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GEOMETRIC

These are sequences where we get from one term to the next by MULTIPLYING a fixed amount (could be less than 1 or negative).

Examples:

$$2; 6; 18; 54; \dots \quad a = 2 \quad r = 3$$

$$5; -10; 20; -40; 80 \dots \quad a = 5 \quad r = -2$$

$$5; 5; 5; 5; 5; \dots \quad a = 5 \quad r = 1$$

$$1000; 100; 10; 1; \frac{1}{10}; \frac{1}{100}; \dots \quad a = 1000 \quad r = \frac{1}{10}$$

There are two parameters which affect a geometric sequence

a = where does it start?

r = what do we $\times$ by to get from 1 term to the next??

If three numbers a , b and c are in arithmetic sequence then: $\frac{b}{a} = \frac{c}{b}$.

geometric

$$r = \frac{T_n}{T_{n-1}}$$

The formula for the n^{th} term of an arithmetic sequence is $T_n = ar^{n-1}$



$$2; 6; 18; 54$$

Find T_8

$$\begin{aligned} T_8 &= 2 \times 3^{8-1} \\ &= 2 \times 3^7 \\ &= 4374 \end{aligned}$$

$$(2 \times 3)^7$$

- 1) Find the sixteenth term of the following sequence:
2 ; 6 ; 18 ; 54

$$T_n = ar^{n-1} *$$

$$T_{16} = 2 \times 3^{15}$$

$$= \underline{28697814}$$

- 2) A sequence has $T_7 = 12$ and $r = -2$. Write down the first three terms of the sequence.

$$12 = a(-2)^{7-1}$$

$$\frac{3}{16} = \frac{12}{64} = a$$

$$\frac{3}{16}; -\frac{3}{8}; \frac{3}{4}$$

- 3) A sequence has $T_7 = 192$ and $T_2 = 6$. Give the first three terms.

$$T_n = ar^{n-1}$$

$$\textcircled{1} \quad 192 = ar^6$$

$$\textcircled{2} \quad 6 = ar^1$$

$$192 = ar^{7-1} \quad \textcircled{1}$$

$$6 = ar^{2-1} \quad \textcircled{2}$$

$$\textcircled{1} \div \textcircled{2} \quad 32 = r^5$$

$$\sqrt[5]{32} \text{ or } 32^{\frac{1}{5}} = (r^5)^{\frac{1}{5}}$$

$$2 = r$$

by substituting $r = 2$
into equation $\textcircled{2}$ get $a = 3$
3; 6; 12

A **GEOMETRIC MEAN** of two numbers is a number that lies between them so that the three numbers form a G.P.

Consider the following example:

Insert two geometric means between 11 and 88.

$$T_n = ar^{n-1}$$

$$11; -; -; 88$$

$$88 = 11r^{4-1}$$

$$88 = 11r^3$$

$$8 = \frac{88}{11} = r^3$$

$$r = 2$$

$$\begin{aligned} a &= 11 \\ T_4 &= 88 \end{aligned}$$

$$11; \underline{22}; \underline{44}; 88$$

4. The terms 2; x ; y and 9 are such that the first three terms form an arithmetic sequence and the last three terms form a geometric sequence. Calculate the values of x and y .

A.P. $x - 2 = y - x$
 $2x = y + 2$ (1)

G.P. $\frac{y}{x} = \frac{9}{y} \rightarrow y^2 = 9x$ (2)

from (1) $y = 2x - 2$ * Substitute this into (2)

$(2x - 2)^2 = 9x$ Aha!!
 $4x^2 - 8x + 4 = 9x \therefore 4x^2 - 17x + 4 = 0 \rightarrow \text{CALC}$

5. If $\frac{1}{y-x}, \frac{1}{2y}, \frac{1}{z-y}$ is an arithmetic sequence then show that x, y, z form a geometric sequence.

$x = 4$ or $\frac{1}{4}$

Sub. these into *

$y = 6$ $y = -\frac{3}{2}$

Either $2; 4; 6; 9$

OR $2; \frac{1}{4}; -\frac{3}{2}; 9$