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MATHEMATICS

MATERIAL FOR GRADE 12

Trigonometry

QUESTIONS

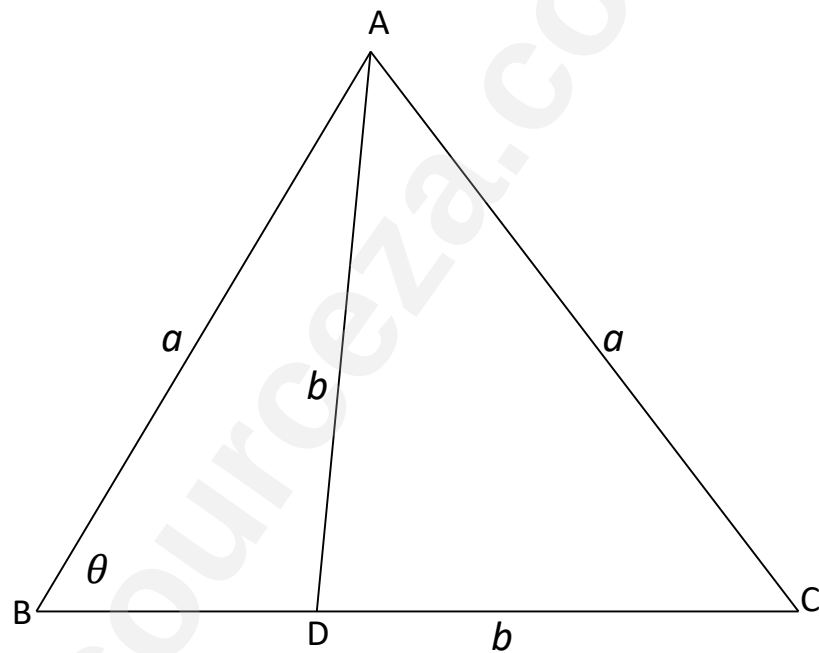
QUESTION 1

In the diagram below, ABC is an isosceles triangle. D lies on BC.

$$AB = AC = a \text{ units}$$

$$AD = DC = b \text{ units}$$

$$\hat{B} = \theta.$$



1.1 Determine, **without** reasons, the size of $\hat{ADC}$ in terms of θ . (2)

1.2 Prove that:

$$\cos 2\theta = \frac{a^2}{2b^2} - 1 \quad (4)$$

1.3 Hence, determine the value of θ if $a = 3$ and $b = 2$
(Rounded off to two decimal digits.) (3)

[9]

QUESTION 2

Simplify the following **without** using a calculator.

2.1 $\cos 56^\circ \cos 26^\circ + \cos 146^\circ \sin(-26^\circ)$ (4)

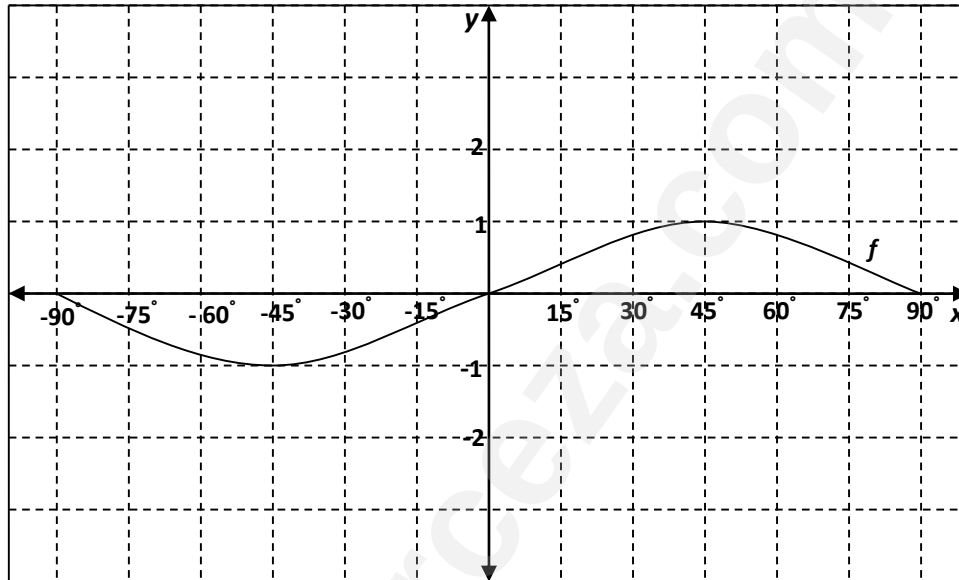
2.2
$$\frac{\tan(180^\circ + x) \cos(360^\circ - x)}{\sin(x - 180^\circ) \cos(90^\circ + x) + \cos(720^\circ + x) \cos(-x)}$$
 (6)

2.3 Prove the identity : $\frac{\cos 2x + \cos^2 x + 3 \sin^2 x}{2 - 2 \sin^2 x} = \frac{1}{\cos^2 x}$ (5)

[15]

QUESTION 3

Consider the function $f(x) = \sin 2x$ for $x \in [-90^\circ; 90^\circ]$



- 3.1 Write down the period of f . (1)
- 3.2 Sketch the graph of $g(x) = \cos(x-15^\circ)$ for $x \in [-90^\circ; 90^\circ]$ on the diagram sheet provided for this sub-question. (5)
- 3.3 Solve the equation: $\sin 2x = \cos(x-15^\circ)$ for $x \in [-90^\circ; 90^\circ]$ (7)
- 3.4 Find the values of x for which $f(x) < g(x)$. (3)

[16]

QUESTION 4

4.1.1 Simplify the following expression to a single trigonometric function:

$$\frac{2 \sin(180^\circ + x) \sin(90^\circ + x)}{\cos^4 x - \sin^4 x} \quad (5)$$

4.1.2 For which value(s) of x , $x \in [0^\circ; 360^\circ]$ is the expression in 4.1 undefined? (3)

4.2 Evaluate, without using a calculator: $\frac{\cos 347^\circ \cdot \sin 193^\circ}{\tan 315^\circ \cdot \cos 64^\circ}$ (5)

4.3 Prove the following identity: $\frac{\cos 3x}{\cos x} = 2\cos 2x - 1$ (5)

[18]

QUESTION 5

The graphs of $f(x) = -2\cos x$ and $g(x) = \sin(x + 30^\circ)$ for $x \in [-90^\circ; 180^\circ]$ are drawn in the diagram below.

- 5.1 Determine the period of g . (1)
- 5.2 Calculate the x -coordinates of P and Q, the points where f and g intersect. (7)
- 5.3 Determine the x -values, $x \in [-90^\circ; 180^\circ]$, for which:
- 5.3.1 $g(x) \leq f(x)$ (3)
- 5.3.2 $f'(x), g(x) > 0$ [14]

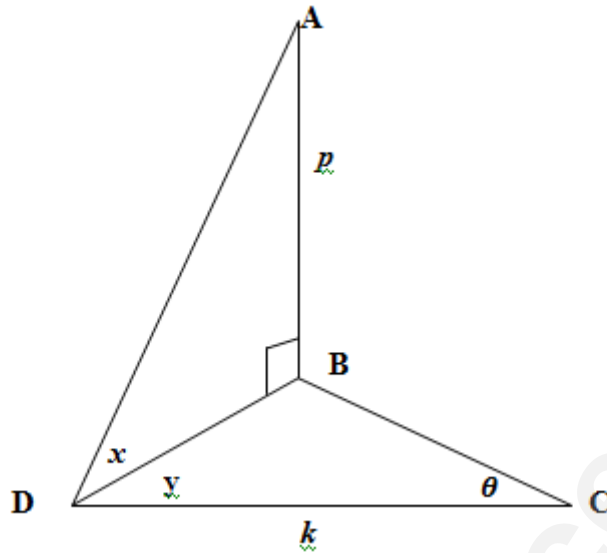
QUESTION 6

AB is a vertical tower of p units high.

D and C are in the same horizontal plane as B, the foot of the tower.

The angle of elevation of A from D is x . $\widehat{BDC} = y$ and $\widehat{DCB} = \theta$.

The distance between D and C is k units.



6.1.1 Express p in terms of DB and x . (2)

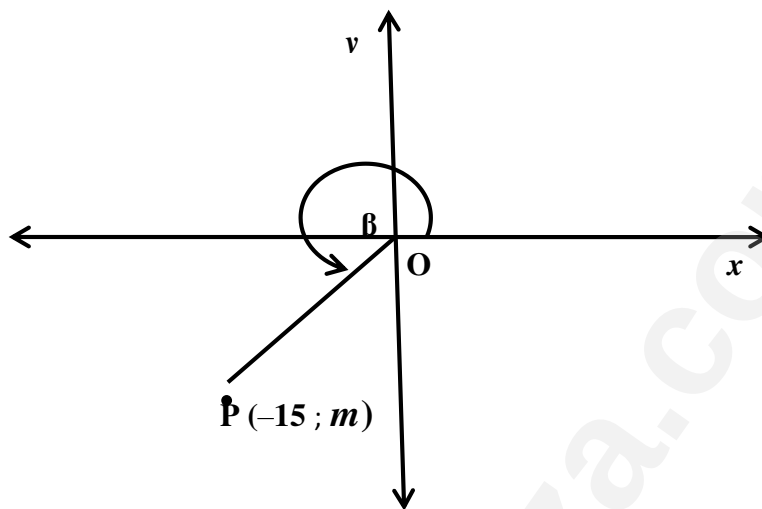
6.1.2 Hence prove that: $p = \frac{k \sin \theta \tan x}{\sin y \cos \theta + \cos y \sin \theta}$ (5)

6.2 Find BC to the nearest meter if $x = 51,7^\circ$, $y = 62,5^\circ$, $p = 80 \text{ m}$ and $k = 95 \text{ m}$. (4)

[11]

QUESTION 7

In the diagram below, $P(-15; m)$ is a point in the third quadrant and $17\cos \beta + 15 = 0$.



7.1 WITHOUT USING A CALCULATOR, determine the value of the following:

7.1.1 m (3)

7.1.2 $\sin \beta + \tan \beta$ (3)

7.1.3 $\cos 2\beta$ (3)

7.2 Simplify:

$$\frac{\sin(180^\circ - x) \cdot \cos(x - 180^\circ) \cdot \tan(360^\circ - x)}{\sin(-x) \cdot \cos(450^\circ + x)} \quad (7)$$

7.3 Consider the identity: $\frac{\sin x + \sin 2x}{1 + \cos x + \cos 2x} = \tan x$

7.3.1 Prove the identity. (5)

7.3.2 Determine the values of x for which this identity is undefined. (4)

[25]

QUESTION 8

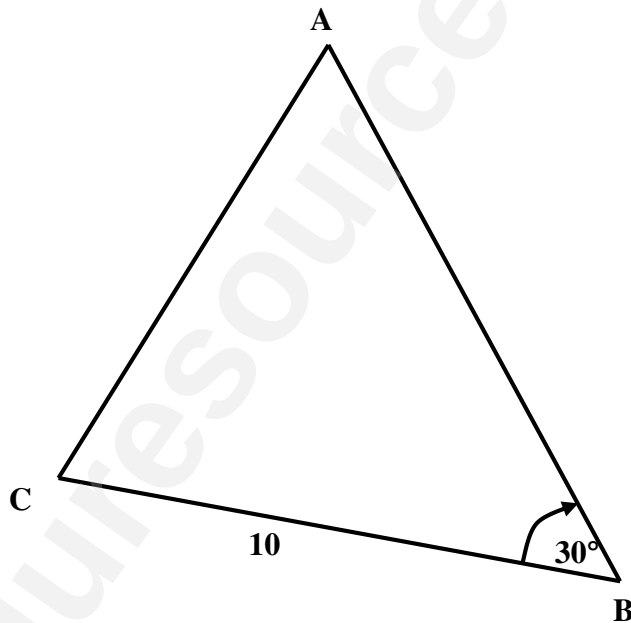
Consider: $f(x) = \cos 2x$ and $g(x) = \sin(x - 60^\circ)$

8.1 Use the grid provided to sketch the graphs of f and g for $x \in [-90^\circ; 180^\circ]$ on the same set of axes. Show clearly all the intercepts on the axes and the coordinates of the turning points. (6)

8.2 Use your graphs to determine the value(s) of x for which $g(x) > 0$. (3)
[9]

QUESTION 9

In the diagram, $\triangle ABC$ is given with $BC = 10$ units, $\hat{B} = 30^\circ$ and $\sin(B + C) = 0,8$.



Determine the length of AC, WITHOUT USING A CALCULATOR. [5]

QUESTION 10

10.1 If $\sin 31^\circ = p$, determine the following, without using a calculator, in terms of p :

10.1.1 $\sin 149^\circ$ (2)

10.1.2 $\cos (-59^\circ)$ (2)

10.1.3 $\cos 62^\circ$ (2)

10.2 Simplify the following expression to a single trigonometric ratio:

$$\tan(180^\circ - \theta) \cdot \sin^2(90^\circ + \theta) + \cos(\theta - 180^\circ) \cdot \sin \theta \quad (6)$$

10.3 Consider: $\frac{\sin 2x + \sin x}{\cos 2x + \cos x + 1} = \tan x$

10.3.1 Prove the identity. (5)

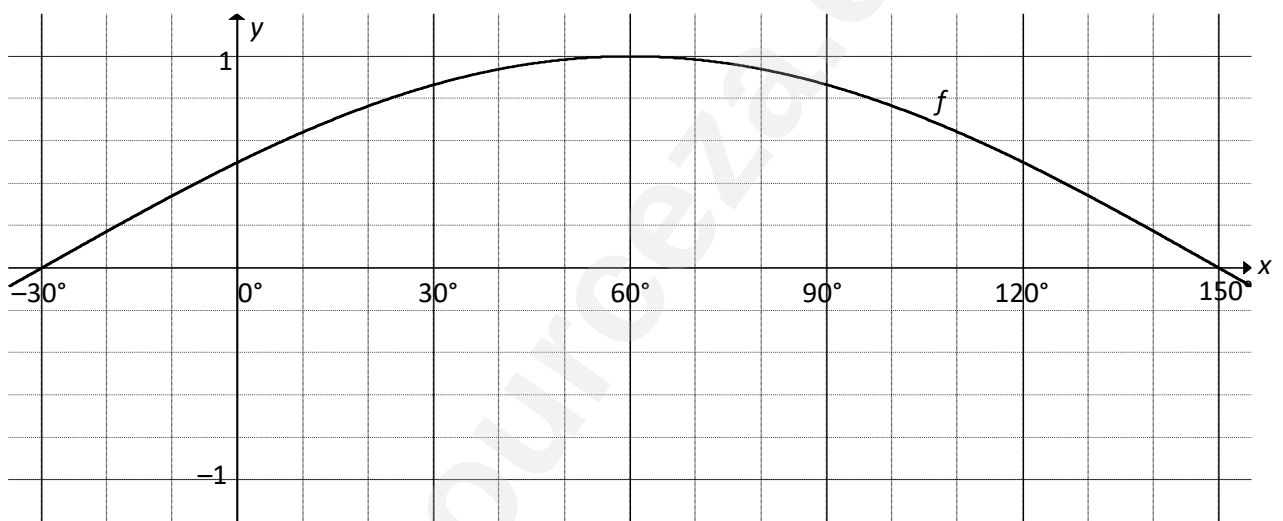
10.3.2 Determine the values of x , where $x \in [180^\circ ; 360^\circ]$, for which the above identity will be invalid. (2)

[19]

QUESTION 11

11.1 Determine the general solution of : $\sin (x + 30^\circ) = \cos 3x$. (6)

11.2 In the diagram below, the graph of $f(x) = \sin (x + 30^\circ)$ is drawn for the interval $x \in [-30^\circ ; 150^\circ]$.



11.2.1 On the same system of axes sketch the graph of g , where $g(x) = \cos 3x$, for the interval $x \in [-30^\circ ; 150^\circ]$. (3)

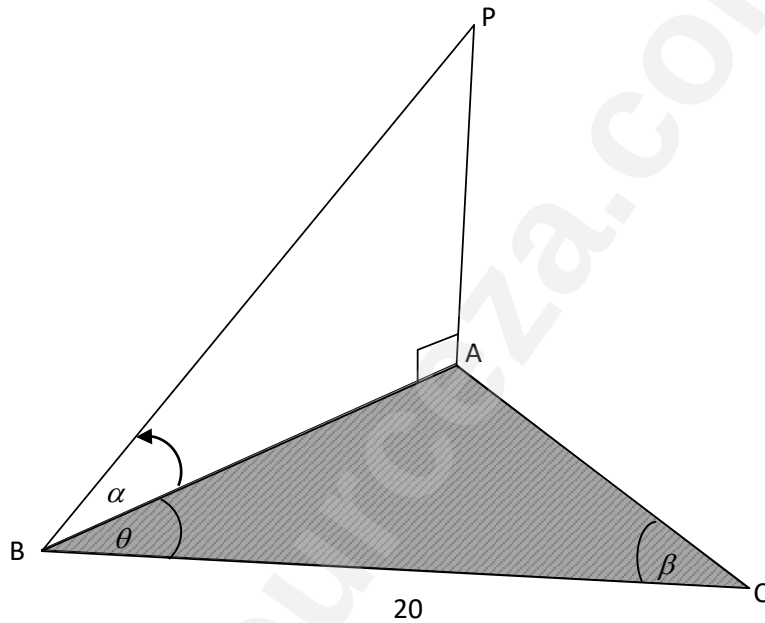
11.2.2 Write down the period of g . (1)

11.2.3 For which values of x will $f(x) \geq g(x)$ in the interval $x \in (-30^\circ ; 150^\circ)$? (3)
[13]

QUESTION 12

In the diagram below, A, B and C are in the same horizontal plane. P is a point vertically above A. The angle of elevation from B to P is α .

$\angle ACB = \beta$, $\angle ABC = \theta$ and $BC = 20$ units.



12.1 Write AP in terms of AB and α . (2)

12.2 Prove that $AP = \frac{20 \sin \beta \tan \alpha}{\sin(\theta + \beta)}$ (3)

12.3 Given that $AB = AC$, determine AP in terms of α and β in its simplest form. (3)
[8]

QUESTION 13

13.1 If $90^\circ < A < 360^\circ$ and $\tan A = \frac{2}{3}$, determine without the use of a calculator.

$$13.1 \quad \sin A = \frac{1}{2} \quad (3)$$

$$13.1. \quad \cos 2A - \sin 2A = \frac{2}{5} \quad (4)$$

13.2 Given that $\sin x = t$, express the following in terms of t , without the use of calculator.

$$13.2. \quad \cos (x - 90^\circ) = \frac{1}{2} \quad (2)$$

$$13.2. \quad \sin 2x = \frac{2}{5} \quad (3)$$

[12]

QUESTION 14

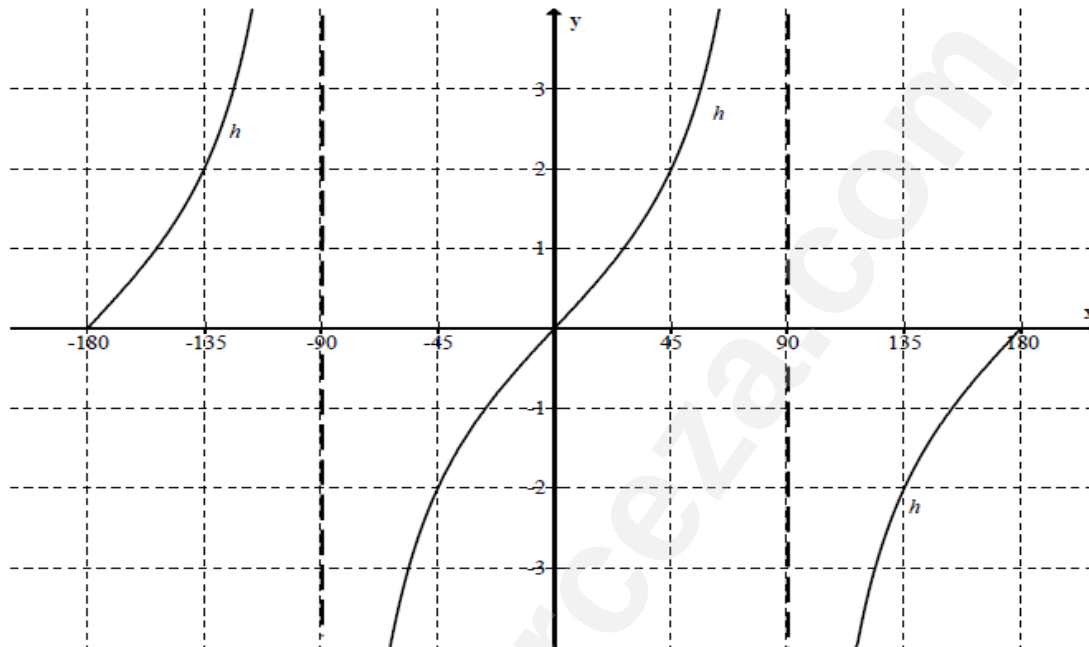
14.1 Calculate without the use of a calculator: $\frac{\cos^2 208^\circ}{\tan 118^\circ \cdot \sin 124^\circ}$ (6)

14.2 Calculate the general solution of θ where $\sin \theta \neq 0$ and $1 - \cos 2\theta = 8 \sin \theta \cdot \sin 2\theta$ (6)

[12]

QUESTION 15

The graph of $h(x) = a \tan x$; for $x \in [-180^\circ; 180^\circ]$, $x \neq -90^\circ$, is sketched below.

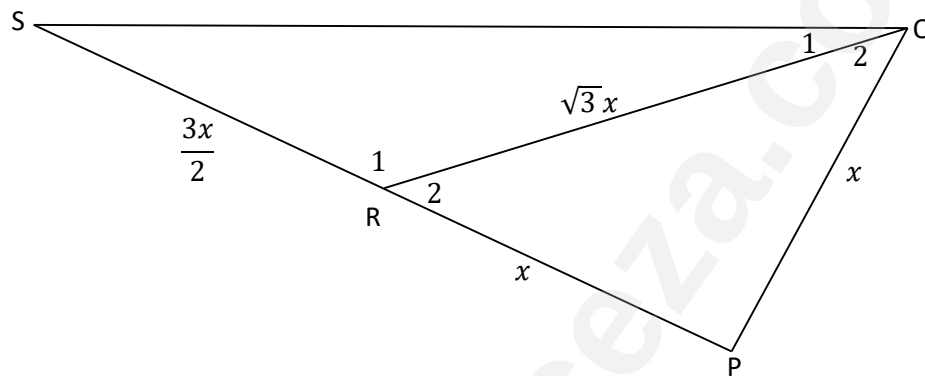


- 15.1 Determine the value of a . (2)
)
- 15.2 If $f(x) = \cos(x + 45^\circ)$, sketch the graph of f for $x \in [-180^\circ; 180^\circ]$, on the diagram provided in your ANSWER BOOK. (4)
- 15.3 How many solutions does the equation $h(x) = f(x)$ have in the domain $[-180^\circ; 180^\circ]$? (1)
- [7]

QUESTION 16

Triangle PQS represents a certain area of a park. R is a point on line PS such that QR divides the area of the park into two triangular parts, as shown below.

$PQ = PR = x$ units, $RS = \frac{3x}{2}$ units and $RQ = \sqrt{3}x$ units.



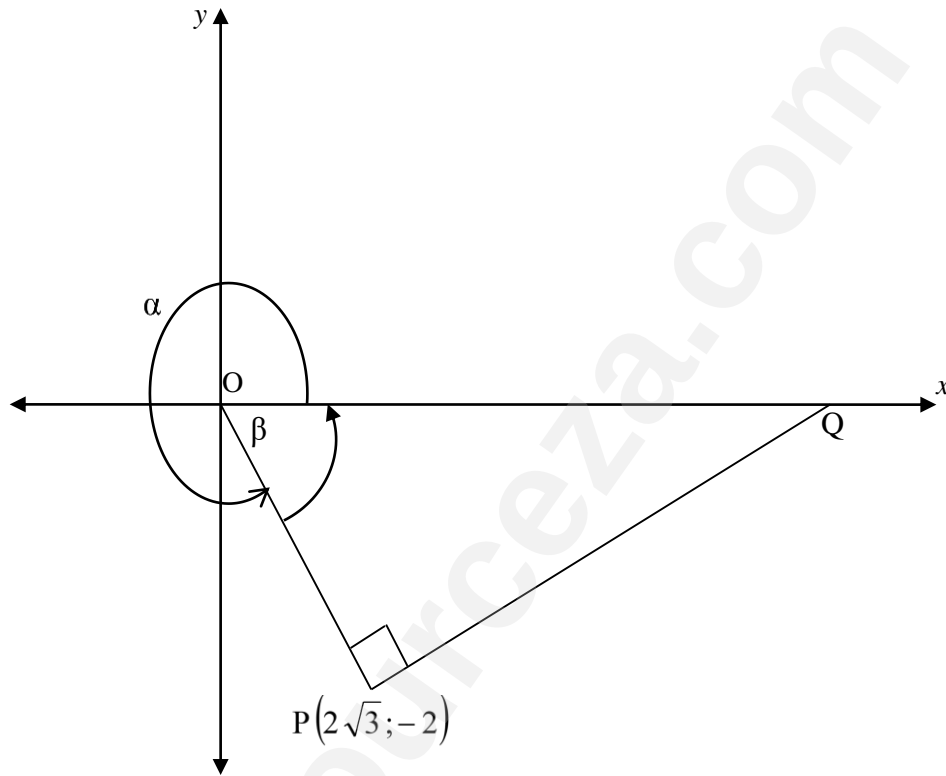
16.1 Calculate the size of $\hat{P}$. (4)

16.2 Determine the area of triangle QRS in terms of x . (5)

[9]

QUESTION 17

- 17.1 In the diagram below, $P(2\sqrt{3}; -2)$ is a point in the Cartesian plane, with reflex angle $\widehat{QOP} = \alpha$. Q is the point on the x -axis so that $\widehat{OPQ} = 90^\circ$



Calculate without measuring:

- 17.1.1 β . (3)
- 17.1.2 the length of OP. (2)
- 17.1.3 the co-ordinates of Q. (3)
- 17.2 If $\cos \alpha + \sqrt{3} \sin \alpha = k \sin (\alpha + \beta)$.

Calculate the values of k and β . (5)

[13]

QUESTION 18

- 18.1 On the same system of axes, sketch the graphs of $f(x) = 3 \cos x$ and $g(x) = \tan \frac{1}{2}x$ for $-180^\circ \leq x \leq 360^\circ$. Clearly show the intercepts with the axes and all turning points. (5)

Use the graphs in 18.1 to answer the following questions.

- 18.2 Determine the period of g . (1)
- 18.3 Determine the co-ordinates of the turning points of f on the given interval. (2)
- 18.4 For which values of x will both functions increase as x increases for $-180^\circ \leq x \leq 360^\circ$? (2)
- 18.5 If the y -axis is moved 45° to the left, then write down the new equation of f in the form $y = \dots$. (1)

[11]

QUESTION 19

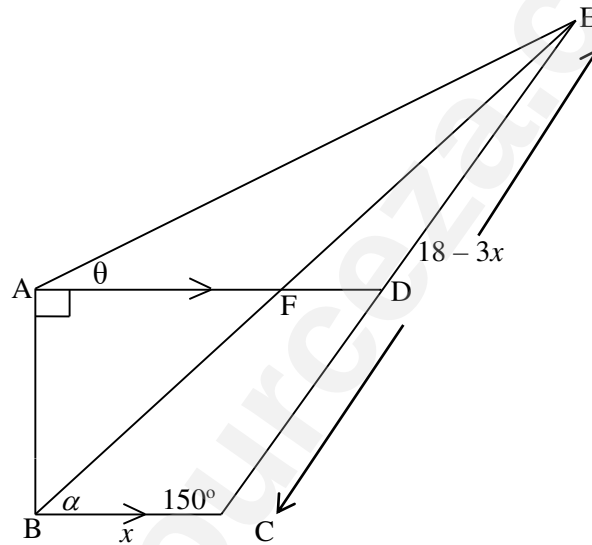
19.1 Determine the general solution of:

$$\cos 54^\circ \cdot \cos x + \sin 54^\circ \cdot \sin x = \sin 2x \quad (5)$$

19.2 ABCD is a trapezium with $AD \parallel BC$, $\hat{BAD} = 90^\circ$ and $\hat{BCD} = 150^\circ$.

CD is produced to E. F is point on AD such that BFE is a straight line, and $\hat{CBE} = \alpha$

The angle of elevation of E from A is θ , $BC = x$ and $CE = 18 - 3x$.



19.2.1 Show that: $BE = \frac{AB \cos \theta}{\sin (\alpha - \theta)}$ (5)

19.2.2 Show that the area of $\triangle BCE = \frac{9}{2}x - \frac{3x^2}{4}$ (3)

19.2.3 Determine, without the use of a calculator, the value of x for which the area of $\triangle BCE$ will be maximum. (3)

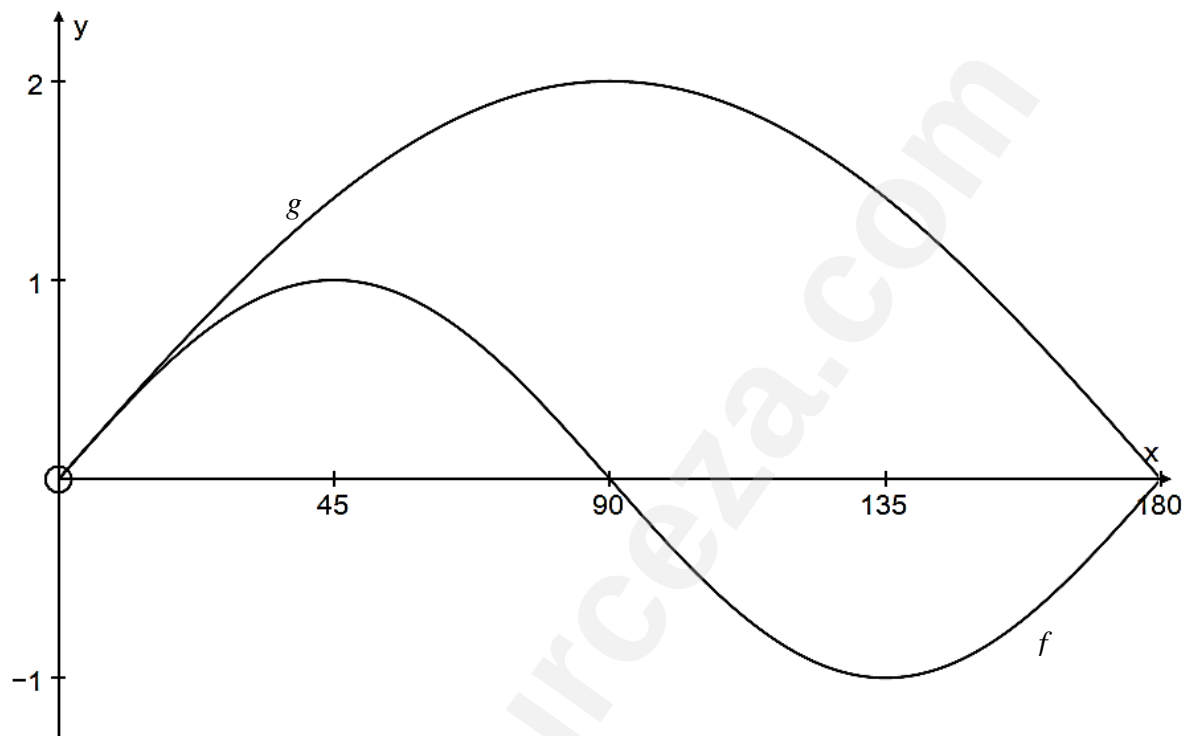
19.2.4 Calculate the length of BE if $x = 3$. (3)

[19]

QUESTION 20

The graphs below represent the functions of f and g .

$$f(x) = \sin 2x \text{ and } g(x) = c \sin dx, x \in [0^\circ; 180^\circ]$$



20.1 Determine the value(s) of x , for $x \in [0^\circ; 180^\circ]$ where:

20.1.1 $g(x) - f(x) = 2$ (1)

20.1.2 $f(x) \leq 0$ (2)

20.1.3 $g(x) \cdot f(x) \geq 0$ (3)

20.2 f in the graph drawn above undergoes transformations to result in g and h as given below. Determine the values of a, b, c and d if

20.2.1 $g(x) = c \sin dx$ (2)

20.2.2 $h(x) = a \cos(x - b)$ (2)

[10]

QUESTION 21

THIS QUESTION HAS TO BE ANSWERED WITHOUT THE USE OF A CALCULATOR:

21.1 Simplify fully: 6.1.1 $\frac{\sin 140^\circ \cdot \tan(-315^\circ)}{\cos 230^\circ \cdot \sin 420^\circ}$ (5)

6.1.2 $\frac{\sin 15^\circ \cdot \cos 15^\circ}{\cos(45^\circ - x) \cos x - \sin(45^\circ - x) \sin x}$ (5)

21.2.1 Express $\cos^2 A$ in terms of $\cos 2A$ (2)

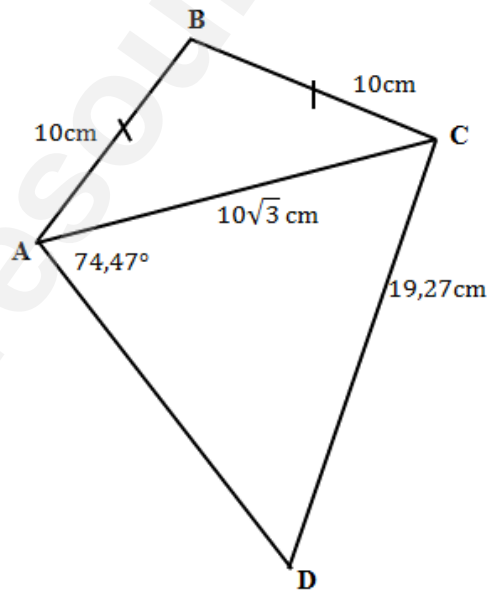
21.2.2 Hence show that $\cos 15^\circ = \frac{\sqrt{\sqrt{3}+2}}{2}$ (4)

21.3 Calculate x when $\sin 2x = \cos(-3x)$ for $x \in [-90^\circ; 90^\circ]$ (6)

[22]

QUESTION 22

Quadrilateral ABCD is drawn with $AB = BC = 10\text{cm}$, $AC = 10\sqrt{3}\text{cm}$, $CD = 19,27\text{cm}$ and $\hat{CAD} = 74,47^\circ$.



22.1 Calculate the size of $\hat{ABC}$. (3)

22.2 Determine whether ABCD is a cyclic quadrilateral. Justify your answer with the necessary calculations and reasons. (5)

[8]

QUESTION 23

23.1 Determine the value of $\frac{\cos(180^\circ + x) \cdot \tan(360^\circ - x) \cdot \sin^2(90^\circ - x)}{\sin(180^\circ - x)} + \sin^2 x$ (6)

23.2 23.2.1 Prove the identity: $\cos(A - B) - \cos(A + B) = 2\sin A \sin B$ (3)

23.2.2 Hence calculate, without using a calculator, the value of $\cos 15^\circ - \cos 75^\circ$ (4)

23.3 Find the value of $\tan \theta$, if the distance between A ($\cos \theta$; $\sin \theta$) and B (6; 7) is $\sqrt{86}$. (4)
[17]

QUESTION 24

Consider : $f(x) = \cos(x - 45^\circ)$ and $g(x) = \tan \frac{1}{2}x$ for $x \in [-180^\circ ; 180^\circ]$

24.1 Use the grid provided to draw sketch graphs of f and g on the same set of axes for $x \in [-180^\circ ; 180^\circ]$. Show clearly all the intercepts on the axes, the coordinates of the turning points and the asymptotes. (6)

24.2 Use your graphs to answer the following questions for $x \in [-180^\circ ; 180^\circ]$

24.2.1 Write down the solutions of $\cos(x - 45^\circ) = 0$ (2)

24.2.2 Write down the equations of asymptote(s) of g . (2)

24.2.3 Write down the range of f . (1)

24.2.4 How many solutions exist for the equation $\cos(x - 45^\circ) = \tan \frac{1}{2}x$? (1)

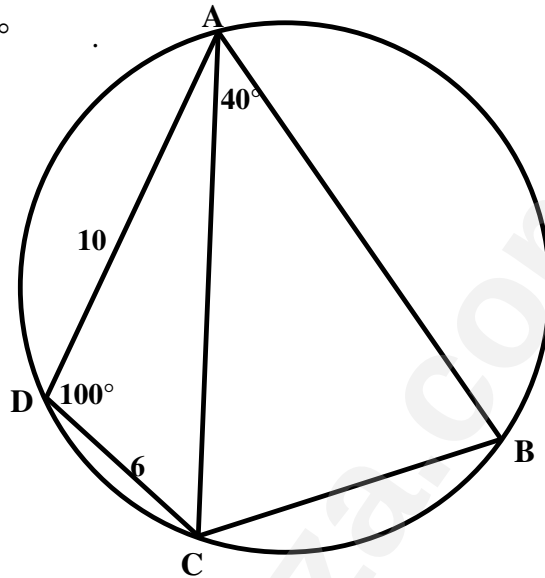
24.2.5 For what value(s) of x is $f(x) \cdot g(x) > 0$ (3)

[15]

QUESTION 25

In the diagram below, ABCD is a cyclic quadrilateral with DC = 6 units, AD = 10 units

$\angle ADC = 100^\circ$ and $\angle CAB = 40^\circ$.



Calculate the following, correct to ONE decimal place:

25.1 The length of BC (6)

25.2 The area of $\triangle ABC$ (3)

[9]

QUESTION 26

26.1 If $\sin 34^\circ = p$, determine the value of each of the following in terms of p , **WITHOUT USING A CALCULATOR.**

26.1.1 $\sin 214^\circ$ (2)

26.1.2 $\cos 34^\circ \cdot \cos(-22^\circ) + \cos 56^\circ \cdot \sin 338^\circ$ (4)

26.1.3 $\cos 68^\circ$ (2)

26.2 Determine the value of each of the following expressions:

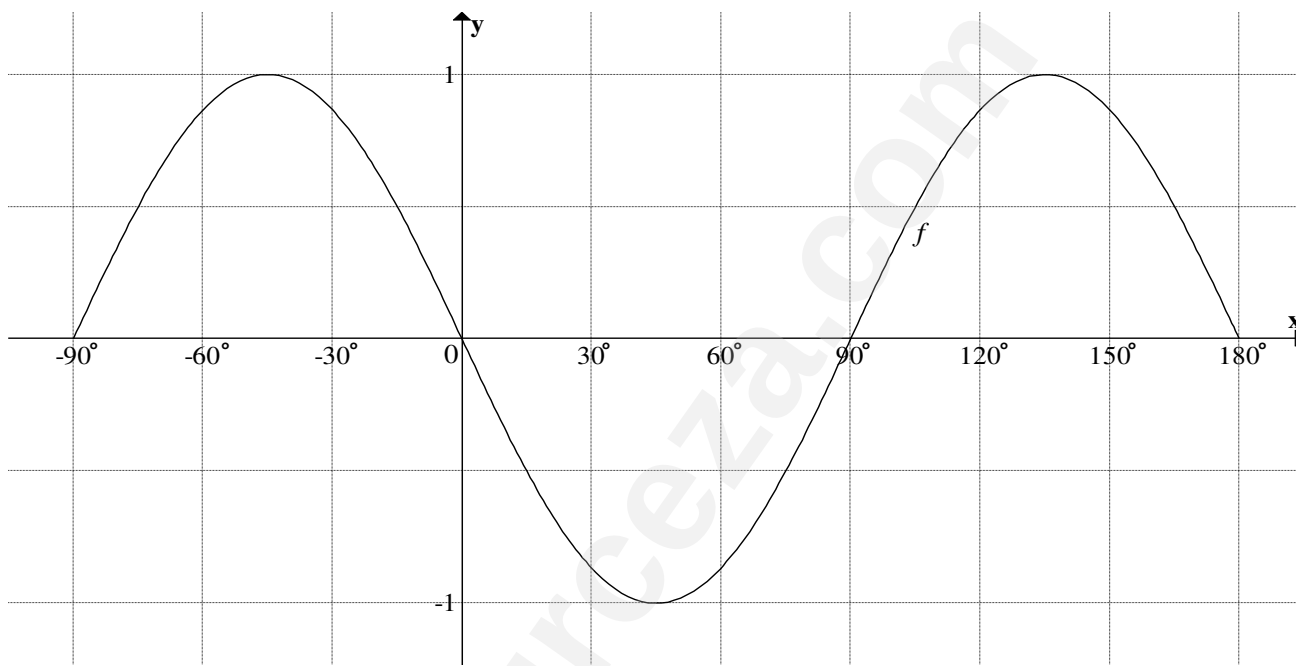
26.2.1 $\frac{\cos(90^\circ - 2\theta) \cdot \sin \theta}{\sin^2(180^\circ + \theta) \cdot \cos(720^\circ + \theta)}$ (6)

26.2.2 $\frac{1}{\sin^2 2x} - \frac{1}{\tan^2 2x}$ (4)

[18]

QUESTION 27

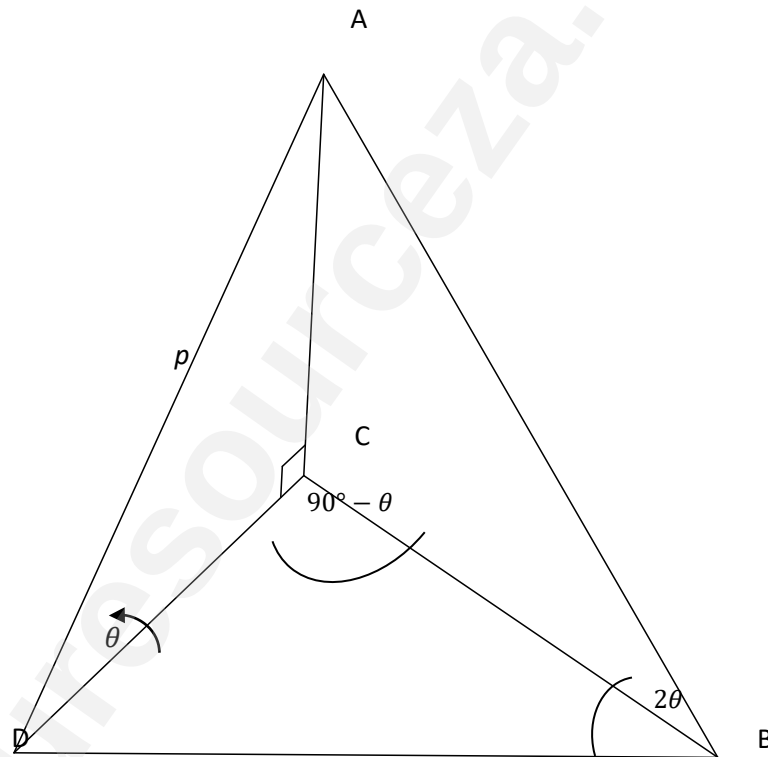
In the diagram, the graph of $f(x) = -\sin 2x$ is drawn for the interval $x \in [-90^\circ; 180^\circ]$.



- 27.1 Draw the graph of g , where $g(x) = \cos(x - 60^\circ)$, on the same system of axes for the interval $x \in [-90^\circ; 180^\circ]$ in the ANSWER BOOK. (3)
- 27.2 Determine the general solution of $f(x) = g(x)$. (5)
- 27.3 Use your graphs to solve x if $f(x) \leq g(x)$ for $x \in [-90^\circ; 180^\circ]$ (3)
- 27.4 If the graph of f is shifted 30° left, give the equation of the new graph which is formed. (2)
- 27.5 What transformation must the graph of g undergo to form the graph of h , where $h(x) = \sin x$? (2)

QUESTION 28

In the diagram below, D, B and C are points in the same horizontal plane. AC is a vertical pole and the length of the cable from D to the top of the pole, A, is p meters. $AC \perp CD$. $\widehat{ADC} = \theta$; $\widehat{DCB} = (90^\circ - \theta)$ and $\widehat{CBD} = 2\theta$.



28.1 Prove that:

$$BD = \frac{p \cos \theta}{2 \sin \theta} \quad (5)$$

28.2 Calculate the height of the flagpole AC if $\theta = 30^\circ$ and $p = 3$ meters. (2)

28.3 Calculate the length of the cable AB if it is further given that $\hat{ADB} = 70^\circ$ (5)

[12]

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