eduresourceza.com

Downloaded from eduresourceza.com

MATHEMATICS

MATERIAL FOR GRADE 12

TRIGONOMETRY

MEMORANDA

eduresourceza.com

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 1

1.1	$\hat{C} = \theta$ $D\hat{A}\,C = \theta$ $\therefore A\hat{D}C = 180° - 2\theta$	✓$expression\ of\ D\hat{A}\,C = \theta$ ✓for answer (2)
1.2	$A\hat{D}C = 180° - 2\theta$ $a^2 = b^2 + b^2 - 2.b.b\cos(180° - 2\theta)$ $= 2b^2 + 2b^2.\cos 2\theta$ $= 2b^2(1 + \cos 2\theta)$ $1 + \cos 2\theta = \frac{a^2}{2b^2}$ $\therefore \cos 2\theta = \frac{a^2}{2b^2} - 1$	✓cosine rule ✓substitution ✓reduction ✓simplification (4)
1.3	$\cos 2\theta = \frac{3^2}{2(2)^2} - 1$ $= \frac{1}{8}$ $2\theta = 82{,}819°$ $\theta = 41{,}41°$	✓substitution ✓correct ref angle ✓correct value of θ (3)
		[9]

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

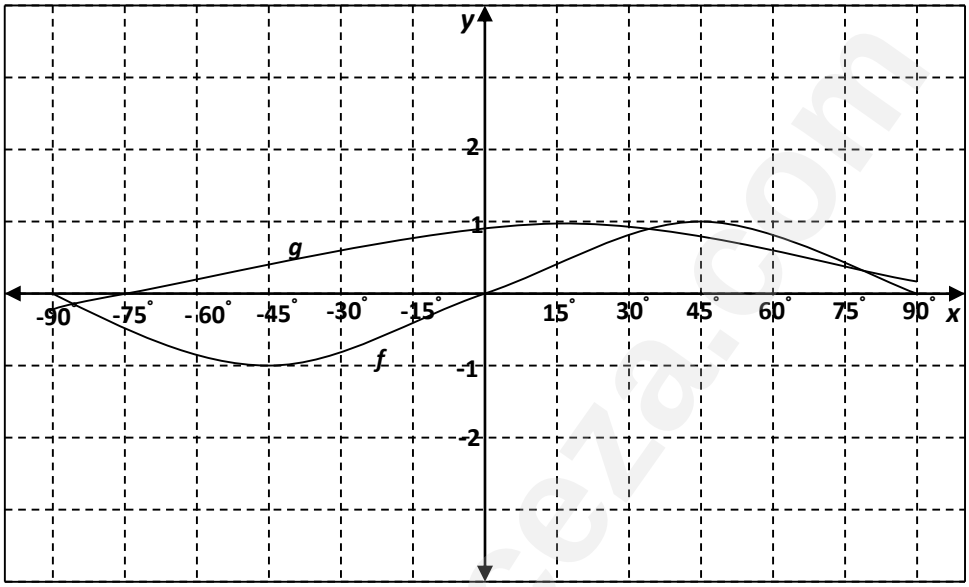
QUESTION 2

2.1	$\sin 34°\cos 26° + \cos 146° \sin(-26°)$ $= \sin 34°\cos 26° + \cos(180°-34°)(-\sin 26°)$ $= \sin 34°\cos 26° + (-\cos 34°)(-\sin 26°)$ $= \sin 34°\cos 26° + \cos 34° \sin 26°$ $= \sin(34°+26°)$ $= \sin 60°$ $= \frac{\sqrt{3}}{2}$	✓$\sin 34°$ ✓$-\cos 34°$ and $-\sin 26°$ ✓$\sin(34°+26°)$ ✓answer (4)
2.2	$\frac{\tan(180°+x)\cos(360°-x)}{\sin(x-180°)\cos(90°+x)+\cos(720°+x)\cos(-x)}$ $= \frac{\tan x.\cos x}{(-\sin x)(-\sin x)+(\cos x)(\cos x)}$ $= \frac{\frac{\sin x}{\cos x}.\cos x}{\sin^2 x+\cos^2 x}$ $= \frac{\sin x}{1}$ $= \sin x$	✓$\tan x$ and $\cos x$ ✓$-\sin x$ and $-\sin x$ ✓$\cos x \cos x$ ✓$\frac{\sin x}{\cos x}$ ✓$\sin^2 x + \cos^2 x = 1$ ✓answer (6)
2.3	LHS $= \frac{\cos 2x+\cos^2 x+3\sin^2 x}{2-2\sin^2 x}$ $= \frac{\cos^2 x-\sin^2 x+\cos^2 x+3\sin^2 x}{2-2\sin^2 x}$ $= \frac{2\cos^2 x+2\sin^2 x}{2(1-\sin^2 x)}$ $= \frac{2(\cos^2 x+\sin^2 x)}{2\cos^2 x}$ $= \frac{1}{\cos^2 x}$ $= RHS$	✓ $\cos^2 x-\sin^2 x$ ✓ $2\cos^2 x+2\sin^2 x$ ✓✓ factorising both Nr &Dn ✓ $1-\sin^2 x=\cos^2 x$ (5)
		[15]

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 3

3.1	Period of f =180°	✓180° (1)
3.2	 $g(x)$: ✓ x -intercepts ✓ y -intercepts ✓ Turning points ✓✓ End points (5)	
3.3	$\sin 2x = \cos(x-15°)$ $\cos(90° - 2x) = \cos(x-15°)$ $90° - 2x = \pm(x-15°) + k360°$ $90° - 2x = x - 15° + k.360°$ or $90° - 2x = -(x-15°) + k360°$ $-3x = -105° + k360°$ or $-x = -75° + k360°$ $(k \in z)$ $x = 35° + k120°$ or $x = 75° + k360°$ $x = -85°; 35°; 75°$	✓cos (90 -2 x) ✓ $90° - 2x = \pm(x-15°) + k360°$ ✓$x = 35° + k120°$ ✓$x = 75° + k360°$ ✓$x = -85°$ ✓$x = 35°$ ✓$x = 75°$ (7)
3.4	$-85° < x < 35°$ or $x > 75°$	✓✓Critical values ✓and notation (3)
		[16]

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 4

4.1.1	$\frac{2\sin(180°+x)\sin(90°+x)}{\cos^4x-\sin^4x}$ $= \frac{-2\sin x.\cos x}{(\cos^2x-\sin^2x)(\cos^2x+\sin^2x)}$ $= \frac{-\sin 2x}{\cos 2x.(1)}$ $= -\tan 2x$	✓$-2\sin x$ ✓$\cos x$ ✓factorisation ✓$-\sin 2x$ ✓$\cos 2x$ (5)
4.1.2	At / *by* $\cos 2x = 0$ $2x = 90°$ or /*of* $2x = 270°$ $x = 45°$ or/*of* $x = 135°$	✓$\cos 2x = 0$ ✓$2x = 90°$ or $2x = 270°$ ✓$x = 45°$ $x = 135°$ (3)
4.2	$= \frac{(\cos 13°)(-\sin 13°)}{(-\tan 45°).(\cos 64°)}$ $= \frac{\cos 13°.-\sin 13°}{-1.\cos 64°}$ $= \frac{2\times\sin 13°\cos 13°}{2\cos 64°}$ $= \frac{\sin 26°}{2\sin 26°}$ $= \frac{1}{2}$	✓$\cos 13°$ ✓$-\sin 13°$ ✓$-\tan 45°$ ✓multiply by 2 in numerator **and** denominator ✓$\frac{\sin 26°}{2\sin 26°}$ (5)
4.3	LHS: $\frac{\cos(2x+x)}{\cos x}$ $= \frac{\cos 2x.\cos x-\sin 2x.\sin x}{\cos x}$ $= \frac{\cos 2x.\cos x-2\sin x\cos x.\sin x}{\cos x}$ $= \frac{\cos x(\cos 2x-2\sin^2x)}{\cos x}$ $= \cos 2x - 1 + 1 - 2\sin^2x$ $= \cos 2x - 1 + \cos 2x$ $= 2\cos 2x - 1$ OR $\frac{\cos(2x+x)}{\cos x}$ $= \frac{\cos 2x.\cos x-\sin 2x.\sin x}{\cos x}$ $= \frac{\cos 2x.\cos x-2\sin x\cos x.\sin x}{\cos x}$	✓$\cos 2x.\cos x - \sin 2x.\sin x$ ✓replacing $\sin 2x$ ✓factorise ✓ $+1-1$ ✓replacing $1-2\sin^2x$ (5) ✓$\cos 2x.\cos x - \sin 2x.\sin x$ ✓replacing $\sin 2x$ ✓factorise ✓ replacing $\cos 2x$ ✓replacing $1-2\sin^2x$ (5)

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

	$= \frac{cosx(cos2x-2sin^2x)}{cosx}$ $= cos2x - 2sin^2x$ $= 2cos^2x - 1 - 2sin^2x$ $= 2(cos^2x - sin^2x) - 1$ $= 2cos2x - 1$	
		[18]

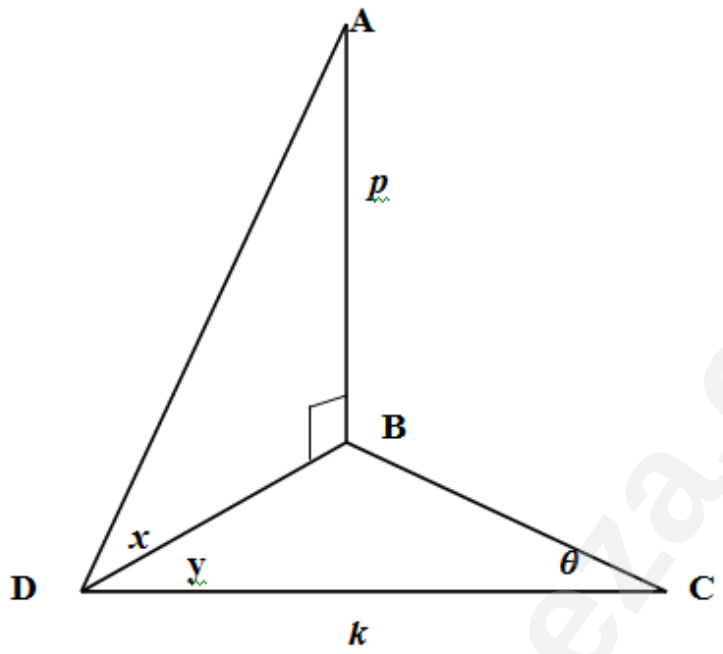
QUESTION 5

5.1	360°	✓ (1)
5.2	$sin(x+30°) = -2cosx$ $sinxcos30° + cosxsin30° = -2cosx$ $\sin x\left(\frac{\sqrt{3}}{2}\right) + \cos x\left(\frac{1}{2}\right) = -2\cos x$ $\sqrt{3}\sin x + \cos x = -4cosx$ $\sqrt{3}\sin x = -5cosx$ $\tan x = -\frac{5}{\sqrt{3}}$ $x = 180° - 70{,}89° + k.180°$ $x = 109.11° + k.180°, k \in Z$ $x = -70{,}89°$ or $x = 109{,}11°$	✓equating f and g ✓expanding $sin(x+30°)$ ✓special angle values ✓$\tan x = -\frac{5}{\sqrt{3}}$ ✓$x = 109.11° + k.180°$ ✓$x = -70{,}89°$ ✓$x = 109{,}11°$ (7)
5.3.1	$x \in [-90°; -70{,}89°] \cup [109{,}11°; 180°]$	✓✓boundaries ✓correct notation (3)
5.3.2	$x \in (-90°; -30°) \cup (90°; 150°)$	✓$(-90°; -30°)$ ✓$(90°; 150°)$ ✓correct notation (3)
		[14]

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 6

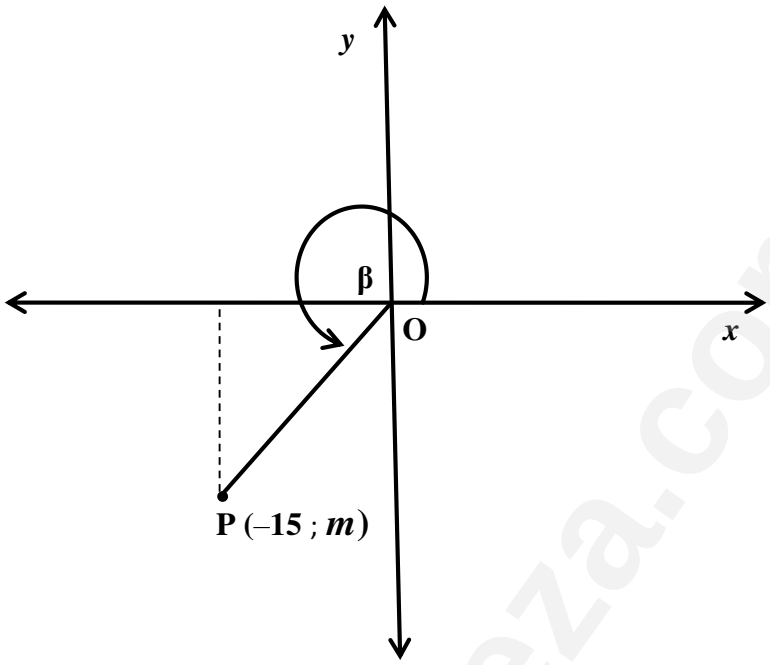


6.1.1	In $\Delta\, ABD$: $\tan x = \frac{p}{DB}$ $p = DB.\tan x$	✓$\tan x = \frac{p}{DB}$ ✓ $p = DB\tan x$ (2)
6.1.2	$\frac{DB}{\sin\theta} = \frac{k}{\sin(180-(y+\theta)}$ $DB = \frac{k.\sin\theta}{\sin(y+\theta)}$ $p = \frac{k.\sin\theta}{\sin(y+\theta)} \times \tan x$ $= \frac{k\sin\theta.\tan x}{\sin y\cos\theta+\cos y.\sin\theta}$	✓$B\hat{D}C = 180 - (y+\theta)$ ✓$\frac{DB}{\sin\theta} = \frac{k}{\sin(180-(y+\theta)}$ ✓reduction formula ✓replacing DB ✓expanding $\sin(y+\theta)$ (5)
6.2	$\tan 51{,}7° = \frac{80}{DB}$ $DB = \frac{80}{\tan 51{,}7°} = 63{,}18\ m$ $BC^2 = (63{,}18)^2 + 95^2 - 2(63{,}18)(95)\cos 62{,}5°$ $= 7473{,}789697\ldots$ $\therefore BC = 86{,}45 \approx 86\ m$	□$\tan 51{,}7° = \frac{80}{DB}$ ✓$DB = 63{,}18\ m$ ✓application of cos. ✓$86m$ (4)
		[11]

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 7

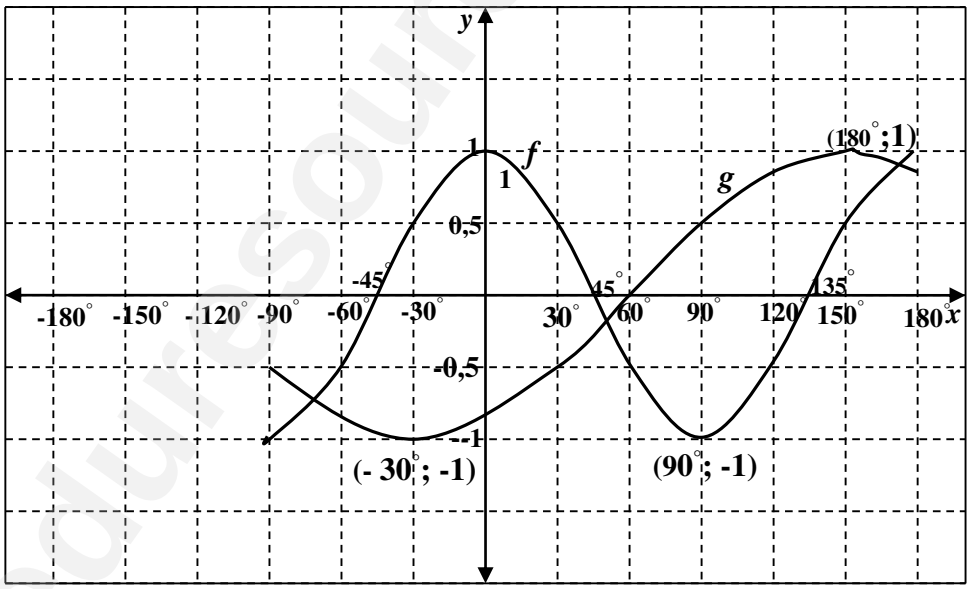
		
7.1.1	$17\cos\beta + 15 = 0$ $\cos\beta = \frac{-15}{17}$ $x^2 + y^2 = r^2$ $m^2 = 17^2 - 15^2$ $= 64$ $m = -8$	✓ $\cos\beta = \frac{-15}{17}$ ✓application of Pythagoras ✓ answer (3)
7.1.2	$\sin\beta + \tan\beta = \frac{-8}{17} + \frac{-8}{-15}$ $= \frac{16}{255}$	✓sub. into sin β ✓sub into tan β ✓answer (3)
7.1.3	$\cos 2\beta = 2\cos^2\beta - 1$ $= 2.\left(\frac{-15}{17}\right)^2 - 1$ $= \frac{161}{289}$	✓identity ✓sub. ✓answer (3)
7.2	$\frac{\sin(180^0 - x).\cos(x - 180^0).\tan(360^0 - x)}{\sin(-x).\cos(450^0 + x)}$ $= \frac{\sin x.(-\cos x).(-\tan x)}{(-\sin x).(-\sin x)}$ $= \frac{\sin x.\cos x.\frac{\sin x}{\cos x}}{\sin x.\sin x}$ $= 1$	✓$\sin x$ ✓$-\cos x$ ✓$-\tan x$ ✓$-\sin x$ ✓$-\sin x$ ✓ $\frac{\sin x}{\cos x}$ ✓answer (7)

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

7.3.1	$\text{LHS} = \dfrac{\sin x + \sin 2x}{1+\cos x+\cos 2x}$ $= \dfrac{\sin x + 2\sin x\cos x}{1+\cos x+2\cos^2 x-1}$ $= \dfrac{\sin x(1+2\cos x)}{\cos x(1+2\cos x)}$ $= \dfrac{\sin x}{\cos x}$ $= \tan x$ $=$ RHS	✓$2\sin x.\cos x$ ✓$2\cos^2 x - 1$ ✓factorise numerator ✓factorise denominator ✓$\dfrac{\sin x}{\cos x} = \tan x$ (5)
7.3.2	$\cos x = 0$ or $\cos x = -\dfrac{1}{2}$ $x = \pm 90° + k360°$ or $x = \pm 120° + k360°$; or $x = 90° + k180°$; $k \in Z$	✓ $\cos x = 0$ or $\cos x = -\dfrac{1}{2}$ ✓$\pm 90° + k180°$ ✓$\pm 120° + k360°$ ✓$k \in Z$ (4) **[25]**

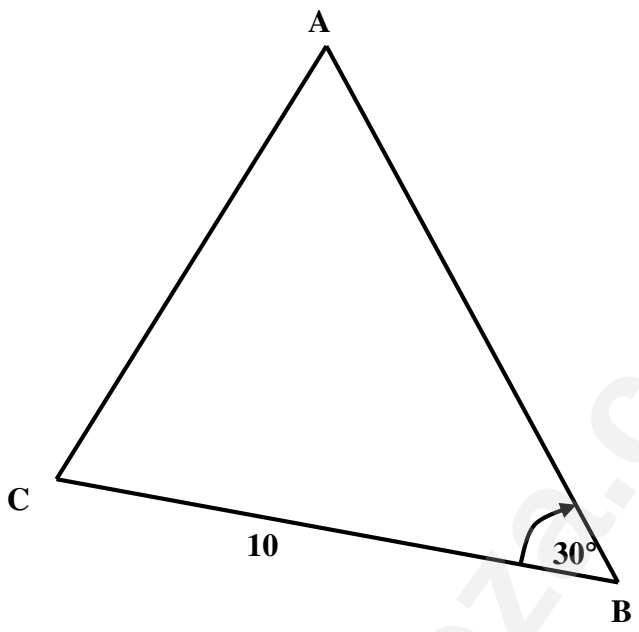
QUESTION 8

8.1	 *f*: ✓*x*-intercept ✓*y*- intercept ✓shape/TP *g*: ✓*x*-intercept ✓*y*- intercept ✓TP (6)	
8.2	$(60° ; 180°]$ or $60° < x \leq 180°$	✓✓end values ✓notation (3) **[9]**

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 9

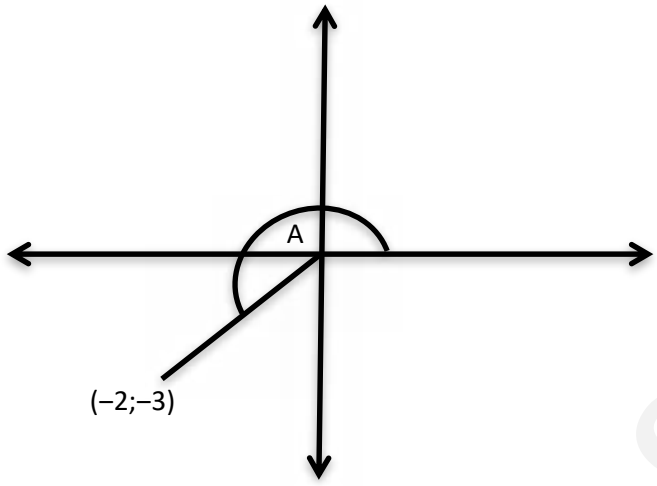


In Δ ABC,	
$\hat{A}+\hat{B}+\hat{C} = 180°$	✓ $\hat{A}+\hat{B}+\hat{C} = 180°$
sin A = sin [180° - (B + C)]	✓sub. for A
= sin (B + C)	✓reduction
= 0,8	
$\frac{AC}{\sin B} = \frac{BC}{\sin A}$	
$\frac{AC}{\sin 30^\circ} = \frac{10}{0,8}$	✓application of sine rule
$AC = 10.\frac{1}{2}.\frac{10}{8}$	
AC = $6\frac{1}{4}$ units	✓answer **[5]**

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 13

13.1.1	 $r^2 = (-2)^2 + (-3)^2$ $= 13$ $\therefore r = \sqrt{13}$ $\sin A = \frac{-2}{\sqrt{13}}$	✓ correct quadrant ✓$r = \sqrt{13}$ ✓$\frac{-2}{\sqrt{13}}$ (3)
13.1.2	$\cos 2A - \sin 2A$ $= \cos^2 A - \sin^2 A - 2\sin A\cos A$ $= \left(\frac{-3}{\sqrt{13}}\right)^2 - \left(\frac{-2}{\sqrt{13}}\right)^2 - 2\left(\frac{-2}{\sqrt{13}}\right)\left(\frac{-3}{\sqrt{13}}\right)$ $= \frac{5}{13} - \frac{12}{13}$ $= \frac{-7}{13}$	✓Expansion of cos double angles ✓ sin double ✓subs ✓Answer (4)
13.2.1	$\cos(x - 90°) = \sin x$ $= t$	✓$\sin x$ ✓t (2)
13.2.2	$\sin 2x = 2\sin x\cos x$ $= 2t\sqrt{1-t^2}$	✓$2\sin x\cos x$ ✓$2t$ ✓$\sqrt{1-t^2}$ (3)
		[12]

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

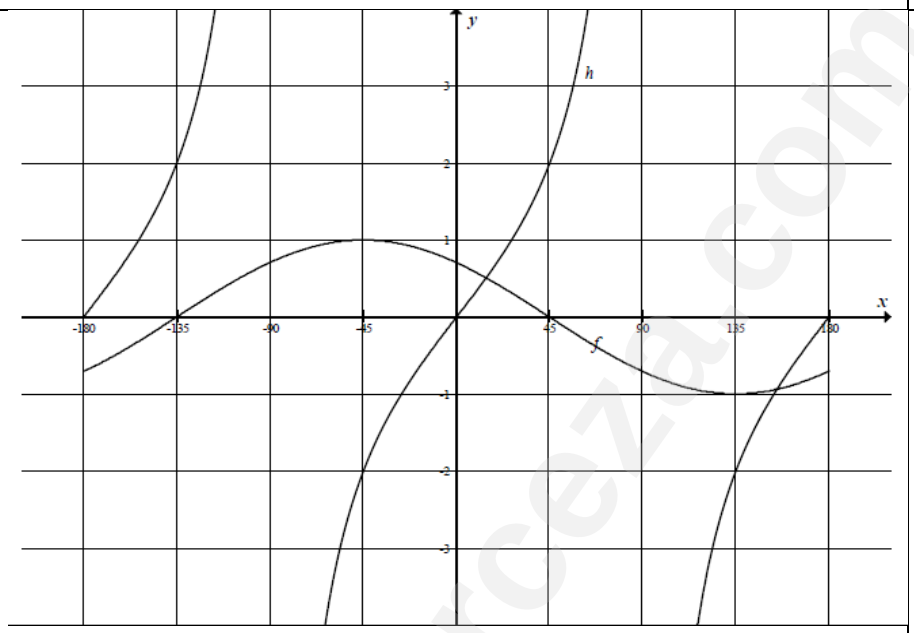
QUESTION 14

14.1	$\frac{\cos^2 208°}{\tan(118°)[\sin(124°)]}$ $= \frac{(-cos28°)^2}{(-\tan 62°)(\sin 124°)}$ $= \frac{\cos^2 28°}{\left(\frac{\sin 62°}{\cos 62°}\right)(2\sin 62°\cos 62°)}$ $= \frac{cos^2 28°}{2sin^2 62°}$ $= \frac{\cos^2 28°}{2\cos^2 28°}$ $= \frac{1}{2}$	✓$\cos^2 28°$ ✓$\sin 124°$ ✓$2\sin 62°\cos 62°$ ✓$\frac{cos^2 28°}{2sin^2 62°}$ ✓$\frac{\cos^2 28°}{2\cos^2 28°}$ ✓$\frac{1}{2}$ (6)
14.2	$1-(1-2\sin^2\theta) = 8\sin\theta(2\sin\theta\cos\theta)$ $2\sin^2\theta = 16\sin^2\theta\cos\theta$ $2sin^2\theta - 8\sin^2\theta\cos\theta = 0$ $2\sin^2\theta(1-4\cos\theta) = 0$ $\sin^2\theta = 0$ or $1-4\cos\theta = 0$ $\sin\theta \neq 0$ $-4\cos\theta = -1$ $\cos\theta = \frac{1}{4}$ $\therefore \theta = \pm 75{,}5° + k.360°$ where $k \in Z$	✓$1-2\sin^2\theta$ ✓$\sin^2\theta(1-8\cos\theta$ ✓both equations ✓$\cos\theta = \frac{1}{4}$ ✓$75{,}5° + k.360°$ ✓$-75{,}5° + k.360°$ (6)
		[12]

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 15

15.1	$a \tan 45° = 2$ $\therefore a = 2$	✓Reading from graph ✓Answer (2)
15.2		✓Amplitude ✓Shape ✓✓Turning points clearly indicated as $(-45°, 1)$ and $(135°, -1)$ (4)
15.3	2 solutions.	✓2 solutions (1)
		[7]

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 16

16.1	$QR^2 = PQ^2 + RP^2 - 2PQ.RP\cos\hat{P}$ $(\sqrt{3x})^2 = x^2 + x^2 - 2.x.x\cos\hat{P}$ $\cos\hat{P} = \frac{x^2 + x^2 - (\sqrt{3x})^2}{2x^2}$ $= \frac{-x^2}{2x^2}$ $= \frac{-1}{2}$ $\hat{P} = 120°$	✓$x^2 + x^2 - (\sqrt{3x})^2$ ✓$2x^2$ ✓Simplification ✓$\hat{P} = 120°$ (4)
16.2	$P\hat{R}Q = P\hat{Q}R = 30°$ $\hat{R}_1 = \hat{Q}_2$ $\hat{R}_2 = 150°$ [angles on str. Line] Area of $\Delta QRS = \frac{1}{2}(QR)(RS)\sin Q\hat{R}S$ $= \frac{1}{2}(\sqrt{3x})\left(\frac{3}{2}x\right)(\sin 150°)$ $= \left(\frac{3\sqrt{3}}{4}x^2\right)\left(\frac{1}{2}\right)$ $= 3\frac{\sqrt{3}}{8}x^2$ or $0{,}65x^2$	✓ S ✓ S ✓Subst into correct formulae ✓Simplification ✓Answer (5)
		[9]

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 17	
17.1.1 $\tan\alpha = \frac{-2}{2\sqrt{3}} = \frac{-1}{\sqrt{3}}$	A✓correct ratio
$\therefore \quad \alpha = 360^\circ - 30^\circ$	
$= \quad 330^\circ$	CA✓ $\alpha = 330^\circ$
$\therefore \quad \beta = \quad 30^\circ$	CA✓ $\beta = 30^\circ$
or	
$\tan(-\beta) = -\tan\beta$	A✓correct ratio
$= -(\frac{-2}{2\sqrt{3}})$	CACA✓✓ $\beta = 30^\circ$
$\tan\beta = \frac{2}{2\sqrt{3}}$	(3)
$\beta = 30^\circ$	
17.1.2 $OP^2 = (2\sqrt{3})^2 + (-2)^2 \quad = 12 + 4$	A✓ using distance for formula
$= 16$	
$\therefore OP = 4$	CA✓answer (2)
17.1.3 $\frac{OP}{OQ} = \cos\beta$	
$\therefore \quad OQ = \frac{OP}{\cos\beta} = \frac{4}{\cos 30^\circ}$	

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

$= \dfrac{4}{\frac{\sqrt{3}}{2}}$	CA✓ $\cos 30^\circ = \dfrac{\sqrt{3}}{2}$
$= \dfrac{8}{\sqrt{3}}$	CA✓ $\dfrac{8}{\sqrt{3}}$
$Q = \left(\dfrac{8\sqrt{3}}{3}\ ;\ 0\right)$	CA✓ co-ordinates
or	
$\dfrac{OP}{OQ} = \cos 30^\circ$	
$OQ = \dfrac{4}{\cos 30^\circ} = \dfrac{4}{\frac{\sqrt{3}}{2}}$	CA✓ $\cos 30^\circ = \dfrac{\sqrt{3}}{2}$
$OQ = \dfrac{8\sqrt{3}}{3}$	
$Q = \left(\dfrac{8\sqrt{3}}{3}\ ;\ 0\right)$	CA✓ $\dfrac{8\sqrt{3}}{3}$
	CA✓ co-ordinates
	(3)

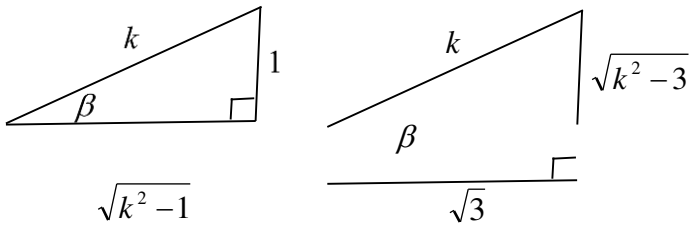
Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

17.2 $2\left[\frac{1}{2}\cos\alpha + \frac{\sqrt{3}}{2}\sin\alpha\right]$	M✓ for introducing 2
$= 2\left[\sin 30^\circ \cos\alpha + \cos 30^\circ \sin\alpha\right]$	A✓ introducing 30° special angle
$= 2\sin\left(30^\circ + \alpha\right)$	A✓ sum compound formula
$k = 2$; $\beta = 30^\circ$	A✓ calculating k
	A✓ calculating β
OR	
$\frac{1}{2}\cos\alpha + \frac{\sqrt{3}}{2}\sin\alpha = \frac{k}{2}\left(\sin\alpha + \beta\right)$	
$= \sin 30^\circ \cos\alpha + \cos 30^\circ \sin\alpha = \frac{k}{2}\sin\left(\alpha + \beta\right)$	A✓ for introducing 2
$= \sin\left(30^\circ + \alpha\right) = \frac{k}{2}\sin\left(\alpha + \beta\right)$	A✓ introducing 30° special angle
$\frac{k}{2} = 1,\ \beta = 30^\circ$	A✓ sum compound formula
$\therefore k = 2,\ \beta = 30^\circ$	A✓ calculating k
OR	A✓ calculating β
	(5)
$\cos\alpha + \sqrt{3}\sin\alpha = k\sin\left(\alpha + \beta\right)$ $\cos\alpha + \sqrt{3}\sin\alpha = k\left(\sin\alpha\cos\beta + \cos\alpha\sin\beta\right)$ $\cos\alpha + \sqrt{3}\sin\alpha = k\sin\alpha\cos\beta + k\cos\alpha\sin\beta$ $\cos\alpha + \sqrt{3}\sin\alpha = \left(k\sin\beta\right)\cos\alpha + \left(k\cos\beta\right)\sin\alpha$ $\therefore 1 = k\sin\beta$ and $\sqrt{3} = k\cos\beta$	A✓ expansion

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

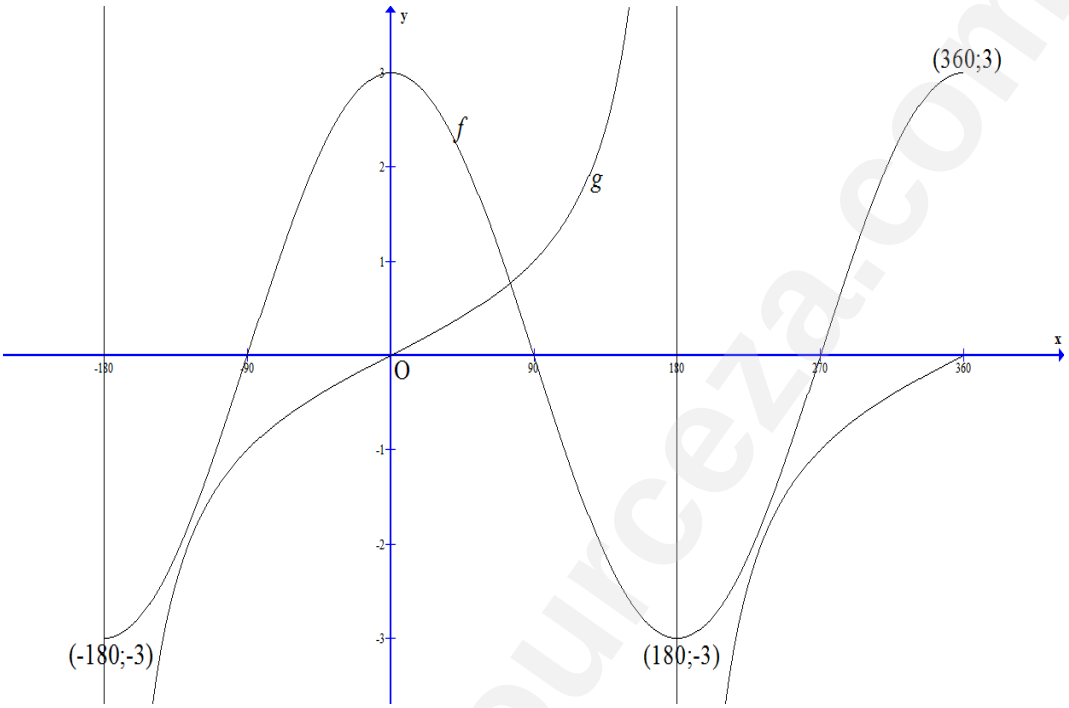
$\therefore \sin\beta = \frac{1}{k}$ $\cos\beta = \frac{\sqrt{3}}{k}$  $\sqrt{k^2-1} = \sqrt{3}$ $k^2 = 4$ $k = 2$ $\sin\beta = \frac{1}{2}$ $\beta = 30^\circ$	A✓ comparing coefficients A✓ values of trig ratios A✓ calculating k A✓ calculating β (5) **[13]**

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 18

18.1



A✓asymptotes

A✓shape of f

A✓shape of g

A✓correct x- intercept of f (-90°; 90°; 270°)

A✓correct x – intercept of g (0°; 360°)

(5)

18.2 360°

A✓360°

(1)

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

18.3	(0; 3) and (180° ; – 3) (–180° ; –3) and (360° ; 3)	CA✓for any two (2)
18.4	$-180^{o} < x < 0^{\circ}$ or $180^{\circ} < x < 360^{\circ}$ OR $-180^{o} < x < 0^{o} \cup 180^{o} < x < 360^{o}$	CA✓$-180^{\circ} < x < 0^{\circ}$ CA✓ $180 < x < 360^{\circ}$ [penalize one mark for incorrect notation] (2)
18.5	$y = 3\cos(x - 45^{\circ})$	A✓$3\cos(x - 45^{\circ})$ (1) **[11]**

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 19	
19.1 $\cos 54^\circ \,.\, \cos x + \sin 54^\circ \,.\, \sin x = \sin 2x$	
$\cos(54^\circ - x) = \sin 2x$	A✓ $\cos(54^\circ - x)$
$= \cos(90^\circ - 2x)$	A✓ $\cos(90^\circ - 2x)$
$\therefore \quad 54^\circ - x = 90^\circ - 2x + n\,.\,360^\circ \;\; or \;\; 54^\circ - x = 360 - (90^\circ - 2x) + n\,.\,360^\circ$	CA✓ $x = 36^\circ + n\,.\,360^\circ$
$x = 36^\circ + n\,.\,360^\circ \quad or \; -3x = 216^\circ + n\,.\,360^\circ$	CA✓
$x = 36^\circ + n\,.\,360^\circ \;\; or \; -x = -72^\circ + n\,.\,120^\circ$	$x = -72^\circ + n\,.\,120^\circ$
$\therefore \quad x = 36^\circ + n\,.\,360^\circ \;\; or \; x = -72^\circ + n\,.\,120^\circ,\; n \in Z$	A✓ $n \in Z$ (5)
19.2.1 $E\hat{F}D = F\hat{B}C = \alpha$ … corresponding $\angle$'s … AD // BC	A✓ S/R
$E\hat{F}D = \theta + A\hat{E}F$ ... ext $\angle$ of Δ AEF	A✓ S/R
$\therefore \; A\hat{E}F = \alpha - \theta$	A✓ $A\hat{E}F = \alpha - \theta$
In Δ ABE : $\dfrac{\sin A\hat{E}B}{AB} = \dfrac{\sin E\hat{A}B}{BE}$	A✓ sine rule application
$\therefore BE = \dfrac{AB \sin(90^\circ + \theta)}{\sin(\alpha - \theta)}$	
$= \dfrac{AB \cos\theta}{\sin(\alpha - \theta)}$	A✓ substitution (5)
19.2.2 Area of Δ BCE $= \dfrac{1}{2}\,.\,x(18 - 3x)\sin 150^\circ$	A✓ area rule
$A(x) = \dfrac{1}{2}x\,.\,(18 - 3x)\,.\,\dfrac{1}{2}$	A✓ $\sin 150^\circ = \dfrac{1}{2}$
$= \dfrac{1}{4}\,.\,x\,.\,18 - \dfrac{1}{4}x\,.\,3x$	

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

$= \frac{18}{4}x - \frac{3}{4}x^2$ $= \frac{9}{2}x - \frac{3}{4}x^2$	A✓ simplifying (3)

eduresourceza.com

Downloaded from eduresourceza.com

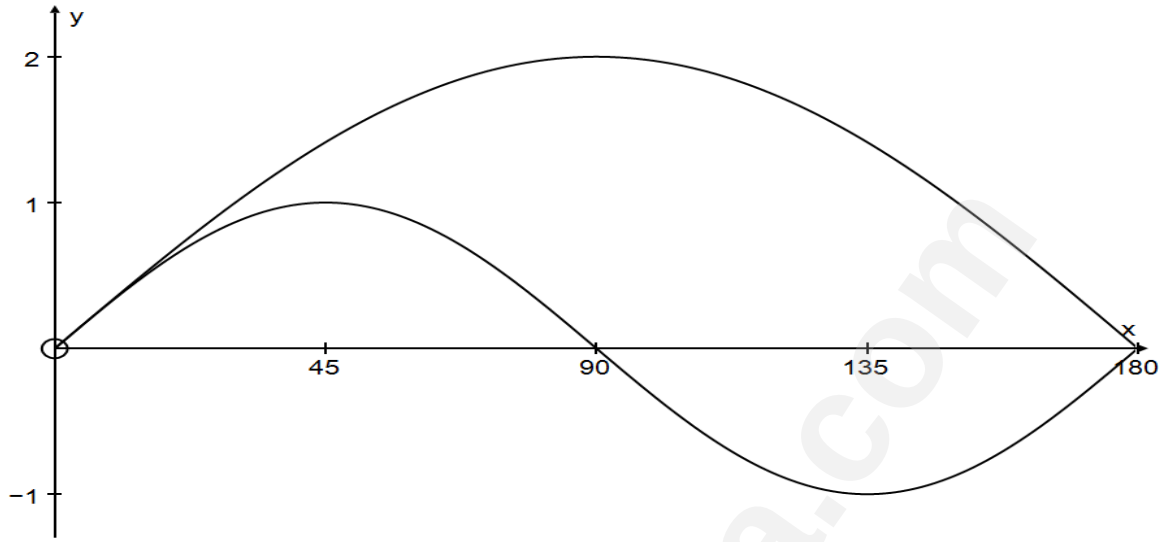
Downloaded from eduresourceza.com

19.2.3 At maximum area: $A'(x)=0$ $A'(x)=\frac{9}{2}-2.\frac{3}{4}x$ $0 = \frac{9}{2}-\frac{3}{2}x$ $3x = 9$ $x = 3$	M✓ $A'(x)=0$ A✓ derivative CA✓ $x = 3$ (3)
19.2.4 BC = 3 CE = 18 – 3 (3) = 18 – 9 = 9 $BE^2 = BC^2 + CE^2 - 2\ BC.\ CE \cos 150^o$ $= (3)^2 + (9)^2 - 2 \times 3 \times 9\ (-\cos 30^o)$ $= 9 + 81 + 54 \cos 30^o$ $= 90 + 54.\left(\frac{\sqrt{3}}{2}\right)$ = 136,765 ∴ BE = 11,695 = 11,69	A✓ for both BC = 3 and CE = 9 CA✓ applying cosine rule and substitution CA✓ answer (3) **[19]**

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 20



20.1.1	$x = 90°$	✓ (1)
20.1.2	$x \in [90°; 180°]$	✓critical points✓notation (2)
20.1.3	$x \in [0°; 90°] \cup x = 180°$	✓✓$[0°; 90°]$ ∪✓$180°$ (3)
20.2.1	$g(x) = 2sinx$ c = 2 d=1	✓c = 2✓ d=1 (2)
20.2.2	$h(x) = 2cos(x - 90°)$ a = 2 b = 90°	✓a = 2✓b = 90° (2)
		[10]

QUESTION 21

21.1.1	$\frac{sin40°.tan45°}{-cos50°.sin60°}$ $= \frac{sin40°.(1)}{-sin40°(\frac{\sqrt{3}}{2})}$ $= -\frac{2}{\sqrt{3}}$	✓$sin40°$ ✓$tan45°$ ✓ $-$ cos50° ✓sin60° ✓answer (5)
21.1.2	$\frac{sin15°cos15°}{\cos(45° - x + x)}$ $= \frac{2sin15°.cos15°}{2cos45°}$ $= \frac{sin30°}{2cos45°}$ $= \frac{\frac{1}{2}}{2\times\frac{\sqrt{2}}{2}}$ $= \frac{1}{2\sqrt{2}}$ or $\frac{\sqrt{2}}{4}$	✓reducing to cos45° ✓manipulating with 2 ✓sin30° ✓substitute ✓answer (5)

Downloaded from eduresourceza.com

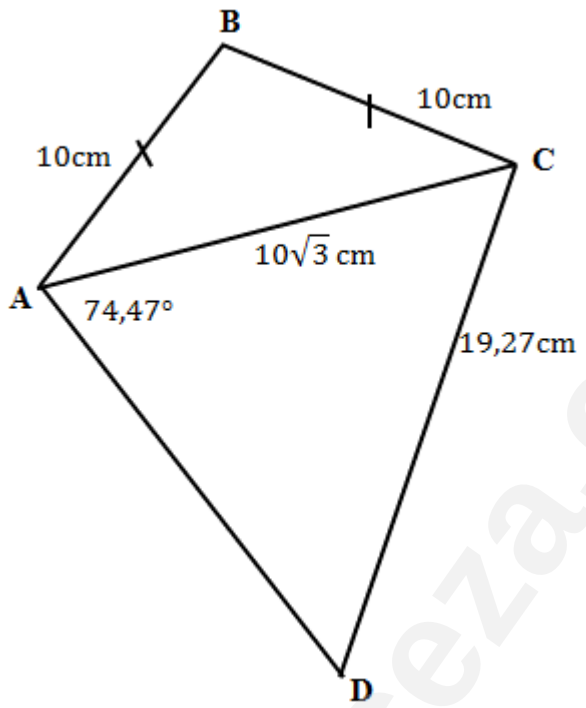
Downloaded from eduresourceza.com

21.2.1	$2cos^2A - 1 = cos2A$ $2cos^2A = cos2A + 1$ $cos^2A = \frac{cos2A+1}{2}$	✓$2cos^2A = cos2A + 1$ ✓$cos^2A = \frac{cos2A+1}{2}$ (2)
21.2.2	$\cos A = \sqrt{\frac{cos2A+1}{2}}$ $\cos 15° = \sqrt{\frac{cos30°+1}{2}}$ $= \sqrt{\frac{\frac{\sqrt{3}}{2}+1}{2}}$ $= \sqrt{\frac{\sqrt{3}+2}{4}}$ $= \frac{\sqrt{\sqrt{3}+2}}{2}$	✓$\cos A = \sqrt{\frac{cos2A+1}{2}}$ ✓cos15°and cos30° ✓substitute ✓$\sqrt{\frac{\sqrt{3}+2}{4}}$ (4)
21.3	$sin2x = cos3x$ $sin2x = sin(90° - 3x)$ $2x = 90° - 3x + k.360°$ $5x = 90° + k.360°$ $x = 18° + k.72°$ or $2x = 90° + 3x + k.360°$ $x = -90° + k.360°$ $x = 18°; 90°; -54°; -90°$ OR $sin2x = \cos(-3x)$ $cos(90° - 2x) = \cos(-3x)$ $90° - 2x = -3x + k.360°$ $x = -90° + k.360°$ or $90° - 2x = 360 - (-3x) + k.360°; k \in Z$ $-5x = 270° + k.360°$ $x = -54° + k.72°$ $x = 18°; 90°; -54°; -90°$	✓$sin2x = cos3x$ ✓$sin(90° - 3x)$ ✓ $x = 18° + k.72°$ ✓$x = -90° + k.360°$ ✓two correct ✓another two correct ✓$cos(90° - 2x)$ ✓ $x = -90° + k.360°$ ✓$90° - 2x = 360 - (-3x)$ ✓$x = -54° + k.72°$ ✓two correct ✓another two correct (6)
		[22]

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 22



22.1	In ΔABC $\cos B = \frac{10^2+10^2-(10\sqrt{3})^2}{2.10.10}$ $\cos B = -0{,}5$ $\hat{B} = 120°$	✓substitute into cosine rule ✓$= -0{,}5$ ✓$\hat{B} = 120°$ (3)
22.2	In ΔACD: $\frac{\sin D}{10\sqrt{3}} = \frac{\sin 74{,}47°}{19{,}27}$ $\sin D = 0{,}866\ldots$ $\hat{D} = 60°$ $\hat{B} + \hat{D} = 180°$ ABCD is a cyclic quadrilateral (opp $\angle s\ suppl.$)	✓subst into sine rule ✓0,866 ✓ $\hat{D} = 60°$ ✓ $\hat{B} + \hat{D} = 180°$ ✓reason (5)
		[8]

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 23

23.1	$\dfrac{\cos(180°+x)\,.\tan(360°-x).\sin^2(90°-x)}{\sin(180°-x)}+\sin^2 x$ $=\dfrac{(-\cos x).(-\tan x).\cos^2 x}{\sin x}+\sin^2 x$ $=\dfrac{\cos x.\dfrac{\sin x}{\cos x}.\cos^2 x}{\sin x}+\sin^2 x$ $=\cos^2 x+\sin^2 x$ $=1$	✓$-\cos x$ ✓$-\tan x$ ✓$\cos^2 x$ ✓$\sin x$ ✓$\cos^2 x+\sin^2 x$ ✓answer (6)
23.2.1	cos (A – B) – cos (A + B) = cos A cos B+ sin A sin B – [cos A cos B – sin A sin B] = cos A cos B + sin A sin B – cos A cos B + sin A sin B = 2sin A sin B	✓expansion of cos(A–B) ✓expansion of cos(A+B) ✓simplification (3)
23.2.2	cos15° – cos 75° = cos (45° – 30°) – cos (45° + 30°) = 2sin 45°.sin 30° $=2\times\dfrac{\sqrt{2}}{2}\times\dfrac{1}{2}$ or $2\times\dfrac{1}{\sqrt{2}}\times\dfrac{1}{2}$ $=\dfrac{\sqrt{2}}{2}$ or $\dfrac{1}{\sqrt{2}}$ **OR** cos15° – cos 75° = cos (45° – 30°) – cos (45° + 30°) = cos 45°cos30°+sin45°sin 30°– [cos 45°cos 30°– sin45°sin30°] = 2sin45° sin 30° $=2\times\dfrac{\sqrt{2}}{2}\times\dfrac{1}{2}$ or $2\times\dfrac{1}{\sqrt{2}}\times\dfrac{1}{2}$ $=\dfrac{\sqrt{2}}{2}$ or $\dfrac{1}{\sqrt{2}}$	✓ 45° – 30° and 45° + 30° ✓2sin 45°.sin 30 ✓$\dfrac{\sqrt{2}}{2}\times\dfrac{1}{2}$ / $\dfrac{1}{\sqrt{2}}\times\dfrac{1}{2}$ ✓answer (4)

Downloaded from eduresourceza.com

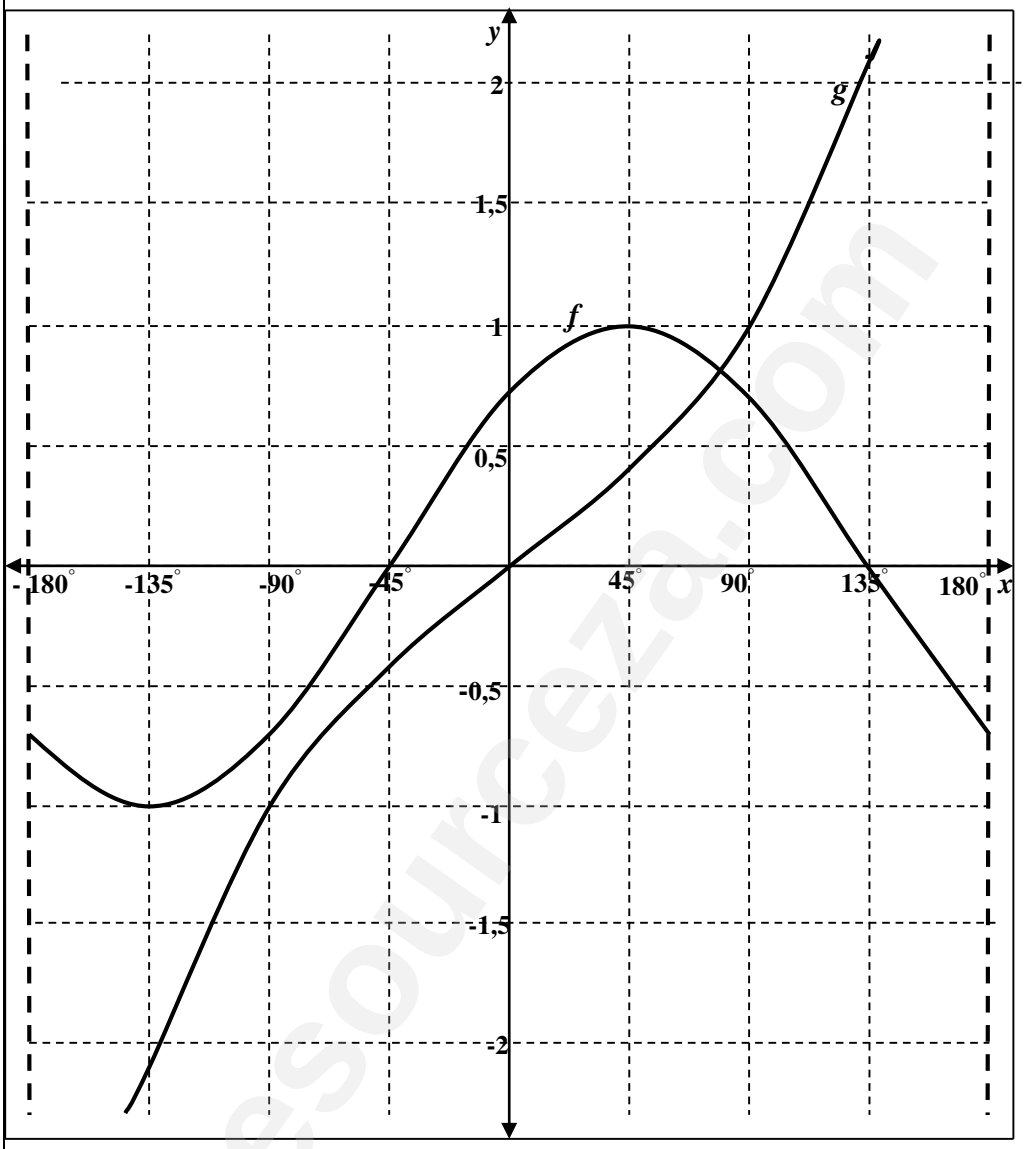
Downloaded from eduresourceza.com

23.4	$AB^2 = (\cos\theta - 6)^2 + (\sin\theta - 7)^2$ $86 = \cos^2\theta - 12\cos\theta + 36 + \sin^2\theta - 14\sin\theta + 49$ $86 = 1 + 36 + 49 - 12\cos\theta - 14\sin\theta$ $0 = -12\cos\theta - 14\sin\theta$ $14\sin\theta = -12\cos\theta$ $\frac{\sin\theta}{\cos\theta} = \frac{-12}{14}$ $\tan\theta = -\frac{6}{7}$ /-0,86	✓ sub. into the distance formula ✓ simplification ✓ $14\sin\theta = -12\cos\theta$ ✓ $\tan\theta = -\frac{6}{7}$ / 0,86 (4) **[17]**

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

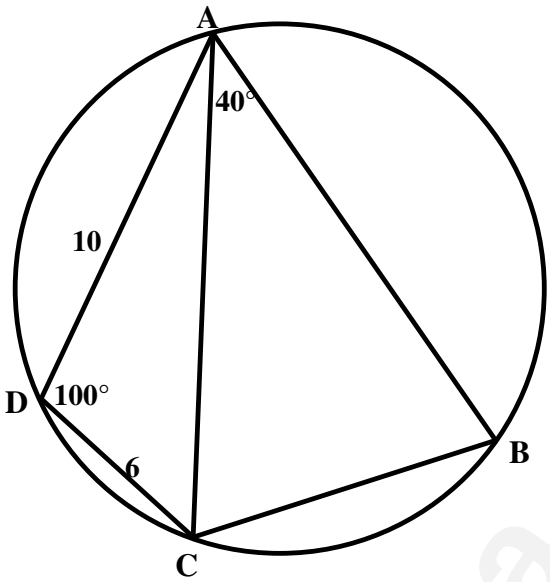
QUESTION 24

24.1	 f: ✓ x-int ✓ y-int ✓turning points g: ✓asymptotes, ✓ passing (0,0), ✓ (90;1)	(6)
24.2.1	$x = -45°$ or x = 135°	✓ $x = -45°$ ✓ $x = 135°$ (2)
24.2.2	$x = 180°$ *or / of* $x = -180°$	✓$x = 180°$ ✓ $x = -180°$ (2)
24.2.3	$y \in [-1;1]$ or $-1 \leq y \leq 1$	✓answer (1)
24.2.4	1	✓answer (1)
24.2.5	$x \in (-180°;-45°)$ *or / of* $(0°\,;135°)$ **OR** $-180° < x < -45°$ or $0° < x < 135°$	✓ – 180°; – 45° ✓ 0°; 135° ✓notation (3) **[15]**

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

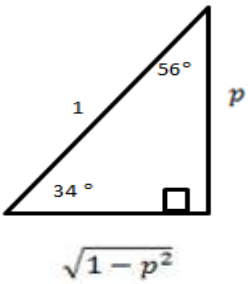
QUESTION 25

		
25.1	$AC^2 = 10^2 + 6^2 - 2(10)(6)\cos 100°$ $= 156{,}8377813$ $AC = 12{,}5$ units $\hat{B} = 80°$ (opp angles of a cyclic quad) In ΔABC, $\frac{BC}{\sin 40°} = \frac{AC}{\sin B}$ $\frac{BC}{\sin 40} = \frac{12{,}5}{\sin 80°}$ $BC = \frac{12{,}5 \times \sin 40°}{\sin 80°}$ $= 8{,}2$ units	✓sub. into cosine rule ✓ 156,8377813 ✓12,5 units ✓ $\hat{B} = 80°$ ✓sub. into sine rule ✓ answer (6)
25.2	Area of $\Delta ABC = \frac{1}{2}AC.BC.\sin 60°$ $= \frac{1}{2}(12{,}5)(8{,}2)\sin 60°$ $= 44{,}4$ square units	✓ 60° ✓ sub. into the area rule ✓ answer (3) **[9]**

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 26

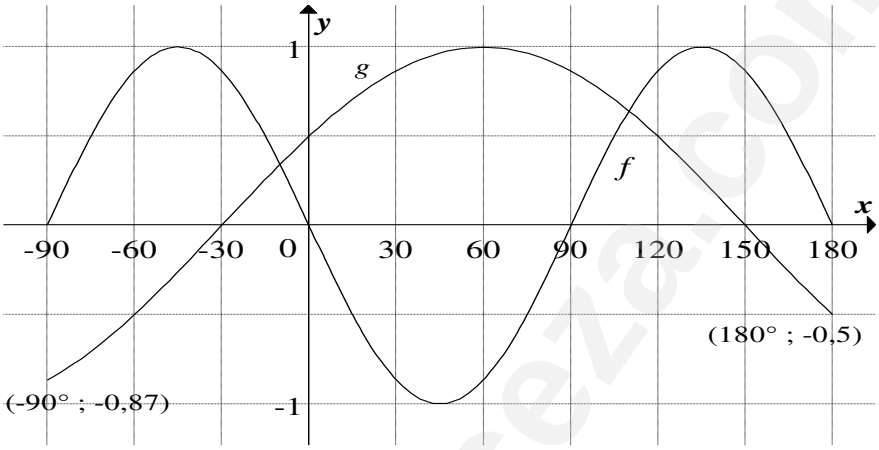
Q26	SUGGESTED ANSWER	DESCRIPTORS	MA
	$(1)^2 = p^2 + (opp)^2$ $(1)^2 - (p)^2 = x^2$ $\therefore x = \sqrt{1-p^2}$  		
26.1.1	$\sin 214°$ $= \sin(180° + 34°)$ $= -\sin 34°$ $= -p$	✓ $-\sin 34°$ ✓ Ans	
26.1.2	$\cos 34°.\cos(-22°) + \sin 34°.\sin 338°$ $= \cos 34°.\cos 22° + \sin 34°(-\sin 22°)$ $= \cos 34°.\cos 22° - \sin 34°.\sin 22°$ $= \cos(34° + 22°)$ $= \cos 56°$ $\therefore \quad = p$	✓✓ reduction ✓ compound angles ✓ $\sqrt{1-p^2}$	
26.1.3	$\cos 68°$ $= \cos 2(34°)$ $= 2\cos^2 34° - 1$ $= 2\left(\sqrt{1-p^2}\right)^2 - 1$ $= 2(1-p^2) - 1$ $= 1 - p^2$	✓ Double angle ✓ Ans	
26.2.1	$\dfrac{\cos(90° - 2\theta).\sin(\theta).}{\sin^2(180° + \theta)\cos(720° + \theta).}$ $= \dfrac{\sin 2\theta.(\sin\theta)}{(-\sin\theta)^2.(\cos\theta)}$ $= \dfrac{2\sin\theta\cos\theta.\sin\theta}{\sin^2\theta.\cos\theta.}$ $= 2$	✓ $\sin 2\theta$ ✓ $(-\sin\theta)^2$ ✓ $\sin^2\theta$ ✓ $\cos\theta$ ✓ $2\sin\theta\cos\theta$ ✓ 2	(6
26.2.2	$\dfrac{1}{\sin^2 2x} - \dfrac{1}{\tan^2 2x}$ $= \dfrac{1}{\sin^2 2x} - \dfrac{\cos^2 2x}{\sin^2 2x}$ $= \dfrac{1-\cos^2 2x}{\sin^2 2x}$	$\dfrac{\cos^2 x}{\sin^2 x}$ ✓ $\dfrac{1-\cos^2 2x}{\sin^2 2x}$ ✓	

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

	$= \frac{sin^2 2x}{sin^2 2x} = 1$	✓ $\sin^2 2x$ ✓Ans	(4
			[

QUESTION 27

Q27	SUGGESTED ANSWER	DESCRIPTORS	MA
27.1		✓ shape ✓ endpoints ✓ intercepts with axes	(3)
27.2	$-\sin 2x = \cos(x - 60°)$ $-\sin 2x = \sin(90° - (x - 60°))$ $-\sin 2x = \sin(150° - x)$ $\sin(180° + 2x) = \sin(150° - x)$ $180° + 2x = 150° - x + n(360°), n \epsilon Z$ $3x = -30 + n(360), n \in Z$ $x = -10° + n(120°)$	✓ equate the equations ✓ co-functions $\sin(180° + 2x)$✓ ✓ ✓ Ans	(5
27.4	$-10° \leq x \leq 110°$	✓ Inequality ✓✓Ans	(3
27.4	$y = -\sin(2x + 30°)$	✓ ✓ Ans	(2
27.5	g must shift 30° right	✓ 30° ✓ right	(2
			[1

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 28

Q28	SUGGESTED ANSWER	DESCRIPTORS	MA
28.1	$\cos\theta = \frac{DC}{p}$ $p.\cos\theta = DC$ $\frac{BD}{\sin(90° - \theta)} = \frac{DC}{\sin 2\theta}$ $\frac{BD}{\cos\theta} = \frac{DC}{2\sin\theta.\cos\theta}$ $\frac{BD}{1} = \frac{DC.\cos\theta}{2\sin\theta.\cos\theta}$ but $p\cos\theta = DC$ $\frac{BD}{1} = \frac{p\cos\theta.\cos\theta}{2\sin\theta.\cos\theta}$ $BD = \frac{p\cos\theta}{2\sin\theta}$	✓ DC ✓ Subst in Sine Rule ✓Simplification ✓Subst DC ✓ Simplification	(5)
28.2	$\sin 30° = \frac{AC}{p}$ $3\sin 30° = AC$ $\therefore AC = 1{,}5\ m$	✓Substitution ✓Ans	(2)
28.3	$BD = \frac{3\cos 30°}{2\sin 30°} = \frac{3}{2}\left(\frac{1}{\sqrt{3}}\right) = \frac{3\sqrt{3}}{2}$ $AB^2 = AD^2 + BD^2 - 2AD.BD\cos ADB$ $= 3^2 + \left(\frac{3\sqrt{3}}{2}\right)^2 - 2(3)\left(\frac{3\sqrt{3}}{2}\right)\cos 70°$ $= 9 + \frac{27}{4} - 9\sqrt{3}\cos 70°$ $= 10{,}418\ldots$ $AB = 3{,}23$ metres	✓BD ✓Cos rule ✓Subst ✓AB^2 ✓AB	(5
			[1

QUESTION 26

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

eduresourceza.com

Downloaded from eduresourceza.com