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MATHEMATICS

MATERIAL FOR GRADE 12

TRIGONOMETRY

MEMORANDA

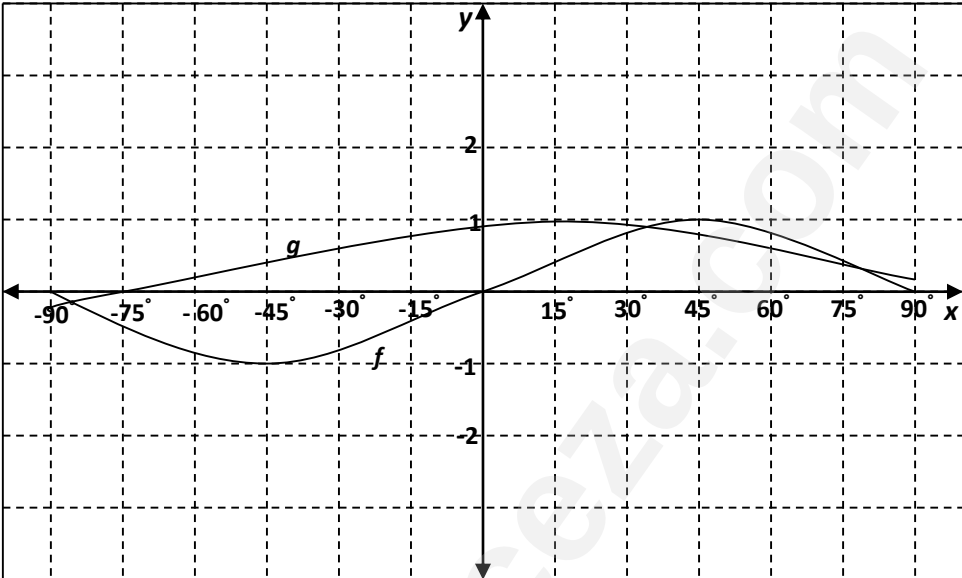
QUESTION 1

1.1	$\hat{C} = \theta$ $D\hat{A}C = \theta$ $\therefore A\hat{D}C = 180^\circ - 2\theta$	$\checkmark$ expression of $D\hat{A}C = \theta$ $\checkmark$ for answer (2)
1.2	$A\hat{D}C = 180^\circ - 2\theta$ $a^2 = b^2 + b^2 - 2 \cdot b \cdot b \cos(180^\circ - 2\theta)$ $= 2b^2 + 2b^2 \cdot \cos 2\theta$ $= 2b^2(1 + \cos 2\theta)$ $1 + \cos 2\theta = \frac{a^2}{2b^2}$ $\therefore \cos 2\theta = \frac{a^2}{2b^2} - 1$	$\checkmark$ cosine rule $\checkmark$ substitution $\checkmark$ reduction $\checkmark$ simplification (4)
1.3	$\cos 2\theta = \frac{3^2}{2(2)^2} - 1$ $= \frac{1}{8}$ $2\theta = 82,819^\circ$ $\theta = 41,41^\circ$	$\checkmark$ substitution $\checkmark$ correct ref angle $\checkmark$ correct value of θ (3)
		[9]

QUESTION 2

2.1	$\begin{aligned} & \sin 34^\circ \cos 26^\circ + \cos 146^\circ \sin (-26^\circ) \\ &= \sin 34^\circ \cos 26^\circ + \cos (180^\circ - 34^\circ) (-\sin 26^\circ) \\ &= \sin 34^\circ \cos 26^\circ + (-\cos 34^\circ) (-\sin 26^\circ) \\ &= \sin 34^\circ \cos 26^\circ + \cos 34^\circ \sin 26^\circ \\ &= \sin (34^\circ + 26^\circ) \\ &= \sin 60^\circ \\ &= \frac{\sqrt{3}}{2} \end{aligned}$	$\checkmark \sin 34^\circ$ $\checkmark -\cos 34^\circ \text{ and } -\sin 26^\circ$ $\checkmark \sin (34^\circ + 26^\circ)$ $\checkmark \text{ answer}$ (4)
2.2	$\begin{aligned} & \frac{\tan(180^\circ + x) \cos(360^\circ - x)}{\sin(x - 180^\circ) \cos(90^\circ + x) + \cos(720^\circ + x) \cos(-x)} \\ &= \frac{\tan x \cdot \cos x}{(-\sin x)(-\sin x) + (\cos x)(\cos x)} \\ &= \frac{\frac{\sin x}{\cos x} \cdot \cos x}{\sin^2 x + \cos^2 x} \\ &= \frac{\sin x}{1} \\ &= \sin x \end{aligned}$	$\checkmark \tan x \text{ and } \cos x$ $\checkmark -\sin x \text{ and } -\sin x$ $\checkmark \cos x \cos x$ $\checkmark \frac{\sin x}{\cos x}$ $\checkmark \sin^2 x + \cos^2 x = 1$ $\checkmark \text{ answer}$ (6)
2.3	$\begin{aligned} \text{LHS} &= \frac{\cos 2x + \cos^2 x + 3\sin^2 x}{2 - 2\sin^2 x} \\ &= \frac{\cos^2 x - \sin^2 x + \cos^2 x + 3\sin^2 x}{2 - 2\sin^2 x} \\ &= \frac{2\cos^2 x + 2\sin^2 x}{2(1 - \sin^2 x)} \\ &= \frac{2(\cos^2 x + \sin^2 x)}{2\cos^2 x} \\ &= \frac{1}{\cos^2 x} \\ &= \text{RHS} \end{aligned}$	$\checkmark \cos^2 x - \sin^2 x$ $\checkmark$ $2\cos^2 x + 2\sin^2 x$ $\checkmark \checkmark \text{ factorising both Nr \& Dn}$ $\checkmark$ $1 - \sin^2 x = \cos^2 x$ (5)
		[15]

QUESTION 3

3.1	Period of $f = 180^\circ$	$\checkmark 180^\circ$	(1)
3.2	 $g(x)$: $\checkmark$ x -intercepts $\checkmark$ y -intercepts $\checkmark$ Turning points $\checkmark\checkmark$ End points	(5)	
3.3	$\sin 2x = \cos(x - 15^\circ)$ $\cos(90^\circ - 2x) = \cos(x - 15^\circ)$ $90^\circ - 2x = \pm(x - 15^\circ) + k360^\circ$ $90^\circ - 2x = x - 15^\circ + k.360^\circ$ or $90^\circ - 2x = -(x - 15^\circ) + k360^\circ$ $-3x = -105^\circ + k360^\circ$ or $-x = -75^\circ + k360^\circ$ ($k \in \mathbb{Z}$) $x = 35^\circ + k120^\circ$ or $x = 75^\circ + k360^\circ$ $x = -85^\circ; 35^\circ; 75^\circ$	$\checkmark \cos(90 - 2x)$ $\checkmark$ $90^\circ - 2x = \pm(x - 15^\circ) + k360^\circ$ $\checkmark x = 35^\circ + k120^\circ$ $\checkmark x = 75^\circ + k360^\circ$ $\checkmark x = -85^\circ$ $\checkmark x = 35^\circ$ $\checkmark x = 75^\circ$	(7)
3.4	$-85^\circ < x < 35^\circ$ or $x > 75^\circ$	$\checkmark\checkmark$ Critical values $\checkmark$ and notation	(3)
			[16]

QUESTION 4

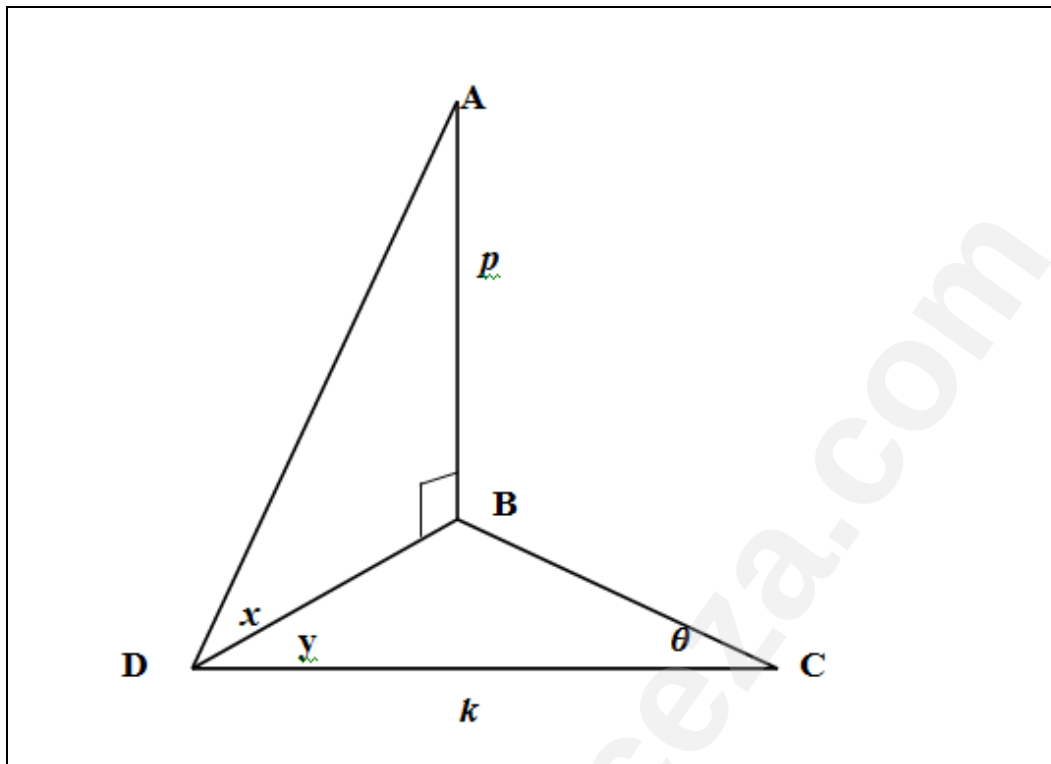
4.1.1	$\frac{2 \sin(180^\circ+x)\sin(90^\circ+x)}{\cos^4 x - \sin^4 x}$ $= \frac{-2 \sin x \cos x}{(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)}$ $= \frac{-\sin 2x}{\cos 2x \cdot (1)}$ $= -\tan 2x$	$\checkmark -2 \sin x$ $\checkmark \cos x$ $\checkmark$ factorisation $\checkmark -\sin 2x$ $\checkmark \cos 2x$ (5)
4.1.2	At / by $\cos 2x = 0$ $2x = 90^\circ$ or /of $2x = 270^\circ$ $x = 45^\circ$ or/of $x = 135^\circ$	$\checkmark \cos 2x = 0$ $\checkmark 2x = 90^\circ$ or $2x = 270^\circ$ $\checkmark x = 45^\circ$ $x = 135^\circ$ (3)
4.2	$= \frac{(\cos 13^\circ)(-\sin 13^\circ)}{(-\tan 45^\circ)(\cos 64^\circ)}$ $= \frac{\cos 13^\circ \cdot -\sin 13^\circ}{-1 \cdot \cos 64^\circ}$ $= \frac{2 \times \sin 13^\circ \cos 13^\circ}{2 \cos 64^\circ}$ $= \frac{\sin 26^\circ}{2 \sin 26^\circ}$ $= \frac{1}{2}$	$\checkmark \cos 13^\circ$ $\checkmark -\sin 13^\circ$ $\checkmark -\tan 45^\circ$ $\checkmark$ multiply by 2 in numerator and denominator $\checkmark \frac{\sin 26^\circ}{2 \sin 26^\circ}$ (5)
4.3	LHS: $\frac{\cos(2x+x)}{\cos x}$ $= \frac{\cos 2x \cos x - \sin 2x \sin x}{\cos x}$ $= \frac{\cos 2x \cos x - 2 \sin x \cos x \sin x}{\cos x}$ $= \frac{\cos x (\cos 2x - 2 \sin^2 x)}{\cos x}$ $= \cos 2x - 1 + 1 - 2 \sin^2 x$ $= \cos 2x - 1 + \cos 2x$ $= 2 \cos 2x - 1$ OR $\frac{\cos(2x+x)}{\cos x}$ $= \frac{\cos 2x \cos x - \sin 2x \sin x}{\cos x}$ $= \frac{\cos 2x \cos x - 2 \sin x \cos x \sin x}{\cos x}$	$\checkmark \cos 2x \cos x - \sin 2x \sin x$ $\checkmark$ replacing $\sin 2x$ $\checkmark$ factorise $\checkmark +1-1$ $\checkmark$ replacing $1 - 2 \sin^2 x$ (5)

	$= \frac{\cos x (\cos 2x - 2\sin^2 x)}{\cos x}$ $= \cos 2x - 2\sin^2 x$ $= 2\cos^2 x - 1 - 2\sin^2 x$ $= 2(\cos^2 x - \sin^2 x) - 1$ $= 2\cos 2x - 1$	
		[18]

QUESTION 5

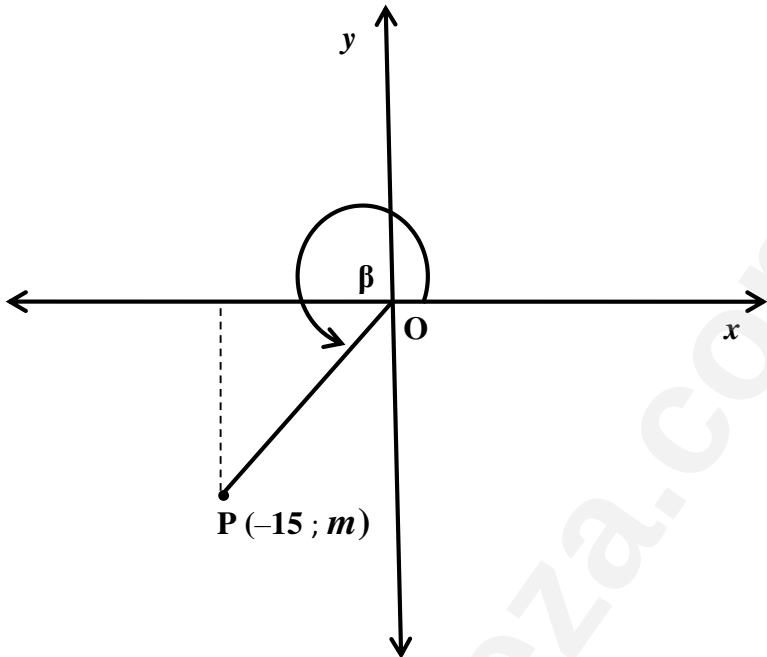
5.1	360°	✓ (1)
5.2	$\sin(x + 30^\circ) = -2\cos x$ $\sin x \cos 30^\circ + \cos x \sin 30^\circ = -2\cos x$ $\sin x \left(\frac{\sqrt{3}}{2} \right) + \cos x \left(\frac{1}{2} \right) = -2\cos x$ $\sqrt{3}\sin x + \cos x = -4\cos x$ $\sqrt{3}\sin x = -5\cos x$ $\tan x = -\frac{5}{\sqrt{3}}$ $x = 180^\circ - 70,89^\circ + k \cdot 180^\circ$ $x = 109,11^\circ + k \cdot 180^\circ, k \in \mathbb{Z}$ $x = -70,89^\circ \text{ or } x = 109,11^\circ$	$\begin{aligned} &\checkmark \text{equating f and g} \\ &\checkmark \text{expanding } \sin(x + 30^\circ) \\ &\checkmark \text{special angle values} \\ &\checkmark \tan x = -\frac{5}{\sqrt{3}} \\ &\checkmark x = 109,11^\circ + k \cdot 180^\circ \\ &\checkmark x = -70,89^\circ \\ &\checkmark x = 109,11^\circ \end{aligned} \quad (7)$
5.3.1	$x \in [-90^\circ; -70,89^\circ] \cup [109,11^\circ; 180^\circ]$	$\begin{aligned} &\checkmark \checkmark \text{boundaries} \\ &\checkmark \text{correct notation} \end{aligned} \quad (3)$
5.3.2	$x \in (-90^\circ; -30^\circ) \cup (90^\circ; 150^\circ)$	$\begin{aligned} &\checkmark (-90^\circ; -30^\circ) \\ &\checkmark (90^\circ; 150^\circ) \\ &\checkmark \text{correct notation} \end{aligned} \quad (3)$
		[14]

QUESTION 6



6.1.1	$\text{In } \triangle ABD: \tan x = \frac{p}{DB}$ $p = DB \cdot \tan x$	$\checkmark \tan x = \frac{p}{DB}$ $\checkmark p = DB \tan x \quad (2)$
6.1.2	$\frac{DB}{\sin \theta} = \frac{k}{\sin(180 - (y + \theta))}$ $DB = \frac{k \cdot \sin \theta}{\sin(y + \theta)}$ $p = \frac{k \cdot \sin \theta}{\sin(y + \theta)} \times \tan x$ $= \frac{k \sin \theta \cdot \tan x}{\sin y \cos \theta + \cos y \cdot \sin \theta}$	$\checkmark \widehat{BDC} = 180 - (y + \theta)$ $\checkmark \frac{DB}{\sin \theta} = \frac{k}{\sin(180 - (y + \theta))}$ $\checkmark \text{reduction formula}$ $\checkmark \text{replacing DB}$ $\checkmark \text{expanding } \sin(y + \theta) \quad (5)$
6.2	$\tan 51,7^\circ = \frac{80}{DB}$ $DB = \frac{80}{\tan 51,7^\circ} = 63,18 \text{ m}$ $BC^2 = (63,18)^2 + 95^2 - 2(63,18)(95)\cos 62,5^\circ$ $= 7473,789697 \dots$ $\therefore BC = 86,45 \approx 86 \text{ m}$	$\square \tan 51,7^\circ = \frac{80}{DB}$ $\checkmark DB = 63,18 \text{ m}$ $\checkmark \text{application of cos.}$ $\checkmark 86 \text{ m} \quad (4)$
[11]		

QUESTION 7

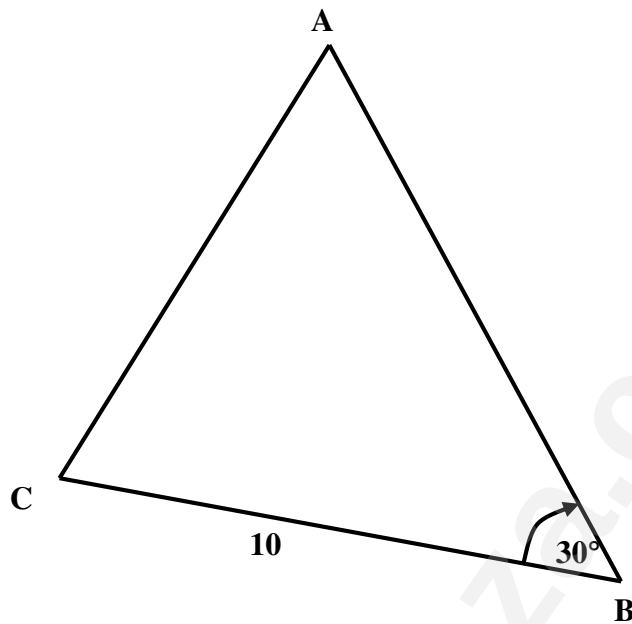
		
7.1.1	$17\cos \beta + 15 = 0$ $\cos \beta = \frac{-15}{17}$ $x^2 + y^2 = r^2$ $m^2 = 17^2 - 15^2$ $= 64$ $m = -8$	$\checkmark \cos \beta = \frac{-15}{17}$ $\checkmark$ application of Pythagoras $\checkmark$ answer (3)
7.1.2	$\sin \beta + \tan \beta = \frac{-8}{17} + \frac{-8}{-15}$ $= \frac{16}{255}$	$\checkmark$ sub. into $\sin \beta$ $\checkmark$ sub into $\tan \beta$ $\checkmark$ answer (3)
7.1.3	$\cos 2\beta = 2\cos^2 \beta - 1$ $= 2\left(\frac{-15}{17}\right)^2 - 1$ $= \frac{161}{289}$	$\checkmark$ identity $\checkmark$ sub. $\checkmark$ answer (3)
7.2	$\frac{\sin(180^\circ - x) \cdot \cos(x - 180^\circ) \cdot \tan(360^\circ - x)}{\sin(-x) \cdot \cos(450^\circ + x)}$ $= \frac{\sin x \cdot (-\cos x) \cdot (-\tan x)}{(-\sin x) \cdot (-\sin x)}$ $= \frac{\sin x \cdot \cos x \cdot \frac{\sin x}{\cos x}}{\sin x \cdot \sin x}$ $= 1$	$\checkmark \sin x$ $\checkmark -\cos x$ $\checkmark -\tan x$ $\checkmark -\sin x$ $\checkmark -\sin x$ $\checkmark \frac{\sin x}{\cos x}$ $\checkmark$ answer (7)

7.3.1	$\begin{aligned} \text{LHS} &= \frac{\sin x + \sin 2x}{1 + \cos x + \cos 2x} \\ &= \frac{\sin x + 2 \sin x \cos x}{1 + \cos x + 2 \cos^2 x - 1} \\ &= \frac{\sin x(1 + 2 \cos x)}{\cos x(1 + 2 \cos x)} \\ &= \frac{\sin x}{\cos x} \\ &= \tan x \\ &= \text{RHS} \end{aligned}$	$\checkmark 2 \sin x \cdot \cos x$ $\checkmark 2 \cos^2 x - 1$ $\checkmark$ factorise numerator $\checkmark$ factorise denominator $\checkmark \frac{\sin x}{\cos x} = \tan x$ (5)
7.3.2	$\cos x = 0$ or $\cos x = -\frac{1}{2}$ $x = \pm 90^\circ + k360^\circ$ or $x = \pm 120^\circ + k360^\circ$; or $x = 90^\circ + k180^\circ$; $k \in \mathbb{Z}$	$\checkmark \cos x = 0$ or $\cos x = -\frac{1}{2}$ $\checkmark \pm 90^\circ + k180^\circ$ $\checkmark \pm 120^\circ + k360^\circ$ $\checkmark k \in \mathbb{Z}$ (4) [25]

QUESTION 8

8.1	f: $\checkmark$ x-intercept $\checkmark$ y-intercept $\checkmark$ shape/TP g: $\checkmark$ x-intercept $\checkmark$ y-intercept $\checkmark$ TP	(6)
8.2	$(60^\circ ; 180^\circ]$ or $60^\circ < x \leq 180^\circ$	$\checkmark \checkmark$ end values $\checkmark$ notation (3) [9]

QUESTION 9

In $\triangle ABC$,

$$\hat{A} + \hat{B} + \hat{C} = 180^\circ$$

$$\begin{aligned}\sin A &= \sin [180^\circ - (B + C)] \\ &= \sin (B + C) \\ &= 0,8\end{aligned}$$

$$\frac{AC}{\sin B} = \frac{BC}{\sin A}$$

$$\frac{AC}{\sin 30^\circ} = \frac{10}{0,8}$$

$$AC = 10 \cdot \frac{1}{2} \cdot \frac{10}{8}$$

$$AC = 6\frac{1}{4} \text{ units}$$

$$\checkmark \hat{A} + \hat{B} + \hat{C} = 180^\circ$$

$$\checkmark \text{sub. for A}$$

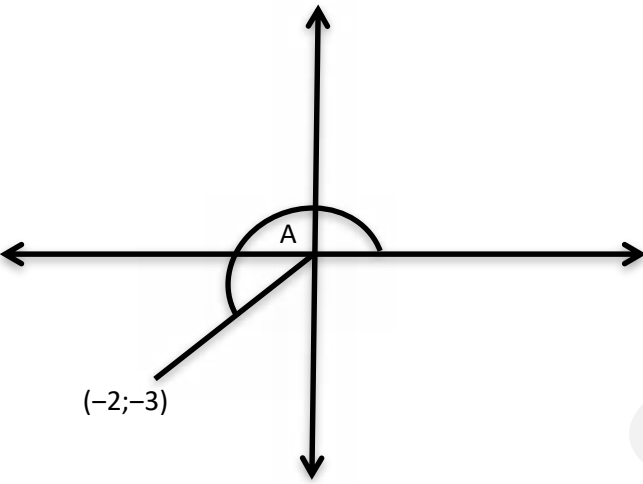
$$\checkmark \text{reduction}$$

$$\checkmark \text{application of sine rule}$$

$$\checkmark \text{answer}$$

[5]

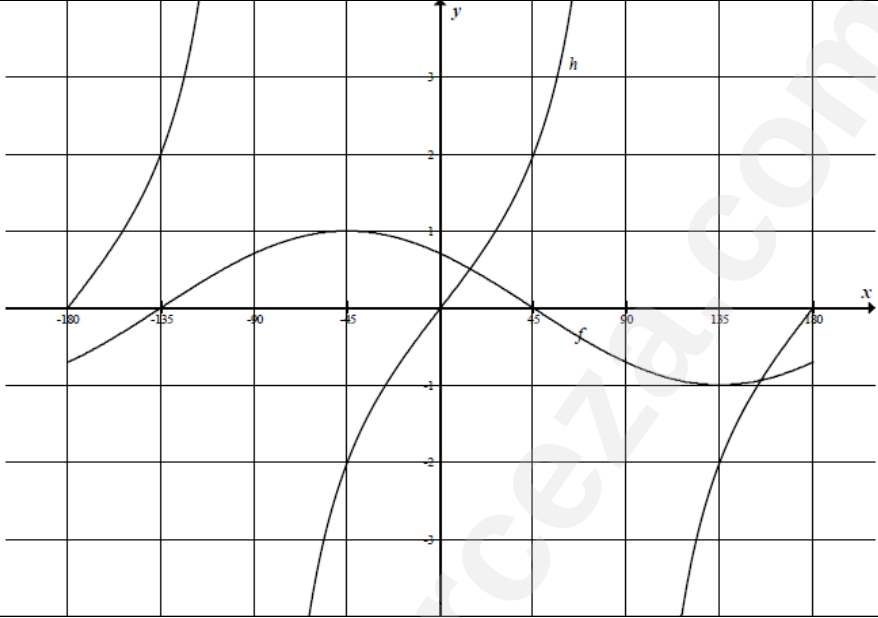
QUESTION 13

13.1.1	 $r^2 = (-2)^2 + (-3)^2$ $= 13$ $\therefore r = \sqrt{13}$ $\sin A = \frac{-2}{\sqrt{13}}$	✓ correct quadrant ✓ $r = \sqrt{13}$ ✓ $\frac{-2}{\sqrt{13}}$
13.1.2	$\cos 2A - \sin 2A$ $= \cos^2 A - \sin^2 A - 2 \sin A \cos A$ $= \left(\frac{-3}{\sqrt{13}}\right)^2 - \left(\frac{-2}{\sqrt{13}}\right)^2 - 2\left(\frac{-2}{\sqrt{13}}\right)\left(\frac{-3}{\sqrt{13}}\right)$ $= \frac{5}{13} - \frac{12}{13}$ $= \frac{-7}{13}$	✓ Expansion of cos double angles ✓ sin double ✓ subs ✓ Answer
13.2.1	$\cos(x - 90^\circ) = \sin x$ $= t$	✓ $\sin x$ ✓ t
13.2.2	$\sin 2x = 2 \sin x \cos x$ $= 2t\sqrt{1 - t^2}$	✓ $2 \sin x \cos x$ ✓ $2t$ ✓ $\sqrt{1 - t^2}$
		[12]

QUESTION 14

14.1	$\frac{\cos^2 208^\circ}{\tan(118^\circ)[\sin(124^\circ)]}$ $= \frac{(-\cos 28^\circ)^2}{(-\tan 62^\circ)(\sin 124^\circ)}$ $= \frac{\cos^2 28^\circ}{\left(\frac{\sin 62^\circ}{\cos 62^\circ}\right)(2 \sin 62^\circ \cos 62^\circ)}$ $= \frac{\cos^2 28^\circ}{2 \sin^2 62^\circ}$ $= \frac{\cos^2 28^\circ}{2 \cos^2 28^\circ}$ $= \frac{1}{2}$	$\checkmark \cos^2 28^\circ$ $\checkmark \sin 124^\circ$ $\checkmark 2 \sin 62^\circ \cos 62^\circ$ $\checkmark \frac{\cos^2 28^\circ}{2 \sin^2 62^\circ}$ $\checkmark \frac{\cos^2 28^\circ}{2 \cos^2 28^\circ}$ $\checkmark \frac{1}{2}$ (6)
14.2	$1 - (1 - 2 \sin^2 \theta) = 8 \sin \theta (2 \sin \theta \cos \theta)$ $2 \sin^2 \theta = 16 \sin^2 \theta \cos \theta$ $2 \sin^2 \theta - 16 \sin^2 \theta \cos \theta = 0$ $2 \sin^2 \theta (1 - 8 \cos \theta) = 0$ $\sin^2 \theta = 0 \quad \text{or} \quad 1 - 8 \cos \theta = 0$ $\sin \theta \neq 0$ $-8 \cos \theta = -1$ $\cos \theta = \frac{1}{8}$ $\therefore \theta = \pm 75,5^\circ + k \cdot 360^\circ \quad \text{where } k \in \mathbb{Z}$	$\checkmark 1 - 2 \sin^2 \theta$ $\checkmark \sin^2 \theta (1 - 8 \cos \theta)$ $\checkmark \text{both equations}$ $\checkmark \cos \theta = \frac{1}{8}$ $\checkmark 75,5^\circ + k \cdot 360^\circ$ $\checkmark -75,5^\circ + k \cdot 360^\circ$ (6)
		[12]

QUESTION 15

15.1	$a \tan 45^\circ = 2$ $\therefore a = 2$	✓ Reading from graph ✓ Answer (2)
15.2		✓ Amplitude ✓ Shape ✓✓ Turning points clearly indicated as $(-45^\circ, 1)$ and $(135^\circ, -1)$ (4)
15.3	2 solutions.	✓ 2 solutions (1)
		[7]

QUESTION 16

16.1	$QR^2 = PQ^2 + RP^2 - 2PQ \cdot RP \cos \hat{P}$ $(\sqrt{3}x)^2 = x^2 + x^2 - 2 \cdot x \cdot x \cos \hat{P}$ $\cos \hat{P} = \frac{x^2 + x^2 - (\sqrt{3}x)^2}{2x^2}$ $= \frac{-x^2}{2x^2}$ $= \frac{-1}{2}$ $\hat{P} = 120^\circ$	$\checkmark x^2 + x^2 - (\sqrt{3}x)^2$ $\checkmark 2x^2$ $\checkmark \text{Simplification}$ $\checkmark \hat{P} = 120^\circ$ (4)
16.2	$\hat{P}\hat{R}Q = \hat{P}\hat{Q}R = 30^\circ \quad \hat{R}_1 = \hat{Q}_2$ $\hat{R}_2 = 150^\circ \quad [\text{angles on str. Line}]$ $\text{Area of } \Delta QRS = \frac{1}{2}(QR)(RS) \sin \hat{Q}\hat{R}S$ $= \frac{1}{2}(\sqrt{3}x) \left(\frac{3}{2}x\right) (\sin 150^\circ)$ $= \left(\frac{3\sqrt{3}}{4}x^2\right) \left(\frac{1}{2}\right)$ $= 3\frac{\sqrt{3}}{8}x^2 \text{ or } 0,65x^2$	$\checkmark S$ $\checkmark S$ $\checkmark \text{Subst into correct formulae}$ $\checkmark \text{Simplification}$ $\checkmark \text{Answer}$ (5)
		[9]

QUESTION 17 17.1.1 $\tan \alpha = \frac{-2}{2\sqrt{3}} = \frac{-1}{\sqrt{3}}$ $\therefore \alpha = 360^\circ - 30^\circ$ $= 330^\circ$ $\therefore \beta = 30^\circ$ or $\tan(-\beta) = -\tan \beta$ $= -\left(\frac{-2}{2\sqrt{3}}\right)$ $\tan \beta = \frac{2}{2\sqrt{3}}$ $\beta = 30^\circ$	A✓ correct ratio CA✓ $\alpha = 330^\circ$ CA✓ $\beta = 30^\circ$ A✓ correct ratio CACA✓✓ $\beta = 30^\circ$ (3)
17.1.2 $OP^2 = (2\sqrt{3})^2 + (-2)^2 = 12 + 4$ $= 16$ $\therefore OP = 4$	A✓ using distance for formula CA✓ answer (2)
17.1.3 $\frac{OP}{OQ} = \cos \beta$ $\therefore OQ = \frac{OP}{\cos \beta} = \frac{4}{\cos 30^\circ}$	

$$= \frac{4}{\frac{\sqrt{3}}{2}}$$

$$= \frac{8}{\sqrt{3}}$$

$$Q = \left(\frac{8\sqrt{3}}{3} ; 0 \right)$$

or

$$\frac{OP}{OQ} = \cos 30^\circ$$

$$OQ = \frac{4}{\cos 30^\circ} = \frac{4}{\frac{\sqrt{3}}{2}}$$

$$OQ = \frac{8\sqrt{3}}{3}$$

$$Q = \left(\frac{8\sqrt{3}}{3} ; 0 \right)$$

$$CA \checkmark \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$CA \checkmark \frac{8}{\sqrt{3}}$$

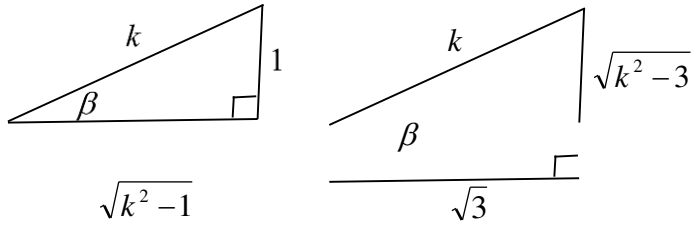
CA $\checkmark$ co-ordinates

$$CA \checkmark \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$CA \checkmark \frac{8\sqrt{3}}{3}$$

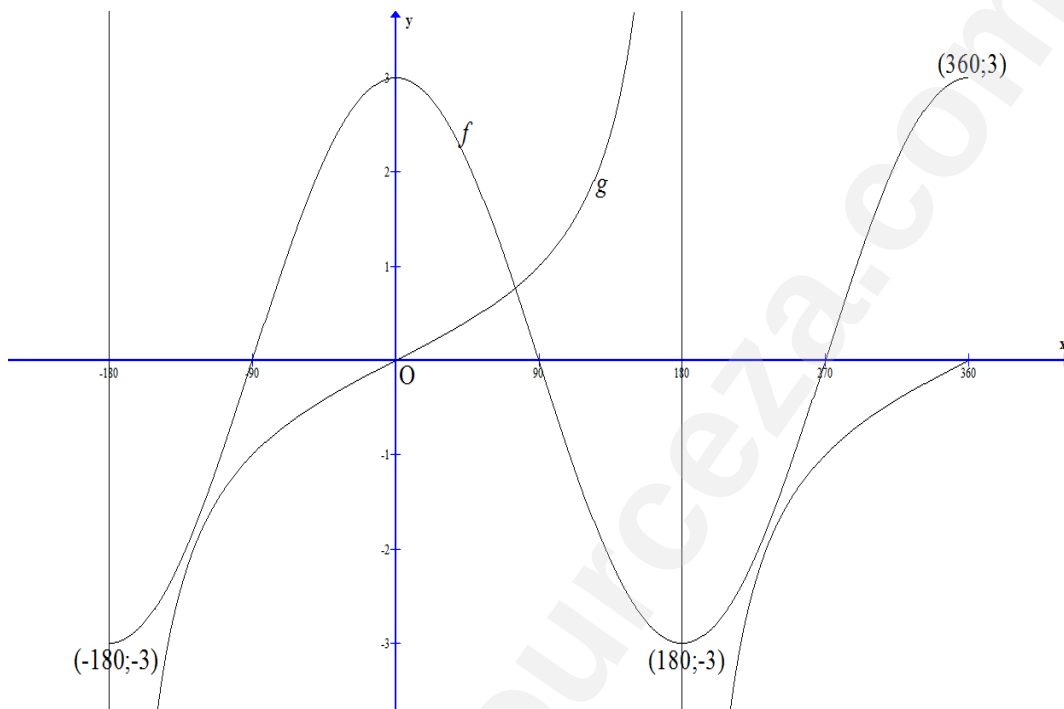
CA $\checkmark$ co-ordinates

17.2 $2 \left[\frac{1}{2} \cos \alpha + \frac{\sqrt{3}}{2} \sin \alpha \right]$ $= 2 [\sin 30^\circ \cos \alpha + \cos 30^\circ \sin \alpha]$ $= 2 \sin (30^\circ + \alpha)$ $k = 2 ; \beta = 30^\circ$ OR $\frac{1}{2} \cos \alpha + \frac{\sqrt{3}}{2} \sin \alpha = \frac{k}{2} (\sin \alpha + \beta)$ $= \sin 30^\circ \cos \alpha + \cos 30^\circ \sin \alpha = \frac{k}{2} \sin (\alpha + \beta)$ $= \sin (30^\circ + \alpha) = \frac{k}{2} \sin (\alpha + \beta)$ $\frac{k}{2} = 1, \beta = 30^\circ$ $\therefore k = 2, \beta = 30^\circ$ OR	M✓ for introducing 2 A✓ introducing 30° special angle A✓ sum compound formula A✓ calculating k A✓ calculating β A✓ for introducing 2 A✓ introducing 30° special angle A✓ sum compound formula A✓ calculating k A✓ calculating β (5)
$\cos \alpha + \sqrt{3} \sin \alpha = k \sin (\alpha + \beta)$ $\cos \alpha + \sqrt{3} \sin \alpha = k (\sin \alpha \cos \beta + \cos \alpha \sin \beta)$ $\cos \alpha + \sqrt{3} \sin \alpha = k \sin \alpha \cos \beta + k \cos \alpha \sin \beta$ $\cos \alpha + \sqrt{3} \sin \alpha = (k \sin \beta) \cos \alpha + (k \cos \beta) \sin \alpha$ $\therefore 1 = k \sin \beta \quad \text{and} \quad \sqrt{3} = k \cos \beta$	A✓ expansion

$\therefore \sin \beta = \frac{1}{k}$ $\cos \beta = \frac{\sqrt{3}}{k}$  $\sqrt{k^2 - 1} = \sqrt{3}$ $k^2 = 4$ $k = 2$ $\sin \beta = \frac{1}{2}$ $\beta = 30^\circ$	A✓ comparing coefficients A✓ values of trig ratios A✓ calculating k A✓ calculating β (5) [13]
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QUESTION 18

18.1



A✓ asymptotes

A✓ shape of f

A✓ shape of g

A✓ correct x - intercept
of f (-90° ; 90° ; 270°)

A✓ correct x – intercept
of g (0° ; 360°)

(5)

18.2 360°

A✓ 360°

(1)

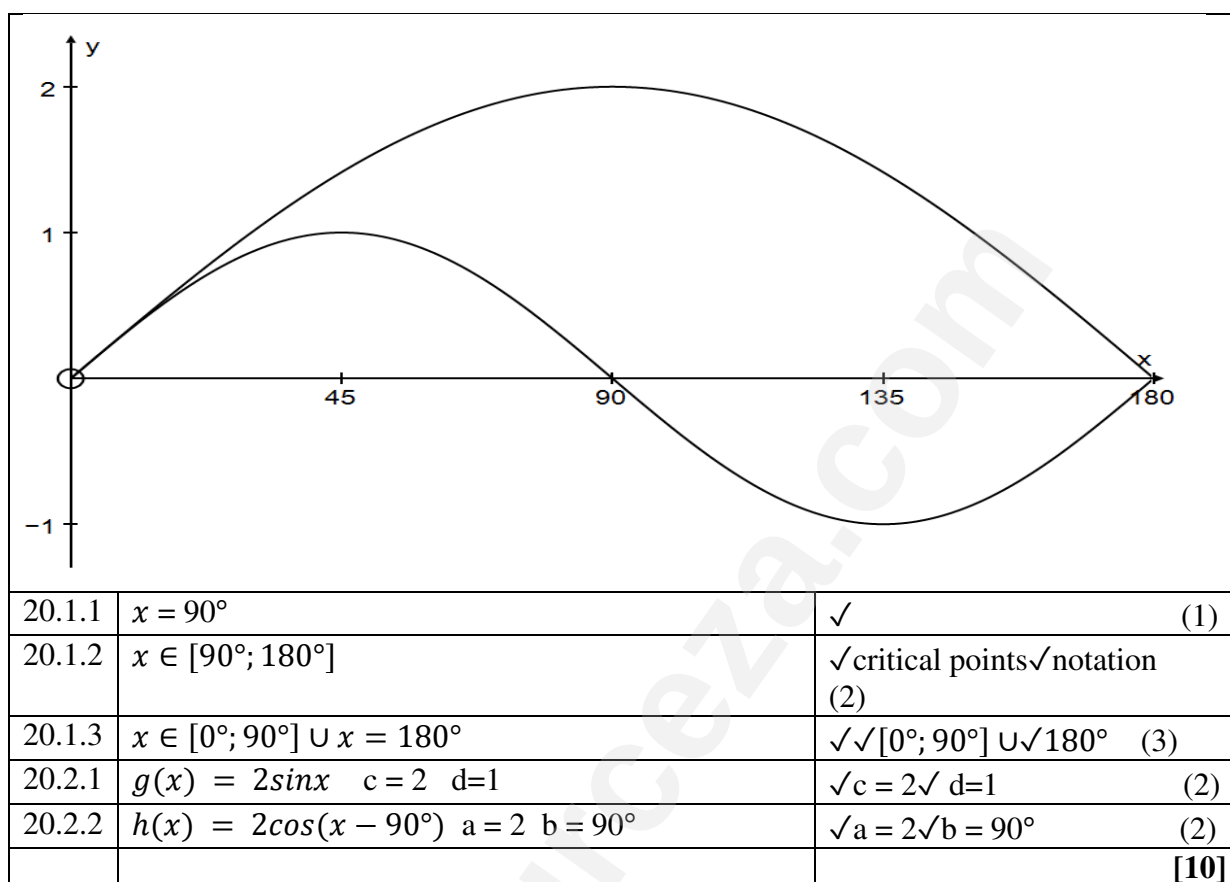
18.3	$(0; 3)$ and $(180^\circ; -3)$ $(-180^\circ; -3)$ and $(360^\circ; 3)$	CA✓ for any two (2)
18.4	$-180^\circ < x < 0^\circ$ or $180^\circ < x < 360^\circ$ OR $-180^\circ < x < 0^\circ \cup 180^\circ < x < 360^\circ$	CA✓ $-180^\circ < x < 0^\circ$ CA✓ $180^\circ < x < 360^\circ$ [penalize one mark for incorrect notation] (2)
18.5	$y = 3\cos(x - 45^\circ)$	A✓ $3\cos(x - 45^\circ)$ (1) [11]

QUESTION 19 19.1 $\cos 54^\circ \cdot \cos x + \sin 54^\circ \cdot \sin x = \sin 2x$ $\cos(54^\circ - x) = \sin 2x$ $= \cos(90^\circ - 2x)$ $\therefore 54^\circ - x = 90^\circ - 2x + n \cdot 360^\circ$ or $54^\circ - x = 360^\circ - (90^\circ - 2x) + n \cdot 360^\circ$ $x = 36^\circ + n \cdot 360^\circ$ or $-3x = 216^\circ + n \cdot 360^\circ$ $x = 36^\circ + n \cdot 360^\circ$ or $-x = -72^\circ + n \cdot 120^\circ$ $\therefore x = 36^\circ + n \cdot 360^\circ$ or $x = -72^\circ + n \cdot 120^\circ, n \in \mathbb{Z}$	A✓ $\cos(54^\circ - x)$ A✓ $\cos(90^\circ - 2x)$ $n \cdot 360^\circ$ CA✓ $x = 36^\circ + n \cdot 360^\circ$ CA✓ $x = -72^\circ + n \cdot 120^\circ$ A✓ $n \in \mathbb{Z}$ (5)
19.2.1 $\hat{EFD} = \hat{FBC} = \alpha$... corresponding $\angle$'s ... $AD \parallel BC$ $\hat{EFD} = \theta + \hat{AEF}$... ext $\angle$ of $\triangle AEF$ $\therefore \hat{AEF} = \alpha - \theta$ In $\triangle ABE$: $\frac{\sin \hat{AEB}}{AB} = \frac{\sin \hat{EAB}}{BE}$ $\therefore BE = \frac{AB \sin(90^\circ + \theta)}{\sin(\alpha - \theta)}$ $= \frac{AB \cos \theta}{\sin(\alpha - \theta)}$	A✓ S/R A✓ S/R A✓ $\hat{AEF} = \alpha - \theta$ A✓ sine rule application A✓ substitution (5)
19.2.2 Area of $\triangle BCE = \frac{1}{2} \cdot x(18 - 3x) \sin 150^\circ$ $A(x) = \frac{1}{2} x \cdot (18 - 3x) \cdot \frac{1}{2}$ $= \frac{1}{4} \cdot x \cdot 18 - \frac{1}{4} x \cdot 3x$	A✓ area rule A✓ $\sin 150^\circ = \frac{1}{2}$

$= \frac{18}{4}x - \frac{3}{4}x^2$ $= \frac{9}{2}x - \frac{3}{4}x^2$	A✓ simplifying (3)
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19.2.3 At maximum area: $A'(x) = 0$ $A'(x) = \frac{9}{2} - 2 \cdot \frac{3}{4}x$ $0 = \frac{9}{2} - \frac{3}{2}x$ $3x = 9$ $x = 3$	M✓ $A'(x) = 0$ A✓ derivative CA✓ $x = 3$ (3)
19.2.4 $BC = 3$ $CE = 18 - 3(3)$ $= 18 - 9$ $= 9$ $BE^2 = BC^2 + CE^2 - 2 BC \cdot CE \cos 150^\circ$ $= (3)^2 + (9)^2 - 2 \times 3 \times 9 (-\cos 30^\circ)$ $= 9 + 81 + 54 \cos 30^\circ$ $= 90 + 54 \cdot \left(\frac{\sqrt{3}}{2}\right)$ $= 136,765$ $\therefore BE = 11,695$ $= 11,69$	A✓ for both $BC = 3$ and $CE = 9$ CA✓ applying cosine rule and substitution CA✓ answer (3) [19]

QUESTION 20

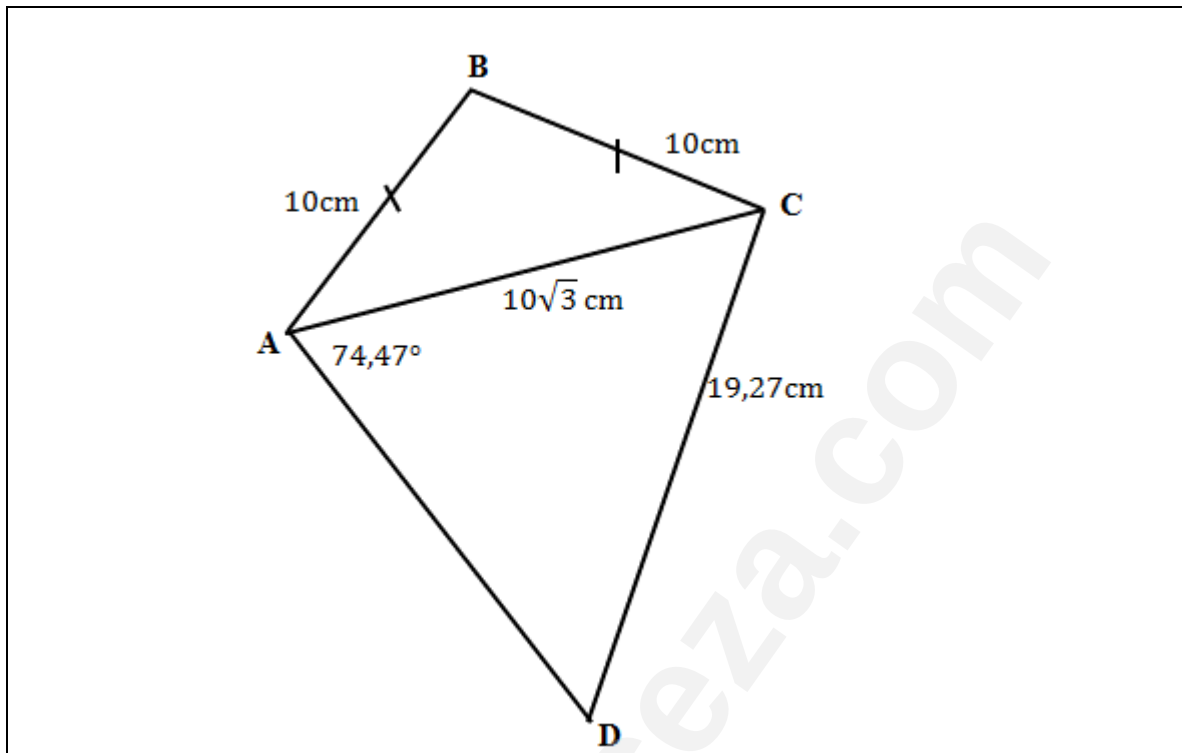


QUESTION 21

21.1.1	$\frac{\sin 40^\circ \cdot \tan 45^\circ}{-\cos 50^\circ \cdot \sin 60^\circ}$ $= \frac{\sin 40^\circ \cdot (1)}{-\sin 40^\circ \left(\frac{\sqrt{3}}{2}\right)}$ $= -\frac{2}{\sqrt{3}}$	✓ $\sin 40^\circ$ ✓ $\tan 45^\circ$ ✓ $-\cos 50^\circ$ ✓ $\sin 60^\circ$ ✓ answer (5)
21.1.2	$\frac{\sin 15^\circ \cos 15^\circ}{\cos(45^\circ - x + x)}$ $= \frac{2\sin 15^\circ \cos 15^\circ}{2\cos 45^\circ}$ $= \frac{\sin 30^\circ}{2\cos 45^\circ}$ $= \frac{\frac{1}{2}}{2 \times \frac{\sqrt{2}}{2}}$ $= \frac{1}{2\sqrt{2}} \text{ or } \frac{\sqrt{2}}{4}$	✓ reducing to $\cos 45^\circ$ ✓ manipulating with 2 ✓ $\sin 30^\circ$ ✓ substitute ✓ answer (5)

21.2.1	$2\cos^2 A - 1 = \cos 2A$ $2\cos^2 A = \cos 2A + 1$ $\cos^2 A = \frac{\cos 2A + 1}{2}$	$\sqrt{2\cos^2 A = \cos 2A + 1}$ $\sqrt{\cos^2 A = \frac{\cos 2A + 1}{2}}$ (2)
21.2.2	$\cos A = \sqrt{\frac{\cos 2A + 1}{2}}$ $\cos 15^\circ = \sqrt{\frac{\cos 30^\circ + 1}{2}}$ $= \sqrt{\frac{\frac{\sqrt{3}}{2} + 1}{2}}$ $= \sqrt{\frac{\sqrt{3} + 2}{4}}$ $= \frac{\sqrt{\sqrt{3} + 2}}{2}$	$\sqrt{\cos A = \sqrt{\frac{\cos 2A + 1}{2}}}$ $\sqrt{\cos 15^\circ \text{ and } \cos 30^\circ}$ $\sqrt{\text{substitute}}$ $\sqrt{\sqrt{\frac{\sqrt{3} + 2}{4}}}$ (4)
21.3	$\sin 2x = \cos 3x$ $\sin 2x = \sin(90^\circ - 3x)$ $2x = 90^\circ - 3x + k.360^\circ$ $5x = 90^\circ + k.360^\circ$ $x = 18^\circ + k.72^\circ$ or $2x = 90^\circ + 3x + k.360^\circ$ $x = -90^\circ + k.360^\circ$ $x = 18^\circ; 90^\circ; -54^\circ; -90^\circ$ OR $\sin 2x = \cos(-3x)$ $\cos(90^\circ - 2x) = \cos(-3x)$ $90^\circ - 2x = -3x + k.360^\circ$ $x = -90^\circ + k.360^\circ$ or $90^\circ - 2x = 360 - (-3x) + k.360^\circ; k \in \mathbb{Z}$ $-5x = 270^\circ + k.360^\circ$ $x = -54^\circ + k.72^\circ$ $x = 18^\circ; 90^\circ; -54^\circ; -90^\circ$	$\sqrt{\sin 2x = \cos 3x}$ $\sqrt{\sin(90^\circ - 3x)}$ $\sqrt{x = 18^\circ + k.72^\circ}$ $\sqrt{x = -90^\circ + k.360^\circ}$ $\sqrt{\text{two correct}}$ $\sqrt{\text{another two correct}}$ $\sqrt{\cos(90^\circ - 2x)}$ $\sqrt{x = -90^\circ + k.360^\circ}$ $\sqrt{90^\circ - 2x = 360 - (-3x)}$ $\sqrt{x = -54^\circ + k.72^\circ}$ $\sqrt{\text{two correct}}$ $\sqrt{\text{another two correct}}$ (6)
		[22]

QUESTION 22



22.1	In $\triangle ABC$ $\cos B = \frac{10^2 + 10^2 - (10\sqrt{3})^2}{2 \cdot 10 \cdot 10}$ $\cos B = -0,5$ $\hat{B} = 120^\circ$	✓ substitute into cosine rule $\checkmark = -0,5$ $\checkmark \hat{B} = 120^\circ$ (3)
22.2	In $\triangle ACD$: $\frac{\sin D}{10\sqrt{3}} = \frac{\sin 74,47^\circ}{19,27}$ $\sin D = 0,866\dots$ $\hat{D} = 60^\circ$ $\hat{B} + \hat{D} = 180^\circ$ ABCD is a cyclic quadrilateral (opp $\angle$s <i>suppl.</i>)	✓ subst into sine rule $\checkmark 0,866$ $\checkmark \hat{D} = 60^\circ$ $\checkmark$ $\hat{B} + \hat{D} = 180^\circ$ $\checkmark$ reason (5)
		[8]

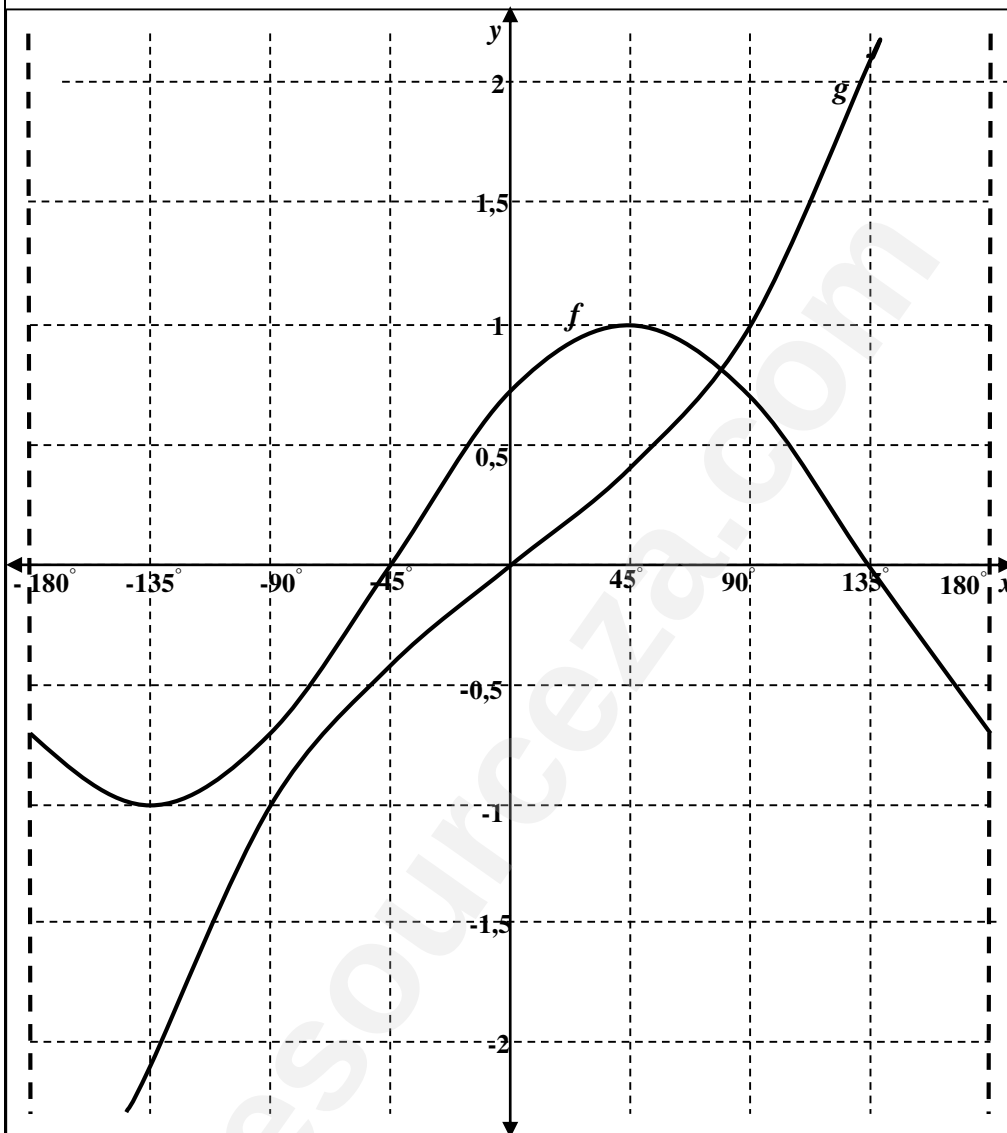
QUESTION 23

23.1	$\frac{\cos(180^\circ + x) \cdot \tan(360^\circ - x) \cdot \sin^2(90^\circ - x)}{\sin(180^\circ - x)} + \sin^2 x$ $= \frac{(-\cos x) \cdot (-\tan x) \cdot \cos^2 x}{\sin x} + \sin^2 x$ $= \frac{\cos x \cdot \frac{\sin x}{\cos x} \cdot \cos^2 x}{\sin x} + \sin^2 x$ $= \cos^2 x + \sin^2 x$ $= 1$	✓ $-\cos x$ ✓ $-\tan x$ ✓ $\cos^2 x$ ✓ $\sin x$ ✓ $\cos^2 x + \sin^2 x$ ✓ answer (6)
23.2.1	$\cos(A - B) - \cos(A + B)$ $= \cos A \cos B + \sin A \sin B - [\cos A \cos B - \sin A \sin B]$ $= \cos A \cos B + \sin A \sin B - \cos A \cos B + \sin A \sin B$ $= 2\sin A \sin B$	✓ expansion of $\cos(A - B)$ ✓ expansion of $\cos(A + B)$ ✓ simplification (3)
23.2.2	$\cos 15^\circ - \cos 75^\circ = \cos(45^\circ - 30^\circ) - \cos(45^\circ + 30^\circ)$ $= 2\sin 45^\circ \cdot \sin 30^\circ$ $= 2 \times \frac{\sqrt{2}}{2} \times \frac{1}{2} \quad \text{or} \quad 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{2}$ $= \frac{\sqrt{2}}{2} \quad \text{or} \quad \frac{1}{\sqrt{2}}$ OR $\cos 15^\circ - \cos 75^\circ$ $= \cos(45^\circ - 30^\circ) - \cos(45^\circ + 30^\circ)$ $= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ - [\cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ]$ $= 2\sin 45^\circ \sin 30^\circ$ $= 2 \times \frac{\sqrt{2}}{2} \times \frac{1}{2} \quad \text{or} \quad 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{2}$ $= \frac{\sqrt{2}}{2} \quad \text{or} \quad \frac{1}{\sqrt{2}}$	✓ $45^\circ - 30^\circ$ and $45^\circ + 30^\circ$ ✓ $2\sin 45^\circ \cdot \sin 30^\circ$ ✓ $\frac{\sqrt{2}}{2} \times \frac{1}{2} / \frac{1}{\sqrt{2}} \times \frac{1}{2}$ ✓ answer (4)

23.4	$AB^2 = (\cos \theta - 6)^2 + (\sin \theta - 7)^2$ $86 = \cos^2 \theta - 12 \cos \theta + 36 + \sin^2 \theta - 14 \sin \theta + 49$ $86 = 1 + 36 + 49 - 12 \cos \theta - 14 \sin \theta$ $0 = -12 \cos \theta - 14 \sin \theta$ $14 \sin \theta = -12 \cos \theta$ $\frac{\sin \theta}{\cos \theta} = \frac{-12}{14}$ $\tan \theta = -\frac{6}{7} / -0,86$	✓ sub. into the distance formula ✓ simplification ✓ $14 \sin \theta = -12 \cos \theta$ ✓ $\tan \theta = -\frac{6}{7} / 0,86 \quad (4)$ [17]
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QUESTION 24

24.1



f : ✓ x-int ✓ y-int ✓ turning points

g : ✓ asymptotes, ✓ passing $(0,0)$, ✓ $(90;1)$

(6)

24.2.1

$$x = -45^\circ \text{ or } x = 135^\circ$$

$$\checkmark x = -45^\circ$$

$$\checkmark x = 135^\circ$$

(2)

24.2.2

$$x = 180^\circ \text{ or/ of } x = -180^\circ$$

$$\checkmark x = 180^\circ$$

$$\checkmark x = -180^\circ$$

(2)

24.2.3

$$y \in [-1;1] \text{ or } -1 \leq y \leq 1$$

✓ answer

(1)

24.2.4

$$1$$

✓ answer

(1)

24.2.5

$$x \in (-180^\circ; -45^\circ) \text{ or/ of } (0^\circ; 135^\circ)$$

OR

$$-180^\circ < x < -45^\circ \text{ or } 0^\circ < x < 135^\circ$$

$$\checkmark -180^\circ; -45^\circ$$

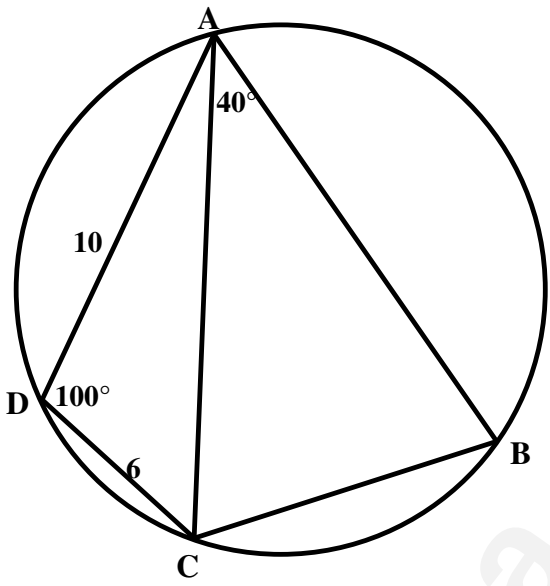
$$\checkmark 0^\circ; 135^\circ$$

✓ notation

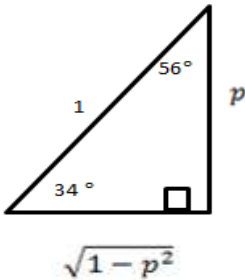
(3)

[15]

QUESTION 25

		
25.1	$AC^2 = 10^2 + 6^2 - 2(10)(6) \cos 100^\circ$ $= 156,8377813$ $AC = 12,5 \text{ units}$ $\hat{B} = 80^\circ \text{ (opp angles of a cyclic quad)}$ In $\triangle ABC$, $\frac{BC}{\sin 40^\circ} = \frac{AC}{\sin 80^\circ}$ $\frac{BC}{\sin 40^\circ} = \frac{12,5}{\sin 80^\circ}$ $BC = \frac{12,5 \times \sin 40^\circ}{\sin 80^\circ}$ $= 8,2 \text{ units}$	✓ sub. into cosine rule ✓ 156,8377813 ✓ 12,5 units ✓ $\hat{B} = 80^\circ$ ✓ sub. into sine rule ✓ answer (6)
25.2	$\text{Area of } \triangle ABC = \frac{1}{2} AC \cdot BC \cdot \sin 60^\circ$ $= \frac{1}{2} (12,5)(8,2) \sin 60^\circ$ $= 44,4 \text{ square units}$	✓ 60° ✓ sub. into the area rule ✓ answer (3) [9]

QUESTION 26

Q26	SUGGESTED ANSWER	DESCRIPTORS	MA
	$(1)^2 = p^2 + (opp)^2$ $(1)^2 - (p)^2 = x^2$ $\therefore x = \sqrt{1 - p^2}$ 		
26.1.1	$\sin 214^\circ$ $= \sin(180^\circ + 34^\circ)$ $= -\sin 34^\circ$ $= -p$	✓ $-\sin 34^\circ$ ✓ Ans	
26.1.2	$\cos 34^\circ \cdot \cos(-22^\circ) + \sin 34^\circ \cdot \sin 338^\circ$ $= \cos 34^\circ \cdot \cos 22^\circ + \sin 34^\circ \cdot (-\sin 22^\circ)$ $= \cos 34^\circ \cdot \cos 22^\circ - \sin 34^\circ \cdot \sin 22^\circ$ $= \cos(34^\circ + 22^\circ)$ $= \cos 56^\circ$ $\therefore = p$	✓✓ reduction ✓ compound angles ✓ $\sqrt{1 - p^2}$	
26.1.3	$\cos 68^\circ$ $= \cos 2(34^\circ)$ $= 2 \cos^2 34^\circ - 1$ $= 2(\sqrt{1 - p^2})^2 - 1$ $= 2(1 - p^2) - 1$ $= 1 - p^2$	✓ Double angle ✓ Ans	
26.2.1	$\frac{\cos(90^\circ - 2\theta) \cdot \sin(\theta)}{\sin^2(180^\circ + \theta) \cos(720^\circ + \theta)}$ $= \frac{\sin 2\theta \cdot (\sin \theta)}{(-\sin \theta)^2 \cdot (\cos \theta)}$ $= \frac{2 \sin \theta \cos \theta \cdot \sin \theta}{\sin^2 \theta \cdot \cos \theta}$ $= 2$	✓ $\sin 2\theta$ ✓ $(-\sin \theta)^2$ ✓ $\sin^2 \theta$ ✓ $\cos \theta$ ✓ $2 \sin \theta \cos \theta$ ✓ 2	(6)
26.2.2	$\frac{1}{\sin^2 2x} - \frac{1}{\tan^2 2x}$ $= \frac{1}{\sin^2 2x} - \frac{\cos^2 2x}{\sin^2 2x}$ $= \frac{1 - \cos^2 2x}{\sin^2 2x}$	$\frac{\cos^2 x}{\sin^2 x}$ ✓ $\frac{1 - \cos^2 2x}{\sin^2 2x}$ ✓	

	$= \frac{\sin^2 2x}{\sin^2 2x} = 1$	✓ $\sin^2 2x$ ✓ Ans	(4)
			[

QUESTION 27

Q27	SUGGESTED ANSWER	DESCRIPTORS	MA
27.1	The graph shows two functions, f and g, plotted on a coordinate system. The x-axis is labeled with angles from -90 to 180 in increments of 30. The y-axis is labeled with values -1, 0, and 1. Function f is a sine wave that starts at $(-90^\circ; -0.87)$, passes through the origin $(0, 0)$, and ends at $(180^\circ; -0.5)$. Function g is a sine wave that starts at $(-90^\circ; 0)$, reaches a maximum at $(0, 1)$, and ends at $(180^\circ; 0)$.	✓ shape ✓ endpoints ✓ intercepts with axes	(3)
27.2	$-\sin 2x = \cos(x - 60^\circ)$ $-\sin 2x = \sin(90^\circ - (x - 60^\circ))$ $-\sin 2x = \sin(150^\circ - x)$ $\sin(180^\circ + 2x) = \sin(150^\circ - x)$ $180^\circ + 2x = 150^\circ - x + n(360^\circ), n \in \mathbb{Z}$ $3x = -30 + n(360), n \in \mathbb{Z}$ $x = -10^\circ + n(120^\circ)$	✓ equate the equations ✓ co-functions $\sin(180^\circ + 2x)$ ✓ ✓ ✓ Ans	(5)
27.4	$-10^\circ \leq x \leq 110^\circ$	✓ Inequality ✓ ✓ Ans	(3)
27.4	$y = -\sin(2x + 30^\circ)$	✓ ✓ Ans	(2)
27.5	g must shift 30° right	✓ 30° ✓ right	(2)
			[1.

QUESTION 28

Q28	SUGGESTED ANSWER	DESCRIPTORS	MA
28.1	$\cos \theta = \frac{DC}{p}$ $p \cdot \cos \theta = DC$ $\frac{BD}{\sin(90^\circ - \theta)} = \frac{DC}{\sin 2\theta}$ $\frac{BD}{\cos \theta} = \frac{DC}{2 \sin \theta \cdot \cos \theta}$ $\frac{BD}{1} = \frac{DC \cdot \cos \theta}{2 \sin \theta \cdot \cos \theta} \text{ but } p \cos \theta = DC$ $\frac{BD}{1} = \frac{p \cos \theta \cdot \cos \theta}{2 \sin \theta \cdot \cos \theta}$ $BD = \frac{p \cos \theta}{2 \sin \theta}$	✓ DC ✓ Subst in Sine Rule ✓ Simplification ✓ Subst DC ✓ Simplification	(5)
28.2	$\sin 30^\circ = \frac{AC}{p}$ $3 \sin 30^\circ = AC$ $\therefore AC = 1,5 \text{ m}$	✓ Substitution ✓ Ans	(2)
28.3	$BD = \frac{3 \cos 30^\circ}{2 \sin 30^\circ} = \frac{3}{2} \left(\frac{1}{\sqrt{3}} \right) = \frac{3\sqrt{3}}{2}$ $AB^2 = AD^2 + BD^2 - 2AD \cdot BD \cos ADB$ $= 3^2 + \left(\frac{3\sqrt{3}}{2} \right)^2 - 2(3) \left(\frac{3\sqrt{3}}{2} \right) \cos 70^\circ$ $= 9 + \frac{27}{4} - 9\sqrt{3} \cos 70^\circ$ $= 10,418 \dots$ $AB = 3,23 \text{ metres}$	✓ BD ✓ Cos rule ✓ Subst ✓ AB² ✓ AB	(5)
			[1]

QUESTION 26

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