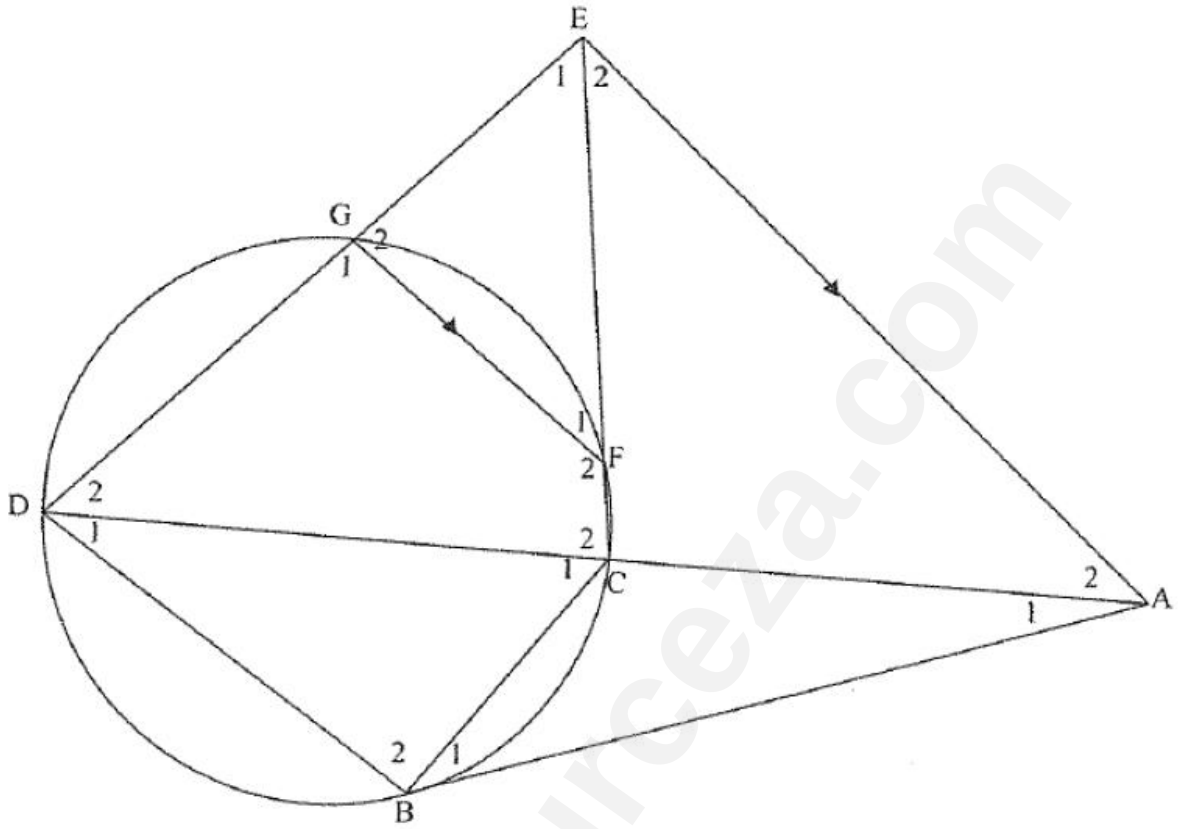
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9.3	$\hat{A} = x$ ($\angle$at centre $= 2 \times \angle$at circum) $\hat{C}_1 + \hat{C}_2 = x$ ($\angle$s opposite = sides) $\hat{C}_2 + \hat{C}_3 = x$ (ext. $\angle$ of cyclic quad) $\therefore \hat{C}_1 + \hat{C}_2 = \hat{C}_2 + \hat{C}_3$ $\therefore \hat{C}_1 = \hat{C}_3$ **OR** $\hat{A} = x$ ($\angle$at centre $= 2 \times \angle$at circum) $\hat{C}_1 + \hat{C}_2 = \hat{A}$ ($\angle$s opposite = sides) $\hat{A} = \hat{F}_2$ (exterior $\angle$ of cyclic quad) $\hat{F}_2 = \hat{C}_2 + \hat{C}_3$ (corresponding $\angle$s, CE $\parallel$ BF) $\therefore \hat{C}_1 + \hat{C}_2 = \hat{C}_2 + \hat{C}_3$ $\therefore \hat{C}_1 = \hat{C}_3$	✓S ✓S ✓S ✓R (4) ✓S ✓R ✓S ✓R (4)
9.4	CG = FB, CG $\parallel$ BF (given) $\therefore$ CGBF is a parm (one pair opp. sides equal and parallel) BG = FC = 2 (opp. sides of parm.) CF : FD = 2 : 1 $\frac{BD}{ED} = \frac{FD}{CD}$ (line $\parallel$ to one side of Δ or proportionality theorem) $\frac{BD}{ED} = \frac{1}{3}$	✓ R ✓S/R ✓S ✓S\R ✓$\frac{BD}{ED} = \frac{1}{3}$ (5)
		[16]

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QUESTION/*VRAAG 9*



9.1	tangent-chord theorem / *raaklyn-koordstelling*	✓ R (1)
9.2	In ΔABC and/*en* ΔADB: $\hat{A}_1 = \hat{A}_1$ [common/*gemeenskaplik*] $\hat{B}_1 = \hat{D}_1$ [proven/*bewys* in 9.1] ∴ ΔABC ||| ΔADB [∠ ; ∠ ; ∠] **OR/*OF*** In ΔABC and/*en* ΔADB: $\hat{A}_1 = \hat{A}_1$ [common/*gemeenskaplik*] $\hat{B}_1 = \hat{D}_1$ [proven/*bewys* in 9.1] $B\hat{C}A = A\hat{B}D$ [∠s of Δ = 180°/∠*e van* Δ = 180°] ∴ ΔABC ||| ΔADB	✓S ✓S ✓R ✓S ✓S ✓S (3)

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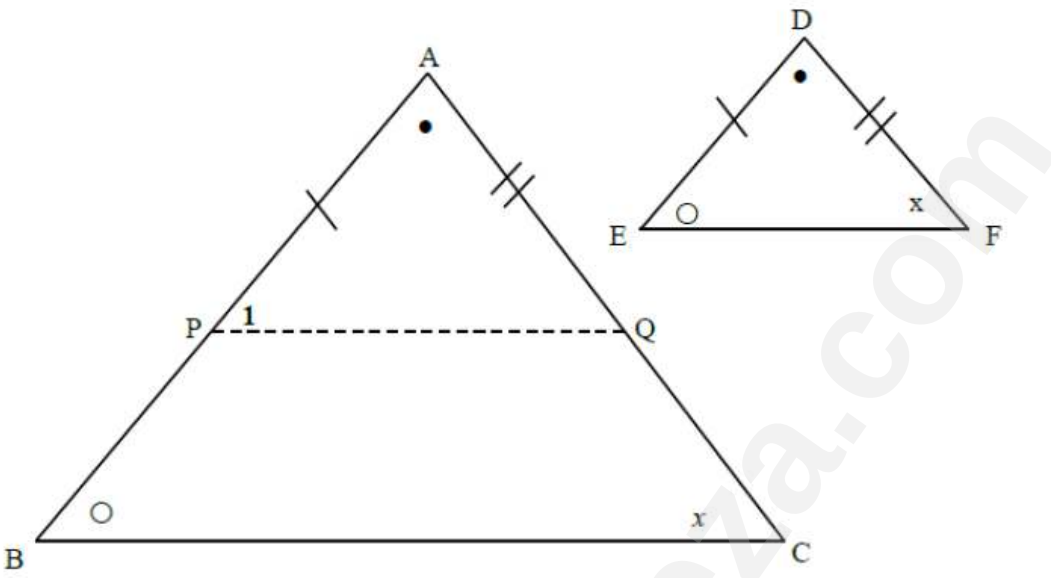
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9.3	$\hat{E}_2 = \hat{F}_1$ [alternate ∠s/ *verwiss ∠e* ; EA || GF] $\hat{F}_1 = \hat{D}_2$ [ext ∠ of cyc quad DGFC/*buite ∠v kdvh DGFC*] $\therefore \hat{E}_2 = \hat{D}_2$	✓S ✓R ✓S ✓R (4)
9.4	In ΔAEC and/*en* ΔADE: $\hat{A}_2 = \hat{A}_2$ [common/*gemeenskaplik*] $\hat{E}_2 = \hat{D}_2$ [proven/*bewys* in 9.3] $\therefore$ ΔAEC ||| ΔADE [∠ ; ∠ ; ∠] $\therefore \frac{AE}{AD} = \frac{AC}{AE}$ [from ||| Δs / *uit* ||| Δe] $\therefore AE^2 = AD \times AC$ $\therefore AE = \sqrt{AD \times AC}$ **OR/*OF*** In ΔAEC and/*en* ΔADE: $\hat{A}_2 = \hat{A}_2$ [common/*gemeenskaplik*] $\hat{E}_2 = \hat{D}_2$ [proven/*bewys* in 9.3] $A\hat{C}E = A\hat{E}D$ [∠s of Δ = 180°/*∠e van* Δ = 180°] $\therefore$ ΔAEC ||| ΔADE $\therefore \frac{AE}{AD} = \frac{AC}{AE}$ [from ||| Δs / *uit* ||| Δe] $\therefore AE^2 = AD \times AC$ $\therefore AE = \sqrt{AD \times AC}$	✓S ✓S ✓R ✓S ✓S ✓S ✓S ✓S ✓S ✓S (5)
9.5	$\frac{AB}{AD} = \frac{AC}{AB}$ [from |||Δs / uit ||| Δe] from/*van* 9.2 $AB^2 = AD \times AC$ $= AE^2$ [from/*van* 9.4] $\therefore AB = AE$	✓S ✓S ✓S (3) [16]

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QUESTION/*VRAAG* 10

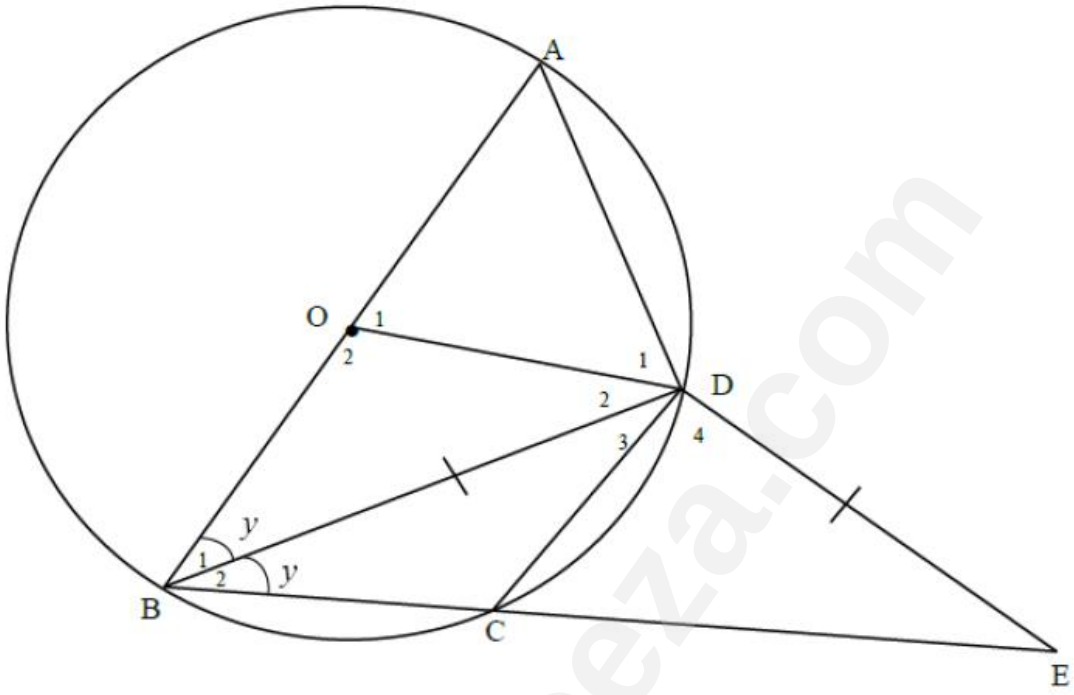


10.1	$\hat{A} = \hat{D}$ [given/*gegee*] AQ = DF [construction/*konstruksie*] AP = DE [construction/*konstruksie*] $\therefore \Delta APQ \equiv \Delta DEF$ [S∠S] $\hat{P}_1 = \hat{E}$ $\hat{P}_1 = \hat{B}$ [$\hat{E} = \hat{B}$] PQ ∥ BC [corresp ∠'s/*ooreenkomstige ∠e*] $\frac{AP}{AB} = \frac{AQ}{AC}$ [PQ ∥ BC] But/*maar* AP = DE and/*en* AQ = DF $\therefore \frac{DE}{AB} = \frac{DF}{AC}$	✓construction/*konstruksie* ✓S/R ✓$\hat{P}_1 = \hat{E}$ ✓S/R ✓S (5)

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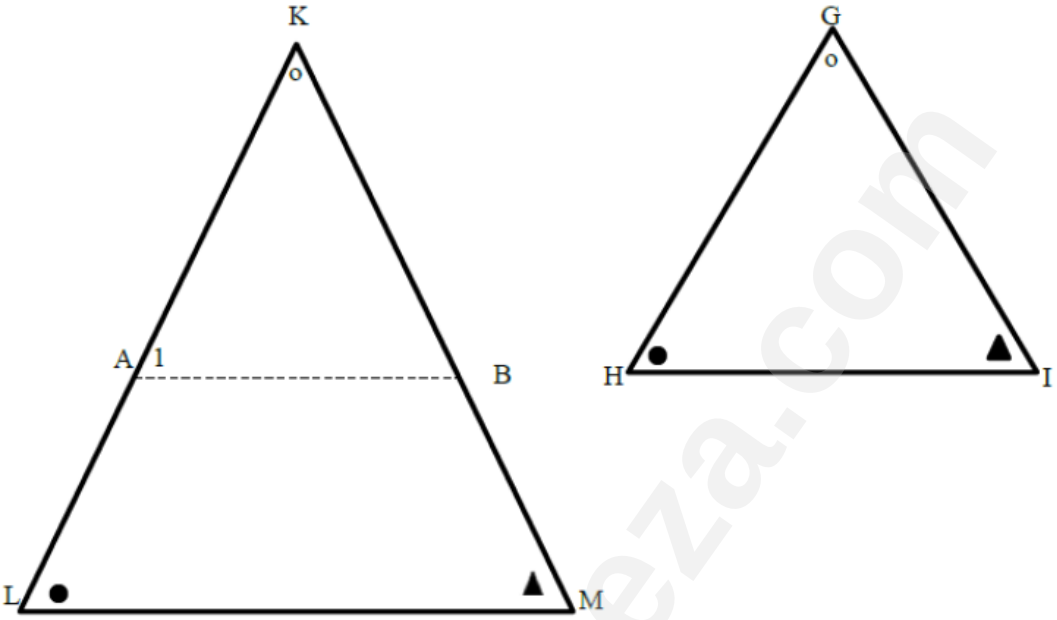
10.2.1	$A\hat{D}B = 90°$ [∠ in semi-circle/*semi-sirkel*] $\hat{A} = 90° - y$ [∠ in semi-circle/*semi-sirkel*] $\hat{B}_2 = \hat{E}$ [$\angle^s$ opp = sides/*∠e teenoor = sye*] $D\hat{C}E = \hat{A} = 90° - y$ [ext. ∠ of a cyclic quad = *int. opp ∠* / *verlengde ∠ van 'n Kdvh = binne teenoorg ∠*] $\therefore \hat{D}_4 = 90°$ [sum of $\angle^s$ in Δ / *som van* $\angle^e$ in Δ]	✓S/R ✓S ✓S/R ✓S/R ✓S/R (5)
10.2.2	$\hat{B}_1 = \hat{D}_2 = y$ [∠ opp = sides/*∠e teenoor = sye*] $\hat{B}_1 = \hat{B}_2$ [given/*gegee*] $\hat{E} = \hat{D}_2$ [proven/*bewys*] $\therefore \Delta BOD \,\|\|\, \Delta BDE$ [∠∠∠]	✓S/R ✓S ✓R (3)
10.2.3	$\frac{DE}{OD} = \frac{BE}{BD}$ [$\|\|\|$ Δ] but/*maar* BD = DE [given/*gegee*] $\therefore \frac{DE}{OD} = \frac{BE}{DE}$ $DE^2 = BE.OD$	✓S ✓R ✓S ✓substitution/ *vervanging* (4)
		[17]

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QUESTION / *VRAAG* 10

10.1



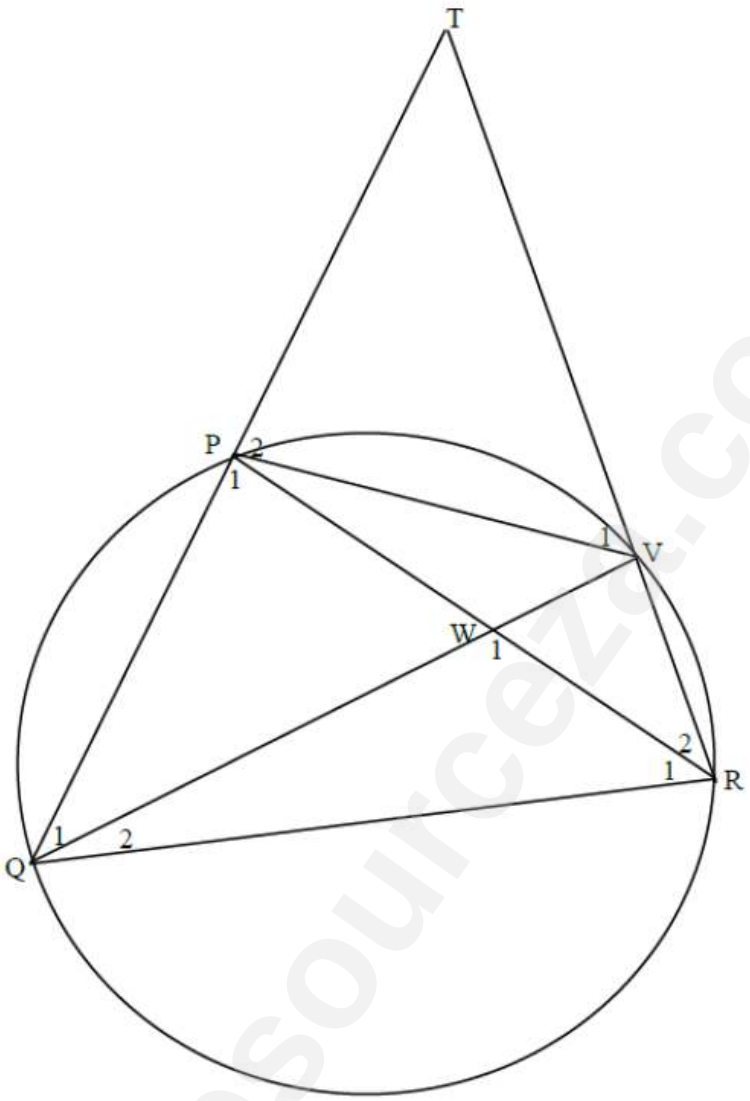
NB: NO construction 0 / 5 / *GEEN konstruksie 0 / 5*

	On sides KL and KM of ΔKLM mark points A and B respectively such that KA = GH and KB = GI. Draw AB *Op sye KL en KM van ΔKLM plaas A en B onderskeidelik sodat KA = GH en KB = GI. Trek lyn AB.*	✓construction / *konstruksie*
	Proof / Bewys In ΔGHI and/*en* ΔKAB	
	$KA = GH$ construction / *konstruksie*	
	$\hat{K} = \hat{G}$ given / *gegee*	
	$KB = GI$ construction / *konstruksie*	
	$\therefore \Delta GHI \equiv \Delta KAB$ S∠S	✓S / R $\Delta GHI \equiv \Delta KAB$ S∠S
	$\therefore \hat{A}_1 = \hat{H}$	✓S
	but $\hat{L} = \hat{H}$ given / *gegee*	
	$\therefore \hat{A}_1 = \hat{L}$	
	$\therefore AB \parallel LM$ corr.∠s = / *ooreenkoms.∠e=*	✓R
	$\frac{KL}{KA} = \frac{KM}{KB}$ line ∥ one side *Δ/lyn∥een sy v Δ*	✓S / R
	$\therefore \frac{KL}{GH} = \frac{KM}{GI}$	(5)

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10.2



10.2.1	$\hat{R}_1 = 60°$	equilateral Δ / *gelyksydige* Δ	✓S
	$\hat{W}_1 = \hat{P}_1 + \hat{Q}_1$	ext. $\angle$ of a Δ / *buite* $\angle$ *v* Δ	✓S
	$= 60° + \hat{Q}_1$		
	$= \hat{R}_1 + \hat{Q}_1$		
	$\hat{Q}_1 = \hat{R}_2$	$\angle$s in the same segment/ *$\angle$e in dieselfde segment*	✓S / R
	$\therefore \hat{W}_1 = T\hat{R}Q$		(3)
10.2.2	In ΔTQR and / *en* ΔQRV		
	1. $\hat{W}_1 = T\hat{R}Q$	proved / *reedsbewys*	✓S
	2. $\hat{R}_1 = T\hat{Q}R$	equilateral Δ / *gelyksydige* Δ	✓S
	3. $\hat{Q}_2 = \hat{T}$	sum $\angle$s of Δ / *som van* $\angle$*e v* Δ	
	$\therefore \Delta WRQ \| \| \| \Delta RQT$	$\angle\angle\angle$	✓R
			(3)

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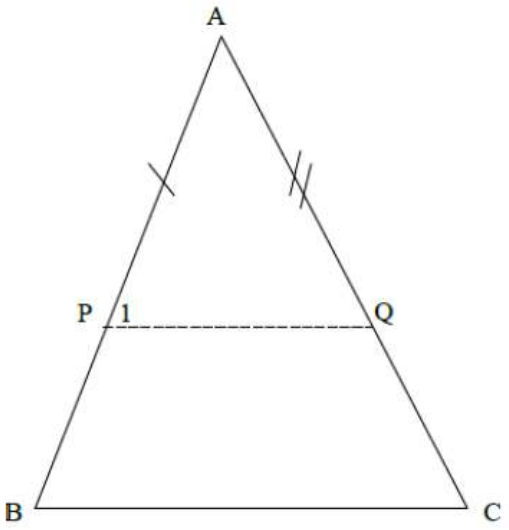
10.2.3	In ΔTPV and / *en* ΔWQR	
	1. $P\hat{Q}R = \hat{R}_1$ both 60° / *albei* 60°	✓S
	$P\hat{Q}R = \hat{V}_1$ ext.∠ of a cyclicquad. / *buite∠ v kvh*	✓S / R
	$\hat{V}_1 = \hat{R}_1$	✓S
	2. $\hat{P}_2 = T\hat{R}Q$ ext.∠ of a cyclicquad. / *buite∠ v kvh*	✓S
	but / *maar* $\hat{W}_1 = T\hat{R}Q$ proved / *reedsbewys*	
	$\hat{P}_2 = \hat{W}_1$	
	3. $\hat{T} = \hat{Q}_2$ sum of ∠s of Δ / *somv ∠e v Δ*	✓S
	$\Delta VPT \text{ ||| } \Delta RWQ$ ∠∠∠	✓ R
	$\frac{VP}{RW} = \frac{PT}{WQ} = \frac{VT}{RQ}$ corresponding sides in prop/ *ooreenkomstige sye in verhouding*	
	$\therefore \frac{PT}{WQ} = \frac{PV}{WR}$	(6)
		[17]

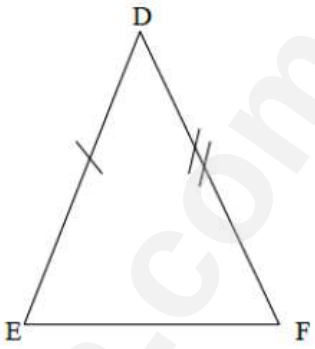
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QUESTION 10

10.1





Construct AP = DE and AQ = DF and draw PQ In Δ APQ and ΔDEF: AP = DE AQ = DF $\hat{A} = \hat{D}$ ∴ΔAPQ ≡ ΔDEF (SAS) $\hat{P}_1 = \hat{E}$, But $\hat{B} = \hat{E}$ … (given) $\hat{P}_1 = \hat{B}$ PQ ∥ BC (corresponding angles =) $\therefore \frac{AB}{AP} = \frac{AC}{AQ}$ (proportionality theorem; PQ ∥ BC) $\therefore \frac{AB}{DE} = \frac{AC}{DF}$ (construction)	A✓ construction A✓ ΔAPQ ≡ ΔDEF A✓ SAS A✓S A✓R A✓S A✓R	(7)

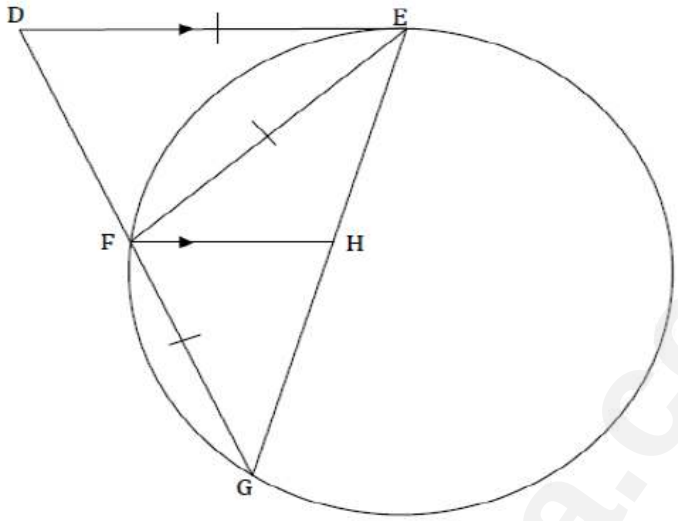
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10.2



10.2.1	$\hat{G}$ (tan chord theorem) $E\hat{F}H$ (alt ∠'s, DE||FH) $F\hat{E}H = \hat{G}$ (∠'s opposite = sides)	A✓ S/R A✓ S ✓R A✓ S/R	(4)
10.2.2(a)	In Δ's DEF and DGE: 1) $\hat{D}$ is common 2) $D\hat{E}F = \hat{G}$ (proven) 3) $D\hat{F}E = D\hat{E}G$ (sum of angles in triangle) ΔDEF ||| ΔDGE (∠∠∠)	 A✓ S A✓ S A✓ R	(3)
10.2.2(b)	Let $D\hat{E}F = x$: $E\hat{F}D = 2x$ (ext . angle of Δ) $\hat{D} = 2x$ (∠s opposite = sides) $\therefore 2x + 2x + x = 180°$ $x = 36°$ $\therefore \hat{D} = 2x = 72°$	 A✓ S A✓ R A✓ S A✓ Equation A✓ value of x	(5)

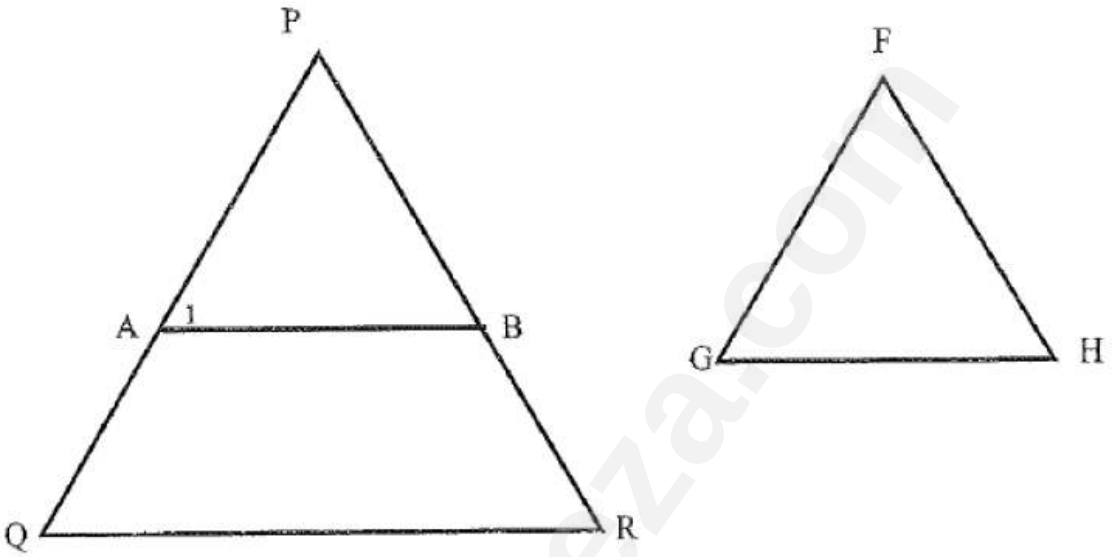
10.2.3	$\frac{DE}{DG} = \frac{DF}{DE}$ (ΔDEF ||| ΔDGE) $\frac{FG}{DG} = \frac{DF}{FG}$ (DE = FG) $\frac{2}{k+2} = \frac{k}{2}$ $k^2 + 2k = 4$	A✓ S A✓ S A✓ substitution	(3)
10.2.4	$\frac{GH}{GE} = \frac{GF}{GD} = \frac{2}{k+2}$ (Prop. Intercept theorem DE//FH)	A✓ S A✓R	(2)
			[24]

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QUESTION/*VRAAG* 10

10.1

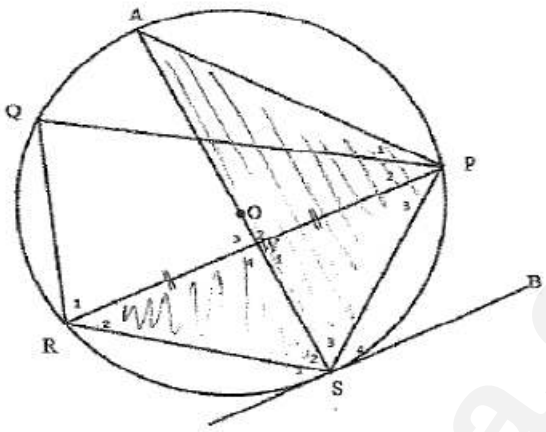


10.1	Construction: Mark off PA = FG and PB = FH and join AB. Konstruksie: PA = FG en PB = FH en verbind AB. $\Delta PAB \equiv \Delta FGH$ (S∠S) $\hat{A}_1 = \hat{G}$ (congruency) $\hat{G} = \hat{Q}$ (given) $\therefore \hat{A}_1 = \hat{Q}$ $\therefore AB \parallel QR$ (corresponding ∠'s =) $\frac{PQ}{PA} = \frac{PR}{PB}$ (line ∥ to one side of Δ) $\therefore \frac{PQ}{FG} = \frac{PR}{FH}$ (construction)	✓construction konstruksie ✓S/R ✓ ✓S/R ✓S ✓R (6)

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10.2



10.2.1	$\hat{W}_2 = 90°$ (line from centre to midpoint/ Lyn uit mdpt van sirkel na mdpt van krd) $\hat{S}_3 + \hat{S}_4 = 90°$ (tangent ⊥to radius/rkl ⊥r) $\hat{W}_2 = \hat{S}_3 + \hat{S}_4$ $\therefore$ SB ∥ RP (corresponding ∠'s = Ooreenkomstige ∠'e =)	✓S ✓R ✓S ✓R ✓R (5)
10.2.2	In Δ APS and ΔRWS: $\hat{A} = \hat{R}_2$ (∠'s same segment) $A\hat{P}S = \hat{W}_4$ ($A\hat{P}S = 90°$;∠ in semi-circle) $\hat{S}_3 = \hat{S}_2$ ($\angle' s of \Delta$) $\therefore$ Δ APS ||| Δ RWS (∠∠∠)	✓ S/R ✓S ✓R ✓S (4)
10.2.3	$\frac{PS}{WS} = \frac{AS}{RS}$ (||| Δ's) $\therefore$ PS.RS = AS.WS. But $\Delta RWS \equiv \Delta PWS$ (S∠S) $RS = PS$ $\therefore RS^2 = AS.WS$	✓S ✓S ✓ S/R ✓S (4)
10.2.4	$RS^2 = RW^2 + WS^2$ (Pythagoras) $AS.WS = RW^2 + WS^2$ ($RS^2 = AS.WS$)	✓S ✓substitution/

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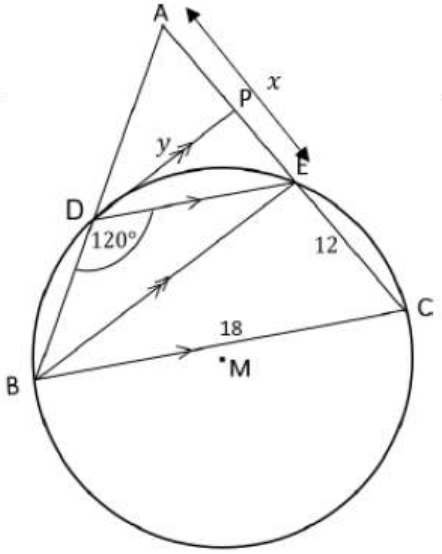
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	$\frac{AS.WS}{WS} = \frac{RW^2}{WS} + \frac{WS^2}{WS}$ $\therefore AS = \frac{RW^2}{WS} + WS$	vervanging ✓S ✓S (4)
		[23]

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QUESTION 10



10.1.1	$\frac{AD}{DB} = \frac{AE}{EC}$ [line ∥ one side of Δ] or [prop theorem; DE ∥ BC] [*lyn ∥ een sy van Δ*] *of* [*eweredighst; DE ∥ BC*] $\frac{5}{4} = \frac{x}{12}$ $4x = 60$ $x = 15$ *units*	✓ *S* ✓ *R* ✓ *answer*	(3) R
10.1.2	$\frac{\text{area of } \Delta AEB}{\text{area of } \Delta ECB} = \frac{½AEh_B}{½ECh_B}$ [Same height B / *dieselfde hoogtepunt B*] $\frac{\text{area of } \Delta AEB}{\text{area of } \Delta ECB} = \frac{AE}{EC}$ $\frac{\text{area of } \Delta AEB}{\text{area of } \Delta ECB} = \frac{15}{12}$ $\frac{\text{area of } \Delta AEB}{\text{area of } \Delta ECB} = \frac{5}{4}$	✓ *S* ✓ *answer* CA from 10.1.1	(2) R
	OR		
	$\hat{B} = 60°$ [co-interior ∠s DE ∥ BC] / [*ko – binne ∠e DE ∥ BC*] $\hat{C} = 60°$ [opp. ∠s cyclic quad/ *teenoorst ∠e kdvh*] $\hat{A} = 60°$ [∠s of Δ ; *∠e v.Δ*] $\therefore AB = BC = AC = 27$		
	$\frac{\text{area of } \Delta AEB}{\text{area of } \Delta ECB} = \frac{½AB.AE\sin 60°}{½EC.BC\sin 60°}$ $\frac{\text{area of } \Delta AEB}{\text{area of } \Delta ECB} = \frac{27 \times 27}{12 \times 18}$ *or* $\frac{27 \times 27}{12 \times 27}$ $\frac{\text{area of } \Delta AEB}{\text{area of } \Delta ECB} = \frac{27}{8}$ *or* $\frac{27}{12}$	✓ *S* ✓ *answer*	(2) R

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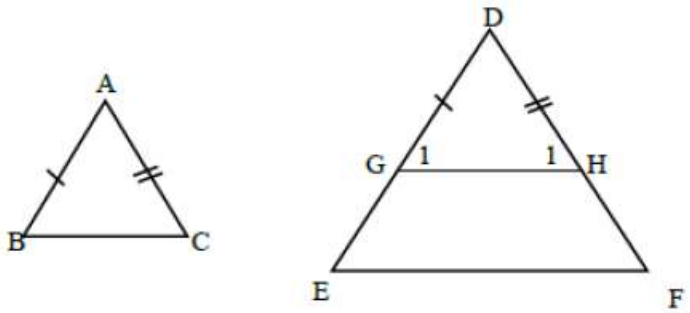
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10.2	$\hat{C} = 60°$ [opp. ∠s cyclic quad/ *teenoorst ∠e kdvh*]	✓ *S*	
	$BE^2 = 12^2 + 18^2 - 2(12)(18)cos60°$ $BE^2 = 144 + 324 - 432(\frac{1}{2})$ $BE^2 = 252$ $BE = \sqrt{252}$ $BE = 6\sqrt{7}$	✓ *Subst* ✓ $\sqrt{252}$	
	$\frac{AD}{AB} = \frac{DP}{BE}$ [ΔADP ||| ΔABE] $\frac{5}{9} = \frac{y}{6\sqrt{7}}$ $9y = 5 \times 6\sqrt{7}$ $y = \frac{10\sqrt{7}}{3}$ *or* 8,82	✓ *S* ✓ *R* ✓ *answer*	(6) P
	OR		
	$\hat{C} = 60°$ [opp. ∠s cyclic quad/ *teenoorst ∠e kdvh*]	✓ *S*	
	$BE^2 = 12^2 + 27^2 - 2(12)(27)cos60°$ $BE^2 = 144 + 324 - 648(\frac{1}{2})$ $BE^2 = 792$ $BE = \sqrt{792}$ $BE = 6\sqrt{22}$	✓ *Subst* ✓ $\sqrt{729}$	
	$\frac{AD}{AB} = \frac{DP}{BE}$ [ΔADP ||| ΔABE] $\frac{5}{9} = \frac{y}{6\sqrt{22}}$ $9y = 5 \times 6\sqrt{22}$ $y = \frac{10\sqrt{22}}{3}$ *or* 15,63	✓ *S* ✓ *R* ✓ *answer*	
			[11]

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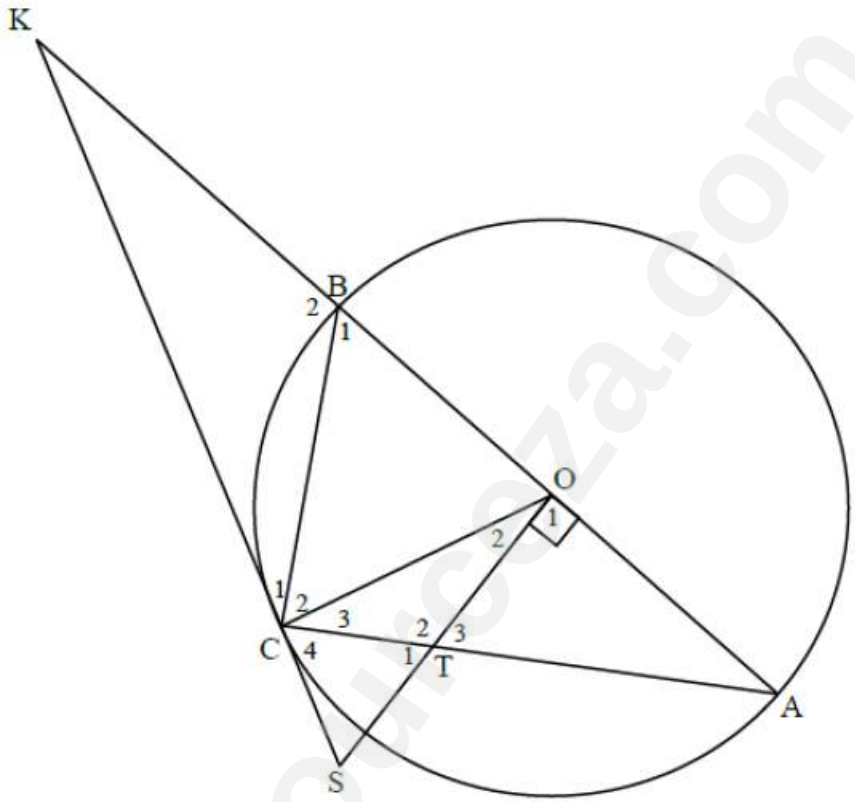
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QUESTION 10

		
10.1	Construction : DG = AB and DH = AC In ΔABC and ΔDEF 1. $\hat{A} = \hat{D}$ (given) 2. AB = DG (construction) 3. AC = DH (construction) ∴ ΔABC ≡ ΔDEF (S∠S) ∴ $\hat{B} = \hat{G}_1$ and $\hat{B} = \hat{E}$ (given) ∴ $\hat{E} = \hat{G}_1$ ∴ EF ∥ GH (corresponding ∠s =) ∴ $\frac{DG}{DE} = \frac{DH}{DF}$ (line ∥ to side of Δ) but AB = DG and AC = DH ∴ $\frac{AB}{DE} = \frac{AC}{DF}$	✓construction ✓S/R ✓S ✓S/R ✓S ✓R (6)

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10.2		
10.2.1	In ΔCKB and ΔAKC $\hat{C}_1 = \hat{A}$ (tan-chord) $\hat{K} = \hat{K}$ (common $\angle$) $K\hat{C}T = \hat{B}_2$ (3^{rd} $\angle$ in Δ) $\therefore \Delta CKB \,\|\|\, \Delta AKC$ ($\angle\angle\angle$)	✓S/R ✓S ✓R (3)
10.2.2	$\hat{T}_2 = \hat{O}_1 + \hat{A}$ (ext $\angle$ of Δ) $K\hat{C}T = \hat{C}_1 + \hat{C}_2 + \hat{C}_3$ But $\hat{C}_2 + \hat{C}_3 = 90°$ ($\angle$s in semi-circle) But $\hat{C}_1 = \hat{A}$ (proven) $K\hat{C}T = \hat{A} + 90°$ $\therefore \hat{T}_2 = K\hat{C}T = \hat{A} + 90°$	✓S/R ✓S ✓R ✓S

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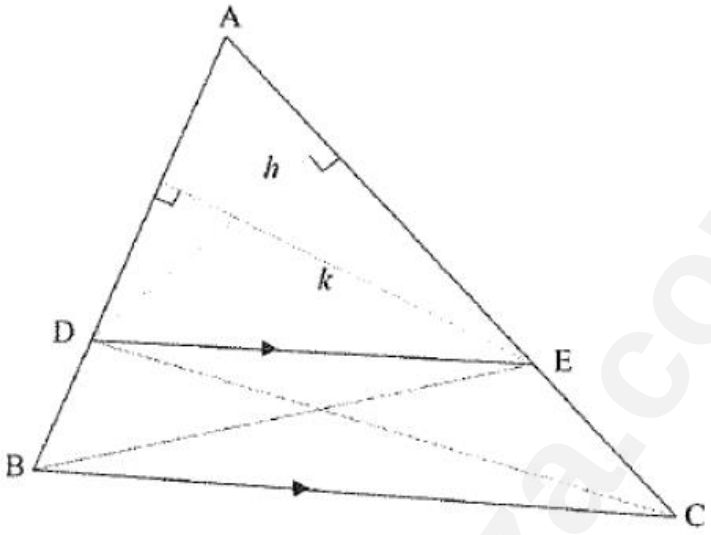
		(4)
10.2.3	$\Delta COT \text{ ||| } \Delta AKC$ $T\hat{C}O = \hat{A}$ ($\angle$s opp equal sides; CO = AO) $\hat{T}_2 = K\hat{C}A$ (proved) $\hat{O}_2 = \hat{K}$ (3^{rd} $\angle$ in Δ) $\Delta COT \text{ ||| } \Delta AKC$ ($\angle\angle\angle$)	✓S/R ✓S ✓R (3)
10.2.4	$\Delta CKB \text{ ||| } \Delta AKC$ $\therefore \frac{BK}{KC} = \frac{CK}{AK}$ $\therefore BK.AK = KC^2$ $\Delta COT \text{ ||| } \Delta AKC$ $\therefore \frac{KC}{OT} = \frac{AC}{CT}$ $\therefore KC.CT = AC.OT$ $\therefore KC = \frac{AC.OT}{CT}$ $\therefore KC^2 = \frac{AC^2.OT^2}{CT^2}$	✓ratio ✓S ✓ratio ✓S (4)
		[20]

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QUESTION/*VRAAG* 10

10.1

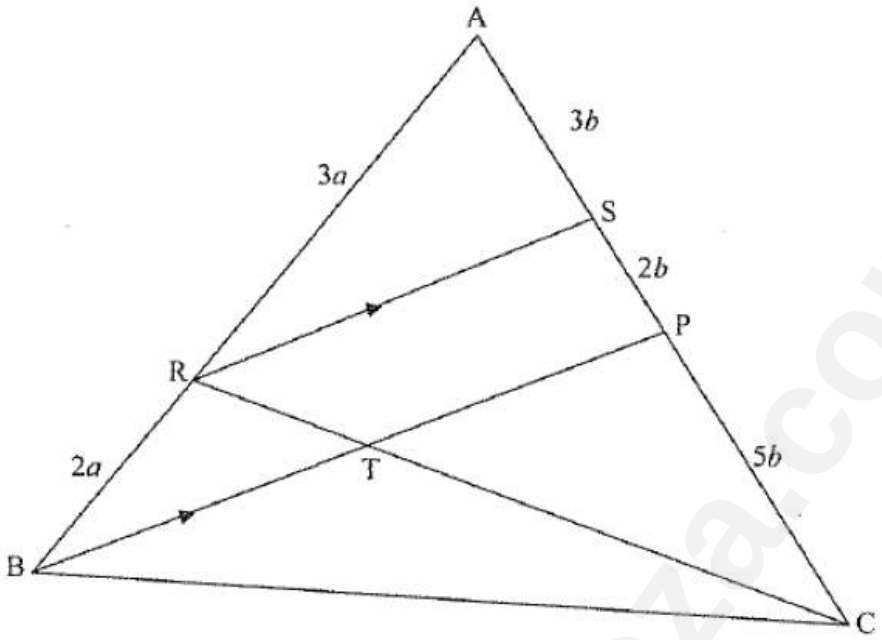


	Construction: Connect DC and BE and draw the altitudes k and h *Konstruksie: Verbind DC en BE en trek hoogtelyne k en h.*	✓ constr/ *konstr*
	$\frac{\text{Area } \Delta ADE}{\text{Area } \Delta BDE} = \frac{\frac{1}{2} \times AD \times k}{\frac{1}{2} \times BD \times k} = \frac{AD}{BD}$ $\frac{\text{Area } \Delta ADE}{\text{Area } \Delta DEC} = \frac{\frac{1}{2} \times AE \times h}{\frac{1}{2} \times EC \times h} = \frac{AE}{EC}$ but/*maar*: Area ΔBDE = Area ΔDEC [DE common base/ *gemeenskaplike basis* with/*met* DE || BC] $\therefore \frac{\text{Area } \Delta ADE}{\text{Area } \Delta BDE} = \frac{\text{Area } \Delta ADE}{\text{Area } \Delta DEC}$ $\therefore \frac{AD}{BD} = \frac{AE}{EC}$	✓S ✓S ✓S ✓R ✓conclusion/ *afleiding* (6)

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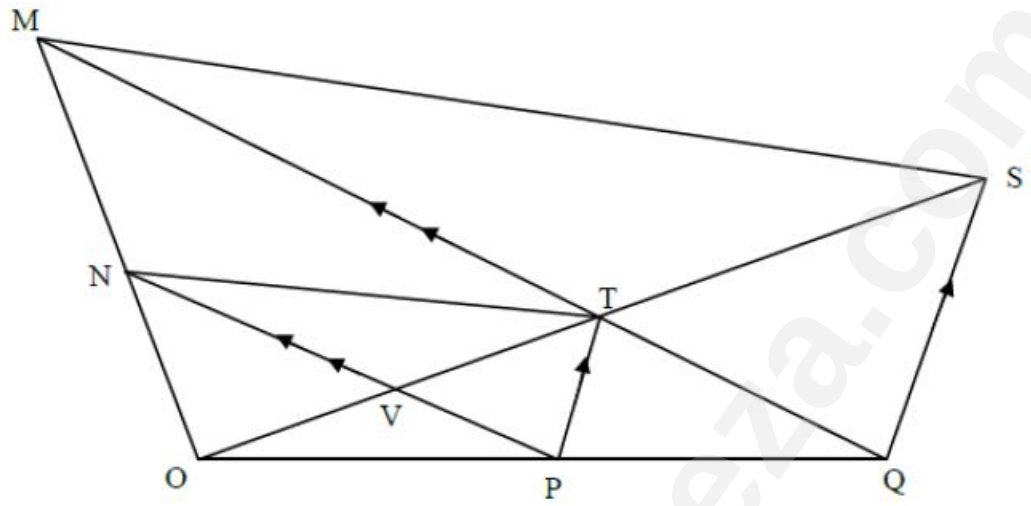


10.2.1	$\frac{AS}{SP}=\frac{AR}{RB}$ [line || side of Δ/ *lyn* || *sy van* Δ] **OR/*OF*** [RS || BP] $=\frac{3}{2}$ $\frac{AS}{SC}=\frac{3}{7}$ [AP = PC]	✓S ✓R ✓$\frac{3}{2}$ ✓$\frac{3}{7}$ (4)
10.2.2	$\frac{RT}{TC}=\frac{SP}{PC}$ [line || side of Δ/ *lyn* || *sy van* Δ] **OR/*OF*** [RS || TP] $=\frac{2}{5}$	✓S ✓R ✓answer/*antwoord* (3)
10.2.3	$\frac{\text{Area of } \Delta RAS}{\text{Area of } \Delta RSC}=\frac{AS}{SC}=\frac{3}{7}$ [common height / *gemeenskaplike hoogte*]	✓$\frac{3}{7}$ ✓R (2)
10.2.4	$\frac{\text{Area } \Delta TPC}{\text{Area } \Delta RSC}=\frac{\frac{1}{2}TC.PC.\sin T\hat{C}P}{\frac{1}{2}RC.SC.\sin T\hat{C}P}$ $=\frac{TC}{RC}\times\frac{PC}{SC}$ $=\frac{5}{7}\times\frac{5}{7}$ $=\frac{25}{49}$	✓Correct subst into area rule/ *Korrekte vervang. in area-reël* ✓$\frac{5}{7}$ ✓answer/*antwoord* (3) **[18]**

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QUESTION/*VRAAG* 11



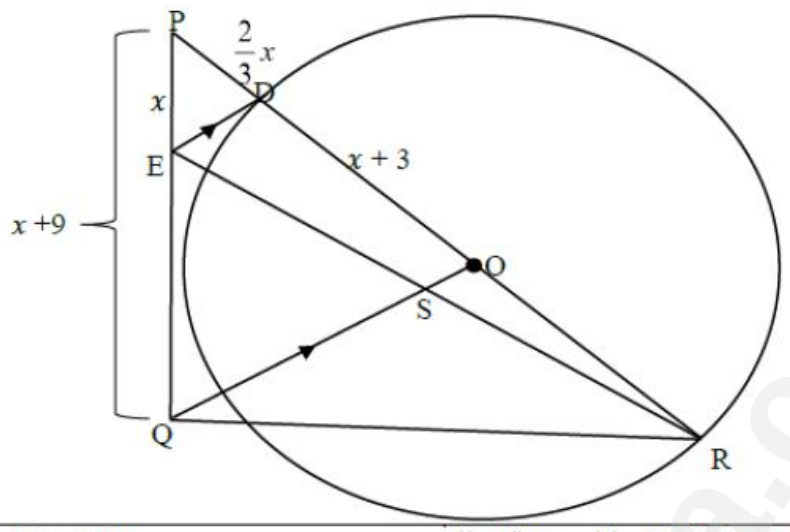
11.1	$\frac{ON}{NM}=\frac{OT}{TS}$ [prop theorem/*eweredigh*] $\frac{ON}{NM}=\frac{OP}{PQ}$ [prop theorem/*eweredigh*] $\therefore \frac{OT}{TS}=\frac{OP}{PQ}$ $\therefore NT \parallel MS$	✓S ✓R ✓S ✓S (4)
11.2	$\frac{ON}{NM}=\frac{OT}{TS}=\frac{3}{5}$ [prop theorem/*eweredigh*] $OT=\frac{3}{8}\times 32$ $=12$ $VT=\frac{5}{8}\times 12$ $=\frac{15}{2}$ or 7,5	✓S/R ✓OT ✓method/*metode* ✓ answer/*antwoord* (4)
		[8]

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QUESTION / *VRAAG* 11



11.1	$\frac{PE}{EQ}=\frac{PD}{DO}$	line ‖ one side ΔPOQ **OR** prop theorem ED‖OQ / *lyn ‖ een sy* ΔPOQ ***OF*** *eweredigheid stelling ED‖OQ*	✓S ✓R
	$\frac{x}{9}=\frac{\frac{2}{3}x}{x+3}$		
	$x^2+3x=6x$		
	$x^2-3x=0$		
	$x(x-3)=0$		
	$x=0$ or/*of* $x=3$		
	N.A / *n.v.t*		✓ $x=3$
	$DO=6$		
	$DO=OR$	radii / *radiusse*	
	$OR=6$ units / *eenhede*		✓OR = 6 (4)
11.2	S is the midpoint of RE / *S is die middelpunt van RE*		
	$DE=2OS$	midpoint theorem / *middelpunt stelling*	✓R
	DE = 2,8 units / *eenhede*		✓answer (2)
11.3	$\frac{\text{Area }\Delta PED}{\text{Area }\Delta PER}=\frac{PD}{PR}$	same height (DE) / *dieselfde hoogte (DE)*	✓S ✓R
	$=\frac{2}{14}$		
	$=\frac{1}{7}$		✓ $\frac{1}{7}$
	Area ΔPER = 7 × Area ΔPED		
	= 18,9 units^2 / *eenhede*^{2}		✓18,9 (4)
			[10]

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