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MATHEMATICS GRADE 12

exam school 2012

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INTRODUCTION

Have you heard about Mindset? Mindset Network, a South African non-profit organisation, was founded in 2002. We develop and distribute quality and contextually relevant educational resources for use in the schooling, health and vocational sectors. We distribute our materials through various technology platforms like TV broadcasts, the Internet (www.mindset.co.za/learn) and on DVDs. The materials are made available in video, print and in computer-based multimedia formats.

At Mindset we are committed to innovation. In the last three years, we have successfully run a series of broadcast events leading up to and in support of the Grade 12 NSC examinations

Now we are proud to announce our 2012 edition of Exam School. From 15th October till 20th November will bring you revision lessons in nine subjects - Mathematics, Physical Sciences, Life Sciences, Mathematical Literacy, English 1st Additional Language, Accounting, Geography, Economics and Business Studies.

In this exam revision programme we have selected Questions mainly from the Nov 2011 Papers and have tried to cover as many topics as we can. Each topic is about an hour long and if you work through the selected questions you will certainly have increased confidence to face your exams. In addition to the topics and questions in this booklet, we have schedule 1½ hour live shows a day or two before you write your exams. To get the most out of these shows, we need you to participate by emailing us questions, calling in or posting on twitter, peptxt or facebook.

Since you asked us for late night study sessions and that's what we've planned! You'll find repeats of our Live shows at 10:30pm every evening. Then from midnight to 6:00 am there are revision lessons too. So if you can't watch during the day, you can record or watch early in the morning! .

GETTING THE MOST FROM EXAM SCHOOL

You must read this booklet! You'll find the exam overviews and lots of study tips and hints here. in Start your final revision by working through the questions for a topic fully without looking up the solutions. If you get stuck and can't complete the answer don't panic. Make a note of any questions you have. Now you are ready to watch a Learn Xtra session. When watching the session, compare the approach you took to what the teacher does. Don't just copy the answers down but take note of the method used. Also make a habit of marking your work by checking the memo. Remember, there are usually more than one way to answer a question.If you still don't understand post your question on Facebook – you'll get help from all the other Mindsetters on the page. You can also send an email to helpdesk@learnxtra.co.za and we'll get back to you within 48 hours.

Make sure you keep this booklet. You can re-do the questions you did not get totally correct and mark your own work. Exam preparation requires motivation and discipline, so try to stay positive, even when the work appears to be difficult. Every little bit of studying, revision and exam practice will pay off. You may benefit from working with a friend or a small study group, as long as everyone is as committed as you are.

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We are pleased to announce that we'll continue to run our special radio broadcasts on community radio stations in Limpopo, Eastern Cape and KZN. This programme is called MTN Learn. Find out more details at www.mtnlearning.co.za. You can also listen online or download radio broadcasts of previous shows. Tuning into radio will give you the chance to learn extra! Look out for additional notes in Newspaper supplements too.

Mindset believes that the 2012 Learn Xtra Spring School will help you achieve the results you want. All the best to the Class of 2012!

CONTACT US

We want to hear from you. So let us have your specific questions or just tell us what you think through any of the following:

LearnXtra

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www.learnxtra.co.za

helpdesk@learnxtra.co.za

086 105 8262

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EXAM SCHOOL (DSTV AND TOPTV 319)

MATHEMATICS PAPER 1

DATE	TIMES	TOPICS
16 October	22:30 - 12:00 12:00 - 01:30 01:30 – 02:30 02:30 - 03:30 03:30 - 04:30 04:30 – 06:00	Live: Functions (Repeat) Linear Programming Sequences & Series Finance Functions: Linear & Quadratic Live: Calculus (Repeat)
29 October	17:00– 18:00 18:00 - 19:00	Sequences & Series Finance
31 October	17:00– 18:00 18:00 - 19:00	Calculus Calculus Applications
1 November	9.00 - 10.00 10.00 - 11.00 11.00 - 12:00 12:00 - 13:00 13:30 – 14:30 14.30 - 15.30 15.30 – 17:00 17.00 - 18.00 18.00 – 19:00 22.30 – 24:00 24:00 – 06:00	Sequences & Series Finance Functions: Linear & Quadratic Functions: Hyperbolic, Exponential and Inverses Calculus Calculus Applications Exam Revision Live Functions: Linear & Quadratic Functions: Hyperbolic, Exponential and Inverses Exam Revision Live (Repeat) (Repeat of all Topics)

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MATHEMATICS PAPER 2

DATE	TIMES	TOPICS
2 November	22:30 – 12:00	Live: Trigonometry (Repeat)
	12:00 – 01:00	Data Handling
	01:00 – 02:00	Co-ordinate Geometry: Lines & Transformations
	02:00 – 03:00	Co-ordinate Geometry: Circles
	03:00 – 04:00	Trigonometry: Expressions, Equations & Graphs
	04:00 – 05:00	Trigonometry: Identities
	05:00 – 06:00	Trigonometry: 2D & 3D problems
4 November	9.00 - 10.00	Data Handling
	10.00 - 11.00	Co-ordinate Geometry: Lines & Transformations
	11.00 - 12:00	Co-ordinate Geometry: Circles
	12:00 - 13:00	Trigonometry: Expressions, Equations & Graphs
	13:30 – 14:30	Trigonometry: Identities
	14.30 - 15.30	Trigonometry: 2D & 3D problems
	15.30 – 17:00	Exam Revision Live
	17.00 - 18.00	Co-ordinate Geometry: Lines & Transformations
	18.00 – 19:00	Co-ordinate Geometry: Circles
	22.30 – 24:00	Exam Revision Live (Repeat)
	24:00 – 06:00	(Repeat of all Topics)

MTN LEARN RADIO SCHEDULE

EXAM SCHOOL: MATHEMATICS

DATE	TIME	TOPICS
8-Oct	17:00 -18:00	Algebra
9-Oct	17:00 -18:00	Analytical Geometry
10-Oct	17:00 -18:00	Functions
11-Oct	17:00 -18:00	Trigonometry
13-Oct	09:00 -10:00	Calculus
31-Oct	17:00 -18:00	Algebra
	18:00 -19:00	Functions
1-Nov	17:00 -18:00	Calculus
	18:00 -19:00	Calculus Applications
3-Nov	09:00 -10:00	Data Handling
	10:00 -11:00	Analytical Geometry
4-Nov	17:00 -19:00	Trigonometry

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MTN LEARN: PARTICIPATING COMMUNITY RADIO STATIONS

KwaZulu Natal:

Hindvani Radio	91.5 fm – Durban 102.3 fm – Rest or KZN
Maputaland Radio	170.6 fm

Limpopo Province:

Sekgosese Radio	100.3 fm
Greater Tzaneen Radio	104.8 fm
Mohodi FM	98.8 fm
Moletsi	98.6 fm
Univen	99.8 fm

Eastern Cape:

Vukani fm	90.6 – 99.9 fm
Fort Hare Community Radio	88.2 fm
Mdantsane fm	89.5 fm
Nkqubela fm	97.0 fm
Graaff Reinet	90.2 fm

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PREPARING FOR EXAMINATIONS

1. Prepare well in advance for all your papers and subjects. You need to start your planning for success in the final examination now. You cannot guarantee success if you only study the night before an exam.
2. Write down the date of your prelim and final exam so that you can plan and structure a study time table for all your subjects.
3. Set up a study time-table according to your prelim and final Grade 12 exam time-table and stick to your study schedule. If you study a small section every day, you will feel you have achieved something and you will not be as nervous by the time you have to go and write your first paper.
4. Your study programme should be realistic. You need to spend no more than 2 hours per day on one topic. Do not try to fit too much into one session. When you cover small sections of work often, you will master them more quickly. The broadcast schedule may help you to make sure you have covered all the topics that are in the exam.
5. When studying don't just read through your notes or textbook. You need to be active by making summary checklists or mind maps. Highlight the important facts that you need to memorise. You may need to write out definitions and formulae a few times to make sure you can remember these. Check yourself as often as you can. You may find it useful to say the definitions out aloud.
6. Practise questions from previous examination papers. Follow these steps for using previous exam papers effectively:
   - Take careful note of all instructions - these do not usually change.
   - Try to answer the questions without looking at your notes or the solutions.
   - Time yourself. You need to make sure that you complete a question in time. To work out the time you have, multiply the marks for a question by total time and then divide by the total number of marks. In most exams you need to work at a rate of about 1 mark per minute.
   - Check your working against the memo. If you don't understand the answer given, contact the Learn Xtra Help desk (email: helpdesk@learnxtra.co.za).
   - If you did not get the question right, try it again after a few days.
7. Preparing for, and writing examinations is stressful. You need to try and stay healthy by making sure you maintain a healthy lifestyle. Here are some guidelines to follow:
   - Eat regular small meals including breakfast. Include fruit, fresh vegetables, salad and protein in your diet.
   - Drink lots of water while studying to prevent dehydration.
   - Plan to exercise regularly. Do not sit for more than two hours without stretching or talking a short walk.
   - Make sure you develop good sleeping habits. Do not try to work through the night before an exam. Plan to get at least 6 hours sleep every night.

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EXAM TECHNIQUES

1. Make sure you have the correct equipment required for each subject. You need to have at least one spare pen and pencil. It is also a good idea to put new batteries in your calculator before you start your prelims or have a spare battery in your stationery bag.
2. Make sure you get to the exam venue early - don't be late.
3. While waiting to go into the exam venue, don't try to cram or do last minute revision. Try not to discuss the exam with your friends. This may just make you more nervous or confused.
4. Here are some tips as to what to do when you receive your question papers:
   Don't panic, because you have prepared well.
   - You are always given reading time before you start writing. Use this time to take note of the instructions and to plan how you will answer the questions. You can answer questions in any order.
   - Time management is crucial. You have to make sure that you answer all questions. Make notes on your question paper to plan the order for answering questions and the time you have allocated to each one.
   - It is a good idea always to underline the key words of a question to make sure you answer it correctly.
   - Make sure you look any diagrams and graph carefully when reading the question. Make sure you check the special answer sheet too.
   - When you start answering your paper, it is important to read every question twice to make sure you understand what to do. Many marks are lost because learners misunderstand questions and then answer incorrectly.
   - Look at the mark allocation to guide you in answering the question.
   - When you start writing make sure you number your answers exactly as they are in the questions.
   - Make sure you use the special answer sheet to answer selected questions.
   - Think carefully before you start writing. It is better to write an answer once and do it correctly than to waste time rewriting answers.
   - DO NOT use correction fluid (Tippex) because you may forget to write in the correct answer while you are waiting for the fluid to dry. Rather scratch out a wrong answer lightly with pencil or pen and rewrite the correct answer.
   - Check your work. There is usually enough time to finish exam papers and it helps to look over your answers. You might just pick up a calculation error.

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EXAM OVERVIEW

MATHEMATICS PAPER 1 3HRS TOTAL MARKS: 150

Number & Number Relations

Patterns & Sequences	±30 marks
Annuities & Finance	±15 marks

Functions & Algebra

Functions & Graphs	±35 marks
Algebra, Equations & Inequalities	±20 marks
Calculus	±35 marks
Linear Programming	±15 marks

MATHEMATICS PAPER 2 3HRS TOTAL MARKS: 150

Space, Shape and Measurement

Co-ordinate Geometry	±40 marks
Transformations	±25 marks
Trigonometry	±60 marks

Data Handling and Probability

Data Handling	±25 marks

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SEQUENCES AND SERIES

Check List

Make sure you can:

- Use identify arithmetic, geometric and quadratic sequences, write an expression for the general term and hence calculate the value of any term in the sequence
- Interpret sigma notation and find the sum of both arithmetic and geometric series
- Calculate the sum to infinity for geometric series

STUDY NOTES

Arithmetic Sequences and Series

An arithmetic sequence or series is the linear number pattern.

We have a formula to help us determine any specific term of an arithmetic sequence. We also have formulae to determine the sum of a specific number of terms of an arithmetic series.

The formulae are as follows:

$T_n = a + (n-1)d$ where $a =$ first term and $d =$ constant difference

$S_n = \frac{n}{2}[2a + (n-1)d]$ where $a =$ first term and $d =$ constant difference

$S_n = \frac{n}{2}[a + l]$ where l is the last term

Geometric Sequences and Series

A geometric sequence or series is an exponential number pattern.
We have a formula to help us determine any specific term of a geometric sequence. We also have formulae to determine the sum of a specific number of terms of a geometric series.

The formulae are as follows:

$T_n = ar^{n-1}$

$S_n = \frac{a(r^n - 1)}{r - 1}$ where $r \neq 1$

Convergent Geometric Series

Consider the following geometric series:

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ...$

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We can work out the sum of progressive terms as follows:

$S_1 = \frac{1}{2} = 0,5$ (Start by adding in the first term)

$S_2 = \frac{1}{2} + \frac{1}{4} = \frac{3}{4} = 0,75$ (Then add the first two terms)

$S_3 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8} = 0,875$ (Then add the first three terms)

$S_4 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = \frac{15}{16} = 0,9375$ (Then add the first four terms)

If we continue adding progressive terms, it is clear that the decimal obtained is getting closer and closer to 1. The series is said to converge to 1. The number to which the series converges is called the **sum to infinity** of the series.

There is a useful formula to help us calculate the sum to infinity of a convergent geometric series.

The formula is $S_\infty = \frac{a}{1-r}$

If we consider the previous series $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ...$

It is clear that $a = \frac{1}{2}$ and $r = \frac{1}{2}$

$$S_\infty = \frac{a}{1-r}$$

$$\therefore S_\infty = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$$

A geometric series will converge only if the constant ratio is a number between negative one and positive one.

In other words, the sum to infinity for a given geometric series will exist only if $-1 < r < 1$

If the constant ratio lies outside this interval, then the series will not converge.

For example, the geometric series $1 + 2 + 4 + 8 + 16 + ...$ will not converge since the sum of the progressive terms of the series diverges.

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Question 1 (*Adapted from Nov 2011, Paper 1, Question 2*)

1.1 Given the sequence: 4 ; x ; 32

Determine the value(s) of x if the sequence is:

1.1.1 Arithmetic (2)

1.1.2 Geometric (3)

1.2 Determine the value of P if P = $\sum_{k=1}^{13} 3^{k-5}$ (4)

Question 2 *(Adapted from Nov 2011, Paper 1, Question 3)*

The following sequence is a combination of an arithmetic and a geometric sequence:

3 ; 3 ; 9 ; 6 ; 15 ; 12 ; …

2.1 Write down the next TWO terms (2)

2.2 Calculate $T_{52} - T_{51}$. (5)

Question 3 (Adapted from Nov 2011, Paper 1, Question 4)

A quadratic pattern has a second term equal to 1, a third term equal to – 6 and a fifth term equal to – 14.

3.1 Calculate the second difference of this quadratic pattern. (5)

3.2 Hence, or otherwise, calculate the first term of the pattern. (2)

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FINANCE

Check List

Make sure you can:

- Use the formulae give to calculate present and future value of annuities
- Calculate the value of a sinking fund

STUDY NOTES

Future Value Annuity Formula

This formula deals with saving money for the future. Remember that the value of *n* represents the number of payments and not necessarily the duration of the investment.

$$\mathrm{F} = \frac{x\left[(1+i)^n - 1\right]}{i}$$

where:

$x =$ equal and regular payment per period

$n =$ number of payments

$i =$ interest rate as a decimal $= \dfrac{r}{100}$

Present Value Annuity Formula

This formula deals with loans. There must always be a gap between the loan and the first payment for the formula to work.

$$\mathrm{P} = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

where:

$x =$ equal and regular payment per period

$n =$ number of payments

$i =$ interest rate as a decimal $= \dfrac{r}{100}$

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Question 1 *(Adapted from Nov 2011, Paper 1, Question 7)*

1.1 How many years will it take for an article to depreciate to half its value according to the reducing-balance method at 7% per annum? (4)

1.2 Two friends each receive an amount of R6 000 to invest for a period of 5 years. They invest the money as follows:

- Radesh: 8,5% per annum simple interest. At the end of the 5 years, Radesh will receive a bonus of exactly 5% of the principal amount.
- Thandi: 8% per annum compounded quarterly.

Who will have the bigger investment after 5 years? Justify your answer with appropriate calculations. (6)

1.3 Nicky opened a savings account with a single deposit of R1 000 on 1 April 2011. She then makes 18 monthly deposits of R700 at the end of every month. Her first payment is made on 30 April 2011 and her last payment on 30 September 2012. The account earns interest at 15% per annum compounded monthly.

Determine the amount that should be in her savings account immediately after her last deposit is made (that is on 30 September 2012). (6)

Question 2 (Adapted from Feb/Mar 2011, P1, Question 8)

2.1 R1 430,77 was invested in a fund paying i% p.a. compounded monthly. After 18 months the fund had a value of R1 711,41. Calculate i. (4)

2.2 A father decided to buy a house for his family for R800 000. He agreed to pay monthly instalments of R10 000 on a loan which incurred interest at a rate of 14% p.a. compounded monthly. The first payment was made at the end of the first month.

2.2.1 Show that the loan would be paid off in 234 months. (4)

2.2.2 Suppose the father encountered unexpected expenses and was unable to pay any instalments at the end of the 120th, 121st, 122nd and 123rd months. At the end of the 124th month he increased his payment so as to still pay off the loan in 234 months by 111 equal monthly payments. Calculate the value of this new instalment. (7)

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Question 3

Mark's small business, called Postal Emporium, purchases a photocopying machine for R200 000. The photocopy machine depreciates in value at 20% per annum on a reducing balance. Mark's business wants to buy a new machine in five years' time. A new machine will cost much more in the future and its cost will escalate at 16% per annum effective. The old machine will be sold at scrap value after five years. A sinking fund is set up immediately in order to save up for the new machine. The proceeds from the sale of the old machine will be used together with the sinking fund to buy the new machine. The small business will pay equal monthly amounts into the sinking fund, and the interest earned is 18% per annum compounded monthly. The first payment will be made immediately, and the last payment will be made at the end of the five year period.

3.1 Find the scrap value of the old machine. (2)

3.2 Find the cost of the new machine in five years' time. (2)

3.3 Find the amount required in the sinking fund in five years' time. (1)

3.4 Find the equal monthly payments made into the sinking fund. (4)

3.5 Suppose that the business decides to service the machine at the end of each year for the five year period. R3000 will be withdrawn from the sinking fund at the end of each year starting one year after the original machine was bought.

3.5.1 Calculate the reduced value of the sinking fund at the end of the five year period due to these withdrawals (3)

3.5.2 Calculate the increased monthly payment into the sinking fund which will yield the original sinking fund amount as well as allow for withdrawals from the fund for the services. (5)

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FUNCTIONS: LINEAR & QUADRATIC

Check List

Make sure you can:

- Determine if a graph is a function
- Draw a sketch graph of a linear and quadratic function given the equations
- Use a graph to determine the equation of a parabola or straight line
- Find points of intersection

STUDY NOTES

How to tell if a graph is a Function

For every input of a function there is a unique output. To test if a graph is a function, we use a ruler to perform the "vertical line test" on a graph to see whether it is a function or not. Hold a clear plastic ruler parallel to the y-axis, i.e. vertical.

Move it from left to right over the axes.
If the ruler only ever cuts the curve in one place only throughout the movement from left to right, then the graph is a function.
If the ruler ever passes through two or more points on the graph, the graph will not be a function.

Rules for Sketching Parabolas of the Form $y = a*x + p+^4 + q$

- The value of *a* tells us if the graph is concave ($a > 0$) or convex ($a < 0$).
- The equation of the axis of symmetry of the graph is obtained by putting the expression $x + p = 0$ and solving for *x*.
- The axis of symmetry passes through the *x*-coordinate of turning point of the parabola.
- The graph of $y = a(x+p)^2 + q$ is obtained by shifting the graph of $y = ax^2$ by p units to the left or right and then *q* units up or down.
  - If $p > 0$, the shift is left.
  - If $p < 0$, the shift is right.
  - If $q > 0$, the shift is upwards.
  - If $q < 0$, the shift is downwards.
- The *y*-coordinate of the turning point is *q.*
- The *y*-intercept of the graph can be determined by putting $x = 0$.
  The *x*-intercept(s) of the graph can be determined by putting $y = 0$.

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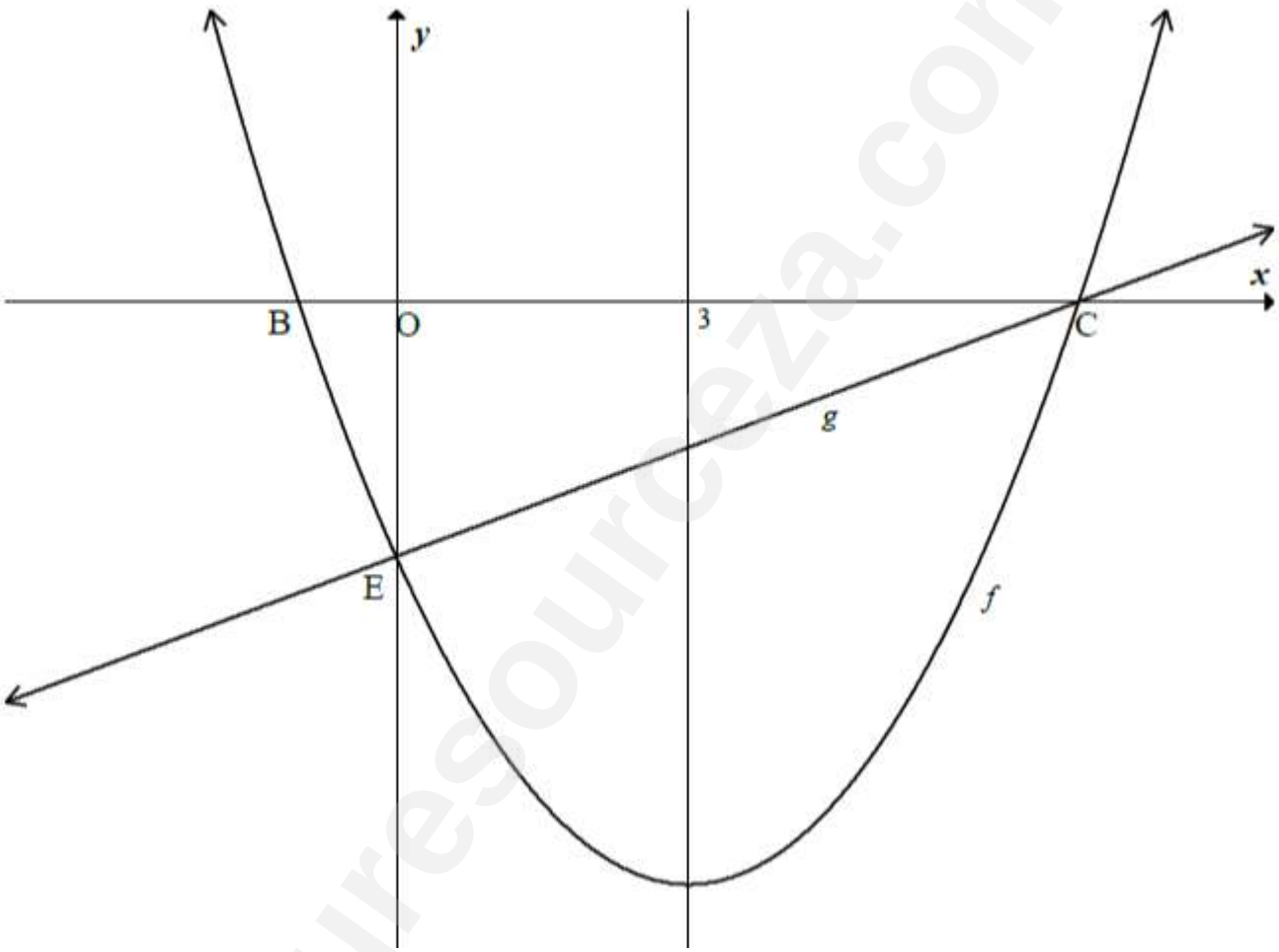
Question 1 (Adapted from Feb/Mar 2011, Paper 1, Question 6)

A parabola f intersects the x-axis at B and C and the y-axis at E. The axis of symmetry of the parabola has equation x = 3. The line through E and C has equation:

$g(x) = \frac{x}{2} - \frac{7}{2}$



1.1 Show that the coordinates of C are (7 ; 0). (1)

1.2 Calculate the x-coordinate of B. (1)

1.3 Determine the equation of f in the form $y = a(x - p)^2 + q$. (6)

1.4 Write down the equation of the graph of h, the reflection of f in the x-axis. (1)

1.5 Write down the maximum value of t(x) if t(x) = 1 – f(x). (2)

1.6 Solve for x if $f(x^2 - 2) = 0$ (4)

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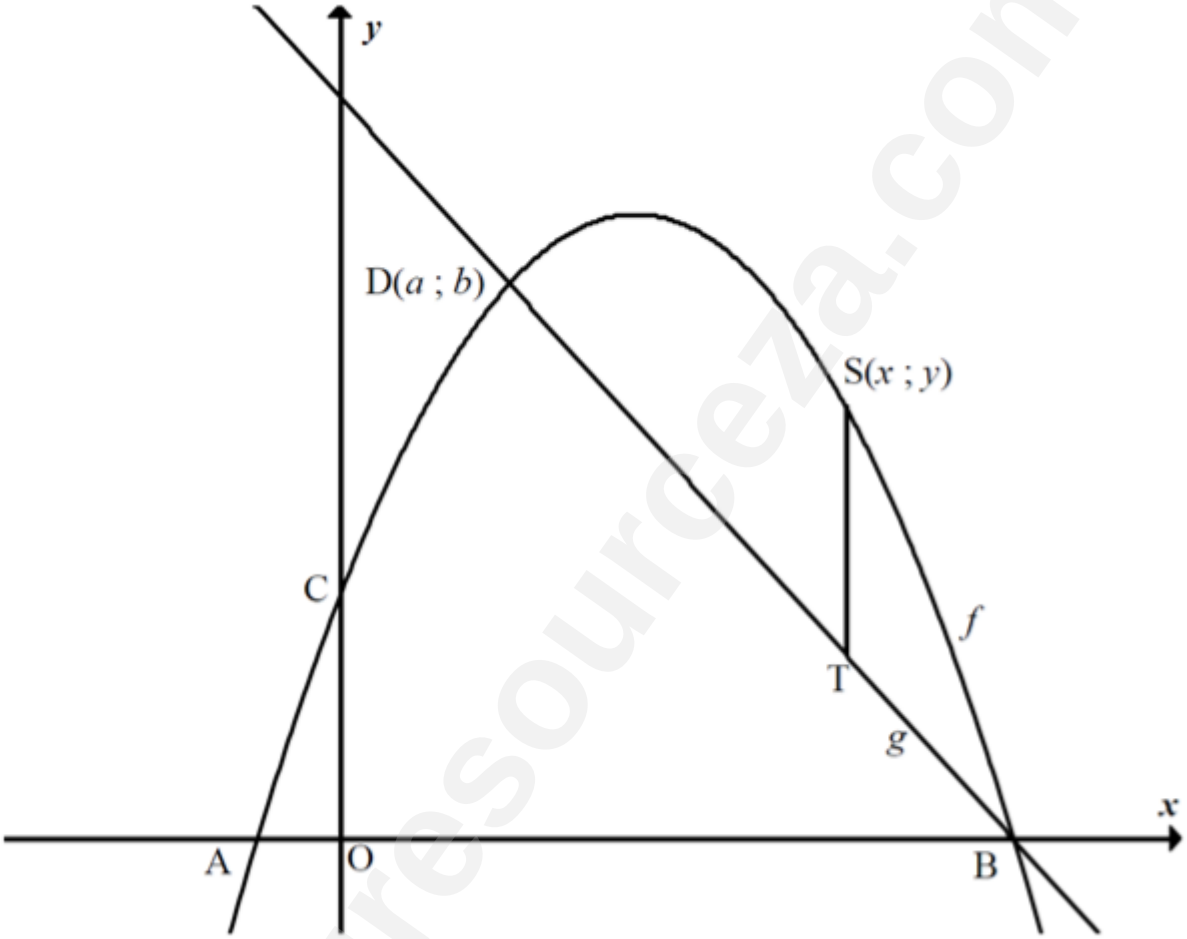
Question 2(Adapted from Feb/Mar 2010, Paper 1, Question 6)

The graphs of $f(x) = -x^2 + 7x + 8$ and

$g(x) = -3x + 24$ are sketched below.

f and g intersect in D and B. A and B are the x-intercepts of f.



2.1 Determine the coordinates of A and B. (4)

2.2 Calculate a, the x-coordinate of D. (4)

2.3 S(x ; y) is a point on the graph of f, where $a \leq x \leq 8$. ST is drawn parallel to the y-axis with T on the graph of g. Determine ST in terms of x. (2)

2.4 Calculate the maximum length of ST. (2)

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FUNCTIONS: HYPERBOLIC, EXPONENTIAL & INVERSES

Check List

Make sure you can:

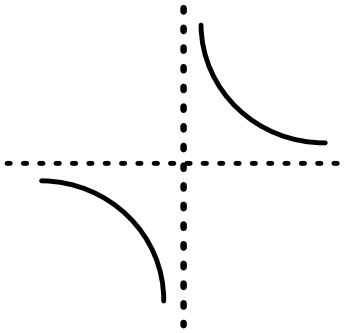
- Draw sketch graphs of hyperbolic and exponential functions
- Determine the inverse of functions
- Calculate points of intersection for different graphs

STUDY NOTES

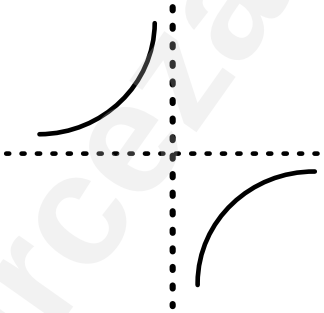
Rules for Sketching Hyperbolas of the Form $y = \dfrac{a}{x+p} + q$

1. Determine the **shape**:

$a > 0$



$a < 0$



(The dotted lines are the asymptotes)

2. Write down the **asymptotes** and draw them on a set of axes.
   Vertical asymptote: $x + p = 0$ Horizontal asymptote: $y = q$
3. Plot **four graph points** on your set of axes.
4. Determine the *y*-intercept: let $x = 0$.
5. Determine the *x*-intercept: let $y = 0$.
6. Draw the newly formed graph.

Rules for Sketching Hyperbolas of the Form $y = ab^{x+p} + q$

1. Write down the **horizontal asymptote** and draw it on a set of axes:
   Horizontal asymptote: $y = q$
2. Plot **three graph points** on your set of axes.
3. Draw the newly formed graph.

Logarithmic Functions

The inverse of the exponential function is called the logarithmic function.

Consider the function $y = a^x$. The inverse of this graph is $x = a^y$

It is now possible to make *y* the subject of the formula in the equation $x = a^y$ by means of the concept of a logarithm.

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If $x = a^y$, then it is clear from the definition of a logarithm that $\log_a x = y$. In other words, we can write the inverse of the function $f(x) = a^x$ as $f^{-1}(x) = \log_a x$.
The inverse function is formed by reflecting the function across the line y=x

Note on Inverses of Functions

The inverse of some functions are not functions but relations. The inverse of a parabola is not a function. However, if the domain of the original function is restricted then the inverse may be a function too. For example f(x) = x^2 ,then the inverse of f, $f^{-1}(x) = \pm\sqrt{x}$, is not a function. But if f(x) has a restricted domain x≥ 0 or x ≤ 0, then the inverse will be a function.

Question 1 (Adapted from Nov 2011, Paper 1, Question 5)

1.1 Consider the function: f (x) = $\frac{-6}{x-3} - 1$

1.1.1 Calculate the coordinates of the y-intercept of f. (2)

1.1.2 Calculate the coordinates of the x-intercept of f. (3)

1.1.3 Sketch the graph of f below, showing clearly the asymptotes and the intercepts with the axes. (4)

1.1.4 For which values of x is f (x) > 0 ? (2)

1.1.5 Calculate the average gradient of f between x = – 2 and x = 0. (4)

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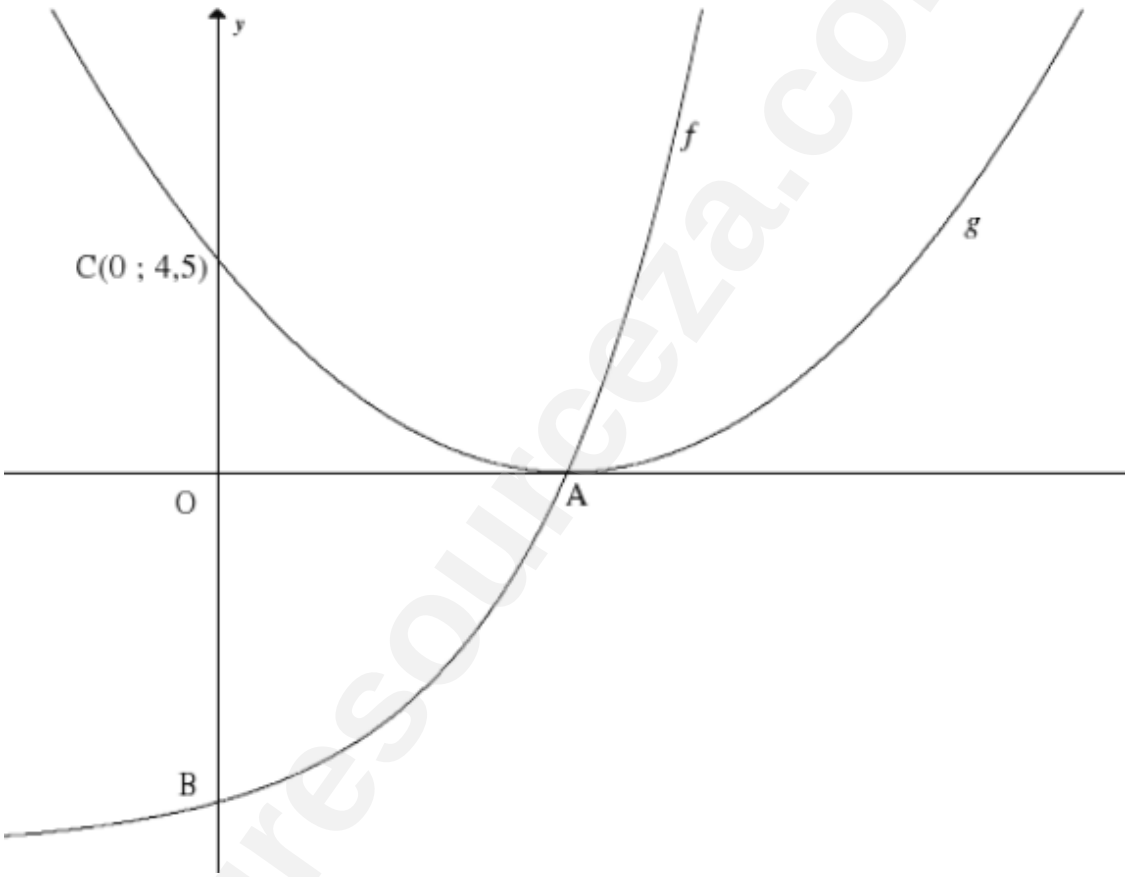
Question 2 (Adapted from Nov 2011, Paper 1, Question 6)

The graphs of $f(x) = 2^x - 8$ and $g(x) = ax^2 + bx + c$ are sketched below.

B and C (0; 4,5) are the y-intercepts of the graphs of f and g respectively.

The two graphs intersect at A, which is the turning point of the graph of g and the x-intercept of the graphs of f and g.



2.1 Determine the coordinates of A and B. (4)

2.2 Write down an equation of the asymptote of the graph of f. (1)

2.3 Determine an equation of h if $h(x) = f(2x) + 8$. (2)

2.4 Determine an equation of h^{-1} in the form y = ... (2)

2.5 Write down an equation of p, if p is the reflection of h^{-1} about the x-axis. (1)

2.6 Calculate

$$\sum_{k=0}^{3} g(k) - \sum_{k=4}^{5} g(k)$$

Show ALL your working. (4)

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Question 3 (Adapted from Feb/Mar 2011, Paper 1, Question 7)

Consider the function $f(x) = \left(\frac{1}{3}\right)^x$

3.1 Is f an increasing or decreasing function? Give a reason for your answer. (2)

3.2 Determine $f^{-1}(x)$ in the form y = … (2)

3.3 Write down the equation of the asymptote of f(x) – 5. (1)

3.4 Describe the transformation from f to g if $g(x) = \log_3 x$. (2)

Question 4 (Adapted from Nov 2010, Paper 1, Question 5)

Consider the function $f(x) = 4^{-x} - 2$.

4.1 Calculate the coordinates of the intercepts of f with the axes. (4)

4.2 Write down the equation of the asymptote of f. (1)

4.3 Sketch the graph of f. (3)

4.4 Write down the equation of g if g is the graph of f shifted 2 units upwards. (1)

4.5 Solve for x if f(x) = 3. (You need not simplify your answer.) (3)

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CALCULUS

Check List
Make sure you can:

- Use first principles to find the derivative of a function
- Use the rules to find derivatives.
- Interpret different notation for derivatives
- Find the tangent to a graphs

STUDY NOTES

Use of first principles and rules to find derivatives

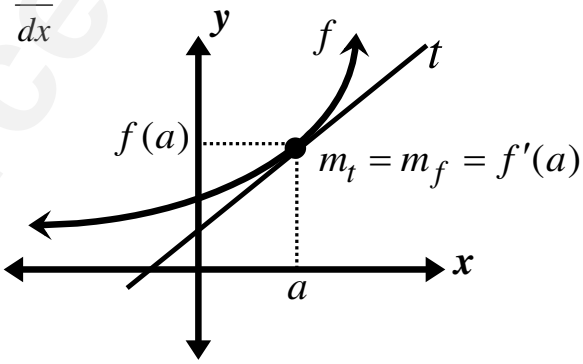
The most important fact in calculus is that the gradient of the tangent to a curve at a given point is the gradient of the curve at that point.

Other words for gradient are: **rate of change, derivative, slope**

Symbols for Gradient are: $f'(x)$ $\quad D_x$ $\quad \frac{dy}{dx}$

$f'(a)$ = "kuvj g'i t cf kgpv'qh'f 'cv'x = a

$f(a)$ = "kuvj g'y/xcnwg'eqt t gur qpf kpi 'vq'x = a



Average Gradient

The average gradient (or average rate of change) of a function f between $x = a$ and $x = b$ is the gradient of the line joining the points on the graph of the function. We say that the average gradient of f over the interval is the gradient of the line ***ab***.

Gradient of a Curve at a Point using First Principles

The formula to determine the gradient of a function from first principles is given by the following limit:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Gradient of a Function using the Rules of Differentiation

You will be required to use the following rules of differentiation to determine the gradient of a function.

Rule 1 If $f(x) = ax^n$, then $f'(x) = a.nx^{n-1}$

Rule 2 If $f(x) = ax$, then $f'(x) = a$

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Rule 3 If $f(x) = \text{number}$, then $f'(x) = 0$

Determining the Equation of the Tangent to a Curve at a point

The gradient of the tangent to a curve at a point is the derivative at that point.
The equation is given by $y - y_1 = m(x - x_1)$ where $(x_1 ; y_1)$ is the point of tangency and $m = f'(x_1)$

Calculus Questions

Question 1 (Adapted from November 2011 P1, Question 8)

1.1 Determine $f'(x)$ from first principles if $f(x) = -4x^2$ (5)

1.2 Evaluate:

1.2.1 $\frac{dy}{dx}$ if $y = \frac{3}{2x} - \frac{x^2}{2}$ (3)

1.2.2 $f'(1)$ if $f(x) = (7x+1)^2$ (4)

Question 2

Given: $f(x) = -2x^2 + 1$

2.1 Determine the gradient of the graph at $x = -2$, i.e. $f'(-2)$ (1)

2.2 Determine $f(-2)$. What does your answer represent? (2)

2.3 Determine the average gradient of f between $x = -2$ and $x = 4$ (4)

Question 3

Determine the following and leave your answer with positive exponents:

3.1 $D_x\left[\frac{1}{\sqrt{x}}\left(x^3 - 2x^2 + 3x\right)\right]$ (4)

3.2 $f'(x)$ if $f(x) = \frac{1}{2\sqrt[4]{x^3}}$ (3)

Question 4

Determine the equation of the tangent to $f(x) = x^2 - 6x + 5$ at $x = 2$. (5)

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CALCULUS APPLICATIONS

Check List

Make sure you can:

- Interpret graphs using principals of differentiation
- Draw a sketch of a cubic function
- Use the graph of the derivative of a function to answer questions about the function
- Use differentiation to solve maximum and minimum problems.

Rules for Sketching the Graph of a Cubic Function

The graph of the form $f(x) = ax^3 + bx^2 + cx + d$ is called a cubic function.
The main concepts involved with these functions are as follows:

Intercepts with the Axes

For the *y*-intercept, let $x = 0$ and solve for *y*
For the *x*-intercepts, let $y = 0$ and solve for *x*
(You might have to use the factor theorem here.)

Stationary Points

Determine $f'(x)$, equate it to zero and solve for x.
Then substitute the x-values of the stationary points into the original equation to obtain the corresponding y-values.
If the function has two stationary points, establish whether they are maximum or minimum turning points.

Points of Inflection

If the cubic function has only one stationary point, this point will be a point of inflection that is also a stationary point.
For points of inflection that are not stationary points, find $f''(x)$, equate it to 0 and solve for *x*. Alternatively, simply add up the *x*-coordinates of the turning points and divide by 2 to get the *x*-coordinate of the point of inflection.

Maxima and Minima Problems

Know the formulas for perimeter, area and volume of the basic shapes, e.g. spheres, cubes, rectangles, triangular prisms, circles and trapeziums. You need to create equations using the given information.

The maximum value is determined if $f'(x) = 0$ and $f''(x) > 0$
The maximum value is determined if $f'(x) = 0$ and $f''(x) < 0$

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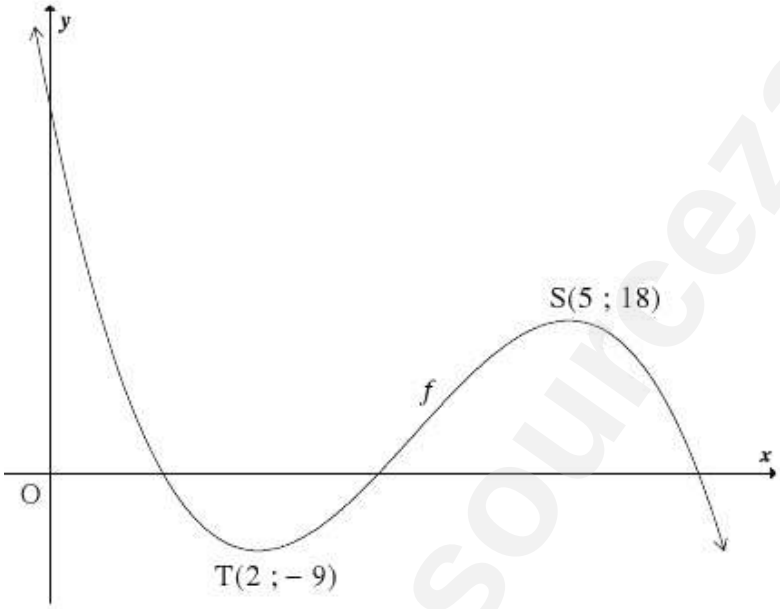
Calculus Applications Questions

Question 1

Sketch the graph of $f(x) = 2x^3 - 6x - 4$
Indicate the coordinates of the stationary points, intercepts with the axes and any points of inflection. (12)

Question 2 (Adapted from November 2011 P1, Question 9)
The function f (x) = − 2 x^3 + ax^2 + bx + c is sketched below.
The turning points of the graph of f are T(2 ; – 9) and S(5 ; 18).



2.1 Show that a = 21 , b = – 60 and c = 43. (7)
2.2 Determine an equation of the tangent to the graph of f at x = 1. (5)
2.3 Determine the x-value at which the graph of f has a point of inflection. (2)

Question 3 (Adapted from November 2011 P1, Question 11)
Water is flowing into a tank at a rate of 5 litres per minute. At the same time water flows out of the tank at a rate of k litres per minute. The volume (in litres) of water in the tank at time t (in minutes) is given by the formula V (t) = 100 − 4t .

3.1 What is the initial volume of the water in the tank? (1)
3.2 Write down TWO different expressions for the rate of change of the volume of water in the tank. (3)
3.3 Determine the value of k (that is, the rate at which water flows out of the tank). (2)

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LINEAR PROGRAMMING

STUDY NOTES

Terminology

Constraints - definition, limitation or restriction on a variable
Implicit constraints – limitations implied by the situation (Integers, reals etc.)
Feasible region – region on the model where a solution exists
Model – A system of definitions, assumptions and equations; graphical interpretation (picture) of the system.
Optimisation- finding the optimal solution, the best solution to our problem

Essential skills

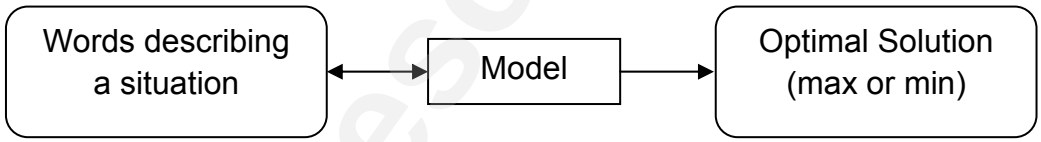
Make sure you can

- sketch straight lines as well as find equations
- create inequalities by interpreting words into inequalities as well as solving inequalities
- substitute co-ordinate pairs into an equation to find a solution
- sketch and interpret a region using inequalities

Types of questions or sub-questions

Key Concepts

Words to inequalities
Equations/inequalities to model
Model to solution: optimisation
Model to equations/inequalities



Question 1 Words into equations or inequalities:

A farmer needs to build more dams to increase the carrying capacity of his farm. He needs at least 12000kg sand, at least 800kg gravel and at lease 480kg cement. He can purchase ready packed crates from the local hardware store. A Strong-build crate has 600kg sand, 200kg gravel, and 60kg cement. A Fast–build crate has 200kg sand, 200kg gravel, and 160kg cement.
Let x be the number of Strong-build crates and y the number of Fast–build crates, write down all the constraints.

Question 2 Words into equations or inequalities:

The owner of a bus takes tourists (adults and children) on a tour of Soweto during the World Cup. He is allowed to provide the tour as long as the group consists of less than 60 people and subject to the following constraints:

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There must be at least 35 persons in the group
There must be at least 6 adults in the group
There must not be more than 14 adults
If x represents the number of children and y the number of adults, write down all the constraints.

Question 3 Words into equations or inequalities:

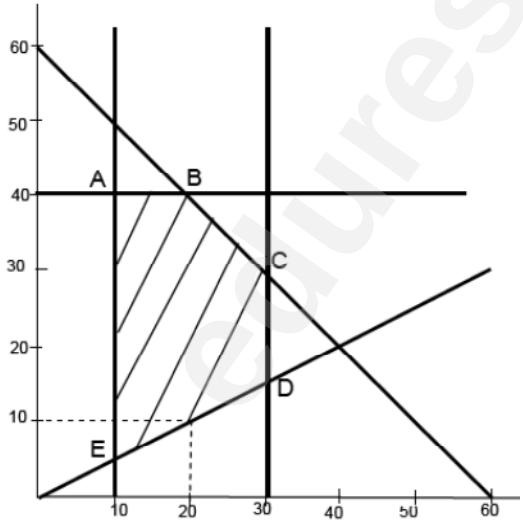
A surfboard company makes two types of surfboards, namely airbrushed and clear.
It takes eight hours to make an airbrushed board and four hours to make a clear board. The company can only work for a maximum of 72 hours per week.
At least 10 boards must be made weekly. No more than 12 clear boards can be produced weekly.
Let x be the number of airbrushed boards and y the number of clear boards, write down all the constraints.

Question 4 Words into equations or inequalities:

An entrepreneur in the low-veld makes two types of braai stands. x Impala-braai sets and y Kudu-braai sets are made every month. The factory cannot manufacture more than 100 Impala and 60 Kudu sets per month. The maximum number of 140 sets of both types can be made each month. A farm produce store has a standing order for 10 Impala and 30 Kudu set per month. The number of Kudu sets made each month is not more than twice the number of Impala sets made each month.

Question 5 Model into a series of Inequalities:

The sketch represents the feasible region for a system of inequalities (constraints).Write down all the inequalities which apply to this graph.



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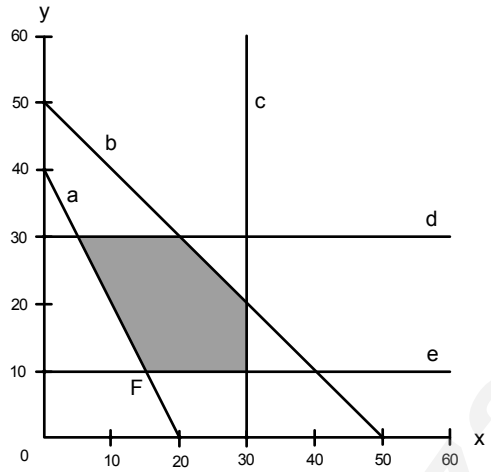
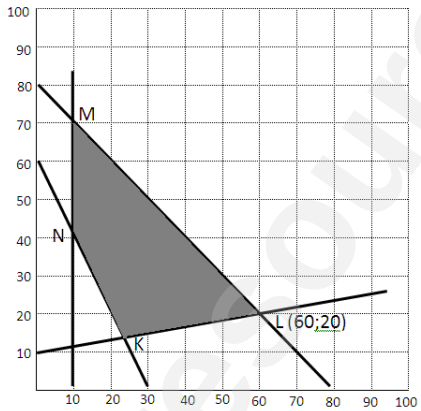
Question 6 Model into a series of Inequalities:

In the sketch alongside, the shaded area represents the feasible region for a set of linear constraints. The lines have been labelled a, b, c, d and e. Find the respective equations.



Question 7 Model into a series of Inequalities:



Give the constraints which are described by the above model.

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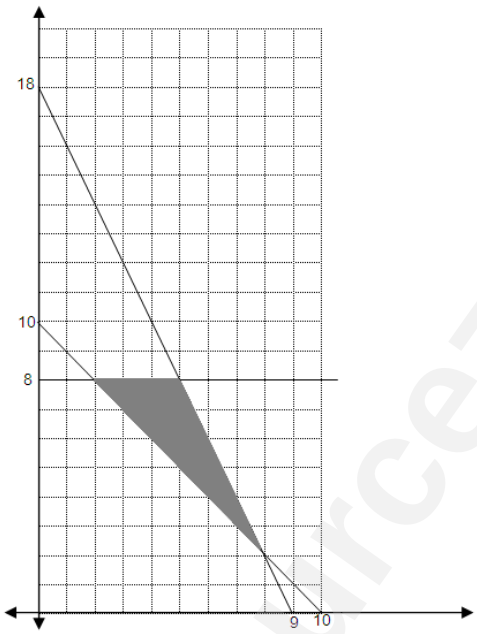
Question 8 Maximising or Minimising from a Model:

A factory produces Plasma(x) and Flat Screen(y) televisions. The profit on one Plasma television is R750 and on one Flat Screen is R1250.

8.1 Write down the objective function representing the profit per week. (1)

8.2 Determine the number of Plasma televisions and the number of Flat Screen televisions that have to be produced to yield a maximum profit using the model below. (4)

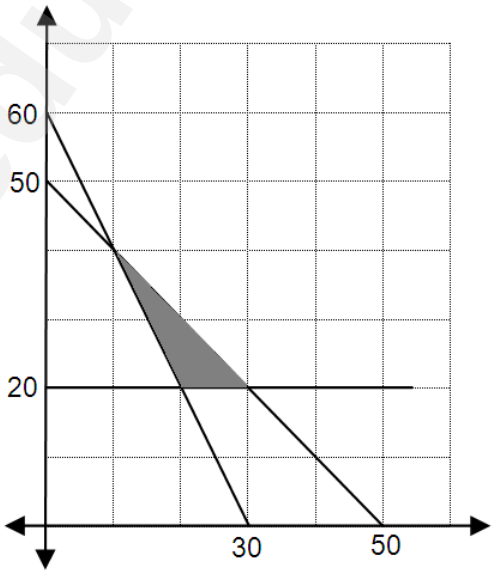


Question 9 Maximising or Minimising from a Model:

Stacey runs a cottage industry that manufacturers a variety of chocolate sauces using local ingredients, vanilla extract imported form Mauritius and some other secret items. Recipe X uses cream and melted Bar One's while recipe Y uses cream, almonds and Lint chocolate.

9.1 If a profit of R200 is made on every litre of recipe X sold and R150 profit on every litre of recipe Y sold, write down an equation for weekly profit. (2)

9.2 Use the model below to determine the maximum profit Stacey could make. (2)



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Question 10 'Complete' Question:

Adapted from DBE November 2010, Paper 1, Question 11

A factory produces two types of braai stands, Type A and Type B.
Type A requires one hour of machine-time and three hours for welding and finishing.
Type B requires two hours of machine-time and one hours for welding and finishing.
In one day the factory has available no more than 28 hours machine-time and no more than 24 hours for welding and finishing.

10.1 The factory produces x Type A and y Type B on a particularly day, write down the constraints terms of x and y. (4)
10.2 Represent these constraints on the grid provided.
Shade the feasible region. (3)
10.3 Determine the largest number of each type that could be manufactured on one day. (2)
10.4 Determine the maximum number of braai stands that could be manufactured. (2)
10.5 If the demand of Type A braai stands is at least as large as Type B braai stands, determine the maximum number of both types that can be manufactured. What is the machine-time requires in this case? (5)

Question 11:

Adapted from DBE Feb-March 2011, Paper 1, Question 12

While preparing for the 2010 Soccer World Cup, a group of investors decided to build a guesthouse with single and double bedrooms to hire out to visitors. They came up with the following constraints for the guesthouse:

* There must be at least one single bedroom.
* They intend to build at least 10 bedrooms altogether, but not more than 15.
* Furthermore, the number of double bedrooms must be at least twice the number of single bedrooms.
* There should not be more than 12 double bedrooms.

Let the number of single bedrooms be x and the number of double bedrooms be y.

11.1 Write down the constraints as a system of inequalities. (6)
11.2 Represent the system of constraints on the graph paper provided on the diagram sheet.
Indicate the feasible region by means of shading. (7)
11.3 According to these constraints, could the guesthouse have 5 single bedrooms and 8 double bedrooms? Motivate your answer (2)
11.4 The rental for a single bedroom is R600 per night and R900 per night for a double bedroom. How many rooms of each type of bedroom should the contractors build so that the guesthouse produces the largest income per night? Use a search line to determine your answer and assume that all bedrooms in the guesthouse are fully occupied. (3)

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Solutions to Sequences & Series

1.1.1

$x-4=32-x$ $2x=36$ $x=18$ **OR** $a=4$ $a+2d=32$ $2d=28$ $d=14$ $x=14+4$ $x=18$ **OR** $x=\frac{4+32}{2}=18$ **Note:** If answer only: award 2/2 marks **Note:** If candidate writes $x-4 \quad 32-x$ only (i.e. omits equality) : 0/2 marks	✓ $T_2-T_1=T_3-T_2$ ✓ answer (2) ✓ $a+2d=32$ and $a=4$ ✓ answer (2) ✓ substitutes correctly into arithmetic mean formula i.e. $\frac{4+32}{2}$ ✓ answers (2)

1.1.2

$\frac{x}{4}=\frac{32}{x}$ $x^2=128$ $x=\pm\sqrt{128}$ $x=\pm 8\sqrt{2}$ OR $x=\pm 11{,}31$ OR $x=\pm 2^{\frac{7}{2}}$ **Note:** If candidate writes $\frac{x}{4} \quad \frac{32}{x}$ only (i.e. omits equality) : 0/2 marks **Note:** If only $x=\sqrt{128}$ then penalty 1 mark	✓ $\frac{T_2}{T_1}=\frac{T_3}{T_2}$ ✓ $x^2=128$ ✓ both answers (surd or decimal or exponential form) (3)

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1.2

$P = \sum_{k=1}^{13} 3^{k-5}$ $= 3^{1-5} + 3^{2-5} + 3^{3-5} + \ldots + 3^{13-5}$ $= 3^{-4} + 3^{-3} + 3^{-2} + \ldots + 3^{8}$ $= \dfrac{3^{-4}\left(3^{13}-1\right)}{3-1}$ $= 9841{,}49$ or $9841\frac{40}{81}$ or $\frac{797161}{81}$	**Note:** Correct answer only: 1/4 marks only	✓ $a = 3^{-4}$ or $\frac{1}{81}$ ✓ $r = 3$ ✓ subs into correct formula ✓ answer (4)

2.1 21; 24

2.2

$T_{2k} = 3.2^{k-1}$ and so $T_{52} = 3.2^{26-1} = 100663296$ $T_{2k-1} = 3 + 6(k-1) = 6k - 3$ and so $T_{51} = 6(26) - 3 = 153$ $T_{52} - T_{51} = 100663296 - 153$ $= 100663143$	**Note:** If candidate writes out all 52 terms **and** gets correct answer: award 5/5 marks **Note:** If candidate used $k = 52$: max 2/5	✓ 3.2^{k-1} ✓ T_{52} ✓ $6k - 3$ ✓ T_{51} ✓ answer (5)

3.1

The second, third, fourth and fifth terms are 1 ; – 6 ; T_4 and – 14 First differences are: – 7 ; $T_4 + 6$; $-14 - T_4$ So $T_4 + 6 + 7 = -14 - 2T_4 - 6$ $T_4 = -11$ d = – 11 + 6 + 7 = 2 or – 14 + 22 – 6 = 2	**Note:** Answer only (i.e. $d = 2$) with no working: 3 marks **Note:** Candidate gives $T_4 = -11$ and $d = 2$ only: award 5/5 marks	✓ – 7 ✓ $T_4 + 6$ ✓ $-14 - T_4$ ✓ setting up equation $T_5 - T_2 = (T_5 - T_4) + (T_4 - T_3) + (T_3 - T_2)$ ✓ answer (5)

3.2

$T_4 + 13 = -8 + T_1$

$T_1 = 10$

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Solutions to Finance

1.1

$A = P(1-i)^n$ $\frac{P}{2} = P(1-0{,}07)^n$ $\frac{1}{2} = 0{,}93^n$ $\log\frac{1}{2} = n\log 0{,}93$ $n = \frac{\log\frac{1}{2}}{\log 0{,}93}$ $= 9{,}55$ years	**OR**	$A = P(1-i)^n$ $\frac{P}{2} = P(1-0{,}07)^n$ $\frac{1}{2} = 0{,}93^n$ $\log_{0{,}93}\frac{1}{2} = n$ $n = 9{,}55$ years	✓ $A = \frac{P}{2}$ ✓ subs into correct formula ✓ log ✓ answer (4)
Note: If candidate interchanges A and P i.e. uses $P = \frac{A}{2}$: max 2/4 marks		**Note:** If candidate uses incorrect formula: max 1/4 marks for $A = \frac{P}{2}$	

1.2

Radesh: $A = P(1+in)$ $= 6\,000(1+0{,}085\times 5)$ $= 8\,550$	OR $A = 6\,000 + 8{,}5\%$ of 6000×5 $= 6000 + 510\times 5$ $= 6000 + 2550$ $= 8\,550$	✓ 8 550
Bonus $= 0{,}05\times 6\,000$ $= 300$ Received $= 8\,550 + 300$ $=$ R8 850		✓ **R8 850**
Thandi: $A = P(1+i)^n$ $= 6\,000\left(1+\frac{0{,}08}{4}\right)^{20}$ $=$ R8 915,68 Thandi's investment is bigger.		✓ $n = 20$ ✓ $i = \frac{0{,}08}{4}$ ✓ answer ✓ choice made (6)

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1.3

F_v = initial deposit with interest + annuity $=1\,000\left(1+\frac{0,15}{12}\right)^{18}+700\left(\frac{\left(1+\frac{0,15}{12}\right)^{18}-1}{\frac{0,15}{12}}\right)$ $=1\,250,58+14\,032,33$ $=\text{R}15\,282,91$	✓ $i=\frac{0,15}{12}$ or $\frac{1}{80}$ or $0,0125$ ✓ $n=18$ ✓ $n=18$ ✓ $1\,000\left(1+\frac{0,15}{12}\right)^{18}$ ✓ $700\left(\frac{\left(1+\frac{0,15}{12}\right)^{18}-1}{\frac{0,15}{12}}\right)$ ✓ answer (6)

2.1

$A=P(1+i)^n$ $1711,41=1430,77\left(1+\frac{i}{12}\right)^{18}$ $\left(1+\frac{i}{12}\right)^{18}=1,196146131...$ **OR** $\left[\frac{1711,41}{1430,77}\right]^{\frac{1}{18}}=1,00999...$ $1+\frac{i}{12}=1,009999937...$ $\therefore i=12(1,01-1)$ $i=0,1199992431...$ $=0,12$ $=12\%$ Rate = 12, 00% p.a. compounded monthly.	✓ substitution ✓ $\left(1+\frac{i}{12}\right)^{18}=1,196146131...$ ✓ $1+\frac{i}{12}=1,009999937...$ ✓ answer (4)

2.2.1

Balance outstanding after 233rd month $=800000\left(1+\frac{0,14}{12}\right)^{233}-\frac{10000\left[\left(1+\frac{0,14}{12}\right)^{233}-1\right]}{\frac{0,14}{12}}$ = R4 660,04 which is less than R10 000 Therefore the loan will be paid off after 234 months.	✓ substitution into P formula ✓ 234 ✓ answer ✓ argument (4)

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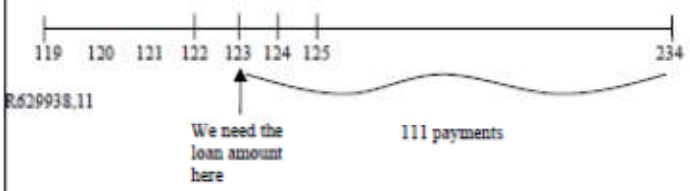
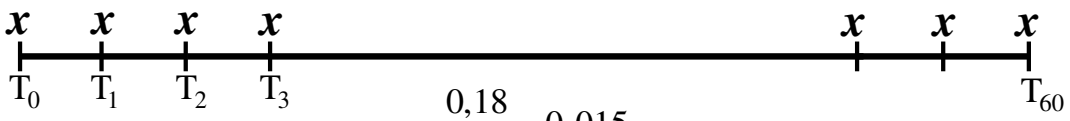
2.2.2

Balance Outstanding after 119 months $= 800000\left(1+\frac{0,14}{12}\right)^{119} - \frac{10000\left[\left(1+\frac{0,14}{12}\right)^{119}-1\right]}{\frac{0,14}{12}}$ $= \text{R}629\ 938,11$  Total Payable at the end of the 123rd month $= 629\ 938,11\left(1+\frac{0,14}{12}\right)^{4}$ $= \text{R}659\ 853,68$ New instalment: $659\ 853,68 = \frac{x\left[1-\left(1+\frac{0,14}{12}\right)^{-111}\right]}{\frac{0,14}{12}}$ $x = \text{R}10\ 632,39$	✓ $800000\left(1+\frac{0,14}{12}\right)^{119}$ ✓ $\frac{10000\left[\left(1+\frac{0,14}{12}\right)^{119}-1\right]}{\frac{0,14}{12}}$ ✓ R629 938,11 ✓ $629938,11\left(1+\frac{0,14}{12}\right)^{4}$ ✓ R659 853,68 ✓ substitution into P_n ✓ answer (7) [15]

3(a)	$A = 200\ 000(1-0,2)^5$ $\therefore A = \text{R}65\ 536$	✓ correct formula ✓ answer (2)
3(b)	$A = 200\ 000(1+0,16)^5$ $\therefore A = \text{R}420\ 068,33$	✓ correct formula ✓ answer (2)
3(c)	Sinking fund = 420 068,33 − 65 536 Sinking fund = 354 532,33	✓ answer (1)

3(d) Draw a time line



$$\frac{0,18}{12} = 0,015$$

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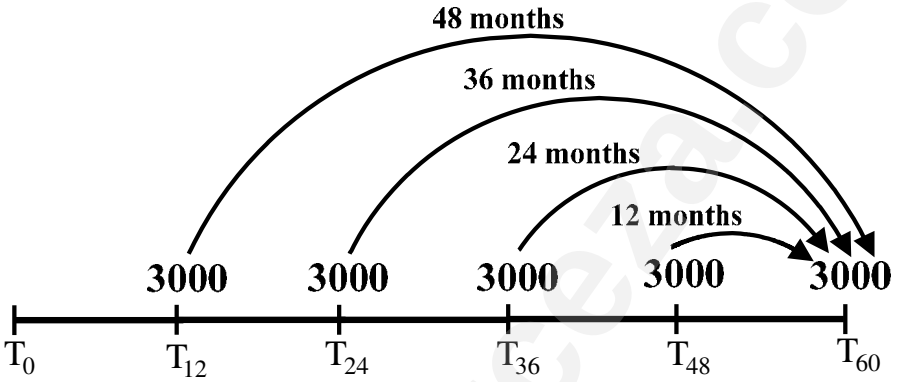
3(d)	$354\ 532{,}33 = \dfrac{x\left[(1{,}015)^{61}-1\right]}{0{,}015}$ $\dfrac{354\ 532{,}33 \times 0{,}015}{\left[(1{,}015)^{61}-1\right]} = x$ $\therefore x = \text{R}3593{,}55$	✓ correct formula ✓ $F = 354\ 532{,}33$ ✓ $n = 61$ ✓ $\dfrac{0{,}18}{12} = 0{,}015$ ✓ $x = \text{R}3593{,}55$ (4)

3(e) (1) Draw a time line



3(e)(1)	Future value of the withdrawals: $3000\left(1+\dfrac{0{,}18}{12}\right)^{48} + 3000\left(1+\dfrac{0{,}18}{12}\right)^{36}$ $+3000\left(1+\dfrac{0{,}18}{12}\right)^{24} + 3000\left(1+\dfrac{0{,}18}{12}\right)^{12} + 3000$ $= \text{R}22\ 133{,}22$ The reduced value of the sinking fund will be: $\text{R}354\ 532{,}33 - \text{R}22\ 133{,}22 = \text{R}332\ 399{,}11$	✓ services ✓ R22 133,22 ✓ reduced value (3)

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3(e)(2)	If we add R22 133,22 to the original sinking fund amount of R354 532,33, then it will be possible not only to receive the sinking fund amount of R354 532,33 at the end of the five year period, but also be able to make the service withdrawals at the end of each year for the five year period. $354\ 532{,}33 + 22\ 133{,}22 = \dfrac{x\left[(1{,}015)^{61} - 1\right]}{0{,}015}$ $\therefore 376\ 665{,}55 = \dfrac{x\left[(1{,}015)^{61} - 1\right]}{0{,}015}$ $\therefore \dfrac{376\ 665{,}55 \times 0{,}015}{\left[(1{,}015)^{61} - 1\right]} = x$ $\therefore x = \text{R}3817{,}90$	✓ correct formula ✓ $\text{F} = 376\ 665{,}55$ ✓ $n = 61$ ✓ $\dfrac{0{,}18}{12} = 0{,}015$ ✓ $x = \text{R}3817{,}90$ (5) **[17]**

Solutions to Functions: Linear & Quadratic

1.1

$\dfrac{x}{2} - \dfrac{7}{2} = 0$ $x = 7$ C(7 ; 0)	$\dfrac{x}{2} - \dfrac{7}{2} = 0$ (1)

1.2

x- coordinate of B is $3 - 4 = -1$	✓ answer (1)

1.3

$f(x) = a(x-3)^2 + q$ At B and C: $0 = 16a + q$ At E: $-\dfrac{7}{2} = 9a + q$ Solving simultaneously gives $a = \dfrac{1}{2}$ and $q = -8$	✓ substitution ✓ substitution ✓ substitution ✓✓ $a = \dfrac{1}{2}$ ✓ $q = -8$ (6)

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1.4

$h(x) = -f(x) = \frac{-1}{2}(x-3)^2 + 8$	✓ answer (1)

1.5

$1 - f(x) = -\frac{1}{2}(x-3)^2 + 9$ $\therefore$ Maximum value is 9.	✓ method ✓ answer (2)

1.6

$f(x^2 - 2) = 0$ $f(x) = 0$ if $x = -1$ or $x = 7$ $\therefore f(x^2 - 2) = 0$ if $x^2 - 2 = -1$ or $x^2 - 2 = 7$ $\therefore x^2 = 1$ or $x^2 = 9$ $\therefore x = 1$ or $x = -1$ or $x = 3$ or $x = -3$	✓ substitution ✓ simplification ✓ answer ✓ answer (4)

2.1

$-x^2 + 7x + 8 = 0$ $x^2 - 7x - 8 = 0$ $(x-8)(x+1) = 0$ $x = 8$ or $x = -1$ A(– 1 ; 0) B(8 ; 0)	✓ = 0 ✓ factors ✓ answer A ✓ answer B (4)

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2.2

$-x^2+7x+8=-3x+24$ $-x^2+10x-16=0$ $x^2-10x+16=0$ $(x-8)(x-2)=0$ $x=8$ or $x=2$ x-value of D is 2. (i.e. $a=2$)	✓ equating ✓ standard form ✓ factors ✓ answer (4)

2.3

$ST=(-x^2+7x+8)-(-3x+24)$ $=-x^2+10x-16$	✓ subtraction ✓ answer (2)

2.4

Maximum length of ST is at $x=\frac{-10}{2(-1)}=5$. Maximum length of ST is $-5^2+50-16=9$.	✓ method ✓ answer (2)

Solutions to Functions: Hyperbolic, Exponential & Inverses

1.1.1	$y=f(0)$ $=\frac{-6}{0-3}-1$ $=1$ $(0\ ;\ 1)$ **OR** $x=0$ and $y=1$	✓ $y=1$ ✓ $x=0$ (2)
1.1.2	$0=\frac{-6}{x-3}-1$ $1=\frac{-6}{x-3}$ $x-3=-6$ $x=-3$ $(-3\ ;\ 0)$	✓ $y=0$ ✓ $x-3=-6$ ✓ answer (3)

Note:
Mark 5.1.1 and 5.1.2 as a single question.
If the intercepts are interchanged:
max 3/5 marks

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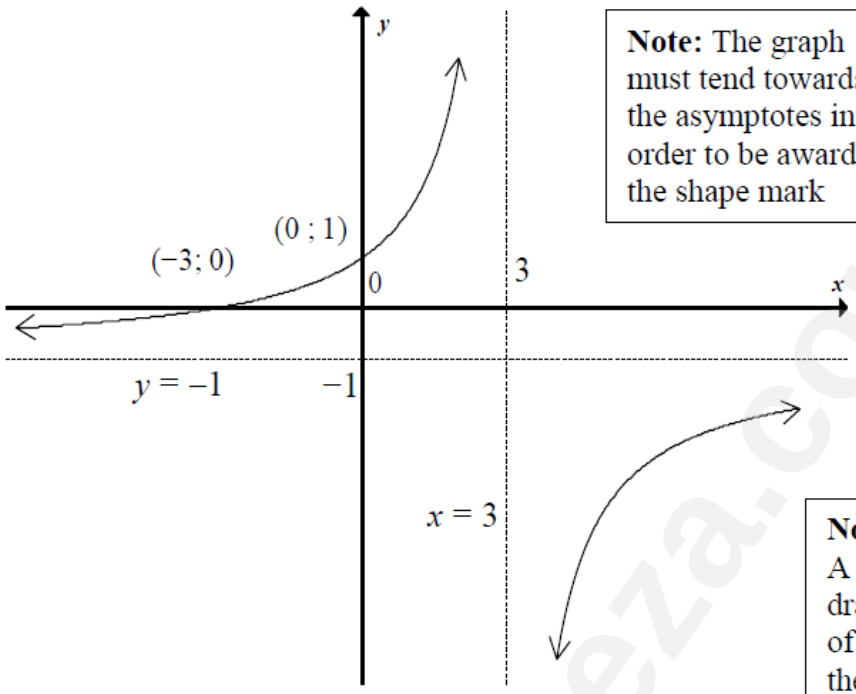
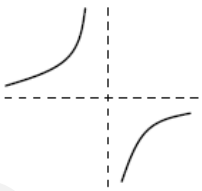
1.1.3	 **Note:** The graph must tend towards the asymptotes in order to be awarded the shape mark **Note:** A candidate who draws only one 'arm' of the hyperbola loses the 'shape' mark i.e. max 3/4 marks	✓ shape  ✓ both intercepts correct ✓ horizontal asymptote ✓ vertical asymptote (4)
1.1.4	$-3 < x < 3$ **OR** $(-3; 3)$ **OR** $-3 < x$ and $x < 3$ **Note**: if candidate writes $-3 < x$ only: 1/2 marks **Note**: if candidate writes $x < 3$ only: 1/2 marks	✓ −3 and 3 ✓ inequality OR interval notation (2)
1.1.5	$y = \frac{-6}{-2-3} - 1$ $= \frac{1}{5}$ $m = \frac{1 - \frac{1}{5}}{0-(-2)}$ $= \frac{2}{5}$	✓ $\frac{1}{5}$ ✓ formula ✓ substitution ✓ answer (4)

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2.1	$0 = 2^x - 8$ $8 = 2^x$ $2^3 = 2^x$ $x = 3$ A(3 ; 0) $f(0) = 2^0 - 8$ $= 1 - 8$ $= -7$ B(0 ; –7)	✓ $y = 0$ ✓ answer for A ✓ $x = 0$ ✓ answer for B (4)
2.2	$y = -8$ **OR** $y + 8 = 0$ **Note**: no CA marks	✓ answer (1)
2.3	$h(x) = f(2x) + 8$ $= \left(2^{2x} - 8\right) + 8$ $= 4^x$ or 2^{2x} **Note**: answer only: award 2/2 marks	✓ $\left(2^{2x} - 8\right)$ ✓ answer of $h(x) = 4^x$ or 2^{2x} (2)
2.4	$x = 4^y$ **OR** $x = 2^{2y}$ $y = \log_4 x$ $2y = \log_2 x$ $y = \frac{1}{2}\log_2 x$ **OR** $y = \log_2 \sqrt{x}$ **OR** $y = \frac{\log x}{\log 4}$ **Note**: answer only award 2/2 marks **Note**: candidate works out f^{-1} and gets $y = \log_2(x+8)$ award 1/2 marks	✓ switch x and y ✓ answer in the form $y = \ldots$ (2)
2.5	$p(x) = -\log_4 x$ **OR** $p(x) = \log_{\frac{1}{4}} x$	✓answer (1)
2.6	$\sum_{k=0}^{3} g(k) - \sum_{k=4}^{5} g(k)$ $= g(0) + g(1) + g(2) + g(3) - g(4) - g(5)$ $x = 3$ is the axis of symmetry of g $\therefore$ by symmetry $g(2) = g(4)$ and $g(1) = g(5)$ Answer $= g(0) + g(3)$ $= 4{,}5 + 0$ $= 4{,}5$	✓ $= g(0) + g(1) + g(2) + g(3) - g(4) - g(5)$ ✓ $g(2) = g(4)$ and $g(1) = g(5)$ ✓ $g(0) + g(3)$ ✓ answer (4)

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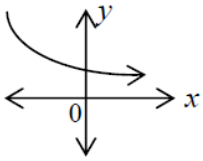
3.1	Decreasing function Since $0 < a < 1$ OR As x increases, $f(x)$ decreases	✓ decreasing ✓ $a < 1$ (2)
3.2	$f^{-1}: \quad x = \left(\frac{1}{3}\right)^y$ $y = \log_{\frac{1}{3}} x$ OR  OR $f^{-1}: \quad x = \left(\frac{1}{3}\right)^y$ $y = -\log_3 x$	✓ $x = \left(\frac{1}{3}\right)^y$ ✓ $y = \log_{\frac{1}{3}} x$ or $y = -\log_3 x$ (2)
3.3	$y = -5$	✓ answer (1)
3.4	Reflection about $y = x$. Reflection about the x-axis.	✓ reflection about $y = x$ ✓ reflection about y-axis (2)

4.1	$f(x) = 4^{-x} - 2$ y-intercept: $x = 0;\ y = 4^0 - 2 = -1$; $(0\,;-1)$ x-intercept: $4^{-x} - 2 = 0$ $4^{-x} = 2$ $2^{-2x} = 2$ $-2x = 1$ $x = -\frac{1}{2}$ **OR** $4^{-x} - 2 = 0$ $4^{-x} = 2$ $\log 4^{-x} = \log 2$ $-x = \frac{\log 2}{\log 4}$ $-x = \frac{\log 2}{2\log 2}$ $x = -\frac{1}{2}$ **OR** $4^{-x} - 2 = 0$ $4^{-x} = 2$ $4^{-x} = 4^{\frac{1}{2}}$ $-x = \frac{1}{2}$ $x = -\frac{1}{2}$ x-intercept is $\left(-\frac{1}{2}\,;0\right)$	✓✓ y-intercept ✓✓ x-intercept (4) **Note:** No penalty if the answer is not left as a coordinate.
4.2	$y = -2$	✓ equation (1)

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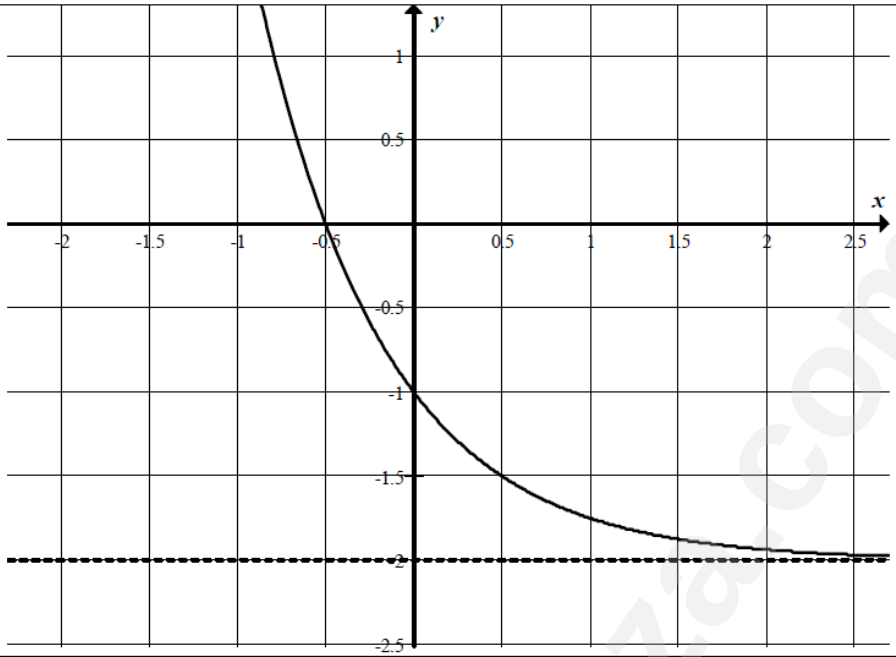
4 3		✓ asymptote ✓ y-intercept or x-intercept ✓ shape (decreasing) (3)
4 4	$g(x) = 4^{-x} - 2 + 2$ $g(x) = 4^{-x}$	✓ equation (1)
4.5	$4^{-x} - 2 = 3$ $4^{-x} = 5$ $-x\log 4 = \log 5$ $x = -\dfrac{\log 5}{\log 4}$ **OR** $x = -\log_4 5$ **OR** $x = \log_{\frac{1}{4}} 5$ **OR** $x = \log_4 \frac{1}{5}$ **OR** $x = -1{,}16$ **OR** $x = \dfrac{\log 5}{\log \frac{1}{4}}$ **OR** $x = \dfrac{\log \frac{1}{5}}{\log 4}$	✓ $4^{-x} = 5$ ✓ $-x\log 4 = \log 5$ ✓ answer (3) **[12]**

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Solutions to Calculus

Question 1

1.1	$f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{-4(x+h)^2-(-4x^2)}{h}$ $=\lim\limits_{h\to 0}\dfrac{-4(x^2+2xh+h^2)+4x^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{-4x^2-8xh-4h^2+4x^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{-8xh-4h^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(-8x-4h)}{h}$ $=\lim\limits_{h\to 0}(-8x-4h)$ $=-8x$	**Note:** Incorrect notation: no lim written: penalty 2 marks lim written before equals sign: penalty 1 mark **Note:** A candidate who gives $-8x$ only: 0/5 marks **Note:** A candidate who omits brackets in the line $\lim\limits_{h\to 0}(-8x-4h)$: NO penalty	✓ formula ✓ substitution ✓ expansion ✓ $-8x-4h$ ✓ answer (5)

1.2.1	$y=\dfrac{3}{2x}-\dfrac{x^2}{2}$ $=\dfrac{3}{2}x^{-1}-\dfrac{1}{2}x^2$ $\dfrac{dy}{dx}=-\dfrac{3}{2}x^{-2}-x$ $=-\dfrac{3}{2x^2}-x$	**Note:** Incorrect notation in 8.2.1 and/or 8.2.2: Penalise 1 mark	✓ $\dfrac{3}{2}x^{-1}$ ✓ $-\dfrac{3}{2}x^{-2}$ ✓ $-x$ (3)
1.2.2	$f(x)=(7x+1)^2$ $=49x^2+14x+1$ $f'(x)=98x+14$ $f'(1)=98(1)+14$ $=112$		✓multiplication ✓ $98x$ ✓14 ✓ answer (4)

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Question 2

2.1	$f'(x) = -4x$ $\therefore f'(-2) = -4(-2) = 8$	✓ answer (1)
2.2	$f(x) = -2x^2 + 1$ $\therefore f(-2) = -2(-2)^2 + 1$ $\therefore f(-2) = -7$ The answer represents the y-value corresponding to $x = -2$	✓ $f(-2) = -7$ ✓ interpretation (2)
2.3	$f(x) = -2x^2 + 1$ $f(-2) = -2(-2)^2 + 1$ $\therefore f(-2) = -7$ $f(4) = -2(4)^2 + 1$ $\therefore f(4) = -31$ $(-2\,;-7)$ and $(4\,;-31)$ Average gradient $= \dfrac{-31-(-7)}{4-(-2)} = \dfrac{-24}{6} = -4$	✓ $f(-2) = -7$ ✓ $f(4) = -31$ ✓ $\dfrac{-31-(-7)}{4-(-2)}$ ✓ -4 (4)

Question 3

3.1	$D_x\left[\dfrac{1}{\sqrt{x}}\left(x^3 - 2x^2 + 3x\right)\right]$ $= D_x\left[\dfrac{x^3}{x^{\frac{1}{2}}} - \dfrac{2x^2}{x^{\frac{1}{2}}} + \dfrac{3x}{x^{\frac{1}{2}}}\right]$ $= D_x\left[x^{\frac{5}{2}} - 2x^{\frac{3}{2}} + 3x^{\frac{1}{2}}\right]$ $= \dfrac{5}{2}x^{\frac{3}{2}} - 3x^{\frac{1}{2}} + \dfrac{3}{2}x^{-\frac{1}{2}}$ $= \dfrac{5}{2}x^{\frac{3}{2}} - 3x^{\frac{1}{2}} + \dfrac{3}{2x^{\frac{1}{2}}}$	✓ $x^{\frac{1}{2}}$ ✓ $D_x\left[x^{\frac{5}{2}} - 2x^{\frac{3}{2}} + 3x^{\frac{1}{2}}\right]$ ✓ $\dfrac{5}{2}x^{\frac{3}{2}} - 3x^{\frac{1}{2}} + \dfrac{3}{2}x^{-\frac{1}{2}}$ ✓ $\dfrac{5}{2}x^{\frac{3}{2}} - 3x^{\frac{1}{2}} + \dfrac{3}{2x^{\frac{1}{2}}}$ (4)

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3.2	$f(x)=\dfrac{1}{2\sqrt[4]{x^3}}$ $\therefore f(x)=\dfrac{1}{2x^{\frac{3}{4}}}$ $\therefore f(x)=\dfrac{1}{2}x^{-\frac{3}{4}}$ $\therefore f'(x)=\dfrac{1}{2}\times-\dfrac{3}{4}x^{-\frac{3}{4}-1}$ $\therefore f'(x)=-\dfrac{3}{8}x^{-\frac{7}{4}}$ $\therefore f'(x)=-\dfrac{3}{8x^{\frac{7}{4}}}$	✓ $\dfrac{1}{2}x^{-\frac{3}{4}}$ ✓ $-\dfrac{3}{8}x^{-\frac{7}{4}}$ ✓ $-\dfrac{3}{8x^{\frac{7}{4}}}$ (3)

Question 4

4.	$f(x)=x^2-6x+5$ $x_T=2$ $y_T=f(2)=(2)^2-6(2)+5$ $\therefore y_T=-3$ $m_T=f'(x)=2x-6$ $\therefore f'(2)=2(2)-6=-2$ $y-y_T=m_t(x-x_T)$ $\therefore y-(-3)=-2(x-2)$ $\therefore y+3=-2x+4$ $\therefore y=-2x+1$	✓ $f(2)=-2$ ✓ $f'(x)=2x-6$ ✓ $f'(2)=2(2)-6=-2$ ✓ $y-(-3)=-2(x-2)$ ✓ $y=-2x+1$ (5)

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Solutions to Calculus Applications

Question 1

***y*-intercept:** $(0;-4)$	✓ $(0;-4)$
***x*-intercepts:**	
$0=2x^3-6x-4$	✓ $0=2x^3-6x-4$
$\therefore 0=x^3-3x-2$	✓ $0=(x+1)(x^2-x-2)$
$\therefore 0=(x+1)(x^2-x-2)$ (using the factor theorem)	✓ $0=(x+1)(x-2)(x+1)$
$\therefore 0=(x+1)(x-2)(x+1)$	✓ $(-1;0)$ $(2;0)$
$\therefore x=-1$ or $x=2$	
$(-1;0)$ $(2;0)$	
Stationary points:	
$f(x)=2x^3-6x-4$	
$\therefore f'(x)=6x^2-6$	
$\therefore 0=6x^2-6$ (At a turning point, $f'(x)=0$)	
$\therefore 0=x^2-1$	
$\therefore x=\pm 1$	✓ $f'(x)=6x^2-6$
$f(1)=-8$	✓ $0=6x^2-6$
$f(-1)=0$	✓ $x=\pm 1$
Turning points are $(1;-8)$ and $(-1;0)$	✓ $(1;-8)$ and $(-1;0)$
Point of inflection:	
$f'(x)=6x^2-6$	
$\therefore f''(x)=12x$	
$\therefore 0=12x$	
$\therefore x=0$	✓ $f''(x)=12x$
$f(0)=-4$	✓ $(0;-4)$
Point of inflection at $(0;-4)$	
Alternatively:	
The *x*-coordinate of the point of inflection can be determined by adding the x-coordinates of the turning points and then dividing the result by 2.	✓ $\frac{(1)+(-1)}{2}$
$x=\frac{(1)+(-1)}{2}=0$	✓ $x=0$

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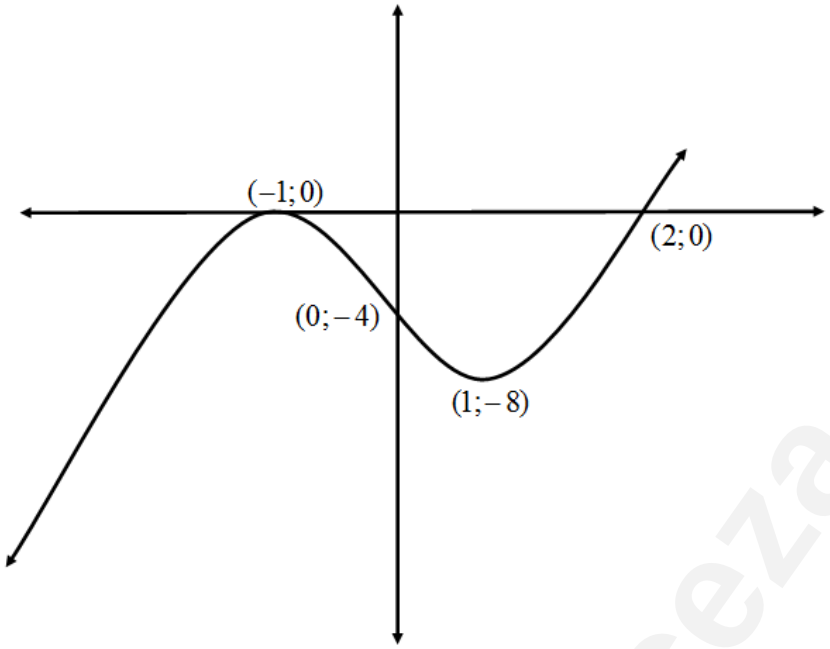
Sketch of $f(x) = 2x^3 - 6x - 4$



Question 2

	$f(x) = -2x^3 + ax^2 + bx + c$ $f'(x) = -6x^2 + 2ax + b$ $= -6(x-5)(x-2)$ $= -6(x^2 - 7x + 10)$ $= -6x^2 + 42x - 60$ $2a = 42$ $a = 21$ $b = -60$ $f(5) = -2(5)^3 + 21(5)^2 - 60(5) + c$ $18 = -25 + c$ $c = 43$	**Note:** A candidate who substitutes the values of a, b and c and then checks (by substitution) that $T(2;-9)$ and $S(5;18)$ lie on the curve: award max 2/7 marks OR $f(2) = -2(2)^3 + 21(2)^2 - 60(2) + c$ $-9 = -52 + c$ $c = 43$	✓ $f'(x) = -6x^2 + 2ax + b$ ✓✓ $-6(x-5)(x-2)$ ✓ $b = -60$ ✓ $2a = 42$ ✓ subs (5 ; 18) or (2 ; -9) ✓ $c = 43$ (7)

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2.2	$f'(x) = -6x^2 + 42x - 60$ $m_{tan} = -6(1)^2 + 42(1) - 60$ $= -24$ $f(1) = -2(1)^3 + 21(1)^2 - 60(1) + 43$ $= 2$ Point of contact is (1 ; 2) $y - 2 = -24(x-1)$ $y = -24x + 26$ **OR** $y = -24x + c$ $2 = -24(1) + c$ $c = 26$ $y = -24x + 26$	✓ $f'(x) = -6x^2 + 42x - 60$ ✓ subs $f'(1)$ ✓ $m_{tan} = -24$ ✓ $f(1) = 2$ ✓ $y - 2 = -24(x-1)$ OR $y = -24x + 26$ (5)
2.3	$f'(x) = -6x^2 + 42x - 60$ $f''(x) = -12x + 42$ $0 = -12x + 42$ $x = \frac{7}{2}$	✓ $f''(x) = -12x + 42$ ✓ $x = \frac{7}{2}$ (2) ✓ $x = \frac{2+5}{2}$

Question 3

3.1	$V(0) = 100 - 4(0)$ $= 100$ litres	✓ answer (1)
3.2	Rate in – rate out $= 5 - k$ l/min $V'(t) = -4$ l/min	✓ $5 - k$ ✓ -4 ✓ units stated once (3)
3.3	$5 - k = -4$ $k = 9$ l/min OR Volume at any time t = initial volume + incoming total – outgoing total $100 + 5t - kt = 100 - 4t$ $5t - kt = -4t$ $9t - kt = 0$ $t(9 - k) = 0$ At 1 minute from start, $t = 1$, $9 - k = 0$, so $k = 9$ OR Since $\frac{dV}{dt} = -4$, the volume of water in the tank is decreasing by 4 litres every minute. So k is greater than 5 by 4, that is, $k = 9$. **Note: Answer only: award 2/2 marks**	✓ $5 - k = -4$ ✓ $k = 9$ (2) ✓ $100 + 5t - kt = 100 - 4t$ ✓ $k = 9$ (2) ✓✓ $k = 9$ (2) [6]

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Solutions to Linear Programming

Question 1 Words into equations or inequalities Answers:

$600x + 200y \geq 12000$
$200x + 200y \geq 800$
$60x + 160y \geq 480$

Implicit constraints: $x, y \in \mathbb{N}_0$

Question 2 Words into equations or inequalities

Answers:
$x + y < 60$
$x + y \geq 35$
$6 \leq y \leq 14$

Implicit constraints: $x, y \in \mathbb{N}_0$

Question 3 Words into equations or inequalities

$x + y \geq 10$
$y \leq 12$
$8x + 4y \leq 72$

Implicit constraints: $x, y \in \mathbb{R}$

Question 4 Words into equations or inequalities

$\boldsymbol{x \leq 100}$
$y \leq 60$
$x + y \leq 140$
$x \geq 10$
$y \geq 30$
$y \leq 2x$

Implicit constraints: $x, y \in \mathbb{R}$

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Question 5 Model into a series of Inequalities:

BC: $\frac{x}{60}+\frac{y}{60}\leq 1$ which can also be written as: $y\leq -x+60$
AE and CD $10\leq x\leq 30$
AB $y\leq 40$
ED $y\geq \frac{1}{2}x$

Question 6 Model into a series of Inequalities:

A: $\frac{x}{20}+\frac{y}{40}\geq 1$ which can also be written as: $y\geq -2x+40$
B: $\frac{x}{50}+\frac{y}{50}\leq 1$ which can also be written as: $y\geq -x+50$
C: $x\leq 30$
D: $y\geq 30$
E: $y\leq 10$

Question 7 Model into a series of Inequalities:

ML $\frac{x}{80}+\frac{y}{80}\leq 1$ which can also be written as: $y\leq -x+80$
NK $\frac{x}{30}+\frac{y}{60}\geq 1$ which can also be written as: $y\geq -2x+60$
MN $x\geq 10$
KL $y\geq \frac{1}{6}x$+10

Question 8 Maximising or Minimising from a Model:

8.1 $P=750x+1250y$

8.2

x	y	Profit
8	2	8 500
2	8	11 500
5	8	13 750

The Maximum profit will be if 5 plasma and 8 flat screen televisions are produced.

Question 9 Maximising or Minimising from a Model:

9.1 $P=200x+150y$

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9.2

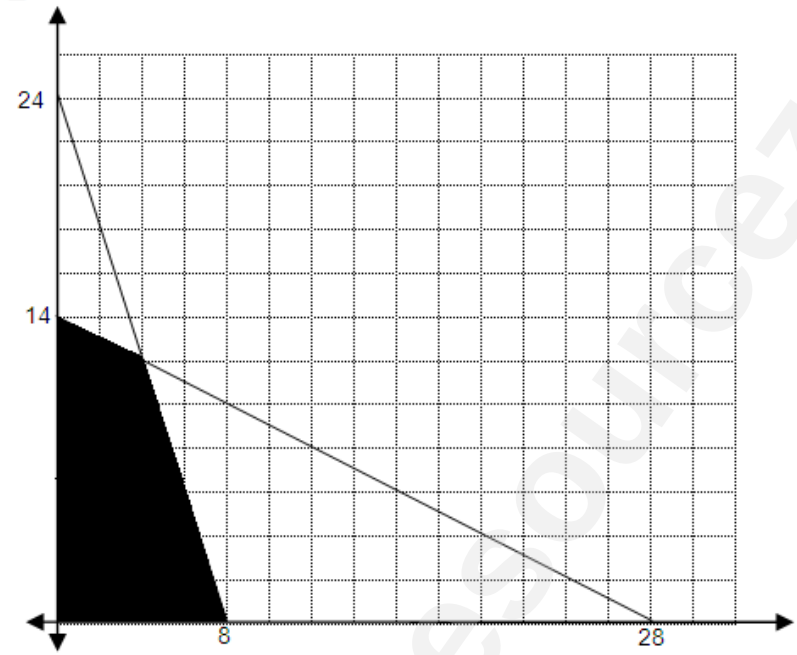
x	y	Profit
10	40	8 000
20	20	7 000
30	20	9 000

Maximum profit if 30 litres of X and 20 litres of Y are sold.

Question 10 'Complete' Question:

10.1 $3x + y \leq 24$
$x + 2y \leq 28$

10.2



10.3 8 x-type and 14 y-type
10.4 4 x-type and 12 y-type. Total 16 braai stands.

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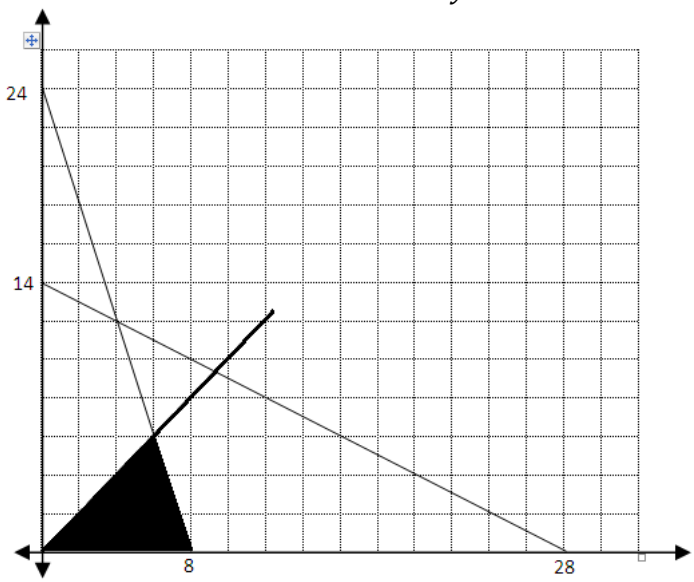
10.5 Additional constraint: $x \geq y$

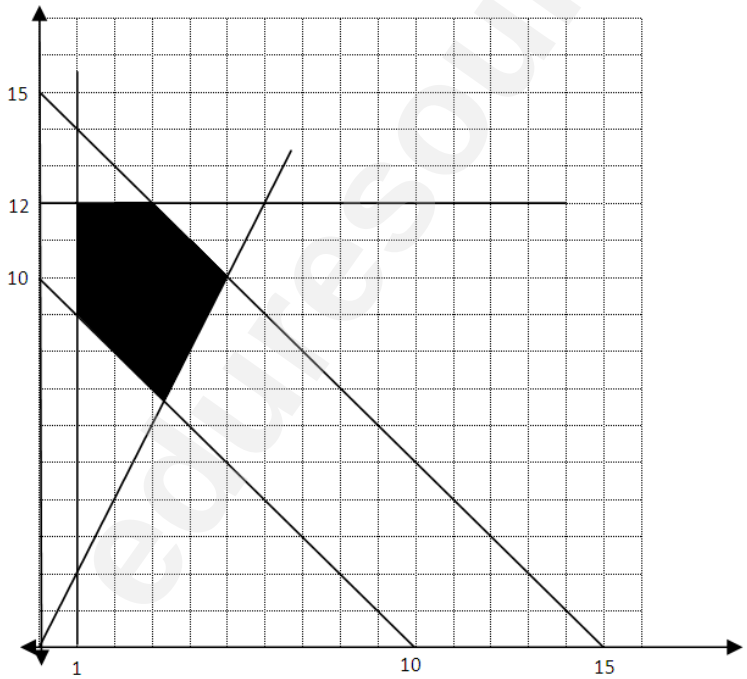


Maximum number is 6 x-type and 6 y-type.
Machine time will be: $6 \cdot (1) + 6 \cdot (2) = 18$ hours

Question 11 'Complete' Question:

11.1 $x \geq 1$ $\quad x + y \geq 10$ $\quad x + y \leq 15$
$y \leq 12$ $\quad x, y \in \mathbb{N}$

11.2



11.3 No it could not. The point (5 ; 8) does not lie within the feasible region.

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11.4 $Income = 600x + 900y$

x	y	Income
1	10	9 600
1	12	11 400
3	12	12 600
15	0	9 000
10	0	6 000

The maximum income per night will come from 3 single and 12 double rooms.

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