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SECONDARY SCHOOL IMPROVEMENT PROGRAMME (SSIP) 2016



GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

GRADE 12

SUBJECT: MATHEMATICS

LEARNER NOTES

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SESSION NO: 6

TOPIC: CALCULUS

Learner note:

Calculus forms a large part of Paper 1 and you can score well in questions involving this topic. Ensure that you don't lose marks by using faulty notation.

SECTION A: TYPICAL EXAM QUESTIONS**QUESTION 1 (17 MARKS) (10 MINUTES) (DoE papers pre-2014)**(a) Given: $f(x) = -2x^2 + 1$ (1) Determine $f'(x)$ from first principles. (5)(2) Determine the gradient of the graph at $x = -2$, i.e. $f'(-2)$ (1)(3) Determine $f(-2)$. What does your answer represent? (2)(4) Determine the average gradient of f between $x = -2$ and $x = 4$ (4)(b) Use first principles to determine the derivative of $f(x) = \frac{1}{x}$ (5)**QUESTION 2 (11 MARKS) (10 MINUTES) (DoE papers pre-2014)**(a) Differentiate f by using first principles if $f(x) = x^2 - 2x$. (5)(b) Determine the gradient of the tangent to the graph of $g(x) = x^3$ at $x = 3$ by using first principles. (6)**QUESTION 3 (14 MARKS) (10 MINUTES) (DoE papers pre-2014)**

Determine the following and leave your answer with positive exponents:

(a) $D_x [(2x - 3)(x + 4)]$ (2)(b) $f'(x)$ if $f(x) = \frac{1}{2\sqrt[4]{x^3}}$ (3)

$$(c) \quad \frac{dy}{dx} \text{ if } y = \left(2\sqrt{x} - \frac{1}{3x}\right)^2 \quad (5)$$

$$(d) \quad D_x \left[\frac{1}{\sqrt{x}} (x^3 - 2x^2 + 3x) \right] \quad (4)$$

QUESTION 4 (11 MARKS) (10 MINUTES) (DoE Nov 2014-2015)

$$(a) \quad \text{If } y = (x^6 - 1)^2, \text{ prove that } \frac{dy}{dx} = 12x^5 \cdot \sqrt{y} \text{ if } x > 1 \quad (3)$$

$$(b) \quad \text{Determine: } D_x \left[\frac{x^3 - 1}{x - 1} \right] \quad (3)$$

QUESTION 5 (11 MARKS) (10 MINUTES) (DoE papers pre-2014)

$$(a) \quad \text{Determine the equation of the tangent to } f(x) = x^2 - 6x + 5 \text{ at } x = 2. \quad (5)$$

$$(b) \quad \text{Find the equation of the tangent to } f(x) = 3x^2 - 5x + 1 \text{ which is parallel to the line } y - 7x + 4 = 0. \quad (6)$$

QUESTION 6 (19 MARKS) (10 MINUTES) (DoE Nov 2014)

$$(a) \quad \text{Sketch the graph of } f(x) = 2x^3 - 6x - 4 \quad (15)$$

$$(b) \quad \text{Given: } f(x) = 2x^3 - 2x^2 + 4x - 1$$

$$\text{Determine the interval on which } f \text{ is concave up.} \quad (4)$$

QUESTION 7 (7 MARKS) (5 MINUTES) (DoE papers pre-2014)

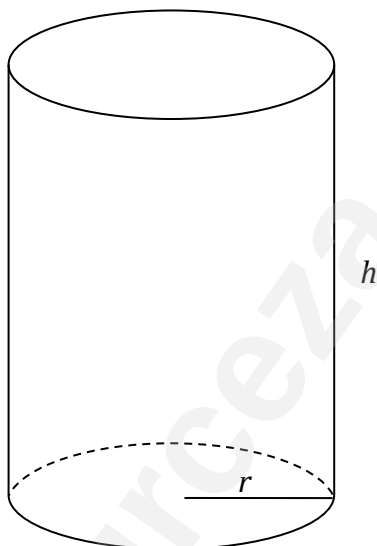
$$(2; 9) \text{ is a turning point on the graph of } f(x) = ax^3 + 5x^2 + 4x + b.$$

$$\text{Determine the value of } a \text{ and } b \text{ and hence the equation of the cubic function.} \quad (7)$$

QUESTION 8 (9 MARKS) (10 MINUTES) (DoE papers pre-2014)

A drinking glass, in the shape of a cylinder, must hold 200 ml of liquid when full.

- (a) Show that the height of the glass, h , can be expressed as $h = \frac{200}{\pi r^2}$. (2)
- (b) Show that the surface area of the glass can be expressed as $S(r) = \pi r^2 + \frac{400}{r}$. (2)
- (c) Hence determine the value of r for which the total surface area of the glass is a minimum. (5)



SECTION B: NOTES ON CONTENT

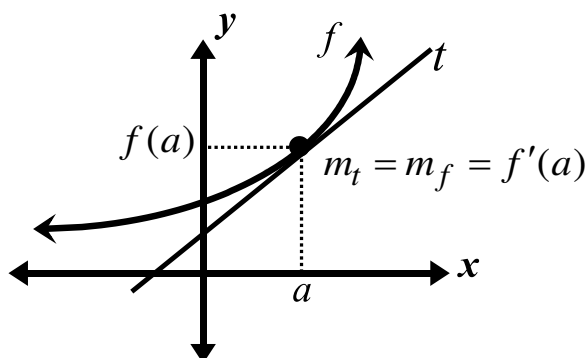
The most important fact in Calculus is that the gradient of the tangent to a curve at a given point is the gradient of the curve at that point.

Other words for gradient are: rate of change, derivative, slope

Symbols for gradient are: $f'(x)$ D_x $\frac{dy}{dx}$

$f'(a)$ is the gradient of f at $x = a$

$f(a)$ is the y-value corresponding to $x = a$



Average gradient

The average gradient (or average rate of change) of a function f between $x = a$ and $x = b$ and is defined to be the gradient of the line joining the points on the graph of the function. We say that the average gradient of f over the interval is the gradient of the line AB.

Gradient of a curve at a point using first principles

The formula to determine the gradient of a function from first principles is given by the following limit:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

The gradient of a function using the rules of differentiation

You will be required to use the following rules of differentiation to determine the gradient of a function.

Rule 1 If $f(x) = ax^n$, then $f'(x) = a.nx^{n-1}$

Rule 2 If $f(x) = ax$, then $f'(x) = a$

Rule 3 If $f(x) = \text{number}$, then $f'(x) = 0$

Determining the equation of the tangent to a curve at a point

The gradient of the tangent to a curve at a point is the derivative at that point.

The equation is given by $y - y_1 = m(x - x_1)$ where $(x_1; y_1)$ is the point of tangency and $m = f'(x_1)$

Rules for sketching the graph of a cubic function

The graph of the form $f(x) = ax^3 + bx^2 + cx + d$ is called a cubic function.

The main concepts involved with these functions are as follows:

Intercepts with the axes:

For the y-intercept, let $x = 0$ and solve for y

For the x-intercepts, let $y = 0$ and solve for x

(you might have to use the factor theorem here)

Stationary points:

Determine $f'(x)$, equate it to zero and solve for x .

Then substitute the x -values of the stationary points into the original equation to obtain the corresponding y -values.

If the function has two stationary points, establish whether they are maximum or minimum turning points.

Points of inflection:

If the cubic function has only one stationary point, this point will be a point of inflection that is also a stationary point.

For points of inflection that are not stationary points, find $f''(x)$, equate it to 0 and solve for x . Alternatively, simply add up the x -coordinates of the turning points and divide by 2 to get the x -coordinate of the point of inflection.

Concavity:

The graph of f is concave up if $f''(x) > 0$

The graph of f is concave down if $f''(x) < 0$

SECTION C: HOMEWORK QUESTIONS

QUESTION 1 (18 MARKS) (15 MINUTES) (DoE papers pre-2014)

- (a) Given: $f(x) = 1 - \frac{1}{4}x^2$
- (1) Determine $f'(x)$ from first principles. (6)
 - (2) Determine the gradient of the graph at $x = -4$, i.e. $f'(-4)$ (1)
 - (3) Determine $f(-2)$. What does your answer represent? (2)
 - (4) Determine the average gradient of f between $x = -2$ and $x = 4$ (4)
- (b) Use first principles to determine the derivative of $f(x) = -\frac{3}{x}$ (5)

QUESTION 2 (10 MARKS) (5 MINUTES) (DoE papers pre-2014)

- (a) Differentiate f by first principles where $f(x) = -2x$. (4)
- (b) Determine the gradient of the tangent to the graph of $g(x) = -2x^3$ at $x = 2$ by using first principles. (6)

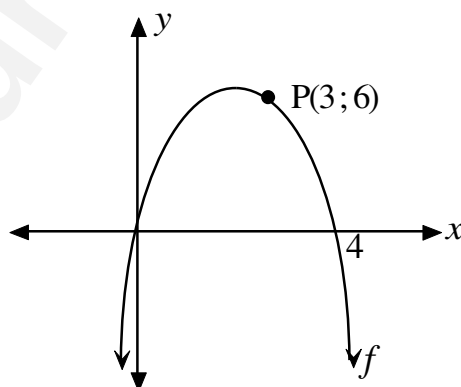
QUESTION 3 (14 MARKS) (10 MINUTES) (DoE papers pre-2014)

Determine the following and leave your answer with positive exponents:

- (a) $f'(x)$ if $f(x) = (4x - 3)^2$ (2)
- (b) $D_x \left[\sqrt[3]{x} + \frac{1}{\sqrt{x}} \right]$ (3)
- (c) $D_x \left[(x^2 - \sqrt{x})^2 \right]$ (5)
- (d) $\frac{dy}{dx}$ if $y = \frac{2x^2 - \sqrt{x} + 5}{\sqrt{x}}$ (4)

QUESTION 4 (11 MARKS) (15 MINUTES) (DoE papers pre-2014)

- (a) Determine the equation of the tangent to the curve $y = 3x^2 - 2x + 2$ at $x = -4$. (5)
- (b) The graph of $f(x) = ax^2 + bx$ passes through the point $P(3; 6)$ and cuts the x -axis at $(4; 0)$. Determine the equation of the tangent to f at P . (6)



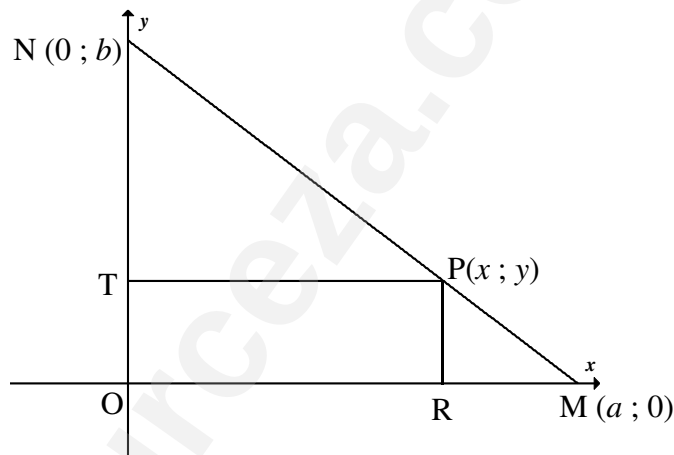
QUESTION 5 (15 MARKS) (10 MINUTES) (DoE papers pre-2014)

Sketch the graph of $f(x) = x^3 - 3x^2 + 4$

Indicate the coordinates of the stationary points, intercepts with the axes and any points of inflection. (15)

QUESTION 6 (8 MARKS) (10 MINUTES) (DoE papers pre-2014)

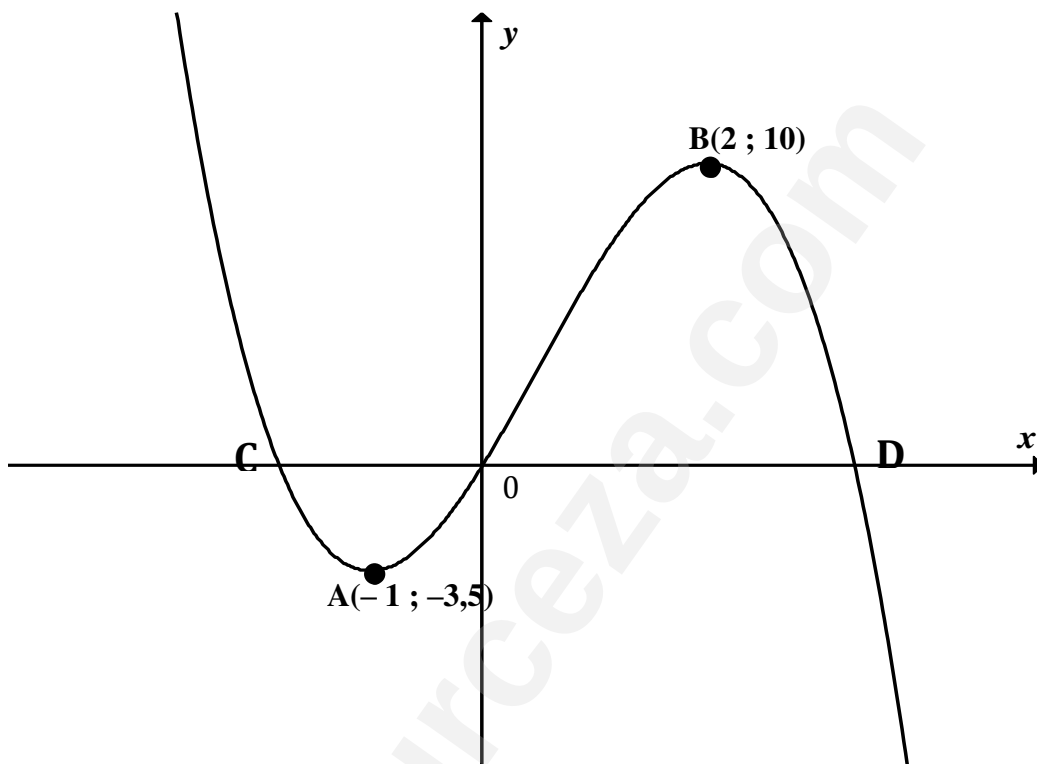
A farmer has a piece of land in the shape of a right-angled triangle OMN, as shown in the figure below. He allocates a rectangular piece of land PTOR to his daughter, giving her the freedom to choose P anywhere along the boundary MN. Let $OM = a$, $ON = b$ and $P(x; y)$ be any point on MN.



- (a) Determine an equation of MN in terms of a and b . (2)
- (b) Prove that the daughter's land will have a maximum area if she chooses P at the midpoint of MN. (6)

QUESTION 7 **(17 MARKS)** **(15 MINUTES)** **(DoE papers pre-2014)**

The graph of $h(x) = -x^3 + ax^2 + bx$ is shown below. $A(-1; -3,5)$ and $B(2; 10)$ are the turning points of h . The graph passes through the origin and further cuts the x -axis at C and D.



- Show that $a = \frac{3}{2}$ and $b = 6$ (7)
- Calculate the average gradient between A and B. (2)
- Determine the equation of the tangent to h at $x = -2$. (5)
- Determine the x -value of the point of inflection of h . (3)

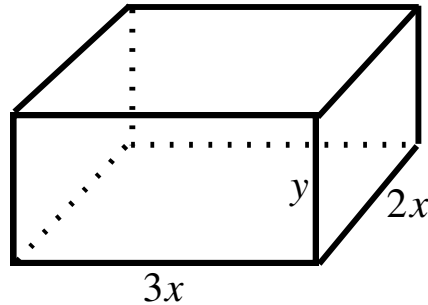
QUESTION 8

(9 MARKS)

(10 MINUTES)

(DoE papers pre-2014)

The sides of the base of a rectangular cardboard box are $3x$ and $2x$ cm respectively. The height is y cm. The box is open on the top (without a lid).



- (a) If the total exterior surface area of the box is 200cm^2 , prove that:
$$y = \frac{20}{x} - \frac{3x}{5} \quad (3)$$
- (b) Express the volume of the box in terms of x . (2)
- (c) Determine the value of x if its volume is to be a maximum. (3)

SESSION NO: 7

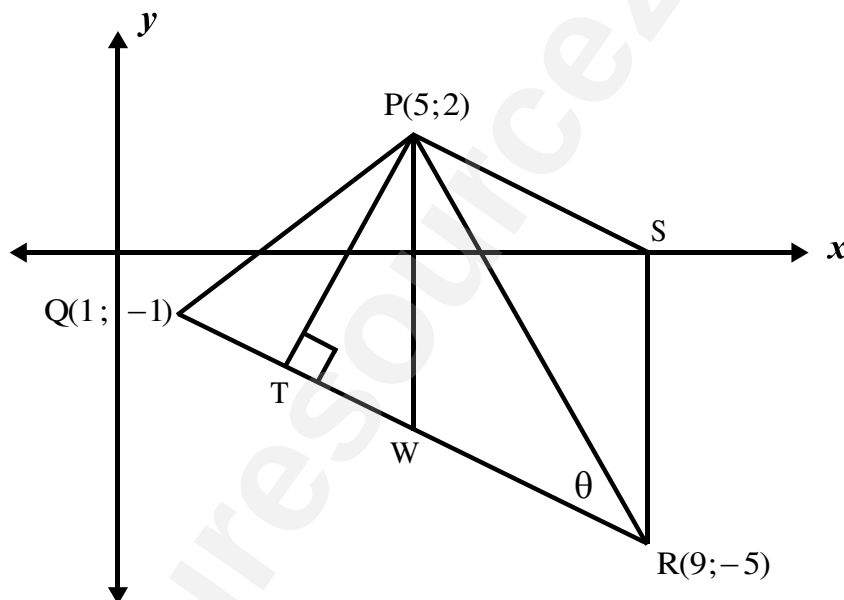
TOPIC: ANALYTICAL GEOMETRY

Learner Note: Analytical Geometry is an important topic that carries a lot of marks in the matric final exam. Make sure that you know the basic formulae and then practise lots of examples involving applications of these formulae. The properties of quadrilaterals are extremely important in Analytical Geometry. Make sure you know how to prove that a quadrilateral is a parallelogram, rectangle, square, rhombus or trapezium by knowing the properties of these quadrilaterals.

SECTION A: TYPICAL EXAM QUESTIONS

QUESTION 1 **(24 MARKS)** **(15 MINUTES)** **(DoE papers pre-2014)**

In the diagram, PQRS is a trapezium with vertices $P(5;2)$, $Q(1;-1)$, $R(9;-5)$ and S . PT is the perpendicular height of PQRS and W is the midpoint of QR . Point S lies on the x -axis and $\widehat{PRQ} = \theta$.



- Determine the equation of PW if W is the midpoint of QR . (2)
- Determine the equation of PS . (4)
- Determine the equation of PT . (3)
- Determine the coordinates of T . (5)

- (e) Show that $QT = \frac{1}{3}TR$. (5)
- (f) Calculate the size of θ rounded off to two decimal places. (5)

QUESTION 2 (17 MARKS) (15 MINUTES) (DoE papers pre-2014)

Consider the following points on a Cartesian plane:

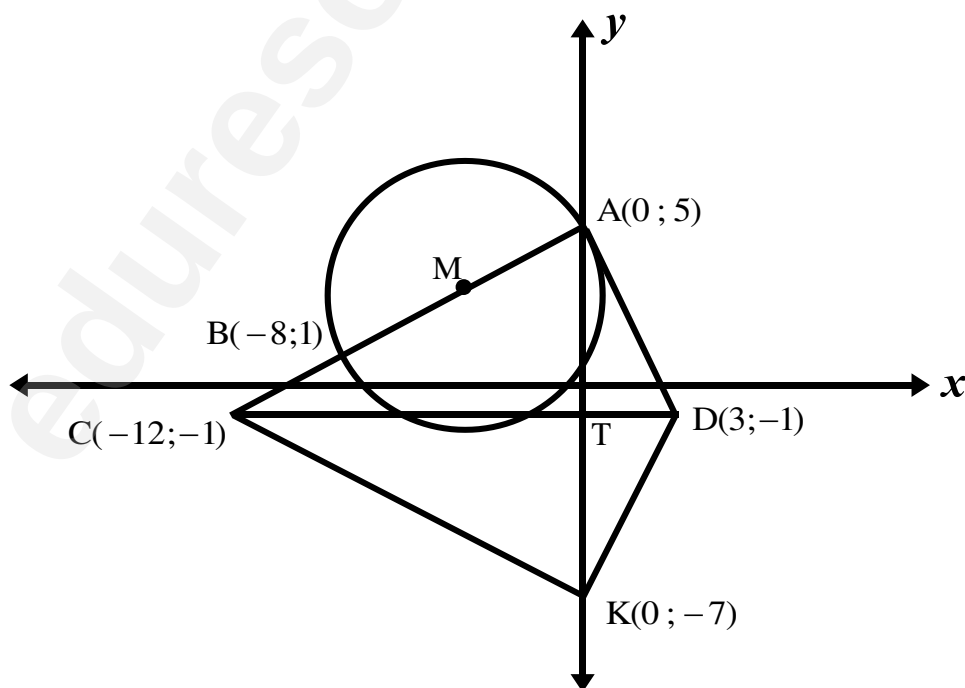
$A(1; 2)$, $A(3; 1)$, $C(-3; k)$ and $C(2; -3)$

Determine the value(s) of k if:

- (a) $(-1; 3)$ is the midpoint of AC. (3)
- (b) AB is parallel to CD. (3)
- (c) $AB \perp CD$. (3)
- (d) A, B and C are collinear. (3)
- (e) $CD = 5\sqrt{2}$ (5)

QUESTION 3 (16 MARKS) (10 MINUTES) (DoE papers pre-2014)

$A(0; 5)$ and $B(-8; 1)$ are two points on the circumference of the circle centre M, in a Cartesian plane. M lies on AB. DA is a tangent to the circle at A. The coordinates of D are $(3; -1)$ and the coordinates of C are $(-12; -1)$. Points C and D are joined. K is the point $(0; -7)$. CTD is a straight line.



- (a) Show that the coordinates of M, the midpoint of AB, are $(-4 ; 3)$. (1)
- (b) Determine the equation of the tangent AD. (4)
- (c) Determine the length of AM. (3)
- (d) Determine the equation of circle centre M in the form $ax^2 + by^2 + cx + dy + e = 0$ (4)
- (e) Quadrilateral ACKD is one of the following: parallelogram ; kite ; rhombus ; rectangle Which one is it? Justify your answer. (4)

QUESTION 4 (7 MARKS) (5 MINUTES) (DoE papers pre-2014)

Determine the coordinates of the centre and the length of the radius of the circle:

$$x^2 + 8x + y^2 - 6y + 5 = 0 \quad (7)$$

SECTION B: NOTES ON CONTENT

If AB is the line segment joining the points $A(x_A ; y_A)$ and $B(x_B ; y_B)$, then the following formulas apply to line segment AB.

The Distance Formula

$$AB^2 = (x_B - x_A)^2 + (y_B - y_A)^2$$

$$\text{or } AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$$

The Midpoint Formula

$$M\left(\frac{x_A + x_B}{2} ; \frac{y_A + y_B}{2}\right) \text{ where M is the midpoint of AB.}$$

The Gradient of a line segment joining two points

$$\text{Gradient of AB} = \frac{y_B - y_A}{x_B - x_A}$$

Parallel lines

Parallel lines have equal gradients. If $AB \parallel CD$ then $m_{AB} = m_{CD}$

Perpendicular lines

The product of the gradients of two perpendicular lines is -1 . If $AB \perp CD$, then $m_{AB} \times m_{CD} = -1$

The equation of the line

$$y - y_A = m(x - x_A)$$

Inclination of a line

$$\tan \theta = m_{AB}$$

If $m_{AB} > 0$, then θ is acute

If $m_{AB} < 0$, then θ is obtuse

Collinear points (A, B and C)

$$m_{AB} = m_{BC}$$

$$\text{OR } m_{AC} = m_{AB}$$

$$\text{OR } m_{AC} = m_{BC}$$

Circles and tangents to circles

The equation of a circle centre the origin is given by: $x^2 + y^2 = r^2$

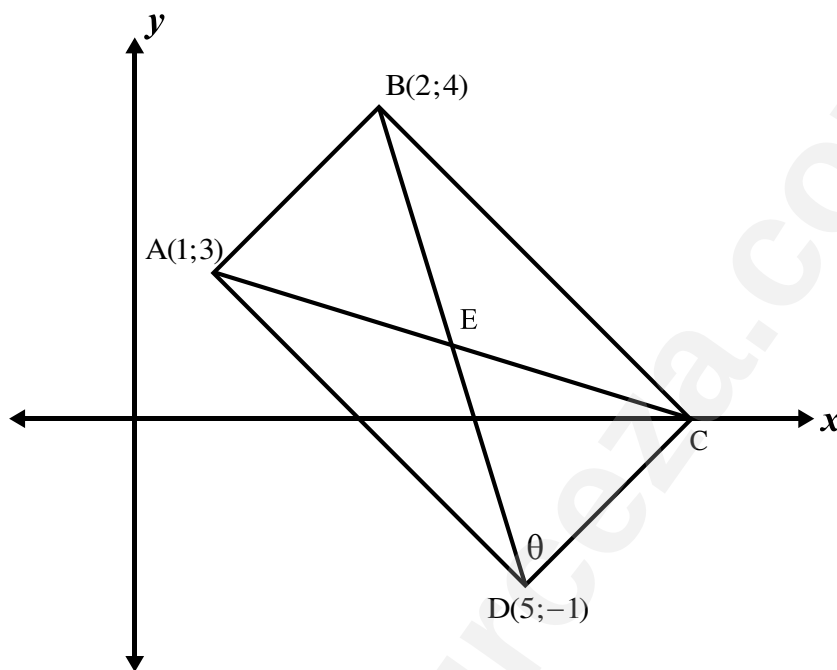
The equation of a circle centre $(a; b)$ is given by: $(x - a)^2 + (y - b)^2 = r^2$

The radius is perpendicular to the tangent: $m_{\text{radius}} \times m_{\text{tangent}} = -1$

SECTION C: HOMEWORK QUESTIONS

QUESTION 1 (31 MARKS) (15 MINUTES) (DoE papers pre-2014)

ABCD is a quadrilateral with vertices $A(1;3)$, $B(2;4)$, C and $D(5;-1)$.



- Determine the coordinates of E, the midpoint of BD. (2)
- Determine the coordinates of C. (5)
- Show that ABCD is a rectangle. (10)
- Determine the area of ABCD. (6)
- Calculate the size of the angle θ rounded off to the nearest degree. (8)

QUESTION 2 (7 MARKS) (5 MINUTES) (DoE papers pre-2014)

- Determine the numerical value of p if the straight line defined by the equation $px + 3y = 6$ has an angle of inclination of 135° with respect to the positive x -axis. (4)
- Calculate the value of k if the points $A(6;5)$, $B(3;2)$ and $C(2k;k+4)$ are collinear. (3)

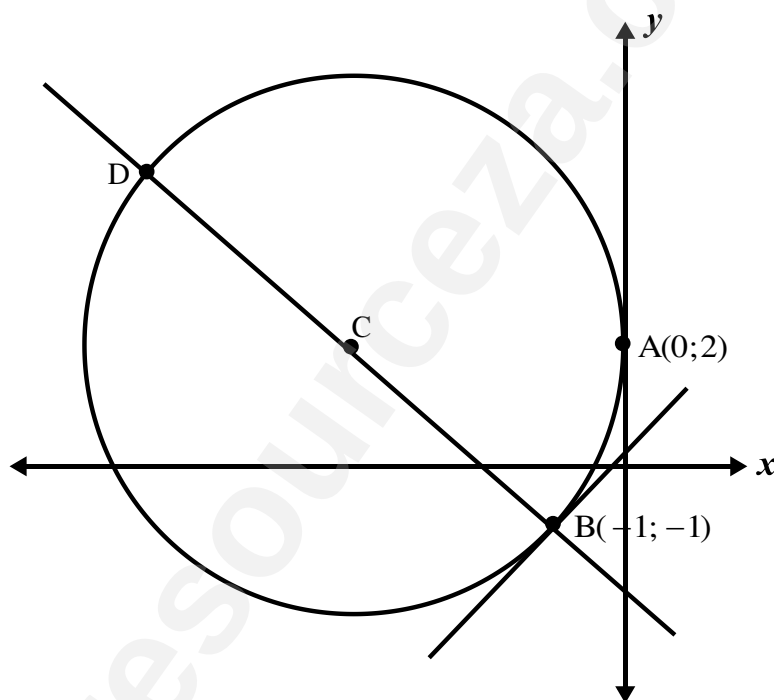
QUESTION 3 (6 MARKS) (5 MINUTES) (DoE papers pre-2014)

The equation of a circle with radius $3\sqrt{2}$ units is $x^2 - 6x + y^2 + 2y - p = 0$.

- (a) Determine the co-ordinates of the centre of the circle. (4)
- (b) Find the value of p . (2)

QUESTION 4 (13 MARKS) (10 MINUTES) (DoE papers pre-2014)

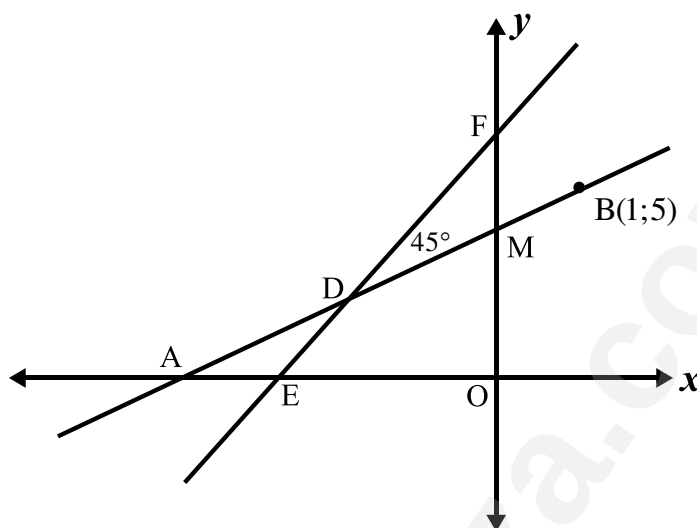
In the diagram below, a circle centre C touches the y -axis at $A(0;2)$. A straight line with equation $3x + 4y = -7$ cuts the circle at $B(-1;-1)$ and D.



- (a) Determine the equation of the tangent to the circle at B. (5)
- (b) Determine the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$. (4)
- (c) Determine the coordinates of D. (4)

QUESTION 5 (19 MARKS) (15 MINUTES) (DoE Nov 2014)

In the diagram below, E and F respectively are the x- and y- intercepts of the line with equation $y = 3x + 8$. The line through B(1;5) making an angle of 45° with EF, as shown, has x- and y- intercepts A and M respectively.



- Determine the coordinates of E. (2)
- Calculate the size of $\hat{DAE}$. (3)
- Determine the equation of AB in the form $y = mx + c$ (4)
- If AB has equation $x - 2y + 9 = 0$, determine the coordinates of D. (4)
- Calculate the area of quadrilateral DEOM. (6)

QUESTION 6 (6 MARKS) (5 MINUTES) (DoE Feb/Mar 2015)

Determine whether or not the circles having equations $(x+2)^2 + (y-4)^2 = 25$ and $(x-5)^2 + (y+1)^2 = 9$ will intersect. (6)

SESSION NO: 8 (CONSOLIDATION)**TOPIC: REVISION OF NUMBER PATTERNS, SEQUENCES AND SERIES****SECTION A: TYPICAL EXAM QUESTIONS****QUESTION 1 (23 MARKS) (15 MINUTES) (DoE papers pre-2014)**

(a) Consider the following series:

$$A: \frac{3}{4} + \frac{3}{2} + 3 + \dots \quad B: \frac{1}{2} + \frac{1}{6} + \frac{1}{18} + \dots$$

(1) Which series converges and has a sum to infinity? Explain. (2)

(2) Calculate the sum to infinity of the convergent geometric series. (3)

(b) The 7th term of an arithmetic sequence has a value of 61 and the 19th term has a value of 97.

(1) Determine the value of the constant difference. (3)

(2) Determine the value of the 15th term. (3)(c) Using an appropriate formula, determine the value of n if:

$$\sum_{k=1}^n (3k - 4) = 2419 \quad (6)$$

(d) Calculate using appropriate formulae: $10368 + 5184 + 2592 + \dots + 10\frac{1}{8}$ (6)**QUESTION 2 (21 MARKS) (15 MINUTES) (DoE papers pre-2014)**(a) Consider the following number pattern: 8; 18; 30; 44;
Determine the next three terms, the general term and the 20th term. (6)

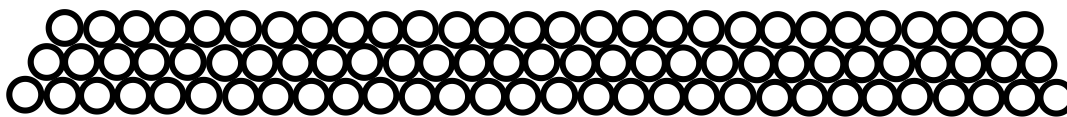
(b) Consider the following number pattern: -2; -3; -6; -11;

(1) Write down the next two terms. (2)

(2) Determine the general term for the number pattern. (4)

(3) Which term of the number pattern is -83? (5)

- (c) In the figure below, a stack of cans is shown. There are 30 cans in the first layer, 29 cans in the second layer (lying on top of the first layer), 28 cans in the third layer. This pattern of stacking continues.



Determine the maximum number of cans that can be stacked in this way. (4)

SECTION B: NOTES ON CONTENT

Arithmetic Sequences and Series

An arithmetic sequence or series is the linear number pattern discussed in Grade 10. We have a formula to help us determine any specific term of an arithmetic sequence. We also have formulae to determine the sum of a specific number of terms of an arithmetic series.

The formulae are as follows:

$$T_n = a + (n-1)d \quad \text{where } a = \text{first term and } d = \text{constant difference}$$

$$S_n = \frac{n}{2}[2a + (n-1)d] \quad \text{where } a = \text{first term and } d = \text{constant difference}$$

$$S_n = \frac{n}{2}[a + l] \quad \text{where } l \text{ is the last term}$$

Geometric Sequences and Series

We have a formula to help us determine any specific term of a geometric sequence. We also have formulae to determine the sum of a specific number of terms of a geometric series. The formulae are as follows:

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \text{where } r \neq 1$$

Convergent geometric series

$$S_\infty = \frac{a}{1-r}$$

A geometric series will converge only if the constant ratio is a number between negative one and positive one.

In other words, the sum to infinity for a given geometric series will exist only if $-1 < r < 1$

If the constant ratio lies outside this interval, then the series will not converge.

Quadratic number patterns

Suppose that the general term of a particular quadratic number pattern is given by

$$T_n = an^2 + bn + c.$$

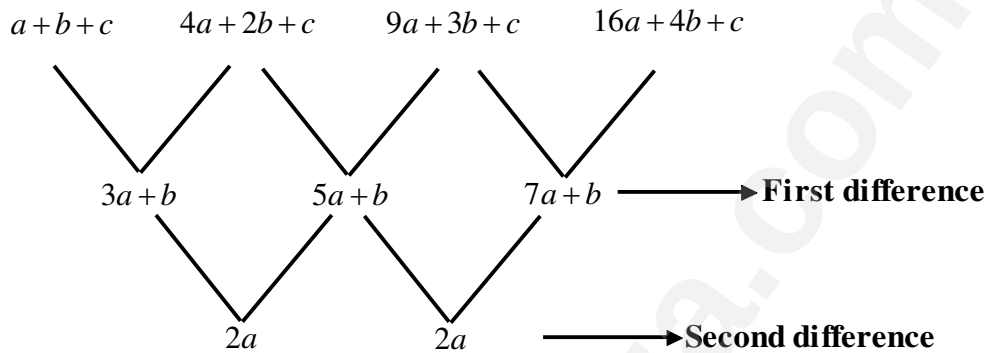
The terms of the number pattern would then be:

$$T_1 = a(1)^2 + b(1) + c = a + b + c$$

$$T_2 = a(2)^2 + b(2) + c = 4a + 2b + c$$

$$T_3 = a(3)^2 + b(3) + c = 9a + 3b + c$$

$$T_4 = a(4)^2 + b(4) + c = 16a + 4b + c$$



You will notice that the constant second difference is given by the expression $2a$.

The first term in the first difference row is given by $3a + b$

and the first term is given by $a + b + c$.

SECTION C: HOMEWORK QUESTIONS

QUESTION 1 (11 MARKS) (10 MINUTES) (DoE papers pre-2014)

Evaluate the sum of each of the following series:

(a) $-1\frac{1}{2} + 3 + 7\frac{1}{2} + \dots + 669$ (5)

(b) $-1\frac{1}{2} + 3 - 6 + \dots + 768$ (6)

QUESTION 2 (24 MARKS) (15 MINUTES) (DoE papers pre-2014)

- (a) The 9th term and the 6th term of a geometric sequence are 80 and 10 respectively.
- (1) Find the first term and the constant ratio. (6)
 - (2) Find the number of terms if the last term is 5120. (4)
- (b) Consider the sequence $8(x-3)^2; 4(x-3)^3; 2(x-3)^4; \dots$ $x \neq 3$
- (1) Determine the values of x for which the sequence will converge? (4)
 - (2) Calculate the sum to infinity if $x = 3,5$ (3)
- (c) Determine the value of m if $\sum_{p=1}^m (2p-7) = 1015$ (7)

QUESTION 3 (14 MARKS) (10 MINUTES) (DoE Nov 2014)

- (a) Given the sequence $-1; -7; -11; p; \dots$
- (1) Write down the value of p . (2)
 - (2) Determine the n th term of the sequence. (4)
 - (3) The first difference between two consecutive terms of the sequence is 96. Calculate the values of these two terms. (4)
- (b) Determine the value of:
- $$\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right)\left(1 + \frac{1}{4}\right)\left(1 + \frac{1}{5}\right)\dots\dots\dots$$
- (4)

QUESTION 4 (9 MARKS) (5 MINUTES) (DoE Feb/Mar 2015)

A quadratic sequence is defined as follows:

$$T_2 - T_1 = 7 \qquad T_3 - T_2 = 13 \qquad T_4 - T_3 = 19$$

- (a) Write down the value of $T_5 - T_4$ and hence $T_{70} - T_{69}$ (4)
- (b) Find T_{69} if $T_{89} = 23\,594$ (5)

QUESTION 5 (12 MARKS) (5 MINUTES) (DoE Nov 2015)

Consider the series: $S_n = -3 + 5 + 13 + 21 + \dots$

- (a) Determine the general term of the series in the form $T_k = bk + c$ (2)
- (b) Write S_n in sigma notation. (2)
- (c) Show that $S_n = 4n^2 - 7n$ (3)
- (d) Another sequence is defined as:

$$Q_1 = -6$$

$$Q_2 = -6 - 3$$

$$Q_3 = -6 - 3 + 5$$

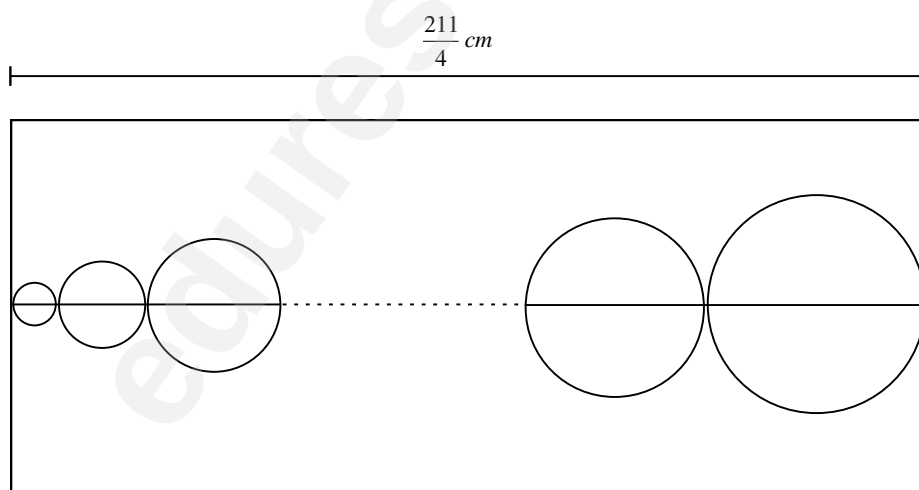
$$Q_4 = -6 - 3 + 5 + 13$$

$$Q_5 = -6 - 3 + 5 + 13 + 21$$

- (1) Write down a numerical expression for Q_6 (2)
- (2) Calculate the value of Q_{129} (3)

QUESTION 6 (10 MARKS) (10 MINUTES) (DoE papers pre-2014)

A number of circles touch each other as follows:



The area of the smallest circle is $4\pi \text{ cm}^2$ and each consecutive circle has an area of $\frac{9}{4}$ times the previous one. How many circles would fit into the rectangle with a length of $\frac{211}{4} \text{ cm}$, as shown in the sketch. (10)

QUESTION 7 (13 MARKS) (10 MINUTES) (DoE papers pre-2014)

Consider the sequence: $3; a; 10; b; 21; \dots$

The sequence has a constant second difference of 1.

- (a) Determine the value of a and b . (4)
- (b) Determine the n th term of the sequence. (5)
- (c) Hence, prove that the sum of any two consecutive numbers in the sequence equals a square number. (4)

SESSION NO: 9 (CONSOLIDATION)

TOPIC: REVISION OF FINANCIAL MATHEMATICS

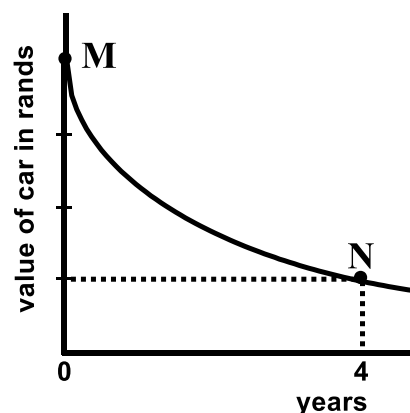
SECTION A: TYPICAL EXAM QUESTIONS

QUESTION 1 (27 MARKS) (30 MINUTES) (DoE papers pre-2014)

- (a) David deposits R65 000 into a savings account paying 15% per annum compounded monthly. He wants to buy a car in three years' time.

- (1) Convert the nominal rate of 15% per annum compounded monthly to the equivalent effective rate (annual). (2)
- (2) Calculate how much money will be available to him in three years' time. (3)
- (3) If he makes a further deposit of R10 000 into the account one year after his first deposit of R65 000, how much money will he have then saved in three years' time. (4)

- (b) Assume that David buys the car for R100 000. He drives the car for four years and then decides to sell the car. Suppose that after four years of depreciation, the car is worth one quarter of its original value. The depreciation of his car is represented in the graph alongside.



- (1) What is the value of the car at M on the graph? (1)
 - (2) What is the value of the car at N on the graph? (1)
 - (3) Based on the graph, what type of depreciation took place? (1)
 - (4) Calculate the rate of depreciation as a percentage. (3)
- (c) Lerato deposits R100 000 into a five-year savings account. Two years later, she withdraws R20 000. Calculate the future value of her investment if the

interest rate for the first two years is 18% per annum compounded monthly and 18% per annum compounded half-yearly for the remaining two years of the savings plan. (6)

- (d) Mandy repays a loan by means of two payments. The first repayment of R13 000 is made after four years. The second and final repayment of R5000 is made three years later. The interest rate during the first four years is 9% per annum compounded semi-annually. For the remaining three years, the interest rate changes to 12% per annum compounded monthly. How much did Mandy originally borrow? (6)

QUESTION 2 (16 MARKS) (20 MINUTES) (DoE papers pre-2014)

- (a) Tumi invested R5 000 and wanted his investment to double as quickly as possible. He settled for an interest rate of 12% per annum compounded quarterly. How long will it take for his investment to double? (4)
- (b) Five years ago, Paul started to save money in a financial institution. At the beginning of the first month he deposited R2000 into the account and made a further deposit of R2000 at the end of that month. He then continued to make month end payments of R2000 into the account. Due to financial difficulty, he did not make the last two payments. The interest rate was 6% per annum compounded monthly. Calculate the future value of his investment at the end of the five-year period. (3)
- (c) Phyllis takes out a twenty-year loan of R400 000. She repays the loan by means of equal monthly payments starting one month after the granting of the loan. The interest rate is 18% per annum compounded monthly.
- (1) Calculate the monthly repayments. (3)
- (2) Calculate the amount owed after the 3rd payment was made. (2)
- (3) Due to financial difficulty, Phyllis misses the 4th, 5th and 6th payments. Calculate her increased monthly payment, which comes into effect from the 7th payment onwards. (4)

SECTION B: NOTES ON CONTENT

Simple interest

The formula for simple interest calculations is:

$$A = P(1 + in)$$

Different compounding periods

Interest can be quoted per annum but calculated over different time periods during a year. Interest can be calculated:

annually:	once per year (usually at the end of the year)
semi-annually:	twice per year (every six months/half yearly) Divide r by 2 and multiply n by 2
quarterly:	four times per year (every three months) Divide r by 4 and multiply n by 4
monthly:	twelve times per year (every one month) Divide r by 12 and multiply n by 12

The formula to find A when given P is as follows:

$$A = P(1 + i)^n$$

The formula to find P when given A is as follows:

$$P = A(1 + i)^{-n}$$

Nominal and annual effective rates

A nominal rate is an annual rate which does not take into consideration the effect of different compounding periods. For example, 15% per annum compounded monthly is a nominal rate.

It is possible to determine an annual effective rate compounded annually that will yield the same accumulated amount as the annual rate compounded monthly.

The formula to calculate the effective annual rate given a nominal rate is:

$$1 + i_{\text{eff}} = \left(1 + \frac{i_{\text{nom}}}{n}\right)^n$$

where:

i_{nom} = nominal rate

i_{eff} = effective rate (annual)

n = number of compoundings per year

Depreciation

When equipment loses its value over a given time period, we say that the equipment is depreciating in value over time. There are two types of depreciation: Linear Depreciation and Reducing Balance Depreciation.

Linear Depreciation

With linear depreciation, equipment is depreciated by a percentage of its original value. It works in the same way as simple interest, but the value decreases rather than increases as with simple interest.

The formula for linear depreciation is

$$A = P(1 - i.n)$$

Reducing Balance Depreciation

With reducing-balance depreciation, equipment is depreciated by a percentage of its previous value. It works in the same way as compound interest, but the value decreases rather than increases as with compound interest.

The formula for Reducing Balance Depreciation is

$$A = P(1 - i)^n$$

Future value annuity formula:

$$F = \frac{x[(1+i)^n - 1]}{i}$$

where:

x = equal and regular payment per period

n = number of payments

i = interest rate as a decimal = $\frac{r}{100}$

Learner note:

This formula deals with saving money for the future. Remember that the value of n represents the number of payments and not necessarily the duration of the investment.

Present value annuity formula:

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

where:

x = equal and regular payment per period

n = number of payments

i = interest rate as a decimal = $\frac{r}{100}$

Learner note:

This formula deals with loans. There must always be a gap between the loan and the first payment for the formula to work.

SECTION C: HOMEWORK QUESTIONS**QUESTION 1 (11 MARKS) (15 MINUTES) (DoE papers pre-2014)**

- (a) Patricia deposited a certain amount of money into a bank account paying 8% per annum compounded half-yearly. After four years, the money has a value of R100 000.
- (1) Convert the nominal interest rate into the equivalent annual effective rate. (2)
 - (2) Hence, or otherwise, calculate the amount of money originally deposited into the bank account by Patricia. (2)
- (b) A motor car bought for R130 000 depreciates on a reducing balance at a rate of 15% per annum for a period of three years. Thereafter, it continues to depreciate on a reducing balance at a rate of 16% per annum for a further two years.
- (1) Calculate the depreciated value of the motor car at the end of the five-year period. (4)
 - (2) What single annual rate of depreciation (reducing balance) will yield the same depreciated value over the five-year period? Round off your answer to one decimal place. (3)

QUESTION 2 (16 MARKS) (15 MINUTES) (DoE papers pre-2014)

- (a) Maria decided to start saving money for her son's future education. She immediately deposited R4000 into a savings account. Three years later, she deposited a further R5000 into the account. One year later, she withdrew R2000 in order to do repairs around the house. Her son needed the money four years after her withdrawal of R2000. The interest rate for the first three years was 15% per annum compound monthly. The interest rate for the remaining five years was 16% per annum compounded quarterly. Calculate the future value of his money at the end of the savings period which lasted eight years. (8)
- (b) Thabo borrowed money in order to finance the purchase of new home. He paid a deposit of R100 000. He then repaid the balance that he owed by means of a payment of R160 000 after three years. His last payment of R170 000 was paid four years after the previous payment. The interest rate for the first three years was 12% per annum compounded monthly. The interest rate

was then 14% per annum compounded half-yearly for the remaining four years. What was the original price of the home? (8)

QUESTION 3 (23 MARKS) (20 MINUTES) (DoE papers pre-2014)

- (a) Aaron opens a savings account and immediately deposits R1 000 into the account. He continues to make monthly payments of R1 000 into the account for a period of one year. The interest rate remains fixed at 18% per annum compounded monthly. Due to financial difficulty, he missed the 4th payment. Calculate the future value of his investment. (8)
- (b) Maria bought a house for R620 000. She paid 35% cash and the balance was paid through a bank loan. The interest paid on the loan was 15% per annum compounded monthly.
- (1) Calculate how much she would pay on a monthly basis if the loan is to be paid off over twenty years assuming that her first payment was made one month after the loan was granted. (7)
 - (2) Calculate the balance outstanding at the end of the sixth year. (4)
 - (3) At the end of the sixth year, the interest rate dropped to 12,25% per annum compounded monthly. Calculate her new monthly repayment one month after the interest rate dropped. (4)

QUESTION 4 (14 MARKS) (10 MINUTES) (DoE Nov 2014)

- (a) Exactly five years ago Mpume bought a new car for R145 000. The current book value of this car is R72 500. If the car depreciates by a fixed annual rate according to the reducing-balance method, calculate the annual rate of depreciation. (3)
- (b) Samuel took out a home loan for R500 000 at an interest rate of 12% per annum compounded monthly. He plans to repay this loan over 20 years and his first payment is made one month after the loan is granted.
- (1) Calculate the value of Samuel's monthly instalment. (4)
 - (2) Melissa took out a loan for the same amount and at the same interest rate as Samuel. Melissa decided to pay R6 000 at the end of every month. Calculate how many months it took for Melissa to settle the loan. (4)
 - (3) Who pays more interest. Samuel or Melissa? Justify your answer. (3)

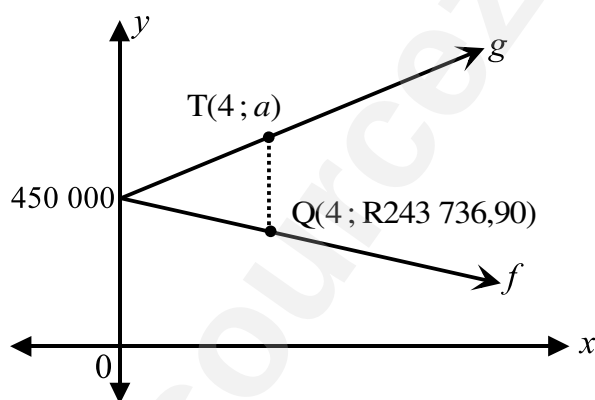
QUESTION 5 (11 MARKS) (15 MINUTES) (DoE Feb/Mar 2015)

Nomsa started working on 1 January 1970. At the end of January 1970 and at the end of each month thereafter, she deposited R400 into an annuity fund. She continued doing this until she retired on 31 December 2013.

- Determine the total amount of money paid into the fund. (2)
- The interest rate on this fund was 8% per annum compounded monthly. Calculate the value of the fund at the time she retired. (5)
- On 1 January 2014, Nomsa invested R2 000 000 in an account paying interest at 10% per annum compounded monthly. Nomsa withdraws a fixed amount from this account at the end of each month, starting on 31 January 2014. If she wishes to make monthly withdrawals from this account for 25 years, what is the maximum amount she could withdraw at the end of each month? (4)

QUESTION 6 (13 MARKS) (15 MINUTES) (DoE Nov 2015)

The graph of f shows the book value of a vehicle x years after the time Joe bought it. The graph of g shows the cost price of a similar vehicle x years later.



- How much did Joe pay for the vehicle? (1)
- Use the reducing-balance method to calculate the percentage annual rate of depreciation of the vehicle that Joe bought. (4)
- If the average rate of the price increase of the vehicle is 8,1% per annum, calculate the value of a . (3)
- A vehicle that costs R450 000 now, is to be replaced at the end of 4 years. The old vehicle will be used as a trade-in. A sinking fund is created to cover the replacement cost of this vehicle. Payments will be made at the end of each month. The first payment will be made at the end of the 13th month and the last payment will be made at the end of the 48th month. The sinking fund earns interest at a rate of 6,2% per annum compounded monthly. Calculate the monthly payment to the fund. (5)