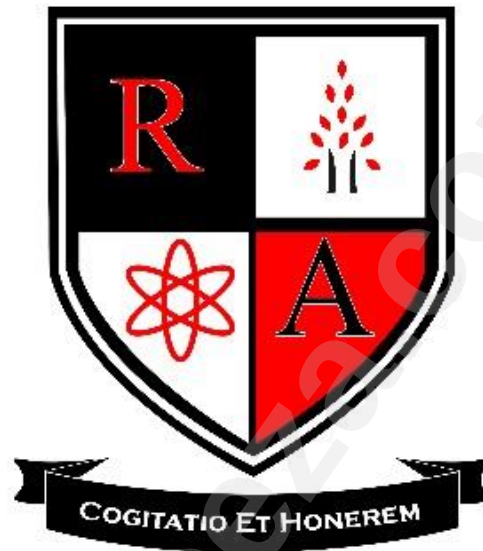




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MATHEMATICS

REDWOOD ACADEMY



JUNE REVISION
NSC, GDE AND KZN
PAST PAPERS AND MEMOS

NAME: _____



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS

MATHEMATICS P1

2018

MARKS: 150

TIME: 3 hours

This question paper consists of 9 pages and 1 information sheet.

QUESTION 11.1 Solve for x :

1.1.1 $(3x - 1)(x + 4) = 0$ (2)

1.1.2 $2x^2 + 9x - 14 = 0$ (correct to TWO decimal places) (4)

1.1.3 $\sqrt{3 - 26x} = 3x$ (4)

1.1.4 $(x - 1)(x - 4) > x + 11$ (5)

1.2 Simplify fully:

$$\frac{\sqrt{16x^7} - \sqrt{25x^7}}{\sqrt{x}}$$
 (3)

1.3 Solve simultaneously for x and y :

$xy = 9$ and $x - 2y - 3 = 0$ (5)

1.4 Prove that $x^2 + 2xy + 2y^2$ cannot be negative for $x, y \in \mathbb{R}$. (4)

[27]

QUESTION 2

2.1 Given the quadratic pattern: 5 ; 10 ; 17 ; 26 ; ...

2.1.1 Write down the next TWO terms of the pattern. (2)

2.1.2 Determine the formula for the n^{th} term of the pattern. (4)

2.1.3 Which term of the pattern will have a value of 1 765? (4)

2.2 The first 24 terms of an arithmetic series are: $35 + 42 + 49 + \dots + 196$.

Calculate the sum of ALL natural numbers from 35 to 196 that are NOT divisible by 7. (5)

[15]

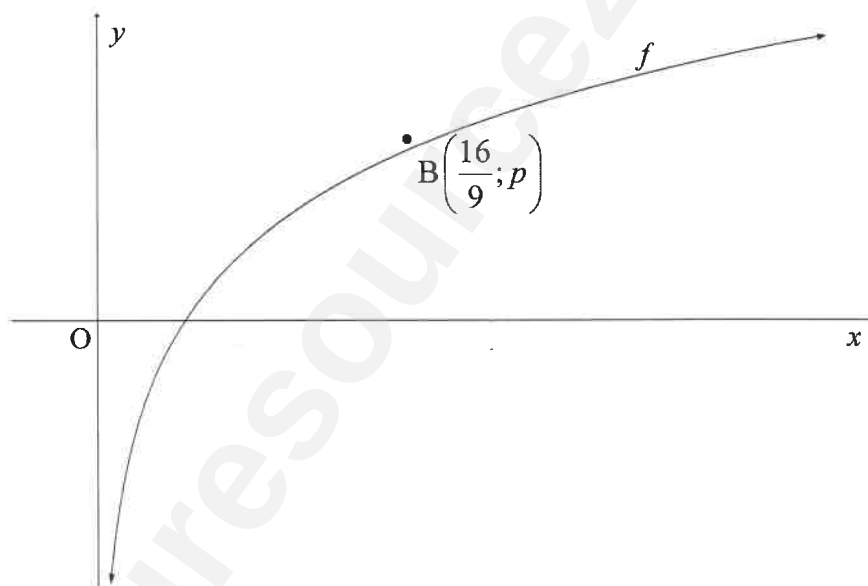
QUESTION 3

Themba is planning a bicycle trip from Cape Town to Pretoria. The total distance covered during the trip will be 1 500 km. He plans to travel 100 km on the first day. For every following day he plans to cover 94% of the distance he covered the previous day.

- 3.1 What distance will he cover on day 3 of the trip? (2)
- 3.2 On what day of the trip will Themba pass the halfway point? (4)
- 3.3 Themba must cover a certain percentage of the previous day's distance to ensure that he will eventually reach Pretoria. Calculate ALL possible value(s) of this percentage. (3)
- [9]

QUESTION 4

The graph of $f(x) = \log_{\frac{4}{3}} x$ is drawn below. $B\left(\frac{16}{9}; p\right)$ is a point on f .

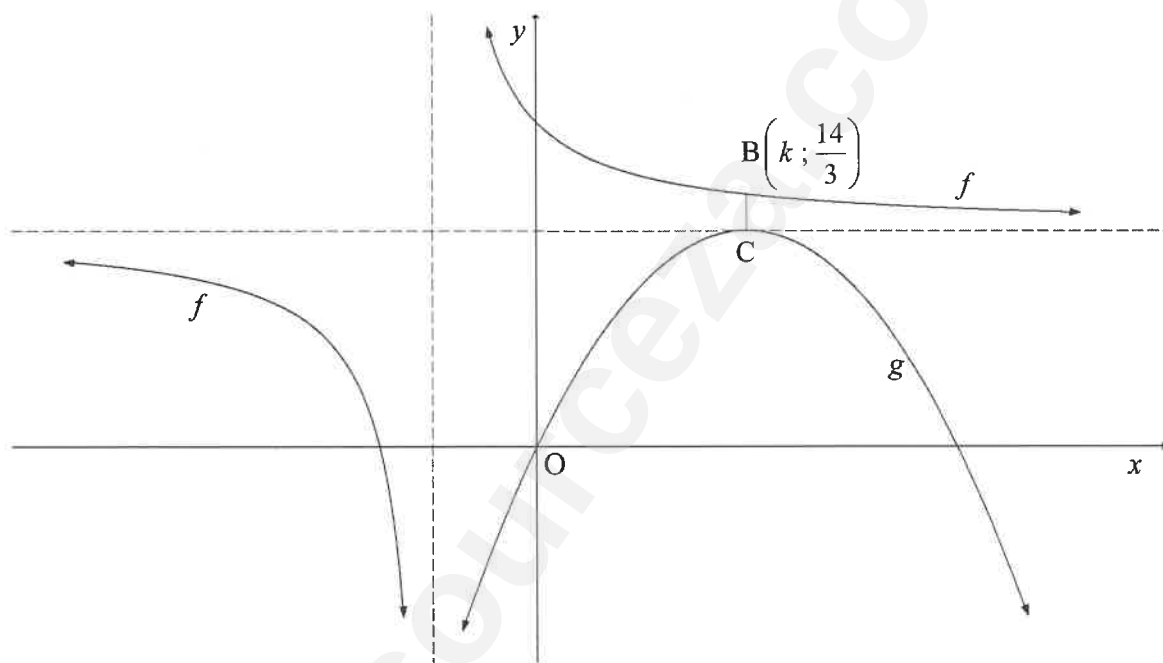


- 4.1 For which value(s) of x is $\log_{\frac{4}{3}} x \leq 0$? (2)
- 4.2 Determine the value of p , without the use of a calculator. (3)
- 4.3 Write down the equation of the inverse of f in the form $y = \dots$ (2)
- 4.4 Write down the range of $y = f^{-1}(x)$. (2)
- 4.5 The function $h(x) = \left(\frac{3}{4}\right)^x$ is obtained after applying two reflections on f .
Write down the coordinates of B'' , the image of B on h . (2)
- [11]

QUESTION 5

The graphs of $f(x) = \frac{2}{x+1} + 4$ and parabola g are drawn below.

- C, the turning point of g , lies on the horizontal asymptote of f .
- The graph of g passes through the origin.
- B $\left(k; \frac{14}{3}\right)$ is a point on f such that BC is parallel to the y -axis.



- 5.1 Write down the domain of f . (2)
- 5.2 Determine the x -intercept of f . (2)
- 5.3 Calculate the value of k . (3)
- 5.4 Write down the coordinates of C. (2)
- 5.5 Determine the equation of g in the form $y = a(x + p)^2 + q$. (3)
- 5.6 For which value(s) of x will $\frac{f(x)}{g(x)} \leq 0$? (4)
- 5.7 Use the graphs of f and g to determine the number of real roots of $\frac{2}{x} - 5 = -(x - 3)^2 - 5$. Give reasons for your answer. (4)

[20]

QUESTION 6

- 6.1 Calculate how many years it will take for the value of a truck to decrease to 50% of its original value if depreciation is calculated at 15% per annum using the reducing-balance method. (4)
- 6.2 Every month Tshepo deposited R1 500 for his retirement into an account that paid interest at a rate of 9,2% per annum, compounded monthly. Tshepo made his first instalment on his 23rd birthday and the last instalment one month before his 55th birthday. Calculate how much money he had in the account on his 55th birthday. (5)
- 6.3 Abram has R150 000 to invest in two separate accounts. One account pays interest at a rate of 8,4% per annum, compounded quarterly, and the other account at a rate of 9,6% per annum, compounded monthly. How much money should he invest in each account so that he will collect the same amount from each account at the end of 12 years? (6)
[15]

QUESTION 7

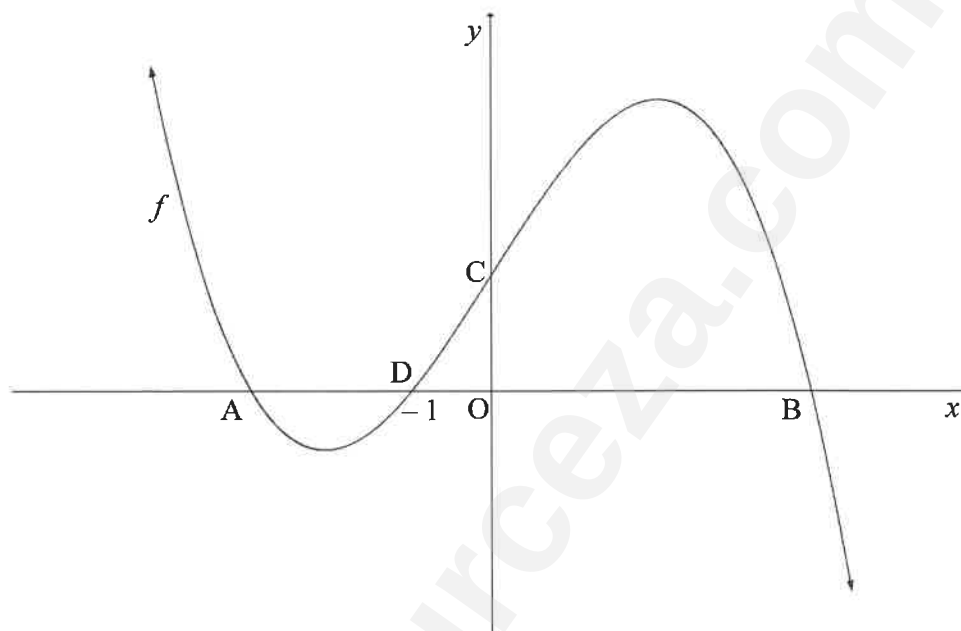
- 7.1 Given: $f(x) = 2 - 3x^2$
Determine $f'(x)$ from first principles. (5)
- 7.2 Determine:
- 7.2.1 $D_x[(4x + 5)^2]$ (3)
- 7.2.2 $\frac{dy}{dx}$ if $y = \sqrt[4]{x} + \frac{x^2 - 8}{x^2}$ (4)
[12]

QUESTION 8

The graph of $f(x) = -x^3 + 13x + 12$ is sketched below.

A, B and D(-1 ; 0) are the x -intercepts of f .

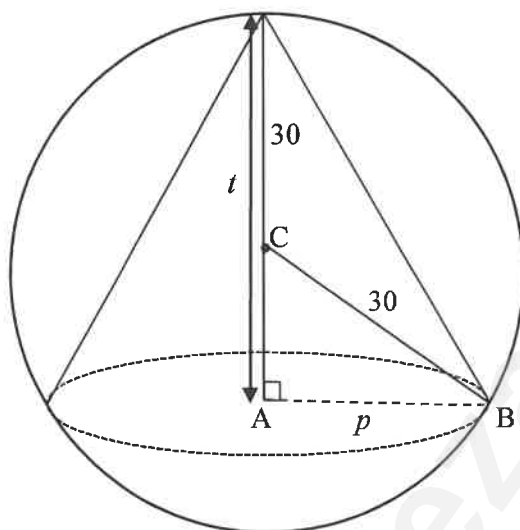
C is the y -intercept of f .



- 8.1 Write down the coordinates of C. (1)
- 8.2 Calculate the coordinates of A and B. (5)
- 8.3 Determine the point of inflection of g if it is given that $g(x) = -f(x)$. (4)
- 8.4 Calculate the value(s) of x for which the tangent to f is parallel to the line $y = -14x + c$. (4)
- [14]

QUESTION 9

A right circular cone with radius p and height t is machined (cut out) from a solid sphere (with centre C) with a radius of 30 cm, as shown in the sketch.



$$\text{Sphere: } V = \frac{4}{3}\pi r^3$$

$$\text{Cone: } V = \frac{1}{3}\pi r^2 h$$

9.1 From the given information, express the following:

9.1.1 AC in terms of t . (1)

9.1.2 p^2 , in its simplest form, in terms of t . (3)

9.2 Show that the volume of the cone can be written as $V(t) = 20\pi t^2 - \frac{1}{3}\pi t^3$. (1)

9.3 Calculate the value of t for which the volume of the cone will be a maximum. (3)

9.4 What percentage of the sphere was used to obtain this cone having maximum volume? (4)
[12]

QUESTION 10

Ben, Nhlanhla, Owen, Derick and 6 other athletes take part in a 100 m race. Each athlete will be allocated a lane in which to run. The athletic track has 10 lanes.

- 10.1 In how many different ways can all the athletes be allocated a lane? (2)
- 10.2 Four athletes taking part in the event insist on being placed in lanes next to each other. In how many different ways can the lanes be allocated to the athletes now? (3)
- 10.3 If lanes are randomly allocated to athletes, determine the probability that Ben will be placed in lane 1, Nhlanhla in lane 3, Owen in lane 5 and Derick in lane 7. (2)
- [7]

QUESTION 11

A survey on their preference of exercise was conducted among 140 people in two age groups. The information is summarised below.

AGE	TENNIS	RUNNING	GYM	TOTAL
35 years and younger	a	28	c	80
Older than 35 years	b	21	d	60
	21	49	70	140

- 11.1 If it is given that preferring to play tennis and age are independent of each other, determine the value of a . (3)
- 11.2 If it is given that $a = 12$, determine the probability that a randomly selected person prefers going to the gym or is in the age group 35 years and younger. (5)
- [8]

TOTAL: 150



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS

MATHEMATICS P2

2018

MARKS: 150

TIME: 3 hours

**This question paper consists of 13 pages, 1 information sheet
and an answer book of 27 pages.**

QUESTION 1

The monthly profit (in thousands of rands) made by a company in a year is given in the table below.

110	112	156	164	167	169
171	176	192	228	278	360

- 1.1 Calculate the:
- 1.1.1 Mean profit for the year (3)
- 1.1.2 Median profit for the year (1)
- 1.2 On the number line provided in the ANSWER BOOK, draw a box and whisker diagram to represent the data. (2)
- 1.3 Hence, determine the interquartile range of the data. (1)
- 1.4 Comment on the skewness in the distribution of the data. (1)
- 1.5 For the given data:
- 1.5.1 Calculate the standard deviation (1)
- 1.5.2 Determine the number of months in which the profit was less than one standard deviation below the mean (2)
- [11]

QUESTION 2

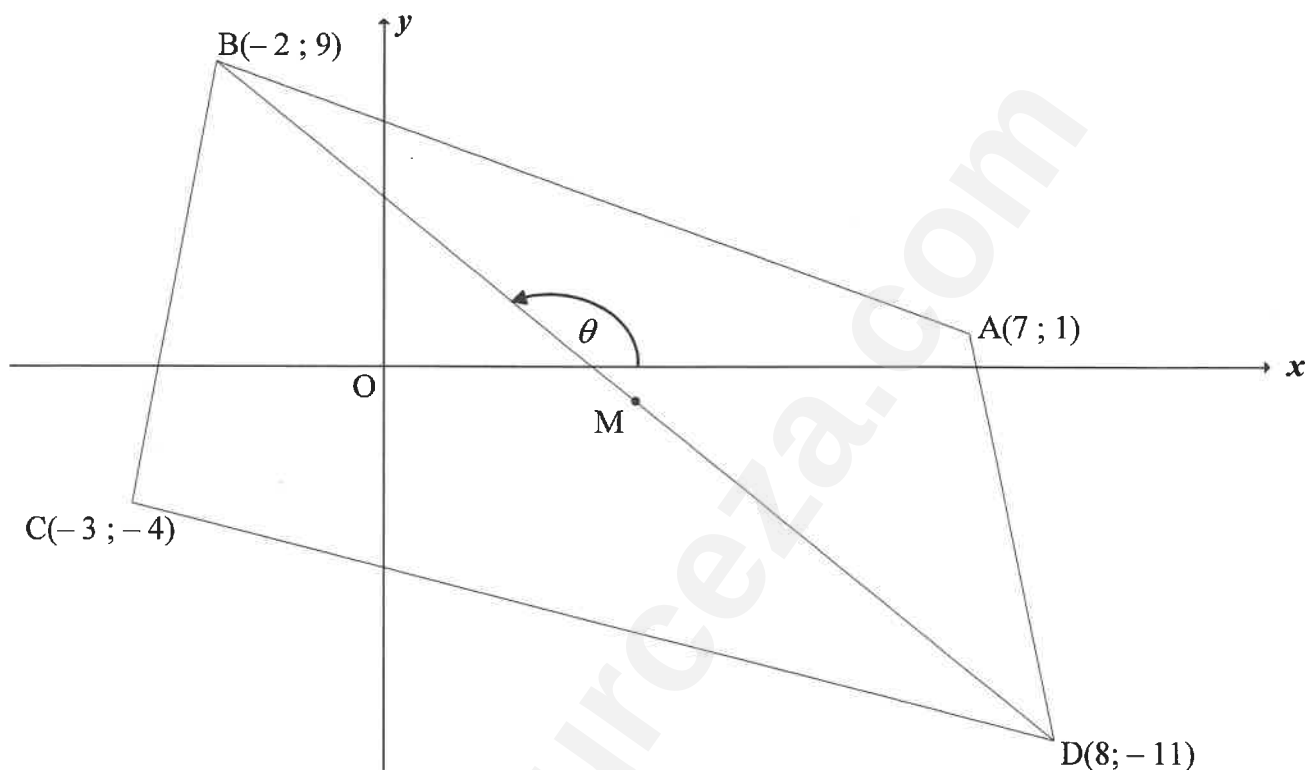
It is said that the number of times that a cricket chirps in a minute gives a very good indication of the air temperature (in °C). The table below shows the information recorded during an observation study.

CHIRPS PER MINUTE	AIR TEMPERATURE IN °C
32	8
40	10
52	12
76	15
92	17
112	20
128	25
180	28
184	30
200	35

- 2.1 Represent the data above on the grid provided in the ANSWER BOOK. (3)
- 2.2 Explain why the claim, 'gives a very good indication', is TRUE. (1)
- 2.3 Determine the equation of the least squares regression line of the data. (3)
- 2.4 Predict the air temperature (in °C) if a cricket chirps 80 times a minute. (2)
- [9]

QUESTION 3

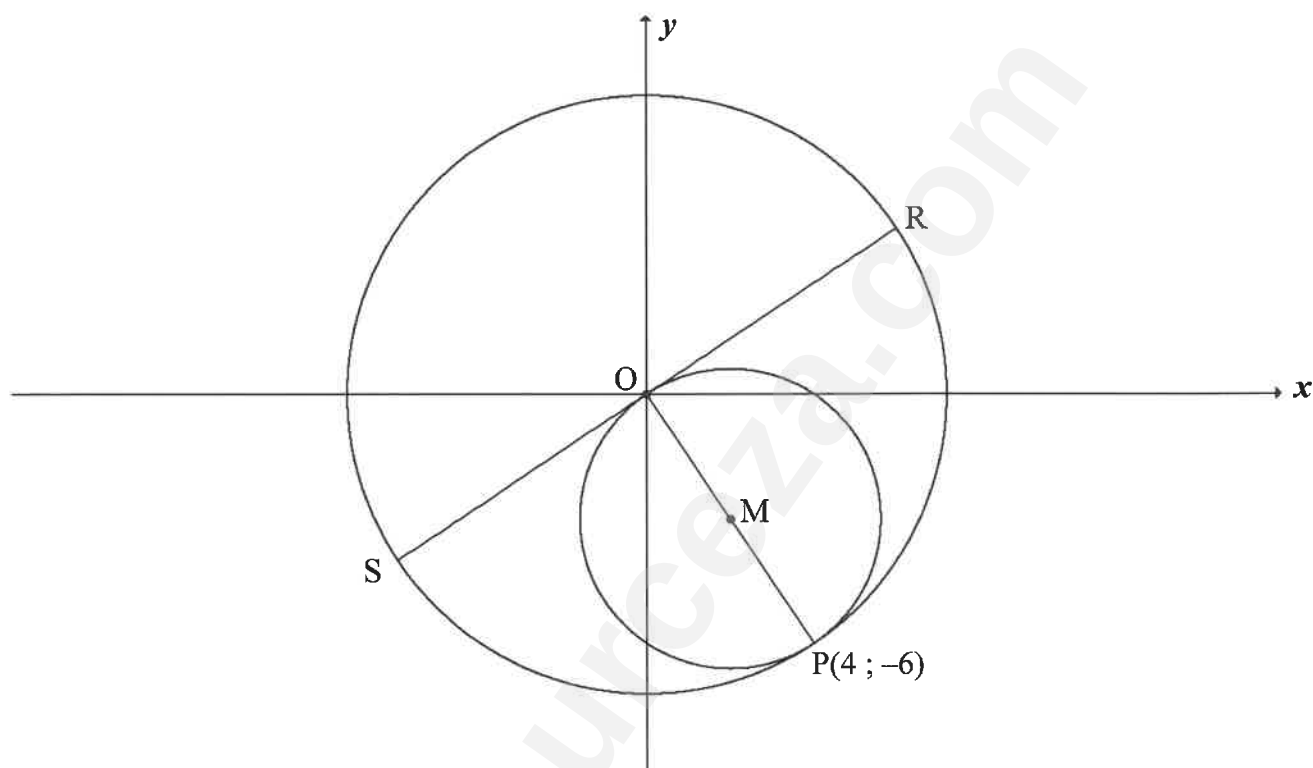
In the diagram, ABCD is a quadrilateral having vertices $A(7; 1)$, $B(-2; 9)$, $C(-3; -4)$ and $D(8; -11)$. M is the midpoint of BD.



- 3.1 Calculate the gradient of AC. (2)
- 3.2 Determine:
- 3.2.1 The equation of AC in the form $y = mx + c$ (2)
- 3.2.2 Whether M lies on AC (4)
- 3.3 Prove that $BD \perp AC$. (3)
- 3.4 Calculate:
- 3.4.1 θ , the inclination of BD (2)
- 3.4.2 The size of $\hat{C}BD$ (3)
- 3.4.3 The length of AC (2)
- 3.4.4 The area of ABCD (5)
- [23]

QUESTION 4

In the diagram, a circle having centre at the origin passes through $P(4 ; -6)$. PO is the diameter of a smaller circle having centre at M . The diameter RS of the larger circle is a tangent to the smaller circle at O .



- 4.1 Calculate the coordinates of M . (2)
- 4.2 Determine the equation of:
- 4.2.1 The large circle (2)
- 4.2.2 The small circle in the form $x^2 + y^2 + Cx + Dy + E = 0$ (3)
- 4.2.3 The equation of RS in the form $y = mx + c$ (3)
- 4.3 Determine the length of chord NR , where N is the reflection of R in the y -axis. (4)
- 4.4 The circle with centre at M is reflected about the x -axis to form another circle centred at K . Calculate the length of the common chord of these two circles. (3)

[17]

QUESTION 5

5.1 In $\triangle MNP$, $\hat{N} = 90^\circ$ and $\sin M = \frac{15}{17}$.

Determine, **without using a calculator**:

5.1.1 $\tan M$ (3)

5.1.2 The length of NP if $MP = 51$ (2)

5.2 Simplify to a single term: $\cos(x - 360^\circ) \cdot \sin(90^\circ + x) + \cos^2(-x) - 1$ (4)

5.3 Consider: $\sin(2x + 40^\circ) \cos(x + 30^\circ) - \cos(2x + 40^\circ) \sin(x + 30^\circ)$

5.3.1 Write as a single trigonometric term in its simplest form. (2)

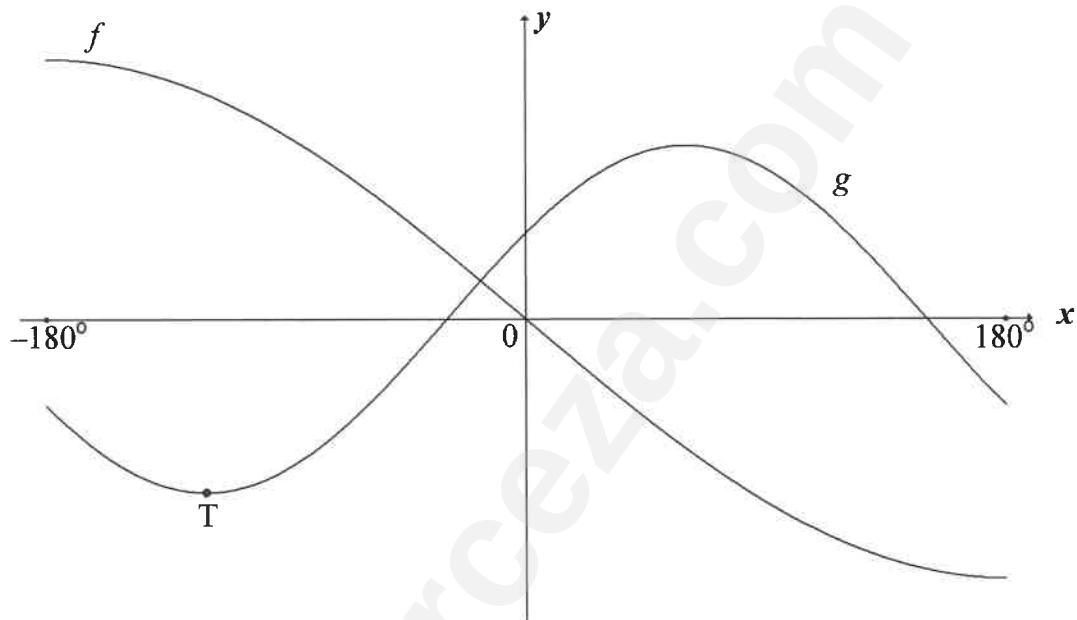
5.3.2 Determine the general solution of the following equation:

$$\sin(2x + 40^\circ) \cos(x + 30^\circ) - \cos(2x + 40^\circ) \sin(x + 30^\circ) = \cos(2x - 20^\circ) \quad (7)$$

[18]

QUESTION 6

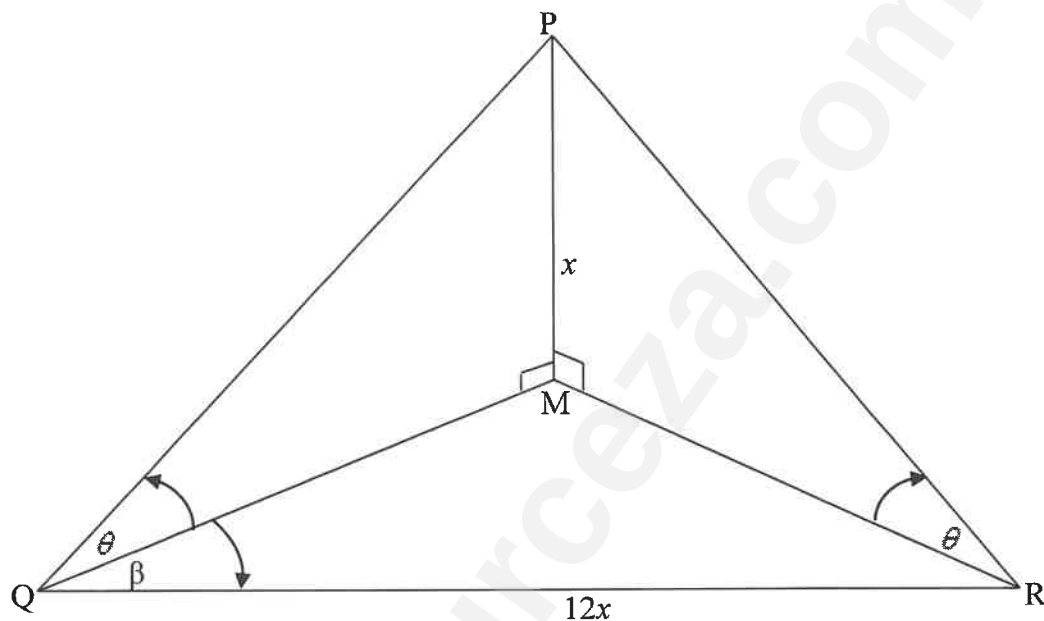
In the diagram, the graphs of $f(x) = -3 \sin \frac{x}{2}$ and $g(x) = 2 \cos(x - 60^\circ)$ are drawn in the interval $x \in [-180^\circ; 180^\circ]$. $T(p; q)$ is a turning point of g with $p < 0$.



- 6.1 Write down the period of f . (1)
- 6.2 Write down the range of g . (2)
- 6.3 Calculate $f(p) - g(p)$. (3)
- 6.4 Use the graphs to determine the value(s) of x in the interval $x \in [-180^\circ; 180^\circ]$ for which:
- 6.4.1 $g(x) > 0$ (3)
- 6.4.2 $g(x) \cdot g'(x) > 0$ (4)
- [13]

QUESTION 7

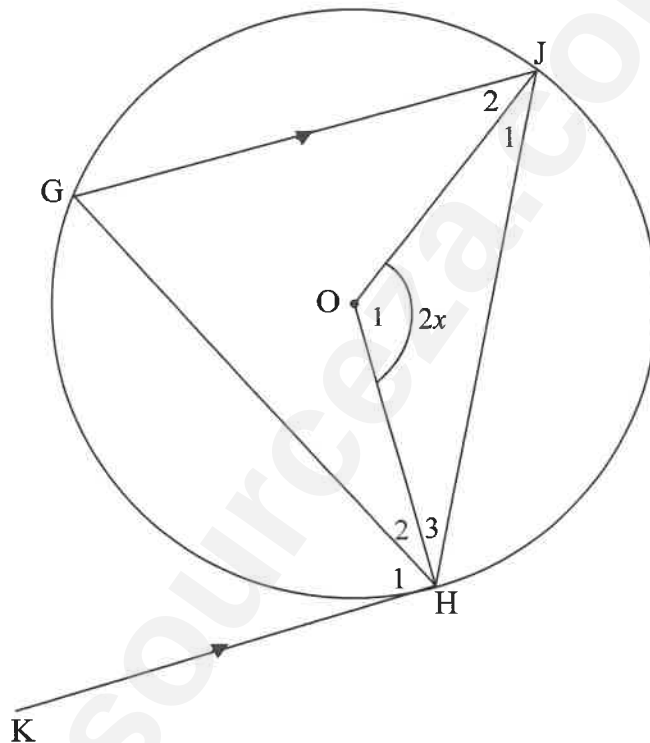
The captain of a boat at sea, at point Q, notices a lighthouse PM directly north of his position. He determines that the angle of elevation of P, the top of the lighthouse, from Q is θ and the height of the lighthouse is x metres. From point Q the captain sails $12x$ metres in a direction β degrees east of north to point R. From point R, he notices that the angle of elevation of P is also θ . Q, M and R lie in the same horizontal plane.



- 7.1 Write QM in terms of x and θ . (2)
- 7.2 Prove that $\tan \theta = \frac{\cos \beta}{6}$. (4)
- 7.3 If $\beta = 40^\circ$ and QM = 60 metres, calculate the height of the lighthouse to the nearest metre. (3)
- [9]

QUESTION 8

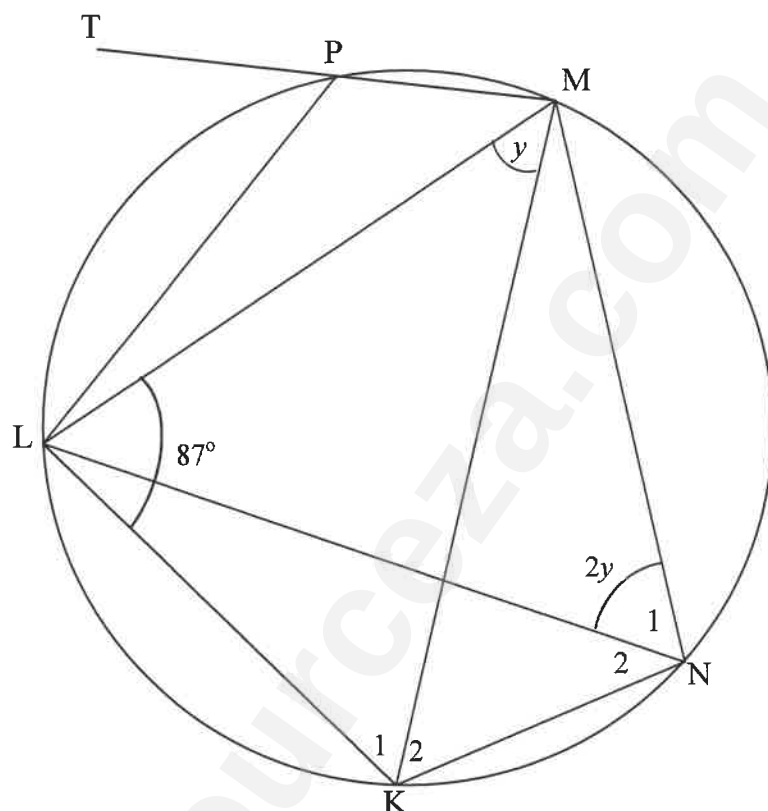
- 8.1 In the diagram, O is the centre of the circle. Radii OH and OJ are drawn. A tangent is drawn from K to touch the circle at H . $\triangle HGJ$ is drawn such that $GJ \parallel KH$. $\hat{O}_1 = 2x$.



- 8.1.1 Name, giving reasons, THREE angles, each equal to x . (5)
- 8.1.2 Prove that $\hat{H}_2 = \hat{H}_3$. (3)

8.2

In the diagram, KLMN is a cyclic quadrilateral with $\angle KLM = 87^\circ$. Diagonals LN and MK are drawn. P is a point on the circle and MP is produced to T, a point outside the circle. Chord LP is drawn. $\angle LMK = y$ and $\angle N_1 = 2y$.



8.2.1 Name, giving a reason, another angle equal to y . (2)

8.2.2 Calculate, giving reasons, the size of:

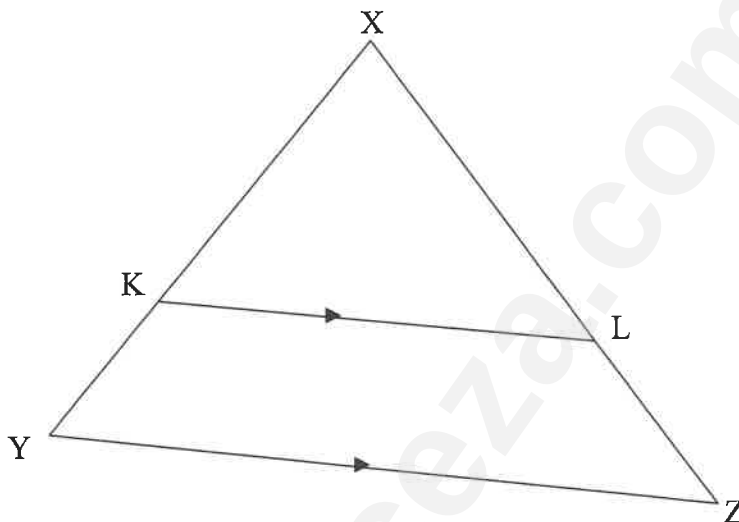
(a) y (3)

(b) $\angle TPL$ (2)

[15]

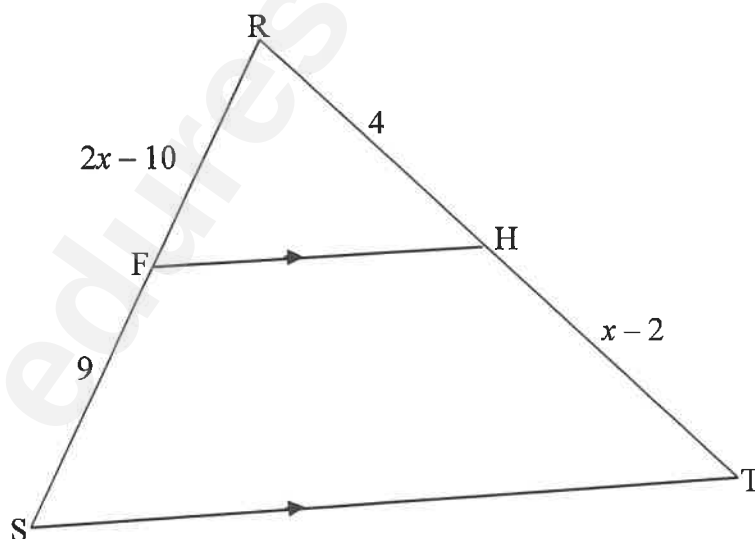
QUESTION 9

- 9.1 Use the diagram to prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally, that is prove that $\frac{XK}{KY} = \frac{XL}{LZ}$.



(5)

- 9.2 In $\triangle RST$, F is a point on RS and H is a point on RT such that $FH \parallel ST$. $RF = 2x - 10$, $FS = 9$, $RH = 4$ and $HT = x - 2$.



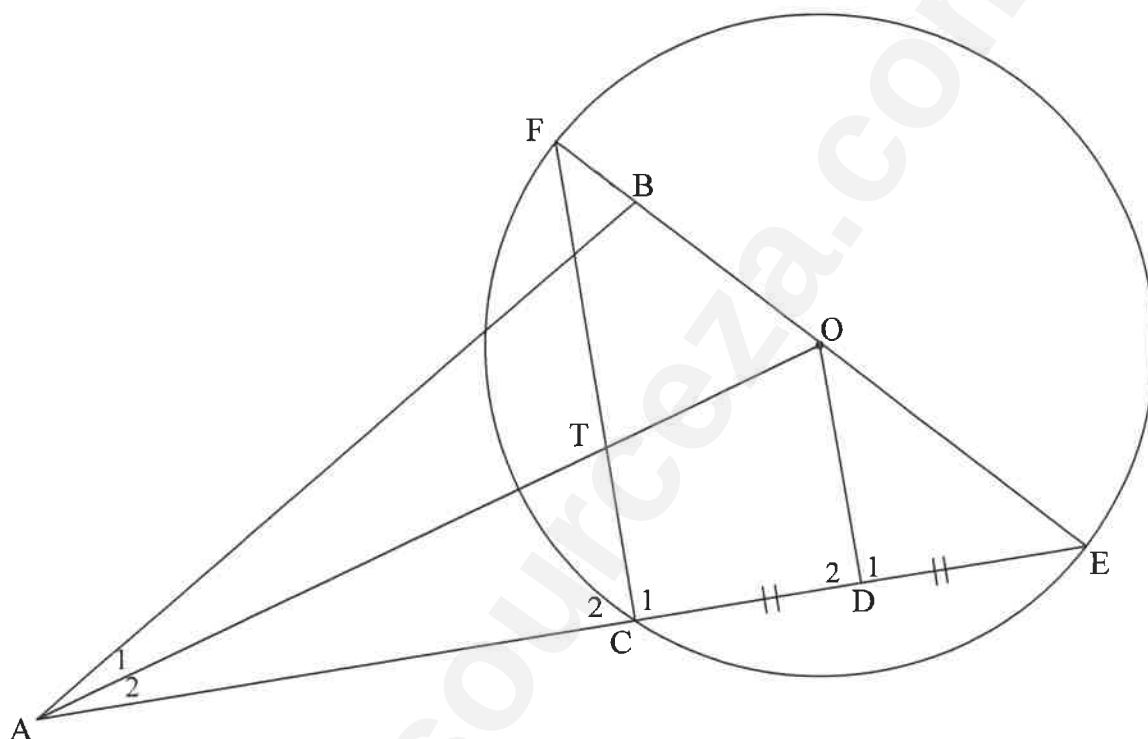
- 9.2.1 Determine, giving a reason, the value of x . (5)

- 9.2.2 Determine the ratio: $\frac{\text{area } \triangle RFH}{\text{area } \triangle RST}$. (4)

[14]

QUESTION 10

In the diagram, $FBOE$ is a diameter of a circle with centre O . Chord EC produced meets line BA at A , outside the circle. D is the midpoint of CE . OD and FC are drawn. $AFBC$ is a cyclic quadrilateral.



10.1 Prove, giving reasons, that:

10.1.1 $FC \parallel OD$ (5)

10.1.2 $\angle DOE = \angle BAE$ (4)

10.1.3 $AB \times OF = AE \times OD$ (7)

10.2 If it is further given that $AT = 3TO$, prove that $5CE^2 = 2BE \cdot FE$ (5)

[21]

TOTAL: 150



education

Department:
Education
PROVINCE OF KWAZULU-NATAL

MATHEMATICS P1

COMMON TEST

JUNE 2018

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

MARKS: 125

TIME: 2.5 hours

N.B. This question paper consists of 6 pages and an information sheet.

QUESTION 1

1.1 Solve for x : (Answer correct to TWO decimal places where necessary)

1.1.1 $x \left(3 - \frac{1}{2}x \right) = 0$ (2)

1.1.2 $x^2 + 8x = 5$ (4)

1.1.3 $2x - \sqrt{10+x} = 1$ (5)

1.2 Solve the following equations simultaneously:

$y - x + 1 = 0$ and $x^2 + xy = 3$ (6)

1.3 How many solutions does the equation $x \cdot 3^x - x + 33x^3 = 0$, have?
Show all working.

(3)
[20]

QUESTION 2

A quadratic sequence has x ; 4; 8; y ; ... as its first four terms.

2.1 Determine the value(s) of x and y if its second difference is 4. (2)

2.2 Determine the n^{th} term of the sequence. (4)

2.3 Which term is equal to 12964? (3)
[9]

QUESTION 3

3.1 In an arithmetic sequence the eighth term is twice the fourth term. The sum of the first three terms is 12. Determine the first term. (5)

3.2 Evaluate: $\sum_{k=1}^{12} 3 \cdot 2^{k-1}$ (3)

3.3 Determine the value(s) of x for which the series

$2 + \frac{2}{3}(x+5) + \frac{2}{9}(x+5)^2 + \dots$ will converge. (3)

[11]

QUESTION 4

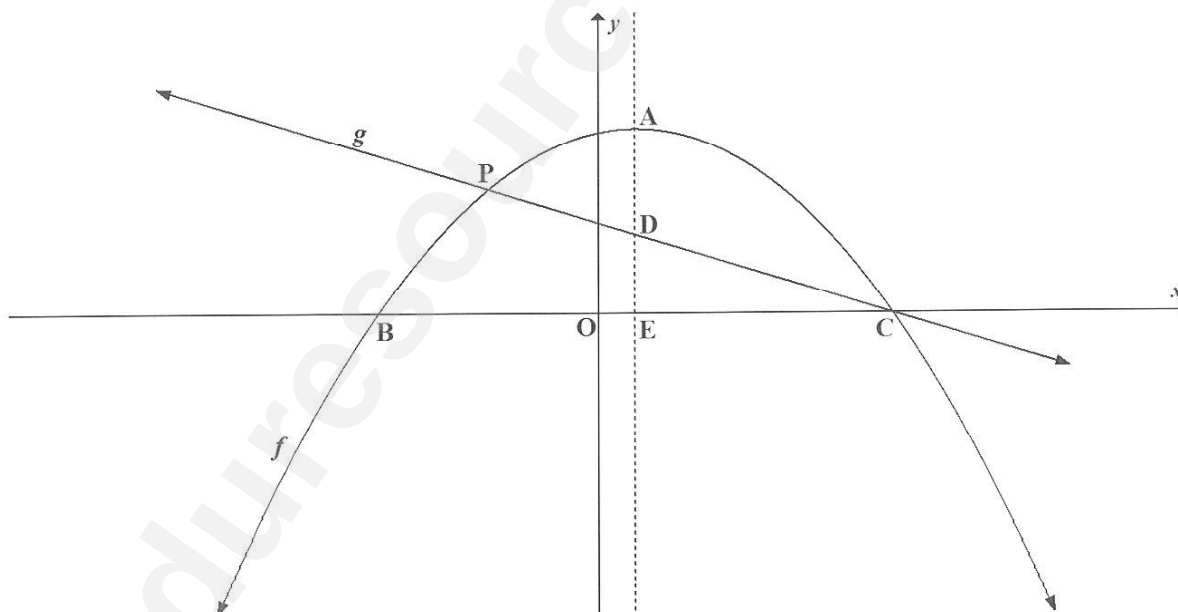
Given: $g(x) = -\frac{2}{x+1} - 3$

- 4.1 Write down the equation(s) of the vertical and horizontal asymptotes of g . (2)
- 4.2 Determine the intercepts of the graph of g with the x and y axes. (2)
- 4.3 Sketch the graph of g . (4)
- [8]**

QUESTION 5

$f(x) = -2x^2 + x + 6$ and $g(x) = -\frac{3}{2}x + 3$ are sketched below. f and g intersect at P and C.

B and C are the x – intercepts of f . ADE is perpendicular to the x – axis, with A on f and D on g . A is the turning point of f .



- 5.1 Determine the co-ordinates of A, the turning point of f . (3)
- 5.2 Write down the range of f . (2)
- 5.3 Calculate the length of BC. (4)
- 5.4 Determine the co-ordinates of P. (5)
- 5.5 Determine the equation of h if $h(x) = f(x - 3)$. Hence, write down the coordinates of the new turning point of h . (3)

[17]

QUESTION 6

Given $h(x) = a^x$ passes through the point $A\left(-2; \frac{1}{9}\right)$.

6.1 Calculate the value of a . (2)

6.2 Write down the equation of h^{-1} , the inverse of h , in the form $y = \dots$ (2)

6.3 Write down the co-ordinates of any point on the graph of the inverse of h . (2)

6.4 Determine the value(s) of x for which $h^{-1}(x) \leq -2$. (2)

6.5 Write down the equation of f if $f(x) = h^{-1}\left(\frac{x}{2}\right)$. (2)

[10]

QUESTION 7

7.1 Office equipment costing R500 000 depreciates on the diminishing balance method at an interest rate of 9,25 % p.a. Determine the value of the office equipment at the end of 7 years. (3)

7.2 A farmer has just bought a new tractor for R900 000.

7.2.1 The farmer wants to replace his present tractor with a new one in 5 years' time by trading the present one for R300 000. The replacement cost of the tractor is expected to increase by 8,25 % p.a. Calculate the amount that he will need to pay for the new tractor. (3)

7.2.2 One month after purchasing his present tractor, he makes monthly deposits of x rands into an account that pays interest at a rate of 12 % p.a. compounded monthly for 5 years. After 60 months he has the exact amount of money to purchase the new tractor after he trades in the present tractor. Calculate the value of x . (5)

7.2.3 Suppose that 12 months after the purchase of the present tractor and every 12 months thereafter he withdraws R8000 from his account to pay for maintenance of the tractor. If he makes 5 such withdrawals, determine the value of the new monthly deposit. (4)

[15]

QUESTION 8

8.1 Determine the derivative of $f(x) = 3x^2 - 5x$ from first principles. (5)

8.2 Determine the derivative with respect to x if:

8.2.1 $f(x) = \frac{x^2 - 4}{x}$ (4)

8.2.2 $g(x) = \sqrt[3]{x^4} + (3x)^{-2}$ (4)
[13]

QUESTION 9

$f(x) = -x^3 + 3x^2 + 9x - 27 = -(x+3)(x-3)^2$ is the equation of a cubic function.

9.1 Write down the intercepts of f . (3)

9.2 Calculate the co-ordinates of the stationary points of f . (5)

9.3 Sketch the graph of f on a system of axes. (Clearly indicate the coordinates of the stationary points and the intercepts with the axes). (4)

9.4 Determine the value(s) of x for which the graph is concave down. (2)

9.5 Determine the equation of the tangent to the graph of f at $x = 0$. (3)

9.6 If $f(x) = k$ has 3 unequal real roots, determine the value(s) of k . (3)

9.7 Write down the equation of f if f is shifted 3 units horizontally to the left. (2)
[22]

TOTAL MARKS: 125



education

Department:
Education
PROVINCE OF KWAZULU-NATAL

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

MATHEMATICS P2

COMMON TEST

JUNE 2018

MARKS: 125

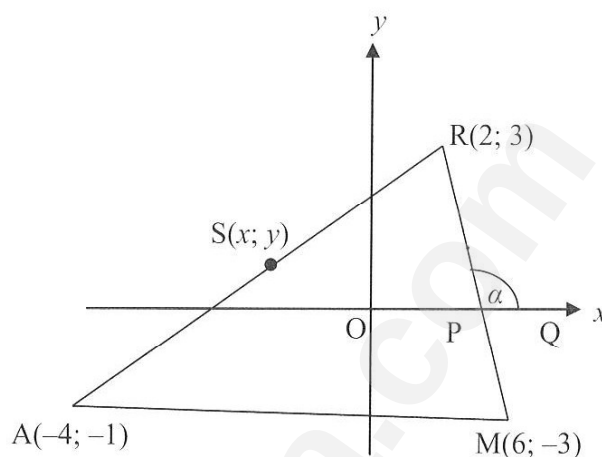
TIME: 2½ hours

This question paper consists of 6 pages and 1 information sheet.

QUESTION 1

In the diagram alongside,
 $A(-4; -1)$ $R(2; 3)$ and
 $M(6; -3)$ are vertices of
 $\triangle ARM$. $S(x; y)$ is the
midpoint of RA .

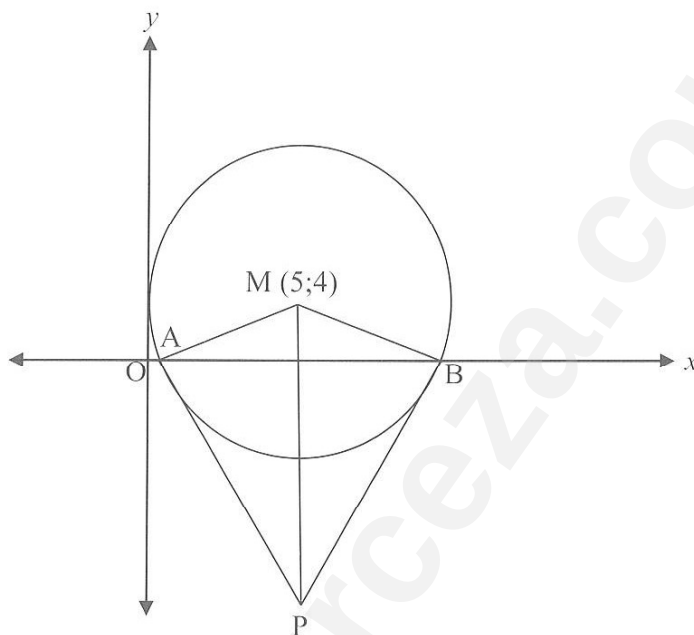
$$\widehat{RPQ} = \alpha$$



- 1.1 Calculate the co ordinates of S, the midpoint of RA . (2)
 - 1.2 Determine the equation of the straight line RM . (5)
 - 1.3 Calculate the co ordinates of P. (3)
 - 1.4 Calculate the length of RA , rounded off to one decimal digit. (3)
 - 1.5 Show that $\triangle ARM$ is right-angled. (4)
 - 1.6 Calculate the value of α rounded off to one decimal place. (4)
 - 1.7 Calculate the area of $\triangle ARM$. (3)
- [24]

QUESTION 2

In the diagram below, a circle with centre $M(5;4)$ and radii AM and MB cut the x -axis at A and B , with A the point closest to the origin.



- 2.1 Determine the equation of the circle. (3)
 - 2.2 Determine the co-ordinates of A and B . (4)
 - 2.3 Determine the equations of tangents to the circle from a point P with points of contact A and B . (6)
 - 2.4 Calculate the co-ordinates of P . (5)
 - 2.5 Calculate the size of $\hat{AMB}$. (6)
- [24]**

QUESTION 3

- 3.1
 - 3.1.1 Show that $\cos(P + Q) + \cos(P - Q) = 2\cos P \cos Q$. (2)
 - 3.1.2 Hence evaluate $\cos 70^\circ + \cos 50^\circ$ without using a calculator if $\cos 10^\circ = 0,984$. (7)
 - 3.2 If $\sin 54^\circ = m$, express the following in terms of m , without using a calculator:
 - 3.2.1 $\sin 594^\circ$ (3)
 - 3.2.2 $\cos 18^\circ$ (5)
- [17]**

QUESTION 4

- 4.1 If $\sin 2\theta = \frac{2\sqrt{6}}{5}$ where $\theta \in [45^\circ; 360^\circ]$ determine, with the aid of a diagram and without using a calculator the value of $\sin \theta$. (6)

- 4.2 Simplify without using a calculator:

$$\frac{\sin 104^\circ (2 \cos^2 15^\circ - 1)}{\tan 38^\circ \cdot \sin 412^\circ} \quad (6)$$

- 4.3 Prove the identity:

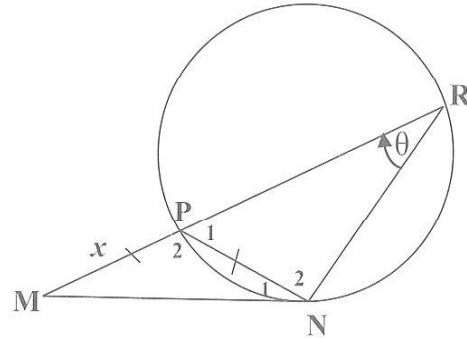
$$\frac{\cos 2x + 1}{\cos 2x \cdot \cos^2 x} = \frac{1}{\left(\cos^2 x - \frac{1}{2}\right)} \quad (5)$$

- 4.4 Determine the solution of:

$$\sin x + \cos x = 1, \text{ for } 0^\circ \leq x \leq 360^\circ \quad (6)$$

- 4.5 In the figure alongside, RP is a diameter of the circle. RPM is a straight line and $PM = PN = x$, $\hat{PRN} = \theta$.

- 4.5.1 Show that:
 $MN^2 = 2x^2 (1 + \sin \theta)$

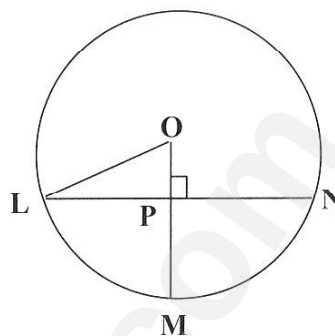


- 4.5.2 If $MN = \sqrt{12}$ units and $x = 2$ units, Show, without using a calculator, that $\theta = 30^\circ$ and $RP = 4$ units. (4)

[32]

QUESTION 5

In the accompanying diagram alongside, LN is a chord of the circle with centre O. OP is drawn perpendicularly onto LN and meets the circle in M.

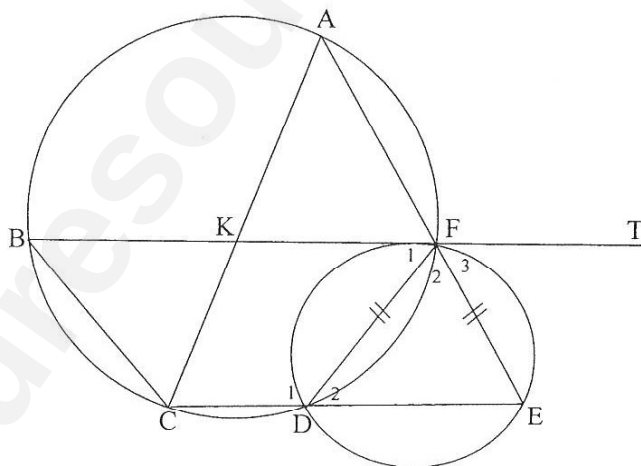


5.1 Prove that $MN^2 = 2 OM \cdot PM$. (4)

5.2 If $OL = 9$ units and $OP = 1$ unit, calculate the length of MN. (2)
[6]

QUESTION 6

In the figure below, two circles cut in points F and D. BFT is a tangent to the smaller circle at F. Straight line AFE is drawn so that $FE = FD$. CDE is a straight line and chords AC and BF cut at K.



Prove that:

6.1 $BT \parallel CE$. (4)

6.2 BCEF is a parallelogram. (5)

6.3 $AC = BF$. (6)

6.4 BF is a diameter if it is given that $AF = FE$. (7)
[22]

TOTAL MARKS: 125

**JOHANNESBURG SOUTH DISTRICT
CLUSTER PAPER
JUNE EXAMINATION 2019**

GRADE 12

**MATHEMATICS
PAPER 1**

MARKS: 150

TIME: 3 hours

This question paper consists of 10 pages, excluding the cover page and an information sheet.

QUESTION 11.1 Solve for x :

1.1.1 $x^2 - x - 30 = 0$ (2)

1.1.2 $3x^2 - 2x = 14$ (correct to TWO decimal places) (4)

1.1.3 $x^2 + 2x + 1 < 0$ (3)

1.1.4 $\sqrt{x^2 - 10} = 3\sqrt{x}$ (5)

1.2 Solve for x and y simultaneously:

$3^{x+1} = 9^y$ and $xy = 3$ (7)

1.3 Given $f(x) = x^2 - 5x + p$

Determine the value of p if $f(x) = 0$ and $x = \frac{5 \pm \sqrt{41}}{2}$. (3)

[24]**QUESTION 2**2.1 Given the geometric series : $9 + 3 + 1 + \frac{1}{3} + \dots$

2.1.1 Determine the next terms of the series (1)

2.1.2 Explain why this sequence will converge (1)

2.1.3 Calculate the sum to infinity of the convergent series. (3)

2.2 The following consists of a constant and an arithmetic sequences.

 $6; x; 6; 12; 6; y; 6; 24; 6; \dots$ 2.2.1 Determine the values of x and y (3)2.2.2 $3T_{13}$ (1)

2.2.3 Calculate the sum of the first 30 terms of the sequence (4)

2.3 Given the quadratic sequence : $10; 1; -5; -8; -8$

2.3.1 Determine which term of the first differences will be equal to 81. (3)

2.3.2 Determine the n th term of the given sequence. (4)

2.4 If $\sum_{n=1}^5 5(3)^{n-1} = p$

Write down $\sum_{n=1}^5 45(3)^n$ in terms p . (3)

[23]

QUESTION 3

Given : $f(x) = \frac{3}{x+2} - 1$

3.1 Write down the equation of asymptotes of f . (2)

3.2 Calculate the x and y intercepts (3)

3.3 Sketch the graph of f . (3)

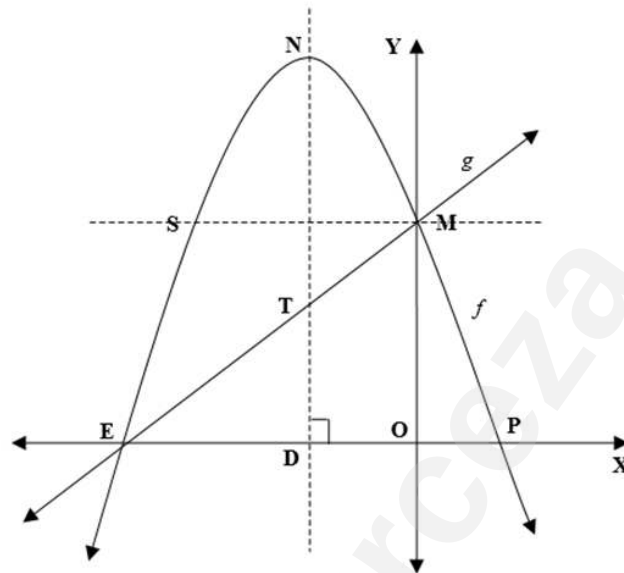
3.4 The graph of $f(x)$ is shifted horizontally 2 units to the right to form $m(x)$.
Write down the domain of $m(x)$. (2)

[10]

QUESTION 4

Given: $f(x) = -x^2 - 4x + 5$ and $g(x) = mx + c$

E, M and P are the intercepts of the graphs with the axes. N is the turning point and NTD the axis of symmetry of f . T is a point on the graph of g .



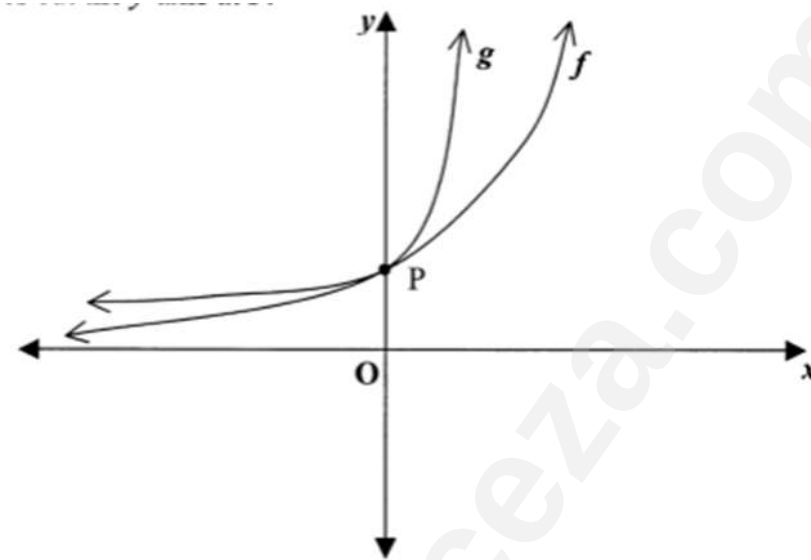
Determine :

- 4.1 The coordinates of M, E and P (4)
- 4.2 The coordinates of N (3)
- 4.3 The equation of g . (3)
- 4.4 The length of NT (3)
- 4.5 The coordinates of S if S is the reflection of M with respect to the axis of symmetry. (2)
- 4.6 For which values of x will :
 - 4.6.1 $f(x) - g(x) < 0$ (2)
 - 4.6.2 $f(x) \cdot g(x) < 0$ (2)

[19]

QUESTION 5

The sketch below shows the curves of $f(x) = a^x$ and $g(x) = 5^x$.
The curves cut the y-axis at P.



- 5.1 Write down the co-ordinates of P. (1)
- 5.2 Determine ALL possible values of a . (2)
- 5.3 Write down the equation of g^{-1} the inverse of g in the form $y = \dots\dots$ (2)
- 5.4 Draw a sketch graph of g^{-1} , the inverse of g . indicate the co-ordinates of any intercepts with the axes. (2)
- 5.5 Write down the values of x for which $\log_5 x < 0$ (2)

[9]

QUESTION 6

6.1 Given: $f(x) = 5 - 2x^2$

6.1.1 Determine: $f'(x)$ from the first principles. (5)

6.1.2 The line $g(x) = -2x + p$ is a tangent to the graph of f at point A.
Determine the coordinates of A. (3)

6.2 Determine:

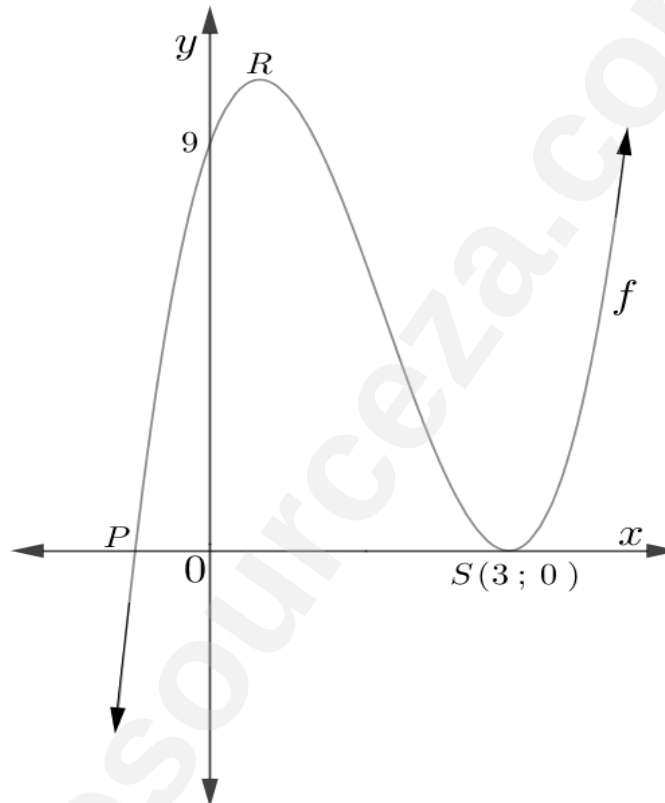
6.2.1 $D_x \left[3x - 3x^2 - \frac{3}{x} \right]$ (3)

6.2.2 $\frac{dy}{dx}$ if $y = \sqrt[3]{x^2} + \frac{x}{3} + \pi^3$ (3)

[14]

QUESTION 7

- 7.1 The graph of $f(x) = x^3 + bx^2 + cx + d$ is sketched below. The graph of f intersects the x -axis at P and $S(3;0)$, and the y -axis at $(0;9)$. R and S are the turning points of f .



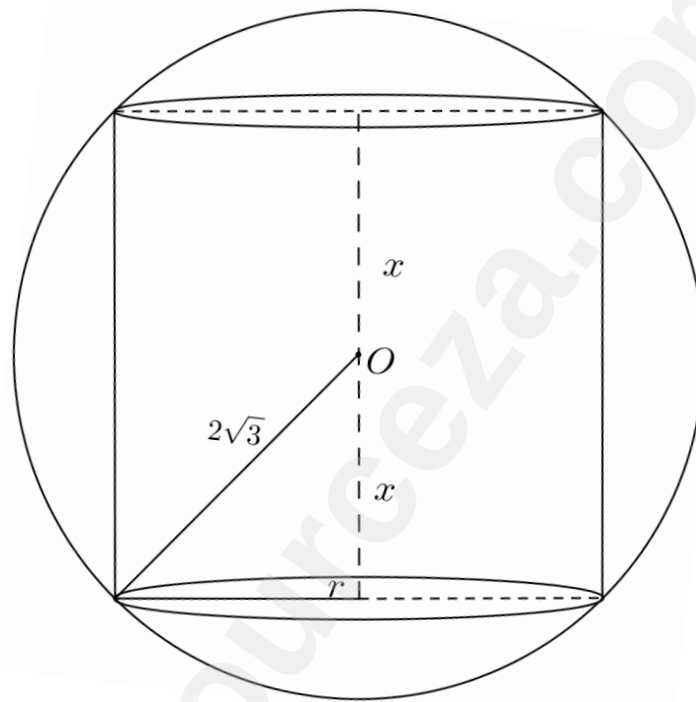
- 7.1.1 Show that: $b = -5$, $c = 3$ and $d = 9$. (4)
- 7.1.2 Calculate the coordinates of R . (5)
- 7.1.3 The x -coordinate of the point of inflection of f . (2)
- 7.2 Use the graph to determine the values of x for which :
- 7.2.1 $f'(x) < 0$ (2)
- 7.2.2 The graph is concave down. (1)
- 7.3 For which value(s) of t will $f(x) = t$ have only one root. (2)

[16]

QUESTION 8

The diagram below shows a cylindrical can that fits into circle O with radius

$R = 2\sqrt{3}$ units.



8.1 Express r in terms of x . (2)

8.2 Calculate the value of x for which the volume of the cylinder, $V = \pi r^2 h$ is a maximum. (4)

8.3 Calculate the maximum volume of the can in terms of π . (2)

[8]

QUESTION 9

- 9.1 Convert the effective interest rate of 10,5 % p.a, to a nominal interest rate compounded quarterly. (4)
- 9.2 Chauke invests R12 500 for k years at a compound interest rate of 9% p.a., compounded quarterly. At the end of the k years his investment is worth R30 440. Calculate the value of k . (4)
- 9.3 Jabulile invests R25 000.00 at an interest rate of 7,5% p.a, compounded monthly for the first 3 years and thereafter she deposits another R10 000.00 at 6,7% p.a, compounded semi-annually. Calculate the total amount of her investment after 7 years. (4)

[12]**QUESTION 10**

- 10.1 The probability that a person will order coffee at a cafe is 0, 6.
The probability that a person will order tea is 0, 3.
The probability that a person will order neither coffee nor tea is 0, 2.
Using a venn diagram, calculate the probability that a person:
- 10.1.1 Will order both coffee and tea. (5)
- 10.1.2 Will order only one of the two. (2)

- 10.2 In a survey, 250 grade 11 & 12 learners were polled to see which brands of soft drink they prefer. The results are shown in the contingency table below.

	Grade 11	Grade 12	Total
Likes Brand A	a	30	150
Likes Brand B	b	c	d
Total	200		250

10.2.1 Determine the value of a , b , c , and d . (2)

10.2.2 Determine the probability of selecting a grade 12 (1)

10.2.3 Determine the probability of selecting a learner in grade 11 that likes Brand A. (1)

10.2.4 Is the learner's being in grade 11 independent of liking brand A? (4)
Show all relevant calculations motivate your answer.

[15]

TOTAL:150

**JOHANNESBURG SOUTH DISTRICT
CLUSTER PAPER
JUNE EXAMINATION 2019**

GRADE 12

**MATHEMATICS
PAPER 2**

MARKS: 150

TIME: 3 hours

This question paper consists of 12 pages, excluding the cover page, 1 information sheet and 7 diagram sheets

QUESTION 1

The distance (in metres) covered by Eric during mountain climbing for a two weeks period is represented in the table below.

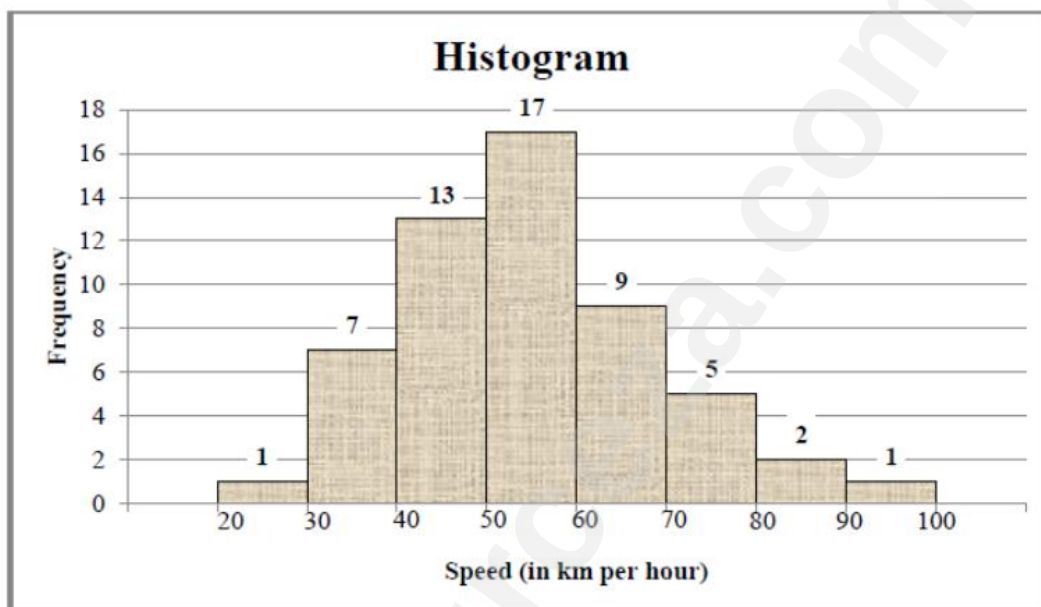
170	191	239	240	246	259	260
260	276	296	310	322	354	490

- 1.1 Calculate the :
 - 1.1.1 Mean distance covered for two weeks (2)
 - 1.1.2 Median distance (1)
- 1.2 Use DIAGRAM SHEET 1 provided to draw a box and whisker diagram to represent the data (3)
- 1.3 Hence determine the interquartile range of the data (2)
- 1.4 Comment on the skewness in the distribution of the data (1)
- 1.5 For the given data:
 - 1.5.1 Calculate the standard deviation (2)
 - 1.5.2 Determine the number of days in which the distance climbed was less than one standard deviation below the mean (2)

[13]

QUESTION 2

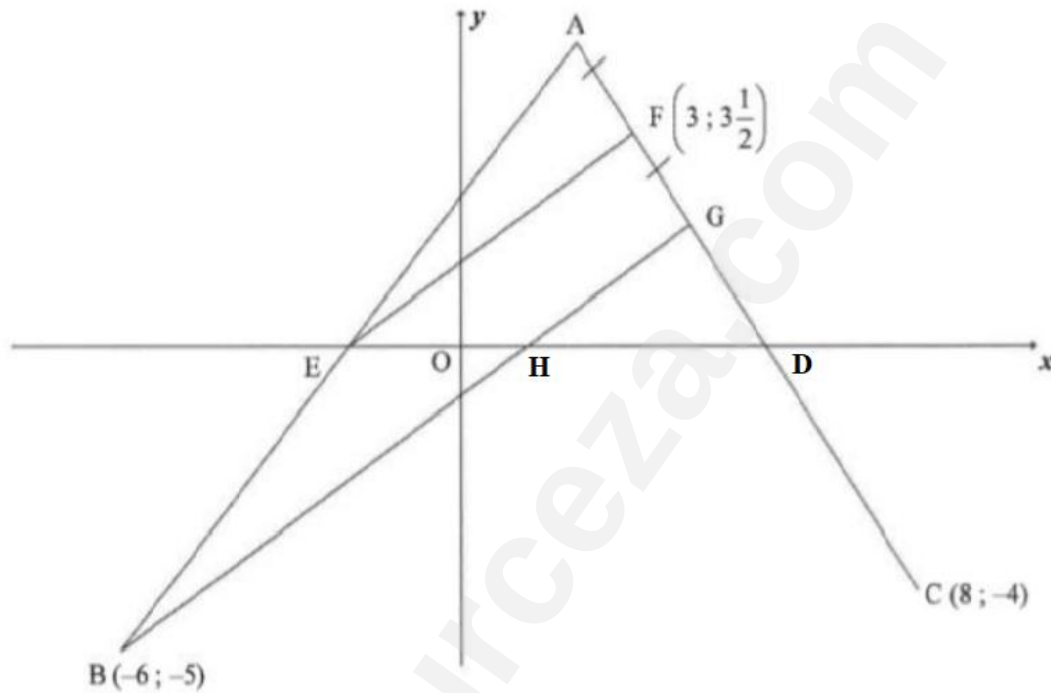
The speeds of 55 cars passing through a certain section of a road are monitored for an hour. The speed limit on this section of the road is 60 km per hour. A histogram is drawn to represent this data.



- 2.1 Identify the modal class of the data (1)
- 2.2 Use the histogram to :
- 2.2.1 Complete the cumulative frequency column in the table using DIAGRAM SHEET 1 (2)
- 2.2.2 USE DIAGRAM SHEET 1 to draw the ogive (cumulative frequency graph) of the above data (3)
- 2.3 The traffic department sends speeding fines to all motorists whose speed exceeds 66km per hour. Estimate the number of motorists who will receive a speeding fine. (2)
- [8]**

QUESTION 3

In the diagram A ; $B(-6; -5)$ and $C(8; -4)$ are points on a Cartesian plane. $F(3; 3\frac{1}{2})$ and G are points on the line AC such that $AF = FG$. E is the x intercept.

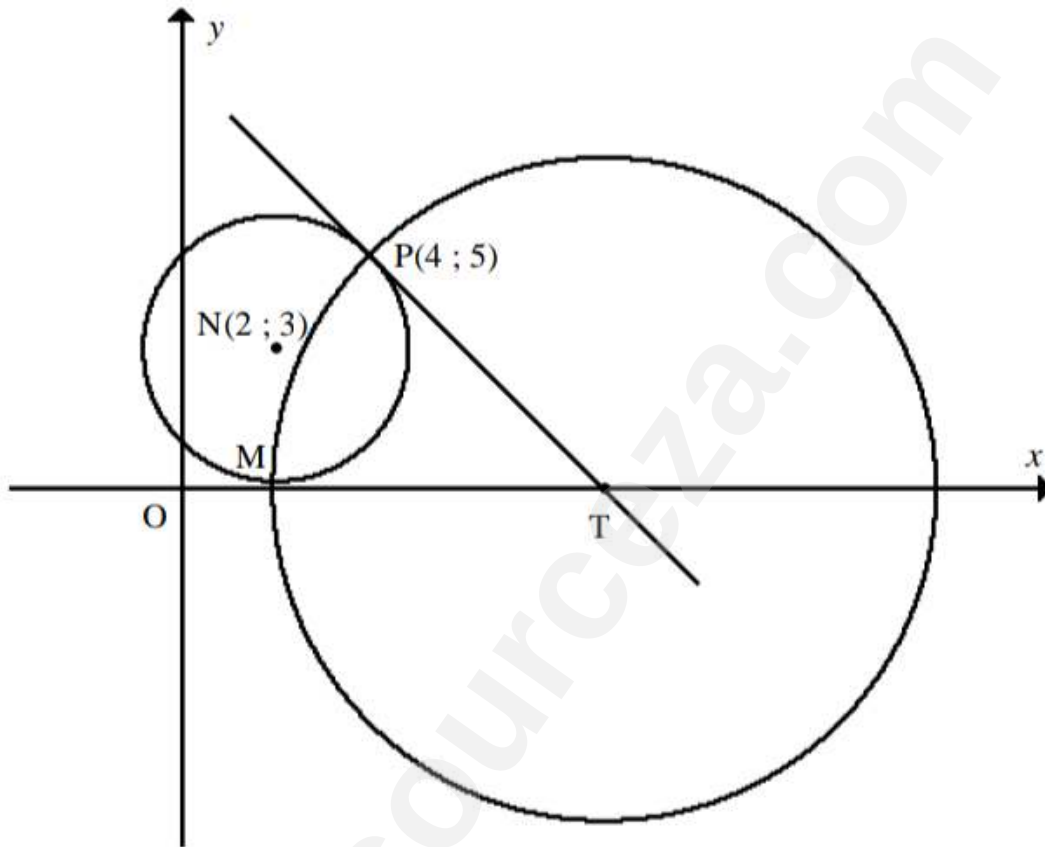


- 3.1 Determine:
- 3.1.1 The equation of AC in the form $y = mx + c$ (3)
- 3.1.2 The co-ordinates of G if the equation of BG is $7x - 10y = 8$ (3)
- 3.2 Show by calculation that the co-ordinates of A is $(2; 5)$ (2)
- 3.3 Calculate the size of $\hat{AED}$ (3)
- 3.4 Calculate the area of $\triangle AED$ (7)
- 3.5 Determine the equation of a circle with centre at the origin passes through point C (2)

[20]

QUESTION 4

In the figure below, T is a point on the x-axis. A circle having T as its centre intersects a circle having N (2; 3) as its centre at P (4; 5) and M. TP is a tangent to the circle centre N at P.

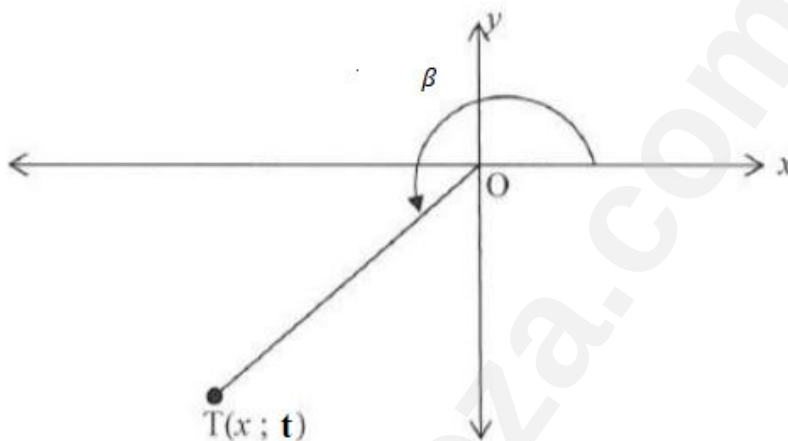


- 4.1 Determine the equation of the circle centre N in the form $(x - a)^2 + (y - b)^2 = r^2$ (3)
- 4.2 Hence write the equation in 4.1 in the form $Ax^2 + Bx + Cy^2 + Dy + E = 0$ (1)
- 4.3 Determine the equation of PT in the form $y = mx + c$ (4)
- 4.4 Determine the equation of the circle with centre T (4)
- 4.5 Prove that NPTM is a Kite if it is given that MT is a tangent to the circle centre N at M. (3)
- 4.6 Determine the equations of tangents to the circle with centre T, which are parallel to the y-axis. (2)

[17]

QUESTION 5

- 5.1 In the diagram below, $T(x; t)$ is a point in the third quadrant and it is given that $\sin \beta = \frac{t}{\sqrt{1+t^2}}$



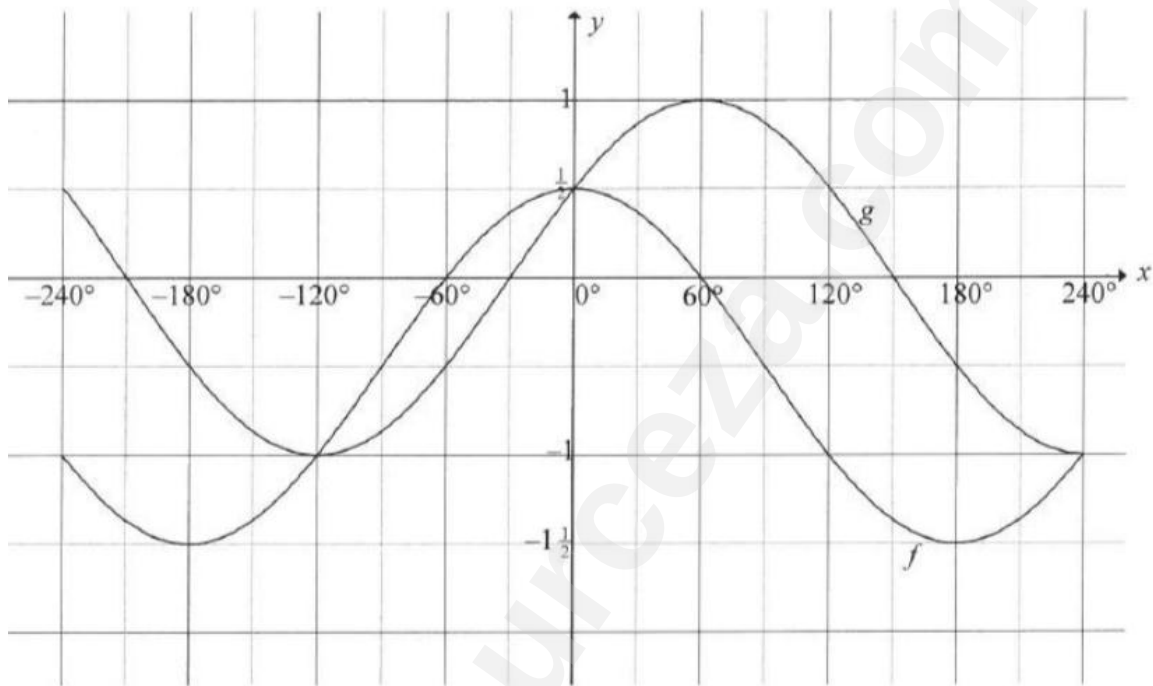
- 5.1.1 Show that $x = -1$ (3)
- 5.1.2 Write $\cos(60^\circ + \beta)$ in terms of t in its simplest form (4)
- 5.1.3 Show that $\cos 2\beta$ can be written as $\frac{1-t^2}{1+t^2}$ (3)
- 5.2 Simplify the following expression to a single trigonometric ratio:
 $\sin(-x) \cdot \sin(540^\circ - x) - \sin(90^\circ + x) \cdot \cos(180^\circ - x)$ (6)
- 5.3 Given the identity:

$$\frac{2\tan x - \sin 2x}{2\sin^2 x} = \tan x$$
- 5.3.1 For which value(s) of x will the identity be undefined in the interval $0^\circ \leq x \leq 180^\circ$ (2)
- 5.3.2 Prove the identity (5)
- 5.4 Determine the general solution of $\cos 2x - 7\cos x - 3 = 0$ (6)

[29]

QUESTION 6

In the diagram below, the graphs of $f(x) = \cos x + q$ and $g(x) = \sin(x + p)$ are drawn on the same system of axes for $-240^\circ \leq x \leq 240^\circ$. The graphs intersect at $(0^\circ; \frac{1}{2})$, $(-120^\circ; -1)$ and $(240^\circ; -1)$.

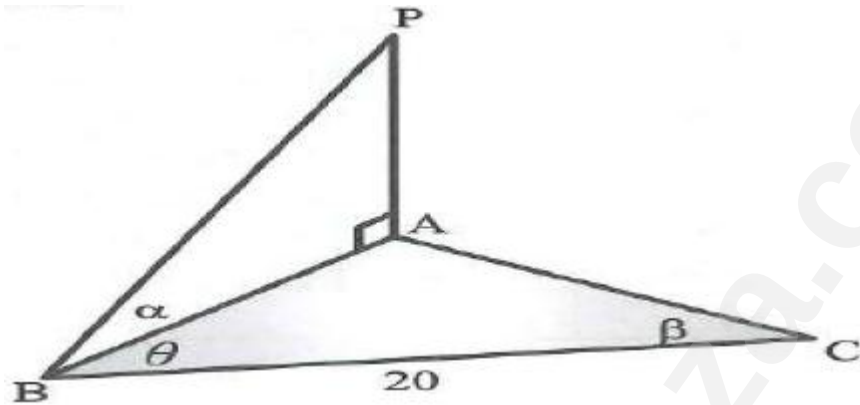


- 6.1 Determine the values of p and q . (2)
- 6.2 Determine the values of x in the interval $-240^\circ \leq x \leq 240^\circ$, for $f(x) > g(x)$ (2)
- 6.3 Describe a transformation that the graph of g has to undergo to form the graph h , where $h(x) = -\cos x$ (2)

[6]

QUESTION 7

In the Figure below A, B and C are in the same horizontal plane P is a point vertically above A. B and C are 20 units apart. The angle of elevation from B to P is α
 $\hat{A}BC = \theta$ and $\hat{ACB} = \beta$



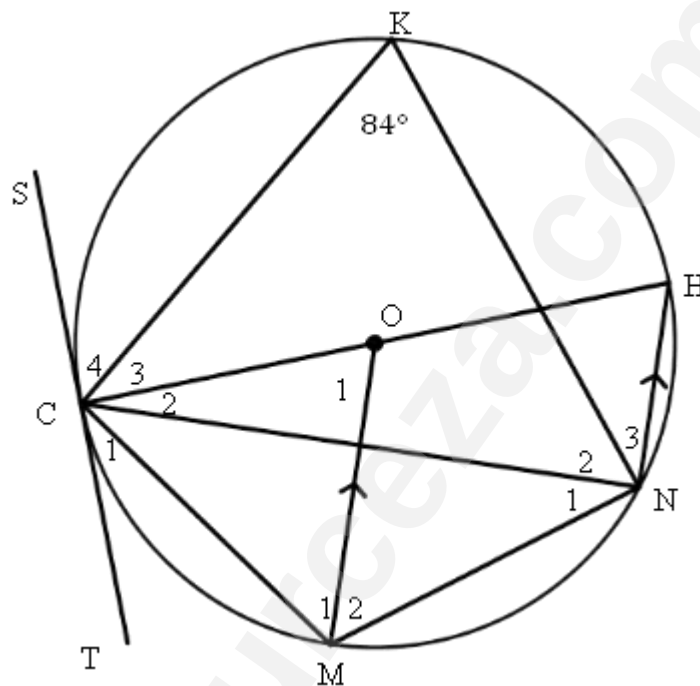
7.1 Prove that $AP = \frac{20 \sin \beta \tan \alpha}{\sin(\theta + \beta)}$ (4)

7.2 Given that $AB = AC$, determine AP if $\beta = 30^\circ$ and $\alpha = 60^\circ$ (3)

[7]

QUESTION 8

O is the centre of the circle and diameter CH. ST is a tangent to the circle at C.
OM // HN and $\hat{K} = 84^\circ$.



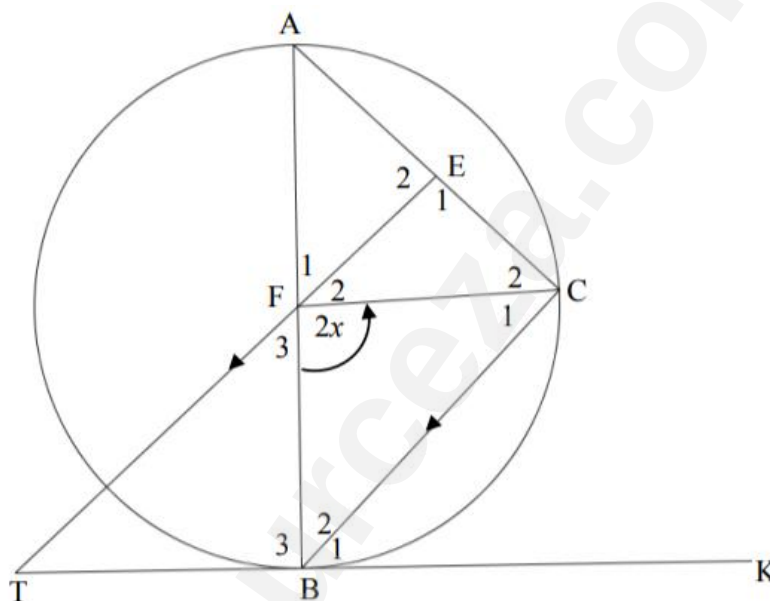
Calculate, with reasons, the size of each of the following angles:

- 8.1 $\hat{H}$ (2)
- 8.2 $\hat{O}_1$ (2)
- 8.3 $\hat{N}_1$ (2)
- 8.4 $\hat{C}_1$ (2)
- 8.5 $\hat{M}_1 + \hat{M}_2$ (2)
- 8.6 Prove why CONM is not a cyclic quadrilateral (2)

[12]

QUESTION 9

In the diagram below, F is the centre of the circle. TBK is a tangent to the circle at B. E and F are points on AC and AB respectively. EF is produced to T. $BC \parallel TE$ and $\widehat{BFC} = 2x$.



9.1 Name with reasons, FOUR angles equal to x (6)

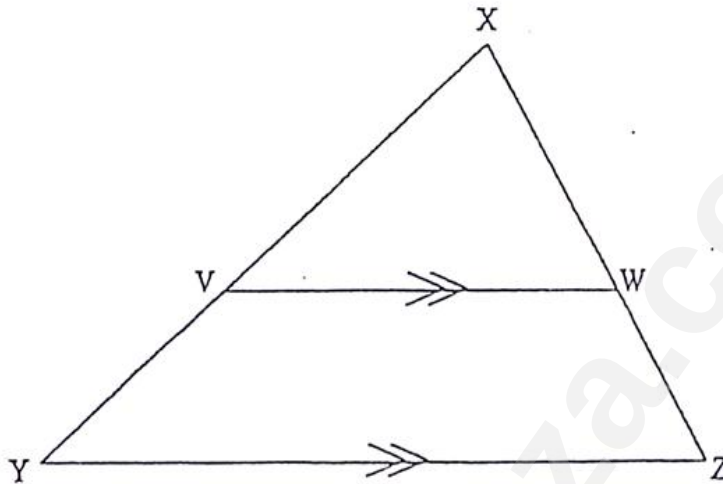
9.2 Is FCBT a parallelogram? Motivate your answer (3)

[9]

QUESTION 10

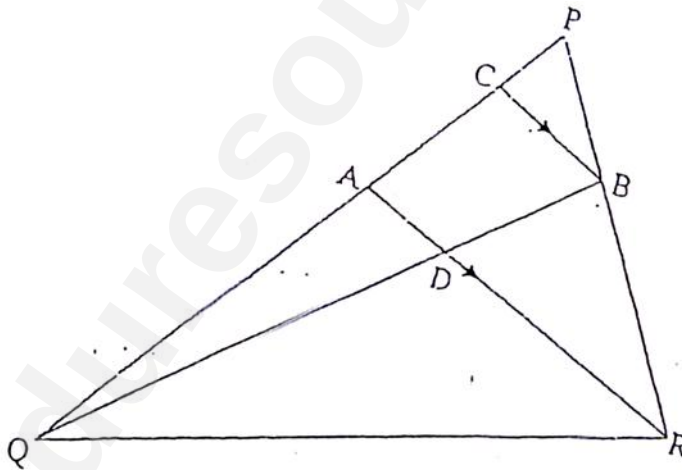
10.1 Use diagram sheet 6, to prove the theorem which states that if

$VW \parallel YZ$, then $\frac{XV}{VY} = \frac{XW}{WZ}$.



(6)

10.2 In $\triangle PQR$ below, B lies on PR such that $2PB = BR$. point A lies on PQ such that $PA: PQ = 3:8$. CB is drawn parallel to AR.



10.2.1 Write down the value of $\frac{\text{area } \triangle PRA}{\text{area } \triangle QRA}$.

(2)

10.2.2 Calculate the value of the ratio $\frac{BD}{BQ}$.

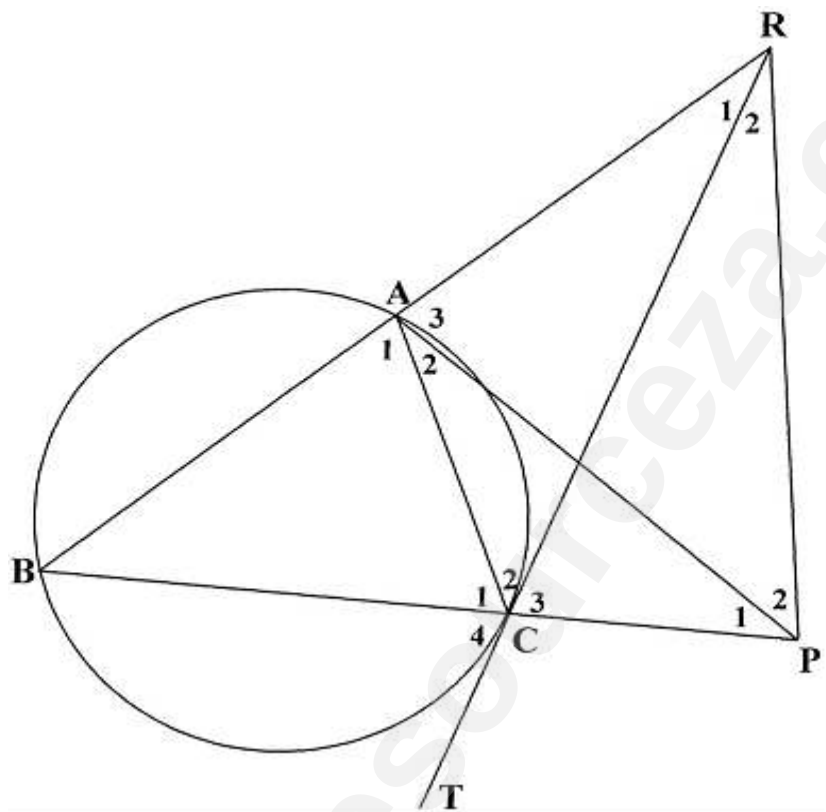
(4)

Show all your working

[12]

QUESTION 11

In the diagram below, chord BA and tangent TC of circle ABC are produced to meet at R.
BC is produced to P with $RC = RP$.
AP is not a tangent.



Prove that:

11.1 ACPR is a cyclic quadrilateral (5)

11.2 $\triangle CBA \sim \triangle RPA$ (6)

11.3 $RC = \frac{CB \cdot RA}{AC}$ (2)

11.4 $RB \cdot AC = RC \cdot CB$ (4)

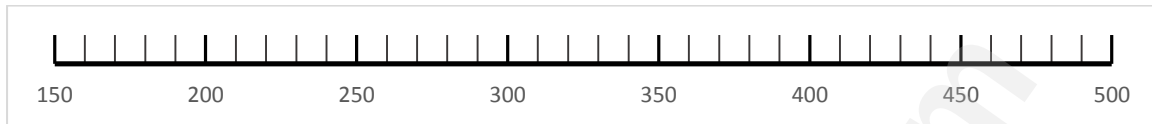
[17]

GRAND TOTAL: 150 MARKS

DIAGRAM SHEET 1

SURNAME AND NAME OF LEARNER

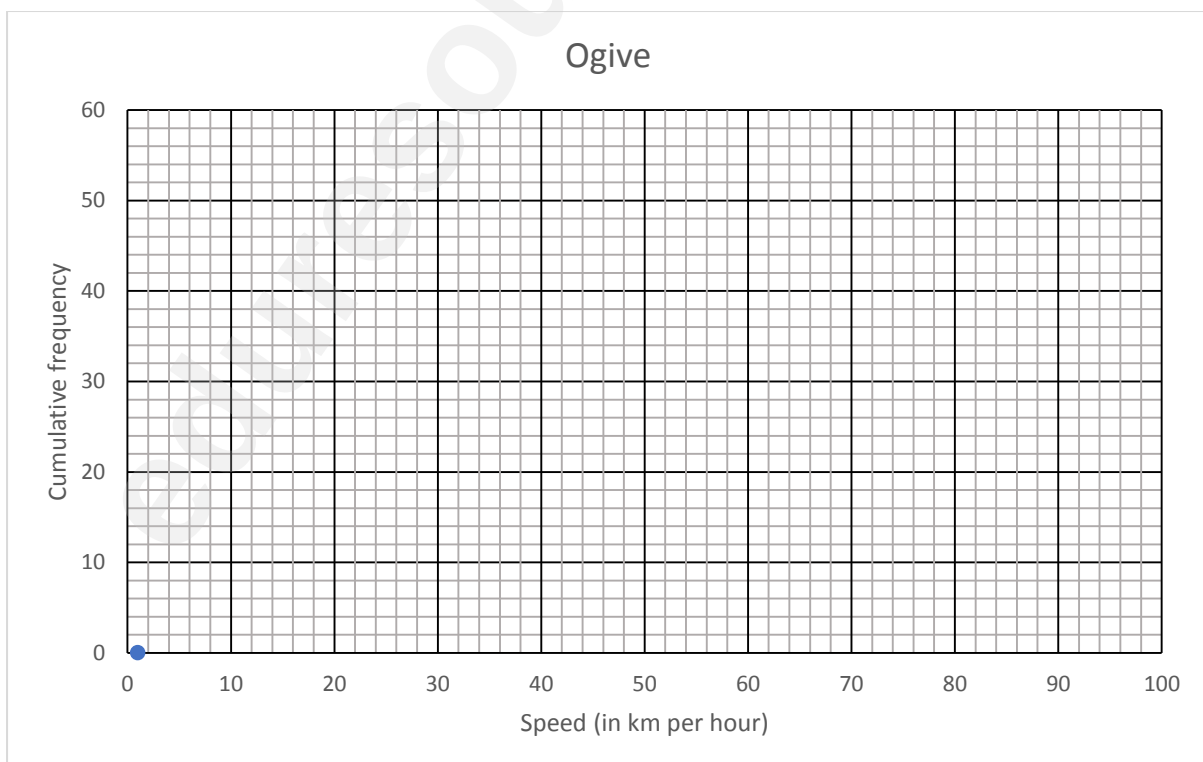
Question 1.2



Question 2.2.1

Class	Frequency	Cumulative frequency
$20 < x \leq 30$	1	
$30 < x \leq 40$	7	
$40 < x \leq 50$	13	
$50 < x \leq 60$	17	
$60 < x \leq 70$	9	
$70 < x \leq 80$	5	
$80 < x \leq 90$	2	
$90 < x \leq 100$	1	

Question 2.2.2





education

Department:
Education

PROVINCE OF KWAZULU-NATAL

GRADE 12

**NATIONAL
SENIOR CERTIFICATE**

MATHEMATICS P1

COMMON TEST

JUNE 2019

MARKS: 150

TIME: 3 hours

N.B. This question paper consists of 9 pages and an information sheet.

QUESTION 11.1 Solve for x :

1.1.1 $(x+3)(9+x)=0$ (2)

1.1.2 $-4x^2+9x=3$, to TWO decimal places. (4)

1.1.3 $2^{x+2}-3.2^{x-1}=80$ (5)

1.1.4 $\frac{x-4}{2}-\sqrt{x-3}=1$ (5)

1.2 Solve for x and y simultaneously given $x-3y=1$ and $x^2-2xy+9y^2=17$. (6)

1.3 Solve $x^2-5x+4<0$ (3)

[25]**QUESTION 2**

The first four terms of the first difference of a quadratic sequence are 5; 9; 13; 17; ... and the 61st term of the quadratic sequence is 7383.

2.1 Calculate the n^{th} term of the quadratic sequence. (5)

2.2 Determine between which two consecutive terms is the 1st difference equal to 2021? (3)

[8]

QUESTION 3

Given $-3 ; -1 ; 1 ; 3 ; \dots$ are the first four terms of an arithmetic sequence.

3.1 Write down the value of the next term of the sequence. (1)

3.2 If the last term of the sequence is 7997, calculate the value of n . (2)

3.3 Given the last term is 7997, determine the sum of the terms that are divisible by 5. (4)
[7]

QUESTION 4

4.1 $1 ; 4 ; 16 ; 64 ; \dots$ are the first four terms of a geometric sequence.

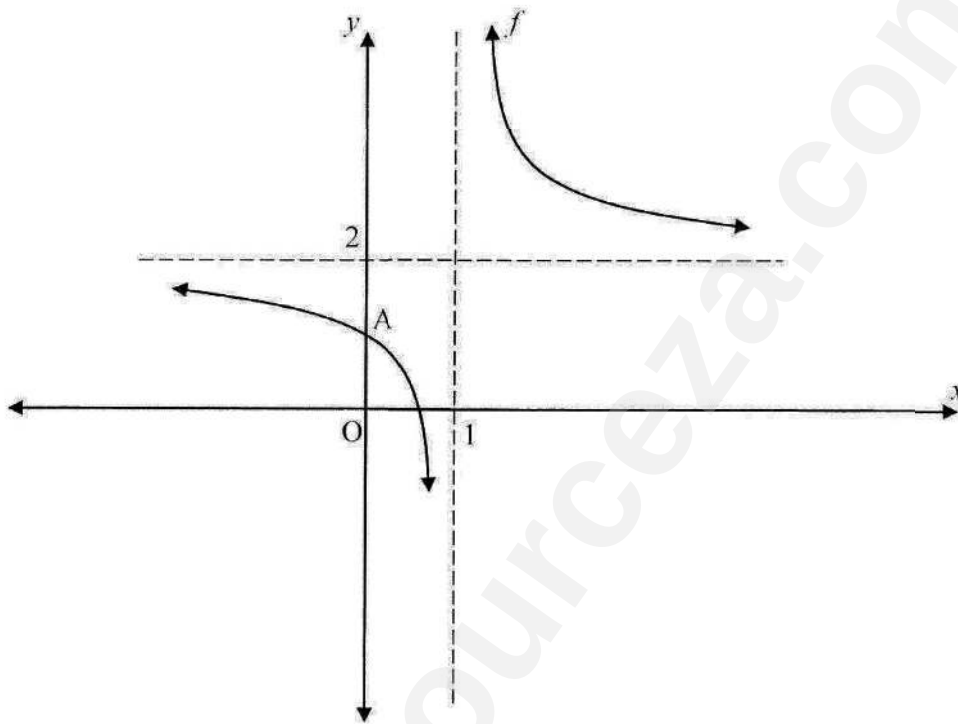
4.1.1 Write down the value of the common ratio. (1)

4.1.2 Calculate the sum to 16 terms of the sequence. (3)

4.2 The sum of the first 3 terms of geometric series is $1\frac{8}{49}$. If the first term is 1, then calculate (5)
the value of the common ratio, r ($r > 0$).
[9]

QUESTION 5

Given $f(x) = \frac{a}{x-b} + c$; $A\left(0; \frac{1}{2}\right)$ is the y -intercept of the graph. The asymptotes to the graph intersect the x -axis at 1 and the y -axis at 2 respectively.

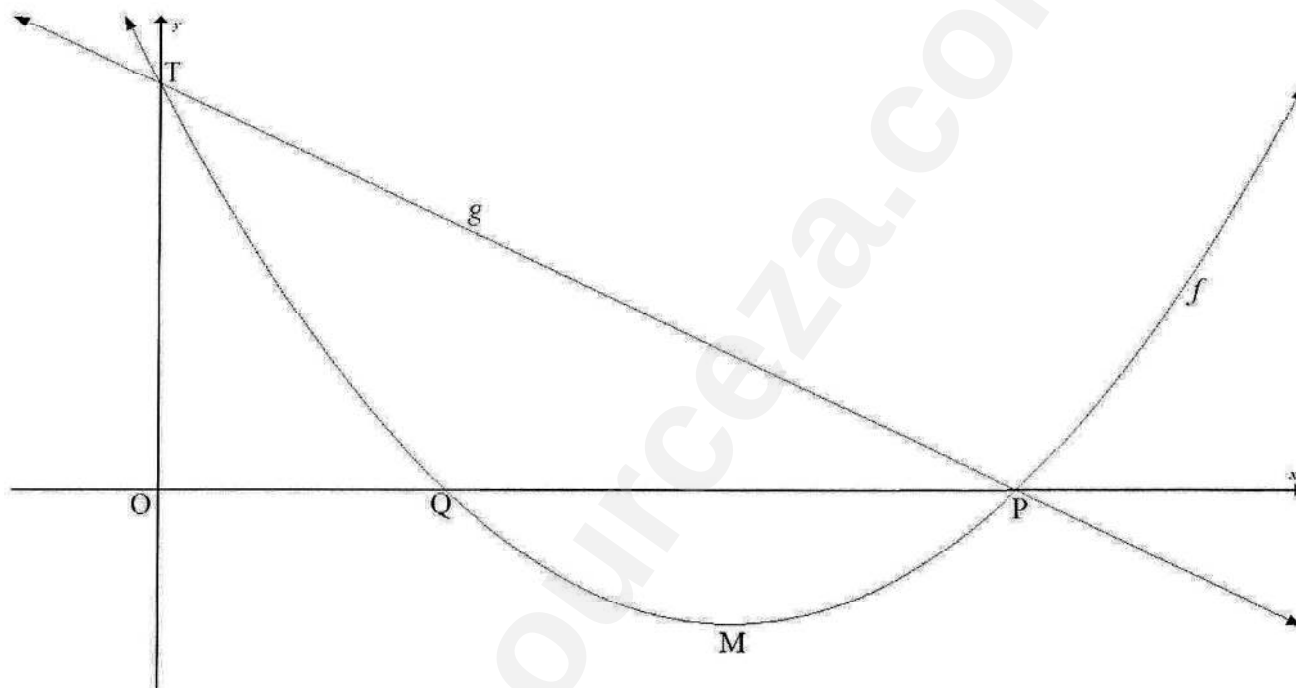


- 5.1 Write down the equations of the vertical and horizontal asymptotes of f . (2)
- 5.2 Calculate the value of a . (3)
- 5.3 Determine the coordinates of A' , the image of A , if it is reflected about $(1; 2)$. (4)
- 5.4 Determine the equation of g if $g(x) = f(x-3)$. (2)

[11]

QUESTION 6

The graphs of $f(x) = ax^2 + bx + c$; $a \neq 0$ and $g(x) = mx + k$ are sketched below. The x -intercepts of f are $Q(2;0)$ and $P(6;0)$, and the y -intercept is $T(0;12)$. The graph of g intersects f at T and P . M is the turning point of f .



- 6.1 Show that the values of $a = 1$, $b = -8$ and $c = 12$. (4)
- 6.2 Calculate the co-ordinates of M , the turning point of f . (2)
- 6.3 Write down the range of f . (1)
- 6.4 Write down the value(s) of x for which $\frac{f(x)}{g(x)} \leq 0$. (3)
- 6.5 Determine the values of k for which $f(x) = k$ has two positive unequal real roots. (2)
- 6.6 Determine the values of x for which the graph of g is a tangent to f . (3)

[15]

QUESTION 7Given $t(x) = 8^x$ 7.1 Write down the equation of t^{-1} , the inverse of t , in the form $y = \dots$ (2)7.2 Show that $t\left(x + \frac{1}{3}\right) = 2 \cdot t(x)$. (3)7.3 Sketch t and t^{-1} in the same system of axes, showing the line of reflection and intercepts with the axes). (4)

[9]

QUESTION 8

8.1 R80 000 was invested in an off-shore company at r % per annum interest rate compounded half yearly. After 5 years the investment grew to R146 338,09. Calculate the interest rate. (4)

8.2 Samuel invested R200 000 with the bank at 11 % per annum, compounded quarterly.

- After two years he withdrew R60 000 from the investment to buy a car.
- After a further three years later he withdrew R45 000 to begin his studies.
- After the last withdrawal the bank increases the interest rate to 12 % per annum compounded monthly.
- Two years later Samuel decides now he wants to withdraw the entire amount of his investment.

Calculate the total amount of his final withdrawal. (8)

[12]

QUESTION 99.1 Determine $f'(3)$ from first principles given $f(x) = 5x^2 + 4$. (5)

9.2 Differentiate:

$$9.2.1 \quad g(x) = \left(2x - \frac{1}{2x}\right)^2 \quad (4)$$

$$9.2.2 \quad D_y \left[\frac{x^3 - 1}{1 - x} \right] \quad (4)$$

[13]

QUESTION 10

Given: $f(x) = (x + 2)^2(x - 3)$ is a cubic function.

10.1 Write down the x - intercepts and the y - intercept of f . (3)

10.2 Determine the local maximum and minimum turning points of f . (4)

10.3 Sketch f showing the coordinates of the turning points and the intercepts with the axes. (4)

10.4 If $f(x) = k$ has one root equal to 0, write down the value of k . (2)

10.5 Given $g(x) = f(x-4)$, write down the coordinates of the new maximum point. (2)

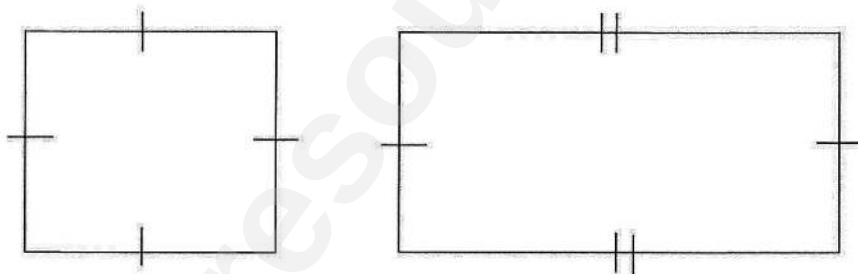
10.6 Write down the values of x for which f is concave up. (2)

10.7 Determine the equation of $g(x)$ if $g(x) = f\left(x + \frac{1}{2}\right)$. (2)

[19]

QUESTION 11

A piece of wire 10 metres long is cut into two pieces. One piece is bent into a square and the other is bent into the shape of a rectangle. The rectangle has the width the same length as the square.



11.1 The length of the wire used to make the square is x metres. Write down in terms of x the length of the side of the square. (1)

11.2 Show that the sum of the areas (S) of the square and the rectangle is given by

$$S = -\frac{1}{8}x^2 + \frac{5}{4}x \quad (4)$$

11.3 Determine the value of x for which the sum of the areas is a maximum. (3)

[8]

QUESTION 12

An airplane company transports passengers to a holiday resort daily. Upon boarding this flight beverages are given. Each passenger chooses exactly one beverage from the given list.

	MALE	FEMALE	TOTAL
TEA	35	45	80
COFFEE	x	y	90
FRUIT JUICE	z	w	30
TOTAL	80	120	p

12.1 Write down the value of p . (1)

12.2 Calculate the probability that a random selected passenger is a female. (2)

12.3 Given the event of choosing fruit juice is independent of being a male, calculate the value of z . (6)

[9]

QUESTION 13

If it is given that $P(A) = 0,34$, $P(B) = 0,26$ and $P(A \text{ or } B) = 0,55$, determine:

13.1 $P(A \text{ and } B)$ (3)

13.2 Are the events A and B mutually exclusive? Explain. (2)

[5]

TOTAL: 150



education

Department:
Education
PROVINCE OF KWAZULU-NATAL

GRADE 12

**NATIONAL
SENIOR CERTIFICATE**

MATHEMATICS P2

COMMON TEST

JUNE 2019

MARKS: 150

TIME: 3 hours

N.B. This question paper consists of 10 pages, 1 information sheet and an answer book with 18 pages.

QUESTION 1

The time taken, in minutes, to complete a 5 km race by 10 athletes of an Athletic Club is given below.

16	18	19	20	21	22	23	24	28	29
----	----	----	----	----	----	----	----	----	----

- 1.1 Calculate the mean time taken to complete the race. (2)
 - 1.2 Calculate the standard deviation. (2)
 - 1.3 Draw a box and whisker diagram to represent the five number summary of the above information. (4)
 - 1.4 How many runners completed the race within one standard deviation of the mean time? (3)
- [11]**

QUESTION 2

A certain company develops a new product and does some market research. The table below is a summary of the ages of people who say they will buy the product.

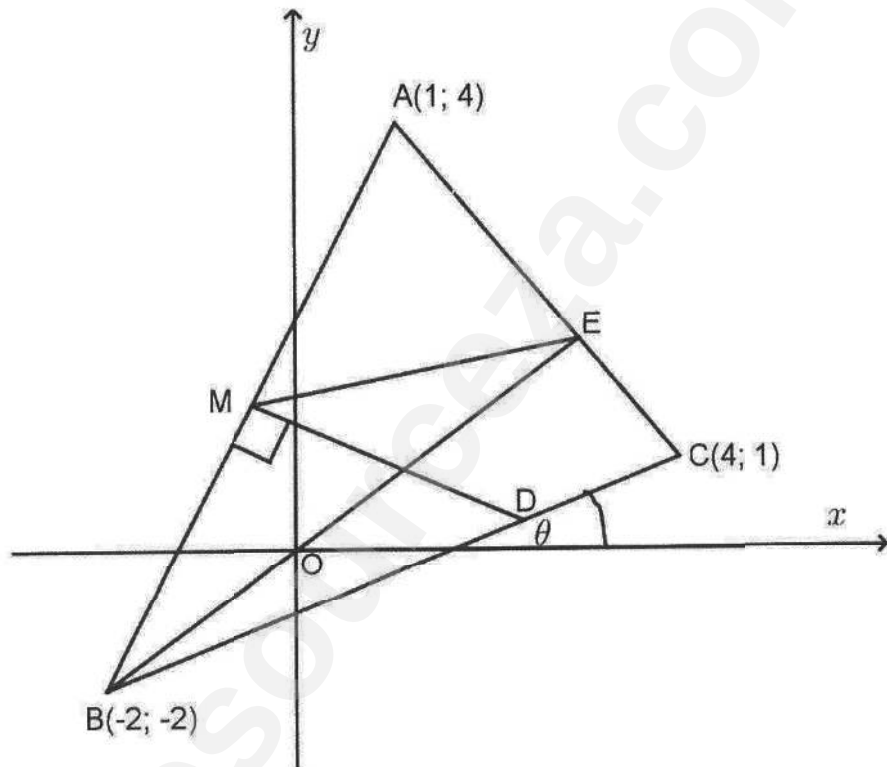
Age	Frequency	Cumulative Frequency
$5 < x \leq 15$	200	200
$15 < x \leq 25$	A	450
$25 < x \leq 35$	20	470
$35 < x \leq 45$	32	B
$45 < x \leq 55$	23	525
$55 < x \leq 65$	300	825
$65 < x \leq 75$	475	1300

- 2.1 Calculate the values of A and B in the table above. (2)
 - 2.2 Calculate the estimated mean age of the people who say they will buy the new product. (3)
 - 2.3 Find the modal class interval. (2)
 - 2.4 Sketch the ogive on the grid provided on the diagram sheet. (3)
 - 2.5 Use your sketch to answer the following question: (2)
- Is the data normally distributed? Give a reason for your answer.

[12]

QUESTION 3

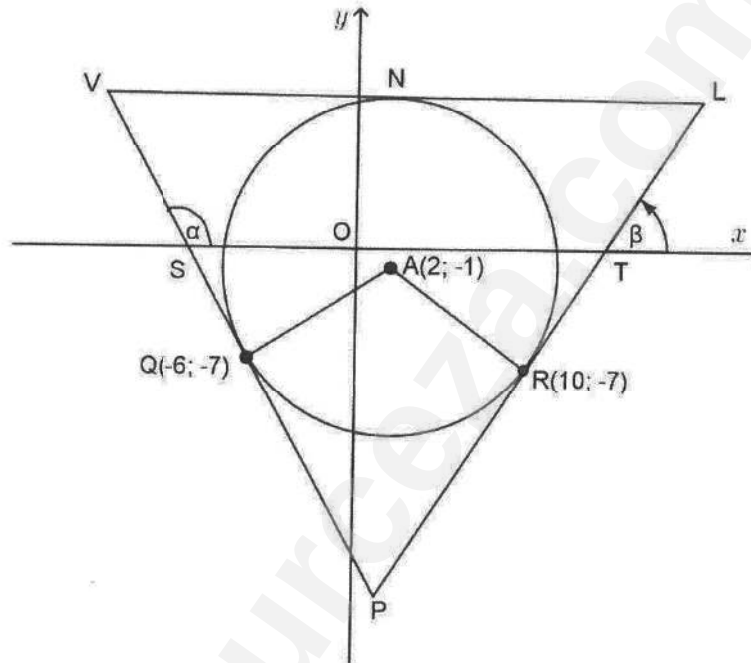
In the diagram, $A(1; 4)$, $B(-2; -2)$ and $C(4; 1)$ are vertices of $\triangle ABC$ in a Cartesian plane. M is the midpoint of AB . E is on AC , D is on BC such that $DM \perp AB$. ME and BE are joined. The inclination of line CB is θ .



- 3.1 Calculate the length of BC , in simplified surd form. (2)
- 3.2 Determine the co-ordinates of M , the midpoint of AB . (2)
- 3.3 Determine the equation of MD . (4)
- 3.4 If $ME \parallel BC$, calculate the co-ordinates of E , with reasons. (3)
- 3.5 Determine the equation of BE . (3)
- 3.6 Calculate the size of θ . (4)
- [18]**

QUESTION 4

PV, PL and VL are tangents to the circle, centre $A(2; -1)$ at Q, R and N respectively. The co-ordinates of Q and R are $(-6; -7)$ and $(10; -7)$ respectively. The inclination of PL is β and PV is α .



- 4.1 Calculate the length of AQ. (2)
- 4.2 Write down the equation of the circle whose centre is A in the form $(x - a)^2 + (y - b)^2 = r^2$. (2)
- 4.3 Calculate the gradients of:
- 4.3.1 QP. (2)
- 4.3.2 PR. (2)
- 4.4 Determine the equations of:
- 4.4.1 QP. (2)
- 4.4.2 PR. (2)
- 4.5.1 Calculate the co-ordinates of P. (4)
- 4.5.2 Express $\hat{P}$ in terms of α and β . (2)
- 4.5.3 If $\tan(\alpha - \beta) = \frac{\sin(\alpha - \beta)}{\cos(\alpha - \beta)}$, show that $\tan P = \frac{24}{7}$. (4)

[22]

QUESTION 5

- 5.1 If $4 \tan A = 3$ and $3 \sin B - 1 = 0$, where $180^\circ \leq A \leq 360^\circ$, $0^\circ \leq B \leq 90^\circ$. Use a sketch to determine the value of the following without using a calculator.

5.1.1 $\cos 2A$ (3)

5.1.2 $\sin (A + B)$ (3)

- 5.2 Simplify, without the use of a calculator:

$$\sin 20^\circ \cos 320^\circ + \cos (-20^\circ) \sin (400^\circ)$$

(3)

- 5.3 Prove the identity:

$$\frac{\cos^2 (90^\circ + \theta)}{\cos (-\theta) + \sin (90^\circ - \theta) \cos \theta} = \frac{1}{\cos \theta} - 1$$

(5)

- 5.4 It is given that

$$\begin{aligned} p &= \cos \alpha + \sin \alpha \\ q &= \cos \alpha - \sin \alpha \end{aligned}$$

deduce the following trigonometric ratios in terms of p and q .

5.4.1 $\cos 2\alpha$ (2)

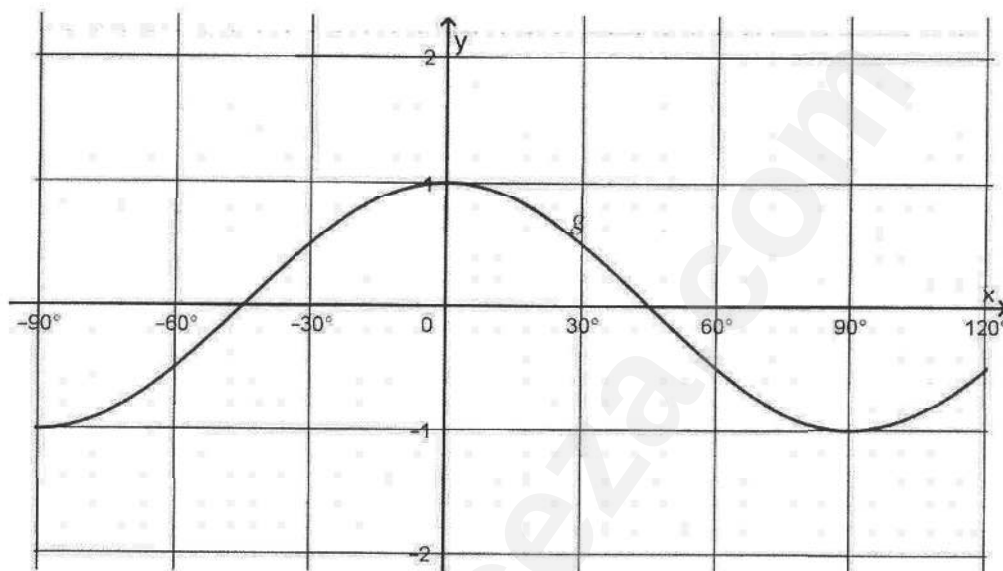
5.4.2 $\frac{1 + \sin 2\alpha}{\cos 2\alpha}$ (5)

- 5.5 Determine the general solution of $6 \cos^2 x + \sin x - 5 = 0$. (6)

[27]

QUESTION 6

In the diagram below, the graph of $g(x) = \cos 2x$, for $x \in [-90^\circ; 120^\circ]$ is drawn.



- 6.1 Draw the graph of $f(x) = \sin(x + 30^\circ)$ for $x \in [-90^\circ; 120^\circ]$ on the set of axes provided in the ANSWER BOOK. (3)
- 6.2 Determine the value(s) of x , $x \in [-90^\circ; 120^\circ]$ for which both graphs are decreasing (2)
- 6.3 Consider $h(x) = f(x + 60^\circ)$.
Describe the transformation the graph of f to obtain the graph of h . (2)
- [7]

QUESTION 7

A, B and C are three points in the same horizontal plane.

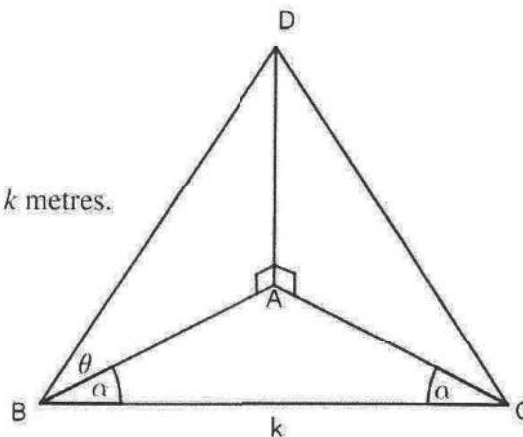
DA is a vertical cliff.

The angle of elevation of the top of the cliff from D is θ .

If $\hat{ABC} = \hat{ACB} = \alpha$ and the distance between B and C is k metres.

Prove that

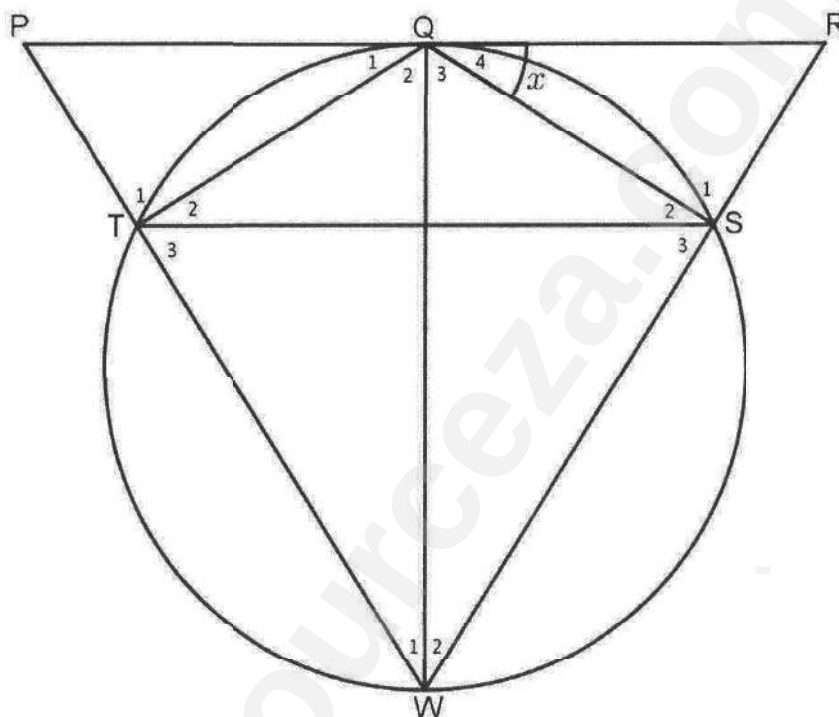
$$AD = \frac{k \tan \theta}{2 \cos \alpha}$$



[6]

QUESTION 8

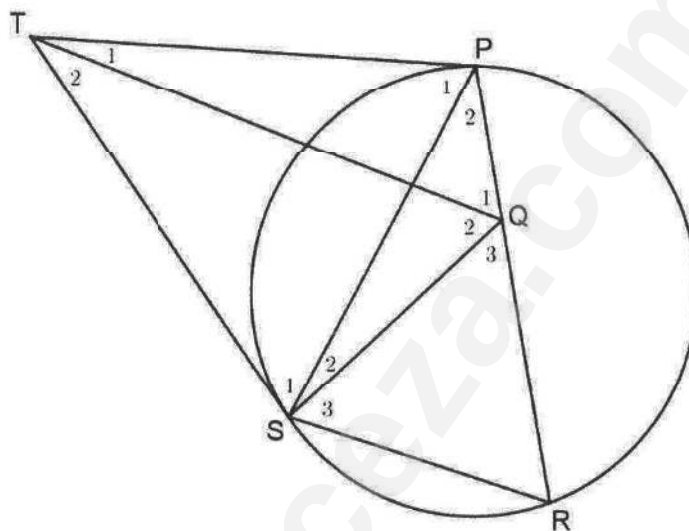
In the figure, TQSW is a cyclic quadrilateral with tangent PR touching the circle at Q.
WQ bisects $\hat{PWR}$. $\hat{Q}_4 = x$



- 8.1 Name with reasons 5 other angles each equal to x . (5)
- 8.2 Prove that:
- 8.2.1 $TS \parallel PR$ (2)
- 8.2.2 $\hat{Q}_3 = \hat{P}$ (3)
- 8.2.3 ΔTQS is an isosceles triangle (4)
- 8.2.4 $\hat{WQP} = \hat{T}_1$ (3)
- [17]

QUESTION 9

- 9.1 In the figure, TP and TS are tangents to the circle. R is a point on the circle and RS are drawn. Q is a point on PR such that $\widehat{TQP} = \widehat{TPS}$. SQ is drawn.



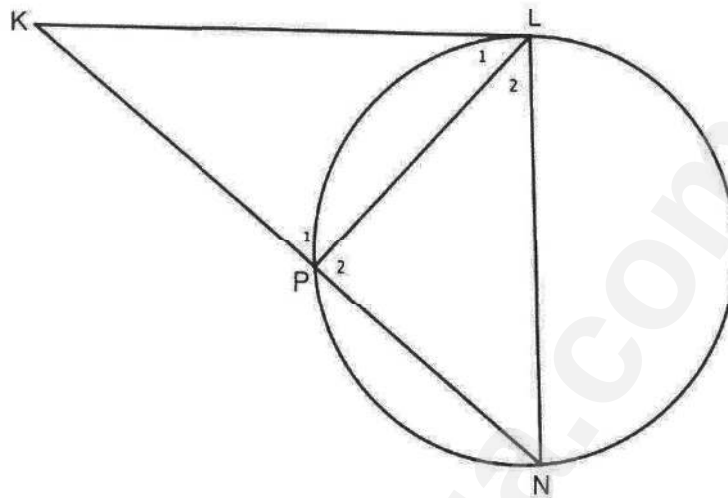
Prove that:

9.1.1 $TQ \parallel SR$ (3)

9.1.2 QPTS is a cyclic quadrilateral (3)

9.1.3 TQ bisects $\widehat{SQP}$. (3)

9.2 In the figure, LN is the diameter of the circle. KL is the tangent to the circle at L.



9.2.1 Prove that $\triangle LPK \sim \triangle NPL$ (4)

9.2.2 Show that $PL^2 = NP \cdot PK$ (3)

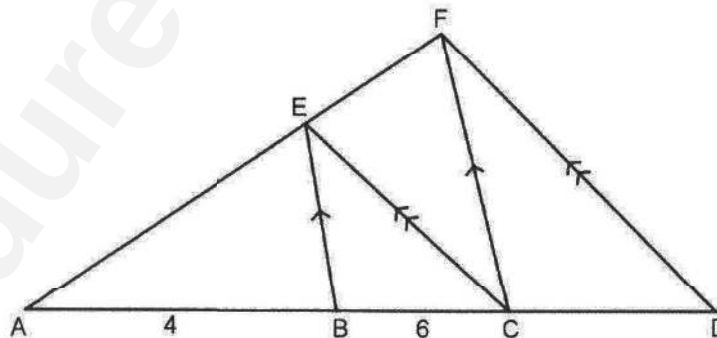
9.2.3 Name another triangle which is similar to $\triangle NPL$ (1)

9.2.4 Calculate the area of the circle if $NP = 10$ cm and $PK = 6$ cm (6)

[23]

QUESTION 10

In $\triangle ADF$, $DF \parallel CE$ and $CF \parallel BE$. If $AB = 4$ units and $BC = 6$ units, then calculate



10.1 the length of CD (3)

10.2 the numerical value of

$$\frac{\text{area of } \triangle FEC}{\text{area of } \triangle FAD} \quad (4)$$

[7]

TOTAL: 150



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS/ NATIONAL SENIOR CERTIFICATE EXAMINATIONS

MATHEMATICS P2

2019

MARKS: 150

TIME: 3 hours

MATHEMATICS: Paper 2



10612E

X05



This question paper consists of 14 pages, 1 information sheet
and an answer book of 25 pages.

QUESTION 1

Each child in a group of four-year-old children was given the same puzzle to complete. The time taken (in minutes) by each child to complete the puzzle is shown in the table below.

TIME TAKEN (t) (IN MINUTES)	NUMBER OF CHILDREN
$2 < t \leq 6$	2
$6 < t \leq 10$	10
$10 < t \leq 14$	9
$14 < t \leq 18$	7
$18 < t \leq 22$	8
$22 < t \leq 26$	7
$26 < t \leq 30$	2

- 1.1 How many children completed the puzzle? (1)
- 1.2 Calculate the estimated mean time taken to complete the puzzle. (2)
- 1.3 Complete the cumulative frequency column in the table given in the ANSWER BOOK. (2)
- 1.4 Draw a cumulative frequency graph (ogive) to represent the data on the grid provided in the ANSWER BOOK. (3)
- 1.5 Use the graph to determine the median time taken to complete the puzzle. (2)

[10]

QUESTION 2

Learners who scored a mark below 50% in a Mathematics test were selected to use a computer-based programme as part of an intervention strategy. On completing the programme, these learners wrote a second test to determine the effectiveness of the intervention strategy. The mark (as a percentage) scored by 15 of these learners in both tests is given in the table below.

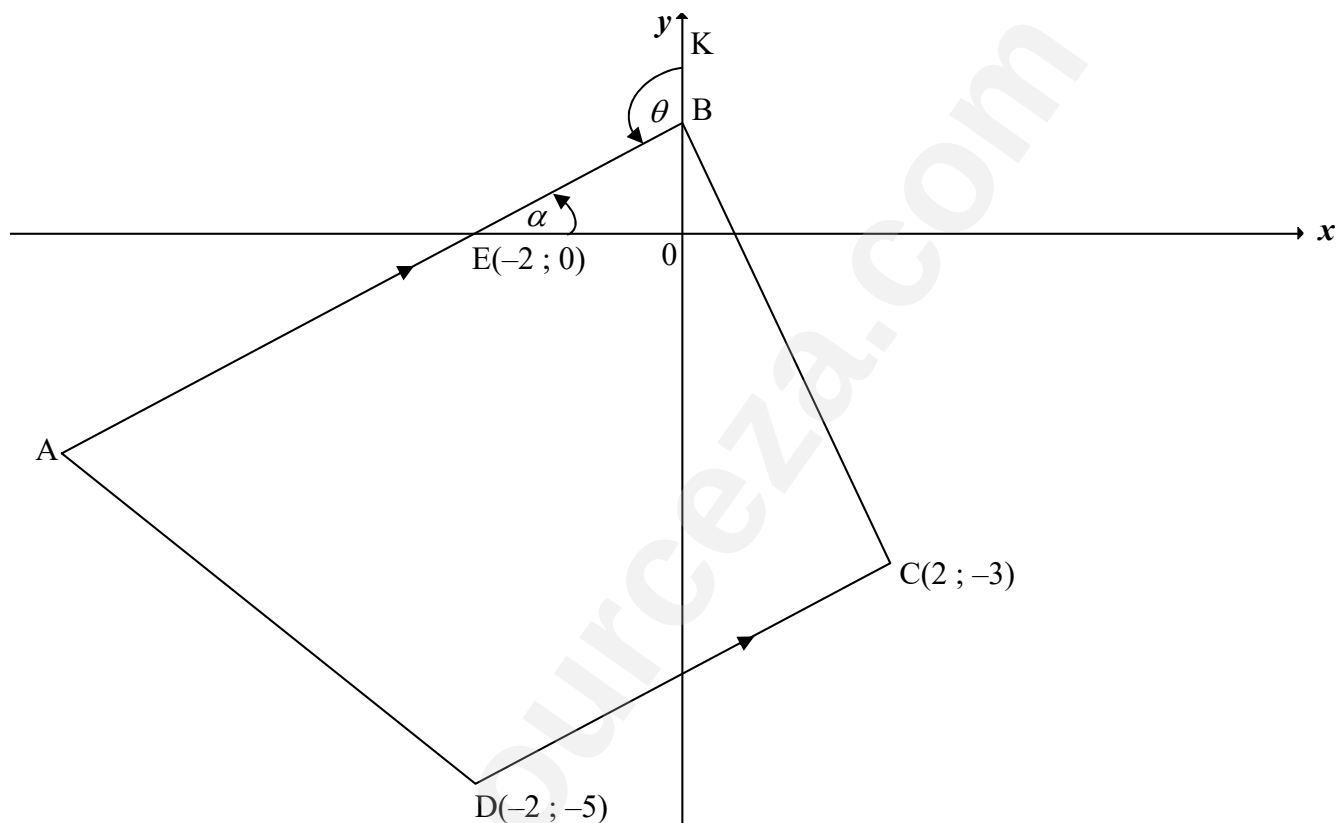
LEARNER	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15
TEST 1 (%)	10	18	23	24	27	34	34	36	37	39	40	44	45	48	49
TEST 2 (%)	33	21	32	20	58	43	49	48	41	55	50	45	62	68	60

- 2.1 Determine the equation of the least squares regression line. (3)
- 2.2 A learner's mark in the first test was 15 out of a maximum of 50 marks.
- 2.2.1 Write down the learner's mark for this test as a percentage. (1)
- 2.2.2 Predict the learner's mark for the second test. Give your answer to the nearest integer. (2)
- 2.3 For the 15 learners above, the mean mark of the second test is 45,67% and the standard deviation is 13,88%. The teacher discovered that he forgot to add the marks of the last question to the total mark of each of these learners. All the learners scored full marks in the last question. When the marks of the last question are added, the new mean mark is 50,67%.
- 2.3.1 What is the standard deviation after the marks for the last question are added to each learner's total? (2)
- 2.3.2 What is the total mark of the last question? (2)
- [10]**



QUESTION 3

In the diagram, A, B, C(2 ; -3) and D(-2 ; -5) are vertices of a trapezium with $AB \parallel DC$. E(-2 ; 0) is the x-intercept of AB. The inclination of AB is α . K lies on the y-axis and $\angle KBE = \theta$.



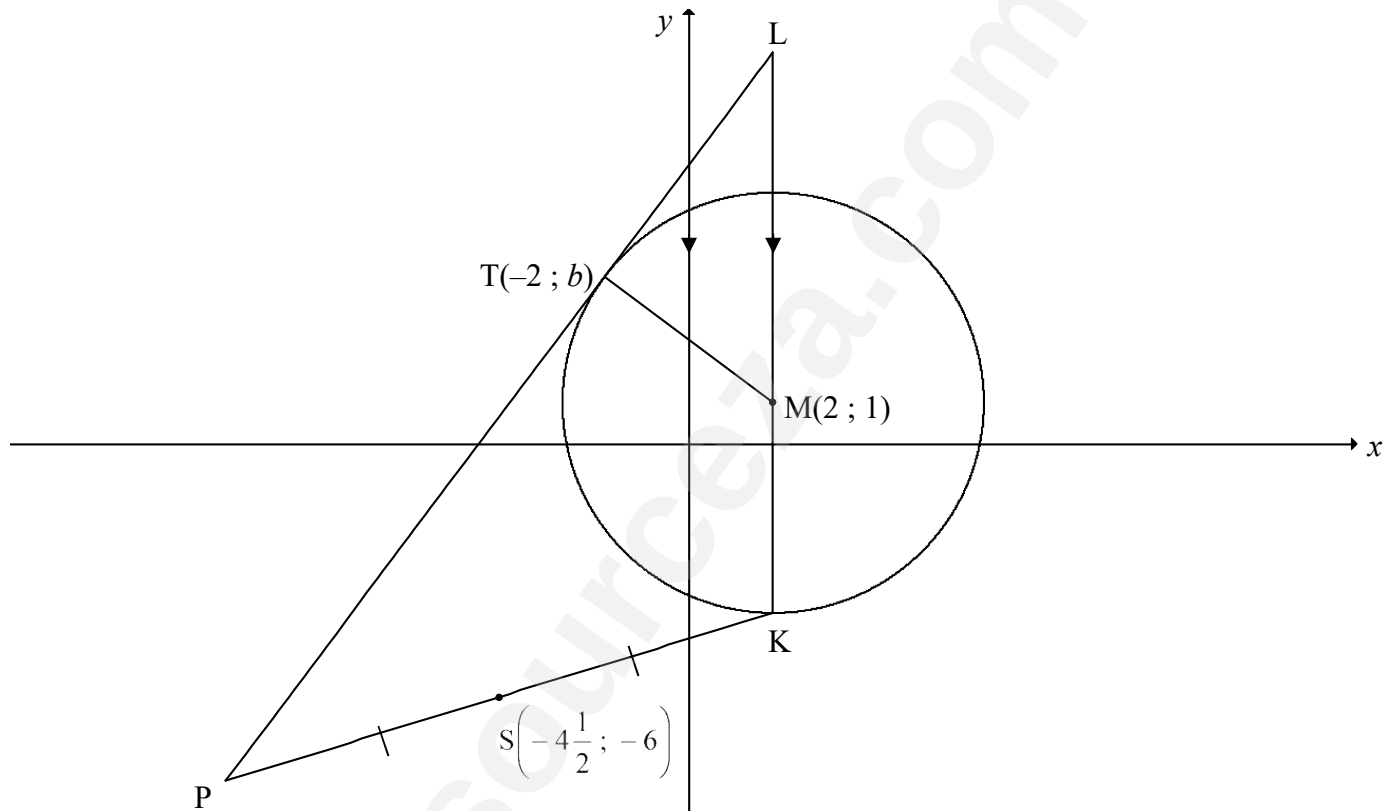
- 3.1 Determine:
- 3.1.1 The midpoint of EC (2)
 - 3.1.2 The gradient of DC (2)
 - 3.1.3 The equation of AB in the form $y = mx + c$ (3)
 - 3.1.4 The size of θ (3)
- 3.2 Prove that $AB \perp BC$. (3)
- 3.3 The points E, B and C lie on the circumference of a circle. Determine:
- 3.3.1 The centre of the circle (1)
 - 3.3.2 The equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$ (4)
- [18]



QUESTION 4

In the diagram, the circle is centred at $M(2; 1)$. Radius KM is produced to L , a point outside the circle, such that $KML \parallel y$ -axis. LTP is a tangent to the circle at $T(-2; b)$.

$S\left(-4\frac{1}{2}; -6\right)$ is the midpoint of PK .



- 4.1 Given that the radius of the circle is 5 units, show that $b = 4$. (4)
- 4.2 Determine:
- 4.2.1 The coordinates of K (2)
- 4.2.2 The equation of the tangent LTP in the form $y = mx + c$ (4)
- 4.2.3 The area of $\triangle LPK$ (7)
- 4.3 Another circle with equation $(x - 2)^2 + (y - n)^2 = 25$ is drawn. Determine, with an explanation, the value(s) of n for which the two circles will touch each other externally. (4)
- [21]



QUESTION 5

5.1 **Without using a calculator**, write the following expressions in terms of $\sin 11^\circ$:

5.1.1 $\sin 191^\circ$ (1)

5.1.2 $\cos 22^\circ$ (1)

5.2 Simplify $\cos(x - 180^\circ) + \sqrt{2} \sin(x + 45^\circ)$ to a single trigonometric ratio. (5)

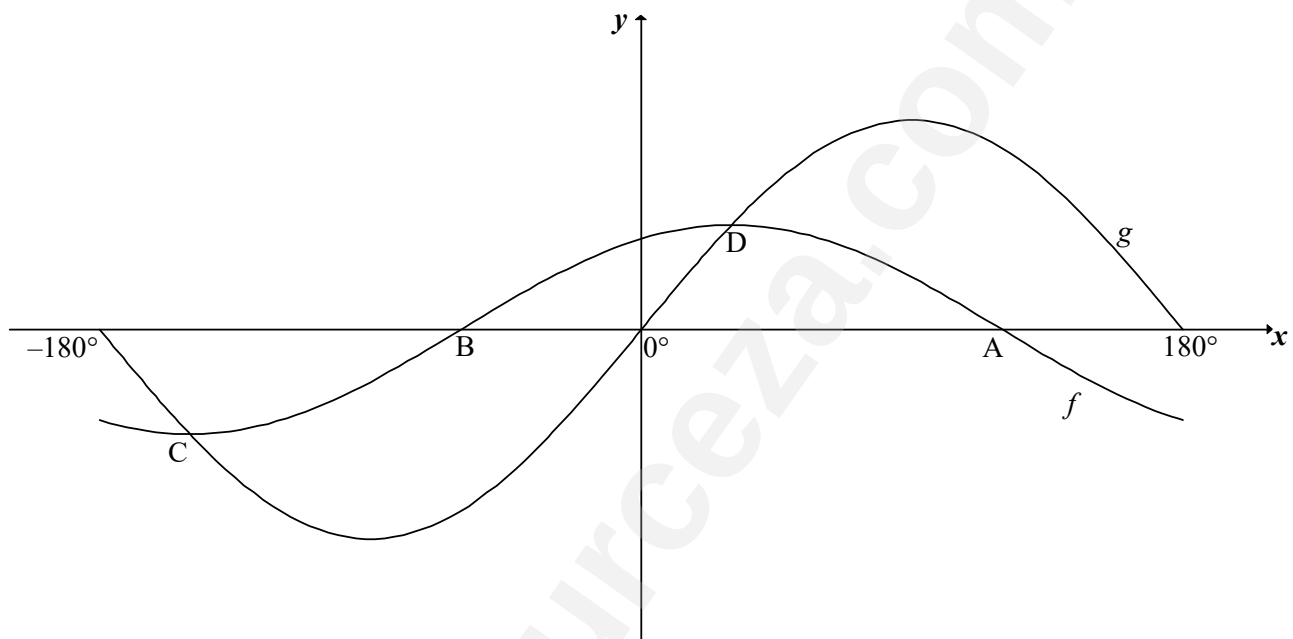
5.3 Given: $\sin P + \sin Q = \frac{7}{5}$ and $\hat{P} + \hat{Q} = 90^\circ$
Without using a calculator, determine the value of $\sin 2P$. (5)
[12]



QUESTION 6

6.1 Determine the general solution of $\cos(x - 30^\circ) = 2 \sin x$. (6)

6.2 In the diagram, the graphs of $f(x) = \cos(x - 30^\circ)$ and $g(x) = 2 \sin x$ are drawn for the interval $x \in [-180^\circ; 180^\circ]$. A and B are the x -intercepts of f . The two graphs intersect at C and D, the minimum and maximum turning points respectively of f .



6.2.1 Write down the coordinates of:

(a) A (1)

(b) C (2)

6.2.2 Determine the values of x in the interval $x \in [-180^\circ; 180^\circ]$, for which:

(a) Both graphs are increasing (2)

(b) $f(x + 10^\circ) > g(x + 10^\circ)$ (2)

6.2.3 Determine the range of $y = 2^{\sin x + 3}$ (5)

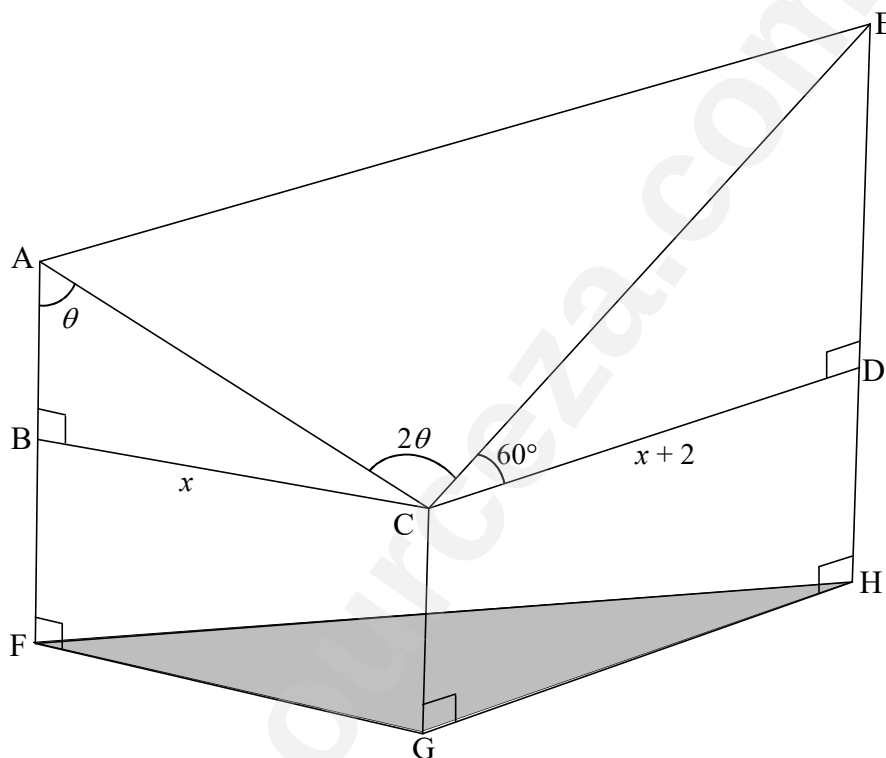
[18]



QUESTION 7

In the diagram below, CGFB and CGHD are fixed walls that are rectangular in shape and vertical to the horizontal plane FGH. Steel poles erected along FB and HD extend to A and E respectively. $\triangle ACE$ forms the roof of an entertainment centre.

$BC = x$, $CD = x + 2$, $\angle BAC = \theta$, $\angle ACE = 2\theta$ and $\angle ECD = 60^\circ$

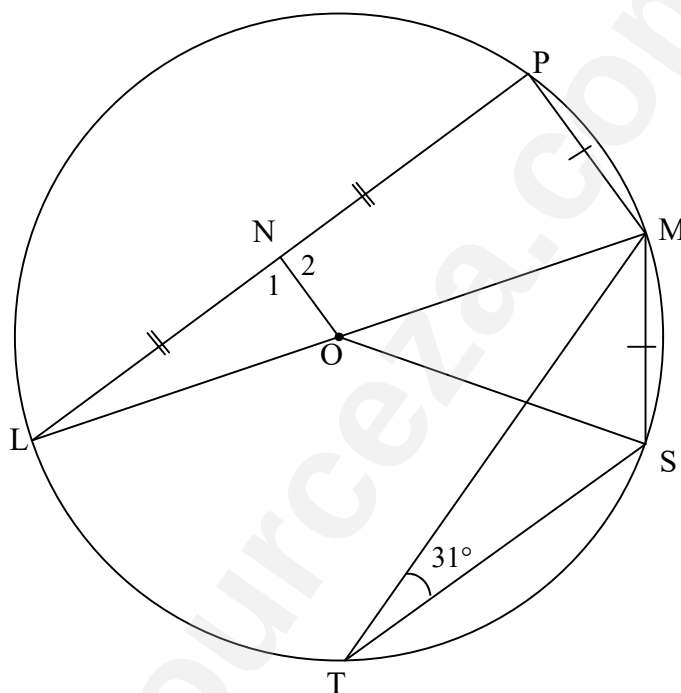


- 7.1 Calculate the length of:
- 7.1.1 AC in terms of x and θ (2)
- 7.1.2 CE in terms of x (2)
- 7.2 Show that the area of the roof $\triangle ACE$ is given by $2x(x+2)\cos\theta$. (3)
- 7.3 If $\theta = 55^\circ$ and $BC = 12$ metres, calculate the length of AE. (4)
- [11]



QUESTION 8

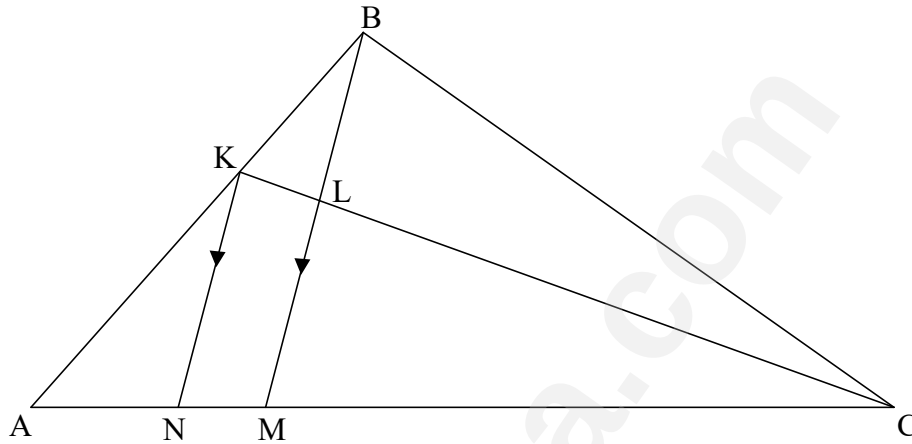
- 8.1 In the diagram, O is the centre of the circle and LOM is a diameter of the circle. ON bisects chord LP at N. T and S are points on the circle on the other side of LM with respect to P. Chords PM, MS, MT and ST are drawn. $PM = MS$ and $\hat{M}TS = 31^\circ$



- 8.1.1 Determine, with reasons, the size of each of the following angles:
- (a) $\hat{MOS}$ (2)
- (b) $\hat{L}$ (2)
- 8.1.2 Prove that $ON = \frac{1}{2} MS$. (4)



- 8.2 In $\triangle ABC$ in the diagram, K is a point on AB such that $AK : KB = 3 : 2$. N and M are points on AC such that $KN \parallel BM$. BM intersects KC at L . $AM : MC = 10 : 23$.



Determine, with reasons, the ratio of:

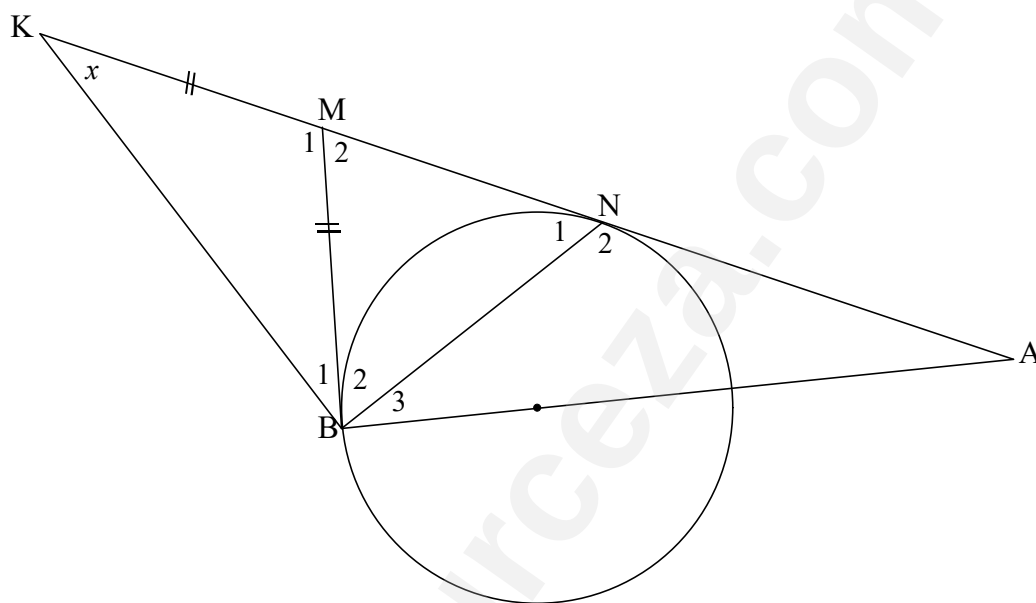
- 8.2.1 $\frac{AN}{AM}$ (2)
- 8.2.2 $\frac{CL}{LK}$ (3)
- [13]



QUESTION 9

In the diagram, tangents are drawn from point M outside the circle, to touch the circle at B and N . The straight line from B passing through the centre of the circle meets MN produced in A . NM is produced to K such that $BM = MK$. BK and BN are drawn.

Let $\hat{K} = x$.

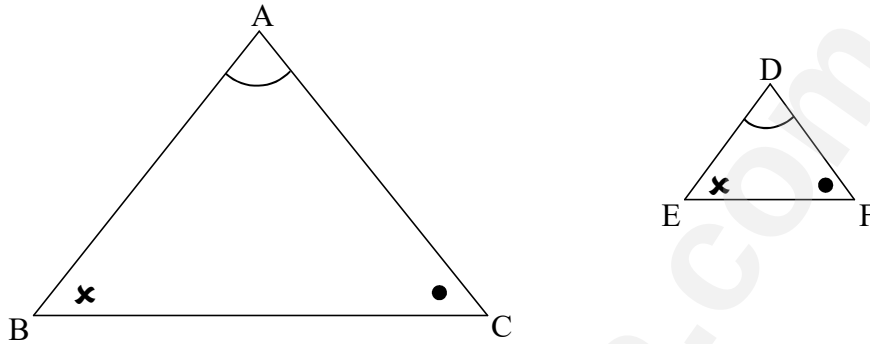


- 9.1 Determine, with reasons, the size of $\hat{N}_1$ in terms of x . (6)
- 9.2 Prove that BA is a tangent to the circle passing through K , B and N . (5)
- [11]



QUESTION 10

10.1 In the diagram, $\triangle ABC$ and $\triangle DEF$ are drawn such that $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.

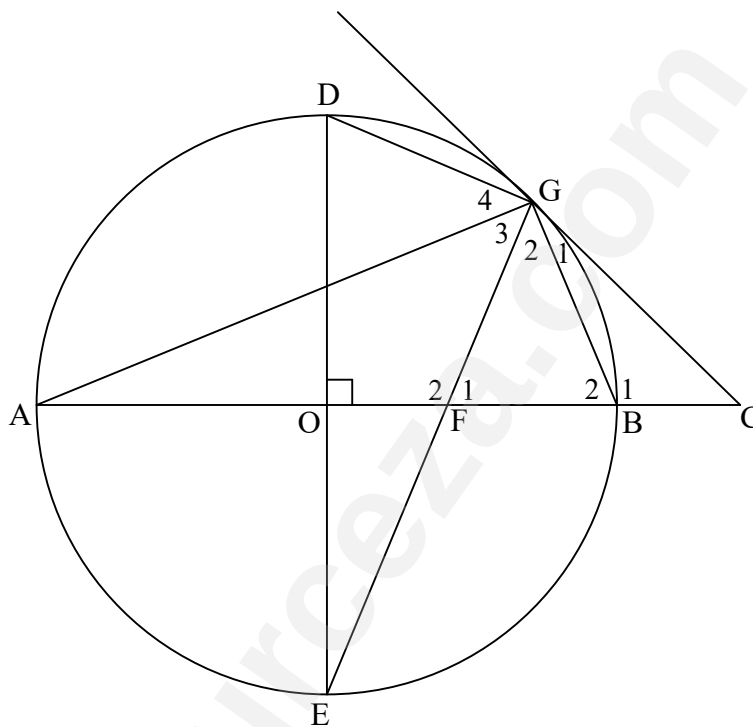


Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, that is $\frac{AB}{DE} = \frac{AC}{DF}$.

(6)



- 10.2 In the diagram, O is the centre of the circle and CG is a tangent to the circle at G . The straight line from C passing through O cuts the circle at A and B . Diameter DOE is perpendicular to CA . GE and CA intersect at F . Chords DG , BG and AG are drawn.



10.2.1 Prove that:

- (a) $DGFO$ is a cyclic quadrilateral (3)
- (b) $GC = CF$ (5)

10.2.2 If it is further given that $CO = 11$ units and $DE = 14$ units, calculate:

- (a) The length of BC (3)
- (b) The length of CG (5)
- (c) The size of $\hat{E}$. (4)

[26]

TOTAL: 150





basic education

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Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS/ NATIONAL SENIOR CERTIFICATE EXAMINATIONS

MATHZ.1

MATHEMATICS P1

MAY/JUNE 2019

MARKS: 150

TIME: 3 hours

This question paper consists of 8 pages and 1 information sheet.

AFTERNOON SESSION

QUESTION 11.1 Solve for x :

1.1.1 $x^2 - 5x - 6 = 0$ (2)

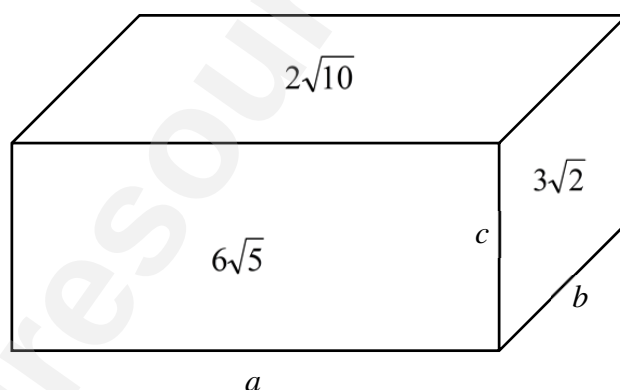
1.1.2 $(3x - 1)(x - 4) = 16$ (correct to TWO decimal places) (4)

1.1.3 $4x - x^2 \geq 0$ (3)

1.1.4 $\frac{5^{2x} - 1}{5^x + 1} = 4$ (3)

1.2 Solve simultaneously for x and y :

$x + 3y = 2$ and $x^2 + 4xy - 5 = 0$ (5)

1.3 A rectangular box has dimensions a , b and c . The area of the surfaces are $2\sqrt{10}$; $3\sqrt{2}$ and $6\sqrt{5}$, as shown in the diagram below.Calculate, **without using a calculator**, the volume of the rectangular box.(5)
[22]**QUESTION 2**

2.1 The first FOUR terms of a quadratic pattern are: 15 ; 29 ; 41 ; 51

2.1.1 Write down the value of the 5th term. (1)2.1.2 Determine an expression for the n^{th} term of the pattern in the form $T_n = an^2 + bn + c$. (4)2.1.3 Determine the value of T_{27} (2)

2.2 Given a geometric sequence: $36 ; -18 ; 9 ; \dots$

2.2.1 Determine the value of r , the common ratio. (1)

2.2.2 Calculate n if $T_n = \frac{9}{4\,096}$ (3)

2.2.3 Calculate S_∞ (2)

2.2.4 Calculate the value of $\frac{T_1 + T_3 + T_5 + T_7 + \dots + T_{499}}{T_2 + T_4 + T_6 + T_8 + \dots + T_{500}}$ (4)

[17]

QUESTION 3

3.1 The first three terms of an arithmetic sequence are: $2p + 3 ; p + 6$ and $p - 2$.

3.1.1 Show that $p = 11$. (2)

3.1.2 Calculate the smallest value of n for which $T_n < -55$. (3)

3.2 Given that $\sum_{k=1}^6 (x - 3k) = \sum_{k=1}^9 (x - 3k)$, prove that $\sum_{k=1}^{15} (x - 3k) = 0$. (5)

[10]

QUESTION 4

Given the exponential function: $g(x) = \left(\frac{1}{2}\right)^x$

4.1 Write down the range of g . (1)

4.2 Determine the equation of g^{-1} in the form $y = \dots$ (2)

4.3 Is g^{-1} a function? Justify your answer. (2)

4.4 The point $M(a ; 2)$ lies on g^{-1} .

4.4.1 Calculate the value of a . (2)

4.4.2 M' , the image of M , lies on g . Write down the coordinates of M' . (1)

4.5 If $h(x) = g(x + 3) + 2$, write down the coordinates of the image of M' on h . (3)

[11]

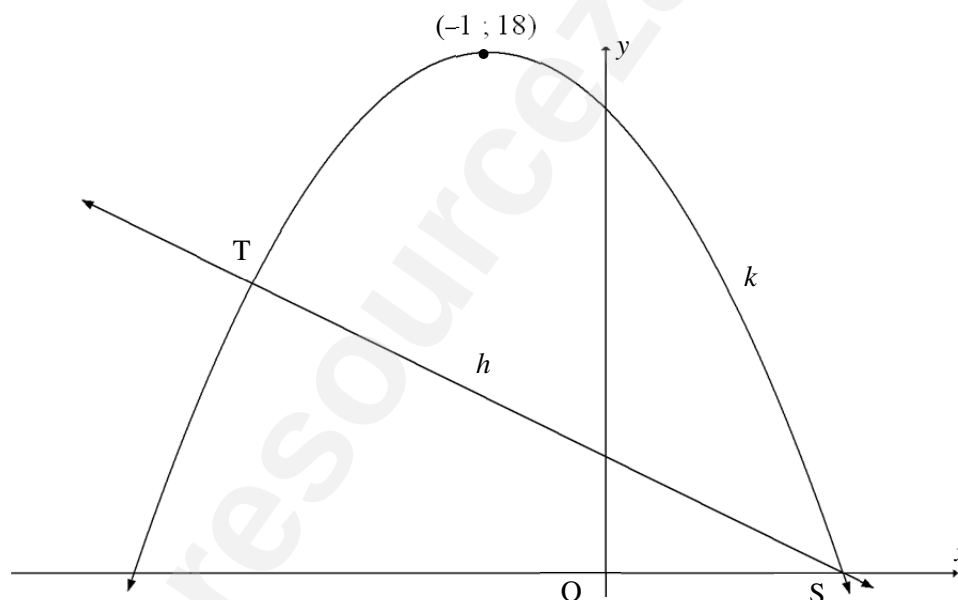


QUESTION 5

5.1 Given: $f(x) = \frac{1}{x+2} + 3$

- 5.1.1 Determine the equations of the asymptotes of f . (2)
- 5.1.2 Write down the y -intercept of f . (1)
- 5.1.3 Calculate the x -intercept of f . (2)
- 5.1.4 Sketch the graph of f . Clearly label ALL intercepts with the axes and any asymptotes. (3)

- 5.2 Sketched below are the graphs of $k(x) = ax^2 + bx + c$ and $h(x) = -2x + 4$. Graph k has a turning point at $(-1 ; 18)$. S is the x -intercept of h and k . Graphs h and k also intersect at T.



- 5.2.1 Calculate the coordinates of S. (2)
- 5.2.2 Determine the equation of k in the form $y = a(x + p)^2 + q$. (3)
- 5.2.3 If $k(x) = -2x^2 - 4x + 16$, determine the coordinates of T. (5)
- 5.2.4 Determine the value(s) of x for which $k(x) < h(x)$. (2)
- 5.2.5 It is further given that k is the graph of $g'(x)$.
- (a) For which values of x will the graph of g be concave up? (2)
- (b) Sketch the graph of g , showing clearly the x -values of the turning points and the point of inflection. (3)

[25]



QUESTION 6

- 6.1 Sandile bought a car for R180 000. The value of the car depreciated at 15% per annum according to the reducing-balance method. The book value of Sandile's car is currently R79 866,96.
- 6.1.1 How many years ago did Sandile buy the car? (3)
- 6.1.2 At exactly the same time that Sandile bought the car, Anil deposited R49 000 into a savings account at an interest rate of 10% p.a., compounded quarterly. Has Anil accumulated enough money in his savings account to buy Sandile's car now? (3)
- 6.2 Exactly 10 months ago, a bank granted Jane a loan of R800 000 at an interest rate of 10,25% p.a., compounded monthly.
The bank stipulated that the loan:
- Must be repaid over 20 years
 - Must be repaid by means of monthly repayments of R7 853,15, starting one month after the loan was granted
- 6.2.1 How much did Jane owe immediately after making her 6th repayment? (4)
- 6.2.2 Due to financial difficulties, Jane missed the 7th, 8th and 9th payments. She was able to make payments from the end of the 10th month onwards. Calculate Jane's increased monthly payment in order to settle the loan in the original 20 years. (5)
[15]

QUESTION 7

- 7.1 Given $f(x) = x^2 + 2$.
Determine $f'(x)$ from first principles. (4)
- 7.2 Determine $\frac{dy}{dx}$ if:
- 7.2.1 $y = 4x^3 + \frac{2}{x}$ (3)
- 7.2.2 $y = 4\sqrt[3]{x} + (3x^3)^2$ (4)
- 7.3 If g is a linear function with $g(1) = 5$ and $g'(3) = 2$, determine the equation of g in the form $y = \dots$ (3)
[14]



QUESTION 8

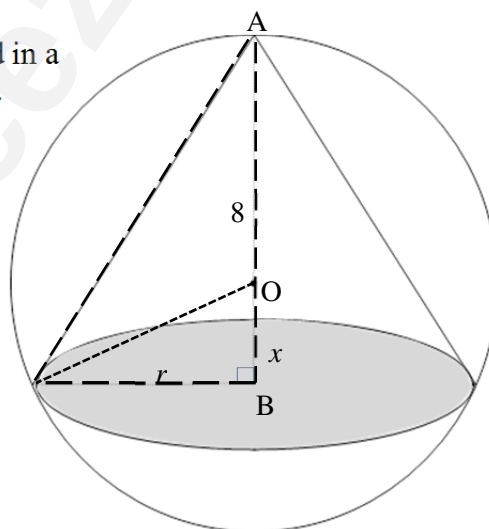
A cubic function $h(x) = -2x^3 + bx^2 + cx + d$ cuts the x -axis at $(-3; 0)$; $\left(-\frac{3}{2}; 0\right)$ and $(1; 0)$.

- 8.1 Show that $h(x) = -2x^3 - 7x^2 + 9$. (3)
- 8.2 Calculate the x -coordinates of the turning points of h . (3)
- 8.3 Determine the value(s) of x for which h will be decreasing. (2)
- 8.4 For which value(s) of x will there be a tangent to the curve of h that is parallel to the line $y - 4x = 7$. (4)
- [12]

QUESTION 9

A cone with radius r cm and height AB is inscribed in a sphere with centre O and a radius of 8 cm. $OB = x$.

$\text{Volume of sphere} = \frac{4}{3}\pi r^3$ $\text{Volume of cone} = \frac{1}{3}\pi r^2 h$



- 9.1 Calculate the volume of the sphere. (1)
- 9.2 Show that $r^2 = 64 - x^2$. (1)
- 9.3 Determine the ratio between the largest volume of this cone and the volume of the sphere. (7)
- [9]



QUESTION 10

- 10.1 A bag contains 7 yellow balls, 3 red balls and 2 blue balls. A ball is chosen at random from the bag and not replaced. A second ball is then chosen. Determine the probability that of the two balls chosen, one is red and the other is blue. (4)
- 10.2 Learners at a hostel may choose a meal and a drink for lunch. Their selections on a certain day were recorded and shown in the partially completed table below.

		MEAL		TOTAL
		SANDWICH (S)	PASTA (P)	
DRINK	Fruit Juice (F)	a	30	b
	Bottled Water (W)			
TOTAL		200		250

The probability of a learner choosing fruit juice and a sandwich on that day was 0,48.

- 10.2.1 Calculate the number of learners who chose fruit juice and a sandwich for lunch on that day. (1)
- 10.2.2 Is the choice of fruit juice independent of the choice of a sandwich for lunch on that day? Show ALL calculations to motivate your answer. (4)
- [9]

QUESTION 11

Two learners from each grade at a high school (Grades 8, 9, 10, 11 and 12) are elected to form a sports committee.

- 11.1 In how many different ways can the chairperson and the deputy chairperson of the sports committee be elected if there is no restriction on who may be elected? (2)
- 11.2 A photographer wants to take a photograph of the sports committee. In how many different ways can the members be arranged in a straight line if:
- 11.2.1 Any member may stand in any position? (1)
- 11.2.2 Members from the same grade must stand next to each other and the Grade 12 members must be in the centre? (3)
- [6]

TOTAL: 150





basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS SENIORSERTIFIKAAT-EKSAMEN

MATHEMATICS P1/WISKUNDE V1

2018

MARKING GUIDELINES/NASIENRIGLYNE

MARKS: 150
PUNTE: 150

These marking guidelines consist of 19 pages.
Hierdie nasienriglyne bestaan uit 19 bladsye.

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1.1.4	$(x-1)(x-4) > x+11$ $x^2 - 5x + 4 > x + 11$ $x^2 - 6x - 7 > 0$ $(x-7)(x+1) > 0$	$\checkmark x^2 - 5x + 4$ $\checkmark$ standard form $\checkmark$ factors
	$x < -1$ or $x > 7$	$\checkmark \checkmark x < -1$ or $x > 7$ (5)
1.2	$\frac{4\sqrt{x^7} - 5\sqrt{x^7}}{\sqrt{x}}$ $= \frac{-\sqrt{x^7}}{\sqrt{x}}$ $= \frac{-x^{\frac{7}{2}}}{x^{\frac{1}{2}}}$ $= -x^3$	$\checkmark 4\sqrt{x^7} - 5\sqrt{x^7}$ $\checkmark -\sqrt{x^7}$ $\checkmark -x^3$ (3)
	OR/OF $\frac{\sqrt{x^7}(4-5)}{\sqrt{x}}$ $= \sqrt{x^6}(-1)$ $= -x^3$	OR/OF $\checkmark \sqrt{x^7}(4-5)$ $\checkmark \sqrt{x^6}(-1)$ $\checkmark -x^3$ (3)
	OR/OF $\frac{(16x^7)^{\frac{1}{2}} - (25x^7)^{\frac{1}{2}}}{x^{\frac{1}{2}}}$ $= \frac{4x^{\frac{7}{2}} - 5x^{\frac{7}{2}}}{x^{\frac{1}{2}}}$ $= \frac{-x^{\frac{7}{2}}}{x^{\frac{1}{2}}}$ $= -x^3$	OR/OF $\checkmark \frac{4x^{\frac{7}{2}} - 5x^{\frac{7}{2}}}{x^{\frac{1}{2}}}$ $\checkmark -x^{\frac{7}{2}} \cdot \frac{1}{x^{\frac{1}{2}}}$ $\checkmark -x^3$ (3)

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NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- Consistent accuracy applies in ALL aspects of the marking guidelines.

LET WEL:

- Indien 'n kandidaat 'n vraag TWEE KEER beantwoord, merk slegs die EERSTE poging.
- Volgehoue akkuraatheid is op ALLE aspekte van die nasienriglyne van toepassing.

QUESTION/VRAAG 1

1.1.1	$(3x-1)(x+4) = 0$ $x = \frac{1}{3}$ or $x = -4$	$\checkmark x = \frac{1}{3}$ $\checkmark x = -4$ (2)
1.1.2	$2x^2 + 9x - 14 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-9 \pm \sqrt{9^2 - 4(2)(-14)}}{2(2)}$ $= \frac{-9 \pm \sqrt{193}}{4}$ $x = 1,22$ or $x = -5,72$	$\checkmark$ substitution into correct formula $\checkmark$ simplification $\checkmark x = 1,22$ $\checkmark x = -5,72$ (4)
	OR/OF $x^2 + \frac{9}{2}x + \frac{81}{16} = 7 + \frac{81}{16}$ $\left(x + \frac{9}{4}\right)^2 = \frac{193}{16}$ $x + \frac{9}{4} = \pm \sqrt{\frac{193}{16}}$ $x = \frac{-9 \pm \sqrt{193}}{4}$ $x = 1,22$ or $x = -5,72$	OR/OF $\checkmark$ for adding $\frac{81}{16}$ on both sides $\checkmark$ simplification $\checkmark x = 1,22$ $\checkmark x = -5,72$ (4)
1.1.3	$\sqrt{3-26x} = 3x$ $3-26x = 9x^2$ $9x^2 + 26x - 3 = 0$ $(9x-1)(x+3) = 0$ $x = \frac{1}{9}$ or $x = -3$ N/A	$\checkmark 3-26x = 9x^2$ $\checkmark$ standard form $\checkmark$ factors $\checkmark$ answer with selection

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1.3	$x - 2y - 3 = 0$ $x = 2y + 3$(1) $xy = 9$(2) Substitute (1) into (2) $(2y+3)y = 9$ $2y^2 + 3y = 9$ $2y^2 + 3y - 9 = 0$ $(2y-3)(y+3) = 0$ $y = \frac{3}{2}$ or $y = -3$ $x = 6$ or $x = -3$	$\checkmark x = 2y + 3$ $\checkmark$ substitution $\checkmark$ standard form $\checkmark y$ -values $\checkmark x$ -values (5)
	OR/OF $y = \frac{x-3}{2}$(1) $xy = 9$(2) Substitute (1) into (2) $x\left(\frac{x-3}{2}\right) = 9$ $x^2 - 3x = 18$ $x^2 - 3x - 18 = 0$ $(x-6)(x+3) = 0$ $x = 6$ or $x = -3$ $y = \frac{3}{2}$ or $y = -3$	$\checkmark$ substitution $\checkmark$ standard form $\checkmark x$ -values $\checkmark y$ -values (5)
	OR/OF $x - 2y - 3 = 0$ $x = 2y + 3$(1) $y = \frac{9}{x}$(2) Substitute (2) into (1) $x = 2\left(\frac{9}{x}\right) + 3$ $x^2 - 2(9) - 3x = 0$ $x^2 - 3x - 18 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-3 \pm \sqrt{(-3)^2 - 4(1)(-18)}}{2(1)}$ $= \frac{-3 \pm \sqrt{81}}{2}$ $x = 6$ or $x = -3$	OR/OF $\checkmark y = \frac{x-3}{2}$ $\checkmark$ substitution $\checkmark$ standard form $\checkmark x$ -values $\checkmark y$ -values (5)

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1.4	$x^2 + 2xy + 2y^2$ $= x^2 + 2xy + y^2 + y^2$ $= (x+y)^2 + y^2$ $(x+y)^2 \geq 0$ and $y^2 \geq 0$ Therefore $(x+y)^2 + y^2 \geq 0$	✓ $x^2 + 2xy + y^2 + y^2$ ✓ $(x+y)^2$ ✓ $(x+y)^2 \geq 0$ and $y^2 \geq 0$ ✓ $(x+y)^2 + y^2 \geq 0$ (4) [27]
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QUESTION/VRAAG 2

2.1.1	37; 50	✓ 37 ✓ 50 (2)
2.1.2	$a = \frac{\text{second difference}}{2} = \frac{2}{2} = 1$ $3a + b = 5$ $3 + b = 5$ $b = 2$ $a + b + c = 5$ $1 + 2 + c = 5$ $c = 2$ $T_n = an^2 + bn + c$ $= n^2 + 2n + 2$	✓ second difference of 2 ✓ $a = 1$ ✓ $b = 2$ ✓ $c = 2$ (4)
2.1.3	$n^2 + 2n + 2 = 1765$ $n^2 + 2n - 1763 = 0$ $(n + 43)(n - 41) = 0$ $n = -43$ or $n = 41$ N/A OR/OF $n^2 + 2n + 2 = 1765$ $n^2 + 2n - 1763 = 0$ $n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-2 \pm \sqrt{2^2 - 4(1)(-1763)}}{2(1)}$ $= \frac{-2 \pm \sqrt{7056}}{2}$ $n = -43$ or $n = 41$ N/A	✓ equating T_n to 1765 ✓ standard form ✓ factors ✓ answer with rejection (4) OR/OF ✓ equating T_n to 1765 ✓ standard form ✓ sub in correct formula ✓ answer with rejection (4)

2.2	Sum of multiples of 7 from 35 to 196: <i>Som van meervoudes van 7 vanaf 35 tot by 196:</i> $a = 35$; $d = 7$ $S_n = \frac{n}{2}[a + \ell]$ $= \frac{24}{2}[35 + 196]$ $= 12[231]$ $= 2772$ Sum of all natural numbers from 35 to 196: <i>Som van alle natuurlike getalle vanaf 35 tot by 196:</i> $a = 35$; $d = 1$; $n = 162$ $S_n = \frac{n}{2}[a + \ell]$ $= \frac{162}{2}[35 + 196]$ $= 81[231]$ $= 18\,711$ Sum of numbers not divisible by 7/ <i>Som van getalle nie deelbaar deur 7</i> $= 18\,711 - 2772$ $= 15\,939$	✓ correct a , d and n substitution into correct formula ✓ answer ✓ 162 ✓ answer ✓ answer (5) [15]
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QUESTION/VRAAG 3

3.1	$r = 0,94$; $a = 100$ $T_3 = ar^2$ $= 100(0,94)^2$ $= 88,36$ km	✓ $r = 0,94$ ✓ answer (2)
3.2	$S_n = \frac{a(r^n - 1)}{r - 1}$ $750 = \frac{100(0,94^n - 1)}{0,94 - 1}$ $\frac{750(-0,06)}{100} = 0,94^n - 1$ $0,94^n = 1 - \frac{9}{20}$ or $\left(\frac{47}{50}\right)^n = \frac{11}{20}$ $0,94^n = 0,55$ $n = \frac{\log 0,55}{\log 0,94}$ $= 9,66$ He will pass the halfway point on the 10 th day <i>Hy sal die halfpadmerk verbystek op die 10^{de} dag</i>	✓ substitution into correct formula ✓ $0,94^n = 0,55$ ✓ use of logarithms ✓ answer (4)
3.3	$S_\infty = \frac{a}{1 - r}$ $1500 < \frac{100}{1 - r}$ $1 - r < \frac{100}{1500}$ $r > \frac{14}{15}$ or 93,33%	✓ use of S_∞ formula ✓ substitution ✓ answer (3) [9]

QUESTION/VRAAG 4

4.1	$0 < x \leq 1$ or $(0; 1]$	✓✓ answer (2)
4.2	$p = \log_{\frac{4}{3}} \frac{16}{9}$ $\left(\frac{4}{3}\right)^p = \frac{16}{9}$ $\left(\frac{4}{3}\right)^p = \left(\frac{4}{3}\right)^2$ $p = 2$	✓ substitution ✓ $\left(\frac{4}{3}\right)^2$ ✓ answer (3)
4.3	$f: y = \log_{\frac{4}{3}} x$ $f^{-1}: x = \log_{\frac{4}{3}} y$ $y = \left(\frac{4}{3}\right)^x$	✓ $x = \log_{\frac{4}{3}} y$ ✓ $y = \left(\frac{4}{3}\right)^x$ (2)
4.4	$y > 0$ or $y \in (0; \infty)$	✓✓ answer (2)
4.5	$\left(-2; \frac{16}{9}\right)$	✓ -2 ✓ $\frac{16}{9}$ (2) [11]

QUESTION/VRAAG 5

5.1	$x \in R; x \neq -1$	✓ $x \in R$ ✓ $x \neq -1$ (2)
5.2	x-intercept of f : $0 = \frac{2}{x+1} + 4$ $\frac{2}{x+1} = -4$ $2 = -4x - 4$ $4x = -6$ $x = -\frac{3}{2}$	✓ equating to 0 ✓ answer (2)
5.3	$y = \frac{2}{x+1} + 4$ $\frac{14}{3} = \frac{2}{k+1} + 4$ $\frac{2}{k+1} = \frac{14}{3} - 4$ $\frac{2}{k+1} = \frac{2}{3}$ $2k + 2 = 6$ $k + 1 = 3$ $k = 2$	✓ substitution ✓ simplification ✓ answer (3)
5.4	$C(2; 4)$	✓ 2 ✓ 4 (2)
5.5	$y = a(x+p)^2 + q$ $= a(x-2)^2 + 4$ Substitute $(0; 0)$: $0 = a(0-2)^2 + 4$ $0 = 4a + 4$ $a = -1$ $y = -(x-2)^2 + 4$	✓ $a(x-2)^2 + 4$ ✓ Substitute $(0; 0)$ ✓ $a = -1$ (3)
5.6	$x \leq -\frac{3}{2}$ or $-1 < x < 0$ or $x > 4$	✓ $x \leq -\frac{3}{2}$ ✓✓ $-1 < x < 0$ ✓ $x > 4$ (4)

5.7	$\frac{2}{x} - 5$: f shifted 1 unit to the right and 9 units down. f is 1 eenheid na regs en 9 eenhede afgeskuif. $-(x-3)^2 - 5$: g shifted 1 unit to the right and 9 units down. g is 1 eenheid na regs en 9 eenhede afgeskuif. Therefore the shift of both graphs took place relative to each other/Dus het die skuif van die grafieke relatief tot mekaar plaasgevind. They only intersect in the third quadrant. Hulle sny mekaar slegs in die derde kwadrant. Therefore there is only one point of intersection. Daar is dus slegs een snypunt.	✓ both shifted 1 unit to the right ✓ both shifted 9 units down ✓ relative shift ✓ one real root (4) [20]
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QUESTION/VRAAG 6

6.1	$A = P(1-i)^n$ $0,5P = P(1-0,15)^n$ $(1-0,15)^n = 0,5$ $(0,85)^n = 0,5$ $n = \frac{\log 0,5}{\log 0,85}$ or $\log_{0,85} 0,5$ $= 4,27$ years	✓ $A = 0,5P$ ✓ substitution into correct formula ✓ use of logs ✓ answer (4)
6.2	In account one month before his 55 th birthday: In rekening een maand voor sy 55 ^{ste} verjaardag: $F = \frac{x(1+i)^n - 1}{i}$ $1500 \left[\frac{1 + \frac{0,092}{12} \cdot 384}{\frac{0,092}{12}} - 1 \right]$ $= 3478620,49$ In account on his 55 th birthday: In rekening op sy 55 ^{ste} verjaardag: $A = P(1+i)^n$ $= 3478620,49 \left(1 + \frac{0,092}{12} \right)^1$ $= R3505289,91$ OR/OF $F = \frac{x(1+i)(1+i)^n - 1}{i}$ $1500 \left(1 + \frac{0,092}{12} \right) \left[\frac{1 + \frac{0,092}{12} \cdot 384}{\frac{0,092}{12}} - 1 \right]$ $= R3505289,91$	✓ value of i ✓ value of n ✓ substitution into correct formula ✓ adding last month's interest ✓ answer OR/OF ✓ value of i ✓ value of n ✓ substitution into correct formula ✓ adding last month's interest ✓ answer (5)

6.3	Invest Rx in account A paying 8,4% p.a. compounded quarterly./Belê Rx in rekening A wat 8,4% p.a rente betaal, kwartaaliks saamgestel. $A = P(1+i)^n$ $= x \left(1 + \frac{0,084}{4}\right)^{48}$ $= 2,711662406x$ Invest (R150 000 – x) in Account B paying 9,6% compounded monthly./Belê (R150 000 – x in rekening A wat 9,6% p.a rente betaal, maandeliks saamgestel. After 12 years, the amounts are equal: $x \left(1 + \frac{0,084}{4}\right)^{48} = (150\,000 - x) \left(1 + \frac{0,096}{12}\right)^{144}$ $2,711662406x = 3,150044027(150\,000 - x)$ $2,711662406x = 472506,6041 - 3,150044027x$ $5,861706433x = 472506,6041$ $x = R80\,609,05$ Invest R80 609 in Account A and R150 000 – R80 609,05 = R69 390,95 in Account B OR/OF a = amount invested at 8,4% p.a. compounded quarterly <i>bedrag belê teen 8,4% p.a. kwartaaliks saamgestel</i> b = amount invested at 9,6% p.a. compounded monthly <i>bedrag belê teen 9,6% p.a. maandeliks saamgestel</i> a + b = 150 000 a = 150 000 – b $(150\,000 - b) \left(1 + \frac{0,084}{4}\right)^{48} = b \left(1 + \frac{0,096}{12}\right)^{144}$ $150\,000 \left(1 + \frac{0,084}{4}\right)^{48} = b \left[\left(1 + \frac{0,096}{12}\right)^{144} + \left(1 + \frac{0,084}{4}\right)^{48} \right]$ b = R69 390,95 a = R80 609,05	✓ $\left(1 + \frac{0,084}{4}\right)^{48}$ ✓✓ ✓ $(150\,000 - x) \left(1 + \frac{0,096}{12}\right)^{144}$ ✓ equation ✓ R80 609,05 ✓ R69 390,95 OR/OF ✓ $\left(1 + \frac{0,096}{12}\right)^{144}$ ✓✓ $(150\,000 - b) \left(1 + \frac{0,084}{4}\right)^{48}$ ✓ equation ✓ b ✓ a (6) (6) [15]
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7.2.1	$D_x \left[(4x + 5)^2 \right]$ $= D_x (16x^2 + 40x + 25)$ $= 32x + 40$	✓ $16x^2 + 40x + 25$ ✓ $32x$ ✓ $+ 40$ (3)
7.2.2	$y = \sqrt[3]{x} + \frac{x^2 - 8}{x^2}$ $y = x^{\frac{1}{3}} + 1 - 8x^{-2}$ $\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}} + 16x^{-3}$	✓ $x^{\frac{1}{3}}$ ✓ $1 - 8x^{-2}$ ✓ $\frac{1}{3}x^{-\frac{2}{3}}$ ✓ $16x^{-3}$ (4) [12]

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QUESTION/VRAAG 7**Penalize 1 mark for incorrect notation in the whole question.**

7.1	$f(x+h) = 2 - 3(x+h)^2$ $= 2 - 3(x^2 + 2xh + h^2)$ $= 2 - 3x^2 - 6xh - 3h^2$ $f(x+h) - f(x) = 2 - 3x^2 - 6xh - 3h^2 - (2 - 3x^2)$ $= -6xh - 3h^2$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{-6xh - 3h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(-6x - 3h)}{h}$ $= \lim_{h \rightarrow 0} (-6x - 3h)$ $= -6x$ OR/OF $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{2 - 3(x+h)^2 - (2 - 3x^2)}{h}$ $= \lim_{h \rightarrow 0} \frac{2 - 3x^2 - 6xh - 3h^2 - (2 - 3x^2)}{h}$ $= \lim_{h \rightarrow 0} \frac{-6xh - 3h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(-6x - 3h)}{h}$ $= \lim_{h \rightarrow 0} (-6x - 3h)$ $= -6x$	✓ $2 - 3x^2 - 6xh - 3h^2$ ✓ $-6xh - 3h^2$ ✓ subst. into formula ✓ factorisation ✓ answer OR/OF ✓ subst. into formula ✓ simplification ✓ $-6xh - 3h^2$ ✓ common factor ✓ answer (5) (5)
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QUESTION/VRAAG 8

8.1	C(0;12)	✓ C(0;12)	(1)
8.2	$-x^3 + 13x + 12 = 0$ $x^3 - 13x - 12 = 0$ $(x+1)(x^2 - x - 12) = 0$ $(x+1)(x-4)(x+3) = 0$ A(-3;0) B(4;0)	✓ $f(x) = 0$ ✓ $(x+1)$ ✓ $(x^2 - x - 12)$ ✓ $x = -3$ or 4 ✓ clearly indicating A and B (5)	
8.3	$f'(x) = -3x^2 + 13$ $f''(x) = -6x$ $-6x = 0$ $x = 0$ For $f(x)$, point of inflection will be at (0 ; 12). <i>Vir $f(x)$, sal buigpunt wees by (0 ; 12)</i> For $g(x)$, point of inflection will be at (0 ; -12). <i>Vir $g(x)$, sal buigpunt wees by (0 ; -12).</i> OR/OF $g'(x) = x^3 - 13x - 12$ $g''(x) = 3x^2 - 13$ $g'''(x) = 6x$ $6x = 0$ $x = 0$ (0; -12) OR/OF $f'(x) = -3x^2 + 13$ TP's where $-3x^2 + 13 = 0$ $x^2 = \frac{13}{3}$ $x = \pm \sqrt{\frac{13}{3}}$ $= \pm 2,08$ x-value of point of inflection: $\frac{-2,08 + 2,08}{2} = 0$ For $f(x)$, point of inflection will be at (0 ; 12). <i>Vir $f(x)$, sal buigpunt wees by (0 ; 12)</i> For $g(x)$, point of inflection will be at (0 ; -12). <i>Vir $g(x)$, sal buigpunt wees by (0 ; -12).</i>	✓ $f'(x) = -3x^2 + 13$ ✓ $f''(x) = -6x$ ✓ equating to zero ✓ (0; -12) (4) OF/OR ✓ $g'(x) = 3x^2 - 13$ ✓ $g''(x) = 6x$ ✓ equating to zero ✓ (0; -12) (4) OR/OF ✓ $f'(x) = -3x^2 + 13$ ✓ $-3x^2 + 13 = 0$ ✓ x-values of TP's ✓ (0; -12) (4)	

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8.4	$f'(x) = -3x^2 + 13$ $-3x^2 + 13 = -14$ $-3x^2 = -27$ $x^2 = 9$ $x = 3$ or $x = -3$	✓ equating derivative to -14 ✓ simplification ✓✓ answers (4) [14]
-----	---	---

QUESTION/VRAAG 10

10.1	$10!$ $= 3\,628\,800$	✓ $10!$ ✓ answer (2)
10.2	$4! \times 7!$ $= 120\,960$ OR/OF $4! \times 6! \times 7$ $= 120\,960$	✓ $4!$ ✓ $7!$ ✓ $4! \times 7!$ or $120\,960$ (3) OR/OF ✓ $4!$ ✓ $6! \times 7$ ✓ $4! \times 6! \times 7$ or $120\,960$ (3)
10.3	$\frac{6!}{10!}$ $= \frac{1}{5040}$ or $0,000198$	✓ $6!$ ✓ $\frac{6!}{10!}$ or $\frac{1}{5040}$ or $0,000198$ (2) [7]

QUESTION/VRAAG 11

11.1	$P(\text{tennis}) \times P(\leq 35 \text{ years}) = P(\text{tennis and } \leq 35 \text{ years})$ $\frac{21}{140} \times \frac{80}{140} = \frac{a}{140}$ $a = 12$	✓ statement ✓ substitution ✓ answer (3)
11.2	$P(\text{gym or } \leq 35 \text{ years})$ $= P(\text{gym}) + P(\leq 35 \text{ years}) - P(\text{gym and } \leq 35 \text{ years})$ $= \frac{70}{140} + \frac{80}{140} - \frac{40}{140}$ $= \frac{110}{140}$ $= \frac{11}{14}$ or $0,79$	✓ statement ✓ $\frac{70}{140}$ ✓ $\frac{80}{140}$ ✓ $\frac{40}{140}$ ✓ $\frac{110}{140}$ or $\frac{11}{14}$ or $0,79$ (5) [8]

TOTAL/TOTAAL: 150

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QUESTION/VRAAG 9

9.1.1	$AC = t - 30$	✓ answer (1)
9.1.2	$30^2 = (t - 30)^2 + p^2$ [Pythagoras] $p^2 = 900 - (t - 30)^2$ $p^2 = 900 - (t^2 - 60t + 900)$ $p^2 = 900 - t^2 + 60t - 900$ $p^2 = 60t - t^2$	✓ $p^2 = 900 - (t - 30)^2$ ✓ $(t^2 - 60t + 900)$ ✓ $p^2 = 60t - t^2$ (3)
9.2	$V(t) = \frac{1}{3}\pi r^2 t$ $= \frac{1}{3}\pi(60t - t^2)t$ $= 20\pi t^2 - \frac{1}{3}\pi t^3$	✓ substitution (1)
9.3	$V(t) = 20\pi t^2 - \frac{1}{3}\pi t^3$ $V'(t) = 40\pi t - \pi t^2$ $40\pi t - \pi t^2 = 0$ $t(40\pi - t\pi) = 0$ $t = 0$ OR $t = 40$ cm N/A	✓ $40\pi t$ ✓ $-\pi t^2$ ✓ answer with selection (3)
9.4	Volume of cone/keël $= 20(\pi)(40)^2 - \frac{1}{3}\pi(40)^3$ $= 10\,666,67\pi$ or $33510,33211$ Volume of sphere/sfeer $= \frac{4}{3}\pi^3$ $= \frac{4}{3}\pi(30)^3$ $= 36\,000\pi$ or $113\,097,3355$ $\frac{10\,666,67\pi}{36\,000\pi}$ $= 0,296296$ $\approx 29,63\%$	✓ volume of cone ✓ volume of sphere ✓ $\frac{10\,666,67\pi}{36\,000\pi}$ ✓ % cut out (4) [12]



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS SENIORSERTIFIKAT-EKSAMEN

MATHEMATICS P2/WISKUNDE V2

2018

MARKING GUIDELINES/NASIENRIGLYNE

MARKS: 150

PUNTE: 150

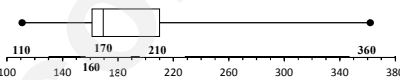
These marking guidelines consist of 21 pages.
Hierdie nasienriglyne bestaan uit 21 bladsye.

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QUESTION/VRAAG 1

110	112	156	164	167	169
171	176	192	228	278	360

1.1.1	Mean/Gemiddelde = $\frac{2283}{12}$ = 190,25 Mean profit/Gemiddelde wins = R190 250,00 or 190,25 thousand rands	✓ sum/som ✓ answer ✓ answer in thousands of rands (3)
1.1.2	Median = $\frac{169 + 171}{2}$ = 170 thousand rands = R170 000	✓ answer (1)
1.2		✓ whiskers ✓ quartiles (2)
1.3	IQR = $Q_3 - Q_1$ = 210 – 160 thousand rands = R50 000	✓ answer (1)
1.4	Skewed to the right or positively skewed.	✓ answer (1)
1.5.1	σ = 67,04118759 thousand rands = R67 041,19	✓ answer (1)
1.5.2	$\bar{x} - \sigma$ = 123,21 thousand rands For 2 months the profit was less than one standard deviation below the mean.	✓ lower limit ✓ answer (2)
		[11]

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NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

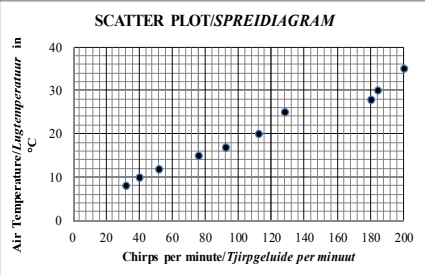
LET WEL:

- As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.
- As 'n kandidaat 'n antwoord van 'n vraag doordruk en nie oordoen nie, sien die doodgetrekte poging na.
- Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.
- Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.

GEOMETRY	
S	A mark for a correct statement (A statement mark is independent of a reason.)
	'n Punt vir 'n korrekte bewering ('n Punt vir 'n bewering is onafhanklik van die rede.)
R	A mark for a correct reason (A reason mark may only be awarded if the statement is correct.)
	'n Punt vir 'n korrekte rede ('n Punt word slegs vir die rede toegeken as die bewering korrek is.)
S/R	Award a mark if the statement AND reason are both correct.
	Ken 'n punt toe as beide die bewering EN rede korrek is.

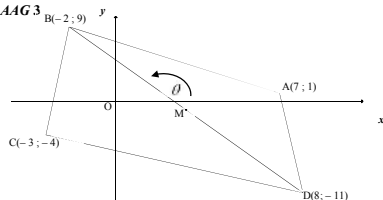
QUESTION/VRAAG 2

CHIRPS/TJIRPGELUIDE PER MINUTE/ PER MINUUT	AIR TEMPERATURE/ LUGTEMPERATUUR IN °C
32	8
40	10
52	12
76	15
92	17
112	20
128	25
180	28
184	30
200	35

2.1		3 marks: All points correct 2 marks: 6 – 9 points correct 1 mark: 3 – 5 points correct (3)
2.2	The points lie almost in a straight line. This suggests a very strong positive relationship between the number of chirps per minute and the temperature of the air. Die punte lê amper in 'n reguitlyn, wat beteken dat daar 'n baie sterk positiewe verband tussen die aantal tjirpgeluide per minuut en die lugtemperatuur is. OR/OF $r = 0,99$ so there is a very strong positive relationship between the number of chirps per minute and the temperature of the air. $r = 0,99$, dus is daar 'n baie sterk positiewe verband tussen die aantal kriekegeluide per minuut en die lugtemperatuur.	✓ justify with straight line / Motivering mbv reguitlyn (1) ✓ link with / gebruik $r = 0,99$ om te motiveer (1)

2.3	$a = 3,97$ $b = 0,15$ $\hat{y} = 3,97 + 0,15x$	$\checkmark a = 3,97$ $\checkmark b = 0,15$ $\checkmark$ equation (3)
2.4	Air temperature $\approx 15,67^\circ\text{C}$ (calculator) OR $\hat{y} \approx 3,97 + 0,15(80)$ $\approx 15,97^\circ\text{C}$ OR Air temperature $\approx 16^\circ\text{C}$ (graph: Accept between 15°C and 17°C)	$\checkmark\checkmark$ answer (2) $\checkmark$ substitution $\checkmark$ answer (2) $\checkmark\checkmark$ answer (2)

QUESTION/VRAAG 3



3.1	$m_{AC} = \frac{1 - (-4)}{7 - (-3)} \text{ OR } \frac{-4 - 1}{-3 - 7}$ $= \frac{5}{10} = \frac{1}{2}$	$\checkmark$ substitution $\checkmark$ answer (2)
3.2.1	$y = \frac{1}{2}x + c$ $1 = \frac{1}{2}(7) + c$ $c = -\frac{5}{2}$ $y = \frac{1}{2}x - 2\frac{1}{2}$ OR/OF $y = \frac{1}{2}x + c$ $-4 = \frac{1}{2}(-3) + c$ $c = -\frac{5}{2}$ $y = \frac{1}{2}x - 2\frac{1}{2}$	$y - y_1 = \frac{1}{2}(x - x_1)$ $y - 1 = \frac{1}{2}(x - 7)$ $y - 1 = \frac{1}{2}x - \frac{7}{2}$ $y = \frac{1}{2}x - 2\frac{1}{2}$ $y - y_1 = \frac{1}{2}(x - x_1)$ $y - (-4) = \frac{1}{2}(x - (-3))$ $y + 4 = \frac{1}{2}x + \frac{3}{2}$ $y = \frac{1}{2}x - 2\frac{1}{2}$ $\checkmark$ substitution M and A(7; 1) $\checkmark$ equation $\checkmark$ substitution M and C(-3; -4) $\checkmark$ equation (2)

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3.2.2	$M\left(\frac{-2+8}{2}; \frac{9+(-11)}{2}\right)$ $\therefore M(3; -1)$ Equation of AC: $y = \frac{1}{2}x - 2\frac{1}{2}$ OR/OF $y = \frac{1}{2}x - 2\frac{1}{2}$ $y = \frac{1}{2}(3) - 2\frac{1}{2}$ $y = -1$ $\therefore M$ lies on AC OR/OF $M\left(\frac{-2+8}{2}; \frac{9+(-11)}{2}\right)$ $\therefore M(3; -1)$ $m_{CM} = \frac{-4+1}{-3-3} = \frac{1}{2}$ $\therefore m_{CM} = m_{AC}$ and C a common point $\therefore M$ lies on AC	$\checkmark$ x coordinate $\checkmark$ y coordinate $\checkmark$ substitution of x $\checkmark$ conclusion (4)
3.3	$m_{BD} = \frac{9-(-11)}{-2-8} \text{ OR } \frac{(-11)-9}{8-(-2)}$ $= -2$ $m_{BD} \times m_{AC} = \frac{1}{2} \times -2$ $= -1$ $\therefore BD \perp AC$	$\checkmark$ correct substitution $\checkmark m_{BD}$ $\checkmark$ product of gradients = -1 (3)
3.4.1	$\tan \theta = m_{BD} = -2$ $\therefore \theta = 116,57^\circ$	$\checkmark \tan \theta = m_{BD}$ $\checkmark$ answer (2)
3.4.2	$\tan \beta = m_{BC}$ $m_{BC} = \frac{9-(-4)}{-2-(-3)} \text{ OR } \frac{-4-9}{-3-(-2)}$ $= 13$ $\beta = 85,6^\circ$ $\therefore \hat{C}\hat{B}\hat{D} = 116,57^\circ - 85,60^\circ$ [ext $\angle$ of Δ] $= 30,97^\circ$ OR/OF $BD = \sqrt{500}$; $BC = \sqrt{170}$ & $CD = \sqrt{170}$ $CD^2 = BD^2 + BC^2 - 2BD \cdot BC \cdot \cos \hat{C}\hat{B}\hat{D}$ $170 = 500 + 170 - 2\sqrt{500} \cdot \sqrt{170} \cdot \cos \hat{C}\hat{B}\hat{D}$ $\cos \hat{C}\hat{B}\hat{D} = \frac{\sqrt{500}}{2\sqrt{170}} = 0,85749...$ $\hat{C}\hat{B}\hat{D} = 30,96^\circ$	$\checkmark m_{BC} = 13$ $\checkmark$ value of β $\checkmark$ answer $\checkmark$ subst into cos rule $\checkmark$ value of $\cos \hat{C}\hat{B}\hat{D}$ $\checkmark$ answer (3)

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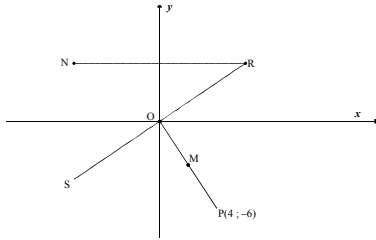
3.4.3	$AC = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ $= \sqrt{(7 - (-3))^2 + (1 - (-4))^2} \text{ OR } \sqrt{((-3) - 7)^2 + ((-4) - 1)^2}$ $= \sqrt{100 + 25}$ $= \sqrt{125} = 5\sqrt{5} = 11,58$	$\checkmark$ correct substitution into distance formula $\checkmark$ answer (2)
3.4.4	$BM = \sqrt{((-2) - 3)^2 + (9 - (-1))^2} \text{ OR } \sqrt{(3 - (-2))^2 + ((-1) - 9)^2}$ $= \sqrt{125} = 5\sqrt{5}$ Area of $\Delta ABC = \frac{1}{2} \text{ base} \times \perp \text{ height}$ $= \frac{1}{2} (\sqrt{125})(\sqrt{125})$ $= 62,5 \text{ square units}$ Area of ABCD = $2 \times 62,5$ $= 125 \text{ square units}$	$\checkmark$ correct substitution into distance formula $\checkmark$ BM $\checkmark$ substitution into area formula $\checkmark 62,5$ $\checkmark 2 \times \Delta ABC$ (5)

[23]

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QUESTION/VRAAG 4



4.1	$M\left(\frac{0+4}{2}; \frac{0+(-6)}{2}\right)$ $\therefore M(2; -3)$	$\checkmark 2 \checkmark -3$ (2)
4.2.1	$x^2 + y^2 = 4^2 + (-6)^2$ $= 52$ $\therefore x^2 + y^2 = 52$	$\checkmark$ substitution $\checkmark$ equation (2)
4.2.2	$(x-2)^2 + (y+3)^2 = \left(\frac{\sqrt{52}}{2}\right)^2 = 13$ $x^2 - 4x + 4 + y^2 + 6y + 9 - 13 = 0$ $x^2 + y^2 - 4x + 6y = 0$	$\checkmark$ substitution of M $\checkmark$ substitution of radius = $\frac{\sqrt{52}}{2}$ $\checkmark$ answer (3)
4.2.3	$m_{OP} = \frac{-6}{4} = -\frac{3}{2}$ $m_{RS} \times m_{OP} = -1$ [radius $\perp$ tangent / raaklyn] $\therefore m_{RS} = \frac{2}{3}$ $\therefore y = \frac{2}{3}x$	$\checkmark m_{OP}$ $\checkmark m_{RS}$ $\checkmark$ equation (3)

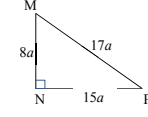
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4.3	$x^2 + y^2 = 52$ and $y = \frac{2}{3}x$ $x^2 + \left(\frac{2}{3}x\right)^2 = 52$ $x^2 + \frac{4}{9}x^2 = 52$ $1\frac{4}{9}x^2 = 52$ $x^2 = 36$ $x = 6$ $\therefore R(6; 4)$ and $N(-6; 4)$ $\therefore NR = 12$ units	$\checkmark$ substitution $\checkmark$ simplification $\checkmark$ value of x $\checkmark$ length of NR (4)
4.4	Let T(x; 0) be the other x intercept of the small circle Then OT is the common chord $\therefore (x-2)^2 + (0+3)^2 = 13$ $(x-2)^2 = 13 - 9 = 4$ $x - 2 = \pm 2$ $x = 2 \pm 2$ $x = 4$ or 0 $\therefore$ length of common chord = OT = 4 units	$\checkmark y = 0$ $\checkmark$ x-values $\checkmark$ answer (3)

[17]

QUESTION/VRAAG 5

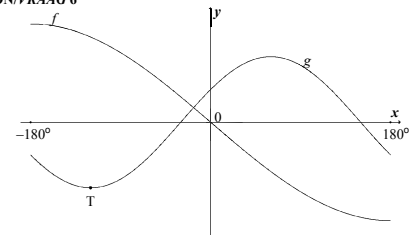
5.1.1	Given: $\sin M = \frac{15}{17}$ $MN^2 = 17^2 - 15^2$ $= 64$ $MN = 8$ OR $\therefore \tan M = \frac{15}{8}$	 $\checkmark$ sketch or Pyth $\checkmark MN = 8$ $\checkmark$ answer (3)
5.1.2	$\sin M = \frac{NP}{MP}$ $\frac{NP}{51} = \frac{15a}{17a}$ $\therefore NP = 45$	$\checkmark$ equating trig ratios $\checkmark$ answer (2)
5.2	$\cos(x - 360^\circ) \cdot \sin(90^\circ + x) + \cos^2(-x) - 1$ $= \cos x \cdot \cos x + \cos^2 x - 1$ $= \cos^2 x + \cos^2 x - 1$ $= 2\cos^2 x - 1$ $= \cos 2x$	$\checkmark \cos x \checkmark \cos x$ $\checkmark \cos^2 x$ $\checkmark$ identity (4)
5.3.1	$\sin(2x + 40^\circ) \cos(x + 30^\circ) - \cos(2x + 40^\circ) \sin(x + 30^\circ)$ $= \sin[(2x + 40^\circ) - (x + 30^\circ)]$ $= \sin(x + 10^\circ)$	$\checkmark$ reduction $\checkmark$ answer (2)
5.3.2	$\sin(2x + 40^\circ) \cos(x + 30^\circ) - \cos(2x + 40^\circ) \sin(x + 30^\circ) = \cos(2x - 20^\circ)$ $\therefore \cos(2x - 20^\circ) = \sin(x + 10^\circ)$ $\cos(2x - 20^\circ) = \cos[90^\circ - (x + 10^\circ)]$ $2x - 20^\circ = 80^\circ - x + k \cdot 360^\circ$ or $2x - 20^\circ = 360^\circ - (80^\circ - x) + k \cdot 360^\circ$ $3x = 100^\circ + k \cdot 360^\circ$ or $2x - 20^\circ = 280^\circ + x + k \cdot 360^\circ$ $x = 33,33^\circ + k \cdot 120^\circ$ or $x = 300^\circ + k \cdot 360^\circ$; $k \in \mathbb{Z}$ OR/OF $\therefore \cos(2x - 20^\circ) = \sin(x + 10^\circ)$ $\sin[90^\circ - (2x - 20^\circ)] = \sin(x + 10^\circ)$ $110^\circ - 2x = x + 10^\circ + k \cdot 360^\circ$ or $110^\circ - 2x = 180^\circ - (x + 10^\circ) + k \cdot 360^\circ$ $3x = 100^\circ - k \cdot 360^\circ$ or $110^\circ - 2x = 170^\circ - x + k \cdot 360^\circ$ $x = 33,33^\circ - k \cdot 120^\circ$ or $x = -60^\circ - k \cdot 360^\circ$; $k \in \mathbb{Z}$	$\checkmark$ equating $\checkmark$ co ratio $\checkmark 80^\circ - x \checkmark 280^\circ + x$ $\checkmark$ simplification/vereenv $\checkmark x = 33,33^\circ + k \cdot 120^\circ$ $\checkmark x = 300^\circ + k \cdot 360^\circ$; $k \in \mathbb{Z}$ $\checkmark$ equating $\checkmark$ co ratio $\checkmark x + 10^\circ \checkmark 170^\circ - x$ $\checkmark$ simplification/vereenv $\checkmark x = 33,33^\circ - k \cdot 120^\circ$ $\checkmark x = -60^\circ - k \cdot 360^\circ$; $k \in \mathbb{Z}$ (7)
		(7)

[18]

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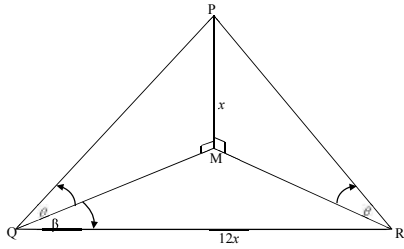
QUESTION/VRAAG 6



6.1	Period = 720°	$\checkmark$ answer (1)
6.2	$y \in [-2; 2]$ OR/OF $-2 \leq y \leq 2$	$\checkmark \checkmark$ answer (2) $\checkmark \checkmark$ answer (2)
6.3	$f(-120^\circ) - g(-120^\circ)$ $= -3 \sin\left(-\frac{120^\circ}{2}\right) - 2 \cos(-120^\circ - 60^\circ)$ $= \frac{4 + 3\sqrt{3}}{2}$ or 4,60 (4,5980...)	$\checkmark x = -120^\circ$ $\checkmark$ substitution $\checkmark$ answer (3)
6.4.1	x-intercepts of g at $-90^\circ + 60^\circ = -30^\circ$ and $90^\circ + 60^\circ = 150^\circ$ $\therefore x \in (-30^\circ; 150^\circ)$ OR/OF x-intercepts of g at $-90^\circ + 60^\circ = -30^\circ$ and $90^\circ + 60^\circ = 150^\circ$ $-30^\circ < x < 150^\circ$	$\checkmark$ value $\checkmark$ value $\checkmark$ answer (3) $\checkmark$ value $\checkmark$ value $\checkmark$ answer (3)
6.4.2	$x \in [-180^\circ; -120^\circ] \cup (-30^\circ; 60^\circ) \cup (150^\circ; 180^\circ]$ OR/OF $-180^\circ \leq x < -120^\circ$ or $-30^\circ < x < 60^\circ$ or $150^\circ < x \leq 180^\circ$	$\checkmark [-180^\circ; -120^\circ]$ $\checkmark (-30^\circ; 60^\circ)$ $\checkmark (150^\circ; 180^\circ]$ $\checkmark$ notation for inclusive in the first/last interval (4) $\checkmark -180^\circ \leq x < -120^\circ$ $\checkmark -30^\circ < x < 60^\circ$ $\checkmark 150^\circ < x \leq 180^\circ$ 1 mark: each interval $\checkmark$ notation for inclusive in the first/last interval (4)

[13]

QUESTION/VRAAG 7



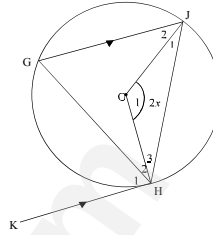
7.1	In $\triangle PMQ$: $\tan \theta = \frac{x}{QM}$ $\therefore QM = \frac{x}{\tan \theta}$ OR/OF $\frac{x}{\sin \theta} = \frac{MQ}{\sin P}$ $MQ = \frac{x \sin P}{\sin \theta}$ $= \frac{x \cos \theta}{\sin \theta}$ $= \frac{x}{\tan \theta}$	✓ trig ratio ✓ answer (2) ✓ sine rule ✓ answer (2)
7.2	In $\triangle PMR$: $\tan \theta = \frac{x}{MR}$ OR $\triangle PMQ \cong \triangle PMR$ [AAS/HHS] $\therefore MR = \frac{x}{\tan \theta} = QM$ $\angle QMR = 180^\circ - 2\theta$ $\frac{\sin \beta}{MR} = \frac{\sin \angle QMR}{12x}$ $\sin \beta \times \frac{12x}{x} = \frac{\sin(180^\circ - 2\theta)}{12x}$ $\tan \theta = \frac{\sin 2\theta}{12x} \times \frac{x}{\sin \beta}$ $\tan \theta = \frac{2 \sin \theta \cos \theta}{12x} \times \frac{x}{\sin \beta}$ $\tan \theta = \frac{\cos \theta}{6}$ OR	✓ $MR = QM$ ✓ correct substitution into the sine rule in $\triangle QMR$ ✓ reduction ✓ double angle (4)

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QUESTION/VRAAG 8

8.1



8.1.1	$\hat{G} = x$ [$\angle$ centre = $2 \times$ circumference / midpts $\angle = 2 \times$ omtreks $\angle$] $\hat{H}_1 = x$ [alt $\angle$s / verwiss $\angle$e; $KH \parallel GJ$] $\hat{G} \hat{J} \hat{H} = x$ [tan chord theorem / raaklyn koordstelling]	✓ S ✓ R ✓ S ✓ S ✓ R
8.1.2	$\hat{J}_1 + \hat{H}_3 = 180^\circ - 2x$ [sum of $\angle$s in Δ / som van $\angle$e in Δ] $\therefore \hat{J}_1 = \hat{H}_3 = 90^\circ - x$ [$\angle$s opp equal sides / $\angle$e teenoor gelyke sye] $\therefore x + \hat{H}_2 = 90^\circ$ OR [tan $\perp$ radius / raaklyn $\perp$ radius] $\hat{H}_2 = 90^\circ - x$ $\therefore \hat{H}_2 = \hat{H}_3$	✓ S ✓ S ✓ R

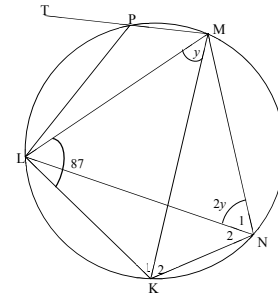
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	In $\triangle PMR$: $\tan \theta = \frac{x}{MR}$ OR $\triangle PMQ \cong \triangle PMR$ [AAS/HHS] $MR^2 = QM^2 + QR^2 - 2QM \cdot QR \cos \beta$ $MR^2 = \left(\frac{x}{\tan \theta}\right)^2 + (12x)^2 - 2\left(\frac{x}{\tan \theta}\right)(12x)(\cos \beta)$ $\frac{x^2}{\tan^2 \theta} = \frac{x^2}{\tan^2 \theta} + 144x^2 - 24\left(\frac{x^2}{\tan \theta}\right)(\cos \beta)$ $24\left(\frac{x^2}{\tan \theta}\right)(\cos \beta) = 144x^2$ $\cos \beta = 6 \tan \theta$ $\tan \theta = \frac{\cos \beta}{6}$	✓ correct substitution into the cosine rule in $\triangle QMR$ ✓ substitution ✓ $MR = QM$ ✓ simplification (4)
7.3	$\frac{x}{QM} = \frac{\cos \beta}{6}$ [both equal $\tan \theta$] $\frac{60 \cos 40}{x} = \frac{60 \cos 40}{6}$ $x = 7.66$ The height of the lighthouse is 8 metres	✓ equating ✓ subst. $QM = 60$ and $\beta = 40^\circ$ ✓ answer (3)

[9]

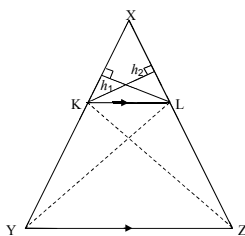
8.2



8.2.1	$\hat{N}_2 = y$ [$\angle$ s in the same seg / $\angle$ e in dieselfde segment]	✓ S ✓ R
8.2.2(a)	$2y + y + 87^\circ = 180^\circ$ [opp $\angle$s of cyclic quad / teenoorst $\angle$e v kvh] $3y = 93^\circ$ $y = 31^\circ$	✓ S ✓ R
8.2.2(b)	$\hat{T} \hat{P} \hat{L} = 62^\circ$ [ext. $\angle$ of cyclic quad / buite $\angle$ v kvh]	✓ S ✓ R
		[15]

QUESTION/VRAAG 9

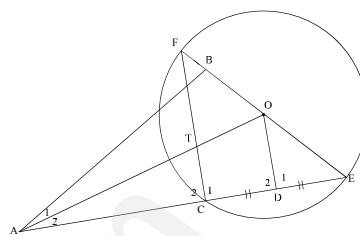
9.1



9.1	Constr: Join KZ and LY and draw h_1 from $K \perp XL$ and h_2 from $L \perp XK$ Konstr: Verbind KZ en LY en trek h_1 vanaf K $\perp XL$ en h_2 vanaf L $\perp XK$ Proof / Bewys: $\frac{\text{area } \Delta XKL}{\text{area } \Delta LYK} = \frac{\frac{1}{2} XK \times h_1}{\frac{1}{2} KY \times h_1} = \frac{XK}{KY}$ $\frac{\text{area } \Delta XKL}{\text{area } \Delta K LZ} = \frac{\frac{1}{2} XL \times h_2}{\frac{1}{2} LZ \times h_2} = \frac{XL}{LZ}$ area ΔXKL = area ΔXKL [common / <i>gemeenskaplik</i>] But area ΔLYK = area $\Delta K LZ$ [same base & height ; $LK \parallel YZ$ / <i>dies basis & hoogte ; $LK \parallel YZ$</i>] $\therefore \frac{\text{area } \Delta XKL}{\text{area } \Delta LYK} = \frac{\text{area } \Delta XKL}{\text{area } \Delta K LZ}$ $\therefore \frac{XK}{KY} = \frac{XL}{LZ}$	✓ constr / konstr $\checkmark \frac{\text{area } \Delta XKL}{\text{area } \Delta LYK} = \frac{\frac{1}{2} XK \times h_1}{\frac{1}{2} KY \times h_1}$ ✓ S ✓ R ✓ S
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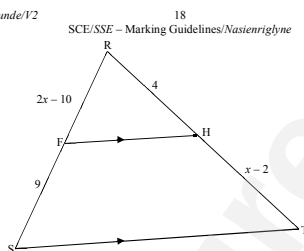
(5)

QUESTION/VRAAG 10



10.1.1	$\hat{C}_1 = 90^\circ$ [$\angle$ in semi circle / $\angle$ in <i>halfsirkel</i>] $\hat{D}_1 = 90^\circ$ [line from centre to midpt of chord / <i>lyn vanaf midpt na midpt van koord</i>] $\therefore \hat{C}_1 = \hat{D}_1$ $\therefore FC \parallel OD$ [corresp $\angle$ s = / <i>ooreenkomstige $\angle$e</i> =] OR/OF $FO = OE$ [radii] $CD = DE$ [given / <i>gegegee</i>] $\therefore FC \parallel OD$ [midpoint theorem / <i>midelpuntstelling</i>]	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R $\checkmark$ R $\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ S $\checkmark$ R	 (5)
10.1.2	$\hat{D}OE = \hat{F}$ [corresp $\angle$ s = ; $FC \parallel OD$] $\hat{B}AE = \hat{F}$ [$\angle$ s in the same seg] $\therefore \hat{D}OE = \hat{B}AE$	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R	 (4)
10.1.3	In $\triangle ABE$ and $\triangle FCE$: $\hat{E}$ is common $\hat{B}AE = \hat{F}$ [proved in 10.1.2] $\therefore \hat{ABE} = \hat{C}_1$ [sum of $\angle$ s in Δ] $\therefore \triangle ABE \parallel \triangle FCE$ [$\angle \angle \angle$] $\frac{AB}{FC} = \frac{AE}{FE}$ [$\parallel \Delta$ s] $AB \times FE = AE \times FC$ But $FE = 2 \text{ OF}$ [$d = 2r$] And $FC = 2 \text{ OD}$ [midpoint theorem] $AB \times 2\text{OF} = AE \times 2\text{OD}$ $\therefore AB \times \text{OF} = AE \times \text{OD}$	$\checkmark$ S $\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ S $\checkmark$ S/R $\checkmark$ S	 (7)

9.2



9.2.1	$\frac{RF}{FS} = \frac{RH}{HT}$ [line $\parallel$ one side of $\triangle OR$ prop theorem; FH $\parallel$ ST] $\frac{2x-10}{9} = \frac{4}{x-2}$ $(2x-10)(x-2) = 4 \times 9$ $2x^2 - 14x - 16 = 0$ $x^2 - 7x - 8 = 0$ $(x-8)(x+1) = 0$ $\therefore x = 8 \quad (x \neq -1)$ OR/OR $\frac{RF}{RS} = \frac{RH}{RT}$ [line $\parallel$ one side of $\triangle OR$ prop theorem; FH $\parallel$ ST] $\frac{2x-10}{2x-1} = \frac{4}{x+2}$ $(2x-10)(x+2) = 4(2x-1)$ $2x^2 - 14x - 16 = 0$ $x^2 - 7x - 8 = 0$ $(x-8)(x+1) = 0$ $\therefore x = 8 \quad (x \neq -1)$	✓ S/R ✓ substitution ✓ standard form ✓ factors ✓ answer with rejection (5) ✓ S/R ✓ substitution ✓ standard form ✓ factors ✓ answer with rejection (5)
9.2.2	$\frac{\text{area } \triangle RHF}{\text{area } \triangle RST} = \frac{\frac{1}{2} RF \times RH \sin \hat{R}}{\frac{1}{2} RS \times RT \sin \hat{R}}$ $= \frac{\frac{1}{2} \times 6 \times 4 \times \sin \hat{R}}{\frac{1}{2} \times 15 \times 10 \times \sin \hat{R}}$ $= \frac{24}{150} = \frac{4}{25}$	✓ numerator/teller ✓ denominator/noemer ✓ substitution ✓ answer

	OR/OF In $\triangle ODE$ and $\triangle ABE$ 1. $\hat{E}$ is common 2. $\hat{DOE} = \hat{EAB}$ (proved in 10.1.2) 3. $\hat{D}_1 = \hat{ABE}$ ($\angle$ sum Δ) $\triangle ODE \parallel \triangle ABE$ ($\angle \angle \angle$) $\frac{EO}{EA} = \frac{OD}{AB} = \frac{ED}{EB}$ ($\parallel \Delta s$) $\therefore AB \cdot EO = OD \cdot EA$ but $OE = FO$ (radii) $\therefore AB \times OF = OD \times EA$	$\checkmark$ S $\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ S $\checkmark$ S $\checkmark$ R	(7)
10.2	$\frac{AT}{TO} = \frac{AC}{CD} = \frac{3}{1}$ [line $\parallel$ one side of $\triangle OR$ prop theorem; $FC \parallel OD$] But $CD = DE$ $\frac{AE}{CE} = \frac{5}{2} \therefore AE = \frac{5}{2} CE$ $\frac{BE}{CE} = \frac{AE}{FE}$ [$\parallel \Delta s$] $\frac{BE}{CE} = \frac{\frac{5}{2} CE}{FE}$ $BE \times FE = \frac{5}{2} CE^2$ $\therefore 5CE^2 = 2BE \cdot FE$	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ S $\checkmark$ substitute $AE = \frac{5}{2} CE$	(5)
			[21]

**MATHEMATICS P2: JUNE 2018
MARKING GUIDELINES NOTES**

QUESTION 1

1.1.1	If left as 190, 25 then penalise 1 mark.
1.1.2	If the position is used: $\left[\frac{1}{4}(n+1) + \frac{3}{4}(n+1) \right] \div 2$ $= \frac{158 + 219}{2}$ $= \frac{377}{2}$ $= 188,5$

QUESTION 2

2.4	Do not accept estimation from the table.
-----	--

QUESTION 3

3.1	No ca if $\frac{x_2 - x_1}{y_2 - y_1}$	
3.3	$MD^2 + AM^2$ $= [(3-8)^2 + (-1+1)^2] + [(3-7)^2 + (-1-1)^2]$ $= 125 + 20$ $= 145$ AD^2 $= (7-8)^2 + (1+1)^2$ $= 145$ $MD^2 + AM^2 = AD^2$	✓ $AM^2 + MD^2$ ✓ AD^2 ✓ $MD^2 + AM^2 = AD^2$ (3)

QUESTION 4

4.3	Candidates can use the rotation of P through 90° to get to R(6 ; 4) If the candidate assumes that R(4 ; 6) : 1/4 marks
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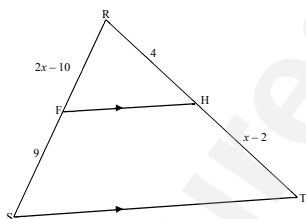
QUESTION 6

6.2	$y \in (-2 ; 2)$ 1/2 marks $-2 < y < 2$ 1/2 marks
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QUESTION 7

7.3	There is NO penalty for incorrect rounding.
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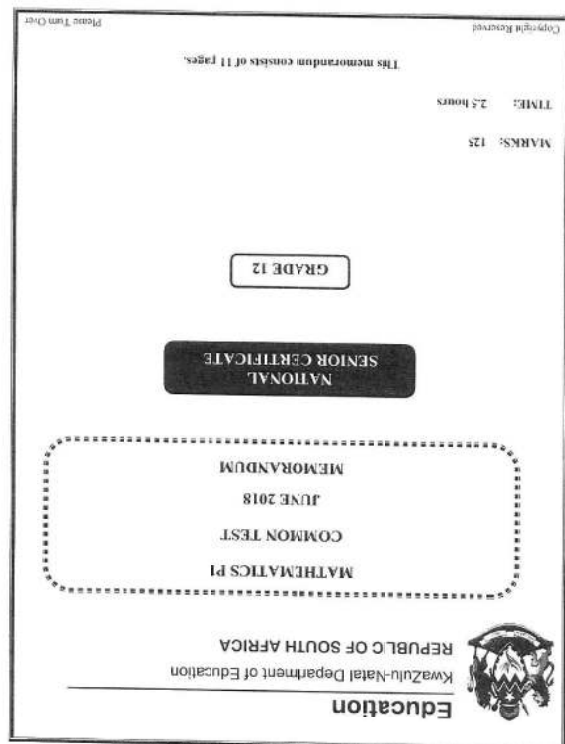
QUESTION 9

9.2.2	Join FT. $\text{area } \triangle RFH = \frac{4}{10} \times (\text{area } \triangle RFT)$ But $\text{area } \triangle RFT = \frac{6}{15} \times (\text{area } \triangle RST)$ (common vertex; = heights) $\text{area } \triangle RFH = \frac{4}{10} \times \frac{6}{15} \times (\text{area } \triangle RST)$ $\frac{\text{area } \triangle RFH}{\text{area } \triangle RST} = \frac{4}{25}$
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2.1	8 4 4 8 T_1 T_2 T_3 T_4 J_1 J_2 J_3 J_4 x $4x$ 4 $4x-8$ x $4x$ 4 $4x-8$ $p-12$	$x = 4$ and $p = 16$ $A \times y$ - value	(2)	
2.2	$2a + 4$ $a = 2$ $3a + b + c = 6$ $b = -6$ $a + b + c = 4$ $c = 8$ $T_1 = 2a^2 - 6a + 8$	$A \times a$ value $A \times b$ value $A \times c$ value $C \times$ answer	(4)	
2.3	$T_1 = 2a^2 - 6a + 8 = 12064$ $2a^2 - 6a - 12956 = 0$ $a^2 - 3a - 6478 = 0$ $(a - 82)(a + 79) = 0$ $\therefore a = 82$ for $a \neq -79$	$A \times$ quadratic factors $C \times$ answer	(3)	91

1111	(3)	A correct formula A substitution C✓ answer	1.3 1.4 1.5
1112	(3)	A correct formula A substitution C✓ answer	1.3 1.4 1.5

1.1.2	$x^2 + 8x - 5 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-8 \pm \sqrt{64 - 4(-5)}}{2(1)}$ $= \frac{-8 \pm \sqrt{76}}{2}$ $= -0.58 \quad \text{or} \quad -8.58$	A✓ formula CAC✓ answers IF possible for rounding off here only in the entire paper!	(4)	A✓ isolating and term CA✓ squaring both sides CA✓ standard form CA✓ factors CA✓ answers and rejecting	(5)	1.2 $x^2 - x + 1 = 0$ $(1) : x = x - 1 \dots (5)$ $x^2 - x + 1 = 0$ $(2) : -5x + 1 = 0$ $x^2 - x - 3 = 0$ $2x^2 - 5x - 3 = 0$ $x^2 - x - 3 = 0$ $x^2 - x^2 - x - x = 3$ $-2x = 3$ $x = -\frac{3}{2}$ $x = -\frac{3}{2} \quad \text{or} \quad x = 1$ $x = \frac{2}{1} \quad \text{or} \quad x = -2$	1.3 $2x - \sqrt{3} + x = 1$ $3x - \sqrt{3} = 1$ $3x = 1 + \sqrt{3}$ $x = \frac{1 + \sqrt{3}}{3}$	1.4 $x^2 - x + 3x^2 = 0$ $4x^2 - 1 + 3x^2 = 0$ $7x^2 - 1 + 3x^2 = 0$ $10x^2 - 1 = 0$ $10x^2 = 1$ $x^2 = \frac{1}{10}$ $x = \pm \sqrt{\frac{1}{10}}$ $x = \pm \frac{1}{\sqrt{10}}$	A✓ factorization A✓ reasoning A✓ conclusion	(3)	100
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6.1	$h(x) = a^x$ $\frac{1}{9} = a^2$ $3 = a^1$ $a = 3$	CA✓ answer	(2)
6.2	$y = \log_3 x$	CAC/CA✓ answer	(2)
6.3	$\left(\frac{1}{9}, -2\right)$ that satisfies the equation.	AA✓ answer or any other point	(2)
6.4	$h(x) \leq -2$ $\log_3 x \leq -2$ $x = 3^{-2} = \frac{1}{9}$ $0 < x \leq \frac{1}{9}$	CA✓ x-value	(2)
6.5	$f(x) = h\left(\frac{x}{2}\right)$ $= \log_3\left(\frac{x}{2}\right)$	CAC/CA✓ answer	(2)

Mathematics P1
QUESTION 6

7 NSC Memo

June Comm Test 2018

7.1	$A = P(1 - i)^t$ $= 500\,000(1 - 0.0925)^t$ $= R253\,152.42$	CA✓ substitution into formula	(3)
7.2.1	$A = P(1 + i)^t$ $= 900\,000(1 + 0.0825)^t$ $= R131\,777.79$ He will pay R133 777.79 + R300 000	CA✓ value	(3)
7.2.2	$F_t = \frac{Rt}{(1 + i)^t - 1}$ $= \frac{R(12706.94)}{\left(1 + \frac{0.12}{12}\right)^{12} - 1}$ $= R12\,706.94$	CA✓ substitution of t CA✓ formula	(5)
7.2.3	$F_t = \frac{Rt}{i(1 + i)^t - 1}$ $= \frac{8000}{\frac{0.12}{12} \left(1 + \frac{0.12}{12}\right)^{12} - 1}$ $= R630.79$ New monthly deposit = R 1337.75	CA✓ substitution of t and i values CA✓ formula	(4)

Mathematics P1
QUESTION 7

8 NSC Memo

June Comm Test 2018

4.1	$x = -1$ and $y = -3$	CA✓ shape CA✓ x-intercept CA✓ y-intercept CA✓ asymptotes	(4)
4.2	y-intercept: $y = -5$ x-intercept: $x = -\frac{5}{3}$	CA✓ x-intercept value CA✓ y-intercept value	(2)
4.3			

Mathematics P1
QUESTION 4

5 NSC Memo

June Comm Test 2018

5.1	$y = -\frac{2}{3}x + \frac{1}{3}$ $\left(\frac{1}{4}, -\frac{1}{6}\right)$ $\left(\frac{1}{4}, -\frac{1}{6}\right) = 0$	CA✓ axis of symmetry value CA✓ y-intercept value CA✓ answer [coordinate form]	(3)
5.2	$y \leq 6\frac{1}{2}$	CAC/CA✓ answer	(2)
5.3	$2x^2 - 3x + 6 = 0$ $(2x + 3)(x - 2) = 0$ $x = -\frac{3}{2}$ or $x = 2$ $ BC = 3\frac{1}{2}$ units	CA✓ factors CA✓ x-values CA✓ answer	(4)
5.4	$-2x^2 + 3x + 6 = \frac{3}{2}x + \frac{3}{2}$ $-4x^2 + 2x + 12 = -3x + 6$ $4x^2 - 5x - 6 = 0$ $(4x + 3)(x - 2) = 0$ $x = -\frac{3}{4}$ or $x = 2$ $P\left(-\frac{3}{4}, \frac{41}{8}\right)$	CA✓ standard form CA✓ factors CA✓ x-values CA✓ answer	(5)
5.5	$h(x) = -2(x - 3)^2 + (x - 3) - 3$ $= -2(x^2 - 6x + 9) + x - 6$ $= -2x^2 + 13x - 24$ New turning point $\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$ $\left(-\frac{13}{-4}, \frac{169 - 288}{-16}\right)$	CA✓ substituting $(x - 3)$ CA✓ x-value CA✓ y-value	(3)

Mathematics P1
QUESTION 5

6 NSC Memo

June Comm Test 2018

Mathematics P1
June Comm Test 2018

8.1	$f(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{3(x+h)^2 - 5(x+h) - (3x^2 - 5x)}{h}$ $= \lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 - 5x - 5h - 3x^2 + 5x}{h}$ $= \lim_{h \rightarrow 0} \frac{6xh + 3h^2 - 5h}{h}$ $= \lim_{h \rightarrow 0} \frac{h(6x + 3h - 5)}{h}$ $= \lim_{h \rightarrow 0} (6x + 3h - 5)$ $= 6x - 5$	CA✓ formula A✓ substitution CA✓ simplification of numerator CA✓ factors CA✓ answer	8.2.1 $f(x) = x - 4x^2$ $A^{\vee}x \quad A^{\vee} - 4x^2$ CA✓ answer CA✓ answer 8.2.2 $g(x) = \sqrt{x^2 + (3x)^2}$ $= \sqrt{x^2 + 9x^2}$ $= \sqrt{10x^2}$ $= \frac{\sqrt{10}}{2}x^2$ CA✓ answer CA✓ answer	(4)
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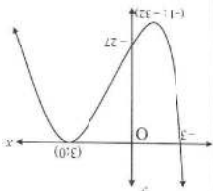
Mathematics P1
Test 2018
June Comm

9.5	$f(x) = -3x^2 + 6x + 9$ $f(10) = 5$ $y = 9x - 27$	A✓ substituting $x = 10$ into derivative CA✓ gradient value CA✓ equation CA✓ $-32 < 8 < 0$ CA✓ 0 A✓ inequality A✓ $f(x+3)$ CA✓ substitution of $(x+3)$ into f	9.7 $f(x) = f(x+3)$ $= -(x+3)^2 + 3(x+3) + 9(x+3) - 27$ CA✓ substitution of $(x+3)$ into f	(2)
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Total Marks : 125

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Mathematics P1
Test 2018
June Comm

9.1	$x = -3$ or $x = 3$ $y = -27$	AA✓ x - intercept / point form A✓ y - intercept / point form	(3)
9.2	$f(x) = x^2 + 3x^2 + 9x - 27$ $f(x) = -3x^2 + 6x + 9 = 0$ $x^2 - 2x - 3 = 0$ $(x+3)(x-3) = 0$ $x = -3$ or $x = 3$ $y = -3x$ or $y = 0$ $x = 1$ or $x = 3$	A✓ derivative and equal to 0 CA✓ standard form CA✓ factors CA✓ x - values CA✓ y - values	(3)
9.3		A✓ shape CA✓ x - intercept CA✓ y - intercept CA✓ turning points	(4)
9.4	$f(x) = -3x^2 + 6x + 9$ $f'(x) = -6x + 6 = 0$ $x = 1$ $x > 1$	CA✓ second derivative CA✓ answer	(2)

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2.1	$[x - 5]^2 + (y - 4)^2 = 25$	✓✓ correct substitution	
2.2	$[x - 5]^2 + (y + 4)^2 = 25$	✓✓ answer	
(d)	$A(2, 0)$ and $B(8, 0)$ $x = 8$ or $x = 2$ $x - 5 = x - 3$ $[x - 5]^2 = 9$ $[x - 5]^2 + 16 = 25$	✓✓ Correct substitution $x - 5 = \pm 3$ $A(2, 0)$ $B(8, 0)$	

QUESTION 2

6	$\tan \alpha = \frac{-3}{2}$ $\tan \alpha = \text{gradient of RM}$ $\therefore \alpha = 56.3^\circ$ $\therefore \text{ref angle} = 180^\circ - 56.3^\circ$ $= 123.7^\circ$	(4) ✓ correct quadrant ✓ answer 56.3° $\therefore \text{ref angle} = 180^\circ - 56.3^\circ$ $\therefore \text{int.} = 123.7^\circ$
1.7	$\text{RM}^2 = (3 + 3)^2 + (6 - 6)^2$ $= 36 + 16$ $= 52$ $\text{RM} = \sqrt{52}$ $= 7.2$ $\text{RA} = 7.2$ $\therefore \text{Area of a ARM} = \frac{1}{2} \times \text{base} \times \text{height}$ $= \frac{1}{2} \times 7.2 \times 7.2$ $= 25.9 \text{ units}^2$	(3) ✓ answer ✓ substitution $\therefore \text{RM} = 7.2$

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education

Department of Education
PROVINCE OF KWAZULU-NATAL



**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

MATHEMATICS p2
MEMORANDUM
COMMON TEST
JUNE 2018

MARKS: 125 TIME: 2½ hours

This memorandum consists of pages.

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4.4	$\sin x + \cos x = 1$ $\sin x = 1 - \cos x$ $(\sin x)^2 = (1 - \cos x)^2$ $\sin^2 x = 1 - 2\cos x + \cos^2 x$ $1 - \cos^2 x = 1 - 2\cos x + \cos^2 x$ $- \cos^2 x = - 2\cos x + \cos^2 x$ $- 2\cos^2 x + 2\cos x = 0$ $- 2\cos x (\cos x - 1) = 0$ $\therefore \cos x = 0$ $\therefore x = 0^\circ$ $\text{OR } 270^\circ$ $\therefore x \in [0^\circ, 360^\circ]$	$\sin x + \cos x = 1$ $\cos x = 1 - \sin x$ $\cos^2 x = (1 - \sin x)^2$ $\cos^2 x = 1 - 2\sin x + \sin^2 x$ $1 - \sin^2 x = 1 - 2\sin x + \sin^2 x$ $- \sin^2 x = - 2\sin x + \sin^2 x$ $- 2\sin^2 x + 2\sin x = 0$ $- 2\sin x (\sin x - 1) = 0$ $\therefore \sin x = 0$ $\therefore x = 0^\circ$ $\text{OR } 360^\circ$ $\therefore x \in [0^\circ, 360^\circ]$	4.5
4.6	$\sin x + \cos x = 1$ $\cos x = 1 - \sin x$ $\cos^2 x = (1 - \sin x)^2$ $\cos^2 x = 1 - 2\sin x + \sin^2 x$ $1 - \sin^2 x = 1 - 2\sin x + \sin^2 x$ $- \sin^2 x = - 2\sin x + \sin^2 x$ $- 2\sin^2 x + 2\sin x = 0$ $- 2\sin x (\sin x - 1) = 0$ $\therefore \sin x = 0$ $\therefore x = 0^\circ$ $\text{OR } 360^\circ$ $\therefore x \in [0^\circ, 360^\circ]$	$\sin x + \cos x = 1$ $\cos x = 1 - \sin x$ $\cos^2 x = (1 - \sin x)^2$ $\cos^2 x = 1 - 2\sin x + \sin^2 x$ $1 - \sin^2 x = 1 - 2\sin x + \sin^2 x$ $- \sin^2 x = - 2\sin x + \sin^2 x$ $- 2\sin^2 x + 2\sin x = 0$ $- 2\sin x (\sin x - 1) = 0$ $\therefore \sin x = 0$ $\therefore x = 0^\circ$ $\text{OR } 360^\circ$ $\therefore x \in [0^\circ, 360^\circ]$	4.7
4.8	$\sin x + \cos x = 1$ $\cos x = 1 - \sin x$ $\cos^2 x = (1 - \sin x)^2$ $\cos^2 x = 1 - 2\sin x + \sin^2 x$ $1 - \sin^2 x = 1 - 2\sin x + \sin^2 x$ $- \sin^2 x = - 2\sin x + \sin^2 x$ $- 2\sin^2 x + 2\sin x = 0$ $- 2\sin x (\sin x - 1) = 0$ $\therefore \sin x = 0$ $\therefore x = 0^\circ$ $\text{OR } 360^\circ$ $\therefore x \in [0^\circ, 360^\circ]$	$\sin x + \cos x = 1$ $\cos x = 1 - \sin x$ $\cos^2 x = (1 - \sin x)^2$ $\cos^2 x = 1 - 2\sin x + \sin^2 x$ $1 - \sin^2 x = 1 - 2\sin x + \sin^2 x$ $- \sin^2 x = - 2\sin x + \sin^2 x$ $- 2\sin^2 x + 2\sin x = 0$ $- 2\sin x (\sin x - 1) = 0$ $\therefore \sin x = 0$ $\therefore x = 0^\circ$ $\text{OR } 360^\circ$ $\therefore x \in [0^\circ, 360^\circ]$	4.9
4.9	$\sin x + \cos x = 1$ $\cos x = 1 - \sin x$ $\cos^2 x = (1 - \sin x)^2$ $\cos^2 x = 1 - 2\sin x + \sin^2 x$ $1 - \sin^2 x = 1 - 2\sin x + \sin^2 x$ $- \sin^2 x = - 2\sin x + \sin^2 x$ $- 2\sin^2 x + 2\sin x = 0$ $- 2\sin x (\sin x - 1) = 0$ $\therefore \sin x = 0$ $\therefore x = 0^\circ$ $\text{OR } 360^\circ$ $\therefore x \in [0^\circ, 360^\circ]$	$\sin x + \cos x = 1$ $\cos x = 1 - \sin x$ $\cos^2 x = (1 - \sin x)^2$ $\cos^2 x = 1 - 2\sin x + \sin^2 x$ $1 - \sin^2 x = 1 - 2\sin x + \sin^2 x$ $- \sin^2 x = - 2\sin x + \sin^2 x$ $- 2\sin^2 x + 2\sin x = 0$ $- 2\sin x (\sin x - 1) = 0$ $\therefore \sin x = 0$ $\therefore x = 0^\circ$ $\text{OR } 360^\circ$ $\therefore x \in [0^\circ, 360^\circ]$	4.10

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QUESTIONS

(*)	$\frac{z}{1} = \frac{d\theta}{d\psi} = \theta \quad \text{ms}^{-1}$ $\frac{z}{1} = \theta \quad \text{ms}^{-1}$ $\theta \quad \text{ms}^{-1} = 1$	$\frac{z}{1} = \frac{d\theta}{d\psi} = \theta \quad \text{ms}^{-1}$ $\frac{z}{1} = \theta \quad \text{ms}^{-1}$ $\theta \quad \text{ms}^{-1} = 1$	$\frac{z}{1} = \frac{d\theta}{d\psi} = \theta \quad \text{ms}^{-1}$ $\frac{z}{1} = \theta \quad \text{ms}^{-1}$ $\theta \quad \text{ms}^{-1} = 1$
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QUESTION

[illegible]

(6)	$\begin{matrix} \text{S/R} \\ \text{S/R} \\ \text{S/R} \\ \text{S/R} \\ \text{S/R} \end{matrix}$	$\begin{aligned} \Delta C &= \text{A: subduces the same net CP} \\ &= \text{CE: } \angle \text{ opposite to equal sides} \\ &= \text{BF: opp. sides of a para.} \end{aligned}$
(5)	$\begin{matrix} \text{S/R} \\ \text{S/R} \\ \text{S/R} \\ \text{S/R} \end{matrix}$	$\begin{aligned} E &= \text{D: opp. } \angle \text{ of a para.} \\ &= \text{DEF: is a parallelogram} \\ &= \text{BC: } \angle \text{ FE: } \angle \text{ correct a proved equal} \\ &= \text{B: ext. } \angle \text{ cyclic quad HCDF} \end{aligned}$
(4)	$\begin{matrix} \text{S/R} \\ \text{S/R} \\ \text{S/R} \\ \text{S/R} \end{matrix}$	$\begin{aligned} F_2 &= \text{D: } \angle \text{ BMT, chord EF} \\ &= \text{BMT: } \angle \text{ CE: } \angle \text{ proved equal} \\ &= \text{F: } \angle \text{ } \\ &= \text{F: } \angle \text{ } \\ &= \text{F: } \angle \text{ } \end{aligned}$

1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020, 2021, 2022, 2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038, 2039, 2040, 2041, 2042, 2043, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2057, 2058, 2059, 2060, 2061, 2062, 2063, 2064, 2065, 2066, 2067, 2068, 2069, 2070, 2071, 2072, 2073, 2074, 2075, 2076, 2077, 2078, 2079, 2080, 2081, 2082, 2083, 2084, 2085, 2086, 2087, 2088, 2089, 2090, 2091, 2092, 2093, 2094, 2095, 2096, 2097, 2098, 2099, 2100, 2101, 2102, 2103, 2104, 2105, 2106, 2107, 2108, 2109, 2110, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2137, 2138, 2139, 2140, 2141, 2142, 2143, 2144, 2145, 2146, 2147, 2148, 2149, 2150, 2151, 2152, 2153, 2154, 2155, 2156, 2157, 2158, 2159, 2160, 2161, 2162, 2163, 2164, 2165, 2166, 2167, 2168, 2169, 2170, 2171, 2172, 2173, 2174, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2182, 2183, 2184, 2185, 2186, 2187, 2188, 2189, 2190, 2191, 2192, 2193, 2194, 2195, 2196, 2197, 2198, 2199, 2200, 2201, 2202, 2203, 2204, 2205, 2206, 2207, 2208, 2209, 2210, 2211, 2212, 2213, 2214, 2215, 2216, 2217, 2218, 2219, 2220, 2221, 2222, 2223, 2224, 2225, 2226, 2227, 2228, 2229, 2230, 2231, 2232, 2233, 2234, 2235, 2236, 2237, 2238, 2239, 2240, 2241, 2242, 2243, 2244, 2245, 2246, 2247, 2248, 2249, 2250, 2251, 2252, 2253, 2254, 2255, 2256, 2257, 2258, 2259, 2260, 2261, 2262, 2263, 2264, 2265, 2266, 2267, 2268, 2269, 2270, 2271, 2272, 2273, 2274, 2275, 2276, 2277, 2278, 2279, 2280, 2281, 2282, 2283, 2284, 2285, 2286, 2287, 2288, 2289, 2290, 2291, 2292, 2293, 2294, 2295, 2296, 2297, 2298, 2299, 2300, 2301, 2302, 2303, 2304, 2305, 2306, 2307, 2308, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2329, 2330, 2331, 2332, 2333, 2334, 2335, 2336, 2337, 2338, 2339, 2340, 2341, 2342, 2343, 2344, 2345, 2346, 2347, 2348, 2349, 2350, 2351, 2352, 2353, 2354, 2355, 2356, 2357, 2358, 2359, 2360, 2361, 2362, 2363, 2364, 2365, 2366, 2367, 2368, 2369, 2370, 2371, 2372, 2373, 2374, 2375, 2376, 2377, 2378, 2379, 2380, 2381, 2382, 2383, 2384, 2385, 2386, 2387, 2388, 2389, 2390, 2391, 2392, 2393, 2394, 2395, 2396, 2397, 2398, 2399, 2400, 2401, 2402, 2403, 2404, 2405, 2406, 2407, 2408, 2409, 2410, 2411, 2412, 2413, 2414, 2415, 2416, 2417, 2418, 2419, 2420, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2428, 2429, 2430, 2431, 2432, 2433, 2434, 2435, 2436, 2437, 2438, 2439, 2440, 2441, 2442, 2443, 2444, 2445, 2446, 2447, 2448, 2449, 2450, 2451, 2452, 2453, 2454, 2455, 2456, 2457, 2458, 2459, 2460, 2461, 2462, 2463, 2464, 2465, 2466, 2467, 2468, 2469, 2470, 2471, 2472, 2473, 2474, 2475, 2476, 2477, 2478, 2479, 2480, 2481, 2482, 2483, 2484, 2485, 2486, 2487, 2488, 2489, 2490, 2491, 2492, 2493, 2494, 2495, 2496, 2497, 2498, 2499, 2500, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2508, 2509, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2517, 2518, 2519, 2520, 2521, 2522, 2523, 2524, 2525, 2526, 2527, 2528, 2529, 2530, 2531, 2532, 2533, 2534, 2535, 2536, 2537, 2538, 2539, 2540, 2541, 2542, 2543, 2544, 2545, 2546, 2547, 2548, 2549, 2550, 2551, 2552, 2553, 2554, 2555, 2556, 2557, 2558, 2559, 2560, 2561, 2562, 2563, 2564, 2565, 2566, 2567, 2568, 2569, 2570, 2571, 2572, 2573, 2574, 2575, 2576, 2577, 2578, 2579, 2580, 2581, 2582, 2583, 2584, 2585, 2586, 2587, 2588, 2589, 2590, 2591, 2592, 2593, 2594, 2595, 2596, 2597, 2598, 2599, 2600, 2601, 2602, 2603, 2604, 2605, 2606, 2607, 2608, 2609, 2610, 2611, 2612, 2613, 2614, 2615, 2616, 2617, 2618, 2619, 2620, 2621, 2622, 2623, 2624, 2625, 2626, 2627, 2628, 2629, 2630, 2631, 2632, 2633, 2634, 2635, 2636, 2637, 2638, 2639, 2640, 2641, 2642, 2643, 2644, 2645, 2646, 2647, 2648, 2649, 2650, 2651, 2652, 2653, 2654, 2655, 2656, 2657, 2658, 2659, 2660, 2661, 2662, 2663, 2664, 2665, 2666, 2667, 2668, 2669, 2670, 2671, 2672, 2673, 2674, 2675, 2676, 2677, 2678, 2679, 26

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QUESTION 4

4. 2 $\sin 104^\circ = \sin (180^\circ - 76^\circ) = \sin 76^\circ$ $\therefore \sin \theta = \frac{5}{13}$ $\sin^2 \theta = \frac{25}{169}$ $\cos 2\theta = \frac{1}{5} = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$ $\therefore X = -1$ $X^2 = 1$ $= 25 - 24$ $= 5^2 - 4^2$ $\therefore X^2 = p^2 - q^2$ $p^2 = x^2 + y^2$ $\therefore 20$ is in the second quadrant $0 < \alpha < 2\pi, 2\pi > \theta$	✓ answer ✓ simplifying ✓ $1 - 2 \sin^2 \theta$ ✓ $X = -1$ Pythagoras ✓ correct diagram ✓ applying	✓ $\cos 30^\circ$ ✓ $\sin 76^\circ$ ✓ $\sin 52^\circ$ ✓ $\left(\frac{\sqrt{5}}{3}\right)$ ✓ $\left(\frac{1}{\cos 38^\circ}\right)$ ✓ $2 \sin 38^\circ \cos 38^\circ$ ✓ $\sqrt{3} \sin 31^\circ \cos 38^\circ$ ✓ $\sqrt{3} \sin 31^\circ \cos 38^\circ$	✓ $\cos 30^\circ$ ✓ $\sin 76^\circ$ ✓ $\sin 52^\circ$ ✓ $\left(\frac{\sqrt{5}}{3}\right)$ ✓ $\left(\frac{1}{\cos 38^\circ}\right)$ ✓ $2 \sin 38^\circ \cos 38^\circ$ ✓ $\sqrt{3} \sin 31^\circ \cos 38^\circ$ ✓ $\sqrt{3} \sin 31^\circ \cos 38^\circ$
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TOTAL MARKS: 115

6.4	$KP = \frac{1}{2} CE \dots AF = FK, KP \parallel CE$	$\checkmark$ S/R
	$= \frac{1}{2} BF \dots$ from 1.3	$\checkmark$ S
	$\therefore k$ is the mid point of BF	$\checkmark$ S
	$AK = KC \dots KF = FE, KP \parallel CE$	$\checkmark$ S/R
	$\therefore k$ is the midpoint of AC	$\checkmark$ S
	$\therefore BK = KF = AK = KC = \text{radius}$	$\checkmark$ S
	$\therefore BF$ is the diameter	$\checkmark$ S
121	(7)	

Mathematics P1

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NSC

Common Test June 2018

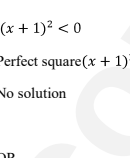
**JOHANNESBURG SOUTH DISTRICT
CLUSTER PAPER
JUNE EXAMINATION 2019**

GRADE 12

**MATHEMATICS
PAPER 1
MEMORANDUM**

MARKS: 150

This memorandum consists of 19 pages, including the cover page.

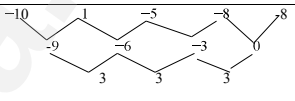
1.1.3	$x^2 + 2x + 1 < 0$ $(x + 1)^2 < 0$ Critical value: $x = -1$ $\begin{array}{c} ++ \quad 0 \quad ++ \\ \hline -1 \end{array}$ No solution OR $(x + 1)^2 < 0$ Perfect square $(x + 1)^2$ is always positive No solution OR 	✓ Factors ✓ critical value $x = -1$ ✓ No solution OR ✓ Factors ✓ Explanation ✓ No solution OR ✓ for correct graph ✓ graph touching x-axis at $x = -1$ ✓ conclusion	(3)
	The graph does not lie below x-axis; therefore, No solution		(3)

QUESTION 1

1.1.1	$(x + 5)(x - 6) = 0$ $x = -5 \text{ or } x = 6$	✓ factors /subst. in quad.formula ✓ answers	(2)
1.1.2	$3x^2 - 2x - 14 = 0$ $x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-14)}}{2(3)}$ $x = \frac{1 \pm \sqrt{172}}{6}$ $x = -1,85 \text{ or } x = 2,52$ If left in simplified surd form; MAX = 3/4 OR/OF $3x^2 - 2x - 14 = 0$ $x^2 - \frac{2}{3}x = \frac{14}{3}$ $x^2 - \frac{2}{3}x + \frac{1}{9} = \frac{14}{3} + \frac{1}{9}$ $(x - \frac{1}{3})^2 = \frac{43}{9}$ $x - \frac{1}{3} = \pm \frac{\sqrt{43}}{3}$ $x = \frac{1}{3} \pm \frac{\sqrt{43}}{3}$ $x = -1,85 \text{ or } x = 2,52$	✓ Standard form ✓ Substitution into correct formula ✓ $x = -1,85$ ✓ $x = 2,52$ PENALIZE -1 MARK FOR INCORRECT ROUNDING OR ✓ Standard form ✓ For adding $\frac{1}{9}$ on both sides ✓ $x = -1,85$ ✓ $x = 2,52$	(4)

1.1.4	$\sqrt{x^2 - 10} = 3\sqrt{x}$ $x^2 - 10 = (3\sqrt{x})^2$ $x^2 - 10 = 9x$ $x^2 - 9x - 10 = 0$ $(x + 1)(x - 10) = 0$ $x = -1 \text{ or } x = 10$ $\therefore x = 10$	✓squaring both sides ✓ standard form ✓ Factors / subst in quad formula ✓Both values of x ✓ correct selection	(5)
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1.2	$3^{x+1} = 9^y \dots\dots\dots (1)$ and $xy = 3 \dots\dots (2)$ $3^{x+1} = 3^{2y}$ $x + 1 = 2y$ $x = 2y - 1 \dots\dots\dots (3)$ Substitute (3) into (2): $(2y - 1)y = 3$ $2y^2 - y - 3 = 0$ $(2y - 3)(y + 1) = 0$ $y = -1$ or $y = \frac{3}{2}$ $x = -3$ or $x = 2$ OR / OF $3^{x+1} = 9^y \dots\dots\dots (1)$ and $xy = 3 \dots\dots (2)$ $3^{x+1} = 3^{2y}$ $x + 1 = 2y$ $y = \frac{x+1}{2} \dots\dots\dots (3)$ Substitute (3) into (2): $x(\frac{x+1}{2}) = 3$ $x^2 + x = 6$ $x^2 + x - 6 = 0$ $(x + 3)(x - 2) = 0$ $x = -3$ or $x = 2$ $y = -1$ or $y = \frac{3}{2}$	$\checkmark 3^{2y}$ $\checkmark x$ subject of the formula $\checkmark$ substitution $\checkmark$ standard form $\checkmark$ factors /quad formula $\checkmark$ values of y $\checkmark$ values of x OR /OF $\checkmark 3^{2y}$ $\checkmark y$ subject of the formula $\checkmark$ substitution $\checkmark$ standard form $\checkmark$ factors /quad formula $\checkmark$ values of x $\checkmark$ values of y	(7)
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2.1.3	$S_{\infty} = \frac{a}{1-r}$ $= \frac{9}{1-\frac{1}{3}} = \frac{27}{2} = 13,5$	$\checkmark$ Correct formula $\checkmark$ Correct Substitution $\checkmark$ Answer	(3)
2.2.1	AS: $x; 12; y; 24$ $12 - x = y - 12 = 24 - y$ $y - 12 = 24 - y$ $2y = 36$ $y = 18$ also $12 - x = y - 12$ $12 - x = 18 - 12$ $x = 6$	$\checkmark y - 12 = 24 - y$ $\checkmark y = 18$ $\checkmark x = 6$	(3)
2.2.2	$3T_{13} = 3(6) = 18$	$\checkmark$ answer	(1)
2.2.3	$S_{15} = 6 \times 15 = 90$ $S_{15} = \frac{15}{2} [2 \times 6 + (15-1) \times 6]$ $S_{15} = 720$ $S_{30} = 90 + 720 = 810$	$\checkmark 90$ $\checkmark$ subst. into correct formula. $\checkmark 720$ $\checkmark$ Answer	(4)
2.3.1	 $T_n = a + (n-1)d$ $81 = -9 + 3(n-1)$ $81 = -9 + 3n - 3$ $81 = 3n - 12$ $93 = 3n$ $n = 31$ $T_{31} = 81$	$\checkmark$ First differences $\checkmark$ substitution $\checkmark n = 31$	(3)

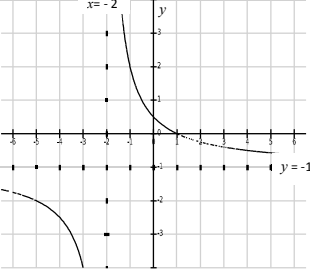
1.3	$\Delta = b^2 - 4ac$ $41 = (-5)^2 - 4(1)(p)$ $41 = 25 - 4p$ $4p = -16$ $p = -4$ OR $x = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(p)}}{2}$ $\frac{5 \pm \sqrt{41}}{2} = \frac{5 \pm \sqrt{25 - 4p}}{2}$ $41 = 25 - 4p$ $4p = -16$ $p = -4$	$\checkmark \Delta = 41$ $\checkmark$ substitution $\checkmark$ Answer OR $\checkmark$ substitution $\checkmark 41 = 25 - 4p$ $\checkmark$ Answer	(3)
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QUESTION 2

2.1.1	$9 + 3 + 1 + \frac{1}{3} + \dots\dots$ Next term = $\frac{1}{9}$	$\checkmark$ Answer	(1)
2.1.2	$r = \frac{1}{3}$ Since $-1 < r < 1$	$\checkmark$ Explanation $-1 < r < 1$	(1)

2.3.2	$2a = 3$ $3a + b = -9$ $a = \frac{3}{2}$ $3\left(\frac{3}{2}\right) + b = -9$ $b = \frac{-27}{2}$ $a + b + c = 10$ $\frac{3}{2} - \frac{27}{2} + c = 10$ $c = 22$ $\therefore T_n = \frac{3}{2}n^2 - \frac{27}{2}n + 22$ OR $T_n = T_1 + (n-1)d_1 + \frac{(n-1)(n-2)d_2}{2}$ $= 10 + (n-1)(-9) + \frac{(n-1)(n-2)(3)}{2}$ $= 10 - 9n + 9 + \frac{3}{2}n^2 - \frac{9}{2}n + 3$ $\therefore T_n = \frac{3}{2}n^2 - \frac{27}{2}n + 22$	$\checkmark$ value of a $\checkmark$ Value of b $\checkmark$ Value of c $\checkmark T_n = \frac{3}{2}n^2 - \frac{27}{2}n + 22$ OR $\checkmark$ substitution in the formula $\checkmark$ value of a $\checkmark$ Value of b $\checkmark$ Value of c	(4)
2.4	$\sum_{n=1}^5 5(3)^{n-1} = p$ $\sum_{n=1}^5 45(3)^n = \sum_{n=1}^5 45 \frac{(3)^n \times 3}{3}$ $\sum_{n=1}^5 9 \times 5(3)^{n-1} \times 3$ $= \sum_{n=1}^5 27.5(3)^{n-1}$ $= 27 \sum_{n=1}^5 5(3)^{n-1}$ $= 27 \times p$ $= 27p$	$\checkmark \sum_{n=1}^5 45 \frac{(3)^n \times 3}{3}$ $\checkmark 27.5(3)^{n-1}$ $\checkmark 27p$	(3)

QUESTION 3

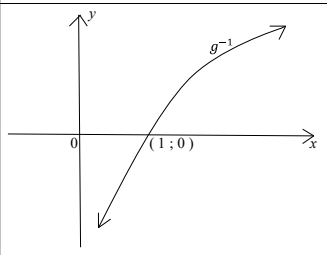
3.1	$x = -2$ and $y = -1$	✓ $x = -2$ ✓ $y = -1$	(2)
3.2	$y = \frac{3}{x+2} - 1$ $y = \frac{1}{2}$ (0; $\frac{1}{2}$) x-intercept: (let $y = 0$) $0 = \frac{3}{x+2} - 1$ $1 = \frac{3}{x+2}$ $x+2 = 3$ $x = 1$ (1; 0)	✓ $y = \frac{1}{2}$ ✓ $y = 0$ ✓ $x = 1$	(3)
3.3		✓ shape in QUAD 1 and 3 ✓ Both asymptotes ✓ Both x and y-intercepts	(3)
3.4	$m(x) = \frac{3}{x+2-2} - 1$ $m(x) = \frac{3}{x} - 1$ Domain: $x \in \mathbb{R}; x \neq 0$ ACCEPT: $x < 0$ or $x > 0$	✓ $m(x)$ ✓ $x \in \mathbb{R}; x \neq 0$ (both must be correct)	(2)
			[10]

QUESTION 4

4.1	$f(x) = -x^2 - 4x + 5$ $g(x) = mx + c$ At M (y-intercept: let $x = 0$) M(0; 5) At E and P (x-intercept: $y = 0$) $-x^2 - 4x + 5 = 0$ $x^2 + 4x - 5 = 0$ $(x-1)(x+5) = 0$ $x = -5$ or $x = 1$ E(-5; 0) and P(1; 0)	✓ M(0; 5) ✓ $y = 0$ ✓ E(-5; 0) ✓ P(1; 0)	(4)
4.2	$x = \frac{-(-4)}{2(-1)} = -2$ $y = -(-2)^2 - 4(-2) + 5 = 9$ N(-2; 9) OR / OF $f(x) = -(x^2 + 4x - 5)$ $= -[x^2 + 4x + (2)^2 - 5 - (2)^2]$ $= -[x^2 + 4x + 4 - 5 - 4]$ $= -[(x+2)^2 - 9]$ $f(x) = -(x+2)^2 + 9$ N(-2; 9) OR $x_N = \frac{-5+1}{2} = -2$ $y = -(-2)^2 - 4(-2) + 5 = 9$ N(-2; 9)	✓ $x = -2$ ✓ substitution ✓ $y = 9$ OR ✓ Adding and subtracting $(2)^2$ ✓ $-(x+2)^2 + 9$ ✓ N(-2; 9) OR ✓ $x = -2$ ✓ substitution ✓ $y = 9$	(3)
			(3)

4.3	$m_{EM} = \frac{5-0}{0-(-5)} = 1$ $c = 5$ (y-intercept) $y = x + 5$	✓ substitution ✓ $m = 1$ ✓ $y = x + 5$	(3)
4.4	$NT = y_N - y_T$ $= [-(-2)^2 - 4(-2) + 5] - [(-2) + 5]$ $= 9 - 3$ $= 6$ units OR $NT = f(x) - g(x)$ $= -x^2 - 4x + 5 - (x + 5)$ $NT = -x^2 - 5x$ When $x = -2$ $NT = -(-2)^2 - 5(-2)$ $NT = 6$ units	✓ substitution ✓ simplification ✓ Answer OR ✓ $NT = -x^2 - 5x$ ✓ subst $x = -2$ ✓ Answer	(3)
4.5	S(-4; 5)	✓ $x = -4$ ✓ $y = 5$	(2)
4.6.1	$f(x) < g(x)$ $x < -5$ or $x > 0$ OR $x \in (-\infty; -5) \cup (0; \infty)$	✓ $x < -5$ ✓ $x > 0$ OR ✓ $(-\infty; -5) \cup (0; \infty)$	(2)
4.6.2	$f(x), g(x) < 0$ $x > 1$ OR $x \in (1; \infty)$	✓ critical value ✓ correct inequality OR ✓ critical value ✓ correct notation	(2)
			[19]

QUESTION 5

5.1	P(0; 1)	✓ P(0; 1)	(1)
5.2	$a > 1$ OR $a \in (1; \infty)$ OR $1 < a < \infty$	✓ critical value ✓ correct notation	(2)
5.3	g: $5^x = y$ g^{-1} : $5^y = x$ $y = \log_5 x$	✓ swapping x and y ✓ Answer	(2)
5.4		✓ correct shape ✓ x-int (1; 0)	(2)
5.5	$\log_5 x = 0$ $x = (5)^0 = 1$ $\therefore 0 < x < 1$ OR $x \in (0; 1)$	✓ critical value $x = 1$ ✓ correct notation/inequality	(2)
			[9]

QUESTION 6

6.1.1	$f(x) = 5 - 2x^2$ $f'(x) = \lim_{h \rightarrow 0} \frac{5 - 2(x+h)^2 - (5 - 2x^2)}{h}$ $= \lim_{h \rightarrow 0} \frac{5 - 2(x^2 + 2xh + h^2) - 5 + 2x^2}{h}$ $= \lim_{h \rightarrow 0} \frac{5 - 2x^2 - 4xh - 2h^2 - 5 + 2x^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(-4x - 2h)}{h}$ $= \lim_{h \rightarrow 0} -4x - 2h$ $= -4x$ OR/OF $f(x+h) = 5 - 2(x+h)^2$ $= 5 - 2x^2 - 4xh - 2h^2$ $f(x+h) - f(x) = 5 - 2x^2 - 4xh - 2h^2 - (5 - 2x^2)$ $= -4xh - 2h^2$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{-4xh - 2h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(-4x - 2h)}{h}$ $= \lim_{h \rightarrow 0} (-4x - 2h)$ $= -4x$	✓ correct formula ✓ subst.in correct formula ✓ $-4xh - 2h^2$ ✓ $-4x - 2h$ ✓ answer penalize (-1 mark incorrect notation) OR ✓ $5 - 2x^2 - 4xh - 2h^2$ ✓ $-4xh - 2h^2$ ✓ correct formula ✓ $\lim_{h \rightarrow 0} (-4x - 2h)$ ✓ Answer	(5)
			(5)

QUESTION 7

7.1.1	$f(x) = x^3 + bx^2 + cx + d$ $y = (x-3)^2(x-x_1)$ Sub (0; 9) $9 = (0-3)^2(0-x_1)$ $9 = -9x_1$ $x = -1$; P (-1; 0) $y = (x-3)^2(x+1)$ $f(x) = x^3 - 5x^2 + 3x + 9$ OR $f(x) = x^3 + bx^2 + cx + d$ Sub S(3;0) $0 = (3)^3 + b(3)^2 + 3c + 9$ $0 = 36 + 9b + 3c$ $12 + 3b + c = 0$(1) AT a turning point, $f'(x) = 0$ $3x^2 + 2bx + c = 0$ Sub S(3;0) $3(3)^2 + 2b(3) + c = 0$ $27 + 6b + c = 0$(2) Eqn (2) - (1) $15 + 3b = 0$ $b = -5$ From eqn(1) $12 + 3(-5) + c = 0$ $c = 3$	✓ $(x-3)^2(x-x_1)$ ✓ Sub (0; 9) ✓ $x = -1$ ✓ $(x^2 - 6x + 9)(x+1)$ OR ✓ equation (1) ✓ equation (2) ✓ method of solving ✓ substitution to get c	(4)
7.1.2	$f'(x) = 0$ $3x^2 - 10x + 3 = 0$ $(3x-1)(x-3) = 0$ $x = \frac{1}{3}$ or $x = 3$ $y = f\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right)^3 - 5\left(\frac{1}{3}\right)^2 + 3\left(\frac{1}{3}\right) + 9 = 9,48$ $R\left(\frac{1}{3}, 9,48\right)$ OR $R\left(\frac{1}{3}, \frac{256}{27}\right)$	✓ $f'(x) = 3x^2 - 10x + 3$ ✓ $f'(x) = 0$ ✓ factors/Quad. Formula ✓ both values of x ✓ $y = 9,48$ or $\frac{256}{27}$	(5)

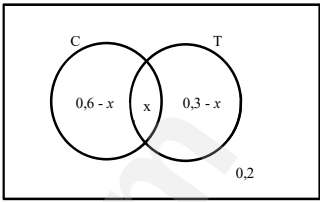
6.1.2	$m_T = f^1(x)$ $-2 = -4x$ $x = \frac{1}{2}$ $y = 5 - 2\left(\frac{1}{2}\right)^2$ $y = \frac{9}{2}$ $A\left(\frac{1}{2}; \frac{9}{2}\right)$	✓ $-2 = -4x$ ✓ $x = \frac{1}{2}$ ✓ $y = \frac{9}{2}$	(3)
6.2.1	$D_x \left[3x - 3x^2 - \frac{3}{x} \right]$ $D_x [3x - 3x^2 - 3x^{-1}]$ $= 3 - 6x + 3x^{-2}$	✓ 3 ✓ $-6x$ ✓ $3x^{-2}$	(3)
6.2.2	$y = x^{\frac{2}{3}} + \frac{1}{3}x + \pi^3$ $\frac{dy}{dx} = \frac{2}{3}x^{-\frac{1}{3}} + \frac{1}{3} + 0$	✓ $x^{\frac{2}{3}}$ ✓ $\frac{2}{3}x^{-\frac{1}{3}}$ ✓ $\frac{1}{3}$	(3)
			[14]

7.1.3	$f''(x) = 6x - 10 = 0$ $6x = 10$ $x = \frac{5}{3}$ OR/OF $x = \frac{-b}{3a} = \frac{-(-5)}{3(1)}$ $= \frac{5}{3}$ OR/OF $x = \frac{x_R + x_S}{2} = \frac{\frac{1}{3} + 3}{2} = \frac{5}{3}$	$\checkmark f''(x) = 0$ $\checkmark$ answer OR $\checkmark x = \frac{-b}{3a}$ $\checkmark$ Answer OR $\checkmark \frac{x_R + x_S}{2}$	(2)
7.2	7.2.1 $x \in \left(\frac{1}{3}; 3\right)$ OR/OF $\frac{1}{3} < x < 3$	$\checkmark$ critical values $\checkmark$ correct notation OR $\checkmark \frac{1}{3} < x$ $\checkmark x < 3$	(2)
	7.2.2 $x < \frac{5}{3}$	$\checkmark x < \frac{5}{3}$	(1)
7.3	$t > 9,48 / \frac{256}{27}$ or $t < 0$	$\checkmark t > 9,48 / \frac{256}{27}$ $\checkmark t < 0$	(2)
			[16]

QUESTION 8

8.1	$x^2 + r^2 = (2\sqrt{3})^2$ Pythagoras $r^2 = 12 - x^2$ $r = \sqrt{12 - x^2}$	$\checkmark x^2 + r^2 = (2\sqrt{3})^2$ $\checkmark r = \sqrt{12 - x^2}$	(2)
8.2	$V = \pi r^2 h$ $= 2\pi x(12 - x^2)$, but $h = 2x$ $V = 24\pi x - 2\pi x^3$ $\therefore \frac{dV}{dx} = \frac{d(24\pi x - 2\pi x^3)}{dx} = 0$ $24\pi - 6\pi x^2 = 0$ $6\pi x^2 = 24\pi$ $x^2 = 4$ $x = \pm 2$ $\therefore x = 2$	$\checkmark$ sub r and h into V formula $\checkmark V = 24\pi x - 2\pi x^3$ $\checkmark \frac{dV}{dx} = 0$ $\checkmark$ answer $\therefore x = 2$	(4)
8.3	$V_{\max} = 24\pi x - 2\pi x^3$ when $x = 2$ $= 24\pi(3) - 2\pi(2)^3$ $= 48\pi - 2 \times 8\pi$ $= (48 - 16)\pi$ $= 32\pi$	$\checkmark$ substitution of $x=2$ into V $\checkmark 32\pi$	(2)
			[8]

QUESTION 10

10.1.1	 $0,6 - x + x + 0,3 - x + 0,2 = 1$ $-x = 1 - 0,6 - 0,3 - 0,2$ $x = 0,1$	$\checkmark$ coffee $\checkmark$ tea $\checkmark 0,2$ $\checkmark$ equation $\checkmark$ answer	(5)
10.1.2	$(0,6 - 0,1) + (0,3 - 0,1) = 0,5 + 0,2 = 0,7$	$\checkmark$ simplify $\checkmark$ answer	(2)
10.2.1	$a = 120$ $b = 80$ $c = 20$ $d = 100$	$\checkmark a$ and b $\checkmark c$ and d	(2)
10.2.2	$P(\text{grade 12}) = \frac{50}{250} = \frac{1}{5} = 0,2 = 20\%$	$\checkmark$ Answer	(1)
10.2.3	$P(\text{grade 11 and brand A}) = \frac{120}{250} = \frac{12}{25} = 0,48 = 48\%$	$\checkmark$ Answer	(1)
10.2.4	$P(\text{grade 11}) \times P(\text{brand A}) = \frac{200}{250} \times \frac{150}{250} = 0,48$ YES, they are independent: since $P(\text{grade 11 and brand A}) = P(\text{grade 11}) \times P(\text{brand A})$	$\checkmark P(\text{grade 11})$ $\checkmark P(\text{brand A})$ $\checkmark$ YES $\checkmark$ Conclusion	(4)
			[15]

TOTAL: 150

QUESTION 9

9.1	$1 + i_{eff} = (1 + \frac{i_{nom}}{n})^n$ $1 + 0,105 = (1 + \frac{i_{nom}}{4})^4$ $\sqrt[4]{1,105} = 1 + \frac{i_{nom}}{4}$ $\sqrt[4]{1,105} - 1 = \frac{i_{nom}}{4}$ $i_{nom} = (\sqrt[4]{1,105} - 1) \times 100$ $= 0,101101904 \times 100$ $= 10,11\%$	$\checkmark$ correct formula $\checkmark$ substitution of i_{eff} and n into correct formula $\checkmark \sqrt[4]{1,105}$ $\checkmark$ answer	(4)
9.2	$A = P(1 + i)^n$ $30\,440 = 12\,500(1 + \frac{0,09}{4})^{4k}$ $\frac{1522}{625} = (1 + \frac{0,09}{4})^{4k}$ $\log \frac{1522}{625} = \log(1 + \frac{0,09}{4})^{4k}$ $4k = \frac{\log \frac{1522}{625}}{\log(1 + \frac{0,09}{4})}$ $4k = 40,00020365$ $k = 10$	$\checkmark$ substitution into correct formula $\checkmark i = \frac{0,09}{4} = 0,0225 = \frac{9}{400}$ $\checkmark$ use of logs $\checkmark k = 10$	(4)
9.3	$A = P(1 + i)^{n_1}(1 + i)^{n_2} + 10000((1 + i)^{n_2})$ $A = 25000(1 + \frac{0,075}{12})^{36}(1 + \frac{0,067}{2})^8 + 10000(1 + \frac{0,067}{2})^8$ $= 40\,722,64871 + 13\,016,18905$ $= R\,53\,738,83776$ $\approx R\,53\,738,84$	$\checkmark (1 + \frac{0,075}{12})^{36}$ $\checkmark (1 + \frac{0,067}{2})^8$ $\checkmark 10000(1 + \frac{0,067}{2})^8$ $\checkmark$ Answer	(4)
			[12]

**JOHANNESBURG SOUTH DISTRICT
CLUSTER PAPER
JUNE EXAMINATION 2019**

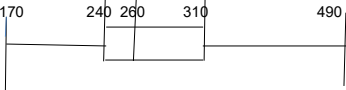
GRADE 12

**MATHEMATICS
PAPER 2
MEMORANDUM**

MARKS: 150

This memorandum consists of 15 pages, including the cover page.

QUESTION 1

	SUGGESTED SOLUTION	REMARKS
1.1.1	$\text{Mean} = \frac{3913}{14}$ $= 279,5\text{m}$ ANSWER ONLY AWARD FULL MARKS	✓ 3913 (2) ✓ Answer
1.1.2	$\text{Median} = \frac{260+260}{2}$ $= 260\text{m}$	✓ Answer (1)
1.2	$Q_1 = 240$ $Q_3 = 310$ 	✓ 240 & 310 ✓ min & max ✓ correct skewness to the right (3)
1.3	$\text{Interquartile range} = Q_3 - Q_1$ $= 310 - 240$ $= 70$	✓ 310 – 240 ✓ Answer (2)
1.4	Positively skewed or data skewed to the right	✓ (1)
1.5.1	Standard deviation = 74,74	✓✓ 74,74
1.5.2	$< \text{Mean} - 1\text{SD}$ $< 279,5 - 74,74$ $< 204,76$ $x < 204,76$ $\therefore 2 \text{ days}$	✓ 279,5 – 74,74 ✓ 2 days (2)
		[13]

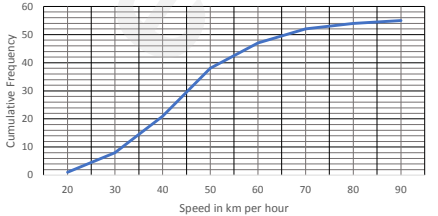
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QUESTION 3

	SUGGESTED SOLUTION	REMARKS
3.1.1	$\text{Gradient of AC} = \frac{-4 - 3\frac{1}{2}}{8 - 3} = -\frac{3}{2}$ $(y - (-4)) = -\frac{3}{2}(x - 8)$ $y = -\frac{3}{2}x + 8$	✓ Gradient of AC ✓ Sub.m and pt (8;-4) ✓ $y = -\frac{3}{2}x + 8$ (3)
3.1.2	G is a point of intersection $y = -1,5x + 8 ; y = \frac{7}{10}x - \frac{4}{5}$ eqn. of AC = eqn. of BG $-1,5x + 8 = \frac{7}{10}x - \frac{4}{5}$ $x = 4$ Substitute x into $y = -1,5(4) + 8$ or $y = \frac{7}{10}(4) - \frac{4}{5}$ $y = 2$ G(4;2)	✓ equating. ✓ $x = 4$ ✓ $y = 2$ (3)
3.2	G(4;2) & F(3;3$\frac{1}{2}$) $\text{Midpoint}_{AG} = F(3;3\frac{1}{2})$ $(3;3\frac{1}{2}) = (\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2})$ $\frac{4+x_2}{2} = 3 ; \frac{2+y_2}{2} = 3\frac{1}{2}$ $x = 2 ; y = 5$ A(2;5)	✓ $\frac{4+x_2}{2} = 3$ ✓ $\frac{2+y_2}{2} = 3\frac{1}{2}$ (2)

4

QUESTION 2

	SUGGESTED SOLUTION	REMARKS																											
2.1	50 < x ≤ 60 ACCEPT 50 – 60	✓ (1)																											
2.2.1	<table border="1"> <thead> <tr> <th>Class</th><th>Frequency</th><th>Cumulative frequency</th></tr> </thead> <tbody> <tr><td>20 < x ≤ 30</td><td>1</td><td>1</td></tr> <tr><td>30 < x ≤ 40</td><td>7</td><td>8</td></tr> <tr><td>40 < x ≤ 50</td><td>13</td><td>21</td></tr> <tr><td>50 < x ≤ 60</td><td>17</td><td>38</td></tr> <tr><td>60 < x ≤ 70</td><td>9</td><td>47</td></tr> <tr><td>70 < x ≤ 80</td><td>5</td><td>52</td></tr> <tr><td>80 < x ≤ 90</td><td>2</td><td>54</td></tr> <tr><td>90 < x ≤ 100</td><td>1</td><td>55</td></tr> </tbody> </table>	Class	Frequency	Cumulative frequency	20 < x ≤ 30	1	1	30 < x ≤ 40	7	8	40 < x ≤ 50	13	21	50 < x ≤ 60	17	38	60 < x ≤ 70	9	47	70 < x ≤ 80	5	52	80 < x ≤ 90	2	54	90 < x ≤ 100	1	55	✓ 8 ✓ 55 (2)
Class	Frequency	Cumulative frequency																											
20 < x ≤ 30	1	1																											
30 < x ≤ 40	7	8																											
40 < x ≤ 50	13	21																											
50 < x ≤ 60	17	38																											
60 < x ≤ 70	9	47																											
70 < x ≤ 80	5	52																											
80 < x ≤ 90	2	54																											
90 < x ≤ 100	1	55																											
2.2.2		✓ Grounding at (20; 0) ✓ Plotting upper limits ✓ Smooth shape (3)																											
2.3	At 55 – 44 = 11 motorists	✓ Accept (43-45) ✓ Accept (10 – 12) motorists (2)																											
		[8]																											

3

3.3	$\tan \hat{AED} = m_{AB}$ $m_{AB} = \frac{-5-5}{-6-2}$ $\tan \hat{AED} = \frac{5}{4}$ $\hat{AED} = 51,34^\circ$	✓ $m_{AB} = \frac{5}{4}$ ✓ $\tan \hat{AED} = \frac{5}{4}$ ✓ $51,34^\circ$ (3)
3.4	Area $\Delta AED = \frac{1}{2} \times AE \times ED \times \sin \hat{AED}$ Equation of AB: Substitute A(2, 5) into $(y - 5) = \frac{5}{4}(x - 2)$ $y = \frac{5}{4}x + \frac{5}{2}$ E is the x intercept ; $y = 0$ $0 = \frac{5}{4}x + \frac{5}{2}$ $x = -2$ E(-2 ; 0) Point D is the y intercept of AC; $x = 0$ $0 = -1,5x + 8$ $x = \frac{16}{3} = 5,33$ D($\frac{16}{3}$; 0) / D(5,33 ; 0) $ED = \frac{16}{3} - (-2) = \frac{22}{3}$ $ED = 7,33 \text{ units}$ $AE = \sqrt{(5-0)^2 + (-2-2)^2} = \sqrt{41} = 6,40 \text{ units}$ $\text{Area } \Delta AED = \frac{1}{2} \times \sqrt{41} \times \frac{22}{3} \times \sin 51,34^\circ$ $= 18,33 \text{ units}^2 \text{ ACCEPT } 18,32 \text{ units}^2$	✓ $y = \frac{5}{4}x + \frac{5}{2}$ ✓ E(-2 ; 0) ✓ D(0 ; $\frac{16}{3}$) ✓ $ED = \frac{22}{3} = 7,33$ ✓ $AE = \sqrt{41} = 6,40$ ✓ Subst.in area formula ✓ 18,33 (7)
3.5	$x^2 + y^2 = r^2$ $8^2 + (-4)^2 = r^2$ $80 = r^2$ $x^2 + y^2 = 80$	✓ $x^2 + y^2$ ✓ 80 (2)
		[20]

5

QUESTION 4

	SUGGESTED SOLUTION	REMARKS
4.1	$(x-a)^2 + (y-b)^2 = r^2$ $(4-2)^2 + (5-3)^2 = r^2$ $(2)^2 + (2)^2 = r^2$ $8 = r^2$ Eqn: $(x-2)^2 + (y-3)^2 = 8$ OR $PN = r = \sqrt{(4-2)^2 + (5-3)^2} = \sqrt{8} = 2\sqrt{2}$ Eqn: $(x-2)^2 + (y-3)^2 = 8$	$\checkmark$ sub.N(2;3)&P(4;5) $\checkmark (x-2)^2 + (y-3)^2$ $\checkmark$ RHS=8 (3) OR $\checkmark r = PN = \sqrt{8}$ $\checkmark (x-2)^2 + (y-3)^2$ $\checkmark$ RHS=8 (3)
4.2	$(x-2)^2 + (y-3)^2 = 8$ $x^2 - 4x + 4 + y^2 - 6y + 9 = 8$ $x^2 - 4x + y^2 - 6y + 5 = 0$	$\checkmark$ (1)
4.3	$m_{NP} = \frac{5-3}{4-2} = 1$ Gradient _{NP} $\times$ Gradient _{PT} = -1 (Tan $\perp$ radius) $m_{PT} = -1$ Sub:P(4;5) $(y-y_1) = m(x-x_1)$ OR $y = -1x + c$ $y-5 = -1(x-4)$ $5 = -1(4) + c$ $y = -x + 9$ $y = -x + 9$	$\checkmark m_{NP} = -1$ $\checkmark m_{PT} = -1$ $\checkmark$ sub. m = -1 & P(4;5) $\checkmark y = -x + 9$ (4)
4.4	T is x -intercept ; $y = 0$ $y = -x + 9$ $0 = -x + 9$ $x = 9$ $T(9;0)$ $(x-9)^2 + (y-0)^2 = r^2$ $(4-9)^2 + (5)^2 = r^2$ $50 = r^2$ $(x-9)^2 + (y)^2 = 50$ OR $PT = r = \sqrt{(4-9)^2 + (5-0)^2} = \sqrt{50} = 5\sqrt{2}$ $(x-9)^2 + (y)^2 = 50$	$\checkmark y = 0$ $\checkmark x = 9$ $T(9;0)$ $\checkmark (x-9)^2 + y^2$ $\checkmark$ RHS = 50 (4) OR $\checkmark y = 0$ $\checkmark x = 9$ $T(9;0)$ $\checkmark (x-9)^2 + y^2$ $\checkmark$ RHS = 50 (4)
4.5	$NM = NP$ (radii) $PT = MT$ (Tans from common pt.) NPTM is a kite (2 adjacent sides equal)	$\checkmark NM = NP$ (radii) $\checkmark PT = MT$ $\checkmark$ Reason (3)
4.6	$x = 9 - 5\sqrt{2} = 1,93$ and $x = 9 + 5\sqrt{2} = 16,07$	$\checkmark x = 9 - 5\sqrt{2}$ $\checkmark x = 9 + 5\sqrt{2}$ (2) [17]

6

QUESTION 5

	SUGGESTED SOLUTION	REMARKS
5.1.1	$x^2 + t^2 = (\sqrt{1+t^2})^2$ $x^2 + t^2 = 1 + t^2$ $x^2 = 1$ $x = \pm 1$ $x = -1$ (2 nd quad)	$\checkmark x^2 + t^2 = (\sqrt{1+t^2})^2$ $\checkmark x^2 = 1$ $\checkmark x = \pm 1$ (3)
5.1.2	$\cos(60^\circ + \beta) = \cos 60^\circ \cos \beta - \sin 60^\circ \sin \beta$ $= \frac{1}{2} \cdot \left(\frac{-1}{\sqrt{1+t^2}}\right) - \frac{\sqrt{3}}{2} \left(\frac{t}{\sqrt{1+t^2}}\right)$ $= \frac{-1-\sqrt{3}t}{2\sqrt{1+t^2}}$	$\checkmark$ correct Expansion $\checkmark \cos \beta = \frac{-1}{\sqrt{1+t^2}}$ $\checkmark$ sub for both special $\angle$ s $\checkmark$ Answer (4)
5.1.3	$\cos 2\beta = \cos^2 \beta - \sin^2 \beta$ $= \left(\frac{-1}{\sqrt{1+t^2}}\right)^2 - \left(\frac{t}{\sqrt{1+t^2}}\right)^2$ $= \frac{1}{1+t^2} - \frac{t^2}{1+t^2}$ $= \frac{1-t^2}{1+t^2}$ OR $\cos 2\beta = 2\cos^2 \beta - 1$ $= 2\left(\frac{-1}{\sqrt{1+t^2}}\right)^2 - 1$ $= \frac{2}{1+t^2} - 1$ $= \frac{1-t^2}{1+t^2}$ OR $\cos 2\beta = 1 - 2\sin^2 \beta$ $= 1 - 2\left(\frac{t}{\sqrt{1+t^2}}\right)^2$ $= 1 - \frac{2t^2}{1+t^2}$ $= \frac{1-t^2}{1+t^2}$	$\checkmark$ double angle expansion $\checkmark$ substitution (3) $\checkmark \frac{1}{1+t^2} - \frac{t^2}{1+t^2}$ (3) OR $\checkmark$ double angle expansion $\checkmark$ substitution (3) $\checkmark \frac{2}{1+t^2} - 1$ (3) OR $\checkmark$ double angle expansion $\checkmark$ substitution (3) $\checkmark 1 - \frac{2t^2}{1+t^2}$ (3)

7

5.2	$= (-\sin x) \cdot \sin(180^\circ - x) - (\cos x) \cdot (-\cos x)$ $= (-\sin x) \cdot (\sin x) - (\cos x) \cdot (-\cos x)$ $= -\sin^2 x + \cos^2 x$ $= \cos 2x$ ACCEPT $= (-\sin x) \cdot (\sin x) - (\cos x) \cdot (-\cos x)$ $= -(\sin x) \cdot (\sin x) + (\cos x) \cdot (\cos x)$ $= \cos(x+x)$ $= \cos 2x$	$\checkmark (-\sin x)$ $\checkmark (\sin x)$ $\checkmark (\cos x)$ $\checkmark (-\cos x)$ $\checkmark -\sin^2 x + \cos^2 x$ $\checkmark \cos 2x$ (6)
5.3.1	$x = 0^\circ$ or $x = 180^\circ$	$\checkmark x = 0^\circ$ $\checkmark x = 180^\circ$ (2)
5.3.2	$2 \frac{\left(\frac{\sin x}{\cos x}\right) - 2 \sin x \cos x}{2 \sin^2 x}$ $\frac{\sin x - \sin^2 x \cos^2 x}{\cos x} \div \sin^2 x$ $\frac{\sin x (1 - \cos^2 x)}{\cos x} \times \frac{1}{\sin^2 x}$ $\frac{\sin x (\sin^2 x)}{\cos x} \times \frac{1}{\sin^2 x}$ $\frac{\sin x}{\cos x} \times \frac{1}{\sin^2 x}$ $\frac{\sin x}{\cos x \tan x}$	$\checkmark \frac{\sin x}{\cos x}$ $\checkmark 2 \sin x \cos x$ $\checkmark \frac{\sin x - \sin^2 x \cos^2 x}{\cos x}$ $\checkmark \sin x (1 - \cos^2 x)$ $\checkmark \sin^2 x$ (5)
5.4	$(2\cos^2 x - 1) - 7\cos x - 3 = 0$ $2\cos^2 x - 7\cos x - 4 = 0$ $(2\cos x + 1)(\cos x - 4) = 0$ $\cos x = -\frac{1}{2}$ or $\cos x = 4$ (no solution) $x = \pm 60^\circ + k \cdot 360^\circ ; k \in \mathbb{Z}$ OR $x = 120^\circ + k \cdot 360^\circ ; k \in \mathbb{Z}$ or $x = 240^\circ + k \cdot 360^\circ ; k \in \mathbb{Z}$	$\checkmark$ double angle substitution $\checkmark$ standard form $\checkmark$ factorisation $\checkmark \cos x = -\frac{1}{2}$ or $\cos x = 4$ $\checkmark$ no solution $\checkmark x = \pm 60^\circ + k \cdot 360^\circ ; k \in \mathbb{Z}$ OR $\checkmark x = 120^\circ + k \cdot 360^\circ ; k \in \mathbb{Z}$ $\checkmark x = 240^\circ + k \cdot 360^\circ ; k \in \mathbb{Z}$ (6)
		[29]

8

QUESTION 6

	SUGGESTED SOLUTION	REMARKS
6.1	$f(x) = \cos x - \frac{1}{2}$ and $g(x) = \sin(x + 30^\circ)$ $p = 30^\circ$ and $q = -\frac{1}{2}$ OR sub. $(60^\circ; 1)$ $\sin(60^\circ + p) = 1$ $60^\circ + p = 90^\circ$ $p = 30^\circ$ Sub. $(0^\circ; \frac{1}{2})$ $\cos 0^\circ + q = \frac{1}{2}$ $1 + q = \frac{1}{2}$ $q = -\frac{1}{2}$	$\checkmark p = 30^\circ$ $\checkmark q = -\frac{1}{2}$ (2) OR $\checkmark p = 30^\circ$ $\checkmark q = -\frac{1}{2}$ (2)
6.2	$x \in (-120^\circ; 0^\circ)$ OR $-120^\circ < x < 0^\circ$	$\checkmark$ end points $\checkmark$ correct notation (2)
6.3	The graph of g has to shift 60° to the left and then Reflected along the x -axis OR The graph of g has to be shifted 120° to the right	$\checkmark$ shift 60° to the left $\checkmark$ reflected along x -axis (2) OR $\checkmark$ shift 120° to the right (2)
		[6]

9

QUESTION 7

	SUGGESTED SOLUTION	REMARKS
7.1	$\hat{B}\hat{A}\hat{C} = 180^\circ - (\theta + \beta)$ (sum of $\angle$ s in a Δ) $\frac{AB}{\sin \beta} = \frac{20}{\sin[180^\circ - (\theta + \beta)]}$ $AB = \frac{20 \cdot \sin \beta}{\sin(\theta + \beta)}$ $\tan \alpha = \frac{AP}{AB}$ (OR USING SINE RULE in ΔAPB) $\tan \alpha = \frac{AP}{\frac{20 \sin \beta}{\sin(\theta + \beta)}}$ $\therefore AP = \frac{20 \sin \beta \cdot \tan \alpha}{\sin(\theta + \beta)}$	$\checkmark 180^\circ - (\theta + \beta)$ $\checkmark$ use of sine rule $\checkmark AB = \frac{20 \sin \beta}{\sin(\theta + \beta)}$ $\checkmark$ definition of $\tan \alpha$
7.2	If $AB = AC$ $\theta = \beta$ ($\angle$ s opp equal sides) $AP = \frac{20 \times \sin 30^\circ \cdot \tan 60^\circ}{\sin(2 \times 30^\circ)}$ $= \frac{10\sqrt{3}}{2}$ $= 20$	$\checkmark \theta = \beta$ $\checkmark$ substitution $\checkmark 20$
		[9]

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QUESTION 9

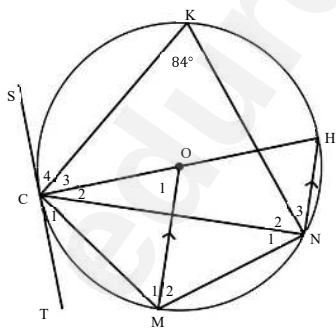
	SUGGESTED SOLUTION	REMARKS
9.1	$\hat{A} = x$ ($\angle$ at centre = $2 \times \angle$ at circumference) $\hat{C}_2 = \hat{A} = x$ ($\angle$ s opp equal sides) $\hat{B}_1 = \hat{A} = x$ (Tan chord Theorem) $\hat{T} = \hat{B}_1 = x$ (Corresp. $\angle$ s ; $TE \parallel BC$)	$\checkmark$ S $\checkmark$ R $\checkmark$ S/R $\checkmark$ S $\checkmark$ S $\checkmark$ R
9.2	$\hat{C}_1 = \hat{B}_2$ ($\angle$ s opp equal sides) In ΔBFC ; $\hat{C}_1 = \frac{180^\circ - 2x}{2}$ ($\angle$ sum in Δ) $\hat{C}_1 = 90^\circ - x$ $\Rightarrow \hat{T} \neq \hat{C}_1$ $\therefore$ FCBT is not a parallelogram (opp. $\angle$ s of quad not equal) OR $\hat{B}_1 = x$ (proven in 9.1) $\hat{B}_2 = 90^\circ - x$ (Tan $\perp$ diameter) $\hat{C}_1 = \hat{B}_2 = 90^\circ - x$ ($\angle$ s opp equal sides) $\Rightarrow \hat{T} \neq \hat{C}_1$ $\therefore$ FCBT is not a parallelogram (opp. $\angle$ s of quad not equal) OR $\hat{C}_2 = x$ (proven in 9.1) $\hat{C}_1 + \hat{C}_2 = 90^\circ$ ($\angle$ s in semi circle) $\hat{C}_1 = 90^\circ - x$ $\Rightarrow \hat{T} \neq \hat{C}_1$ $\therefore$ FCBT is not a parallelogram (opp. $\angle$ s of quad not equal)	$\checkmark$ S/R $\checkmark$ S $\checkmark$ R $\checkmark$ S/R $\checkmark$ S $\checkmark$ R $\checkmark$ S/R $\checkmark$ S $\checkmark$ R $\checkmark$ R
		[9]

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 $\checkmark$ S – Statement $\checkmark$ R – Reason $\checkmark$ S/R – Statement + Reason

QUESTION 8



	SUGGESTED SOLUTION	REMARKS
8.1	$\hat{H} = 84^\circ$ ($\angle$'s in same segment)	$\checkmark$ S $\checkmark$ R
8.2	$\hat{O}_1 = 84^\circ$ (corresponding $\angle$'s; $OM \parallel HN$)	$\checkmark$ S $\checkmark$ R
8.3	$\hat{N}_1 = 42^\circ$ ($\angle$ at centre = $2 \times \angle$ at circumference)	$\checkmark$ S $\checkmark$ R
8.4	$\hat{C}_1 = 42^\circ$ (tan chord Theorem)	$\checkmark$ S $\checkmark$ R
8.5	$\hat{M}_1 + \hat{M}_2 = 96^\circ$ (opp. $\angle$'s of cyclic quad)	$\checkmark$ S $\checkmark$ R
8.6	$\hat{O}_1 = 84^\circ$ AND $\hat{N}_1 = 42^\circ$ CONM is NOT cyclic ($\angle$'s subtended by line segment CM NOT equal)	$\checkmark$ S $\checkmark$ R
		[12]

11

QUESTION 10

10.1	Construction : Draw altitudes $h_1 \perp XV$ and $h_2 \perp XW$. Join VZ and WY $\frac{\text{area} \Delta XVW}{\text{area} \Delta VWY} = \frac{\frac{1}{2} XV \cdot h_1}{\frac{1}{2} VY \cdot h_1} = \frac{XV}{VY} \text{ (same height)}$ $\frac{\text{area} \Delta XVW}{\text{area} \Delta WYZ} = \frac{\frac{1}{2} XW \cdot h_2}{\frac{1}{2} WZ \cdot h_2} = \frac{XW}{WZ} \text{ (same height)}$ $\text{Area} \Delta VWY = \text{Area} \Delta WYZ \text{ (same base, same height, } VW \parallel ZY)$ $\frac{\text{area} \Delta XVW}{\text{area} \Delta VWY} = \frac{\text{area} \Delta XVW}{\text{area} \Delta WYZ}$ $\therefore \frac{XV}{VY} = \frac{XW}{WZ}$	$\checkmark$ Construction Same height $\checkmark$ Area Same height $\checkmark$ Area $\checkmark$ equating areas $\checkmark$ (Same base, same height, $VW \parallel ZY$) $\checkmark$ equating ratios of areas
10.2.1	$PA : AQ = 3 : 5$ $\frac{\text{area} \Delta PRA}{\text{area} \Delta QRA} = \frac{\frac{1}{2} \times 3 \times h}{\frac{1}{2} \times 5 \times h} \text{ (same height)}$ $= \frac{PA}{AQ} = \frac{3}{5}$	$\checkmark$ areas $\checkmark$ answer
10.2.2	From $2PB = BR$ $\frac{PC}{CA} = \frac{PB}{BR} = \frac{1}{2}$ (line parallel to one side of Δ) OR (prop. theorem ; $CB \parallel AR$)	$\checkmark \frac{1}{2}$ $\checkmark$ R

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	$\frac{BD}{BQ} = \frac{CA}{CQ} = \frac{2}{7}$ (line parallel to one side of Δ) OR (prop.theorem, $AD \parallel CB$)	$\checkmark \frac{2}{7} \checkmark R$ (4)
		[12]

QUESTION 11

11.1	$\hat{A}_1 = \hat{C}_4$ (tan chord theorem) $\hat{C}_4 = \hat{C}_3$ (vert.opp $\angle$ s) $\hat{C}_3 = \hat{CPR}$ ($\angle$ s opp.equal sides) $\Rightarrow \hat{A}_1 = \hat{CPR}$ $\therefore ACPR$ is a cyclic quadrilateral(converse ext. $\angle$ of a cyclic quad) or (ext $\angle =$ int opp $\angle$) OR $\hat{C}_3 = \hat{CPR}$ ($\angle$ s opp.equal sides) $\hat{C}_3 + \hat{C}_2 = \hat{A}_1 + \hat{B}$ (ext. $\angle$ of Δ) $\hat{C}_2 = \hat{B}$ (tan chord theorem) $\hat{C}_3 = \hat{A}_1$ $\Rightarrow \hat{A}_1 = \hat{CPR}$ (both $= \hat{C}_3$) $\therefore ACPR$ is a cyclic quadrilateral(converse ext. $\angle$ of a cyclic quad) or (ext $\angle =$ int opp $\angle$) ACCEPT $\hat{CPR}$ or $\hat{P}_1 + \hat{P}_2$ or $\hat{P}$ if candidates use them	$\checkmark s \checkmark R$ $\checkmark S/R$ $\checkmark S$ $\checkmark R$ $\checkmark S/R$ $\checkmark S/R$ $\checkmark S \checkmark R$ $\checkmark R$ (5)
11.2	In ΔCBA and ΔRPA 1. $\hat{C}_1 = \hat{ARP}$ (ext. $\angle$ of cyclic quad) $\hat{C}_2 = \hat{B}$ (tan chord theorem) $\hat{C}_2 = \hat{P}_2$ ($\angle$ s in the same segment) 2. $\Rightarrow \hat{B} = \hat{P}_2$ (proved) 3. $\hat{A}_1 = \hat{A}_3$ ($\angle$ sum in a Δ) (3 rd $\angle$ of Δ) $\therefore \Delta CBA \parallel \Delta RPA$ ($\angle \angle \angle$) / (equiangular Δ s) ACCEPT $\hat{ARP}$ or $\hat{R}_1 + \hat{R}_2$ or $\hat{R}$ if candidates use them	$\checkmark S \checkmark R$ $\checkmark S$ $\checkmark S \checkmark R$ $\checkmark R$ (6)

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education

 Department:
 Education
 PROVINCE OF KWAZULU-NATAL

MATHEMATICS

PAPER 1

JUNE COMMON TEST

MARKING GUIDELINE

 NATIONAL
 SENIOR CERTIFICATE

GRADE 12

MARKS: 150

TIME: 3 hours

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11.3	$\frac{RP}{CB} = \frac{RA}{CA}$ (Δ 's) $RP = \frac{CB \cdot RA}{AC}$ but $RP = RC$ (given) $\therefore RC = \frac{CB \cdot RA}{AC}$	$\checkmark S$ $\checkmark (RC = RP)$ (2)
11.4	In ΔRAC and ΔRCB 1. $\hat{R}_1 = \hat{R}_1$ (common) 2. $\hat{C}_2 = \hat{B}$ (tan chord theorem) 3. $\hat{A}_1 = \hat{CPR}$ ($\angle$ sum in a Δ) / (3 rd $\angle$ of Δ) $\Rightarrow \Delta RAC \parallel \Delta RCB$ ($\angle \angle \angle$) / (equiangular Δ s) $\Rightarrow \frac{AC}{CB} = \frac{RC}{RB}$ (Δ 's) $\therefore RB \cdot AC = RC \cdot CB$	$\checkmark S/R$ $\checkmark S$ $\checkmark R$ $\checkmark S$ (4)
		[17]

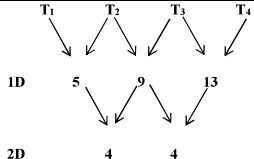
TOTAL: 150

QUESTION 1

1.1.1	$x = -3$ or $x = -9$	$A \checkmark -3 \quad A \checkmark -9$	(2)
1.1.2	$-4x^2 + 9x - 3 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-(-9) \pm \sqrt{(-9)^2 - 4(-4)(-3)}}{2(-4)}$ $= 0,41 \quad \text{or} \quad 1,84$	$A \checkmark$ standard form $CA \checkmark$ substitution in correct formula $CACA \checkmark \checkmark$ answers (penalize 1 mark if rounding off is incorrect-once for entire paper)	(4)
1.1.3	$2^{x+2} - 3 \cdot 2^{x-1} = 80$ $2^x(2^2 - 3 \cdot 2^{-1}) = 80$ $2^x \left(\frac{5}{2} \right) = 80$ $2^x = 80 \times \frac{2}{5} = 32$ $2^x = 2^5$ $x = 5$	$A \checkmark$ factorization $CA \checkmark$ simplifying bracket $CA \checkmark 2^x = 32$ $CA \checkmark$ Exponential form $CA \checkmark$ answer	(5)
1.1.4	$\frac{x-4}{2} - \sqrt{x-3} = 1$ $x-4 - 2\sqrt{x-3} = 2$ $x-6 = 2\sqrt{x-3}$ $x^2 - 12x + 36 = 4x - 12$ $x^2 - 16x + 48 = 0$ $(x-4)(x-12) = 0$ $x = 4 \quad \text{or} \quad x = 12$ n/a	$A \checkmark$ Isolating surd $CA \checkmark$ squaring $CA \checkmark$ standard form $CA \checkmark$ factors $CA \checkmark x$ - values and rejecting	(5)

1.2	$x - 3y = 1 \rightarrow (1)$ $x^2 - 2xy + 9y^2 = 17 \rightarrow (2)$ $(1): x = 3y + 1 \rightarrow (3)$ $(3y + 1)^2 - 2y(3y + 1) + 9y^2 = 17$ $9y^2 + 6y + 1 - 6y^2 - 2y + 9y^2 - 17 = 0$ $12y^2 + 4y - 16 = 0$ $3y^2 + y - 4 = 0$ $(3y + 4)(y - 1) = 0$ $y = \frac{-4}{3} \text{ or } y = 1$ $x = -3 \text{ or } x = 4$	A✓ making x the subject CA✓ substitution into equation (2) CA✓ standard form CA✓ factors CA✓ y values CA✓ x values	(6)
1.3	 $x^2 - 5x + 4 < 0$ $(x - 1)(x - 4) < 0$ $1 < x < 4$	A✓ factors CA✓ Critical Values A✓ Notation	(3)
			[25]

QUESTION 2

2.1	 $2a = 4 \quad a = 2$ $3a + b = 5 \quad b = -1$ $T_n = 2n^2 - n + c$ $T_{61} = 2(61)^2 - (61) + c = 7383$ $c = 7383 - 7381$ $c = 2$ $T_n = 2n^2 - n + 2$	A✓ a value CA✓ b value CA✓ Substitution of 61 and equating to 7383 CA✓ c value CA✓ answer	(5)
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2.2	$4n + 1 = 2021$ $4n = 2020$ $n = 505$ Between T_{505} and T_{506} OR $T_n - T_{n-1} = 2021$ $2n^2 - n + 2 - [2(n-1)^2 - (n-1) + 2]$ $= 2021$ $2n^2 - n + 2 - [2n^2 - 4n + 2 - n + 1 + 2]$ $= 2021$ $4n = 2024$ $n = 506$ Between 505^{th} and 506^{th} OR $T_{n+1} - T_n = 2021$	A✓ equating n^{th} term to 2021 CA✓ value of n CA✓ answer OR A✓ substitution CA✓ value of n CA✓ answer OR	(3)
			[8]

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QUESTION 3

3.1	5	A✓ answer	(1)
3.2	$2n - 5 = 7997$ $2n = 8002$ $n = 4001$	A✓ equating n^{th} term to 7997 CA✓ answer (n must be natural)	(2)
3.3	Pattern: 5 ; 15 ; 25 ; 35 ;; 7995 $10n - 5 = 7995$ $n = 800$ $S_{800} = \frac{800}{2} [5 + 7995]$ $= 3\,200\,000$	A✓ Sequence A✓ n^{th} term CA✓ n - value (n must be natural) CA✓ answer	(4)
			[7]

QUESTION 4

4.1.1	4	A✓ answer	(1)
4.1.2	$S_n = \frac{a(r^n - 1)}{r - 1}$ $S_{16} = \frac{1(4^{16} - 1)}{4 - 1}$ $= 1\,431\,655\,765$	A✓ substitution a into correct formula CA✓ substitution of r into correct formula CA✓ answer	(3)
4.2	$a = 1$ $a + ar + ar^2 = S_3$ $1 + r + r^2 = 1 \frac{8}{49} = \frac{57}{49}$ $49r^2 + 49r - 8 = 0$ $(7r - 1)(7r + 8) = 0$ $r = \frac{1}{7} \text{ or } r = -\frac{8}{7}$ n/a	A✓ substituting $a = 1$ A✓ equating to $1 \frac{8}{49}$ CA✓ standard form CA✓ factors CA✓ answers and rejecting	(5)
			[9]

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2.2	$4n + 1 = 2021$ $4n = 2020$ $n = 505$ Between T_{505} and T_{506} OR $T_n - T_{n-1} = 2021$ $2n^2 - n + 2 - [2(n-1)^2 - (n-1) + 2]$ $= 2021$ $2n^2 - n + 2 - [2n^2 - 4n + 2 - n + 1 + 2]$ $= 2021$ $4n = 2024$ $n = 506$ Between 505^{th} and 506^{th} OR $T_{n+1} - T_n = 2021$	A✓ equating n^{th} term to 2021 CA✓ value of n CA✓ answer OR A✓ substitution CA✓ value of n CA✓ answer OR	(3)
			[8]

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QUESTION 5

5.1	$x = 1$ and $y = 2$	AA✓✓ both asymptote equations	(2)
5.2	$y = \frac{a}{x + p} + q$ $y = \frac{a}{x - b} + c$ $y = \frac{a}{x - 1} + 2$ $A(0; \frac{1}{2})$ $\frac{1}{2} = \frac{a}{0 - 1} + 2$ $-\frac{3}{2} = -a$ $a = \frac{3}{2}$	CA✓ substitution of b and c values A✓ substitution of point A CA✓ a - value (must be positive)	(3)
5.3	By using the gradient method From point A move 1 unit horizontally to the right and 1,5 unit vertically upward. Move 1,5 unit vertically upward and 1 unit horizontally to the left $A(2; 3\frac{1}{2})$ OR By using the midpoint of line segment AA' and that AA' passes through the point of intersection of the asymptotes $A(0; \frac{1}{2})$ and $(1; 2)$ $A'(2(1) - 0; 2(2) - \frac{1}{2})$ $A'(2; 3\frac{1}{2})$	A✓ transformation comment A✓ transformation comment CACA✓✓ coordinates OR A✓ comment A✓ midpoint formula CACA✓✓ coordinates Answer only - full marks	(4)

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5.4	$g(x) = \frac{3}{x-4} + 2$ or $g(x) = \frac{3}{2(x-4)} + 2$	CACA✓✓ answer	(2)
			[11]

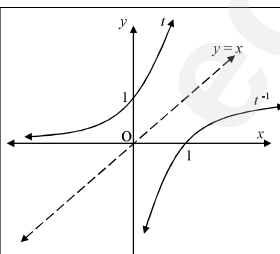
QUESTION 6

6.1	$y = a(x - x_1)(x - x_2)$ $12 = a(0 - 2)(0 - 6)$ $12 = 12a$ $a = 1$ $y = x^2 - 8x + 12$ $b = -8$ and $c = 12$	A✓ formula A✓ substituting x and y intercepts. A✓ simplifying A✓ equation	(4)
6.2	$x = \frac{2+6}{2} = 4$ or $x = -\frac{b}{2a} = -\frac{(-8)}{2(1)} = 4$ or $f'(x) = 2x - 8 = 0$ $x = 4$ $y = (4)^2 - 8(4) + 12 = -4$ $M(4; -4)$	CA✓ axis of symmetry (x must be positive) CA✓ minimum value (y must be negative)	(2)
6.3	$y \in (-\infty; -4)$ OR $y \geq -4$	CA✓ answer	(1)
6.4	$2 \leq x < 6$ or $x > 6$ $x \geq 2; x \neq 6$	A✓✓ $2 \leq x < 6$ ✓ $x > 6$ A✓ $x \neq 6$ ✓✓ $x \geq 2$	(3)
6.5	$-4 < k < 12$	CA✓ critical values A✓ notation	(2)
6.6	$f'(x) = 2x - 8 = -2$ $2x = 6$ $x = 3$	A✓ derivative A✓ equating to -2 CA✓ x - value	(3)
			[15]

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QUESTION 7

7.1	$y = \log_8 x$	AA✓✓ answer	(2)
7.2	$t(x) = 8^x$ $t\left(x + \frac{1}{3}\right) = 8^{x + \frac{1}{3}}$ $= 8^x \cdot 8^{\frac{1}{3}}$ $= 8^x \cdot (2^3)^{\frac{1}{3}}$ $= 8^x \cdot 2$ $= 2t(x)$	A✓ substituting $\left(x + \frac{1}{3}\right)$ A✓ $8^x \cdot 8^{\frac{1}{3}}$ A✓ writing in exponential form	(3)
7.3		Exponential: A✓ shape A✓ intercepts of both graphs with the axes Log graph: A✓ shape A✓ line of reflection	(4)
			[9]

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QUESTION 8

8.1	$A = P(1+i)^n$ $146338,09 = 80000 \left(1 + \frac{i}{2}\right)^{10}$ $\left(1 + \frac{i}{2}\right)^{10} = \frac{146338,09}{80000}$ $i = 2 \left(\sqrt[10]{\frac{146338,09}{80000}} - 1\right)$ $i = 0,1244999989$ Therefore, the interest rate = 12,45 % p.a. compounded half yearly	A✓ substitution into formula CA✓ making i the subject CA✓ decimal value CA✓ answer	(4)
8.2	$A = P_1(1+i_1)^{n_1}(1+i_2)^{n_2}$ $- P_2(1+i_1)^{n_3}(1+i_2)^{n_4}$ $- P_3(1+i_2)^{n_5}$ $A = 200000 \left(1 + \frac{11}{400}\right)^{20} \left(1 + \frac{12}{1200}\right)^{24}$ $- 60000 \left(1 + \frac{11}{400}\right)^{12} \left(1 + \frac{12}{1200}\right)^{24}$ $- 45000 \left(1 + \frac{12}{1200}\right)^{24}$ $= R274\,260,98$ OR $A = P(1+i)^n$ $A = 200\,000 \left(1 + \frac{11\%}{4}\right)^8$ $= R\,248\,476,1104$ $A = 188\,467,1104 \left(1 + \frac{11\%}{4}\right)^{12}$ $R260\,998,6597$ $A = 215\,998,6597 \left(1 + \frac{12\%}{12}\right)^{24}$ $R274\,260,98$	AAAAAAAAA✓✓✓✓✓✓✓✓ OR A✓ substitution of i and n CA✓ answer A✓ substitution of i and n CA✓ P value A✓ substitution of i and n CA✓ P value CA✓ answer	(8)
			[12]

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QUESTION 9 (penalize 1 mark once for incorrect notation in this question)

9.1	$f'(3) = \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$ $= \lim_{h \rightarrow 0} \frac{5(3+h)^2 + 4 - (5(3)^2 + 4)}{h}$ $= \lim_{h \rightarrow 0} \frac{45 + 30h + 5h^2 + 4 - 49}{h}$ $= \lim_{h \rightarrow 0} \frac{h(30 + 5h)}{h}$ $= 30$ OR $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{5(x+h)^2 + 4 - (5x^2 + 4)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{5x^2 + 10xh + 5h^2 + 4 - 5x^2 - 4}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{10xh + 5h^2}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{h(10x + 5h)}{h}$ $f'(x) = 10x$ $f'(3) = 10(3) = 30$	A✓ formula A✓ substitution CA✓ simplification of numerator CA✓ factorization CA✓ answer OR A✓ formula A✓ substitution CA✓ factorization CA✓ value of derivative CA✓ answer	(5)
9.2.1	$g(x) = \left(2x - \frac{1}{2x}\right)^2$ $= 4x^2 - 2 + \frac{1}{4}x^{-2}$ $g'(x) = 8x - \frac{1}{2}x^{-3}$	A✓ first and last terms A✓ middle term CACA✓✓ derivatives	(4)

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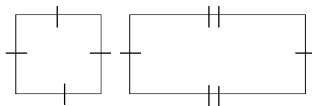
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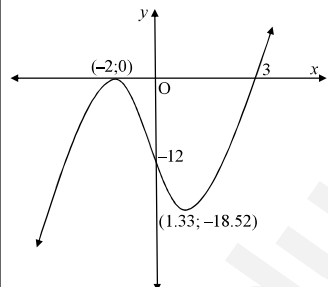
9.2.2	$D_x \left[\frac{x^3 - 1}{1 - x} \right]$ $= D_x \left[\frac{(x-1)(x^2 + x + 1)}{-(x-1)} \right]$ $= D_x [-x^2 - x - 1]$ $= -2x - 1$	A✓ factorizing numerator A✓ factorizing denominator CACA✓✓ each derivative	(4)
			[13]

QUESTION 10

10.1	$x = -2$ or $x = 3$ and $y = -12$ OR $(-2; 0); (3; 0); (0; -12)$	A✓✓ x - intercepts A✓ y - intercept.	(3)
10.2	$f(x) = (x+2)^2(x-3)$ $= (x^2 + 4x + 4)(x-3)$ $= x^3 - 3x^2 + 4x^2 - 12x + 4x - 12$ $= x^3 + x^2 - 8x - 12$ $f'(x) = 3x^2 + 2x - 8 = 0$ $(3x-4)(x+2) = 0$ $x = \frac{4}{3} \text{ or } x = -2$ $y = -18,52 \text{ or } y = 0$ Maximum : $(-2; 0)$ Minimum : $\left(\frac{4}{3}; -18,52\right)$	A✓ expression CA✓ derivative equal to 0 CA✓ x - values CA✓ y - values	(4)

QUESTION 11

11.1	$\frac{x}{4}$	A✓ answer	(1)
11.2	 Length of side of square = $\frac{x}{4}$ Length of rectangle = $\frac{10-x-\frac{x}{2}}{2} = \frac{20-3x}{4}$ $S = \frac{x^2}{16} + \frac{x}{4} \left(\frac{20-3x}{4} \right)$ $= \frac{x^2}{16} + \frac{20x-3x^2}{16}$ $= \frac{-2x^2 + 20x}{16}$ $= -\frac{1}{8}x^2 + \frac{5}{4}x$	A✓ length of rectangle A✓✓ each area A✓ simplifying	(4)
11.3	$S = -\frac{1}{8}x^2 + \frac{5}{4}x$ $S'(x) = -\frac{1}{4}x + \frac{5}{4} = 0$ $x = 5 \text{ metres}$	A✓ derivative A✓ derivative equal to 0 CA✓ answer	(3)
			[8]

10.3		CA✓ x - intercepts CA✓ y - intercept CA✓ turning points A✓ shape	(4)
10.4	$k = -12$	CACA✓✓ answer	(2)
10.5	$(2; 0)$	CA ✓ x - value CA ✓ y - value	(2)
10.6	$f'(x) = 3x^2 + 2x - 8$ $f''(x) = 6x + 2 = 0$ $x > -\frac{1}{3}$	CA✓ second derivative and equal to 0 CA✓ answer	(2)
10.7	$f(x) = (x+2)^2(x-3)$ $f\left(x + \frac{1}{2}\right) = \left(x + 2\frac{1}{2}\right)^2 \left(x - 2\frac{1}{2}\right)$	AA✓✓ for each bracket	(2)
			[19]

QUESTION 12

12.1	200	A✓ answer	(1)
12.2	$\frac{120}{200}$ or $\frac{3}{5}$ or 0,6 or 60%	A✓ 120 A✓ answer	(2)
12.3	$P(\text{Male}) = \frac{80}{200} = \frac{2}{5} = 0,4 = 40\%$ $P(\text{Choosing Fruit Juice}) = \frac{30}{200} = \frac{3}{20} = 0,15 = 15\%$ $P(\text{Male choosing Fruit Juice}) = \frac{z}{200}$ $P(\text{Male choosing Fruit Juice}) = P(\text{Male}) \times P(\text{Choosing Fruit Juice})$ $\frac{z}{200} = \frac{2}{5} \times \frac{3}{20}$ $\frac{z}{200} = \frac{100}{100z} = 1200$ $z = 12$	$A✓ \frac{80}{200} = \frac{2}{5} = 0,4 = 40\%$ $A✓ \frac{30}{200} = \frac{3}{20} = 0,15 = 15\%$ $A✓ \frac{z}{200}$ A✓ Condition of independent events CA✓ substitution in the condition of independent events CA✓ answer	(6)
			[9]

QUESTION 13

13.1	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $0,55 = 0,34 + 0,26 - P(A \text{ and } B)$ $P(A \text{ and } B) = \frac{1}{20} = 0,05$	A✓ Formula A✓ substitution A✓ answer	(3)
13.2	No $P(A \text{ and } B) \neq 0$	A✓ No A✓ Justification	(2)
			[5]



education

 Department:
Education
PROVINCE OF KWAZULU-NATAL

MATHEMATICS P2
MARKING GUIDELINE

COMMON TEST

JUNE 2019

NATIONAL
SENIOR CERTIFICATE

GRADE 12

MARKS: 150

N.B. This marking guidelines consists of 14 pages.

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QUESTION 2

2.1	A = 250 B = 502	✓ (A) A ✓ (A) B	(2)
2.2	$\bar{x} = \frac{2000 + 5000 + 600 + 1280 + 1150 + 18000 + 33250}{1300}$ $\bar{x} = \frac{61280}{1300}$ $\bar{x} = 47,14$ (answer only – full marks)	✓ CA sum ✓ CA 61280 ✓ CA answer	(3)
2.3	$65 < x \leq 75$	✓✓ AA answer	(2)
2.4	AGES OF CONSUMERS 	✓ CA upper limits ✓ CA ✓ CA rounding (5; 0) ✓ CA joining points with a smooth curve	(3)
2.5	Not a normal distribution. Highest frequency is found between the ages 55 to 75. Mean < median, therefore skewed to the left.	✓ A No ✓ A Reason	(2)
			[12]

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Mathematics P2

2
NSC-Marking Guideline

Common Test June 2019

QUESTION 1

1.1	$\bar{x} = \frac{220}{10} = 22$	✓ A 220 ✓ CA answer Answer only full marks	(2)
1.2	$\sigma = 3,95$	✓✓ AA answer If formula is used 1 CA mark for substitution and 1 CA mark for answer.	(2)
1.3		✓ A minimum & maximum value ✓ A quartile 1 value ✓ A median value ✓ A quartile 3 value If No Diagram No marks	(4)
1.4	$(\bar{x} - \sigma; \bar{x} + \sigma)$ $(22 - 3,95; 22 + 3,95)$ $(18,05; 25,95)$ 6 runners (answer only – full marks)	✓ CA 18,05 ✓ CA 25,95 ✓ CA answer	(3)
			[11]

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Mathematics P2

4
NSC-Marking Guideline

Common Test June 2019

QUESTION 3

3.1	$BC = \sqrt{(x^2 - x^1)^2 + (y^2 - y^1)^2}$ $= \sqrt{(-2 - 1)^2 + (-2 - 4)^2}$ $= \sqrt{9 + 36}$ $= \sqrt{45}$ $= 3\sqrt{5}$	✓ A substitution ✓ CA answer	(2)
3.2	$M\left(\frac{1-2}{2}; \frac{4-2}{2}\right)$ $M\left(-\frac{1}{2}; 1\right)$	✓ A $-\frac{1}{2}$ ✓ A 1	(2)
3.3	$m_{AB} = \frac{-2-4}{-2-1} = \frac{-6}{-3}$ $= 2$ $m_{MD} = -\frac{1}{2} \quad (DM \perp AB)$ $y = mx + c$ $1 = -\frac{1}{2}\left(-\frac{1}{2}\right) + c$ $c = 1 - \frac{1}{4}$ $= \frac{3}{4}$ $y = -\frac{1}{2}x + \frac{3}{4}$	✓ A m_{AB} ✓ CA gradient of MD ✓ CA subst. $\left(-\frac{1}{2}; 1\right)$ into eq. ✓ CA answer	(4)

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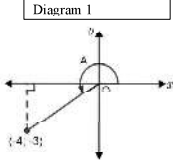
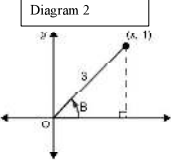
3.4	E is the midpoint since $ME \parallel BC$. $E\left(\frac{1+4}{2}, \frac{4+1}{2}\right)$ $= E\left(\frac{5}{2}, \frac{5}{2}\right)$	✓ A S/R ✓ A substitution ✓ CA answer (provided coordinates are positive) (3)
3.5	$m_{BE} = \frac{-2-5}{-2-2}$ $m_{BE} = 1$ $y - y_1 = m(x - x_1)$ OR $y = mx + c$ $y - (-2) = 1(x - (-2))$ $-2 = 1(-2) + c$ $y + 2 = x + 2$ $0 = c$ $y = x$ $y = x$	✓ CA m_{BE} (must be positive) ✓ CA substitution ✓ CA answer (must be positive) (3)
3.6	$m_{BC} = \frac{-2-1}{-2-4} = \frac{1}{2}$ $m_{BC} = \tan \theta = \frac{1}{2}$ $\theta = 26,57^\circ$	✓ A Substitution ✓ CA $m_{BC} = \frac{1}{2}$ ✓ CA $\tan \theta = \frac{1}{2}$ ✓ CA answer (4)
[18]		
QUESTION 4		
4.1	$AQ = \sqrt{(-6-2)^2 + (-7-(-1))^2}$ $AQ = \sqrt{(-8)^2 + (-6)^2}$ $AQ = \sqrt{64 + 36}$ $AQ = \sqrt{100}$ $\therefore AQ = 10$	✓ A subst. into dist. formula ✓ CA answer (2)

4.2	$(x-a)^2 + (y-b)^2 = r^2$ $(x-2)^2 + (y+1)^2 = 100$	✓ A $(x-2)^2 + (y+1)^2$ ✓ CA 100 (2)
4.3.1	$m_{AQ} = \frac{-7+1}{-6-2} = \frac{3}{4}$ $\therefore m_{QP} = -\frac{4}{3}$ rad. $\perp$ tan	✓ A $\frac{3}{4}$ ✓ CA $-\frac{4}{3}$ (2)
4.3.2	$m_{AR} = \frac{-7+1}{10-2} = -\frac{3}{4}$ $\therefore m_{PR} = \frac{4}{3}$ rad. $\perp$ tan	✓ A $-\frac{3}{4}$ ✓ CA $\frac{4}{3}$ (2)
4.4.1	$m_{QP} = -\frac{4}{3}$ $y - y_1 = m(x - x_1)$ $y + 7 = -\frac{4}{3}(x + 6)$ $y = -\frac{4}{3}x - 15$	✓ CA substitution ✓ CA answer (2)
4.4.2	$m_{PR} = \frac{4}{3}$ $y - y_1 = m(x - x_1)$ $y + 7 = \frac{4}{3}(x - 10)$ $y = \frac{4x}{3} - \frac{40}{3} - 7$ $= \frac{4}{3}x - \frac{61}{3}$	✓ CA substitution ✓ CA answer (2)

4.5.1	$\frac{4}{3}x - \frac{61}{3} = -\frac{4}{3}x - 15$ $\frac{8}{3}x = \frac{16}{3}$ $x = 2$ $y = -\frac{53}{3}$ $P(2; -\frac{53}{3})$ OR The x-co-ordinate of P is 2 (ARPQ is a kite) Subst $x=2$ in $y = -\frac{4}{3}x - 15$ $y = -\frac{4}{3}(2) - 15$ $= -\frac{8}{3} - 15$ $= \frac{-8-45}{3}$ $= \frac{-53}{3}$ $P(2; -\frac{53}{3})$	✓ CA Equating ✓ CA x value ✓ CA y value ✓ CA both co-ordinates ✓ A $x = 2$ ✓ CA substitution ✓ CA y value ✓ CA both co-ordinates (4)
4.5.2	In $\triangle SPR$ $\alpha = \hat{P} + \beta$ (ext $\angle$ of $\triangle SPR$) $\therefore \hat{P} = \alpha - \beta$	✓ A S/R ✓ A $\hat{P} = \alpha - \beta$ (2)

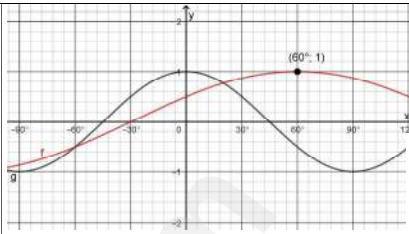
4.5.3	$\tan(\alpha - \beta) = \frac{\sin(\alpha - \beta)}{\cos(\alpha - \beta)}$ $= \frac{\sin \alpha \cos \beta - \cos \alpha \sin \beta}{\cos \alpha \cos \beta + \sin \alpha \sin \beta}$ $= \frac{\left(\frac{4}{5}\right)\left(\frac{3}{5}\right) - \left(-\frac{3}{5}\right)\left(\frac{4}{5}\right)}{\left(-\frac{3}{5}\right)\left(\frac{3}{5}\right) + \left(\frac{4}{5}\right)\left(\frac{4}{5}\right)}$ $= \frac{\frac{12}{25} + \frac{12}{25}}{-\frac{9}{25} + \frac{16}{25}}$ $= \frac{24}{25} \times \frac{25}{7}$ $= \frac{24}{7}$ OR $\tan \beta = \frac{4}{3} \therefore \beta = 53.13^\circ$ $\tan \alpha = -\frac{4}{3} \therefore \alpha = 126.87^\circ$ $\tan(\alpha - \beta) = \frac{\sin(126.87 - 53.13)}{\cos(126.87 - 53.13)}$ $= \frac{3,42857 \dots}{\frac{24}{7}}$	✓ A expansion ✓ A numerator ✓ A denominator ✓ A Simplification ✓ A answer (4) ✓ A answer ✓ A simplification ✓ A Answer (4)
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QUESTION 5

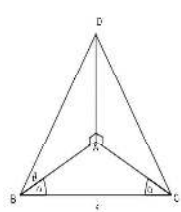
5.1	$\tan A = \frac{3}{4}$ $\sin B = \frac{1}{3}$   $r^2 = (-4)^2 + (-3)^2 \dots$ Pyth $r^2 = 25$ $r = 5$ $x^2 = 3^2 - 1^2 \dots$ Pyth $x^2 = 8$ $x = 2\sqrt{2}$	✓ A diagram 1 ✓ A identity ✓ CA answer	(3)
5.1.1	$\cos 2A$ $= 2\cos^2 A - 1$ $= 2\left(\frac{-4}{5}\right)^2 - 1$ $= \frac{7}{25}$	✓ A identity ✓ CA answer	(3)
5.1.2	$\sin(A+B)$ $= \sin A \cos B + \cos A \sin B$ $= \left(\frac{-3}{5}\right)\left(\frac{2\sqrt{2}}{3}\right) + \left(\frac{-4}{5}\right)\left(\frac{1}{3}\right)$ $= -\frac{15}{15}$	✓ A Diagram 2 ✓ A expansion ✓ CA answer	(3)
5.2	$\sin 20^\circ \cos 320^\circ + \cos(-20^\circ) \sin 400^\circ$ $= \sin 20^\circ \cos 40^\circ + \cos 20^\circ \sin 40^\circ$ $= \sin(20^\circ + 40^\circ)$ $= \sin 60^\circ$ $= \frac{\sqrt{3}}{2}$	✓ A reduction ✓ CA simplify ✓ CA answer (provided special angle ratio) ANSWER ONLY = 0	(3)

5.3	$\cos^2(90^\circ + \theta)$ $\cos(-\theta) + \sin(90^\circ - \theta) \cos \theta$ $= \frac{\sin^2 \theta}{\cos \theta + \cos^2 \theta}$ $= \frac{\sin^2 \theta}{\cos \theta (1 + \cos \theta)}$ $= \frac{1 - \cos^2 \theta}{\cos \theta (1 + \cos \theta)}$ $= \frac{1 - \cos \theta}{\cos \theta}$ $= \frac{1}{\cos \theta} - 1$ $= \text{RHS}$	✓ A numerator ✓ A denominator ✓ A common factor ✓ A difference of squares ✓ A simplification	(5)
5.4.1	$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$ $= (\cos \alpha + \sin \alpha)(\cos \alpha - \sin \alpha)$ $= p \cdot q$	✓ A expansion ✓ A answer	(2)
5.4.2	$\frac{1 + \sin 2\alpha}{\cos 2\alpha}$ $= \frac{\sin^2 \alpha + 2 \sin \alpha \cos \alpha + \cos^2 \alpha}{\cos^2 \alpha - \sin^2 \alpha}$ $= \frac{(\sin \alpha + \cos \alpha)^2}{(\cos \alpha - \sin \alpha)(\cos \alpha + \sin \alpha)}$ $= \frac{p}{q}$	✓ A numerator ✓ A denominator ✓ A factorise ✓ A factorise ✓ CA answer	(5)
5.5	$6 \cos^2 x + \sin x - 5 = 0$ $6(1 - \sin^2 x) + \sin x - 5 = 0$ $6 - 6 \sin^2 x + \sin x - 5 = 0$ $-6 \sin^2 x + \sin x + 1 = 0$ $(3 \sin x + 1)(-2 \sin x + 1) = 0$ $\sin x = -\frac{1}{3}; \sin x = \frac{1}{2}$ $x = 199,47^\circ + k, 360^\circ; k \in \mathbb{Z}$ OR $x = 30^\circ + k, 360^\circ; k \in \mathbb{Z}$ $x = 340,53^\circ + k, 360^\circ; k \in \mathbb{Z}$ OR $x = 150^\circ + k, 360^\circ; k \in \mathbb{Z}$	✓ A identity ✓ CA factors ✓ CA both ratios ✓ A $k \in \mathbb{Z}$ ✓ CA both solutions of $\sin x = -\frac{1}{3}$ ✓ CA both solutions of $\sin x = \frac{1}{2}$	(6)

QUESTION 6

6.1		✓ A x-intercept ✓ A t. pt. (60°; 1) ✓ A shape	(3)
6.2	$60^\circ < x < 90^\circ$	✓ CA end values ✓ CA notation	(2)
6.3	Graph of f moves 60° left.	✓ A shifts 60° ✓ A to the left	(2)
			[7]

QUESTION 7

7	$\tan \theta = \frac{DA}{AB}$ $\therefore AD = AB \tan \theta$ Also $\frac{AB}{\sin \alpha} = \frac{k}{\sin(180^\circ - 2\alpha)}$ $AB \sin 2\alpha = k \sin \alpha$ $AB = \frac{k \sin \alpha}{2 \sin \alpha \cos \alpha}$ $AB = \frac{k}{2 \cos \alpha}$ $\therefore AD = \frac{k \tan \theta}{2 \cos \alpha}$		✓ A trig ratio ✓ AAD value ✓ A substitute into sine rule $\sin 2\alpha = 2 \sin \alpha \cos \alpha$ ✓ A $AB = \frac{k \sin \alpha}{2 \sin \alpha \cos \alpha}$ ✓ A making AB value - simplified	[6]
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QUESTION 8

8.1	$\hat{Q}_1 = \hat{W}_2 = x \dots$ (tan chord theorem) $\hat{W}_2 = \hat{W}_1 = x \dots$ (WQ bisects PWR.) $\hat{Q}_1 = \hat{W}_1 = x \dots$ (tan - chord theorem) $\hat{T}_2 = \hat{W}_2 = x \dots$ ($\angle$ s in same segment) $\hat{S}_2 = \hat{W}_1 = x \dots$ ($\angle$ s in same segment)	A✓S/R A✓S/R A✓S/R A✓S/R	(5)
8.2.1	$\hat{T}_2 = \hat{Q}_1 = x$ $\therefore TS \parallel PR \dots$ (alternate $\angle$ s equal)	A✓S A✓R	(2)
8.2.2	$\hat{T}_3 = \hat{P} \dots$ (corresponding $\angle$ s; $TS \parallel PR$) $\hat{T}_3 = \hat{Q}_3 \dots$ ($\angle$ s in same segment) $\therefore \hat{P} = \hat{Q}_3$	AA✓S✓R A✓S/R	(3)
8.2.3	In ΔTQS $\hat{T}_2 = x$ $\hat{S}_2 = x$ $\therefore \hat{T}_2 = \hat{S}_2 = x$ $\therefore \Delta TQS$ isosceles ($\angle$ s opposite equal sides)	A✓S A✓S A✓S A✓R	(4)
8.2.4	$W\hat{Q}P = W\hat{S}Q \dots$ (tan - chord theorem) $\hat{T}_1 = W\hat{S}Q \dots$ (ext $\angle$ of cyclic quad)	AA✓S✓R A✓S/R	(3)
			[17]

QUESTION 9

9.1.1	$\hat{P}_1 = \hat{Q}_1$ given $\hat{P}_1 = \hat{R}$ tan-chord theorem $\therefore \hat{Q}_1 = \hat{R}$ $\therefore TQ \parallel SR$ (corr $\angle$ s are equal)	AA $\checkmark$ S $\checkmark$ R A $\checkmark$ R	(3)
9.1.2	$\hat{P}_1 = \hat{Q}_1$ given TS = TP tan from same point $\hat{P}_1 = \hat{S}_1$ equal $\angle$ s opp equal sides $\therefore \hat{Q}_1 = \hat{S}_1$ $\therefore QPTS$ is a cyclic quad converse equal $\angle$ s subtended by same chord	A $\checkmark$ S/R A $\checkmark$ S/R A $\checkmark$ R	(3)
9.1.3	QPTS is a cyclic quad $\therefore \hat{P}_1 = \hat{Q}_2$ but $\hat{P}_1 = \hat{Q}_1$ $\therefore \hat{Q}_1 = \hat{Q}_2$ $\therefore TQ$ bisect $S\hat{Q}P$ $\angle$ s in same $\odot$ segm given	AA $\checkmark$ S $\checkmark$ R A $\checkmark$ S	(3)
9.2.1	In ΔLPK and ΔNPL KL $\perp$ P = LN $\perp$ P ... tan chord theorem $\hat{P}_2 = 90^\circ$... $\angle$ in semi $\odot$ $\hat{P}_1 = \hat{P}_2$... both = 90° PKL = NLP ... remaining angle $\therefore \Delta LPK \parallel \Delta NPL$... $\angle \angle \angle$	AA $\checkmark$ S $\checkmark$ R A $\checkmark$ S/R A $\checkmark$ R	(4)
9.2.2	$\frac{PL}{NP} = \frac{KL}{NL} = \frac{PK}{PL}$... $\Delta LPK \parallel \Delta NPL$ $\frac{PL}{NP} = \frac{PK}{PL}$... $\Delta LPK \parallel \Delta NPL$ $\therefore PL^2 = NP \cdot PK$	AA $\checkmark$ S $\checkmark$ R A $\checkmark$ proportionality	(3)
9.2.3	ΔNLK	A $\checkmark$ answer	(1)
9.2.4	$\Delta NLK \parallel \Delta NPL$ $\therefore \frac{KN}{LN} = \frac{LN}{NP}$ [// Δ s] $LN^2 = KN \cdot NP$ $= 16 \times 10$ $= 160$ $LN = \sqrt{160}$ Radius = $\frac{1}{2}\sqrt{160}$ Area of Circle = πr^2 $= \pi \left(\frac{1}{2}\sqrt{160}\right)^2$ $= 125,66 \text{ cm}^2$ OR	A $\checkmark$ S/R A $\checkmark$ Substitution $\checkmark$ CA NL value $\checkmark$ CA radius = $\frac{1}{2}\sqrt{160}$ $\checkmark$ CA Substitution $\checkmark$ CA Answer	(6)

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basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS/ SENIORSERTIFIKAAT-EKSAMEN NATIONAL SENIOR CERTIFICATE EXAMINATIONS/ NASIONALE SENIORSERTIFIKAAT-EKSAMEN

MATHEMATICS P1/
WISKUNDE V1

MARKING GUIDELINES/NASIENRIGLYNE

2019

MARKS: 150
PUNTE: 150

These marking guidelines consist of 15 pages.
Hierdie nasienriglyne bestaan uit 15 bladsye.

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From Question No. 9.2.2: $PL^2 = NP \cdot PK$ $= 10 \text{ cm} \times 6 \text{ cm}$ $= 60 \text{ cm}^2$	$\checkmark$ A $PL^2 = 60 \text{ cm}^2$	
$NL^2 = PL^2 + PN^2$... Pythagoras $= 60 + 100$ (ΔLPN) $= 160 \text{ cm}^2$ $\therefore NL = \sqrt{160} \text{ cm}$ $\frac{1}{2} NL = \frac{1}{2} \text{ diameter} = \frac{1}{2} \sqrt{160} \text{ cm}$	$\checkmark$ A Pythagoras	
Area of circle = πr^2 $= \pi \times \left(\frac{1}{2}\sqrt{160}\right)^2 \text{ cm}^2$ $= 125,66 \text{ cm}^2$ OR $40\pi \text{ cm}^2$	$\checkmark$ CA NL value $\checkmark$ CA radius = $\frac{1}{2}\sqrt{160}$ $\checkmark$ CA substitution	
	$\checkmark$ CA answer	[23]

QUESTION 10

10.1	$\frac{AE}{EF} = \frac{4}{6}$... prop theorem; EB//FC $\frac{AE}{EF} = \frac{AC}{CD}$... prop theorem; EC//FD $\frac{4}{6} = \frac{10}{CD}$ $CD = 15 \text{ units}$	A $\checkmark$ S/R A $\checkmark$ S/R $\checkmark$ CA answer	(3)
10.2	$\frac{\Delta FEC}{\Delta CFA} = \frac{1}{5}$ same height $\frac{\text{Area } \Delta CFA}{\text{Area } \Delta FAD} = \frac{10}{25} = \frac{2}{5}$ same height $\frac{\text{Area } \Delta FEC}{\text{Area } \Delta FAD} = \frac{\text{Area } \Delta FEC}{\text{Area } \Delta CFA} \times \frac{\text{Area } \Delta CFA}{\text{Area } \Delta FAD}$ $= \frac{3}{5} \times \frac{2}{5}$ $= \frac{6}{25}$	A $\checkmark$ S/R A $\checkmark$ S/R $\checkmark$ CA simplify $\checkmark$ CA answer	(4)

TOTAL MARKS: 150

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NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- Consistent Accuracy applies in all aspects of the marking memorandum.

LET WEL:

- Indien 'n kandidaat 'n vraag TWEE keer beantwoord, merk slegs die EERSTE poging.
- Volgehoue akkuraatheid is DEURGAANS op ALLE aspekte van die memorandum van toepassing.

QUESTION/VRAAG 1

1.1.1	$x^2 - 5x - 6 = 0$ $(x-6)(x+1) = 0$ $x = 6$ or $x = -1$ OR/OF $x^2 - 5x - 6 = 0$ $x = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(-6)}}{2(1)}$ $x = \frac{5 \pm \sqrt{49}}{2}$ $x = 6$ or $x = -1$	$\checkmark$ factors $\checkmark$ both answers OR/OF $\checkmark$ correct subst into correct formula $\checkmark$ both answers	(2)
1.1.2	$(3x-1)(x-4) = 16$ $3x^2 - 13x - 12 = 0$ $x = \frac{13 \pm \sqrt{(-13)^2 - 4(3)(-12)}}{2(3)}$ $x = \frac{13 \pm \sqrt{313}}{6}$ $x = 5,12$ or $x = -0,78$ OR/OF $3x^2 - 13x - 12 = 0$ $x^2 - \frac{13}{3}x - 4 = 0$ $x^2 - \frac{13}{3}x + \left(-\frac{13}{6}\right)^2 = 4 + \left(-\frac{13}{6}\right)^2$ $\left(x - \frac{13}{6}\right)^2 = \frac{313}{36}$ $x = \frac{13 \pm \sqrt{313}}{6}$	$\checkmark$ standard form $\checkmark$ correct subst into correct formula $\checkmark \checkmark$ answers OR/OF $\checkmark$ standard form $\checkmark$ adding $\left(-\frac{13}{6}\right)^2$ both sides $\checkmark \checkmark$ answers	(4)

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1.1.3	$x(4-x) \geq 0$ $-x(x-4) \geq 0$ or $-x(x-4) \leq 0$ $x(x-4) \leq 0$  $0 \leq x \leq 4$ or $x \in [0; 4]$	✓ factorisation ✓✓ $0 \leq x \leq 4$ (3)
1.1.4	$\frac{5^{2x}-1}{5^x+1} = 4$ $\frac{(5^x+1)(5^x-1)}{5^x+1} = 4$ $5^x-1 = 4$ $5^x = 5$ $x = 1$ OR/OF $\frac{5^{2x}-1}{5^x+1} = 4$ $5^{2x}-1 = 4 \cdot 5^x + 4$ $5^{2x} - 4 \cdot 5^x - 5 = 0$ $(5^x-5)(5^x+1) = 0$ $5^x = 5$ or $5^x \neq -1$ $x = 1$	✓ factors in numerator ✓ $5^x - 1 = 4$ ✓ answer OR/OF ✓ standard form ✓ factors ✓ answer (3)
1.2	$x = 2 - 3y$(1) $x^2 + 4xy - 5 = 0$(2) Substitute (1) in (2): $(2-3y)^2 + 4y(2-3y) - 5 = 0$ $4 - 12y + 9y^2 + 8y - 12y^2 - 5 = 0$ $-3y^2 - 4y - 1 = 0$ $3y^2 + 4y + 1 = 0$ $(3y+1)(y+1) = 0$ $y = -\frac{1}{3}$ or $y = -1$ $x = 3$ or $x = 5$ OR/OF	✓ $x = 2 - 3y$ ✓ correct subst into correct formula ✓ either standard form ✓ y – values ✓ x – values (5)

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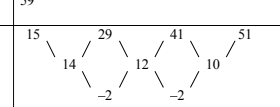
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1.3	$y = \frac{2}{3} - \frac{x}{3}$(1) $x^2 + 4xy - 5 = 0$(2) Substitute (1) in (2): $x^2 + 4x\left(\frac{2}{3} - \frac{x}{3}\right) - 5 = 0$ $3x^2 + 8x - 4x^2 - 15 = 0$ $-x^2 + 8x - 15 = 0$ $x^2 - 8x + 15 = 0$ $(x-5)(x-3) = 0$ $x = 3$ or $x = 5$ $y = -\frac{1}{3}$ or $y = -1$	✓ $y = \frac{2}{3} - \frac{x}{3}$ ✓ correct subst into correct formula ✓ either standard form ✓ x – values ✓ y – values (5)
1.3	$ab = 2\sqrt{10}$ $bc = 3\sqrt{2}$ $ac = 6\sqrt{5}$ $abbcac = 2\sqrt{10} \cdot 6\sqrt{5} \cdot 3\sqrt{2}$ $(abc)^2 = 36\sqrt{100}$ $abc = \sqrt{360} = 6\sqrt{10}$ OR/OF $ac = 6\sqrt{5} \therefore a = \frac{6\sqrt{5}}{c}$ $bc = 3\sqrt{2} \therefore b = \frac{3\sqrt{2}}{c}$ $ab = 2\sqrt{10}$ $\left(\frac{6\sqrt{5}}{c}\right)\left(\frac{3\sqrt{2}}{c}\right) = 2\sqrt{10}$ $18\sqrt{10} = 2\sqrt{10}c^2$ $c^2 = 9$ $c = 3$ Volume = $abc = 2\sqrt{10} \cdot 3 = \sqrt{360} = 6\sqrt{10}$	✓ volume = abc ✓✓ $abbcac = 2\sqrt{10} \cdot 6\sqrt{5} \cdot 3\sqrt{2}$ ✓ $(abc)^2 = 36\sqrt{100}$ ✓ answer OR/OF ✓ $a = \frac{6\sqrt{5}}{c}$ ✓ $b = \frac{3\sqrt{2}}{c}$ ✓ value of c ✓ Volume = abc ✓ answer (5) [22]

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QUESTION/VRAAG 2

2.1.1	59	✓ answer (1)
2.1.2	 $2a = -2$ $a = -1$ $3(-1) + b = 14$ $b = 17$ $(-1) + (17) + c = 15$ $c = -1$ $T_n = -n^2 + 17n - 1$	✓ second difference of -2 ✓ a ✓ b ✓ c (4)
2.1.3	$T_{27} = -(27)^2 + 17(27) - 1$ $= -271$	✓ substitution ✓ answer (2)
2.2.1	$r = \frac{-18}{36} = -\frac{1}{2}$	✓ answer (1)
2.2.2	$T_n = 36\left(-\frac{1}{2}\right)^{n-1}$ $\frac{9}{4096} = 36\left(-\frac{1}{2}\right)^{n-1}$ $\frac{1}{16384} = \left(-\frac{1}{2}\right)^{n-1}$ $\left(-\frac{1}{2}\right)^{14} = \left(-\frac{1}{2}\right)^{n-1}$ $14 = n - 1$ $n = 15$ OR/OF $36; -18; 9; -\frac{9}{2}; \frac{9}{4}; -\frac{9}{8}; \dots; \frac{9}{4096}$ If you look only at the denominator: 2; 4; 8; ...; 4096 $2^k = 4096$ $2^k = 2^{12}$ $k = 12$ $\therefore n = 15$ terms	✓ $T_n = 36\left(-\frac{1}{2}\right)^{n-1}$ ✓ $\frac{1}{16384} = \left(-\frac{1}{2}\right)^{n-1}$ ✓ answer OR/OF ✓ $2^k = 4096$ ✓ $k = 12$ ✓ answer (3)

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2.2.3	$S_{-r} = \frac{a}{1-r}$ $= \frac{36}{1 - \left(-\frac{1}{2}\right)}$ $= 24$	✓ correct subst into correct formula with $-1 < r < 1$ ✓ answer if $-1 < r < 1$ (2)
2.2.4	$S_{250 \text{ even}} = \frac{-18\left(\left(\frac{1}{4}\right)^{250} - 1\right)}{\frac{1}{4} - 1}$ $= -24$ $36\left(\left(\frac{1}{4}\right)^{250} - 1\right)$ $S_{250 \text{ odd}} = \frac{\frac{1}{4} - 1}{\frac{1}{4} - 1}$ $= 48$ $S_{\text{odd}} = 48$ $S_{\text{even}} = -24$ $= -2$ OR/OF $T_1 + T_3 + T_5 + \dots + T_{499}$ $T_2 + T_4 + T_6 + \dots + T_{500}$ $= \frac{a + ar^2 + ar^4 + \dots + ar^{498}}{ar + ar^3 + ar^5 + \dots + ar^{499}}$ $= \frac{a + ar^2 + ar^4 + \dots + ar^{498}}{r(a + ar^2 + ar^4 + \dots + ar^{498})}$ $= \frac{1}{r}$ $= -2$	✓ $r = \frac{1}{4}$ and $n = 250$ ✓ $S_{250 \text{ even}} = -24$ ✓ $S_{250 \text{ odd}} = 48$ ✓ answer OR/OF ✓ $a + ar^2 + ar^4 + \dots + ar^{498}$ ✓ $ar + ar^3 + ar^5 + \dots + ar^{499}$ ✓ ✓ $r(a + ar^2 + ar^4 + \dots + ar^{498})$ ✓ answer (4)

[17]

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QUESTION/VRAAG 3

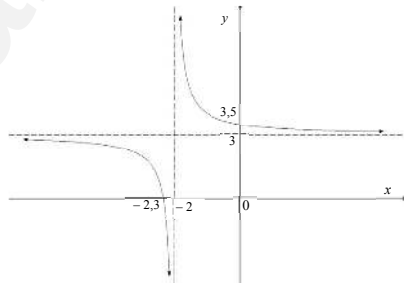
3.1.1	$p + 6 - (2p + 3) = p - 2 - (p + 6)$ $-p + 3 = -8$ $p = 11$	✓ equating i.t.o p ✓ simplifying (2)
3.1.2	$T_n = 25 + (n-1)(-8) = 33 - 8n$ $33 - 8n < -55$ $-8n < -88$ $n > 11$ ∴ Term 12 will be the first term smaller than -55 ∴ Term 12 sal die eerste term kleiner as -55 wees.	✓ subst into T_n formula ✓ $n > 11$ ✓ $n = 12$ (3)
3.2	$S_6 = \frac{n}{2}[a + l] = \frac{6}{2}[(x-3) + (x-18)]$ $= 6x - 63$ $S_9 = \frac{n}{2}[a + l] = \frac{9}{2}[(x-3) + (x-27)]$ $= 9x - 135$ $6x - 63 = 9x - 135$ $3x = 72$ $x = 24$ ∴ $S_{15} = \frac{n}{2}[a + l] = \frac{15}{2}[(x-3) + (x-45)]$ $= \frac{15}{2}[2x - 48]$ $= \frac{15}{2}[2(24) - 48] = 0 = \text{RHS}$ OR/OF $\sum_{k=1}^n (x-3k) = 0$ $(x-21) + (x-24) + (x-27) = 0$ ∴ $3x - 72 = 0$ $3x = 72$ $x = 24$ $\sum_{k=1}^{15} (24-3k)$ $= 21 + 18 + 15 + \dots + -21.$ $S_n = \frac{n}{2}[a + l]$ $= \frac{15}{2}[21 - 21]$ $= 0 = \text{RHS}$ OR/OF	✓ $6x - 63$ ✓ $9x - 135$ ✓ 24 ✓ $\frac{15}{2}[(x-3) + (x-45)]$ ✓ substitution of x OR/OF ✓ expansion ✓ $3x - 72 = 0$ ✓ 24 ✓ substitution of x ✓ sum of 15 terms (5) OR/OF

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4.4.1	$y = -\log_2 x$ $2 = -\log_2 a$ $a = 2^{-2} = \frac{1}{4}$ or $a = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$	✓ correct subst into correct formula ($a; 2$) ✓ answer (2)
4.4.2	$M\left(2; \frac{1}{4}\right)$ or $M'(2; a)$	✓ answer (1)
4.5	$M'\left(-1; \frac{9}{4}\right)$	✓ -1 ✓ $\frac{9}{4}$ (3)
[11]		

QUESTION/VRAAG 5

5.1.1	$x = -2$ $y = 3$	✓ answer ✓ answer (2)
5.1.2	$\left(0; \frac{7}{2}\right)$	✓ answer (1)
5.1.3	$\frac{1}{x+2} + 3 = 0$ $1 + 3(x+2) = 0$ $3x = -7$ $x = -\frac{7}{3}$ x-intercept $\left(-\frac{7}{3}; 0\right)$	✓ $y = 0$ ✓ answer (2)
5.1.4		✓ asymptotes at $y = 3$ and $x = -2$ ✓ intercepts at $y = 3.5$ and $x = -2.3$ ✓ shape (reasonable representation in correct quadrants) (3)

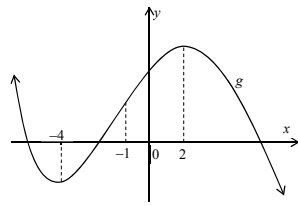
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	$(x-3) + (x-6) + (x-9) + (x-12) + (x-15) + (x-18)$ $= (x-3) + (x-6) + (x-9) + (x-12) + (x-15) + (x-18)$ $+ (x-21) + (x-24) + (x-27)$ ∴ $3x - 72 = 0$ $3x = 72$ $x = 24$ $\sum_{k=1}^{15} (24-3k)$ $= 21 + 18 + 15 + \dots + -21.$ $S_n = \frac{n}{2}[a + l]$ $= \frac{15}{2}[21 - 21]$ $= 0 = \text{RHS}$	✓ expansion ✓ $3x - 72 = 0$ ✓ 24 ✓ substitution of x ✓ sum of 15 terms (5) [10]
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QUESTION/VRAAG 4

4.1	$y > 0$ OR/OF $y \in (0; \infty)$	✓ answer OR/OF ✓ answer (1)
4.2	$g: y = \left(\frac{1}{2}\right)^x$ $g^{-1}: x = \left(\frac{1}{2}\right)^y$ $y = \log_{\frac{1}{2}} x$ or $y = -\log_2 x$ or $y = \log_2 \frac{1}{x}$	✓ $x = \left(\frac{1}{2}\right)^y$ ✓ equation (2)
4.3	Yes. The vertical line test cuts g^{-1} once Ja. Die vertikale lyn toets sny g^{-1} slegs eenkeer. OR/OF Yes. For every x-value there is a unique y-value Ja. Vir elke x-waarde is daar 'n unieke y-waarde OR/OF Yes. g is a one-to-one function / Ja. g is 'n een-tot-een funksie OR/OF Yes. The horizontal line cuts g only once Ja. Die horisontale lyn sny g slegs een keer	✓ yes ✓ valid reason (2) OR/OF ✓ yes ✓ valid reason (2) OR/OF ✓ yes ✓ valid reason (2) OR/OF ✓ yes ✓ valid reason (2)

5.2.1	$-2x + 4 = 0$ $2x = 4$ $x = 2$ ∴ $S(2; 0)$	✓ $y = 0$ ✓ $x = 2$ (2)
5.2.2	Equation of k : $y = a(x+1)^2 + 18$ $0 = a(2+1)^2 + 18$ or $0 = a(-4+1)^2 + 18$ $9a = -18$ $a = -2$ $y = -2(x+1)^2 + 18$	✓ $y = a(x+1)^2 + 18$ ✓ substitute $(2; 0)$ or $(-4; 0)$ ✓ a (3)
5.2.3	$-2x^2 - 4x + 16 = -2x + 4$ $-2x^2 - 2x + 12 = 0$ $x^2 + x - 6 = 0$ $(x+3)(x-2) = 0$ $x = -3$ or $x = 2$ $y = -2(-3) + 4 = 10$ $T(-3; 10)$	✓ equating ✓ standard form ✓ factors ✓ choosing $x = -3$ ✓ answer (5)
5.2.4	$x < -3$ or $x > 2$ OR/OF $(-\infty; -3) \cup (2; \infty)$	✓ ✓ answer (2) OR/OF ✓ ✓ answer (2)
5.2.5(a)	$x < -1$ OR/OF $(-\infty; -1)$	✓ ✓ answer (2) OR/OF ✓ ✓ answer (2)
5.2.5(b)		✓ shape of cubic with local min tp moving to local max tp ✓ turning points at $x = 2$ and $x = -4$ ✓ point of inflection at $x = -1$ (3) [25]

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QUESTION/VRAAG 6

6.1.1	$A = P(1-i)^n$ $79866,96 = 180\,000(1-0,15)^n$ $(1-0,15)^n = \frac{79866,96}{180\,000}$ $\log\left(\frac{79866,96}{180\,000}\right)$ $n = \frac{\log(1-0,15)}{\log\left(\frac{79866,96}{180\,000}\right)}$ $n = 4,999\dots$ $n \approx 5 \text{ years}$	✓ substitution ✓ use of logs ✓ answer	(3)
6.1.2	$A = P(1+i)^n$ $= 49\,000\left(1+\frac{0,1}{4}\right)^{20}$ $= R80\,292,21$ The money will be enough to buy the car. <i>Die geld sal genoeg wees om die motor te koop.</i>	✓ values of i and n ✓ substitution ✓ conclusion (consistent with answer)	(3)
6.2.1	$P = \frac{x[1-(1+i)^{-n}]}{i}$ $7853,15 = \frac{x\left[1-\left(1+\frac{0,1025}{12}\right)^{-234}\right]}{\frac{0,1025}{12}}$ $P = R793\,749,25$ OR/OF Balance Outstanding / Uitstaande balans $= 800\,000\left(1+\frac{0,1025}{12}\right)^6 - \frac{7853,15\left[1-\left(1+\frac{0,1025}{12}\right)^6\right]}{\frac{0,1025}{12}}$ $= 841\,885,56 - 48\,136,62$ $= R793\,748,94$	✓ $n = 234$ ✓ $i = \frac{0,1025}{12}$ ✓ substitution in present value formula ✓ answer OR/OF ✓ $n = 6$ in both ✓ $i = \frac{0,1025}{12}$ ✓ A – F ✓ R793 748,94	(4)

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6.2.2	$A = P(1+i)^n$ $= 793749,25\left(1+\frac{0,1025}{12}\right)^3$ $= R814\,263,3052$ New instalment/Nuwe paaiement: $P = \frac{x[1-(1+i)^{-n}]}{i}$ $814263,3052 = \frac{x\left[1-\left(1+\frac{0,1025}{12}\right)^{-231}\right]}{\frac{0,1025}{12}}$ $x = R8\,089,20$	✓ $793749,25\left(1+\frac{0,1025}{12}\right)^3$ ✓ $n = 231$ ✓ substitution of new P ✓ substitution of n and i into formula ✓ answer	(5) [15]
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QUESTION/VRAAG 7

7.1	$f(x) = x^2 + 2$ $f(x+h) = (x+h)^2 + 2$ $= x^2 + 2xh + h^2 + 2$ $f(x+h) - f(x) = x^2 + 2xh + h^2 + 2 - (x^2 + 2)$ $= 2xh + h^2$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{2xh + h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(2x + h)}{h}$ $= \lim_{h \rightarrow 0} (2x + h)$ $= 2x$ OR/OF $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 2 - (x^2 + 2)}{h}$ $= \lim_{h \rightarrow 0} \frac{2xh + h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(2x + h)}{h}$ $= \lim_{h \rightarrow 0} (2x + h)$ $= 2x$	✓ $x^2 + 2xh + h^2 + 2$ ✓ $\lim_{h \rightarrow 0} \frac{2xh + h^2}{h}$ ✓ $\lim_{h \rightarrow 0} \frac{h(2x + h)}{h}$ ✓ answer OR/OF ✓ $x^2 + 2xh + h^2 + 2$ ✓ $\lim_{h \rightarrow 0} \frac{2xh + h^2}{h}$ ✓ $\lim_{h \rightarrow 0} \frac{h(2x + h)}{h}$	(4)
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7.2.1	$y = 4x^3 + 2x^{-1}$ $\frac{dy}{dx} = 12x^2 - 2x^{-2}$	✓ $+2x^{-1}$ ✓ $12x^2$ ✓ $-2x^{-2}$	(3)
7.2.2	$y = 4\sqrt[3]{x} + (3x^3)^2$ $= 4x^{\frac{1}{3}} + 9x^6$ $\frac{dy}{dx} = \frac{4}{3}x^{-\frac{2}{3}} + 54x^5$	✓ $4x^{\frac{1}{3}}$ ✓ $9x^6$ ✓ $\frac{4}{3}x^{-\frac{2}{3}}$ ✓ $54x^5$	(4)
7.3	Point of contact: (1; 5) $m = 2$ $y - y_1 = m(x - x_1)$ or $y = 2x + c$ $y - 5 = 2(x - 1)$ $5 = 2 + c$ $c = 3$ $y = 2x + 3$ $y = 2x + 3$	✓ $m = 2$ ✓ substitution of (1; 5) ✓ answer	(3) [14]

QUESTION/VRAAG 8

8.1	$h(x) = -2\left(x + \frac{3}{2}\right)(x-1)(x+3)$ $h(x) = -(2x+3)(x^2+2x-3)$ $h(x) = -2x^3 - 7x^2 + 9$ OR/OF $h(x) = -(2x+3)(x-1)(x+3)$ $h(x) = -(2x+3)(x^2+2x-3)$ $h(x) = -2x^3 - 7x^2 + 9$	$\checkmark \checkmark -2\left(x + \frac{3}{2}\right)(x-1)(x+3)$ $\checkmark$ correct simplification (3) OR/OF $\checkmark \checkmark -(2x+3)(x-1)(x+3)$ $\checkmark$ correct simplification (3)
8.2	$h'(x) = -6x^2 - 14x$ $-6x^2 - 14x = 0$ $-2x(3x + 7) = 0$ $x = 0 \text{ or } x = -\frac{7}{3}$	$\checkmark$ first derivative $\checkmark = 0$ $\checkmark$ both answers (3)
8.3	$x < -\frac{7}{3} \text{ or } x > 0$ OR/OF $x \in \left(-\infty; -\frac{7}{3}\right) \cup (0; \infty)$	$\checkmark \checkmark$ answer (2) OR/OF $\checkmark \checkmark$ answer (2)

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8.4	$y = 4x + 7$ $-6x^2 - 14x = 4$ $0 = 6x^2 + 14x + 4$ $0 = 3x^2 + 7x + 2$ $0 = (3x+1)(x+2)$ $x = -\frac{1}{3} \text{ or } x = -2$	✓ $y = 4x + 7$ ✓ $h'(x) = 4$ ✓ standard form ✓ both answers	(4) [12]
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QUESTION/VRAAG 9

9.1	Volume of Sphere $= \frac{4}{3}\pi(8)^3 \text{ or } = \frac{2048\pi}{3} \text{ or } = 2144,66$	✓ answer	(1)
9.2	$r^2 + x^2 = 8^2 \text{ (Pythagoras)}$ $r^2 = 64 - x^2$	✓ substitution or reason Pythagoras	(1)
9.3	$V_{\text{cone}} = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi(64 - x^2)(8 + x)$ $= \frac{\pi}{3}(512 + 64x - 8x^2 - x^3)$ $\frac{dV}{dx} = \frac{64\pi}{3} - \frac{16\pi}{3}x - \frac{3\pi}{3}x^2$ $0 = 64 - 16x - 3x^2$ $0 = (8 - 3x)(x + 8)$ $x = \frac{8}{3} \text{ or } x = -8$ $\frac{V_{\text{cone}}}{V_{\text{sphere}}} = \frac{\frac{1}{3}\pi\left(\frac{512}{9}\right)\left(\frac{32}{3}\right)}{\frac{2048\pi}{3}}$ $= \frac{8}{27} = 0,3$	✓ $h = 8 + x$ ✓ $\frac{1}{3}\pi(64 - x^2)(8 + x)$ ✓ expansion ✓ $\frac{dV}{dx} = \frac{64\pi}{3} - \frac{16\pi}{3}x - \frac{3\pi}{3}x^2$ ✓ $x = \frac{8}{3}$ ✓ volume of the cone ✓ $\frac{8}{27}$ or 0,3	(7) [9]

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QUESTION/VRAAG 10

10.1	$P(\text{One Red and One Blue})$ $= P(\text{Red, Blue}) + P(\text{Blue, Red})$ $= \left(\frac{3}{12} \times \frac{2}{11}\right) + \left(\frac{2}{12} \times \frac{3}{11}\right)$ $= \frac{1}{11}$	✓ $\left(\frac{3}{12}\right) \times \left(\frac{2}{11}\right)$ ✓ $\left(\frac{2}{12}\right) \times \left(\frac{3}{11}\right)$ ✓ addition of products ✓ answer (4)
10.2.1	$a = 0,48 \times 250$ $a = 120$	✓ answer (1)
10.2.2	$b = 150$ $P(S) \times P(F)$ $= \frac{200}{250} \times \frac{150}{250}$ $= 0,48$ $= P(S \text{ and } F)$ These events are independent / Hierdie gebeurtenisse is onafhanklik	✓ b ✓ $P(S) \times P(F)$ ✓ $\frac{200}{250}$ and $\frac{150}{250}$ ✓ conclusion (with realistic probabilities) (4)
		[9]

QUESTION/VRAAG 11

11.1	10×9 $= 90$	✓✓ 10×9 (2)
11.2.1	$10!$ $= 3\,628\,800$	✓ $10!$ (1)
11.2.2	$2! \times 2! \times 2! \times 2! \times 2! \times 4!$ $= 768$	✓ $2! \times 2! \times 2! \times 2! \times 2!$ ✓ $4!$ ✓ $2! \times 2! \times 2! \times 2! \times 2! \times 4!$ or 768 (3)
		[6]

TOTAL/TOTAAL: 150

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NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.
- As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.
- Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.
- Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.

GEOMETRY • MEETKUNDE	
S	A mark for a correct statement (A statement mark is independent of a reason) 'n Punt vir 'n korrekte bewering ('n Punt vir 'n bewering is onafhanklik van die rede)
R	A mark for the correct reason (A reason mark may only be awarded if the statement is correct) 'n Punt vir 'n korrekte rede ('n Punt word slegs vir die rede toegeken as die bewering korrek is)
S/R	Award a mark if statement AND reason are both correct Ken 'n punt toe as die bewering EN rede beide korrek is

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QUESTION/VRAAG 1

1.1	45 children	✓ answer																								
1.2	$\bar{x} = \frac{\sum fx}{n} = \frac{(4 \times 2) + (8 \times 10) + (12 \times 9) + (16 \times 7) + (20 \times 8) + (24 \times 7) + (28 \times 2)}{45}$ $\bar{x} = \frac{692}{45}$ OR $\bar{x} = 15,38$ minutes	✓ 692 ✓ answer																								
1.3	<table border="1"> <thead> <tr> <th>Time taken (t) (in minutes)</th><th>Number of children</th><th>Cumulative frequency</th></tr> </thead> <tbody> <tr> <td>$2 < t \leq 6$</td><td>2</td><td>2</td></tr> <tr> <td>$6 < t \leq 10$</td><td>10</td><td>12</td></tr> <tr> <td>$10 < t \leq 14$</td><td>9</td><td>21</td></tr> <tr> <td>$14 < t \leq 18$</td><td>7</td><td>28</td></tr> <tr> <td>$18 < t \leq 22$</td><td>8</td><td>36</td></tr> <tr> <td>$22 < t \leq 26$</td><td>7</td><td>43</td></tr> <tr> <td>$26 < t \leq 30$</td><td>2</td><td>45</td></tr> </tbody> </table>	Time taken (t) (in minutes)	Number of children	Cumulative frequency	$2 < t \leq 6$	2	2	$6 < t \leq 10$	10	12	$10 < t \leq 14$	9	21	$14 < t \leq 18$	7	28	$18 < t \leq 22$	8	36	$22 < t \leq 26$	7	43	$26 < t \leq 30$	2	45	✓ first 4 cum freq correct ✓ last 3 cum freq correct
Time taken (t) (in minutes)	Number of children	Cumulative frequency																								
$2 < t \leq 6$	2	2																								
$6 < t \leq 10$	10	12																								
$10 < t \leq 14$	9	21																								
$14 < t \leq 18$	7	28																								
$18 < t \leq 22$	8	36																								
$22 < t \leq 26$	7	43																								
$26 < t \leq 30$	2	45																								
1.4	CUMULATIVE FREQUENCY GRAPH (OGIVE)	✓ plotting cum freq at upper limits correctly (all points) ✓ shape (smooth) ✓ grounding (2;0)																								
1.5	On graph at the y-value of 22,5 or 23 Median = 15 minutes.	✓ graph ✓ answer																								
		[10]																								

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MATHEMATICS P2/
WISKUNDE V2

MARKING GUIDELINES/NASIENRIGLYNE

2019

MARKS: 150
PUNTE: 150

These marking guidelines consist of 20 pages.
Hierdie nasienriglyne bestaan uit 20 bladsye..

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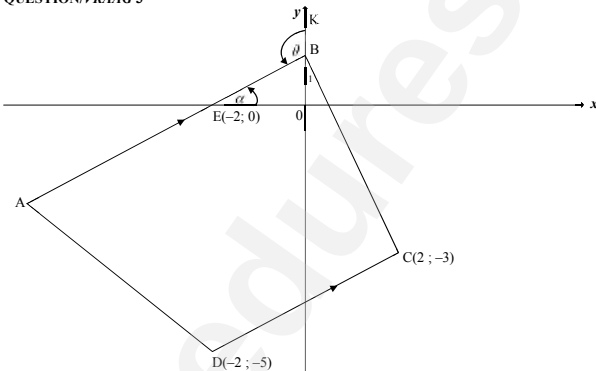
QUESTION/VRAAG 2

2.1	$a = 12,44$ $b = 0,98$ $y = 12,44 + 0,98x$ Answer only: full marks	✓ value of a ✓ value of b ✓ equation (3)
2.2.1	Percentage = $\frac{15}{50} \times 100$ $= 30\%$	✓ answer (1)
2.2.2	$\hat{y} = 12,44 + 0,98x$ $\hat{y} = 12,44 + 0,98(30)$ $\hat{y} = 41,84$ $= 42$ OR $\hat{y} = 41,87$ (if using calculator) $\hat{y} = 42$ OR $\hat{y} = \frac{21}{50}$ Answer only: full marks	✓ substitution of 30 ✓ answer as integer ✓ value of y ✓ answer as integer (2) (2)
2.3.1	standard deviation = 13,88	✓ ✓ answer (2)
2.3.2	$x = 50,67 - 45,67$ $= 5\%$ Answer only: full marks	✓ 50,67 – 45,67 ✓ answer (2)
		[10]

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QUESTION/VRAAG 3



3.1.1	Midpoint of EC: $= \left(\frac{-2+2}{2} ; \frac{0+(-3)}{2} \right) = \left(0 ; -\frac{3}{2} \right)$	✓ x value ✓ y value	(2)
3.1.2	$m_{DC} = \frac{-3-(-5)}{2-(-2)} \text{ OR } \frac{-5-(-3)}{-2-2}$ $= \frac{2}{4} = \frac{1}{2}$ Answer only: full marks	✓ substitution ✓ answer	(2)
3.1.3	$m_{AB} = \frac{1}{2}$ [AB DC] $y = \frac{1}{2}x + c$ $0 = \frac{1}{2}(-2) + c$ OR $y - y_1 = \frac{1}{2}(x - x_1)$ $c = 1$ $\therefore y = \frac{1}{2}x + 1$	$\checkmark m_{AB} = \frac{1}{2}$ $\checkmark$ substitution of $(-2; 0)$ $\checkmark$ equation	(3)
3.1.4	$\tan \alpha = m_{AB} = \frac{1}{2}$ $\alpha = 26,57^\circ$ $\theta = 90^\circ + 26,57^\circ$ [ext $\angle$ of Δ] $= 116,57^\circ$	$\checkmark \tan \alpha = \frac{1}{2}$ $\checkmark$ value of α $\checkmark$ value of θ	

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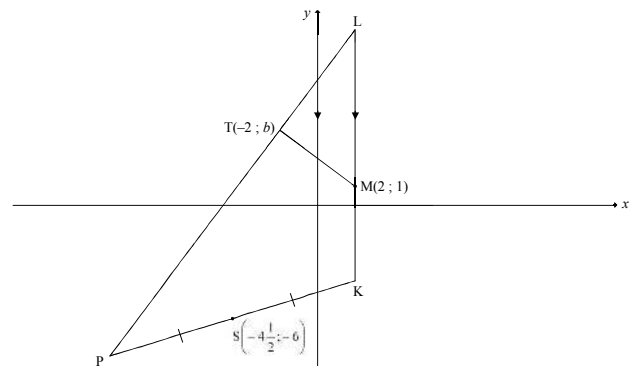
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3.2	$B(0; 1)$ $m_{BC} = \frac{1-(-3)}{0-2}$ OR $m_{BC} = \frac{(-3)-1}{2-0}$ $= -2$ $= -2$ $m_{AB} \times m_{BC} = \frac{1}{2} \times -2$ $= -1$ $\therefore AB \perp BC$	✓ coordinates of B ✓ $m_{BC} = -2$ ✓ product of gradients $= -1$	(3)
3.3.1	$\angle B = 90^\circ$ $\therefore EC$ is diameter [converse: $\angle$ in semi circle] $\therefore$ centre of circle $= \left(0; -\frac{3}{2}\right)$	✓ answer	(1)
3.3.2	$(x-0)^2 + \left(y + \frac{3}{2}\right)^2 = r^2$ $(-2-0)^2 + \left(0 + \frac{3}{2}\right)^2 = r^2$ OR $(2-0)^2 + \left(-3 - \left(-\frac{3}{2}\right)\right)^2 = r^2$ OR $(0-0)^2 + \left(1 - \left(-\frac{3}{2}\right)\right)^2 = r^2$ OR $r = \frac{EC}{2} = \frac{\sqrt{(-2-2)^2 + (0-(-3))^2}}{2}$ OR $r = 1 - \left(-\frac{3}{2}\right)$ $\therefore r^2 = \frac{25}{4}$ or $r = \frac{5}{2}$ $x^2 + \left(y + \frac{3}{2}\right)^2 = \frac{25}{4}$	✓ substitution of centre ✓ correct substitution of $E(-1; 0)$, $B(0; 1)$ or $C(2; -3)$ to calculate r^2 or r ✓ value of r^2 or r ✓ equation	(4)

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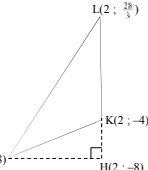
QUESTION/VRAAG 4



4.1	$(x-2)^2 + (y-1)^2 = 25$ $(-2-2)^2 + (b-1)^2 = 25$ $(b-1)^2 = 9$ OF $16 + b^2 - 2b + 1 = 25$ $b-1 = \pm 3$ $\therefore b=4$ or $b \neq -2$	$(x-2)^2 + (y-1)^2 = 25$ $(-2-2)^2 + (b-1)^2 = 25$ $b^2 - 2b - 8 = 0$ $\therefore b=4$ or $b \neq -2$	✓ equation of the circle ✓ substitution of point T ✓ simplification ✓ answer	(4)
4.2.1	$K(2; -1-5)$ $\therefore K(2; -4)$	Answer only: full marks	✓ x value ✓ y value	(2)
4.2.2	$m_{MT} = \frac{4-1}{-2-2} = -\frac{3}{4}$ $m_{PL} = \frac{4}{3}$ [radius $\perp$ tangent] $y = \frac{4}{3}x + c$ $4 = \frac{4}{3}(-2) + c$ $c = \frac{20}{3}$ $y = \frac{4}{3}x + \frac{20}{3}$	$m_{MT} = -\frac{3}{4}$ $m_{PL} = \frac{4}{3}$	✓ substitution of m_{PL} and the point T ✓ equation	(4)

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	OR $m_{MT} = \frac{4-1}{-2-2} = -\frac{3}{4}$ $m_{PL} = \frac{4}{3} \quad [\text{radius} \perp \text{tangent}]$ $y - y_1 = \frac{4}{3}(x - x_1)$ $y - 4 = \frac{4}{3}(x + 2)$ $y = \frac{4}{3}x + \frac{20}{3}$ OR P(-11; -8) $m_{PL} = \frac{4 - (-8)}{-2 - (-11)}$ $= \frac{4}{3}$ $y = \frac{4}{3}x + c$ $-8 = \frac{4}{3}(-11) + c$ $c = \frac{20}{3}$ $y = \frac{4}{3}x + \frac{20}{3}$	✓ m_{MT} ✓ $m_{PL} = \frac{4}{3}$ ✓ substitution of m_{PL} and the point T ✓ equation (4) ✓ coordinates of P ✓ $m_{PL} = \frac{4}{3}$ ✓ substitution of m_{PL} and the point P or T ✓ equation (4)
4.2.3	$y_L = \frac{4}{3}(2) + \frac{20}{3} = \frac{28}{3}$ L(2; $\frac{28}{3}$) and K(2; -4): LK = $\frac{28}{3} - (-4) = \frac{40}{3}$ <u>Coordinates of P:</u> $\frac{x+2}{2} = -4 \frac{1}{2} \quad \text{and} \quad \frac{y-4}{2} = -6$ $\therefore x = -11 \quad y = -8$ $\therefore P(-11; -8)$ $\perp$ height (PH) = $2 - (-11) = 13$ Area $\Delta PKL = \frac{1}{2}(LK)(PH)$ $= \frac{1}{2}\left(\frac{40}{3}\right)(13)$ $= \frac{260}{3}$ OR 86,67 square units 	✓ $y_L = \frac{28}{3}$ ✓ length of LK ✓ x_P ✓ y_P ✓ length of $\perp$ height ✓ substitution into the area formula ✓ answer (7)

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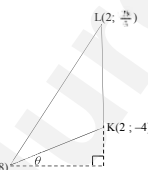
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QUESTION/VRAAG 5

5.1.1	$\sin 191^\circ$ $= -\sin 11^\circ$	$\checkmark -\sin 11^\circ$ (1)
5.1.2	$\cos 22^\circ$ $= \cos(2 \times 11^\circ)$ $= 1 - 2\sin^2 11^\circ$	$\checkmark$ answer (1)
5.2	$\cos(x - 180^\circ) + \sqrt{2} \sin(x + 45^\circ)$ $= -\cos x + \sqrt{2}(\sin x \cos 45^\circ + \cos x \sin 45^\circ)$ $= -\cos x + \sqrt{2}\left(\sin x \left(\frac{1}{\sqrt{2}}\right) + \cos x \left(\frac{1}{\sqrt{2}}\right)\right)$ $= -\cos x + \sin x + \cos x$ $= \sin x$ OR $\cos(x - 180^\circ) + \sqrt{2} \sin(x + 45^\circ)$ $= -\cos x + \sqrt{2}(\sin x \cos 45^\circ + \cos x \sin 45^\circ)$ $= -\cos x + \sqrt{2}\left(\sin x \left(\frac{\sqrt{2}}{2}\right) + \cos x \left(\frac{\sqrt{2}}{2}\right)\right)$ $= -\cos x + \sin x + \cos x$ $= \sin x$	$\checkmark -\cos x$ ✓ expansion $\checkmark$ special angle ratios $\checkmark$ simplification of last 2 terms $\checkmark$ answer (5)
5.3	$\sin P + \sin Q = \sin P + \cos P$ $(\sin P + \cos P)^2 = \left(\frac{7}{5}\right)^2$ $\sin^2 P + 2 \sin P \cos P + \cos^2 P = \frac{49}{25}$ $2 \sin P \cos P = \frac{49}{25} - 1$ $\sin 2P = \left(\frac{49}{25} - \frac{25}{25}\right)$ $= \frac{24}{25}$	$\checkmark \sin Q = \cos P$ $\checkmark$ squaring $\checkmark$ expansion $\checkmark \sin^2 P + \cos^2 P = 1$ $\checkmark$ answer (5)

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4.2.3	OR $y_L = \frac{4}{3}(2) + \frac{20}{3} = \frac{28}{3}$ L(2; $\frac{28}{3}$) and K(2; -4): LK = $\frac{28}{3} - (-4) = \frac{40}{3}$ <u>Coordinates of P:</u> $\frac{x+2}{2} = -4 \frac{1}{2} \quad \text{and} \quad \frac{y-4}{2} = -6$ $\therefore x = -11 \quad y = -8$ $\therefore P(-11; -8)$ $PK^2 = (-11 - 2)^2 + (-8 - (-4))^2$ $PK = \sqrt{185}$ units $m_{PK} = \frac{-8 - (-4)}{-11 - 2} = \frac{4}{13}$ $\tan \theta = \frac{4}{13} \quad \therefore \theta = 17,1027...^\circ$ $\therefore \angle PKL = 90^\circ + 17,1027...^\circ = 107,1^\circ$ Area $\Delta PKL = \frac{1}{2}(PK)(LK) \sin \angle PKL$ $= \frac{1}{2}\left(\sqrt{185}\right)\left(\frac{40}{3}\right) \sin 107,1^\circ$ $= 86,67$ square units 	✓ $y_L = \frac{28}{3}$ ✓ length of LK ✓ x_P ✓ y_P ✓ $\angle PKL$ ✓ substitution into the area rule ✓ answer (7)
4.3	The centres of the two circles lie on the same vertical line $x = 2$, and the sum of the radii = 10 $n - 1 = 10$ or $1 - n = 10$ $n = 11$ or $n = -9$ Answer only: full marks	✓ correct method ✓ sum of radii = 10 ✓ $n = 11$ ✓ $n = -9$ (4)

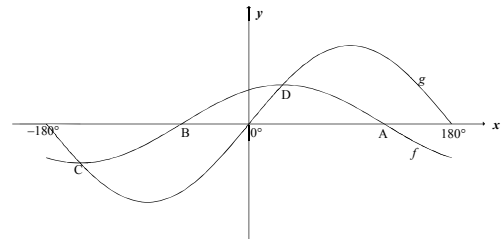
[21]

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QUESTION/VRAAG 6

6.1	$\cos(x - 30^\circ) = 2 \sin x$ $\cos x \cos 30^\circ + \sin x \sin 30^\circ = 2 \sin x$ $\frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x = 2 \sin x$ $\frac{\sqrt{3}}{2} \cos x = \frac{3}{2} \sin x$ $\tan x = \frac{\sqrt{3}}{3}$ $x = 30^\circ + k \cdot 180^\circ; \quad k \in \mathbb{Z}$ OR $x = 30^\circ + k \cdot 360^\circ$ or $x = 210^\circ + k \cdot 360^\circ; \quad k \in \mathbb{Z}$	$\checkmark$ expansion $\checkmark$ special $\angle$ s $\checkmark$ simplification $\checkmark$ equation in tan $\checkmark 30^\circ$ $\checkmark k \cdot 180^\circ; k \in \mathbb{Z}$ OR $\checkmark 30^\circ$ and 210° $\checkmark k \cdot 360^\circ; k \in \mathbb{Z}$ (6)
-----	---	--



6.2.1(a)	A(120°; 0)	$\checkmark$ answer (1)
6.2.1(b)	C(-150°; -1)	$\checkmark$ x value ✓ y value (2)
6.2.2(a)	$x \in (-90^\circ; 30^\circ)$ OR $-90^\circ < x < 30^\circ$	$\checkmark$ endpoints $\checkmark$ correct interval (2)
6.2.2(b)	$x \in (-160^\circ; 20^\circ)$ OR $-160^\circ < x < 20^\circ$	$\checkmark$ endpoints $\checkmark$ correct interval (2)
6.2.3	$y = 2 \sin x + 3$ Range of $y = 2 \sin x$: $y \in [-2; 2]$ OR $-2 \leq y \leq 2$ Range of $y = 2 \sin x + 3$: $y \in [1; 5]$ OR $1 \leq y \leq 5$ Range: $y = 2 \sin x + 3$: $y \in [2; 32]$ OR $2 \leq y \leq 32$	$\checkmark 1 \checkmark 5$ $\checkmark 2 \checkmark 32$ $\checkmark$ correct interval (5)

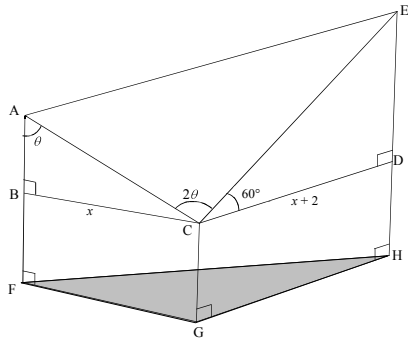
Answer only: full marks

[18]

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QUESTION/VRAAG 7



7.1.1	$\sin \theta = \frac{x}{AC}$ $AC = \frac{x}{\sin \theta}$ OR $\frac{\sin \theta}{x} = \frac{\sin 90^\circ}{AC}$ $AC = \frac{x}{\sin \theta}$	✓ trig ratio ✓ simplification (2)
7.1.2	$\cos 60^\circ = \frac{x+2}{CE}$ $CE = \frac{x+2}{\cos 60^\circ}$ $= \frac{x+2}{\frac{1}{2}} = 2(x+2)$ OR $\frac{\sin 30^\circ}{x+2} = \frac{\sin 90^\circ}{CE}$ $CE = \frac{x+2}{\sin 30^\circ}$ $= \frac{x+2}{\frac{1}{2}} = 2(x+2)$	✓ trig ratio ✓ making CE the subject (2)
7.2	$\text{Area } \triangle ACE = \frac{1}{2} AC \cdot EC \cdot \sin \hat{ACE}$ $= \frac{1}{2} \left(\frac{x}{\sin \theta} \right) (2(x+2)) \sin 2\theta$ $= \frac{x(x+2) \times 2 \sin \theta \cos \theta}{\sin \theta}$ $= 2x(x+2) \cos \theta$	✓ use area rule correctly ✓ substitution of $\frac{x}{\sin \theta} (2(x+2))$ ✓ substitution of $\sin 2\theta$ (3)

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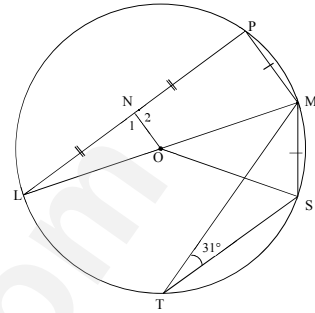
7.3	$EC = 2(12 + 2) = 28$ $AE^2 = AC^2 + EC^2 - 2(AC)(EC) \cos \hat{ACE}$ $= \left(\frac{12}{\sin 55^\circ} \right)^2 + 28^2 - 2 \left(\frac{12}{\sin 55^\circ} \right) (28) \cos 110^\circ$ $AE = 35,77m$	✓ EC ✓ use cosine rule correctly ✓ substitution ✓ answer (4)
[11]		

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QUESTION/VRAAG 8

8.1

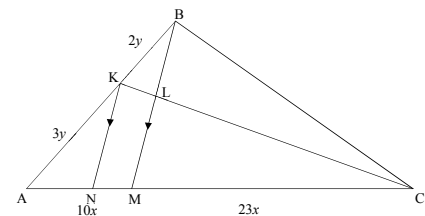


8.1.1(a)	$\hat{MOS} = 62^\circ$ [$\angle$ at centre = $2 \times \angle$ at circumf/middelpts $\angle = 2 \times$ omtreks $\angle$]	✓ S ✓ R (2)
8.1.1(b)	$\hat{L} = 31^\circ$ [equal chords; equal $\angle$ s / = koorde; = $\angle$ e]	✓ S ✓ R (2)
8.1.2	$LN = NP$ and $LO = OM$ $\therefore ON = \frac{1}{2} PM$ [midpoint theorem/middelpuntstelling] $\therefore ON = \frac{1}{2} MS$ [$PM = MS$] OR $\hat{N}_1 = 90^\circ$ [line from centre to midpt chord/lyn v midpt na midpt kd] $\hat{P} = 90^\circ$ [$\angle$ in semi-circle/ $\angle$ in halfsirkel] $\hat{L}$ is common/gemeen $\therefore \triangle NLO \parallel \triangle PLM$ ($\angle \angle \angle$) $\frac{NL}{PL} = \frac{NO}{PM} = \frac{1}{2}$ $\therefore ON = \frac{1}{2} PM$ $\therefore ON = \frac{1}{2} MS$ [$PM = MS$]	✓ $LO = OM$ ✓ S ✓ R ✓ S ✓ S R ✓ S/R ✓ S ✓ S (4)

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8.2

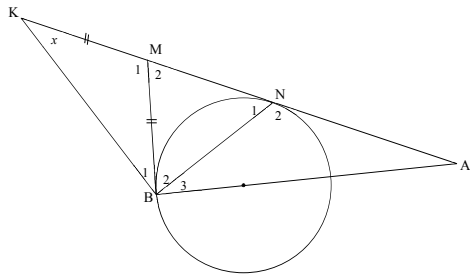


8.2.1	$\frac{AN}{AM} = \frac{AK}{AB}$ [line $\parallel$ one side of $\triangle$ OR prop theorem; $KN \parallel BM$ / $lyn \parallel$ sy van $\triangle$ OR eweredigheidst; $KN \parallel BM$] $\frac{AN}{AM} = \frac{3y}{5y} = \frac{3}{5}$	✓ R ✓ S (2)
8.2.2	$\frac{AM}{MC} = \frac{10x}{23x}$ [given] $AM = 5y = 10x \therefore y = 2x$ $\frac{LC}{KL} = \frac{MC}{NM}$ [line $\parallel$ one side of $\triangle$ OR prop theorem; $KN \parallel LM$ / $lyn \parallel$ sy van $\triangle$ OR eweredigheidst; $KN \parallel LM$] $= \frac{23x}{2y} = \frac{23x}{4x} = \frac{23}{4}$ OR $\frac{AM}{MC} = \frac{10x}{23x}$ [given] $\frac{AN}{MN} = \frac{3y}{2y} = \frac{6x}{4x}$ $\frac{LC}{KL} = \frac{MC}{NM}$ [line $\parallel$ one side of $\triangle$ OR prop theorem; $KN \parallel LM$ / $lyn \parallel$ sy van $\triangle$ OR eweredigheidst; $KN \parallel LM$] $= \frac{23x}{2y} = \frac{23x}{4x} = \frac{23}{4}$	✓ S ✓ R ✓ S ✓ S ✓ R ✓ S (3)
[13]		

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QUESTION/VRAAG 9



9.1	$\hat{B}_1 = x$ [$\angle$'s opp = sides/ $\angle$ e teenoor = sye] $M_1 = 2x$ [ext $\angle$ of Δ] OR $M_1 = 180^\circ - 2x$ [$\angle$ s of Δ] $BM = MN$ [2 tans from a common point/raaklyne vanuit dieselfde punt] $\hat{N}_1 = \frac{180^\circ - 2x}{2} = 90^\circ - x$ [$\angle$'s opp = sides/ $\angle$ e teenoor = sye] OR $NM = BM$ [2 tans from a common point/raaklyne vanuit dieselfde punt] $\hat{B}_2 = \hat{N}_1$ [$\angle$'s opp = sides/ $\angle$ e teenoor = sye] $\hat{B}_1 = x$ [$\angle$'s opp = sides/ $\angle$ e teenoor = sye] In Δ KBN: $x + x + \hat{B}_2 + \hat{N}_1 = 180^\circ$ [sum of $\angle$'s of Δ] $2x + 2\hat{N}_1 = 180^\circ$ $x + \hat{N}_1 = 90^\circ$ $\hat{N}_1 = 90^\circ - x$	$\checkmark$ S $\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R $\checkmark$ answer $\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ answer (6)
9.2	$M\hat{B}A = \hat{B}_2 + \hat{B}_3 = 90^\circ$ [tangent L diameter/raaklyyn L middellyn] $\hat{B}_3 = 90^\circ - \hat{B}_2$ $= 90^\circ - (90^\circ - x) = x$ $\hat{B}_1 = \hat{K} = x$ $\therefore$ AB is a tangent/raaklyyn converse tan-chord theorem/ omgekeerde raakl koordsr[]	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ S $\checkmark$ R (5)

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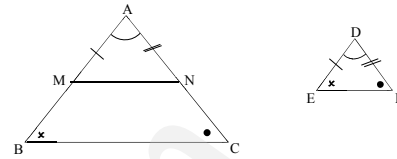
	OR $\hat{B}_2 = \hat{N}_1$ $\hat{B}_1 + \hat{B}_2 = x + (90^\circ - x) = 90^\circ$ $\therefore$ KN is diameter/middellyn [converse $\angle$ in semi-circle/ omgekeerde $\angle$ in halfsirkel] $M\hat{B}A = \hat{B}_2 + \hat{B}_3 = 90^\circ$ [tangent $\perp$ diameter] $\therefore$ AB is a tangent/raaklyn converse tan-chord theorem/ omgekeerde raak koords[1]	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R $\checkmark$ R
		(5)

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QUESTION/VRAAG 10

10.1

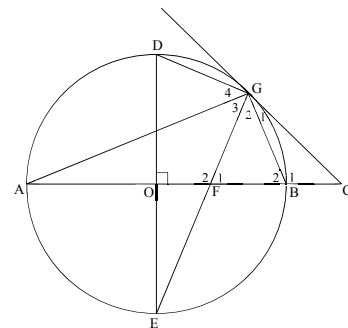


10.1	Constr: Let M and N lie on AB and AC respectively such that $AM = DE$ and $AN = DF$. Draw MN. Konst: Merk M en N op AB en AC onderscheidelijk af zodanig dat $AM = DE$ en $AN = DF$. Verbind MN. Proof: In $\triangle AMN$ and $\triangle DEF$ $AM = DE$ [Constr] $AN = DF$ [Constr] $\hat{A} = \hat{D}$ [Given] $\therefore \triangle AMN \cong \triangle DEF$ (SAS) $\therefore AMN = DEF$ $MN \parallel BC$ [corresp $\angle$'s are equal / overeenkomstige $\angle$e =] $\frac{AB}{AM} = \frac{AC}{AN}$ [line $\parallel$ one side of $\triangle OR$ prop theorem; $MN \parallel BC$] $\therefore \frac{AB}{DE} = \frac{AC}{DF}$ [$AM=DE$ and $AN=DF$]	✓ Constr / Konst ✓ $\triangle AMN \cong \triangle DEF$ ✓ SAS ✓ $MN \parallel BC$ and R ✓ $\frac{AB}{AM} = \frac{AC}{AN}$ ✓ R
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10.2



10.2.1(a)	$\hat{D}\hat{O}B=90^\circ$ $\hat{D}\hat{G}F=\hat{G}_3+\hat{G}_4=90^\circ$ [$\angle$ in semi-circle/ $\angle$ in <i>halsfirkel</i>] $\hat{D}\hat{O}B+\hat{D}\hat{G}F=180^\circ$ $\therefore$ DGFO is a cyclic quad. [converse: opp $\angle$ s of cyclic quad/ <i>omgekeerde teenoort $\angle$e v koordevh</i>] OR $\angle$ s of quad = 180° <i>$\angle$e v koordevh = 180°</i>	$\checkmark$ S $\checkmark$ R $\checkmark$ R	(3)
	OR $\hat{E}\hat{O}B=90^\circ$ $\hat{D}\hat{G}F=\hat{G}_3+\hat{G}_4=90^\circ$ [$\angle$ in semi-circle/ $\angle$ in <i>halsfirkel</i>] $\hat{E}\hat{O}B = \hat{D}\hat{G}F$ $\therefore$ DGFO is a cyclic quad. [converse: ext $\angle$ = opp int $\angle$ / <i>omgekeerde buite $\angle$ = teenoort $\angle$</i>] OR ext $\angle$ of quad = opp int $\angle$ / <i>buite $\angle$ v v h = teenoort $\angle$</i>	$\checkmark$ S $\checkmark$ R $\checkmark$ R	(3)
10.2.1(b)	$\hat{F}_1 = \hat{D}$ $\hat{G}_1 + \hat{G}_2 = \hat{D}$ [ext $\angle$ of cyclic quad/buite $\angle$ v koordevh] $\therefore \hat{F}_1 = \hat{G}_1 + \hat{G}_2$ [tan-chord theorem/taakl koordst] $\therefore GC = CF$ [sides opp equal $\angle$ s/sye teenoor = $\angle$ e]	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R	(5)

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10.2.2(a)	$AB = DE = 14$ $\therefore OB = 7$ units $\therefore BC = OC - OB = 11 - 7$ $= 4$ units	[diameters/middellynne] Answer only: full marks	✓ S ✓ S ✓ S	(3)
10.2.2(b)	In $\triangle CGB$ and $\triangle CAG$ $\hat{G}_1 = \hat{A} = x$ [tan-chord theorem/raakl koordst] $\hat{C} = \hat{C}$ [common] $\triangle CGB \parallel \triangle CAG$ [$\angle, \angle, \angle$] $\frac{CG}{CA} = \frac{CB}{CG}$ $\frac{CG}{18} = \frac{4}{CG}$ $CG^2 = 72$ $CG = \sqrt{72}$ or $6\sqrt{2}$ or 8,49 units		✓ S/R ✓ S ✓ S ✓ CA = 18 ✓ answer	(5)
10.2.2(c)	$OF = OC - FC$ $= 11 - \sqrt{72}$ $\tan E = \frac{OF}{OE}$ $= \frac{11 - \sqrt{72}}{7} = 0,36$ $\hat{E} = 19,76^\circ$ OR $OF = OC - FC$ $= 11 - \sqrt{72}$ $FE^2 = OE^2 + OF^2$ $= 7^2 + (11 - \sqrt{72})^2$ $FE = 7,437.. = 7,44$ $\cos E = \frac{OE}{FE}$ OR $\sin E = \frac{OF}{FE}$ $= \frac{7}{7,44} = 0,94$ OR $= \frac{11 - \sqrt{72}}{7,44} = 0,338$ $\hat{E} = 19,76^\circ$		✓ OF ✓ trig ratio ✓ substitution ✓ answer ✓ OF ✓ trig ratio ✓ substitution ✓ answer	(4)
				[26]
				TOTAL/TOTAAL: 150