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NoteSheet on Rate of Change

When we calculate a derivative we are calculating a rate of change. The gradient of a graph is a measure of the **rate** at which **y changes** as **x changes**.

If for example we plotted the volume of water (in litre) in a leaking tank against time (in minutes) then the derivative (gradient) would give us the instantaneous **rate** at which volume is changing. The answer would be in litres / minute.

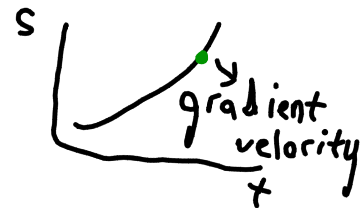
If the height of a stone (H) is a function of time (t) then calculating the derivative $\frac{dH}{dt}$ will give the rate at which the height is changing with respect to time – the speed of the stone through the air.

The most common application of this is in the study of speed, distance and acceleration.

If we have a graph of distance (displacement) vs time then the gradient (derivative) would give us the instantaneous rate of change of distance with respect to time. This would be the speed (velocity). If the displacement is measured in meters and the time in seconds then the velocity would be in meters / second or $m.s^{-1}$

Those of you who do Science know that $s = ut + \frac{1}{2}at^2$

Differentiating this with respect to time gives us $\frac{ds}{dt} = u + at$



This should be familiar to us as our formula for velocity!

Because physicists so often differentiate with respect to time they use $\dot{s}$ for $\frac{ds}{dt}$

Remember that if we use two points on a graph then we are finding the **average rate of change**. In this case we would be finding the average **velocity**.

If we have a graph of velocity vs time then the gradient will be the instantaneous rate of change of velocity with respect to time. This would be the acceleration. If the units are as above then the acceleration would be in meters / second / second or $m.s^{-2}$.

Given $v = u + at$

Differentiating this with respect to time gives us $\frac{dv}{dt} = a$

This is also $\frac{d^2s}{dt^2}$ or $\ddot{s}$

You should not use this dot notation in your exams!

You should also not use your formulae from physical science.

All you need to remember is:

Differentiating distance (displacement) gives speed (velocity).

Differentiating speed (velocity) gives acceleration.

1. The height of a ^{ball}~~stone~~ (in m), thrown vertically up, after t seconds, is given by $h = 15t - 5t^2$. Find the maximum height the ball reaches and the time it takes to reach that height.

At top ^{the} speed = 0

$$\frac{dh}{dt} = 15 - 10t = 0$$

$$15 = 10t$$

$$t = 1,5 \text{ seconds}$$

$$h = 15(1,5) - 5(1,5^2)$$

$$h_{\text{MAX}} = 11,5 \text{ m}$$

2. A stone is thrown vertically upwards. Its height (in meters) at time t (in seconds) is given by $s = 30t - 5t^2$. Determine its:

- a) Average velocity in the first second (use 2 pts.)
- When $t = 0$, $s = 0 \text{ m}$
- When $t = 1$, $s = 25 \text{ m}$
- Avg. speed = $\frac{25-0}{1-0} = 25 \text{ m/s}$
- b) Instantaneous velocity after 1.5 seconds

$$v = \frac{ds}{dt} = 30 - 10t$$

When $t = 1,5$

$$\frac{ds}{dt} = 30 - 10(1,5)$$

$$= 15 \text{ m.s}^{-1}$$

Velocity is 15 m/s or 15 m/s

3. During an experiment, the concentration of glucose in the human body, $M(t)$ varied according to the formula: $M(t) = 4.5 - 0.03t^2$ where t is measured in hours and the concentration of glucose is measured in milligrams (mg).

- a) Determine the average rate of change of glucose concentration in the body over the interval between $t=0$ to $t=3$ hours, giving the correct units for your answer.

$$\begin{aligned}
 M(0) &= 4.5 \text{ mg} \\
 M(3) &= 4.5 - 0.03(3^2) = 4.23 \text{ mg} \\
 \text{Avg} &= \frac{4.5 - 4.23}{0 - 3} = -0.09 \text{ mg/hr}
 \end{aligned}$$

Handwritten notes: $(0; 4\frac{1}{2})$ and $(3; 4.23)$ with arrows pointing to the corresponding values in the calculation.

- b) Determine the instantaneous rate of change of glucose concentration when $t=1$, giving the correct units for your answer.

$$\begin{aligned}
 M'(t) &= -0.06t \\
 M'(1) &= -0.06 \text{ mg} \cdot \text{hr}^{-1}
 \end{aligned}$$

- c) Did the concentration of glucose increase or decrease during the first three hours? (1)

decreased !!

4. A particle covers a distance s metre according to the formula

$$s = \frac{1}{3}t^3 + \frac{1}{2}t^2 - 2t \quad (t \text{ in seconds}).$$

- a) Give an expression for the velocity v in terms of t . $v = \frac{ds}{dt} = t^2 + t - 2$

- b) Determine the velocity after 2 seconds.

metres/sec.

$$4 \text{ m.s}^{-1} \text{ OR } 4 \frac{\text{m}}{\text{s}} = 4 \text{ m/s}$$

- c) After how many seconds will the particle be stationary?

When will $\frac{ds}{dt} = 0$?

$$t^2 + t - 2 = 0$$

$$(t+2)(t-1) = 0 \quad t = \cancel{2} \text{ or } 1$$

- d) Express the acceleration a in terms of t .

$$a = \frac{dv}{dt} \text{ or } \frac{d^2s}{dt^2} = 2t + 1$$

- e) Determine the acceleration after 3 seconds

$$2(3) + 1 = 7 \text{ m.s}^{-2} = 7 \text{ m/s}^2$$

$$= 7 \text{ m/s/s}$$

FROM LAST YEAR'S TRIALS PAPER!

~~bucket~~ ^{tank} has two pipes entering it. One is filling the tank at a variable rate while the other is draining it at a variable rate. The volume of water (in litres) in the tank at time t (in hours) with $(t \in [0;3])$ is given by

$$V = -t^3 - 2t^2 + 15t$$

- (1) What is the average rate of flow in litres/hour in the first hour?

$$\frac{12-0}{1-0} = 12 \quad (2)$$

At 0 hrs 0 litres
At 1 hr 12 litres

$\therefore$ Average rate in 1st hr = 12 l/hr

- (2) Did the volume of water increase or decrease over that time?

increase

- (3) What is the instantaneous rate of flow at 2 hours?

$$\frac{dV}{dt} = -3t^2 - 4t + 15 \quad (2)$$

When $t=2$ $\frac{dV}{dt} = -12 - 8 + 15 = -5$ l/hr

- (4) At what point in the three hour time interval was the bucket fullest?
Give your answer to the nearest minute.

$$\frac{dV}{dt} = 0$$

$$-3t^2 - 4t + 15 = 0$$

$$3t^2 + 4t - 15 = 0 \quad \therefore (3t - 5)(t + 3) = 0$$

$$3t = 5 \quad t = \frac{5}{3} \text{ hrs or } 3 \quad (1)$$

- (5) What is the maximum volume the bucket contained?
Give your answer to the nearest litre.

$$\frac{5}{3} \times 60 = 100 \text{ mins}$$

$$V = -\left(\frac{5}{3}\right)^3 - 2\left(\frac{5}{3}\right)^2 + 15\left(\frac{5}{3}\right)$$

$$= 14,8$$

$$\approx 15 \text{ litres}$$

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