



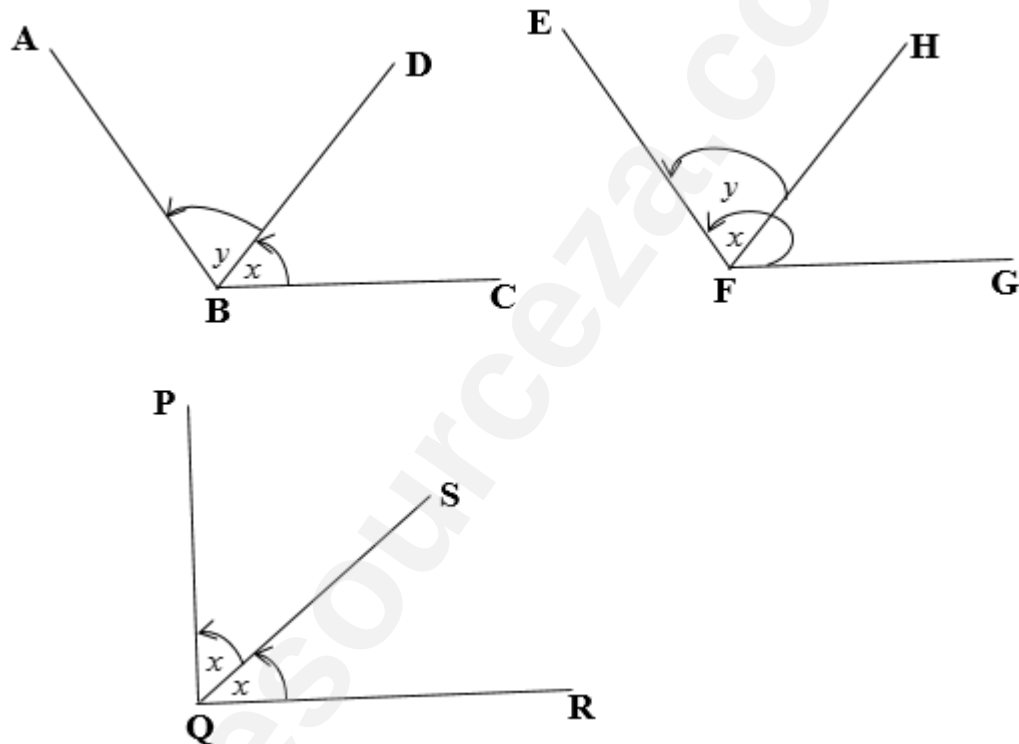
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INVESTIGATING COMPOUND ANGLES AND THEREFORE, DOUBLE ANGLES.

QUESTION 1

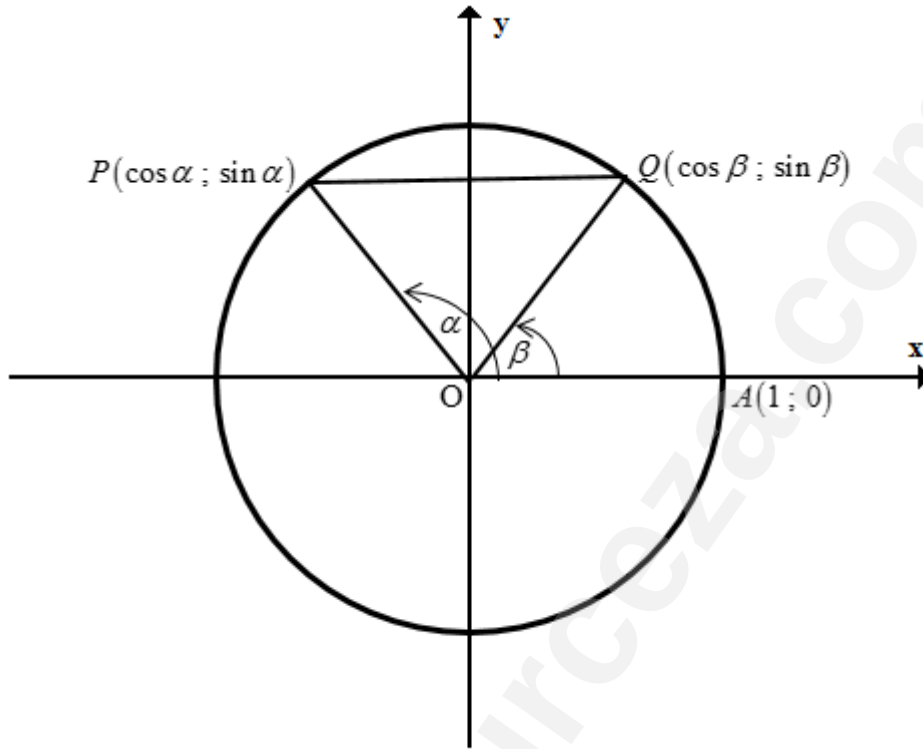
1.2 Compound and double angles?

Consider the following diagrams



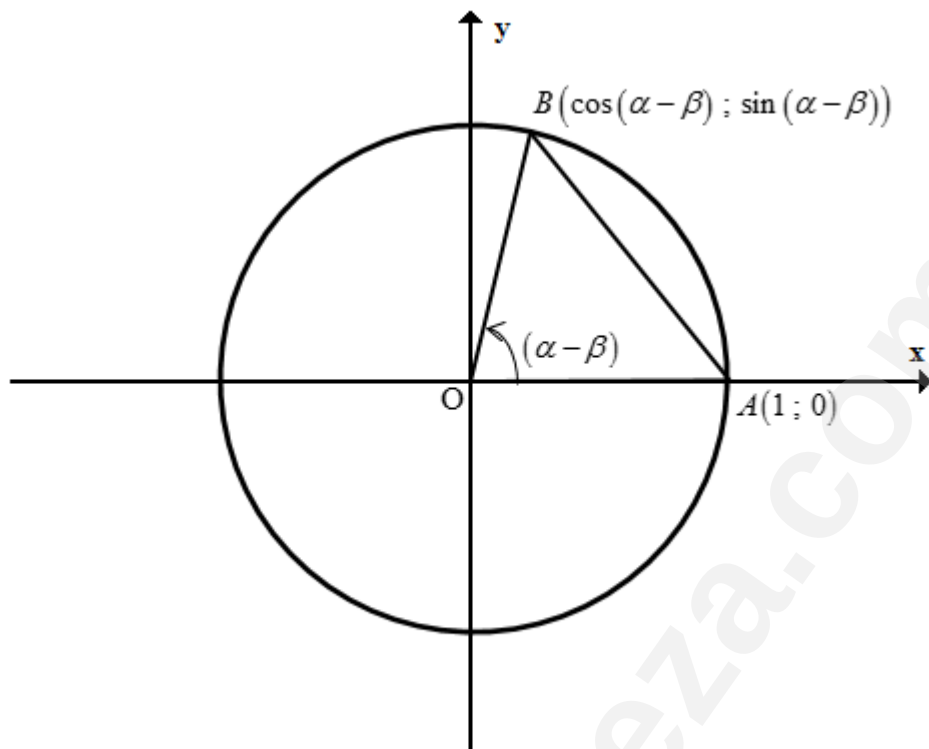
1.1.1	$\angle ABC = x + y$	(1)	✓ answer
1.1.2	$\angle HFG = x - y$	(1)	✓ answer
1.1.3	$\angle PQR = 2x$	(1)	✓ answer

- 1.2 Consider the following unit circle sketched below. By the definition of trigonometric functions, the points P and Q on the terminal sides of angles α and β are labelled as shown in figure below.



1.2	$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ $PQ^2 = (\cos \alpha - \cos \beta)^2 + (\sin \alpha - \sin \beta)^2$ $= \cos^2 \alpha - 2 \cos \alpha \cos \beta + \cos^2 \beta$ $+ \sin^2 \alpha - 2 \sin \alpha \sin \beta + \sin^2 \beta$ $= \cos^2 \alpha + \sin^2 \alpha - 2 \cos \alpha \cos \beta - 2 \sin \alpha \sin \beta$ $+ \cos^2 \beta + \sin^2 \beta$ $= 1 - 2(\cos \alpha \cos \beta + \sin \alpha \sin \beta) + 1$ $= 2 - 2(\cos \alpha \cos \beta + \sin \alpha \sin \beta)$	(5)	✓ substitution ✓ expansion ✓✓ use of $\cos^2 \theta + \sin^2 \theta = 1$ ✓ simplification
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- 1.3 Consider ΔPOQ in the sketched above. Now place angle $\alpha - \beta$ in standard position and label the point B on the unit circle corresponding to the terminal side of angle $(\alpha - \beta)$, as shown in the figure below:



1.3	$AB^2 = (\cos(\alpha - \beta) - 1)^2 + (\sin(\alpha - \beta) - 0)^2$ $= \cos^2(\alpha - \beta) - 2\cos(\alpha - \beta) + 1 + \sin^2(\alpha - \beta)$ $= 1 - 2\cos(\alpha - \beta) + 1$ $= 2 - 2\cos(\alpha - \beta)$	(4)	✓ correct substitution ✓ correct expansion ✓ use of identities ✓ simplification										
1.4	In $\triangle POQ \equiv \triangle BOA$ <table><tr><th>Statement</th><th>Reason</th></tr><tr><td>$OP = OB$</td><td>both radii = 1</td></tr><tr><td>$OQ = OA$</td><td>both radii = 1</td></tr><tr><td>$\hat{POQ} = \hat{BOA}$</td><td>both = $\alpha - \beta$</td></tr><tr><td>$\therefore \triangle POQ \equiv \triangle BOA$</td><td>SAS</td></tr></table>	Statement	Reason	$OP = OB$	both radii = 1	$OQ = OA$	both radii = 1	$\hat{POQ} = \hat{BOA}$	both = $\alpha - \beta$	$\therefore \triangle POQ \equiv \triangle BOA$	SAS	(4)	✓ S/ R ✓ S/ R ✓ S/ R ✓ conclusion
Statement	Reason												
$OP = OB$	both radii = 1												
$OQ = OA$	both radii = 1												
$\hat{POQ} = \hat{BOA}$	both = $\alpha - \beta$												
$\therefore \triangle POQ \equiv \triangle BOA$	SAS												

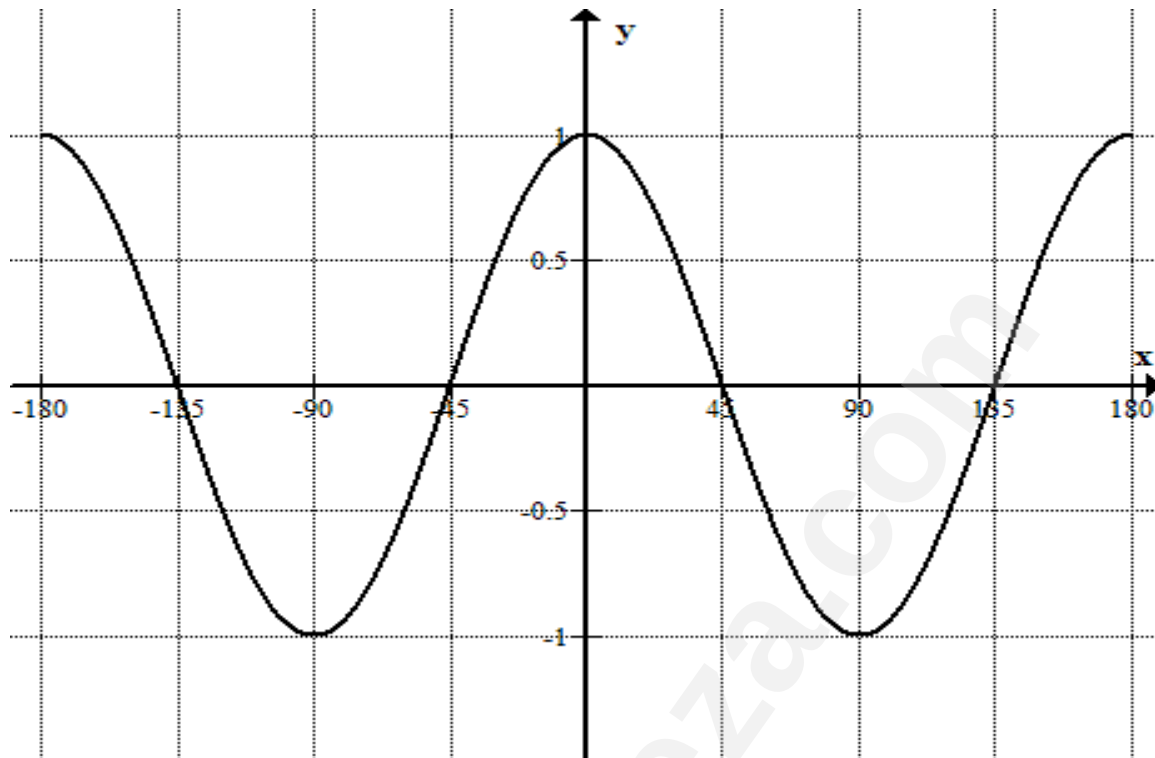
1.5	PQ = AB from congruency Therefore	(4)	✓ S/ R ✓ $PQ^2 = AB^2$
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	$AB^2 = PQ^2$ $2 - 2\cos(\alpha - \beta) = 2 - 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$ $-2\cos(\alpha - \beta) = -2(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$ $\therefore \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$		✓ substitution ✓ simplification
1.6	$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ $\sin(\alpha + \beta) = \cos[(90^\circ - (\alpha + \beta))]$ $= \cos[(90^\circ - \alpha) - \beta]$ $= \cos(90^\circ - \alpha)\cos\beta + \sin(90^\circ - \alpha)\sin\beta$ $= \sin\alpha\cos\beta + \cos\alpha\sin\alpha$	(3)	✓ $\cos(\alpha + \beta) = \cos[\alpha - (-\beta)]$ ✓ expansion ✓ answer
1.7.1	$\cos 15^\circ = \cos(45^\circ - 30^\circ)$ or $\cos 15^\circ = \cos(60^\circ - 45^\circ)$ $= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$ $= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \times \frac{1}{2}$ $= \frac{\sqrt{6} + \sqrt{2}}{4}$	(3)	✓ correct expansion ✓ correct substitution ✓ answer
1.7.2	$\sin 105^\circ = \sin(60^\circ + 45^\circ)$ $= \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$ $= \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} + \frac{1}{2} \times \frac{\sqrt{2}}{2}$ $= \frac{\sqrt{6} + \sqrt{2}}{4}$	(3)	✓ correct expansion ✓ correct substitution ✓ answer
		[29]	

QUESTION 2

2.1.1										
x	-180°	-135°	-90°	-45°	0°	45°	90°	135°	180°	
$2\cos^2 x - 1$	1	0	-1	0	1	0	-1	0	1	✓ answer (1)

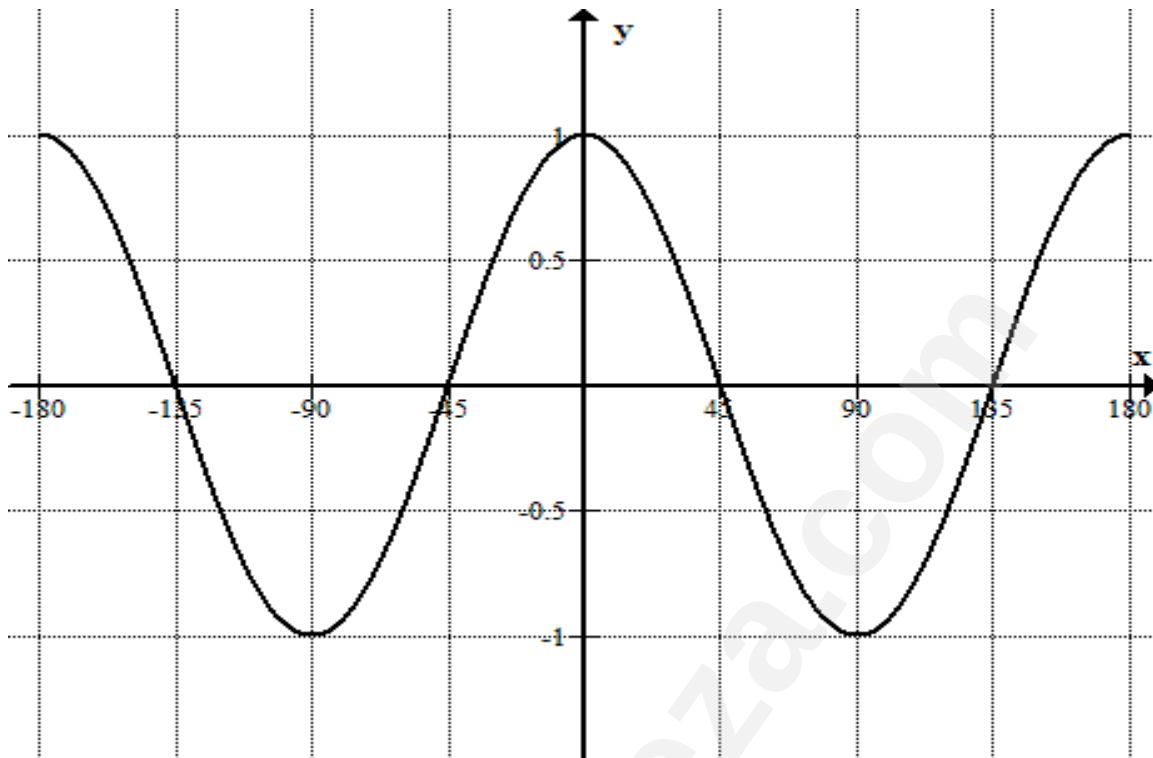
2.1.2 Use the table to sketch the graph of $f(x) = 2\cos^2 x - 1$ where $x \in [-180^\circ; 180^\circ]$.



2.1.2	Refer to diagram	(3)	✓ shape ✓ all x- intercepts ✓ all turning points
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2.2.1										
x	-180°	-135°	-90°	-45°	0°	45°	90°	135°	180°	
$1 - 2\sin^2 x$	1	0	-1	0	1	0	-1	0	1	✓ answer (1)

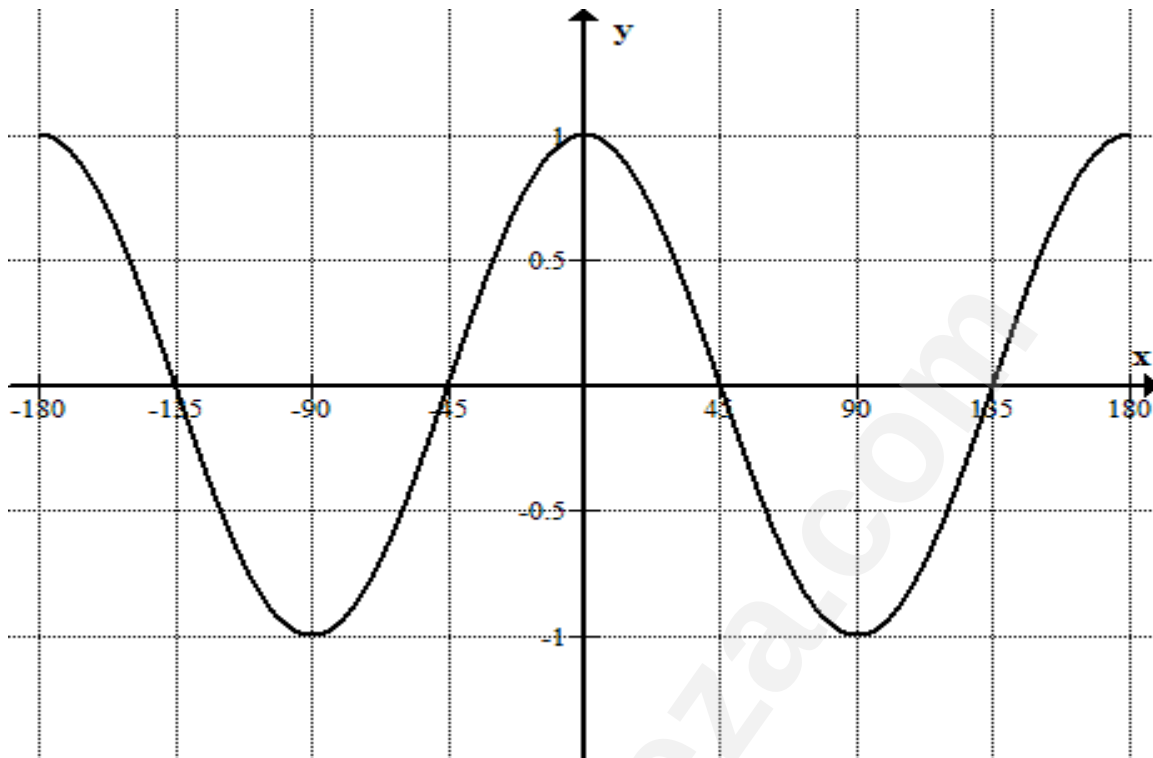
2.2.2 Use the table to sketch the graph of $g(x) = 1 - 2\sin^2 x$ where $x \in [-180^\circ ; 180^\circ]$.



2.2.2	Refer to the diagram	(3)	✓ shape ✓ all x- intercepts ✓ all turning points
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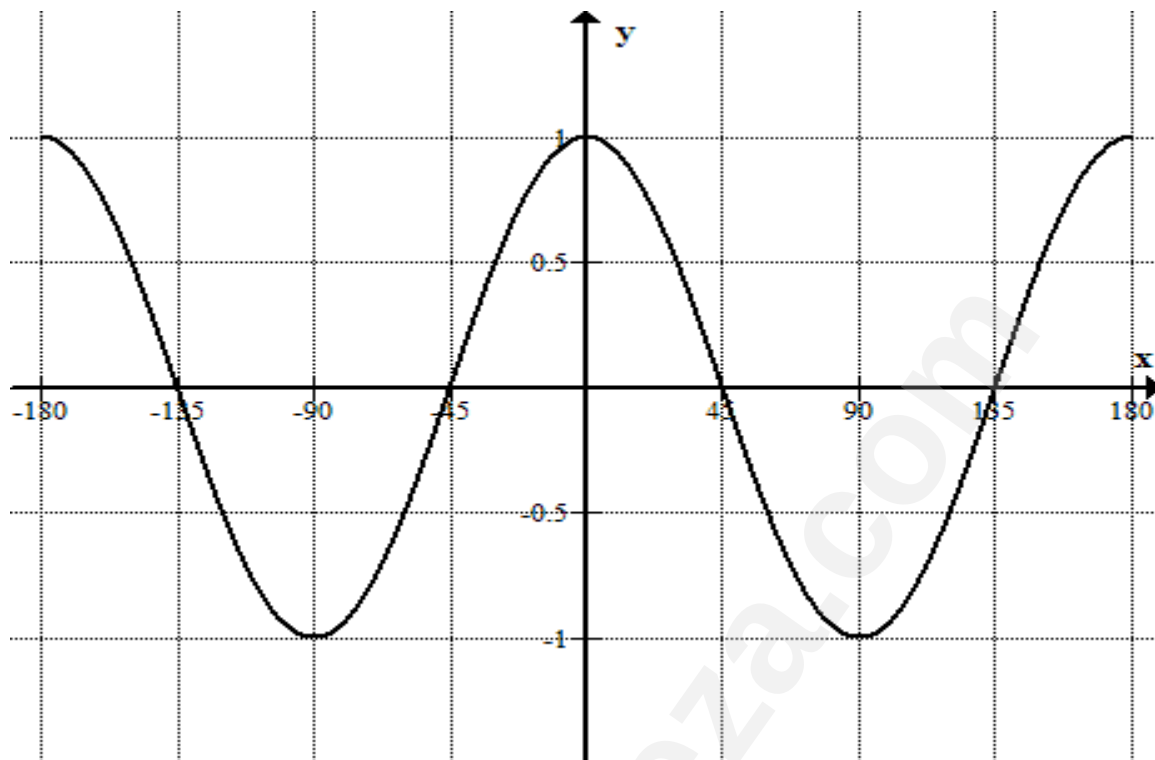
2.3.1										
x	-180°	-135°	-90°	-45°	0°	45°	90°	135°	180°	
$\cos 2x$	1	0	-1	0	1	0	-1	0	1	✓ answer (1)

2.3.2 Use the table to sketch the graph of $h(x) = \cos 2x$ where $x \in [-180^\circ ; 180^\circ]$.



2.4.1										
x	-180°	-135°	-90°	-45°	0°	45°	90°	135°	180°	
$\cos^2 x - \sin^2 x$	1	0	-1	0	1	0	-1	0	1	✓answer
										(1)

2.4.2 Use the table to sketch the graph of $h(x) = \cos 2x$ where $x \in [-180^\circ ; 180^\circ]$.

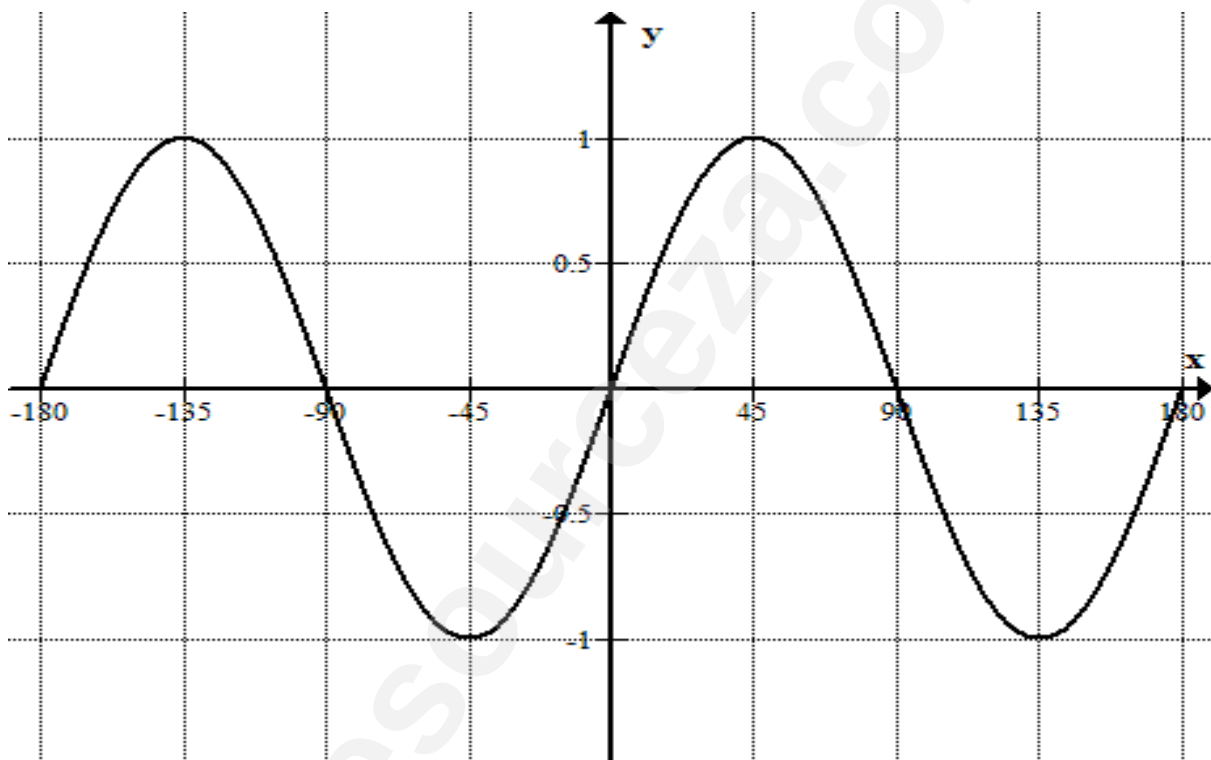


2.4.2	Refer to the diagram	(3)	✓ shape ✓ all x- intercepts ✓ all turning points
2.5	f and g will coincide with h	(1)	✓ answer
2.6	$\cos 2x = 2 \cos^2 x - 1$ $\cos 2x = 1 - 2 \sin^2 x$	(2)	✓ answer ✓ answer
		[19]	

QUESTION 3

3.1.1										
x	-180°	-135°	-90°	-45°	0°	45°	90°	135°	180°	
$2 \sin x \cos x$	0	1	0	-1	0	1	0	-1	0	✓ answer (1)

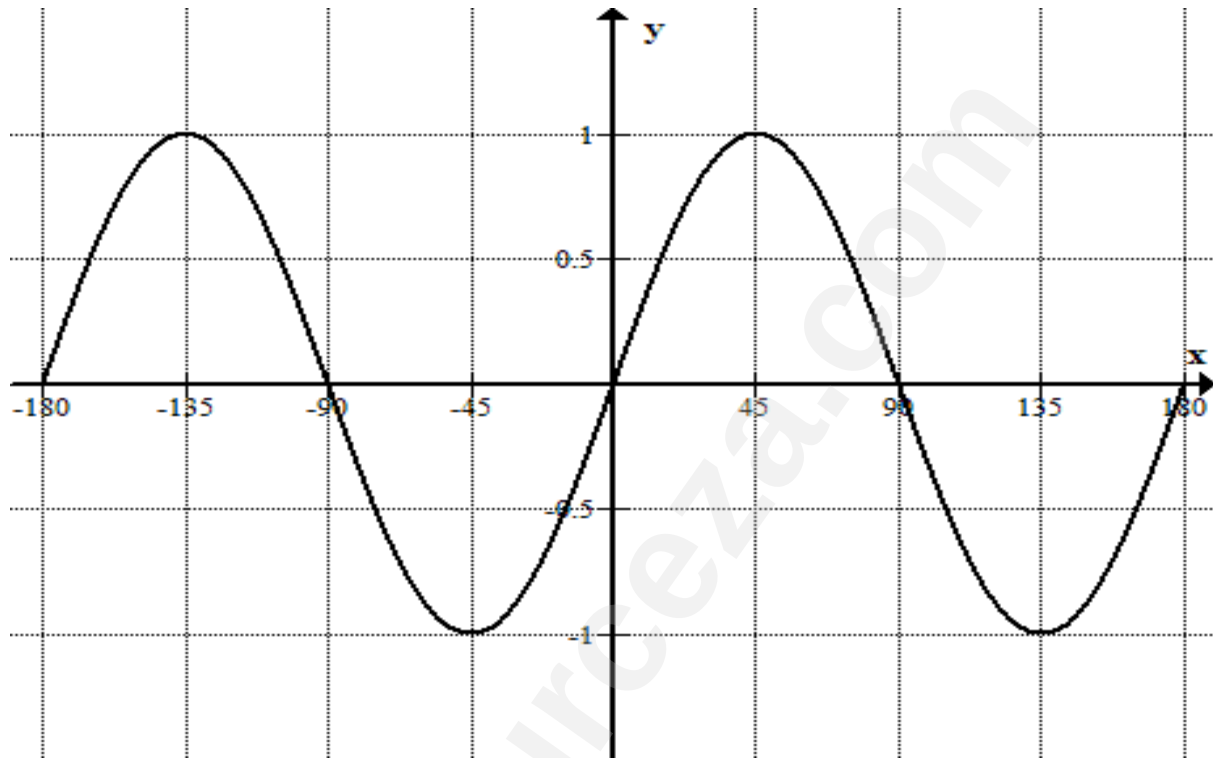
3.1.2 Use the table to sketch the graph of $u(x) = 2 \sin x \cos x$ where $x \in [-180^\circ; 180^\circ]$.



3.1.2	Refer to the diagram	(3)	✓ shape ✓ all x- intercepts ✓ all turning points
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3.2.1										
x	-180°	-135°	-90°	-45°	0°	45°	90°	135°	180°	
$\sin 2x$	0	1	0	-1	0	1	0	-1	0	✓ answer (1)

3.2.2 Use the table to sketch the graph of $v(x) = \sin 2x$ where $x \in [-180^\circ; 180^\circ]$.



3.2.2	Refer to the diagram	(3)	✓ shape ✓ all x - intercepts ✓ all turning points
3.3	u will coincide with v	(1)	✓ answer
3.4	$\sin 2x = 2 \sin x \cos x$	(1)	✓ answer
		[10]	

QUESTION 4

APPLICATION

4.1	$\frac{4 \sin 15^\circ \cos 15^\circ}{\sqrt{2} \cos 405^\circ}$ $= \frac{2 \times 2 \sin 15^\circ \cos 15^\circ}{\sqrt{2} \cos (360^\circ + 45^\circ)}$ $= \frac{2 \times \sin 30^\circ}{\sqrt{2} \cos 45^\circ}$ $= \frac{2 \times \frac{1}{2}}{\sqrt{2} \times \frac{\sqrt{2}}{2}}$ $= 1$	(4)	✓ $\sin 30^\circ$ ✓ $\cos 45^\circ$ ✓ correct substitution ✓ answer
4.2.1	$\text{LHS} = \frac{1 - \cos 2A}{\sin 2A}$ $= \frac{1 - (1 - 2 \sin^2 A)}{2 \sin A \cos A}$ $= \frac{2 \sin^2 A}{2 \sin A \cos A}$ $= \frac{\sin A}{\cos A}$ $= \tan A$ $\therefore \text{LHS} = \text{RHS}$	(3)	✓ $1 - 2 \sin^2 A$ ✓ $2 \sin A \cos A$ ✓ $\frac{\sin A}{\cos A}$
4.2.2	$\tan 15^\circ = \frac{1 - \cos 2(15^\circ)}{\sin 2(15^\circ)}$ $= \frac{1 - \cos 30^\circ}{\sin 30^\circ}$ $= \frac{1 - \frac{\sqrt{3}}{2}}{\frac{1}{2}}$ $= 2 - \sqrt{3}$	(4)	✓ $\frac{1 - \cos 2(15^\circ)}{\sin 2(15^\circ)}$ ✓ $\frac{1 - \cos 30^\circ}{\sin 30^\circ}$ ✓ correct substitution ✓ answer
4.3.1		(3)	✓ correct expansion

	$\text{LHS} = \sqrt{2} \sin(x + 45^\circ)$ $= \sqrt{2} (\sin x \cos 45^\circ + \cos x \sin 45^\circ)$ $= \sqrt{2} \left(\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x \right)$ $= \frac{\sqrt{2}}{1} \times \frac{1}{\sqrt{2}} (\sin x + \cos x)$ $= \sin x + \cos x$ $\therefore \text{LHS} = \text{RHS}$		✓ correct substitution ✓ factoring
4.3.2	Minimum $\sqrt{2}(-1) = -\sqrt{2}$ $\sqrt{2} \sin(x + 45^\circ) = -\sqrt{2}$ $\sin(x + 45^\circ) = -1$ $x + 45^\circ = 270^\circ + 360^\circ.k$ $x = 225^\circ + 360^\circ.k$ $x = 225^\circ \quad x = -135^\circ$	(3)	✓ $-\sqrt{2}$ ✓ -135° ✓ 225°
		[17]	
	TOTAL:	[75]	