



eduresourceza.com



GAUTENG PROVINCE

EDUCATION

REPUBLIC OF SOUTH AFRICA

**PROVINCIAL
ASSESSMENT TASK**

GRADE 12

MATHEMATICS INVESTIGATION

Date: _____

MARKS: 50

TIME: 2 hours

This question paper consists of 8 pages and 6 answer sheets.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

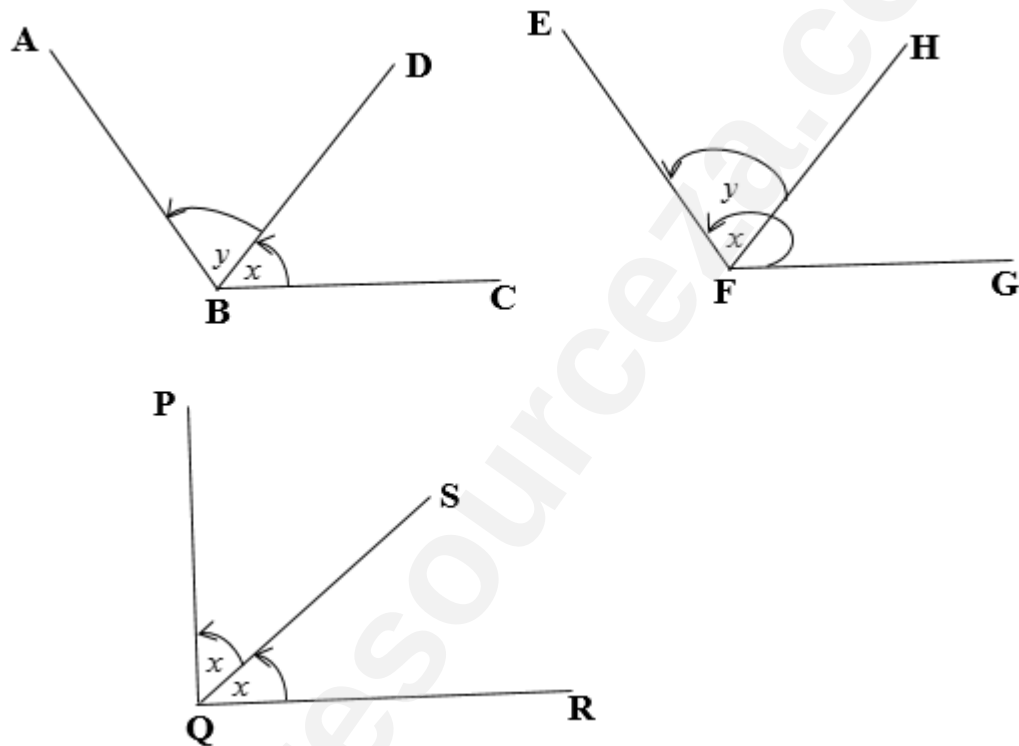
1. This task paper consists of 5 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper
4. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining your answers.
5. Answers only will not necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
10. Write neatly and legibly.

INVESTIGATING COMPOUND ANGLES AND THEREFORE, DOUBLE ANGLES.

QUESTION 1

1.1 Compound and double angles

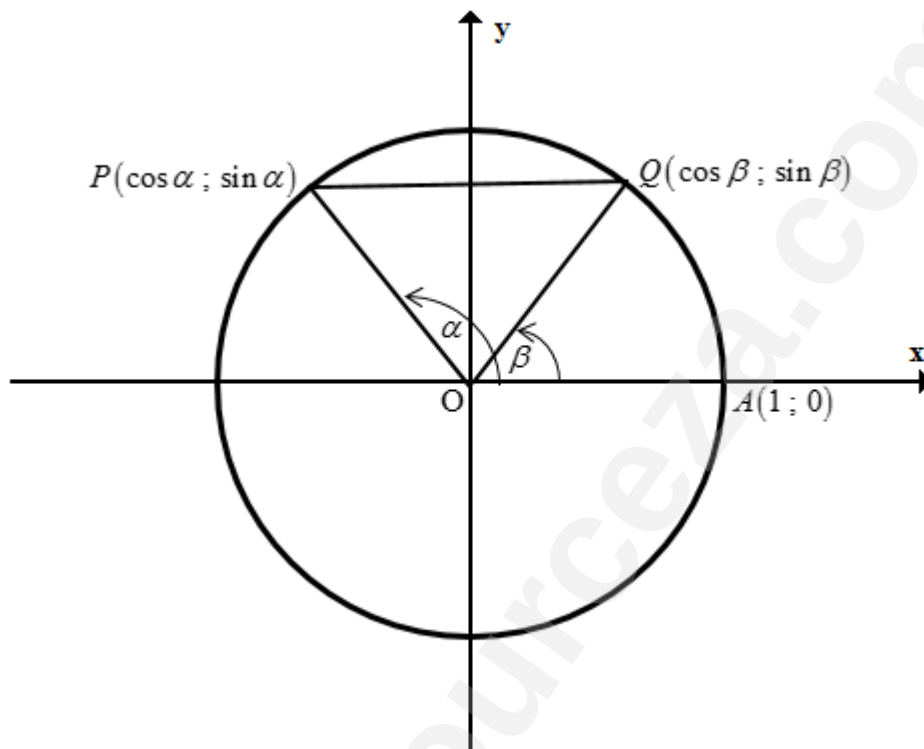
Consider the following diagrams



Write in terms of x and/ or y

- 1.1.1 $\angle ABC$ (1)
- 1.1.2 $\angle HFG$ (1)
- 1.1.3 $\angle PQR$ (1)

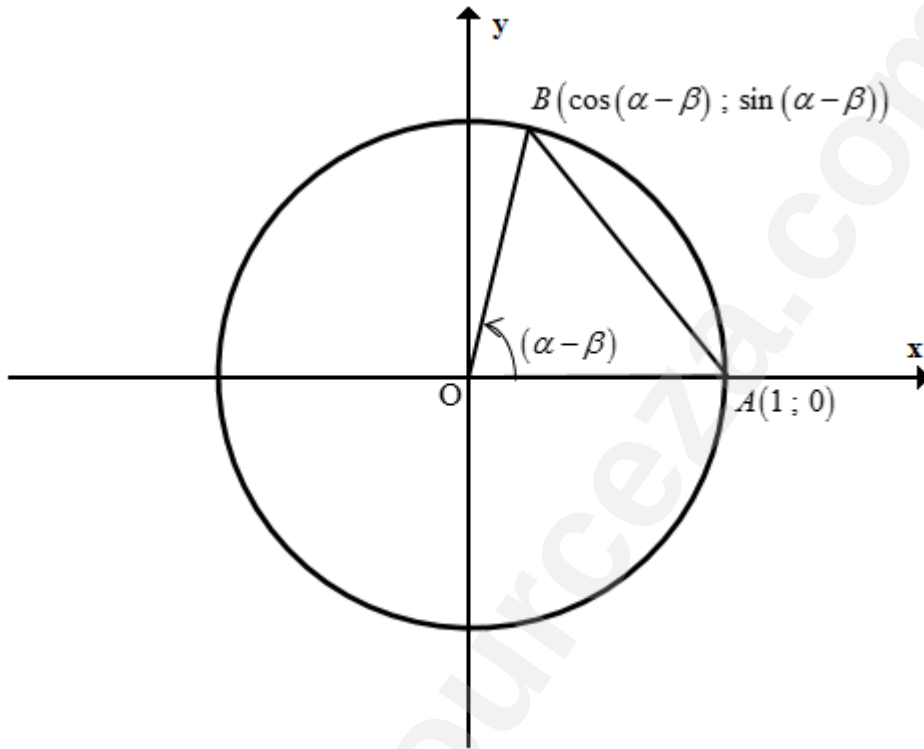
- 1.2 Consider the following unit circle sketched below. By the definition of trigonometric functions, the points P and Q on the terminal sides of angles α and β are labelled as shown in figure below.



Use the distance formula to show that

$$PQ^2 = 2 - 2(\cos \alpha \cos \beta + \sin \alpha \sin \beta) \quad (5)$$

- 1.3 Consider ΔPOQ in the sketched above. Now place angle $(\alpha - \beta)$ in standard position and label the point B on the unit circle corresponding to the terminal side of angle $(\alpha - \beta)$, as shown in the figure below:



Use the distance formula to show that

$$AB^2 = 2 - 2\cos(\alpha - \beta) \quad (4)$$

- 1.4 Use 1.2 and 1.3 above to prove that $\Delta POQ \equiv \Delta BOA$ (4)
- 1.5 Hence, compare sides PQ and AB and write a conclusion about $\cos(\alpha - \beta)$. (4)
- 1.6 Use $\cos(\alpha - \beta)$ to derive a formula for $\sin(\alpha + \beta)$. (3)
- 1.7 By applying the compound angles and without using a calculator. Determine the value of:

1.7.1 $\cos 15^\circ$ (3)

1.7.2 $\sin 105^\circ$ (3)

[29]

QUESTION 2

Using your CALCULATOR, go to TABLE mode.

2.1 Insert the function $f(x) = 2\cos^2 x - 1$

- Start: -180° ; End: 180°
- Step 45°

2.1.1 Complete the following table:

x	-180°	-135°	-90°	-45°	0°	45°	90°	135°	180°
$2\cos^2 x - 1$									

(1)

2.1.2 Use the table to sketch the graph of $f(x) = 2\cos^2 x - 1$ where $x \in [-180^\circ ; 180^\circ]$.

(3)

2.2 Using your calculator insert the following function $g(x) = 1 - 2\sin^2 x$

2.2.1 Complete the following table:

x	-180°	-135°	-90°	-45°	0°	45°	90°	135°	180°
$1 - 2\sin^2 x$									

(1)

2.2.2 Use the table to sketch the graph of $g(x) = 1 - 2\sin^2 x$ where $x \in [-180^\circ ; 180^\circ]$.

(3)

2.3 Using your calculator insert the following function $j(x) = \cos^2 x - \sin^2 x$

2.3.1 Complete the following table:

x	-180°	-135°	-90°	-45°	0°	45°	90°	135°	180°
$\cos^2 x - \sin^2 x$									

(1)

2.3.2 Use the table to sketch the graph of where $h(x) = \cos 2x$ $x \in [-180^\circ ; 180^\circ]$.

(3)

2.4 Using your calculator insert the following function $j(x) = \cos 2x$

2.4.1 Complete the following table:

x	-180°	-135°	-90°	-45°	0°	45°	90°	135°	180°
$\cos 2x$									

(1)

2.4.2 Use the table to sketch the graph of $h(x) = \cos 2x$ where $x \in [-180^\circ; 180^\circ]$.

(3)

2.5 If the graphs f , g and h were drawn on the set of axes as j what will your observation be?

(1)

2.6 Make a comparison of the four expressions.

(2)

[19]

QUESTION 3

Using your CALCULATOR, go to TABLE mode.

3.1 Using your calculator insert the following function $u(x) = 2 \sin x \cos x$

3.1.1 Complete the following table:

x	-180°	-135°	-90°	-45°	0°	45°	90°	135°	180°
$2 \sin x \cos x$									

(1)

3.1.2 Use the table to sketch the graph of $u(x) = 2 \sin x \cos x$ where $x \in [-180^\circ; 180^\circ]$.

(3)

3.2 Using your calculator insert the following function $v(x) = \sin 2x$

3.2.1 Complete the following table:

x	-180°	-135°	-90°	-45°	0°	45°	90°	135°	180°
$\sin 2x$									

(1)

3.2.2 Use the table to sketch the graph of $v(x) = \sin 2x$ where $x \in [-180^\circ; 180^\circ]$.

(3)

3.3 If the graph of u was drawn on the same set of axes as v , what will you observe?

(1)

3.4 Make a comparison of the two expressions.

(1)

[10]

QUESTION 4**APPLICATION**

4.1 Determine without using a calculator, the value of

$$\frac{4 \sin 15^\circ \cos 15^\circ}{\sqrt{2} \cos 405^\circ} \quad (4)$$

4.2 Without the use of tables or a calculator

4.2.1 Prove that $\frac{1 - \cos 2A}{\sin 2A} = \tan A$ (3)

4.2.2 Hence calculate, the value of $\tan 15^\circ$ (4)

4.3 Given that $\sqrt{2} \sin(x + 45^\circ) = \sin x + \cos x$

4.3.1 Prove this identity. (3)

4.3.2 Hence deduce the minimum value of $\sin x + \cos x$ and give the corresponding values of x , for $x \in [-360^\circ; 360^\circ]$. (3)

[17]

TOTAL:

[75]

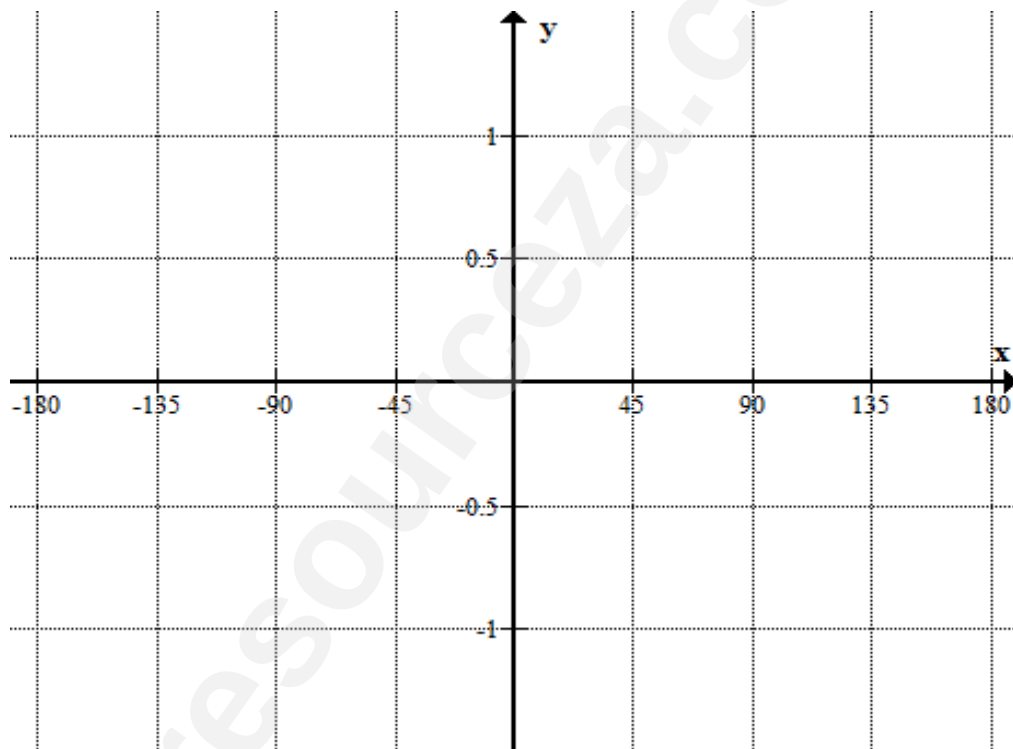
NAME OF LEARNER:

CLASS:

2.1.1 Complete the following table:

x	-180°	-135°	-90°	-45°	0°	45°	90°	135°	180°
$2\cos^2 x - 1$									

2.1.2



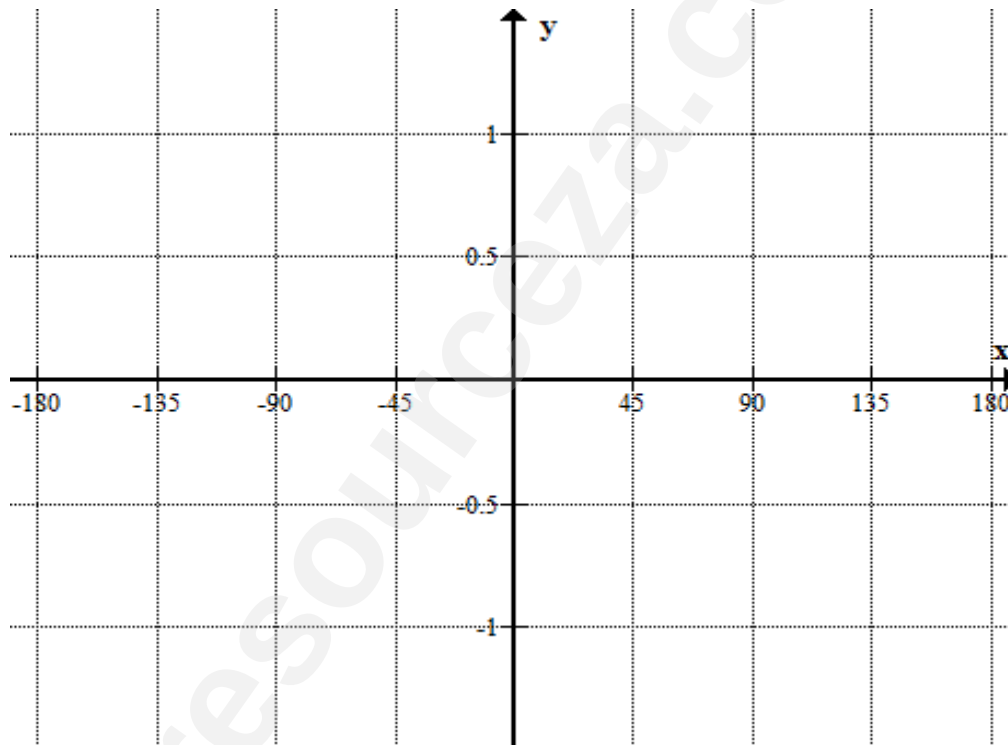
NAME OF LEARNER:

CLASS:

2.2.1 Complete the following table:

x	-180°	-135°	-90°	-45°	0°	45°	90°	135°	180°
$1 - 2\sin^2 x$									

2.2.2



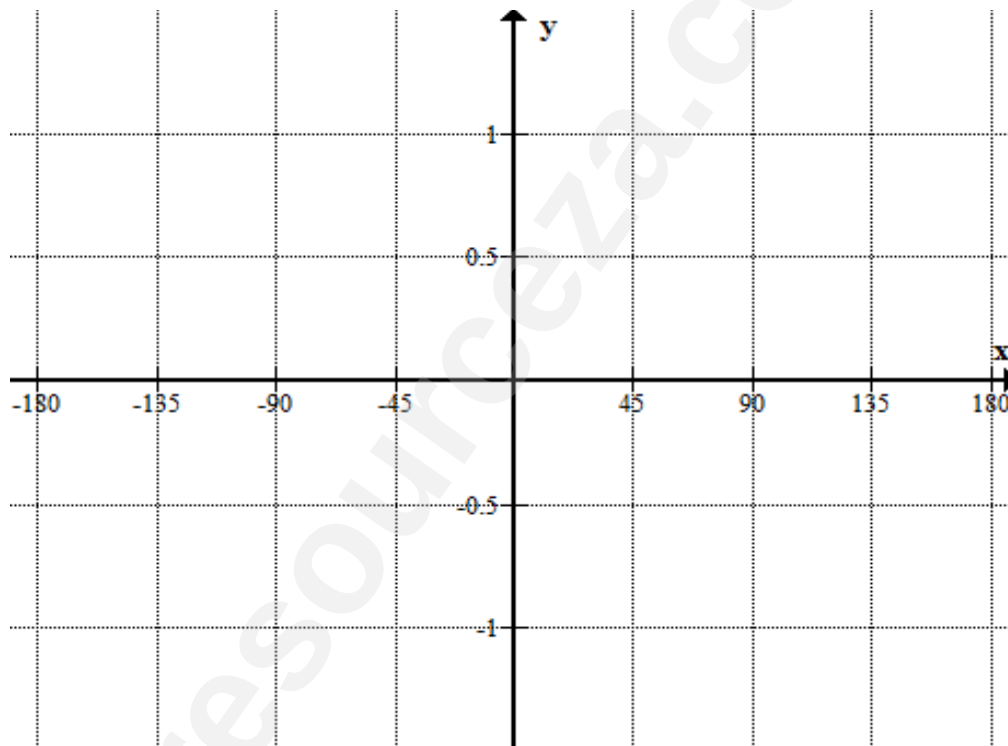
NAME OF LEARNER:

CLASS:

2.3.1 Complete the following table:

x	-180°	-135°	-90°	-45°	0°	45°	90°	135°	180°

2.3.2

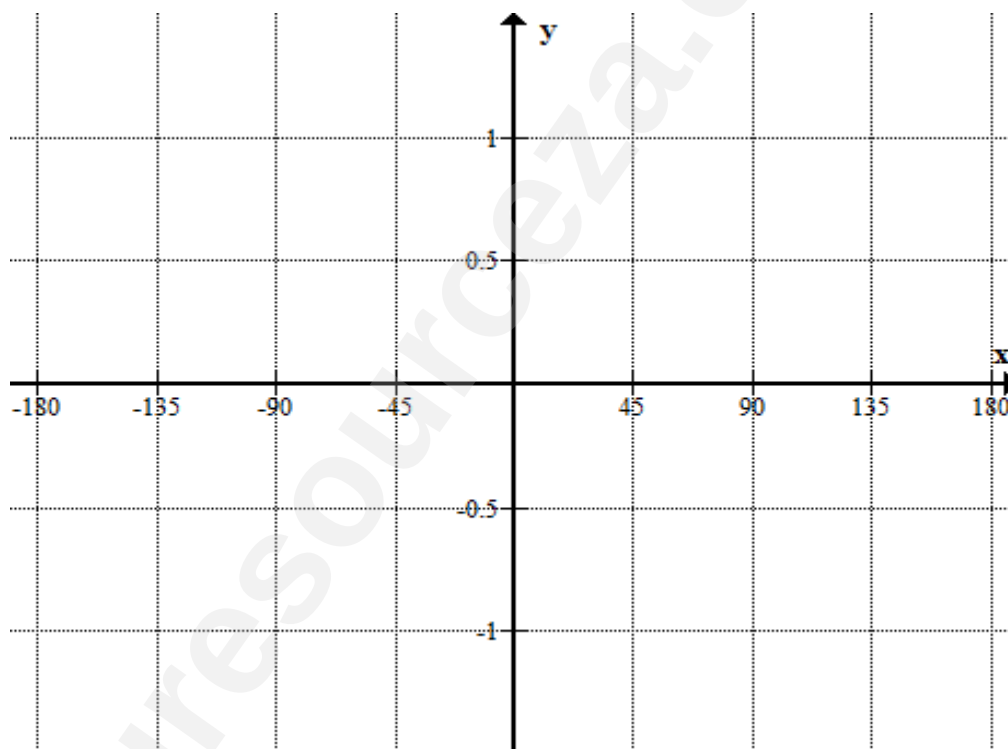


NAME OF LEARNER: CLASS:

2.4.1 Complete the following table:

x	-180°	-135°	-90°	-45°	0°	45°	90°	135°	180°
$\cos 2x$									

2.4.2

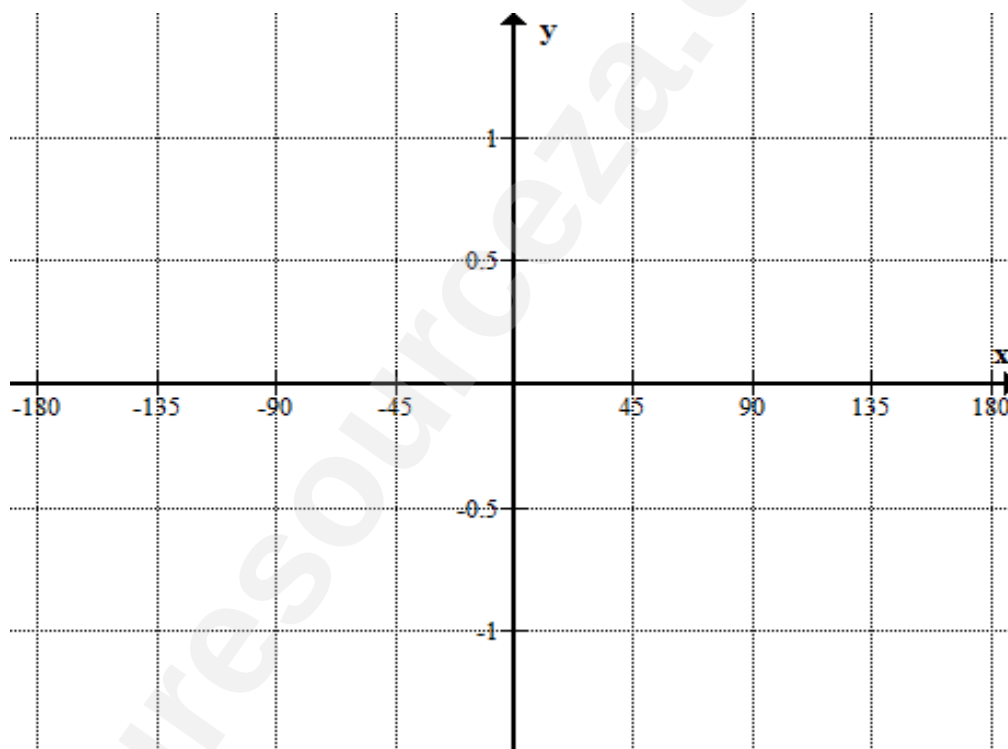


NAME OF LEARNER: CLASS:

3.1.1 Complete the following table:

x	-180°	-135°	-90°	-45°	0°	45°	90°	135°	180°
$2 \sin x \cos x$									

3.1.2



NAME OF LEARNER: CLASS:

3.2.1 Complete the following table:

x	-180°	-135°	-90°	-45°	0°	45°	90°	135°	180°
$\sin 2x$									

3.2.2

