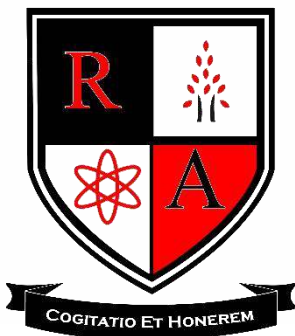




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REDWOOD ACADEMY



MATHEMATICS 12

PAPER 1

SECTION REVISION

BY NAEEM MOHAMED



D. MATRIC HELP ON THE INTERNET

Visit the following websites regularly to download past papers, media publications, workbooks and other valuable mathematics resources:

- **MIND THE GAP STUDY GUIDES:**
[https://www.education.gov.za/Curriculum/LearningandTeachingSupportMaterials\(LTSM\)/MindtheGapStudyGuides.aspx](https://www.education.gov.za/Curriculum/LearningandTeachingSupportMaterials(LTSM)/MindtheGapStudyGuides.aspx)
- **NSC PAST PAPERS:**
[https://www.education.gov.za/Curriculum/NationalSeniorCertificate\(NSC\)Examinations.aspx](https://www.education.gov.za/Curriculum/NationalSeniorCertificate(NSC)Examinations.aspx)
- **EXAM RESOURCES:**
<https://www.ecexams.co.za/>

E. MARK DISTRIBUTION FOR MATHEMATICS CAPS GRADE 12

Paper 1:

Description	% of Paper 1
Patterns & Sequences	± 17%
Annuities & Finance	± 10%
Functions & Graphs	± 23%
Algebra and Equations (and inequalities)	± 17%
Calculus	± 23%
Probability	± 10%
	100%

- **Bookwork** (Theory and proofs) of a maximum 6 marks could be included in the paper.
- Questions will not necessarily be compartmentalised in sections as this table indicates. Various topics can be **integrated** in the same question.

Paper 2:

Description	% of Paper 2
Analytical Geometry	± 27%
Trigonometry	± 27%
Statistics	± 13%
Euclidean Geometry and Measurement	± 33%
	100%

- **Bookwork** (Theorems and trigonometric proofs) of a maximum 12 marks could be included in the paper.
- Questions will not necessarily be compartmentalised in sections as this table indicates. Various topics can be **integrated** in the same question.



NAME:

A. CONTENTS

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GENERAL ADVICE FOR 2021 MATRICULANTS

2021 – The year you write your final school examinations! Those who excel during this vital year will open up your learning and earning opportunities. Use your final year of study well. Attend all your classes in school and any extra tuition offered to you, both privately and by your school. Eat healthy, play sports and and any activity that undermines your studying and preparation. At the same time, remain calm as stressing too much can work against you.

Keep up to date with the latest matric news in newspapers and on the internet. Work through previous year's examination papers, exemplar papers as well as departmental papers and any sample papers prepared for you by your tutors and teachers. Ask your tutors / teachers to work through the questions, answers and examination technique with you.

Do everything you possibly can to convert twelve years of school into a National Senior Certificate with excellent marks, while making the most of probably the most enjoyable year of your schooling life.

Good luck!

Neem Mohamed

C. GUIDE STRUCTURE

This booklet is focused on preparing you for the trial and final examinations. Each module is based on a specific question in the paper and starts off with important aspects in the subject guideline. Theory and questions from national, IEB and exemplar papers are then worked through during the course.



INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + i)^n \quad A = P(1 - i)^n \quad A = P(1 + i)^n$$

$$T_n = a + (n-1)d \quad S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1} \quad S_n = \frac{a(r^n - 1)}{r - 1} \quad ; \quad r \neq 1 \quad S_\infty = \frac{a}{1 - r} \quad ; \quad -1 < r < 1$$

$$F = \frac{x[1 + i]^n - 1}{i} \quad P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \quad y - y_1 = m(x - x_1) \quad m = \frac{y_2 - y_1}{x_2 - x_1} \quad m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A \quad \text{area } \triangle ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \frac{\cos^2 \alpha - \sin^2 \alpha}{1 - 2 \sin^2 \alpha} \quad 2 \cos^2 \alpha - 1$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

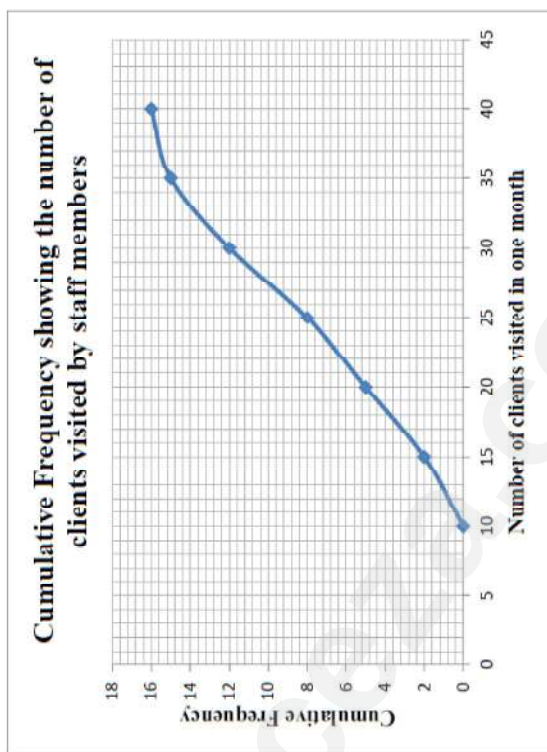


MODULE 1

DATA HANDLING: REGRESSION AND CORRELATION

QUESTION 1

- 1.1 Ros is the sales manager for a print and design company. His staff members must report about the number of clients they visit every month. Below is the ogive representing the collected data in one month.



- 1.1 How many staff members were there? (1)
- 1.2 Determine the semi-interquartile range. (3)
- 1.3 Draw a box whisker diagram for the data. (3)
- 1.4 Comment on the distribution of data using your result in QUESTION 1.3. (1)

[8]





QUESTION 2

In an experiment, a group of 23 girls were presented with a page containing 30 coloured rectangles. They were asked to name the colours of the rectangles correctly as quickly as possible. The time, in seconds, taken by each of the girls is given in the table below.

12	13	13	14	14	16	17	18	18	18	19	20
21	21	22	22	23	24	25	27	29	30	36	

Calculate:

2.1.1 The mean of the data (2)

2.1.2 The interquartile range of the data (3)

The standard deviation of the times taken by the girls is 5,94. How many girls took longer than ONE standard deviation from the mean to name the colours? (2)

Draw a box and whisker diagram to represent the data on the number line provided in the ANSWER BOOK. (3)

The five-number summary of the times taken by a group of 23 boys in naming the colours of the rectangles correctly is (15 ; 21 ; 23,5 ; 26 ; 38).

2.4.1 Which of the two groups, girls or boys, had the lower median time to correctly name the colours of the rectangles? (1)

2.4.2 The first three learners who named the colours of all 30 rectangles correctly in the shortest time will receive a prize. How many boys will be among these three prizewinners? Motivate your answer. (2) [13]

QUESTION 1

The following table shows the test marks (in %) of Grade 11 learners in Frances Baard High School.

INTERVAL OF TEST MARKS	NUMBER OF LEARNERS
$0 \leq x < 20$	4
$20 \leq x < 40$	5
$40 \leq x < 60$	9
$60 \leq x < 80$	13
$80 \leq x < 100$	10
Totals	41

1.1 Write down the modal class. (1)

1.2 Calculate the estimated mean. (3)

1.3 Complete the cumulative frequency table provided in the ANSWER BOOK. (2)

1.4 Draw a cumulative frequency curve (ogive) to represent the data on the grid provided in the ANSWER BOOK. (3)

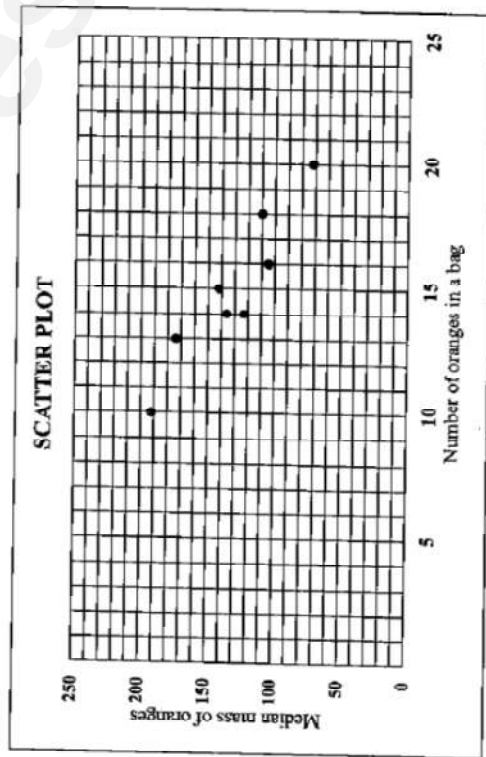
1.5 Use the cumulative frequency curve (ogive) to determine the interquartile range for the data. (3) [12]



QUESTION 1

A student is investigating the number of oranges in a bag in relation to the median mass of the oranges filled in the same bag. The findings are recorded in the table below.

Number of oranges in a bag	18	16	20	15	14	13	14	10
Median mass of oranges in the same bag (to the nearest gram)	110	105	72	142	123	174	136	192



- 1.1 Determine the equation of the least squares regression line for the data. (3)
 - 1.2 Write down the correlation coefficient of the data. (1)
 - 1.3 Draw the least squares regression line on the scatter plot given in your ANSWER BOOK. (2)
 - 1.4 Comment on the strength of the relationship between the number of oranges in the bag and the median mass of the oranges. (1)
 - 1.5 Determine the possible median mass of oranges in a bag, if there are 12 oranges in that bag. (2)
- [9]



QUESTION 1

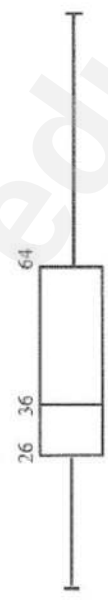
1. The following information represents the amount of maize exported to other countries over 11 years in 1000 tons.

39	42	48	54	62	68	78	82	91	93
----	----	----	----	----	----	----	----	----	----

- 1.1 Calculate the mean amount of maize exported over the 11 years. (2)
 - 1.2 Calculate the standard deviation of the data. (2)
 - 1.3 Calculate the number of years that are within one standard deviation of the mean. (2)
 - 1.4 Draw a box and whisker diagram to represent the data. (4)
 - 1.5 Comment on the skewness of the data. (1)
 - 1.6 There was an error in the data. The mean amount of maize exported over the 11 years should increase by 1,25 thousands of tons. What impact will this error have on the:
 - 1.6.1 yearly data provided in the above table? (1)
 - 1.6.2 on the interquartile range of the given data above? (1)
- [13]

QUESTION 1

Some of the test results of 21 learners are given below. There was only one result of 26 marks and only one result of 64 marks.

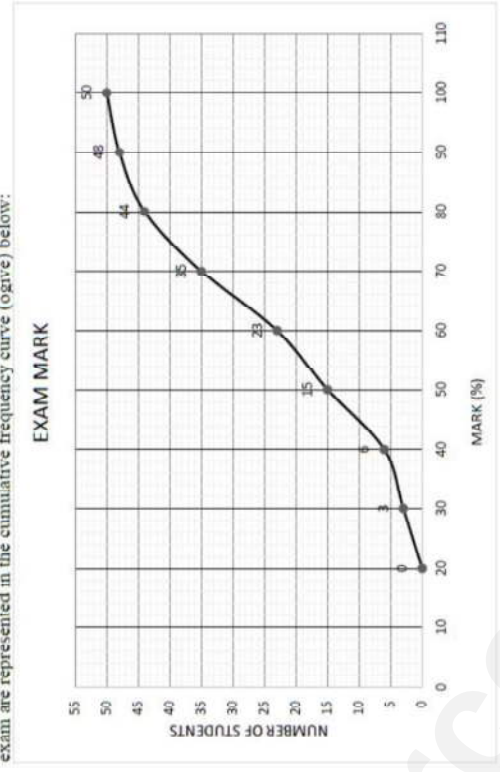


- What information is omitted on the diagram above? (1)
- The results were read to the learners in ascending order. If the 5th learner's result was 26, which learner obtained a result of 64? (2)
- One of the learners was arguing that the distribution of the data wasn't symmetrical. Is the learner correct? Give a reason for the learner's remark. (3)
- The class calculated the following, using the test results:
 $\bar{x} = 45,5$ and $s\bar{x} = 19,2$.
Use the above information and determine the number of learners whose results fall outside ONE standard deviation of the mean. (3)
- If the marks of each learner would increase by 5 marks, what effect would it have on the mean and standard deviation? (2)

[11]

QUESTION 1

- A number of learners' marks for a certain exam were requested in a survey. Marks from the exam are represented in the cumulative frequency curve (ogive) below:

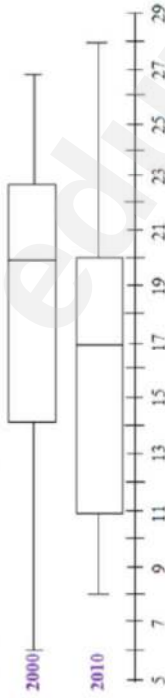


- How many learners participated in this survey? (1)
- Complete the frequency column in the table given in the ANSWER BOOK.

Interval	Frequency	Cumulative frequency
$20 \leq x < 30$		3
$30 \leq x < 40$		6
$40 \leq x < 50$		15
$50 \leq x < 60$		23
$60 \leq x < 70$		35
$70 \leq x < 80$		44
$80 \leq x < 90$		48
$90 \leq x < 100$		50

- Use the graph to determine the median mark. (2)
- Write down the modal class. (1)
- If a learner needs 70% to qualify for a reward, how many learners qualified? (2)

- 1.2 The results from a test for students for the year 2000 and the year 2010 are illustrated in the box and whisker plot. The total mark for the test was 30.

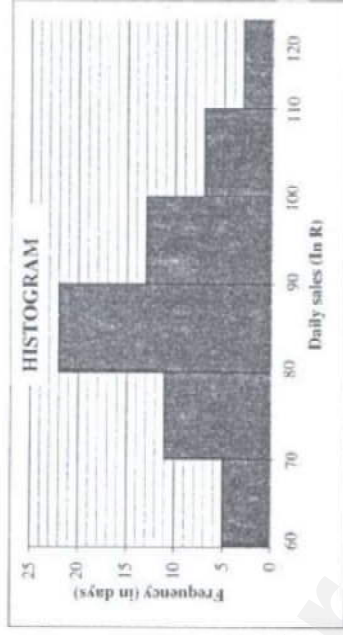


- 1.2.1 Determine the interquartile range for 2010. (2)

- 1.2.2 In which year did students perform better in the test? Motivate your answer. (2) [12]

QUESTION 1

A street vendor has kept record of his daily sales for November and December 2017. The daily sales(in Rand) is shown in the histogram below.



- 1.1 A table summarising the daily sales for November and December 2017 is drawn in the answer book. Complete the cumulative frequency column in the table. (3)
- 1.2 Draw an ogive (cumulative frequency graph) of the daily sales for November and December 2017 on the grid provided IN THE ANSWER BOOK. (4)
- 1.3 On how many days did the daily sales exceed 100? (2)
- 1.4 Use the ogive IN THE ANSWER BOOK to estimate the median value of the daily sales. (2)
- [11]

Unit 2: Bivariate Data and Regression Modelling

QUESTION 1

The time (in seconds) between the consecutive landings of aeroplanes at an airport on day 1 was recorded. The data is given in the Cumulative Frequency table below.

Time in seconds	Number of aeroplanes (Frequency)	Cumulative Frequency
$60 < t \leq 90$	2	2
$90 < t \leq 120$	16	18
$120 < t \leq 150$	28	46
$150 < t \leq 180$	17	63
$180 < t \leq 210$	k	p
$210 < t \leq 240$	7	80

Show that $k = 10$.

Write down the value of p .

Calculate the estimated mean time between the landings of two consecutive aeroplanes.

It is given that $(q ; 186,89)$ is the interval of the landing time between aeroplanes within ONE standard deviation from the estimated mean.

1.4.1 Write down the estimated standard deviation of the time between the consecutive landings of the aeroplanes.

1.4.2 Calculate the value of q .

On day 2, the same number of aeroplanes that landed on day 1, land at the airport. The elapsed time between all the consecutive landings of all the aeroplanes is m seconds shorter than the time that is given in the table above.

If an ogive is to be drawn of the data of day 2, the following will be true:

- The ogive will be grounded at $(57 ; 0)$
- The maximum value of the ogive will be at $(237 ; 80)$

Determine the average time between the landing of two aeroplanes on DAY 2, if it is given that the frequency distribution of the two days are the same.

(2)
[10]

QUESTION 2

The data below shows the marks obtained by ten Grade 12 learners from two different Mathematics classes sitting in the first row of each class.

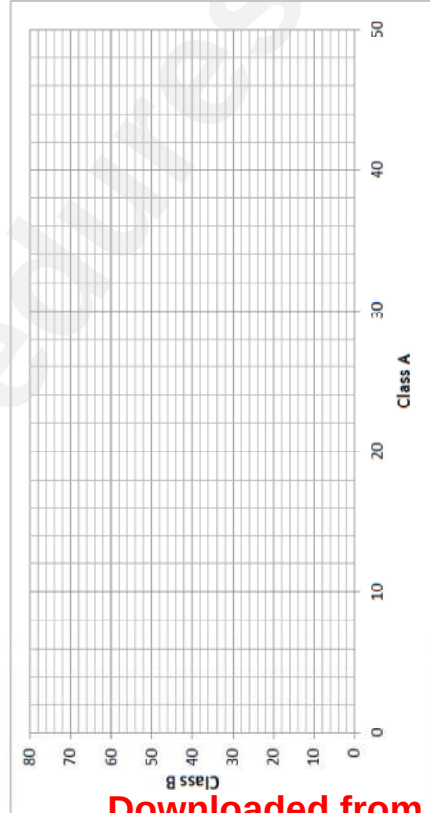
Class A	16	36	20	38	40	30	35	22	40	24
Class B	45	70	44	56	60	48	75	60	63	38

- 2.1 Make use of the grid provided on the DIAGRAM SHEET 1 to draw a scatter plot for the data. (2)
- 2.2 Calculate the equation of the least squares regression line for this data. (2)
- 2.3 Draw the least squares regression line for the data on the scatter plot diagram drawn in QUESTION 2.1 (DIAGRAM SHEET 1). (1)
- 2.4 Calculate the correlation coefficient for the above data. (2)
- 2.5 Calculate the mean and the standard deviation for Class B. (4)
- 2.6 How many scores fall within one standard deviation of the mean? (2)
[13]



DIAGRAM SHEET 1

QUESTIONS 2.1 AND 2.3

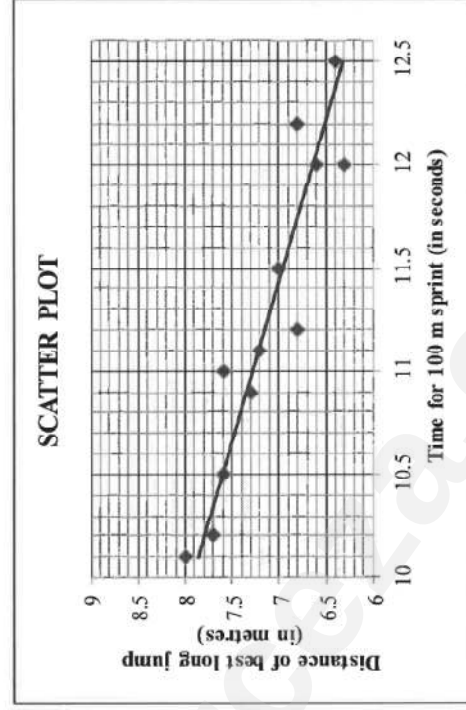


QUESTION 1

The table below shows the time (in seconds, rounded to ONE decimal place) taken by 12 athletes to run the 100 metre sprint and the distance (in metres, rounded to ONE decimal place) of their best long jump.

Time for 100 m sprint (in seconds)	10,1	10,2	10,5	10,9	11	11,1	11,2	11,5	12	12,2	12,5
Distance of best long jump (in metres)	8	7,7	7,6	7,3	7,6	7,2	6,8	7	6,6	6,3	6,4

The scatter plot representing the data above is given below.



The equation of the least squares regression line is $\hat{y} = a + bx$.

- 1.1 Determine the values of a and b . (3)
- 1.2 An athlete runs the 100 metre sprint in 11,7 seconds. Use $\hat{y} = a + bx$ to predict the distance of the best long jump of this athlete. (2)
- 1.3 Another athlete completes the 100 metre sprint in 12,3 seconds and the distance of his best long jump is 7,6 metres. If this is included in the data, will the gradient of the least squares regression line increase or decrease? Motivate your answer without any further calculations. (2)

[7]

QUESTION 2

Research is done to determine if the number of hours reading over a certain period of time has an effect on the results of a candidate's mark in a general knowledge test (out of 120).

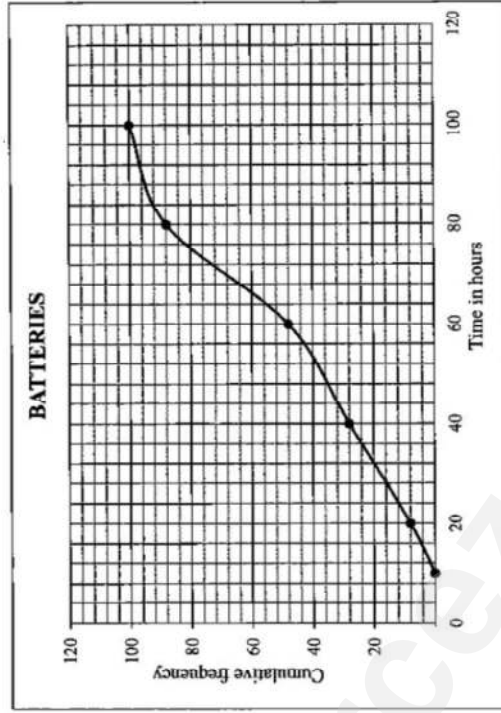
Number of hours	15	20	22	25	32	40	44	50	55	58
Mark out of 120	40	30	55	80	70	75	100	105	98	79

1. Determine the equation of the least squares regression line. (3)
2. Estimate the mark that a person that reads 36 hours in that same period will obtain in the test. (2)
3. What is the correlation between the number of hours reading and the mark a person scores for the test? Motivate your answer. (3) [8]

QUESTION 2

2.1 Batteries are used in everyday life. The Grade 12 Physical Sciences learners investigated the life span of batteries under constant test conditions.

The ogive (cumulative frequency graph) below shows the lifespan (in hours) of the batteries.



- 2.1.1 How many batteries were tested for this investigation? (1)
- 2.1.2 Use the graph to estimate the median time for the life span (in hours) of the batteries. (2)
- 2.1.3 The minimum lifespan of batteries is 10 hours and the maximum lifespan is 100 hours. Use the cumulative frequency graph to draw a box and whisker diagram in your ANSWER BOOK. (3)
- 2.1.4 Comment on the skewness of the distribution of the lifespan of the batteries. (1)



- 2.2 The table below represents values in a data set written in increasing order. None of the values in the data set are repeated.

5	a	19	b	c	d	35
---	-----	----	-----	-----	-----	----

Determine the values of a , b , c , and d if:

- The median is 20.
- The semi interquartile range is 8.
- The upper quartile is twice the lower quartile.
- The mean is 22.

(4)
[11]

QUESTION 2

The following information (in %) represent contributions made by the Agricultural and Mining industries in order to evaluate the GDP(GROSS DOMESTIC PRODUCT) of a certain country.

YEAR	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Agriculture (x)	4.2	3.4	3.1	2.7	2.9	3.0	2.9	3.0	2.6	2.5	2.6
Mining (y)	19.2	19.4	19.2	18.5	17.5	17.0	16.8	15.2	14.2	12.8	12.4

- 2.1 Determine the equation of the least squares regression line. (3)
- 2.2 Estimate the percentage that the mining industry will contribute if the agriculture industry dropped to 1.2 %. (2)
- 2.3 Comment on the strength of the correlation between the contributions made by these two industries. Motivate your answer. (2)
[7]

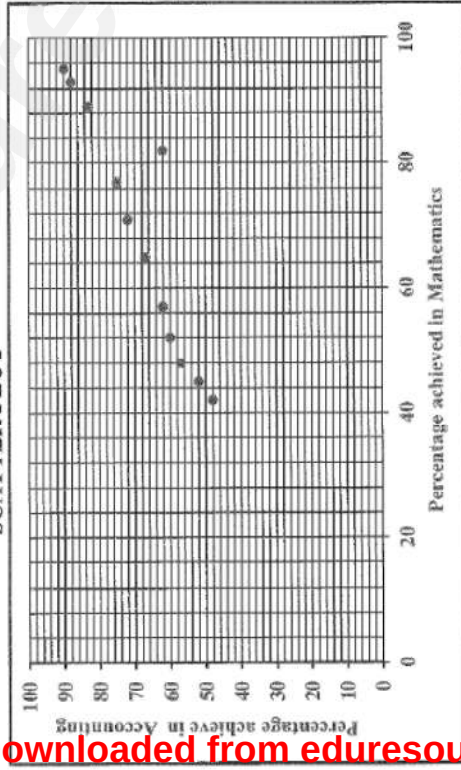


QUESTION 2

At a certain High School only 12 candidates take Mathematics and Accounting. The marks as a percentage scored by these candidates in the Trial Examinations for these two subjects, are shown in the table and scatter plot below.

Mathematics	52	82	93	95	71	65	77	42	89	48	45	57
Accounting	60	62	88	90	72	67	75	48	83	57	52	62

SCATTER PLOT



- Use the scatter plot to comment on the relationship between the learner's performance in Mathematics and Accounting. Provide a reason for your answer. (2)
- Calculate the value of the correlation coefficient. (1)
- Calculate the equation for the least squares regression line for the data. (3)
- If a candidate from this group scored 60% in the Mathematics examination, but was absent for the Accounting examination, predict the percentage he would have scored in the Accounting examination. (2)
- Use the scatter plot and identify any outlier(s) in the data. (1)

[9]



QUESTION 2

A group of 12 learners have been asked to measure their resting heart rate (beats per minute) and the time (in minutes) that they exercise in a week. The data below was gathered.

Minutes of exercise per week	Resting heart rate (BPM)
30	82
40	77
60	75
90	70
140	68
180	67
270	60
350	58
360	52
420	50
440	48
500	45

- 2.1 Represent the data as a scatter plot on the grid provided. (3)
- 2.2 Calculate the correlation coefficient for the given data. (2)
- 2.3 Use the data to calculate the equation of the least squares regression line. (3)
- 2.4 If a learner has a resting heart rate of 65 beats per minute, how many hours would you expect him to exercise per week? (3)

[11]



QUESTION 2

- 2.1 A learner conducted an experiment to investigate the relationship between the age and resting heart rate (in beats per minute). He sought the assistance of the local clinic. The information for 12 people is shown in the table below:

Age	59	32	42	50	22	39	21	20	27	40	29	47
Resting heart rate (beats per minute)	88	74	74	93	85	71	78	82	70	75	95	75

- 2.1.1 Determine the equation of the least squares regression line. (3)
- 2.1.2 Predict the resting heart rate of a person aged 57 years. (2)
- 2.1.3 Write down the correlation coefficient for the data. (1)
- 2.1.4 Use the correlation coefficient to comment on the prediction done in Question 2.1.2. (1)

A group of students wrote a statistics test. The following information was obtained from the marks obtained in this test:

$$\sum_{i=1}^{22} x_i = 1320$$

- 2.2.1 How many students wrote the test? (1)
- 2.2.2 Calculate the mean mark for the test. (2)

[10]

QUESTION 2

The marks, in percentage, obtained in an Accounting and Mathematics test by a group of ten Grade 12 learners is shown in the table below.

Accounting Test	76	65	88	68	70	79	51	66	59	74
Mathematics Test	80	69	93	19	76	85	57	79	62	78

- 2.1 Identify an outlier of the above data. (1)
- 2.2 Determine the equation for the least squares regression line after ignoring the outlier in the above data. (3)
- 2.3 Another learner in the same class obtained 83% in the Accounting test, but due to illness could not write the Mathematics test. Use the equation established in 2.2 to predict the learner's mark for the Mathematics test. (2)
- 2.4 The teacher decided to award the learner who was absent the predicted mark obtained in 2.3 for the Mathematics test. Other learners in the class felt that it was unfair. (2)
- 2.5 Motivate to these learners why the predicted mark is a good indication of what the learner may have scored in the Mathematics test. (2)
- 2.5 After the Mathematics subject advisor has moderated the answer books of the Mathematics tests, she decides to lower every test mark by $p\%$. Explain, without any calculations, what influence the lowering in the marks of the Mathematics test has on the slope of the least squares regression line of the above data when the outlier is ignored. (2)

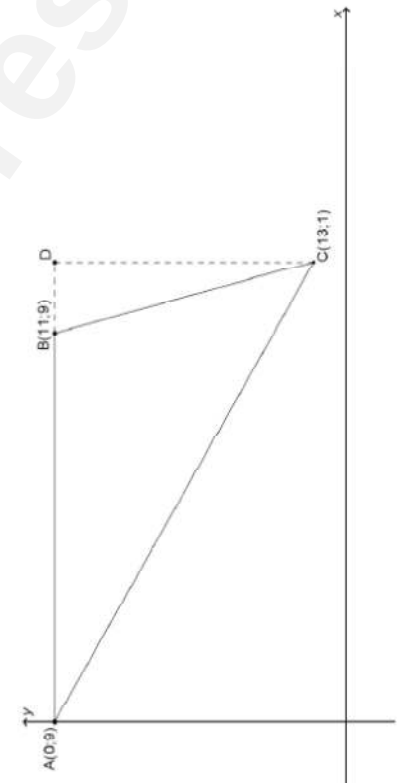
[10]

MODULE 2: ANALYTICAL GEOMETRY

Unit : Straight Lines

QUESTION 3

In the figure, A(0 ; 9), B(11 ; 9) and C(13 ; 1) are the vertices of $\triangle ABC$ with altitude CD.



- Write down the equation of the line AB.

(1)
- Write down the length of AB.

(1)
- Determine the co-ordinates of D.

(1)
- Calculate the area of $\triangle ABC$.

(3)
- Determine the co-ordinates of M, the midpoint of AC.

(2)
- Hence, determine the equation of the perpendicular bisector of AC.

(4)
- Does the perpendicular bisector in QUESTION 3.6 pass through point B or not? Justify your answer using relevant calculations.

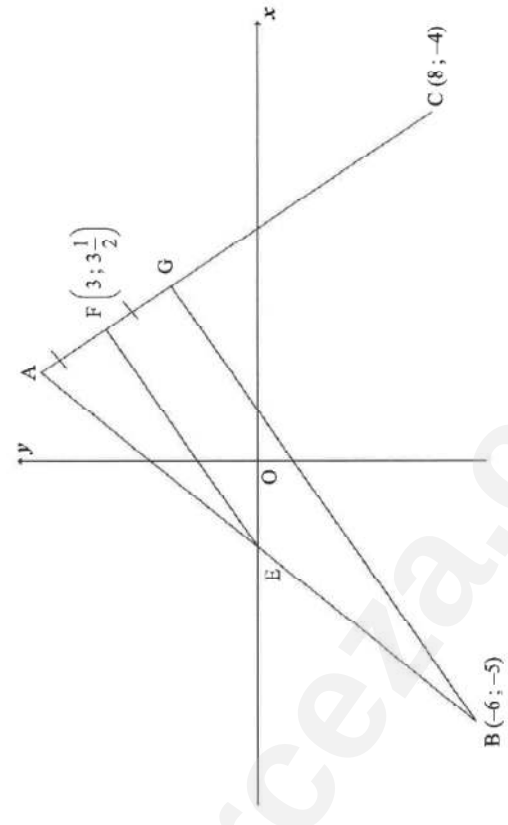
(2)
- Determine $\angle ABC$ correct to the nearest degree.

(3)
- Determine the equation of a line parallel to AC and passing through D.

(2)
- [19]

QUESTION 3

In the diagram, A, B(-6 ; -5) and C(8 ; -4) are points in the Cartesian plane. F $(3; 3\frac{1}{2})$ and G are points on line AC such that AF = FG. E is the x-intercept of AB.



- 3.1 Calculate:

(4)
- 3.1.1 The equation of AC in the form $y = mx + c$

(4)
- 3.1.2 The coordinates of G if the equation of BG is $7x - 10y = 8$

(3)
- 3.2 Show by calculation that the coordinates of A is (2 ; 5).

(2)
- 3.3 Prove that EF $\parallel$ BG.

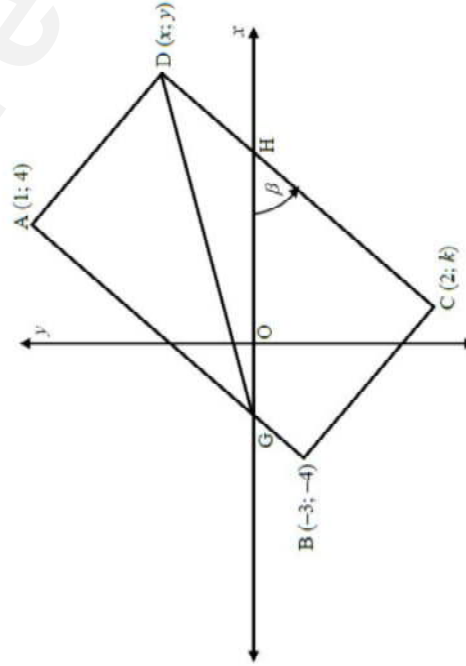
(4)
- 3.4 ABCD is a parallelogram with D in the first quadrant. Calculate the coordinates of D.

(4)
- [17]



QUESTION 3

In the diagram below, A (1; 4), B (-3; -4), C (2; k) and D (x; y) are the vertices of a rectangle. AB and DC cut the x-axis at G and H respectively. GD is drawn. $\angle GHC = \beta$.



Calculate the gradient of BG.

(2)

Determine the equation of AB in the form $y = mx + c$.

(2)

Calculate the:

3.3.1 Value of k (y-coordinate of C).

(4)

3.3.2 Coordinates of D.

(3)

3.3.3 Size of β .

(3)

3.3.4 Area of $\triangle DHG$.

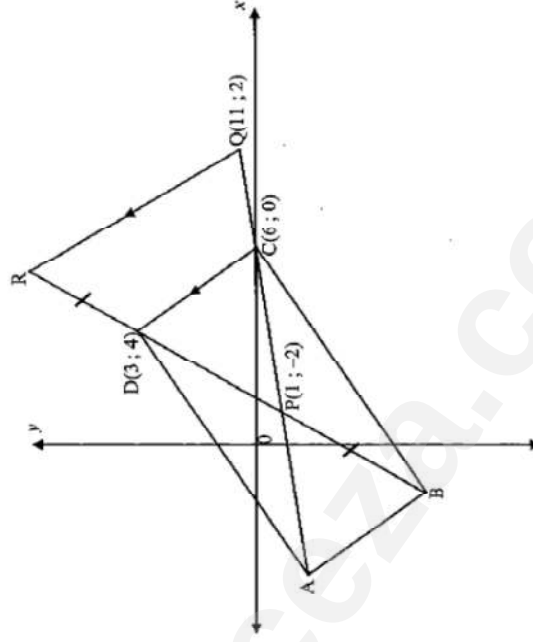
(7)

[21]



QUESTION 3

3.1 In the diagram below, A, B, C (6; 0) and D (3; 4) are the vertices of rectangle ABCD. Diagonals AC and BD bisect each other at P(1; -2). AC is produced to Q(11; 2) and BD is produced to R such that BP = DR and CD $\parallel$ QR.



3.1.1 Calculate the coordinates of B.

(3)

3.1.2 Determine the gradient of CD.

(2)

3.1.3 Show that the equation of QR is $y = -\frac{4}{3}x + \frac{50}{3}$.

(2)

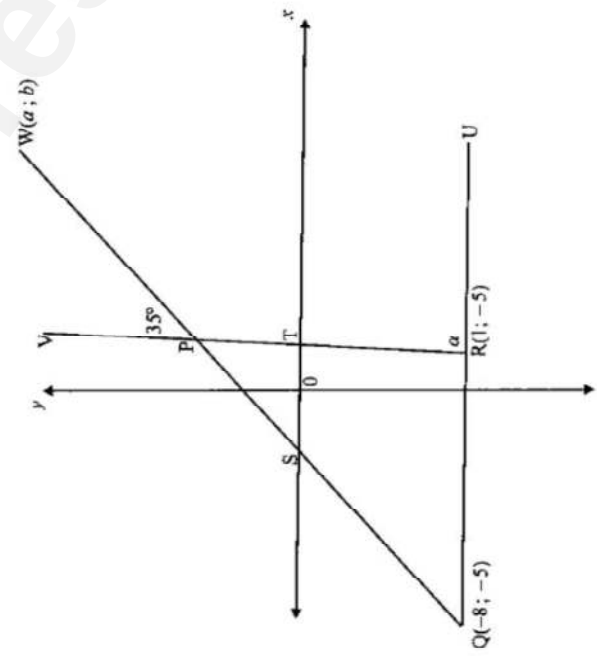
3.1.4 If K(4; y) is a point in the 4th quadrant such that PK = RQ, calculate the value of y.

(6)





- 3.2 In the diagram below, P, Q(-8; -5) and R(1; -5) are the vertices of $\triangle PQR$. RP is produced to V and QP is produced to W(a; b) such that $\angle VPW = 35^\circ$. The equation of QW is $y = x + \frac{2}{3}$. QR is produced to U and URV = α . QW and RV intersect the x-axis at S and T respectively.

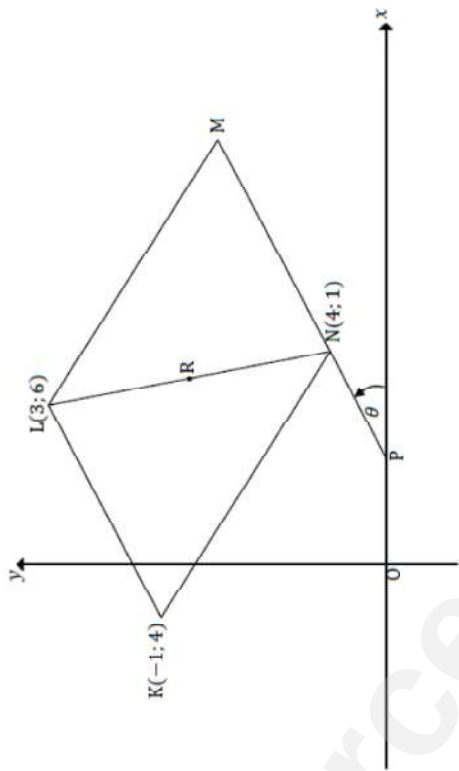


- 3.2.1 Calculate the size of α . (5)
- 3.2.2 It is further given that $QU \perp WU$ and R is the midpoint of QU. Calculate the area of $\triangle QWU$. (6) [24]



QUESTION 3

In the diagram, K(-1; 4); L(3; 6); M and N(4; 1) are vertices of a parallelogram. R is the midpoint of LN. P is the x-intercept of the line MN produced



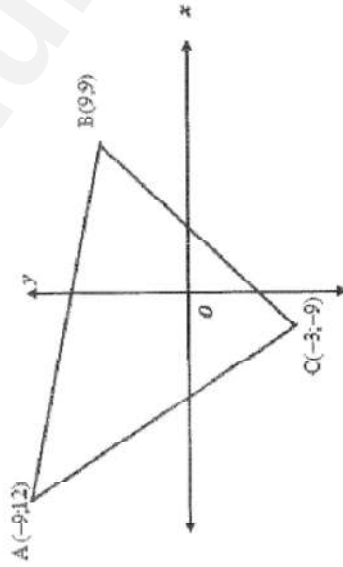
- 3.1 Calculate the:
- 3.1.1 gradient of KL. (2)
- 3.1.2 coordinates of R. (3)
- 3.1.3 coordinates of M. (4)
- 3.2 Determine the equation of NM in the form $y = mx + c$. (3)
- 3.3 Calculate the:
- 3.3.1 coordinates of P. (2)
- 3.3.2 size of θ , the inclination of PM. (2)
- 3.3.3 size of $\angle KPN$. (4) [20]





QUESTION 3

In the diagram $A(-9;12)$, $B(9;9)$ and $C(-3;-9)$ are the vertices of $\triangle ABC$. $K(m;n)$ is a point in the second quadrant.

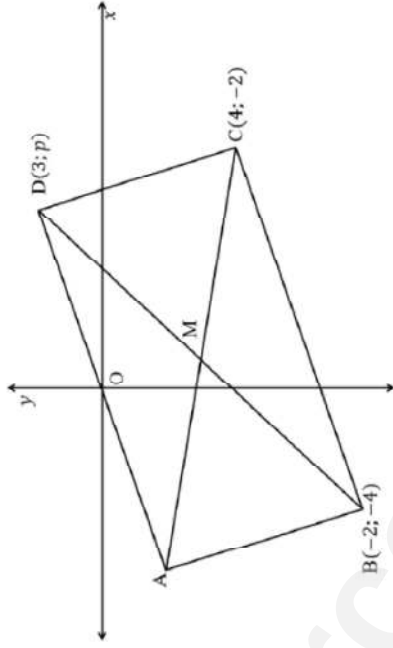


- 1 Calculate the gradient of AB. (2)
- 2 Calculate the size of $\hat{B}$, rounded off to two decimal digits. (5)
- 3 Determine the coordinates of M, the midpoint of BC. (2)
- 4 Determine the equation of the median AM. (3)
- 5 Determine the coordinates of K, if A, K and M are collinear and $BK = 5\sqrt{5}$ units. (8)

[20]

QUESTION 3

In the diagram A, $B(-2;-4)$, $C(4;-2)$ and $D(3;p)$ are the vertices of a rectangle. The diagonals AC and BD intersect at M.



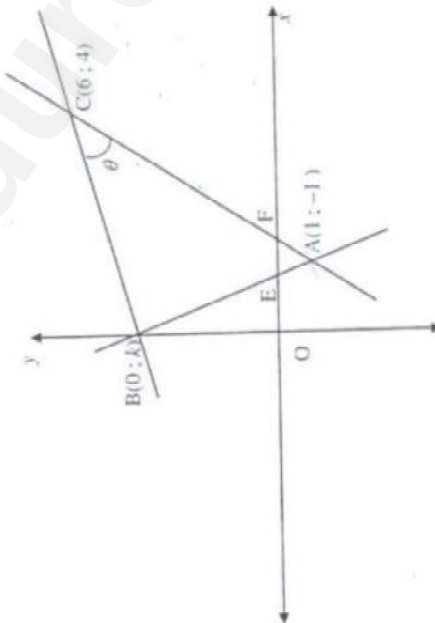
- 3.1 Given that the length of AC is $\sqrt{50}$ units, show that $p = 1$. (4)
- 3.2 Determine the coordinates of M. (3)
- 3.3 Calculate the gradient of DC. (2)
- 3.4 Determine the equation of line AB in the form $y = mx + c$. (2)

[11]



QUESTION 3

In the diagram, $A(1; -1)$, $B(0; k)$ and $C(6; 4)$ are the vertices of $\triangle ABC$. The equations of the sides AB and AC are $y + 3x - 2 = 0$ and $y = x - 2$ respectively. AB cuts the x -axis at E and AC cuts the x -axis at F .

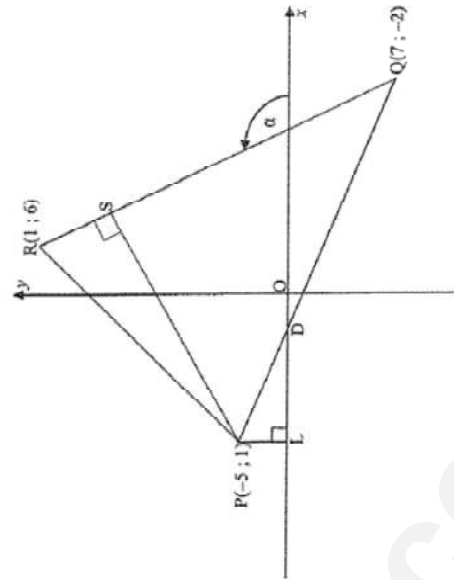


- 1 Write down the value of k . (2)
 - 2 Calculate the length of AC and leave your answer in simplest surd form. (2)
 - 3 Prove that $\angle ABC = 90^\circ$. (3)
 - 4 Calculate the size of θ . (5)
 - 5 Determine the equation of the circle passing through A , B and C in the form $(x-a)^2 + (y-b)^2 = r^2$. (4)
 - 6 If D is a point in the first quadrant, calculate the coordinates of D such that $ABCD$ in that order, forms a parallelogram. (4)
- [20]



QUESTION 3

In the diagram below, $P(-5; 1)$, $Q(7; -2)$ and $R(1; 6)$ are the vertices of $\triangle PQR$. PQ intersects the x -axis at D . The angle of elevation of QR is α . $PS \perp RQ$ and L lies on the x -axis such that $PL \perp x$ -axis.



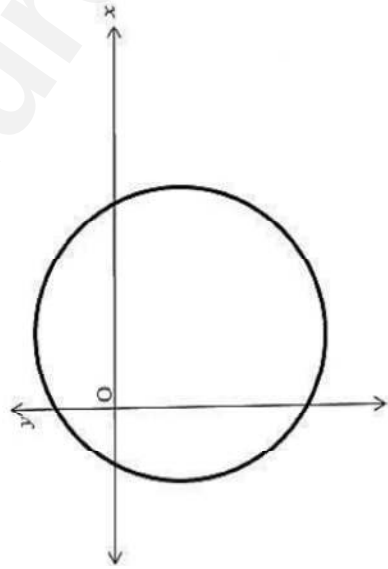
- 3.1 Write down the equation of the line PL . (1)
 - 3.2 Calculate the gradient of QR . (2)
 - 3.3 Determine the equation of the line PS . (4)
 - 3.4 Calculate the size of the angle of inclination of PQ . (3)
 - 3.5 Calculate the size of $\angle PQS$. (4)
 - 3.6 It is given that the areas of $\triangle PRS = 4x^2$ and $\triangle PQS = 16x^2$. Calculate the length of SQ WITHOUT calculating the coordinates of S . (5)
- [19]



Unit 2: Circles

QUESTION 4

The length of the radius of the circle with equation: $x^2 - 2x + y^2 + 4y = a$ is 5 units.

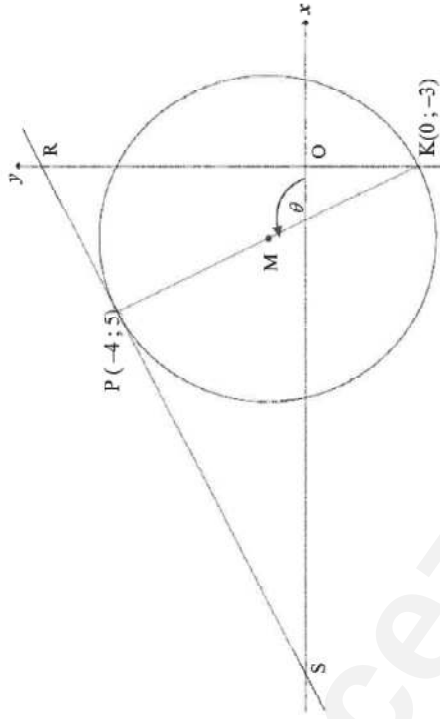


1. Show by means of calculations that $a = 20$ units. (4)
2. Write down the coordinates of the centre M of the circle. (1)
3. A(x; y), with $y > 0$, is one of the points of intersection of the circle and the straight line $x = 4$. Determine the values of x and y. (4)
4. Determine the equation of the tangent to the circle at the point A. (4)
5. Determine whether the point T(-1; -2) lies inside or outside the circle. (3)
6. If the circle is translated 3 units to the left and 1 unit up, determine the equation of the new circle. (3) [19]



QUESTION 4

In the diagram, P(-4; 5) and K(0; -3) are the end points of the diameter of a circle with centre M. S and R are respectively the x- and y-intercept of the tangent to the circle at P. θ is the inclination of PK with the positive x-axis.



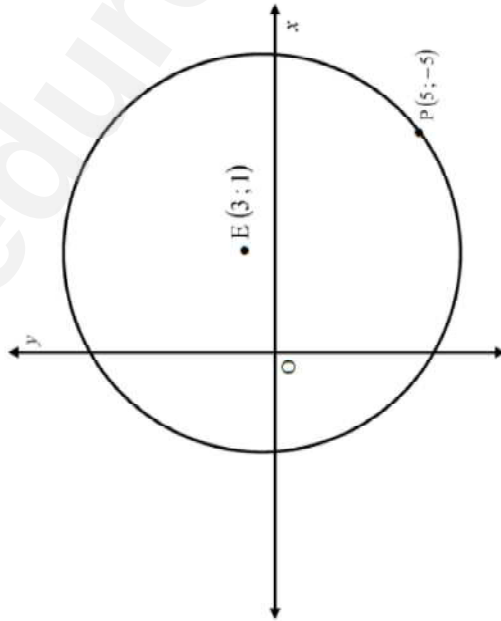
- 4.1 Determine:
 - 4.1.1 The gradient of SR (4)
 - 4.1.2 The equation of SR in the form $y = mx + c$ (2)
 - 4.1.3 The equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$ (4)
 - 4.1.4 The size of $\angle \hat{P}KR$. (3)
 - 4.1.5 The equation of the tangent to the circle at K in the form $y = mx + c$ (2)
- 4.2 Determine the values of t such that the line $y = \frac{1}{2}x + t$ cuts the circle at two different points. (3)
- 4.3 Calculate the area of $\triangle SMK$. (5) [23]





QUESTION 4

In the diagram below, the circle centred at $E(3; 1)$ passes through point $P(5; -5)$.



Determine the equation of:

- 4.1.1 The circle in the form $x^2 + y^2 + Ax + By + C = 0$. (4)
- 4.1.2 The tangent to the circle at $P(5; -5)$ in the form $y = mx + c$. (5)

A smaller circle is drawn inside the circle. Line EP is a diameter of the small circle. Determine the:

- 4.2.1 Coordinates of the centre of the smaller circle. (3)
- 4.2.2 Length of the radius. (3)
- 4.3 Hence, or otherwise, determine whether point $C(9; 3)$ lies inside or outside the circle centre at E. (3) [18]

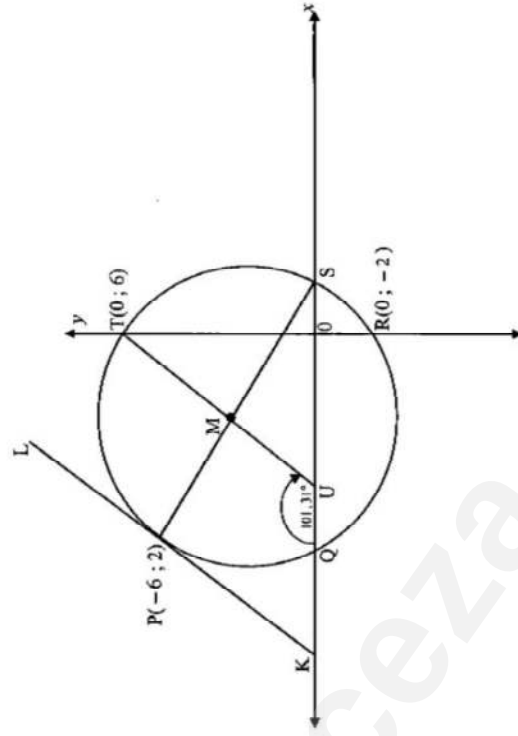


QUESTION 4

In the diagram below, a circle with centre M, cuts the x-axis at Q and S and the y-axis at $T(0; 6)$ and $R(0; -2)$. The equation of diameter SMP is $y = -\frac{1}{5}x + \frac{4}{5}$.

$$y = -\frac{1}{5}x + \frac{4}{5}$$

KPL is a tangent to the circle at $P(-6; 2)$. TM produced cuts the x-axis at U. $\angle QUT = 101,31^\circ$.

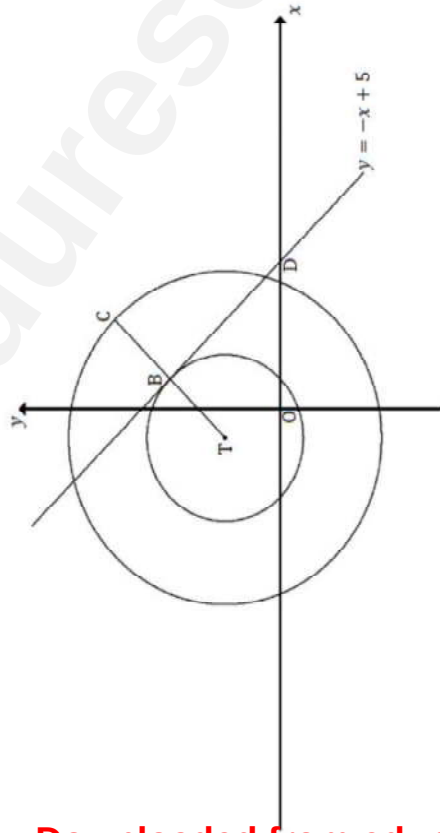


- 4.1 Determine the equation of TU. (3)
- 4.2 Calculate the coordinates of M. (3)
- 4.3 If the coordinates of M are $(-1; 1)$, determine the equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$. (3)
- 4.4 Prove that KL is parallel to TU. (3)
- 4.5 Is the point $V\left(-\frac{1}{2}; 7\right)$ inside the circle? Support your answer with calculations. (3) [15]



QUESTION 4

In the diagram, T is the centre of two concentric circles. The larger circle has equation $x^2 + y^2 - 4y + 2x - 27 = 0$. The smaller circle touches the straight line $y = -x + 5$ at point B. BD is a tangent to smaller circle T. D is the x-intercept of the straight line. C is a point on the larger circle such that TBC is a straight line.

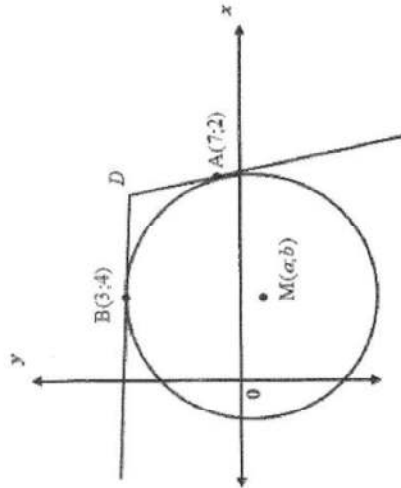


- Calculate the coordinates of T. (4)
 - Show that equation of TB is given by $y = x + 3$. (3)
 - Calculate the coordinates of B. (3)
 - Determine the equation of the smaller circle in the form $(x - a)^2 + (y - b)^2 = r^2$. (3)
 - Calculate the area of quadrilateral OTBD. (7)
- [20]



QUESTION 4

In the diagram a circle $x^2 - 6x + y^2 + 2y = 15$ passes through the points A(7;2) and B(3;4). AD and BD are tangents to the circle, with M the centre of the circle

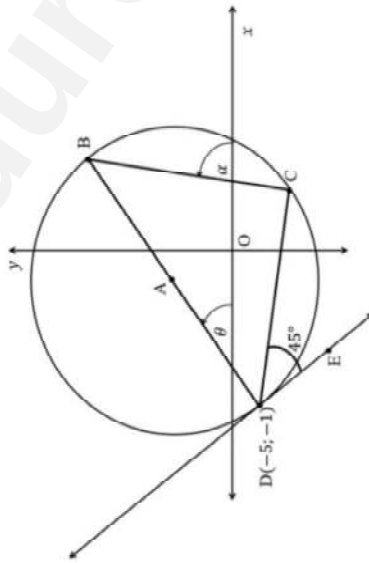


- 4.1 Determine the:
 - 4.1.1 Coordinates of the centre M, and the radius of the circle. (5)
 - 4.1.2 Equation of tangent AD. (4)
 - 4.1.3 Coordinates of D, if BD and AD meet at D. (4)
 - 4.2 Identify the type of quadrilateral ADBM is, and provide a valid reason. (2)
 - 4.3 Determine the area of ADBM. (4)
- [19]



QUESTION 4

In the diagram is the circle with equation $(x + 1)^2 + (y - 2)^2 = 25$. DB is the diameter of the circle and A the centre of the circle. DE is a tangent to the circle at D(-5; -1). The angle $\widehat{EDC} = 45^\circ$. The inclination angles of AD and BC is θ and α respectively. B and C are points on the circumference of the circle.



4.1

Determine:

- 4.1.1 The coordinates of A, the centre of the circle. (2)
- 4.1.2 The coordinates of B (3)
- 4.1.3 The gradient of AD. (2)
- 4.1.4 The value of θ , the inclination angle of AD. (2)
- 4.1.5 The equation of the tangent DE. (3)

4.2

Calculate the gradient of BC. (4)

4.3

Another circle with equation $x^2 + y^2 - 6x + 2y + 8 = 0$ is given. (4)

Show that:

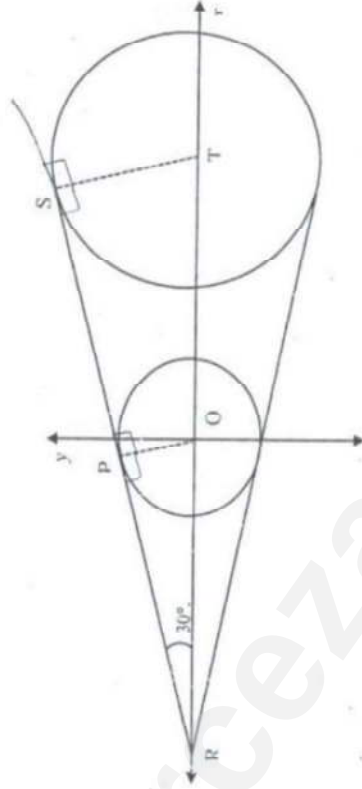
- 4.3.1 The coordinates of the centre of the circle is M(3; -1). (4)
- 4.3.2 The two circles will intersect each other. Show all calculations. (4)

[24]

QUESTION 4

The diagram below shows a representation of the chain of a bicycle attached to two circular cogs as represented in the Cartesian plane. The equations of the circles are given by $x^2 + y^2 = 1$ and $(x - 6)^2 + y^2 = 9$.

RPS is a common tangent to the smaller and larger circles at P and S respectively, with R a point on the negative x-axis. T is the centre of the larger circle. The angle of inclination between the tangent RPS and the horizontal axis is 30° .



4.1 Write down the coordinates of T. (1)

4.2 Write down the length of the radius of the larger circle. (1)

4.3 If R(-3; 0), determine the equation of tangent RPS in the form $y = mx + c$. Leave answer in surd form. (4)

4.4 Determine the equation of the radius ST in the form $y = mx + c$. Leave answer in surd form. (3)

4.5 Determine the coordinates of S. (5)

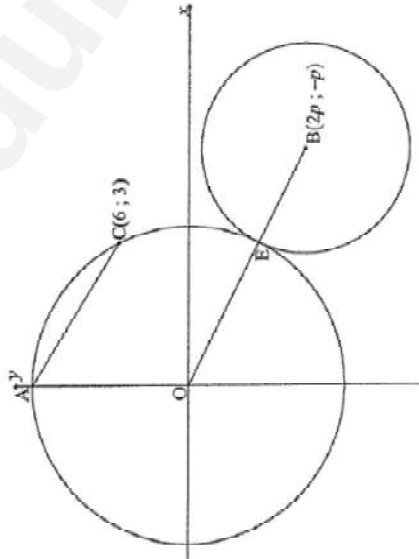
4.6 Determine the distance between the two circular cogs. (4)

[18]



QUESTION 4

In the diagram below, two circles are given. Circle O, having the origin as centre, intersects the y-axis at A and passes through the point C(6; 3). The circle having centre B(2p; -p) touches circle O externally in point E. The centres of the two circles are joined by the line OB.



Determine the equation of the circle having centre O. (2)

Determine the coordinates of A. (2)

Determine the equation of AC. (3)

Calculate the value(s) of k for which the line $y = \frac{1-\sqrt{5}}{2}x + k$ will intersect the circle having centre O at two points, one of which has a positive x-value and the other a negative x-value. (2)

It is given that the length of EB = $\sqrt{20}$.

4.5.1 Write down, in terms of p, the equation of circle B in the form $(x-a)^2 + (y-b)^2 = r^2$. (2)

4.5.2 Determine the value of p if $p > 0$. (5)

4.6 Suppose a third circle with the following equation is given:
 $x^2 + y^2 + 4x \cos \theta + 8y \sin \theta + 3 = 0$

Determine the maximum length that the radius of this circle can be for any value of θ . (6)

[22]

MODULE 3: TRIGONOMETRY

1. GENERAL TRIGONOMETRY

QUESTION 5

5.1 If $\cos 62^\circ = m$, determine the value of each of the following in terms of m.

5.1.1 $\sin 28^\circ$ (2)

5.1.2 $\cos 362^\circ$ (4)

5.2 Simplify to a single ratio:

$$\frac{\tan(360^\circ - x) \cdot \sin(90^\circ + x)}{\sin(-x)}$$

(5)

5.3 If $4 \sin^2 \theta - 3 = 0$, $90^\circ \leq \theta \leq 180^\circ$, calculate the value of:

$$\cos \frac{\theta}{4} \cdot \sin \frac{\theta}{2} - \tan(3\theta - 45^\circ)$$

(6)

[17]

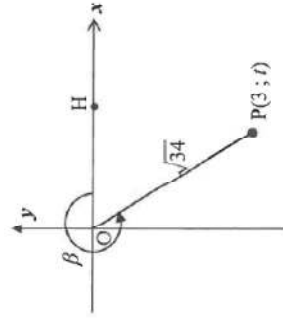


QUESTION 5

5.1 Given: $\frac{\sin(A - 360^\circ) \cos(90^\circ + A)}{\cos(90^\circ - A) \tan(-A)}$

Simplify the expression to a single trigonometric ratio. (6)

5.2 In the diagram, $P(3; t)$ is a point in the Cartesian plane. $OP = \sqrt{34}$ and $\widehat{HOP} = \beta$ is a reflex angle.

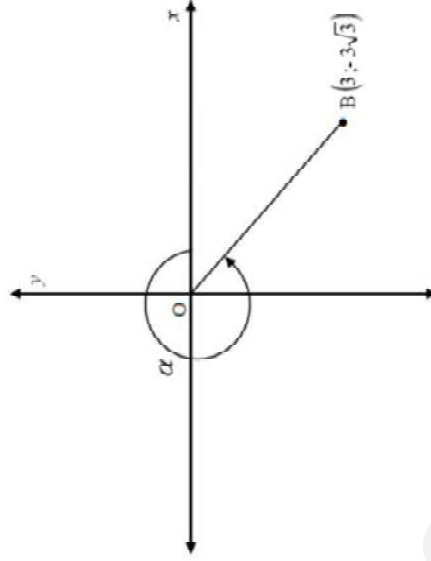


Without using a calculator, determine the value of:

- 5.2.1 t (2)
5.2.2 $\tan \beta$ (1)
5.2.3 $\cos 2\beta$ (4)

QUESTION 5

5.1 In the Cartesian plane below, the point $B(3; -3\sqrt{3})$ and the reflex angle α , are shown.



Determine (without using a calculator) the value of:

- 5.1.1 $\sin \alpha$ (2)
5.1.2 $\cos(\alpha + 30^\circ)$ (4)
5.2 Simplify:
$$\frac{\sin^2(90^\circ - x) \tan(360^\circ - x)}{\sin(-x)}$$
 (4)
5.3 Prove that:
$$\cos(60^\circ + \theta) - \cos(60^\circ - \theta) = -\sqrt{3} \sin \theta$$
 (3)
5.4 Consider the identity: $\frac{1 - \sin 2A}{\sin A - \cos A} = \sin A - \cos A$
5.4.1 Prove the identity. (4)
5.4.2 For which values of A in the interval $0^\circ < A < 180^\circ$ will the identity be undefined? (2)
[19]



QUESTION 5

- 5.1 If $\sin 16^\circ = \frac{1}{\sqrt{1+k^2}}$, express the following in terms of k , without the use of a calculator.

5.1.1 $\tan 16^\circ$

(2)

5.1.2 $\cos 32^\circ$

(3)

Simplify the following expression.

$$\frac{\cos(90^\circ + x) \sin(x - 180^\circ) - \cos^2(180^\circ - x)}{\cos(-2x)}$$

(6)

Calculate the value of the following, without the use of a calculator.

$$\cos 75^\circ \cdot \cos 45^\circ - \cos 15^\circ \cdot \cos 45^\circ$$

(4)

Given: $\tan \theta \left(\sin 2\theta + \frac{3 \cos^2 \theta}{\sin \theta} \right) = -2 \cos^2 \theta + 3 \cos \theta + 2$

5.4.1 Prove the identity.

(3)

5.4.2 Determine the general solution of:

$$\tan \theta \left(\sin 2\theta + \frac{3 \cos^2 \theta}{\sin \theta} \right) = 0$$

(4)

Solve for a and b :

$$\cos(a+b) = -\frac{\sqrt{2}}{2} \text{ if } a+b \in [0^\circ; 180^\circ]$$

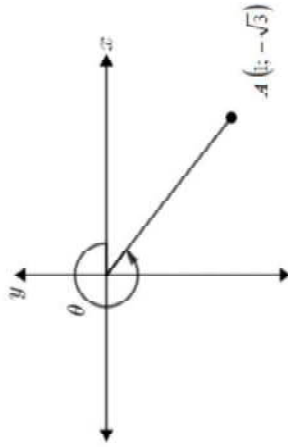
$$\cos(a-2b) = \frac{1}{2} \text{ if } a-2b \in [0^\circ; 180^\circ]$$

(4)

[26]

QUESTION 5

- 5.1 Use the diagram below to calculate, without the use of a calculator, the following



5.1.1 $\tan \theta$

(1)

5.1.2 $\sin(-\theta)$

(3)

5.1.3 $\sin(\theta - 50^\circ)$

(4)

- 5.2 Determine the value of the following trigonometric expression:

$$\frac{\tan(180^\circ - \theta) \sin(90^\circ + \theta)}{\cos 30^\circ \sin(\theta - 360^\circ)}$$

(6)

5.3 Consider: $\frac{\cos 2x - 1}{\sin 2x} = -\tan x$

5.3.1 Prove the identity

(3)

5.3.2 For which value(s) of x , $0^\circ < x < 360^\circ$, is this identity undefined?

(3)

5.3.3 Hence or otherwise, find the general solution of $\frac{\sin 4x}{\cos 4x - 1} = 4$.

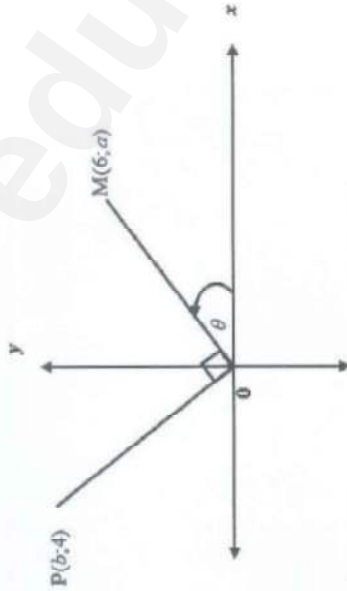
(4)

[24]



QUESTION 5

In the diagram alongside $\widehat{POM} = 90^\circ$, $\widehat{MOX} = \theta$, $M(6;a)$, $P(b;4)$ and $\sqrt{5} \cos \theta - 2 = 0$.



Hint: $\tan(90^\circ + \theta) = -\frac{1}{\tan \theta}$.

Determine, without the use of a calculator, the values of:

5.1.1 a (3)

5.1.2 b (4)

Simplify the following expression to a single trigonometric ratio of θ :

5.2.1 $\frac{\sin 163^\circ}{\cos 73^\circ} - \frac{\sin(90^\circ + \theta) \cdot \tan^2(\theta - 360^\circ)}{\cos(180^\circ - \theta)}$ (7)

5.2.2 $\frac{\sin^2 \theta \cos \theta}{\tan 2\theta(\sin^2 \theta - \cos^2 \theta)}$ (5)

5.3 Determine the general solution of $\cos(x - 60^\circ) = 3 \cos x$. (6)
[25]

QUESTION 5

5.1 If $\sin 40^\circ \cos 22^\circ + \cos 40^\circ \sin 22^\circ = k$, determine without the use of a calculator, the value of the following in terms of k .

5.1.1 $\sin 62^\circ$ (2)

5.1.2 $\tan 118^\circ$ (4)

5.1.3 $\sin 14^\circ \cos 14^\circ$ (3)

5.2 Prove the following identity:

$$\frac{1 - \cos 2\theta}{\sin 2\theta \times \tan \theta} = 1 \quad (4)$$

5.3 For which value(s) of A will the following expression be real?

$$\sqrt{\sin(180^\circ + A) \cdot \cos(90^\circ + A) - \tan 45^\circ} \quad (6)$$

[19]

QUESTION 5

- 5.1 Simplify to one trigonometric ratio:

$$\frac{\sin(A - 180^\circ) \tan(180^\circ - A) \cos A}{\cos(90^\circ + A)} \quad (5)$$

If $\sin 28^\circ = a$ and $\cos 32^\circ = b$, determine the following in terms of a and b .

5.1.1 $\cos 28^\circ$

5.1.2 $\cos 64^\circ$

5.1.3 $\sin 4^\circ$

- Prove the identity:

$$\frac{1}{1 - \sin x} - \frac{1}{1 + \sin x} = \frac{2 \tan x}{\cos x} \quad (5)$$

[18]

QUESTION 5

- 5.1 Simplify each of the following without the use of a calculator. Show ALL calculations.

5.1.1 $\frac{\sin 110^\circ \cdot \tan 60^\circ}{\cos 540^\circ \cdot \tan 250^\circ \cdot \sin 380^\circ} \quad (7)$

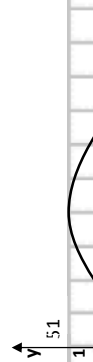
5.1.2 $(1 - \sqrt{2} \sin 22,5^\circ)(\sqrt{2} \sin 22,5^\circ + 1) \quad (4)$

- 5.2 Given the expression:
- $\frac{\cos 2x \cdot \tan x}{\sin^2 x}$

- 5.2.1 For which value(s) of
- x
- , in the interval
- $x \in [0^\circ; 180^\circ]$
- , will this expression be undefined? (3)

5.2.2 Prove that $\frac{\cos 2x \cdot \tan x}{\sin^2 x} = \frac{\cos x}{\sin x} - \tan x \quad (5)$

[19]





Unit 2: Trigonometric Graphs

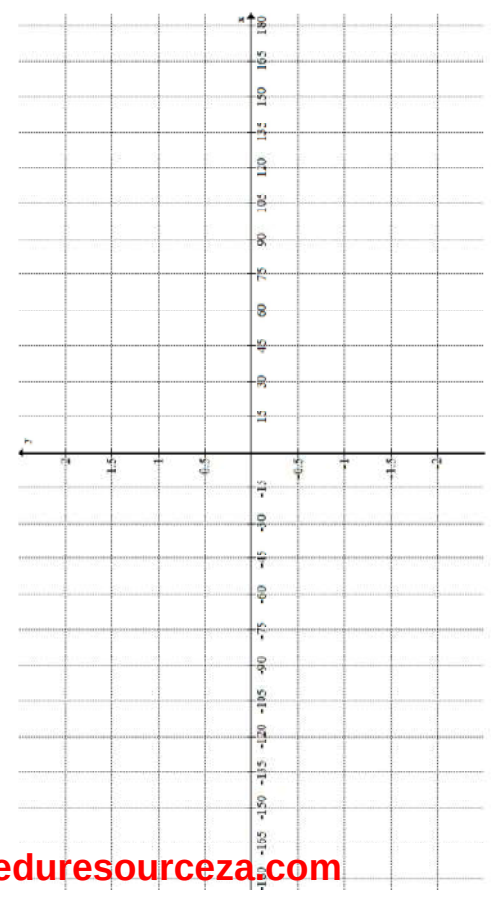
QUESTION 8

Given $f(x) = 2\cos x$ and $g(x) = \sin(x + 30^\circ)$

- (1) Write down the amplitude of f .
- (5) Sketch the graph of f and g on the same set of axes on DIAGRAM SHEET 2.
- (2) Determine the value(s) of x for which $f(x) - g(x) \geq 0$ for $x \in [-180^\circ; 90^\circ]$.
- (2) Give TWO possible ways of transforming the graph of f such that its y -intercept is at the origin.

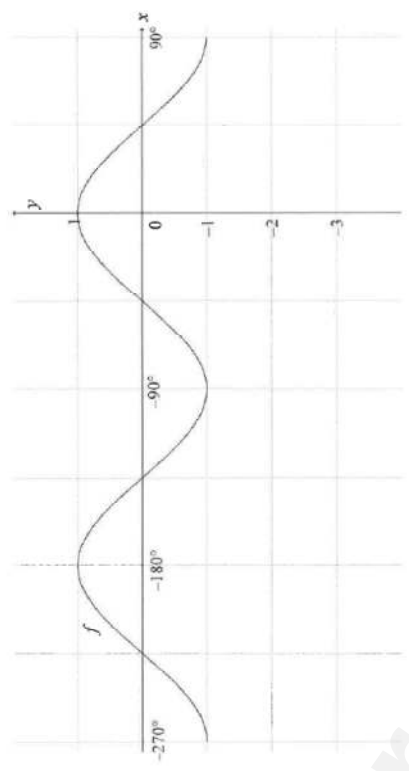
DIAGRAM SHEET 2

QUESTION 8.2



QUESTION 6

In the diagram, the graph of $f(x) = \cos 2x$ is drawn for the interval $x \in [-270^\circ; 90^\circ]$.



- 6.1 Draw the graph of $g(x) = 2\sin x - 1$ for the interval $x \in [-270^\circ; 90^\circ]$ on the grid given in your ANSWER BOOK. Show ALL the intercepts with the axes, as well as the turning points. (4)
- 6.2 Let A be a point of intersection of the graphs of f and g . Show that the x-coordinate of A satisfies the equation $\sin x = \frac{-1 + \sqrt{5}}{2}$. (4)
- 6.3 Hence, calculate the coordinates of the points of intersection of graphs of f and g for the interval $x \in [-270^\circ; 90^\circ]$. (4) [12]

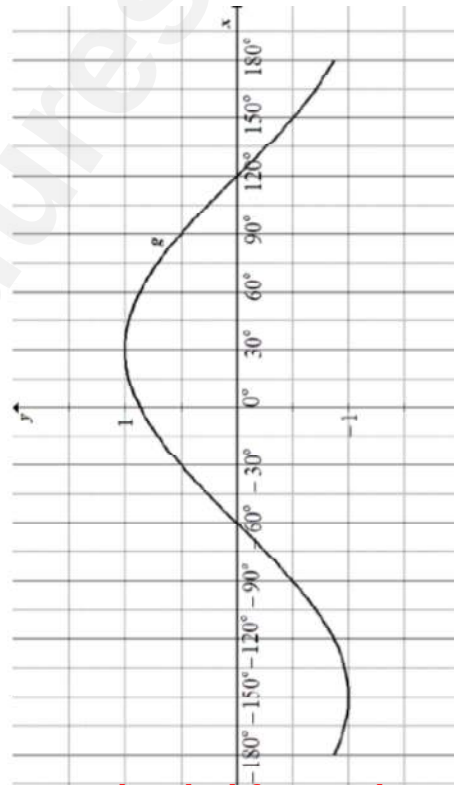




QUESTION 6

6.1 Determine the general solution for $\sin 2x = \cos(x - 30^\circ)$. (5)

6.2 The diagram below shows the graph of $g(x) = \cos(x - 30^\circ)$ for the interval $x \in [-180^\circ; 180^\circ]$.



6.2.1 Write down the period of g . (1)

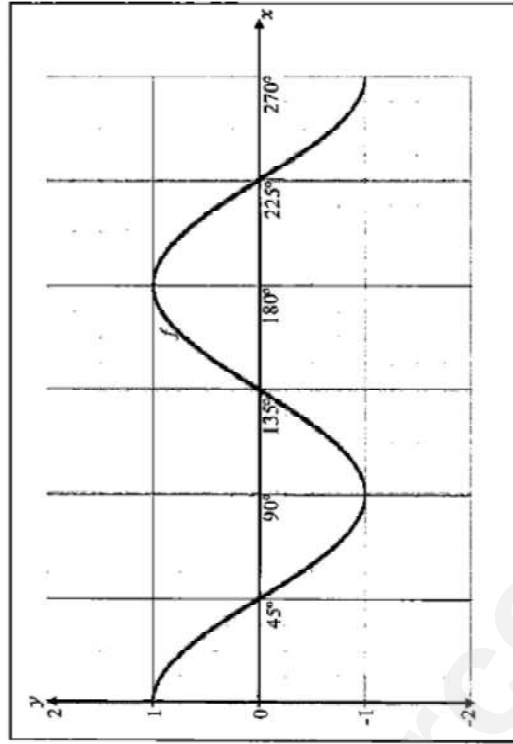
6.2.2 Determine the values of x for which the graph of g is increasing. (2)

6.2.3 On the same system of axes draw the graph of $f(x) = \sin 2x$ for $x \in [-180^\circ; 180^\circ]$ in your ANSWER BOOK. (3)

6.2.4 Hence or otherwise, determine the values of x in the interval $-180^\circ \leq x \leq 180^\circ$ for which $f(x)g(x) < 0$. (3) [14]

QUESTION 6

In the diagram below, the graph of $f(x) = \cos 2x$ is drawn for the interval $x \in [0^\circ; 270^\circ]$.



6.1 Draw the graph of $g(x) = -\frac{1}{2} \tan x$ for the interval $x \in [0^\circ; 270^\circ]$ on the grid provided in the ANSWER BOOK. Show all intercepts with the axes and asymptotes. (4)

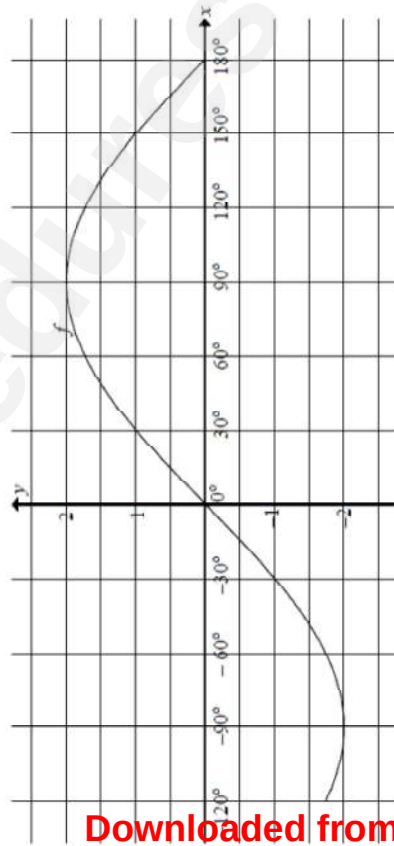
6.2 Write down the range of $h(x) = 3 - f(x)$. (1)

6.3 Use the graph to determine the value(s) of x in the interval $x \in [35^\circ; 270^\circ]$ for which $\frac{f(x)}{g(x)} \geq 0$. (2) [7]



QUESTION 6

In the diagram below, the graph of $f(x) = 2\sin x$ is drawn for the interval $x \in [-120^\circ; 180^\circ]$.



Draw on the same system of axes the graph of $g(x) = \cos(x + 30^\circ)$, for the interval $x \in [-120^\circ; 180^\circ]$. Show all intercepts with the axes as well as the turning and end points of the graph.

Write down the period of f .

For which values of x in the interval $x \in [-120^\circ; 180^\circ]$ is:

6.3.1 The graph of g decreasing?

6.3.2 $f(x) \cdot g(x) > 0$?

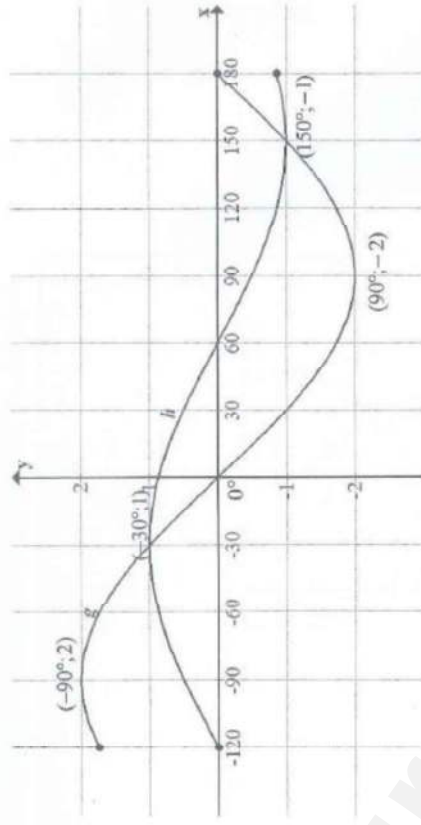
6.4 If the graph of g is moved 60° to the left, determine the equation of the new graph which is formed, in its simplest form.

[11]



QUESTION 6

The graphs of $h(x) = \cos(x + 30^\circ)$ and $g(x) = -2\sin x$, $x \in [-120^\circ; 180^\circ]$ is drawn below.



Use the graphs above to determine for which values of x :

6.1 $2\sin x + \cos x \cos 30^\circ \geq \sin x \sin 30^\circ$, $x \in [-120^\circ; 180^\circ]$

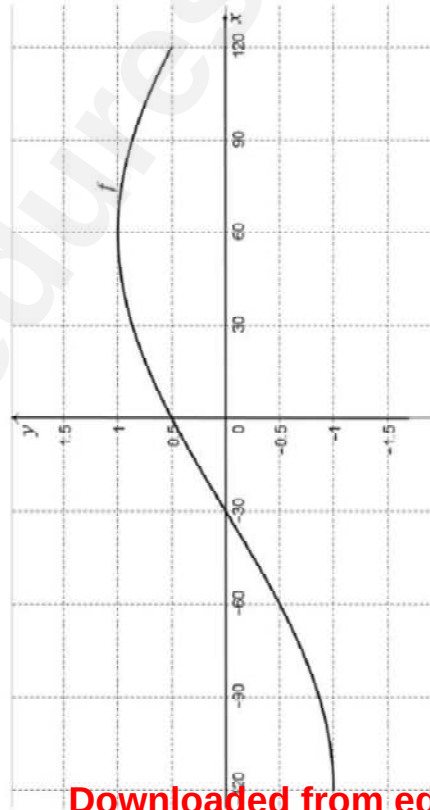
6.2 Will both $g(x)$ and $h(x)$ increase as x increases for $x \in [-120^\circ; 180^\circ]$

[8]



QUESTION 6

In the diagram is the graph of $f(x) = \sin(x + a)$ for the interval $[-120^\circ; 120^\circ]$



Determine the numerical value of a .

(1)

On the grid provided in the ANSWER BOOK, draw the graph of $g(x) = \cos(3x)$ for the interval $x \in [-120^\circ; 120^\circ]$. Clearly show ALL intercepts with the axes, the turning point(s) and endpoint(s) of the graph.

21

Determine the general solution for the following: $f(x) = g(x)$

(5)

Determine the values of x in the interval $x \in [0^\circ; 120^\circ]$, for which $f(x) > g(x)$.

(2)

Describe the transformation from graph g to the graph of $k(x) = \cos(60^\circ - 3x)$.

(2)

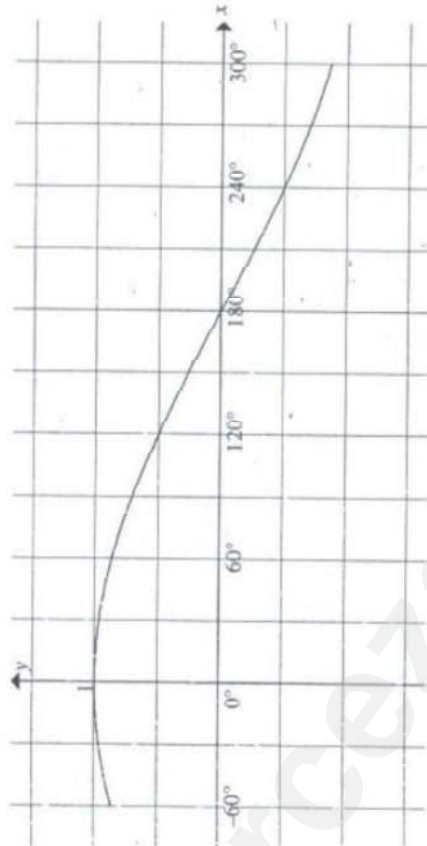
[14]

6.5

QUESTION 6

6.1 Solve for x if $\sin(x + 60^\circ) = \cos \frac{1}{2}x$ and $x \in [-60^\circ; 300^\circ]$. (7)

6.2 In the diagram below, the graph of $f(x) = \cos \frac{1}{2}x$ is drawn for $x \in [-60^\circ; 300^\circ]$.



6.2.1 On the grid provided in THE ANSWER BOOK, draw the graph of $g(x) = \sin(x + 60^\circ)$ for $x \in [-60^\circ; 300^\circ]$. Clearly show all maximum and minimum points, the intercepts with the axes and the endpoints. (3)

6.2.2 Use the solutions obtained in 6.1 as well as the graphs drawn in 6.2.1 to

determine the value(s) of $x \in [-60^\circ; 300^\circ]$ for which:

6.3.1 $f(x) < g(x)$ (3)

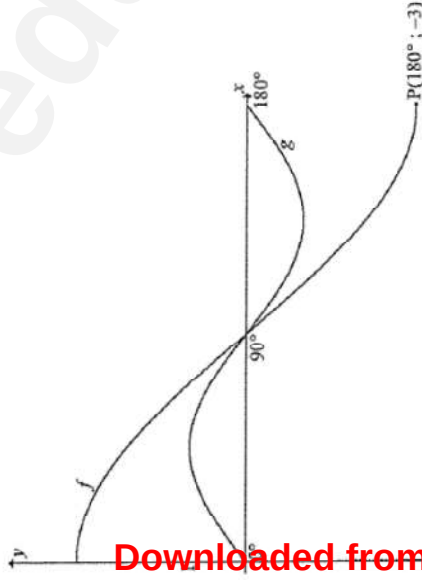
6.3.2 $f(x) \cdot g(x) \leq 0$ (3)

[16]



QUESTION 6

In the diagram below, the graphs of $f(x) = a \cos x$ and $g(x) = \sin bx$ are drawn for the interval $x \in [0^\circ; 180^\circ]$. The point $P(180^\circ; -3)$ is on the graph of f .



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Write down the values of a and b .

(2)

Write down the period of f .

(1)

Write down the range of $g(x) + 3$.

(2)

For which values of x , in the given interval, is $f(x) \cdot g'(x) > 0$?

(3)

6.5 When the graph of g is shifted q° to the left, it coincides with the function $y = \cos^2 x = -\sin^2 x$. Determine the value of q .

(3)

[11]

Unit 3: Identities

QUESTION 6

6.1 Prove that $\frac{\cos 2x}{1 + \sin 2x} = \frac{\cos x - \sin x}{\sin x + \cos x}$ [4]

5.3 Prove:

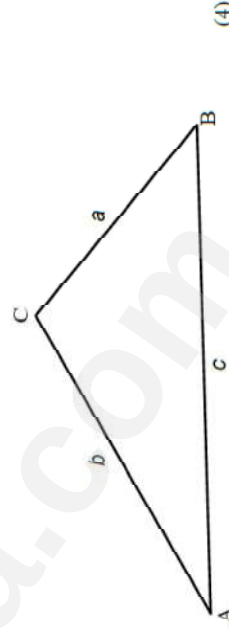
5.3.1 $\sin(A + B) - \sin(A - B) = 2 \cos A \cdot \sin B$ (2)

5.3.2 Without using a calculator, that $\sin 77^\circ - \sin 43^\circ = \sin 17^\circ$ (4) [19]

Unit 4: 2d and 3d Trigonometry

QUESTION 7

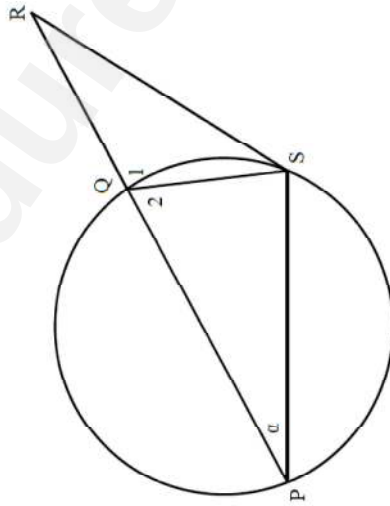
7.1 Use the diagram on DIAGRAM SHEET to prove that:
 $a^2 = b^2 + c^2 - 2bc \cdot \cos A$





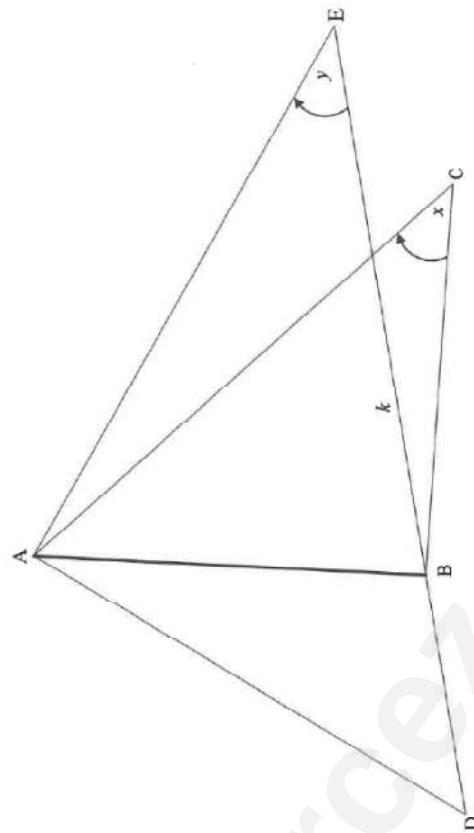
QUESTION 7

- 7.2 In the figure, the diameter PQ of the circle is produced to R. S is a point on the circumference such that $QR = QS = x$. P and S are joined and $\angle QPS = \alpha$.



7.2.1 Prove that : $SR = x\sqrt{2(1 + \sin \alpha)}$ (4)

7.2.2 If $SR = 5\sqrt{3}$ and $x = 5$, show that $PQ = 10$ (4) [12]



7.1 Write down the size of $\angle ABC$. (1)

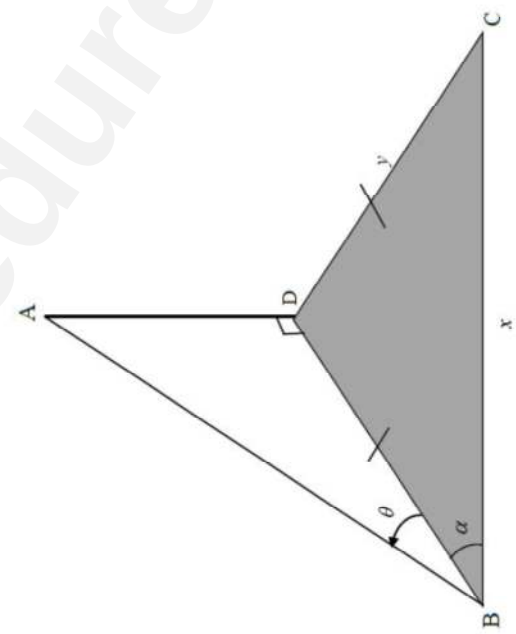
7.2 Show that $AC = \frac{k \tan y}{\sin x}$ (4)

7.3 If it is further given that $\angle DAC = 2x$ and $AD = AC$, show that the distance DC between the players at D and C is $2k \tan y$. (5) [10]



QUESTION 7

In the diagram below, B, C and D are three points on the same horizontal plane such that $BD = DC = y$. $\angle CBD = \alpha$ and $\angle ABD = \theta$. Line $BC = x$.



Prove that $AB = \frac{x}{2 \cos \alpha \cos \theta}$

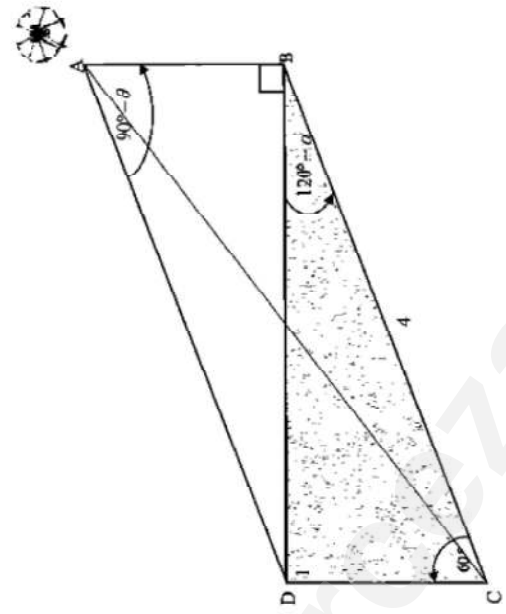
[7]



QUESTION 7

In the diagram below, B, C and D are in the same horizontal plane. A drone positioned at A, captures the images of two objects at B and C. B is directly below the drone and C is 4 units away from B.

$\angle DCB = 60^\circ$; $\angle BDC = 120^\circ - \alpha$ and $\angle DAB = 90^\circ - \theta$.



7.1 Determine $\hat{D}_1$ in terms of α .

(2)

7.2 Without the use of a calculator, determine BD in terms of α .

(3)

7.3 Show that $AB = \frac{2\sqrt{3} \tan \theta}{\sin \alpha}$.

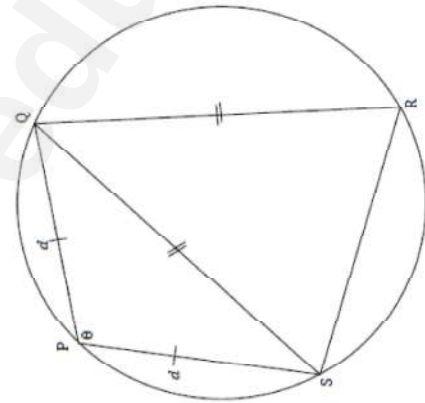
(3)

[8]



QUESTION 7

In the diagram, PQRS is a cyclic quadrilateral with $QS = QR$ and $PQ = PS = d$ units. $\widehat{QPS} = \theta$.



Use the diagram to prove that:

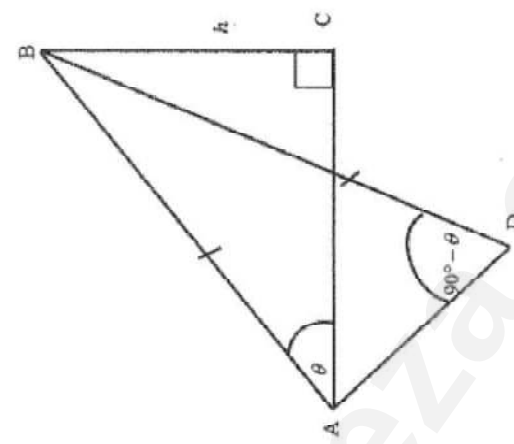
$$QS = d\sqrt{2(1 - \cos \theta)}$$

$$\text{The area of } \triangle QRS = -d^2 \sin 2\theta (1 - \cos \theta)$$

- (2)
(3)
[5]

QUESTION 7

In the diagram below, BC is a pole anchored by two cables at A and D. A, D and C are in the same horizontal plane. The height of the pole is h and the angle of elevation from A to the top of the pole B, is θ . $BA = BD$ and $\widehat{BDA} = 90^\circ - \theta$.

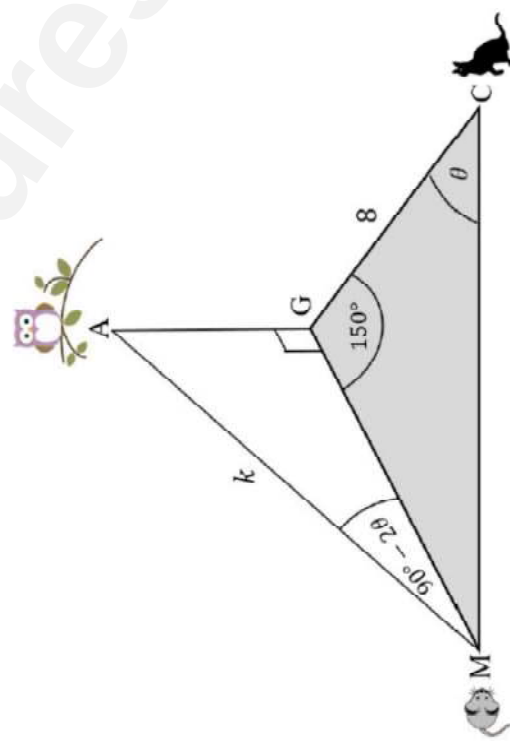


- 7.1 Express AB in terms of h and a trigonometric ratio of θ . (1)
7.2 Determine the magnitude of $\widehat{ABD}$ in terms of θ . (3)
7.3 Determine the distance between the two anchors in terms of h . (3)
[7]



QUESTION 7

A mouse on the ground is looking up to an owl in a tree and a cat to his right on the ground. The angle of elevation from the mouse to the owl is $(90^\circ - 2\theta)$. $AM = k$ units, $GC = 8$ units, $\angle MGC = 150^\circ$ and $\angle MCG = \theta$

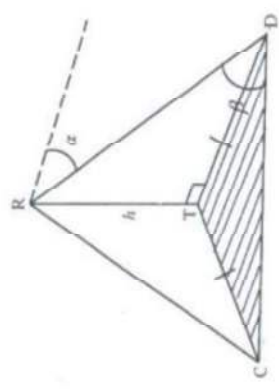


- 7.1 Give the size of $\angle MAG$ in terms of θ . (1)
 - 7.2 Show that $MG = k \sin 2\theta$ (2)
 - 7.3 Show that $MC = k \cos \theta$ (4)
 - 7.4 Show that the area of $\triangle MGC = 2k \sin 2\theta$ (2)
- [9]



QUESTION 7

In the diagram, RT represents the height of a vertical tower. C and D represent two points equidistant from T and which lie on the same horizontal plane as T . The height of the tower is h . The angle of depression of D from R is α . $\angle RDC = \beta$.

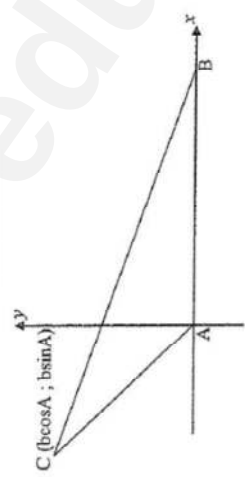


- 7.1 Determine the size of $\angle CRD$ in terms of β . (2)
 - 7.2 Prove that $CD = \frac{2h \cos \beta}{\sin \alpha}$. (5)
 - 7.3 Calculate the height of the tower, rounded off to nearest unit, if $CD = 5,4$ units, $\alpha = 51^\circ$ and $\beta = 65^\circ$. (2)
- [9]



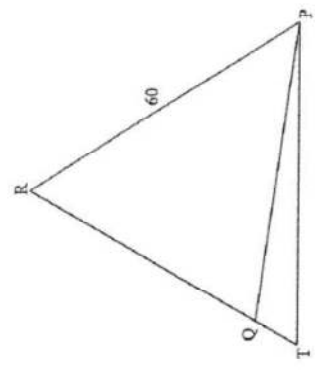
QUESTION 7

7.1 In the diagram below, $\triangle ABC$ is drawn having A at the origin, B on the x-axis and the vertex C has the coordinates $(b \cos A, b \sin A)$.



Use the above diagram to prove that $a^2 = b^2 + c^2 - 2bc \cos A$ (4)

In the diagram below, $\triangle TPR$ is equilateral with $PR = 60$ units. Q is a point on RT such that $RQ:QT = 5:1$.



7.2.1 Show, by calculations, that $PQ = 55,68$ units. (4)

7.2.2 It is given that S is any point on the straight line PQ. Calculate the distance QS when S is the nearest to R. (4) [12]

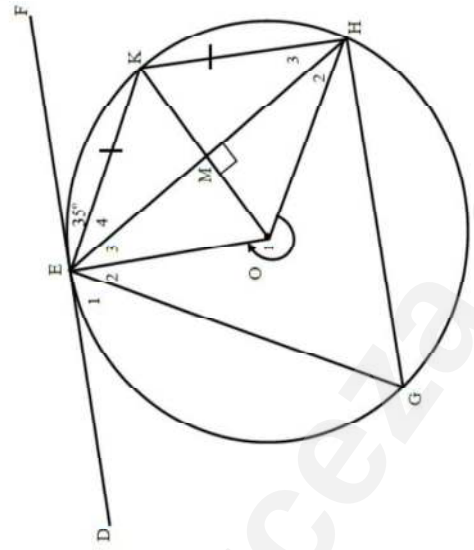


MODULE 4: EUCLIDEAN GEOMETRY

1. CIRCLES

QUESTION 8

DF is a tangent to the circle at E. EKHG is a cyclic quadrilateral. $\angle KEF = 35^\circ$. O is the centre of the circle. $OK \perp EH$ and $EK = HK$.



8.1 Determine, with reasons, the size of each of the following:

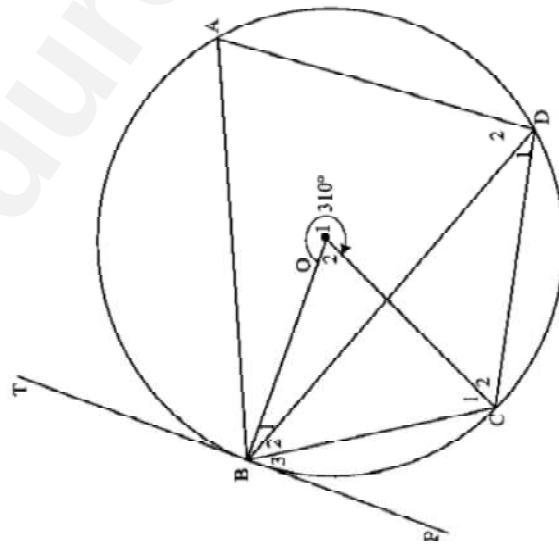
- 8.1.1 $\hat{E}_4$ (3)
- 8.1.2 $\angle EKH$ (2)
- 8.1.3 $\hat{G}$ (2)
- 8.1.4 $\hat{O}_1$ (2)

8.2 It is further given that $EH = 24$ units, $KM = 4$ units and the radius of the circle EKHG is x. Determine the value of x. (4) [13]



QUESTION 8

In the diagram below, A, B, C and D are points on a circle having centre O. PBT is a tangent to the circle at B. Reflex $\angle BOC = \hat{C}_1 = 310^\circ$ as shown in the diagram below.



Calculate, giving reasons, the size of:

$\hat{D}_1$

$\hat{B}_3$

8.3 $\hat{B}_1$, if it is given that $\hat{A} = 60^\circ$.

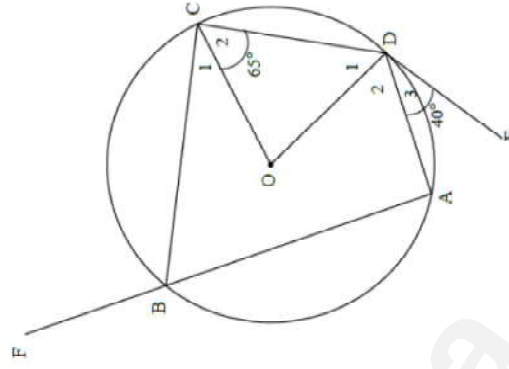
(3)

(2)

(4) [9]

QUESTION 8

8.1 In the diagram, ABCD is a cyclic quadrilateral in the circle centered at O. ED is a tangent to the circle at D. Chord AB is produced to F. Radii OC and OD are drawn. $\hat{A} = 65^\circ$ and $\hat{C}_2 = 65^\circ$.



Determine, giving reasons, the size of each of the following angles:

8.1.1 $\hat{D}_2$

8.1.2 $\hat{FBC}$

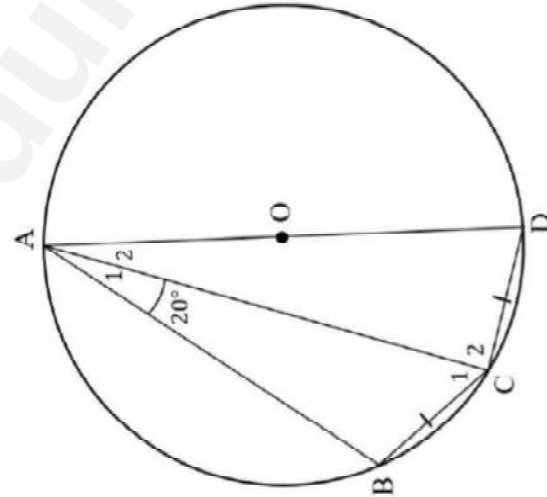
(3)

(4)



QUESTION 8

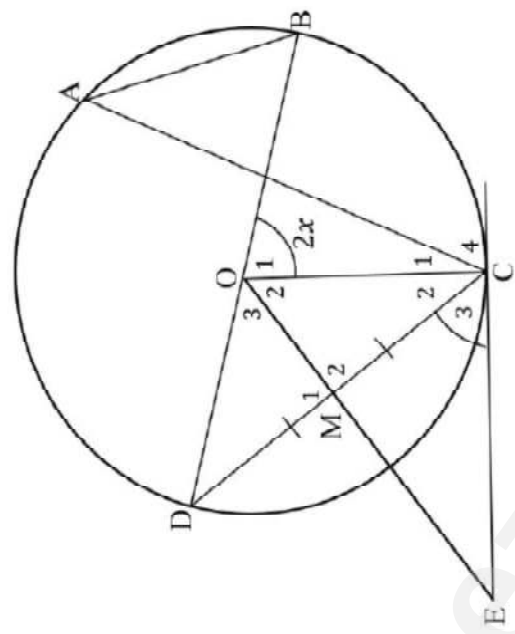
- 8.1 In the diagram, AOD is a diameter of the circle centred at O.
BC = CD and $\hat{A}_1 = 20^\circ$.



Determine, with reasons, the size of each of the following angles:

- 8.1.1 $\hat{A}_2$ (2)
8.1.2 $\hat{C}_2$ (2)
8.1.3 $\hat{A}\hat{B}\hat{C}$ (3)

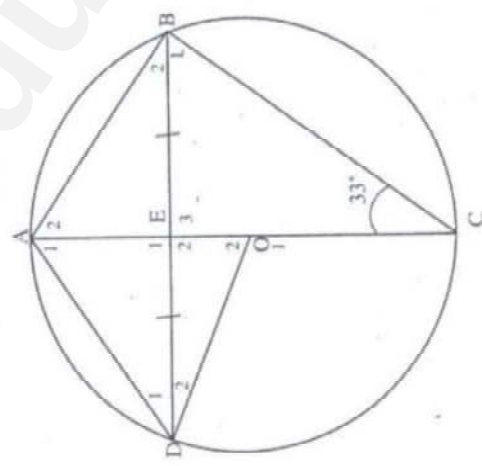
- 8.2 In the diagram, O is the centre of the circle and EC is a tangent to the circle at C.
DM = MC and OME is a straight line.
Let $\hat{O}_1 = 2x$.





QUESTION 8

In the diagram, AC is the diameter of the circle with centre O. AC and chord DB intersect at E. Chords AB, BC and AD and radius OD are drawn. $\angle ACB = 33^\circ$. E such that $DE = EB$. Chords AB, BC and AD and radius OD are drawn. $\angle ACB = 33^\circ$.



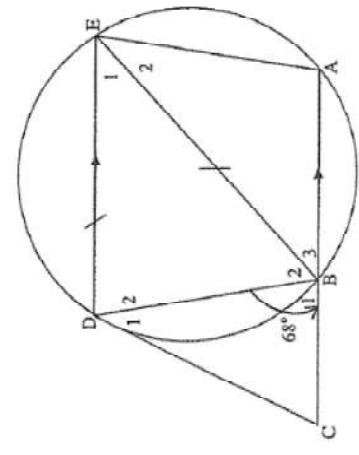
Determine the size of:

- 8.1 $\hat{D}_1$ (2)
 - 8.2 $\hat{A}_1$ (3)
 - 8.3 $\hat{O}_1$ (2)
 - 8.4 $\hat{D}_2$ (2)
 - 8.5 $\hat{A}_2$ (3)
- [12]



QUESTION 8

In the diagram below, BAED is a cyclic quadrilateral with $BA \parallel DE$. $BE = DE$ and $\hat{D}\hat{B}\hat{C} = 68^\circ$. The tangent to the circle at D meets AB produced to C.



8.1 Calculate, with reasons, the size of:

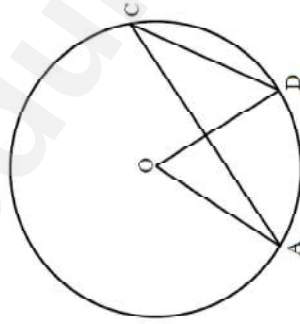
- 8.1.1 $\hat{D}\hat{E}\hat{A}$ (2)
 - 8.1.2 $\hat{A}$ (1)
 - 8.1.3 $\hat{D}_2$ (2)
 - 8.1.4 $\hat{B}_2$ (1)
 - 8.1.5 $\hat{D}_1$ (3)
- 8.2 Prove that $\triangle BDC$ is isosceles. (2)
- 8.3 Prove that DE is a tangent to the circle that passes through the points C, B and D at D. (2)
- [13]





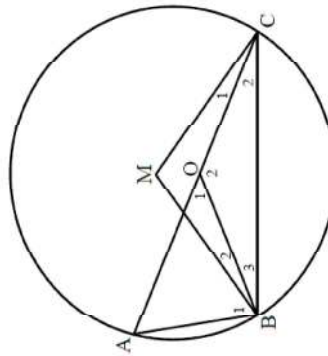
QUESTION 9

- 9.1 O is the centre of the circle and A, D and C are on the circumference of the circle. Use the diagram to prove, USING Euclidean geometry methods, the theorem which states that $\angle AOD = 2\angle ACD$.



(5)

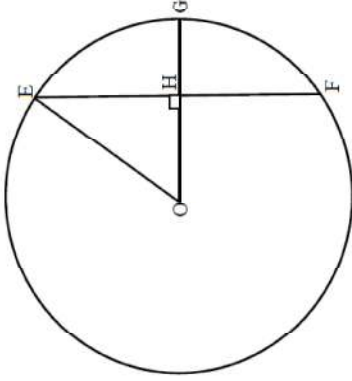
- 9.2 In the figure, M is the centre of the circle through A, B, and C. OB and AC bisect $\angle MBC$ and $\angle MCB$ respectively with O on AC. AB is joined.



Let $\hat{B}_2 = x$

- 9.2.1 Determine the size of $\hat{O}_2$ in terms of x . (4)
9.2.2 Prove that AO is a diameter of circle ABO. (5)

- 9.3 In the diagram below, EF is the chord of a circle with centre O. OG is perpendicular to EF.



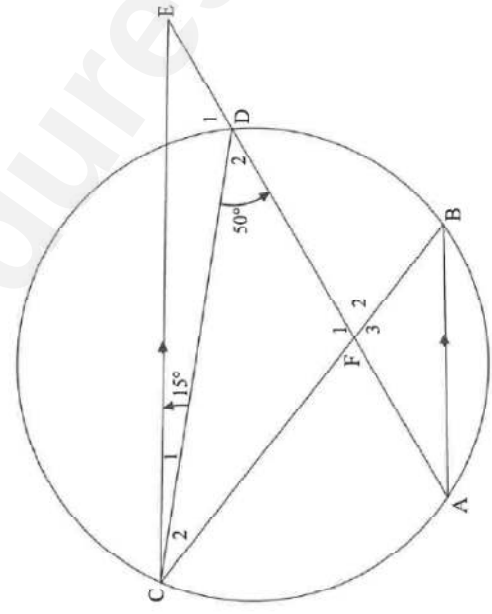
If $OE = x$ cm and $HG = 2$ cm, determine with reasons the size of EF (in its simplest form) in terms of x .

(4)
[18]



QUESTION 8

In the diagram, points A, B, D and C lie on a circle. $CE \parallel AB$ with E on AD produced. Chords CB and AD intersect at F. $\hat{D}_2 = 50^\circ$ and $\hat{C}_1 = 15^\circ$.



Calculate, with reasons, the size of:

8.1.1 $\hat{A}$ (3)

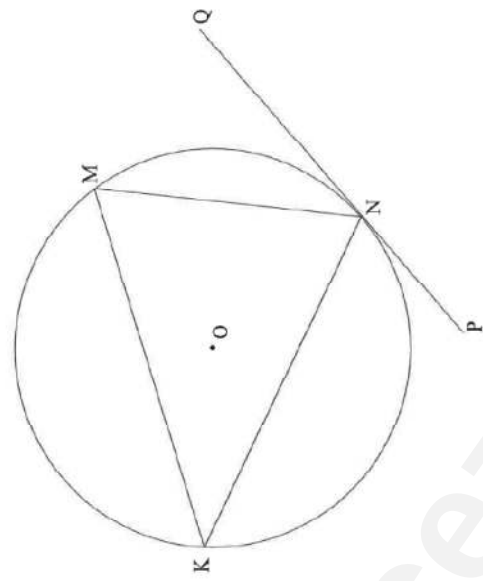
8.1.2 $\hat{C}_2$ (2)

8.2 Prove, with a reason, that CF is a tangent to the circle passing through points C, D and E. (2) [7]



QUESTION 11

11.1 In the diagram, chords KM, MN and KN are drawn in the circle with centre O. PNQ is the tangent to the circle at N.



Prove the theorem which states that $\hat{MNQ} = \hat{K}$.

(5)



Unit 2: Acceptable Reasons in Geometry

4. ACCEPTABLE REASONS: EUCLIDEAN GEOMETRY

In order to have some kind of uniformity, the use of the following shortened versions of the theorem statements is encouraged.

4.1 ACCEPTABLE REASONS: EUCLIDEAN GEOMETRY (ENGLISH)

THEOREM STATEMENT	ACCEPTABLE REASONS(S)
LINES	
The adjacent angles on a straight line are supplementary.	$\angle s$ on a str line
If the adjacent angles are supplementary, the outer arms of these angles form a straight line.	adj $\angle s$ supp
The adjacent angles in a revolution add up to 360° .	$\angle s$ round a pt OR $\angle s$ in a rev
Vertically opposite angles are equal.	vert opp $\angle s =$
If $AB \parallel CD$, then the alternate angles are equal.	alt $\angle s$; $AB \parallel CD$
If $AB \parallel CD$, then the corresponding angles are equal.	corresp $\angle s$; $AB \parallel CD$
If $AB \parallel CD$, then the co-interior angles are supplementary.	co-int $\angle s$; $AB \parallel CD$
If the alternate angles between two lines are equal, then the lines are parallel.	alt $\angle s =$
If the corresponding angles between two lines are equal, then the lines are parallel.	corresp $\angle s =$
If the co-interior angles between two lines are supplementary, then the lines are parallel.	co-int $\angle s$ supp
TRIANGLES	
The interior angles of a triangle are supplementary.	$\angle$ sum in Δ OR sum of $\angle s$ in Δ OR Int $\angle s \Delta$
The exterior angle of a triangle is equal to the sum of the interior opposite angles.	ext $\angle$ of Δ
The angles opposite the equal sides in an isosceles triangle are equal.	$\angle s$ opp equal sides
The sides opposite the equal angles in an isosceles triangle are equal.	sides opp equal $\angle s$
In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.	Pythagoras OR Theorem of Pythagoras
If the square of the longest side in a triangle is equal to the sum of the squares of the other two sides then the triangle is right-angled.	Converse Pythagoras OR Converse Theorem of Pythagoras
If three sides of one triangle are respectively equal to three sides of another triangle, the triangles are congruent.	SSS
If two sides and an included angle of one triangle are respectively equal to two sides and an included angle of another triangle, the triangles are congruent.	SAS OR S, $\angle$ S
If two angles and one side of one triangle are respectively equal to two angles and one side of another triangle, the triangles are congruent.	AAS OR $\angle$ $\angle$ S
If in two right angled triangles, the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of the other, the triangles are congruent.	RHS OR 90°HS
The line segment joining the midpoints of two sides of a triangle is	Midpt Theorem

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Examination Guidelines

THEOREM STATEMENT	ACCEPTABLE REASON(S)
parallel to the third side and equal to half the length of the third side	line through midpt to 2 nd side
The line drawn from the midpoint of one side of a triangle, parallel to another side, bisects the third side.	line one side of Δ OR prop theorem; same lines
A line drawn parallel to one side of a triangle divides the other two sides proportionally.	line divides two sides of Δ in prop
If a line divides two sides of a triangle in the same proportion, then the line is parallel to the third side.	Δs OR equiangular Δs
If two triangles are equiangular, then the corresponding sides are in proportion (and consequently the triangles are similar).	Sides of Δ in prop
If the corresponding sides of two triangles are proportional, then the triangles are equiangular (and consequently the triangles are similar).	same base; same height OR equal bases; equal height
If triangles (or parallelograms) are on the same base (or on bases of equal length) and between the same parallel lines, then the triangles (or parallelograms) have equal areas.	tan $\perp$ radius tan $\perp$ diameter line $\perp$ radius OR converse tan $\perp$ radius OR converse tan $\perp$ diameter
The tangent to a circle is perpendicular to the radius/diameter of the circle at the point of contact.	line from centre to midpt of chord
If a line is drawn perpendicular to a radius/diameter at the point where the radius/diameter meets the circle, then the line is a tangent to the circle.	line from centre $\perp$ to chord
The line drawn from the centre of a circle to the midpoint of a chord is perpendicular to the chord.	perp bisector of chord
The line drawn from the centre of a circle perpendicular to a chord bisects the chord.	$\angle$ at centre = $2 \times \angle$ at circumference
The perpendicular bisector of a chord passes through the centre of the circle;	$\angle s$ in semi circle OR diameter subtends right angle OR $\frac{1}{2} \odot$ $\angle \frac{1}{2} \odot$
The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre)	chord subtends 90° OR converse $\angle s$ in semi circle
The angle subtended by the diameter at the circumference of the circle is 90° .	$\angle s$ in the same seg
If the angle subtended by a chord at the circumference of the circle is 90° , then the chord is a diameter.	line subtends equal $\angle s$ OR converse $\angle s$ in the same seg
Angles subtended by a chord of the circle, on the same side of the chord, are equal	equal chords; equal $\angle s$
If a line segment joining two points subtends equal angles at two points on the same side of the line segment, then the four points are concyclic.	equal chords; equal $\angle s$ equal chords; equal $\angle s$
Equal chords subtend equal angles at the circumference of the circle.	equal chords; equal $\angle s$
Equal chords subtend equal angles at the centre of the circle.	equal chords; equal $\angle s$
Equal chords in equal circles subtend equal angles at the circumference of the circles.	equal chords; equal $\angle s$

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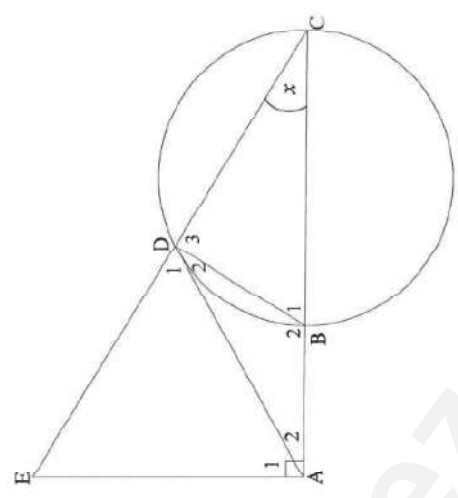
THEOREM STATEMENT	ACCEPTABLE REASON(S)
Equal chords in equal circles subtend equal angles at the centre of the circles.	equal circles; equal chords; equal $\angle$ s
The opposite angles of a cyclic quadrilateral are supplementary	opp $\angle$ s of cyclic quad
If the opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic.	opp $\angle$ s quad are $\perp$ OR converse opp $\angle$ s of cyclic quad
The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.	ext $\angle$ of cyclic quad
If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic.	ext $\angle = \text{int opp } \angle$ OR converse ext $\angle$ of cyclic quad
Two tangents drawn to a circle from the same point outside the circle are equal in length	Tans from common pt OR Tans from same pt
The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.	tan chord theorem
If a line is drawn through the end-point of a chord, making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle.	converse tan chord theorem OR $\angle$ between line and chord

QUADRILATERALS

The interior angles of a quadrilateral add up to 360° .	sum of $\angle$ s in quad
The opposite sides of a parallelogram are parallel.	opp sides of $\parallel$ m
If the opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram.	opp sides of quad are $\parallel$
The opposite sides of a parallelogram are equal in length.	opp sides of $\parallel$ m
If the opposite sides of a quadrilateral are equal, then the quadrilateral is a parallelogram.	opp sides of quad are = OR converse opp sides of a parm
The opposite angles of a parallelogram are equal.	opp $\angle$ s of $\parallel$ m
If the opposite angles of a quadrilateral are equal then the quadrilateral is a parallelogram.	opp $\angle$ s of quad are = OR converse opp angles of a parm
The diagonals of a parallelogram bisect each other.	diag of $\parallel$ m
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.	diags of quad bisect each other OR converse diags of a parm
If one pair of opposite sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram.	pair of opp sides = and $\parallel$
The diagonals of a parallelogram bisect its area.	diag bisect area of $\parallel$ m
The diagonals of a rhombus bisect a right angles.	diags of rhombus
The diagonals of a rhombus bisect the interior angles.	diags of rhombus
All four sides of a rhombus are equal in length.	sides of rhombus
All four sides of a square are equal in length.	sides of square
The diagonals of a rectangle are equal in length.	diags of rect
The diagonals of a kite intersect at right-angles.	diags of kite
A diagonal of a kite bisects the other diagonal.	diag of kite
A diagonal of a kite bisects the opposite angles	diag of kite

Unit 3: Proportionality and Similar Triangles

- 11.2 In the diagram, BC is a diameter of the circle. The tangent at point D on the circle meets CB produced at A. CD is produced to E such that $EA \perp AC$. BD is drawn.
Let $\hat{C} = x$.



11.2.1 Give a reason why:

- (a) $\hat{D}_3 = 90^\circ$ (1)
(b) ABDE is a cyclic quadrilateral (1)



(c) $\hat{D}_2 = x$

Prove that:

(a) $AD = AE$

(b) $\triangle ADB \parallel \triangle ACD$

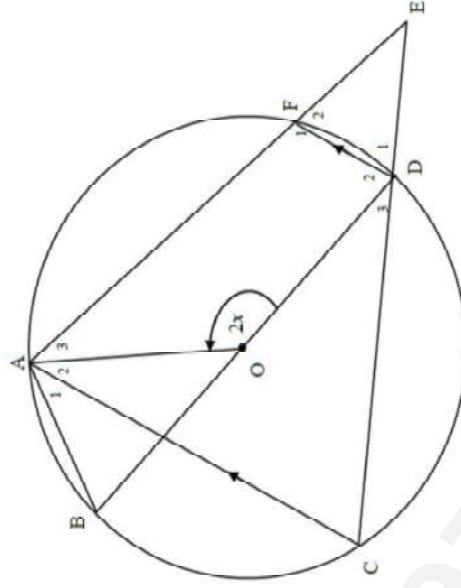
11.2.3 It is further given that $BC = 2AB = 2r$.

(a) Prove that $AD^2 = 3r^2$

(b) Hence, prove that $\triangle ADE$ is equilateral.

QUESTION 9

9.1 A circle with centre O is given below. Lines CD and AF are produced to E. $AOD = 2x$ and BD is the diameter. $AC \parallel FD$.



- 9.1.1 Determine, with reasons, four other angles that are each equal to x . (6)
- 9.1.2 Express $\hat{E}$ in terms of x . (2)
- 9.1.3 Prove that $\triangle AODE$ is a cyclic quadrilateral. (2)
- 9.2 It is further given that $ED : DC = 8 : 12$ and $FE = 10$. Calculate the length of AF. (3)

[13]

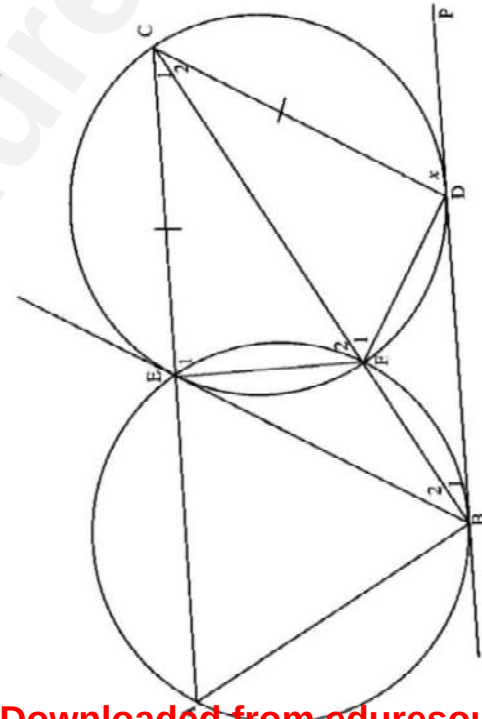




QUESTION 9

- 9.1 Complete the statement so that it is TRUE.
Angles subtended by a chord of a circle, on the same side of a chord, are ... (1)

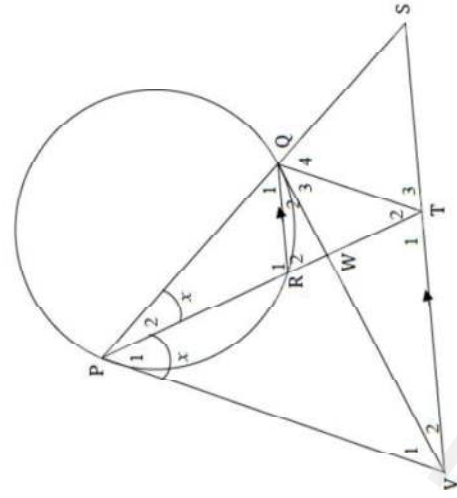
- 9.2 In the diagram below, ABFE and EFDC are cyclic quadrilaterals in two equal circles that intersect at E and F. BFC and AEC are straight lines. BD is a common tangent to the circles at B and D respectively. $EC = CD$.
Let $\hat{CDP} = x$



- Prove, giving reasons, that:
- 9.2.1 $\hat{E}_1 = \hat{E}_2$. (3)
- 9.2.2 ABDC is a cyclic quadrilateral. (3)
- 9.2.3 $BE \parallel CD$. (2)
- 9.2.4 FC is a diameter of circle FDCE if it is given that EBDC is a rhombus. (5)
- [14]

QUESTION 9

In the diagram, PV and VQ are tangents to the circle at P and Q. PQ is produced to S and chord PR is produced to T such that $VTS \parallel RQ$. VQ and RT intersect at W. $\hat{P}_1 = \hat{P}_2 = x$.



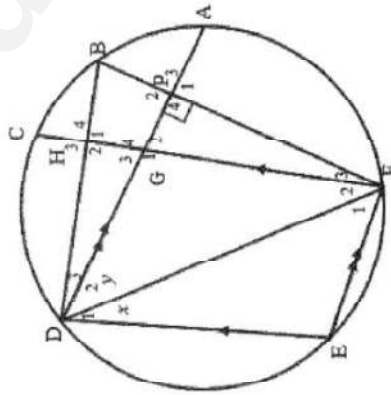
Prove that:

- 9.1 $\hat{S} = x$ (4)
- 9.2 PQTV is a cyclic quadrilateral (5)
- 9.3 TQ is a tangent to the circle passing through Q, W and P (3)
- [12]



QUESTION 9

- 9.1 A, B, C, D, E and F are points on the circumference of the circle. $ED \parallel FC$ and $AD \parallel FE$. $DA \perp FB$. FC intersects DA at G and DB at H.



Let $\angle EDF = x$ and $\angle ADF = y$.

Prove that:

9.1.1 $\hat{B} = x + y$

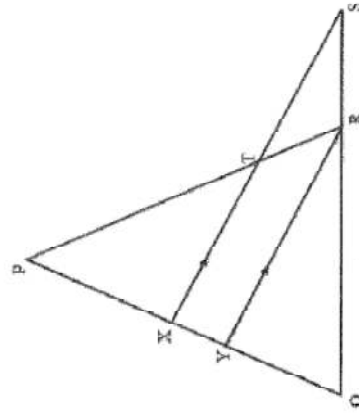
9.1.2 $\hat{H}_1 = 90^\circ$

(4)

(6)

- 9.2 In the diagram QR of $\triangle PQR$ is produced such that $XS \parallel YR$.

$$\frac{PX}{XQ} = \frac{2}{5} \text{ and } \frac{PT}{TR} = \frac{3}{2}.$$



Calculate, giving reasons, the value of $\frac{QR}{RS}$.

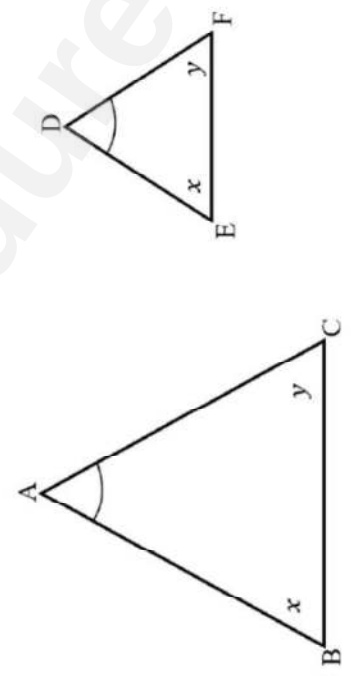
(6)

[16]



QUESTION 9

9.1 In the diagram, $\triangle ABC$ and $\triangle DEF$ are drawn such that $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.

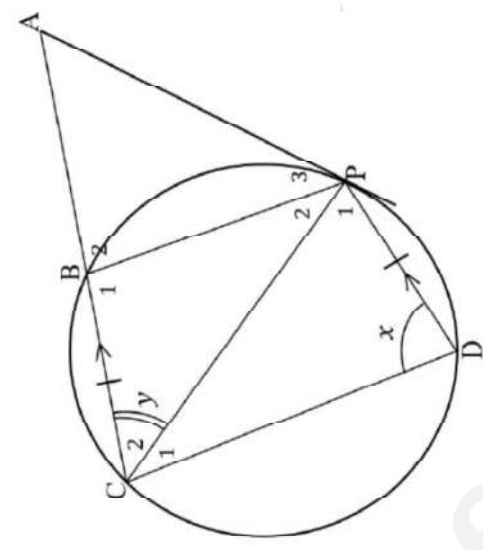


Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, that is

$$\frac{DE}{AB} = \frac{DF}{AC}$$

(6)

9.2 AP is a tangent to the circle at P. $CB \parallel DP$ and $CB = DP$. CBA is a straight line. Let $\hat{D} = x$ and $\hat{C}_2 = y$.



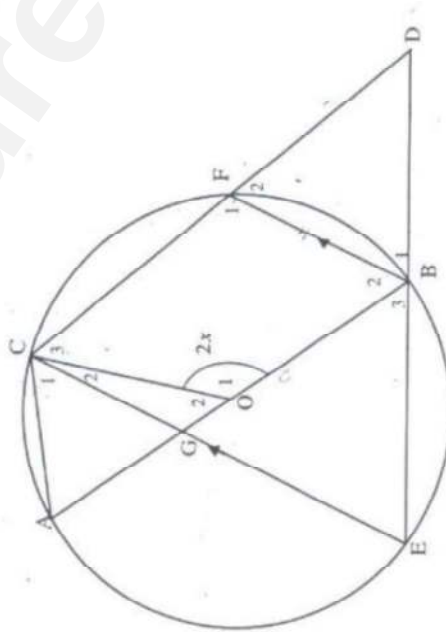
Prove, with reasons that:

- 9.2.1 $\triangle APC \parallel \triangle ABP$ (4)
 - 9.2.2 $AP^2 = AB \cdot AC$ (1)
 - 9.2.3 $\triangle APC \parallel \triangle CDP$ (4)
 - 9.2.4 $AP^2 + PC^2 = AC^2$ (4)
- [19]



QUESTION 9

In the diagram below, AB passes through the centre O of the circle. Chords CF and EB are produced to meet at D. $EC \parallel BF$. Chord AC and radius OC are drawn. Let $\hat{O}_1 = 2x$.



1. Determine, in terms of x , the size of $\hat{F}_2$.

2. Prove that $DF = BD$.

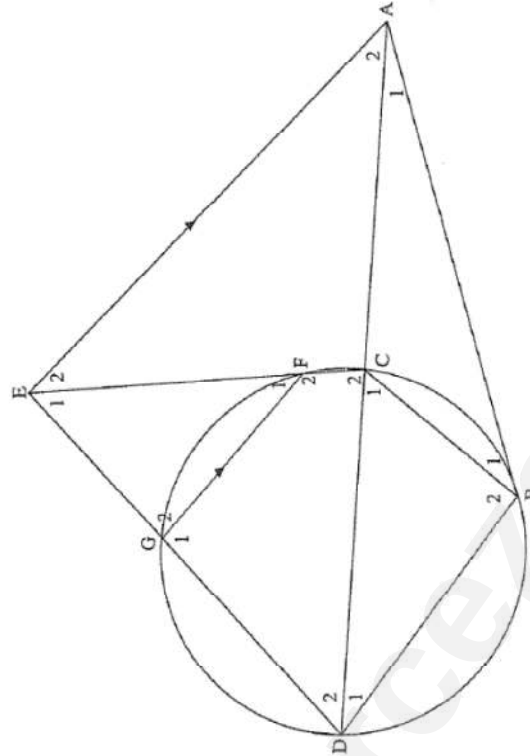
3. Show that $\hat{C}_1 = \hat{C}_3$.

4. If it is further given that $CG = FB$, $FD = 1$ and $BG = 2$, determine the value of $\frac{BD}{ED}$.

[16]

QUESTION 9

In the diagram, DGF C is a cyclic quadrilateral and AB is a tangent to the circle at B. Chords DB and BC are drawn. DG produced and CF produced meet in E and DC is produced to A. $EA \parallel GF$



1. Give a reason why $\hat{B}_1 = \hat{D}_1$.

2. Prove $\triangle ABC \parallel \triangle ADB$.

3. Prove $\hat{E}_2 = \hat{D}_2$.

4. Prove $AE = \sqrt{AD \times AC}$.

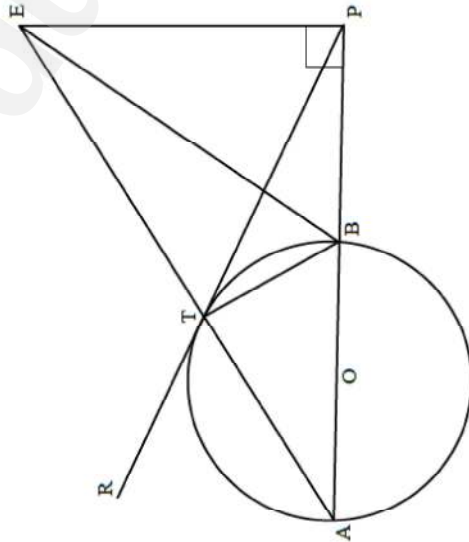
5. Hence, show that $AE = AB$.

[16]



QUESTION 10

In the diagram below, O is the centre with A, B and T on the circumference, BP = OB = AO, PTR is a tangent and EP ⊥ AP.



Prove that:

10.1 TEPB is a cyclic quad

(3)

10.2 ΔATB ∥ ΔAPE

(3)

10.3 TP = PE

(6)

10.4 ΔATB ∥ ΔEPB

(5)

10.5 2BP² = BE · TB

(4)
[21]



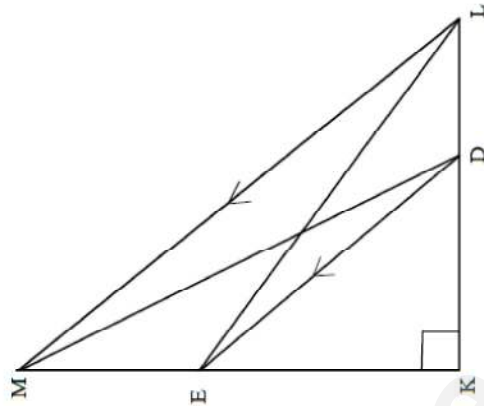
QUESTION 11

11.1 Complete the following statement:

A line parallel to one side of a triangle divides ...

(1)

11.2 In Δ KLM, $\widehat{K} = 90^\circ$ and D is a point on KL so that KD:DL = 2:1 and DE ∥ LM with E on KM.



11.2.1 If DL = x and EM = y, express LM² in terms of x and y.

(4)

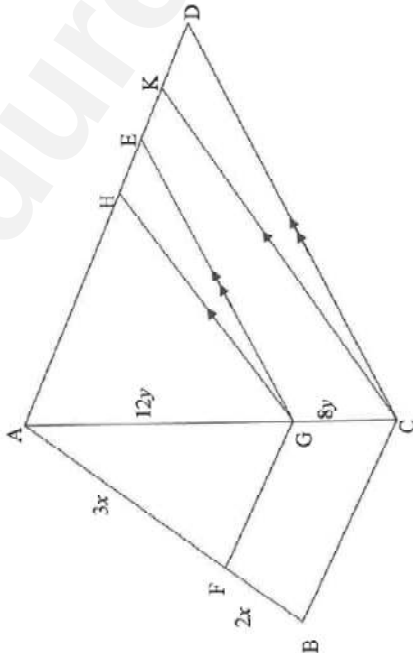
11.2.2 Show that $DM^2 + LE^2 = \frac{13}{9} LM^2$.

(4)
[9]



QUESTION 9

In the diagram, $\triangle ABC$ and $\triangle ACD$ are drawn. F and G are points on sides AB and AC respectively such that $AF = 3x$, $FB = 2x$, $AG = 12y$ and $GC = 8y$. H, E and K are points on side AD such that $GH \parallel CK$ and $GE \parallel CD$.



Prove that:

9.1.1 $FG \parallel BC$ (2)

9.1.2 $\frac{AH}{HK} = \frac{AE}{ED}$ (3)

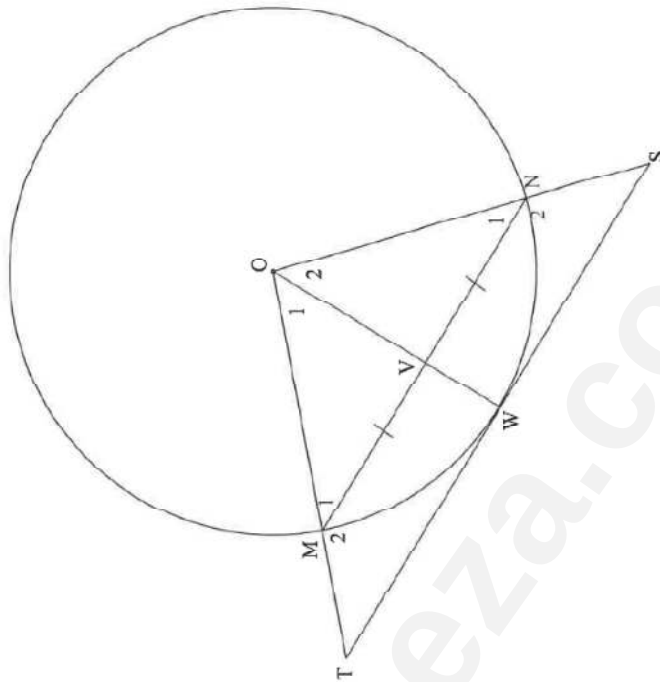
If it is further given that $AH = 15$ and $ED = 12$, calculate the length of EK . (5)

[10]



QUESTION 10

In the diagram, W is a point on the circle with centre O. V is a point on OW. Chord MN is drawn such that $MV = VN$. The tangent at W meets OM produced at T and ON produced at S.



10.1 Give a reason why $OV \perp MN$. (1)

10.2 Prove that: (2)

102.1 $MN \parallel TS$

102.2 TMNS is a cyclic quadrilateral (4)

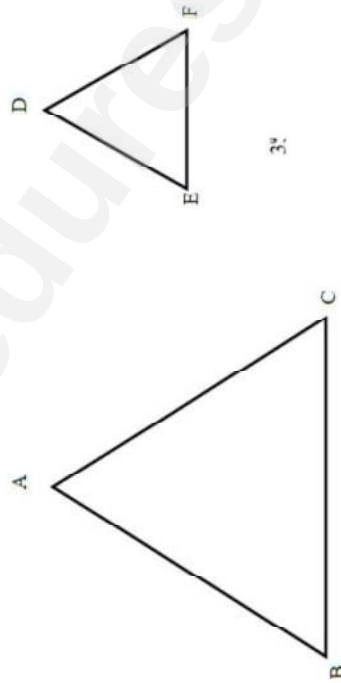
102.3 $OS \cdot MN = 2ON \cdot WS$ (5)

[12]



QUESTION 10

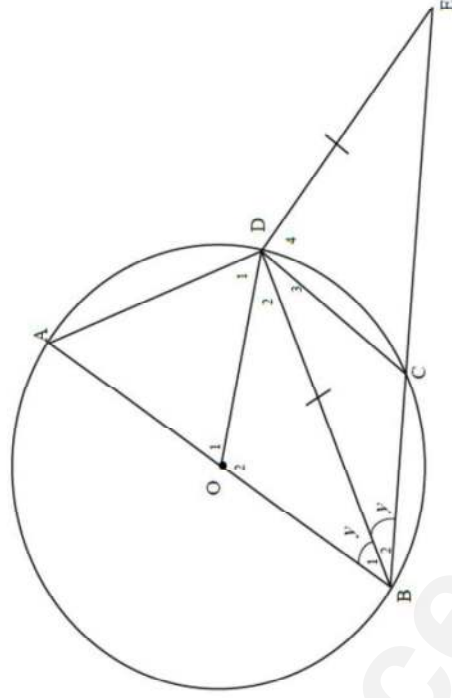
- 10.1 In the diagram below, $\triangle ABC$ and $\triangle DEF$ are drawn with $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.



Prove the theorem that states that if two triangles are similar, then the sides are proportional, i.e. $\frac{DE}{AB} = \frac{EF}{BC} = \frac{DF}{AC}$.

(5)

- 10.2 In the diagram below, AB is the diameter of a circle with centre O. BD and BC are chords. BD = DE. BCE is a line. $B_1 = B_2 = y$.



Prove that:

10.2.1 $\hat{D}_4 = 90^\circ$ (5)

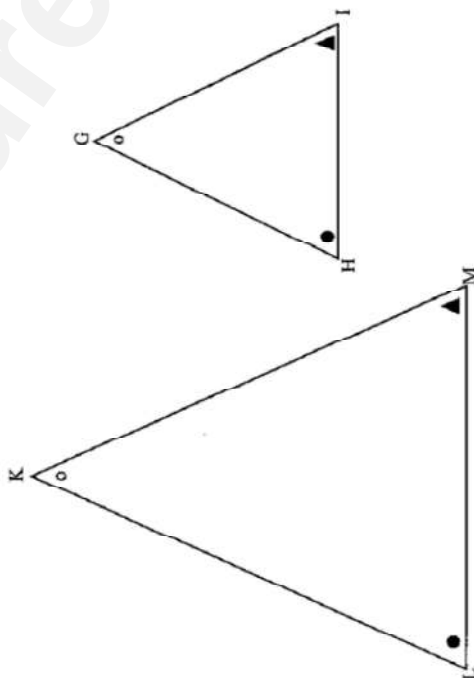
10.2.2 $ABOD \parallel ABDE$ (3)

10.2.3 $DE^2 = BE \cdot OD$ (4)
[17]

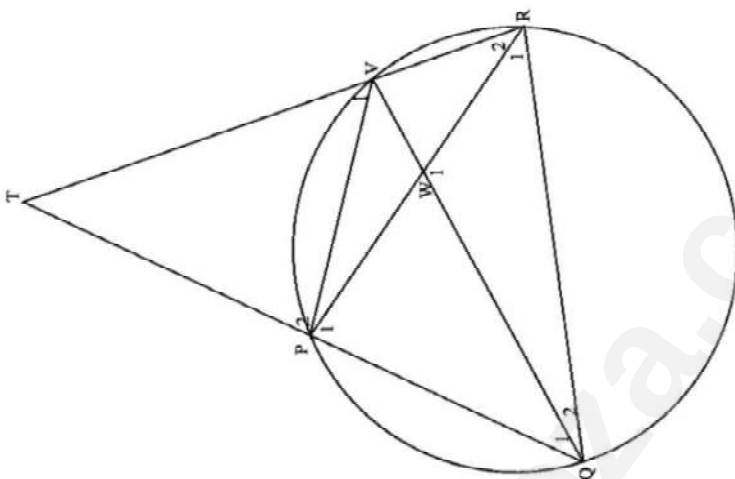
**GRADE 12
MATHEMATICS
REVISION**

10.2 In the diagram below, $\triangle PQR$ is an equilateral triangle inscribed in a circle. V is a point on the circle. QP produced meets RV produced at T . PR and QV intersect at W .

10.1 In the diagram below, $\triangle KLM$ and $\triangle GHI$ are drawn such that $\hat{K} = \hat{G}$, $\hat{L} = \hat{H}$ and $\hat{M} = \hat{I}$. Prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, i.e. prove that $\frac{KL}{GH} = \frac{KM}{GI}$.



(5)



Prove, giving reasons, that:

10.2.1 $\hat{W}_1 = \text{TRQ}$

10.2.2 $\Delta TQR \equiv \Delta QRW$

$$10.2.3 \quad \frac{P_T}{Q^W} = \frac{P_V}{W^R}$$

(E)

(E)

(6) [17]



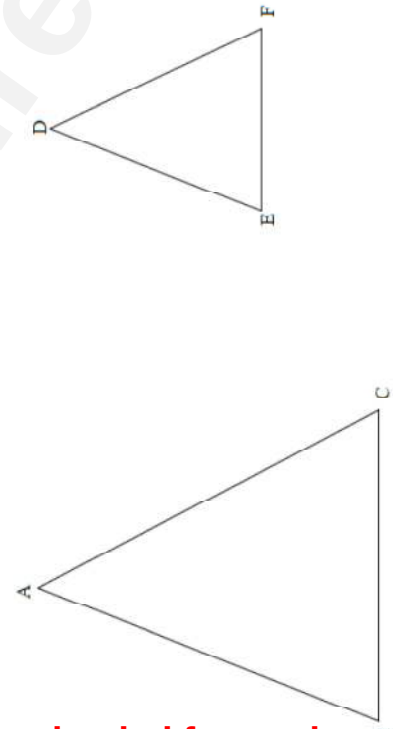
QUESTION 10

10.1 In the diagram below, $\triangle ABC$ and $\triangle DEF$ are drawn with $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.

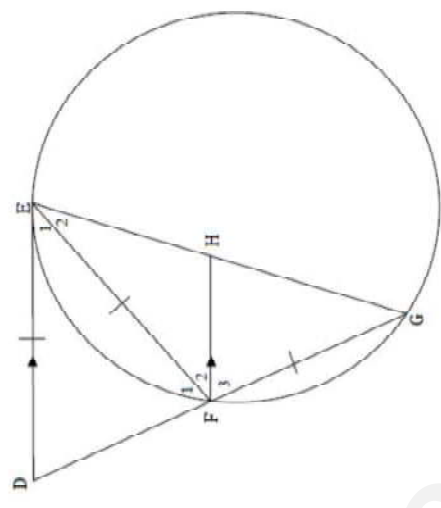
Prove the theorem that states that equiangular triangles are similar and therefore

$$\frac{AB}{DE} = \frac{AC}{DF}$$

(7)



10.2 In the diagram, DE is a tangent to the circle at E and DFG is a secant intersecting the circle at F and G. DE = EF = FG. H is a point on EG such that FH $\parallel$ DE.



10.2.1 Determine, giving reasons, 3 angles each equal to $\hat{DEF}$.

(4)

10.2.2 Prove that:

a) $\triangle DEF \parallel \triangle DGE$

(3)

b) $\hat{D} = 72^\circ$.

(5)

10.2.3 If it is further given that DF = k units and FG = 2 units, prove that $k^2 + 2k = 4$.

(3)

10.2.4 Determine, giving reasons, the ratio of $\frac{GH}{GE}$ in terms of k.

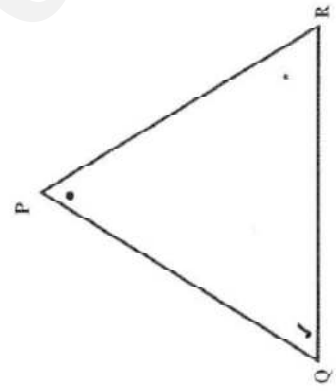
(2)

[24]



QUESTION 10

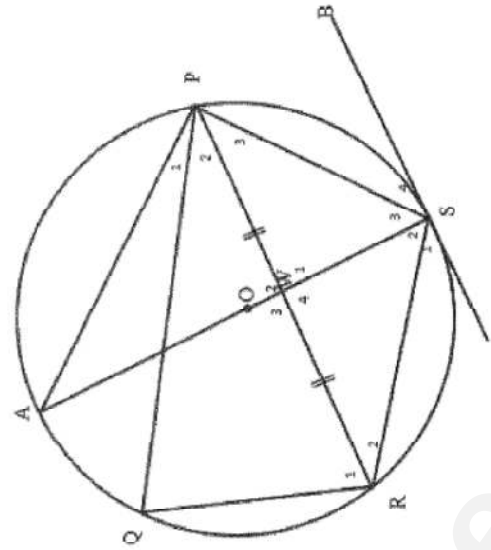
10.1 $\triangle PQR$ and $\triangle FGH$ are given. $\hat{P} = \hat{F}$, $\hat{Q} = \hat{G}$ and $\hat{R} = \hat{H}$.



Prove the theorem which states that if $\triangle PQR \parallel \triangle FGH$, then $\frac{PQ}{FG} = \frac{PR}{FH}$.

(6)

10.2 P, A, Q, R and S lie on the circle with centre O. SB touches the circle at S and RW = WP. AS and RP are straight lines.



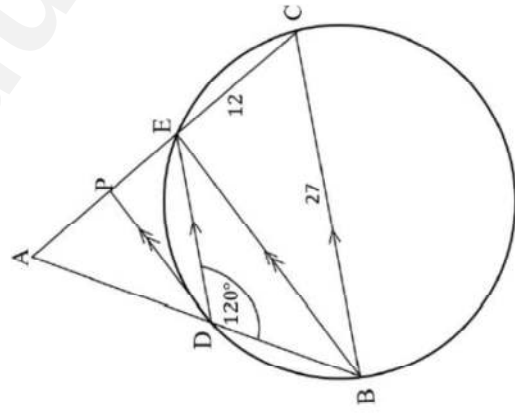
Prove that:

- 10.2.1 $SB \parallel RP$ (5)
 - 10.2.2 $\triangle APS \parallel \triangle RWS$ (4)
 - 10.2.3 $RS^2 = WSAS$ (4)
 - 10.2.4 $AS = \frac{RW^2}{WS} + WS$ (4)
- [23]



QUESTION 10

In $\triangle ABC$ in the diagram, D is a point on AB such that $AD : DB = 5 : 4$. P and E are points on AC such that $DE \parallel BC$ and $DP \parallel BE$. BC is NOT a diameter of the circle.
Given: $\angle BDE = 120^\circ$, $EC = 12$ units and $BC = 27$ units.



10.1 Determine, with reasons:

10.1.1 The length of AE

10.1.2 $\frac{\text{area of } \triangle AEB}{\text{area of } \triangle ECB}$

10.2 Hence, determine the length of DP if $\triangle ADP \parallel \triangle ABE$

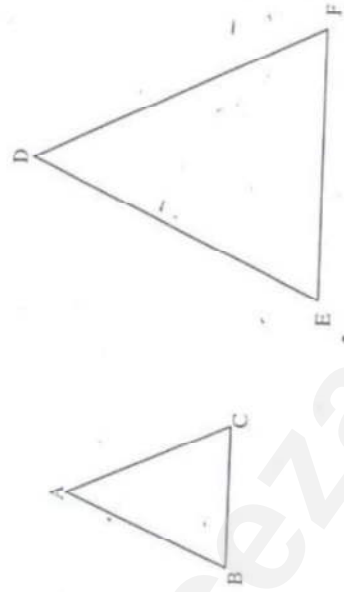
(3)

(2)

(6)
[11]

QUESTION 10

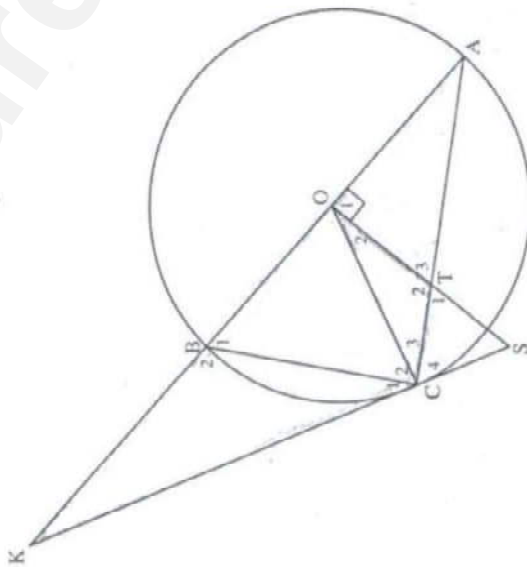
10.1 In the diagram below, $\triangle ABC$ and $\triangle DEF$ are given with $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$. Prove the theorem that states that $\frac{AB}{DE} = \frac{AC}{DF}$.



(6)



- 10.2 In the diagram, AB is the diameter of the circle with centre O. AB is produced to K such that SK is a tangent to the circle at C. SO ⊥ AB. CA and SO intersect at T.



Prove that:

10.2.1 $\triangle CKB \parallel \triangle AKC$ (3)

10.2.2 $\angle KCT = \angle T_2$ (4)

10.2.3 $\triangle COT \parallel \triangle AKC$ (3)

10.2.4 $BK \cdot AK = \frac{OT^2 \cdot CA^2}{CT^2}$ (4)

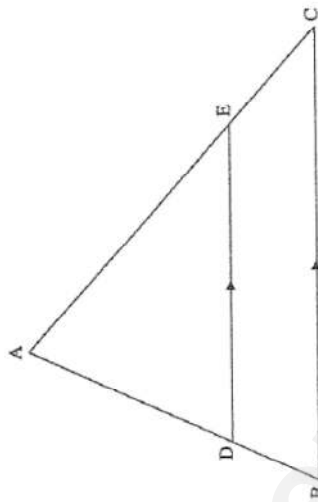
[20]



QUESTION 10

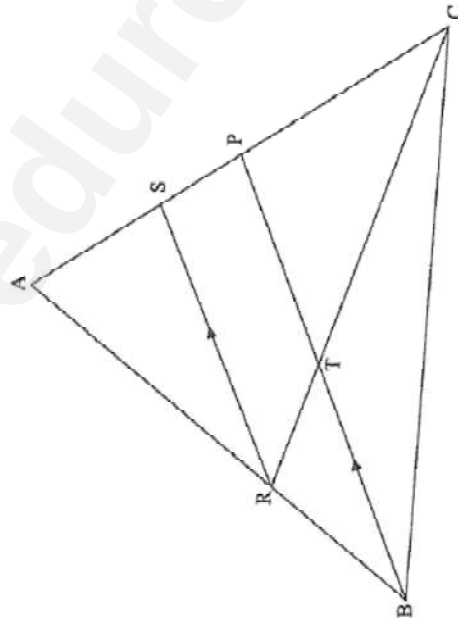
- 10.1 In $\triangle ABC$ below, D and E are points on AB and AC respectively such that $DE \parallel BC$. Prove the theorem which states that $\frac{AD}{DB} = \frac{AE}{EC}$.

(6)





- 10.2 In the diagram below, P is the midpoint of AC in $\triangle ABC$. R is a point on AB such that $RS \parallel BP$ and $\frac{AR}{AB} = \frac{3}{5}$. RC intersects BP in T.



Determine, with reasons, the following ratios:

10.2.1 $\frac{AS}{SC}$

(4)

10.2.2 $\frac{RT}{TC}$

(3)

10.2.3 $\frac{\text{Area of } \triangle RAS}{\text{Area of } \triangle RSC}$

(2)

10.2.4 $\frac{\text{Area of } \triangle TPC}{\text{Area of } \triangle RSC}$

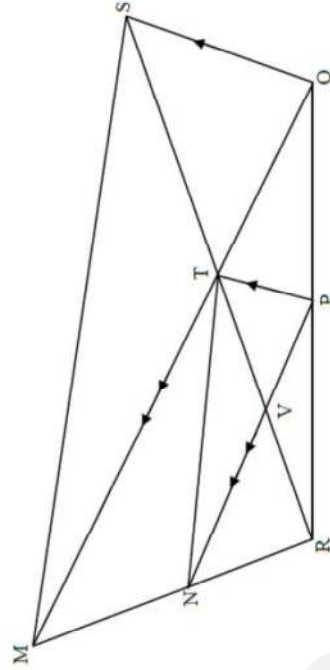
(3)

[18]



QUESTION 11

In the diagram below, PQSM is a quadrilateral. N and P are points on MR and RQ respectively such that $MQ \parallel NP$. The diagonals intersect at T. P is a point on RQ such that $TP \parallel SQ$. TR and NP intersect at V.



- 11.1 Prove that $NT \parallel MS$.

(4)

- 11.2 If $RN = \frac{3}{5} NM$ and $RS = 32$, determine VT.

(4)
[8]

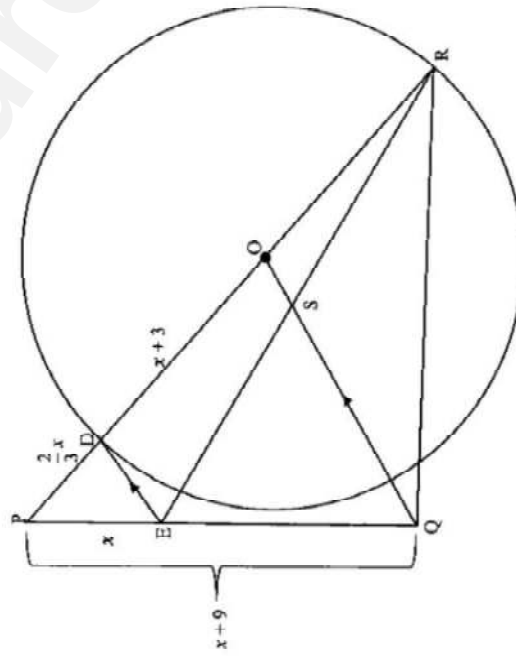




QUESTION 11

In the diagram below, the circle with centre O is drawn. OQ is drawn parallel to a tangent to the circle at D. ER is drawn with S on OQ. RD is produced to P and PQ is joined.

PE = x units, PQ = x + 9 units, PD = $\frac{2}{3}x$ units and DO = x + 3 units.



Calculate the length of RO.

If OS = 1,4 units and S is the midpoint of ER, determine the length of DE.

If the area of $\triangle PED = 2,7$ units², find the area of $\triangle PER$.

(4)
(2)
(4)
[10]

MEMORANDOMS

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REDWOOD



ACADEMY

GRADE 12
MATHEMATICS
REVISION

QUESTION/PRAAG 2

MODULE 1: DATA HANDLING

GENERAL DATA HANDLING

QUESTION/PRAAG 1			
1.1	16	✓ answer/antwoord	(1)
1.2	Semi-IQR = $\frac{30-18,5}{2}$ = 5,75	✓ correct values korrekte waardes ✓ substitution into correct formula / vervanging in korrekte formule ✓ answer/antwoord	(3)
1.3		✓ Min and/en Max/Maks ✓ Q ₁ & Q ₃ ✓ Q ₂	(3)
1.4	The distribution is skewed to the left/Die verspreiding is skeef na links.	✓ answer/antwoord	(1)
[8]			

12	13	13	14	14	16	17	18	18	18	19	20
21	21	22	22	23	24	25	27	29	30	36	

2.1.1	$\bar{x} = \frac{472}{23}$ $\bar{x} = 20,52$ seconds / sekonde	✓ $\frac{472}{23}$ ✓ answer	(2)
2.1.2	Q ₁ = 16 Q ₃ = 24 IQR/KO = Q ₃ - Q ₁ = 24 - 16 = 8	✓ Q ₁ ✓ Q ₃ ✓ answer	(3)
2.2	20,52 + 5,94 = 26,46 ∴ > 26,46 ∴ 4 girls/dogters	✓ 26,46 ✓ answer	(2)
2.3		✓ whiskers ending at 12 & 36 ✓ Q ₁ = 16 & Q ₃ = 24 (box) ✓ Q ₂ = 20	(3)
2.4.1	Girls / Meisies	✓ answer	(1)
2.4.2	Five-number summary of boys: (15 ; 21 ; 23,5 ; 26 ; 38) None of the boys / Nie een van die seuns nie 5 girls completed in less than 15 seconds which was the minimum time taken by the boys. 5 meisies voltooi in minder as 15 sekondes, wat die minimumtyd is wat die seuns geneem het.	✓ answer ✓ reason/rede	(2) [13]



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QUESTION/PRAAG 1

SUGGESTED ANSWER/ VOORGESTELDE ANTWOORD			DESCRIPTORS/ BESKRYWERS																					
1.1	60 ≤ x < 80		✓ answer/antwoord (1)																					
1.2	<table border="1"><thead><tr><th>INTERVAL OF TEST MARKS/ INTERVAL VAN TOETSPUNTE</th><th>NUMBER OF LEARNERS/ AANTAL LEERDERS</th><th>X.f</th></tr></thead><tbody><tr><td>0 ≤ x < 20</td><td>4</td><td>40</td></tr><tr><td>20 ≤ x < 40</td><td>5</td><td>150</td></tr><tr><td>40 ≤ x < 60</td><td>9</td><td>450</td></tr><tr><td>60 ≤ x < 80</td><td>13</td><td>910</td></tr><tr><td>80 ≤ x < 100</td><td>10</td><td>900</td></tr><tr><td>Totals/Totale</td><td>41</td><td>Σ 2450</td></tr></tbody></table> $\bar{x} = \frac{\sum Xf}{n} = \frac{2450}{41} = 59,76$ ANSWER ONLY: FULL MARKS/ SLEGS ANTWOORD: VOLPUNTE		INTERVAL OF TEST MARKS/ INTERVAL VAN TOETSPUNTE	NUMBER OF LEARNERS/ AANTAL LEERDERS	X.f	0 ≤ x < 20	4	40	20 ≤ x < 40	5	150	40 ≤ x < 60	9	450	60 ≤ x < 80	13	910	80 ≤ x < 100	10	900	Totals/Totale	41	Σ 2450	✓ All correct values/Alle korrekte waardes ✓ 2450 ✓ $\bar{x} = 59,76$ (3)
INTERVAL OF TEST MARKS/ INTERVAL VAN TOETSPUNTE	NUMBER OF LEARNERS/ AANTAL LEERDERS	X.f																						
0 ≤ x < 20	4	40																						
20 ≤ x < 40	5	150																						
40 ≤ x < 60	9	450																						
60 ≤ x < 80	13	910																						
80 ≤ x < 100	10	900																						
Totals/Totale	41	Σ 2450																						
1.3	<table border="1"><thead><tr><th>INTERVAL OF TEST MARKS/ INTERVAL VAN TOETSPUNTE</th><th>NUMBER OF LEARNERS/ AANTAL LEERDERS</th><th>CUMULATIVE FREQUENCY/ KUMULATIEWE- FREKWENSIE</th></tr></thead><tbody><tr><td>0 ≤ x < 20</td><td>4</td><td>4</td></tr><tr><td>20 ≤ x < 40</td><td>5</td><td>9</td></tr><tr><td>40 ≤ x < 60</td><td>9</td><td>18</td></tr><tr><td>60 ≤ x < 80</td><td>13</td><td>31</td></tr><tr><td>80 ≤ x < 100</td><td>10</td><td>41</td></tr></tbody></table> Max of 1/2 for 3 correct values in the table/ Maksimum van 1/2 vir 3 korrekte waardes in die tabel		INTERVAL OF TEST MARKS/ INTERVAL VAN TOETSPUNTE	NUMBER OF LEARNERS/ AANTAL LEERDERS	CUMULATIVE FREQUENCY/ KUMULATIEWE- FREKWENSIE	0 ≤ x < 20	4	4	20 ≤ x < 40	5	9	40 ≤ x < 60	9	18	60 ≤ x < 80	13	31	80 ≤ x < 100	10	41	✓✓ ALL correct values in the table/ ALLE korrekte waardes in die tabel (2)			
INTERVAL OF TEST MARKS/ INTERVAL VAN TOETSPUNTE	NUMBER OF LEARNERS/ AANTAL LEERDERS	CUMULATIVE FREQUENCY/ KUMULATIEWE- FREKWENSIE																						
0 ≤ x < 20	4	4																						
20 ≤ x < 40	5	9																						
40 ≤ x < 60	9	18																						
60 ≤ x < 80	13	31																						
80 ≤ x < 100	10	41																						

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1.4	OGIVE/OGIEF		
	✓ using the cumulative frequency/<i>gebruik die kumulatiewefrekvensie</i> ✓ smooth curve/<i>gladde kurwe</i> ✓ using upper limits/<i>gebruik boonste limiete</i>		
1.5	$\begin{aligned} \text{IQR} &= Q_3 - Q_1 \\ &= 68 - 33 \\ &= 35 \end{aligned}$	ACCEPT/AANTVAAR: • 32 OR/OF 34 lower quartile/<i>laer kwartiel</i> • 67 OR/OF 69 upper quartile/<i>boonste kwartiel</i>	✓ L Q ✓ U Q ✓ IQR
			(3)
			(3)

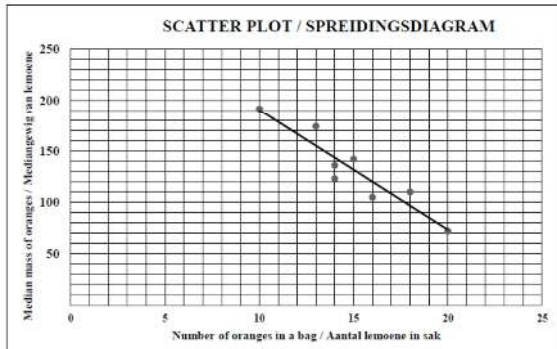
[12]





QUESTION / VRAAG 1

Number of oranges in the bag <i>Aantal lemoene in sak</i>	18	16	20	15	14	13	14	10
Median mass of oranges in the same bag (to the nearest gram) / <i>Mediangeewig van die lemoene in dieselfde sak (tot die naaste gram)</i>	110	105	72	142	123	174	136	192



1.1	$a = 307,20$ $b = -11,70$ $\hat{y} = 307,20 - 11,7x$	$\checkmark a = 307,20$ $\checkmark b = -11,70$ $\checkmark \hat{y} = 307,20 - 11,7x$	(3)
1.2	$r = -0,93$	$\checkmark r = -0,93$	(1)
1.3	See scatter plot above/sien spreidingsdiagram hierbo (10; 190,2) (20; 73,2)	$\checkmark (10; 190,2)$ $\checkmark (20; 73,2)$	(2)
1.4	Negative strong association / <i>Negatiewe sterk assosiasie</i>	$\checkmark$ answer / <i>antwoord</i>	(1)
1.5	$\hat{y} = 307,20 - 11,7(12)$ $= 166,8$	$\checkmark$ substitution / <i>vervang</i> $\checkmark$ answer / <i>antwoord</i>	(2)
			[9]



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QUESTION/VRAAG 1

1.1	Minimum and maximum values/ <i>Minimum en maksimum waardes</i>	$\checkmark$ values both /albei waardes (1)
1.2	$0,75 \times 21 = 15,75$	$\checkmark$ method/metode $\checkmark$ answer (2)
1.3	Yes./Ja The difference between the median and lower quartile is much smaller than the difference between the median and upper quartile. <i>Die verskil tussen die mediaan en onderste kwartiel is baie kleiner as die verskil tussen die mediaan en die boonste kwartiel.</i>	$\checkmark$ yes./Ja $\checkmark$ reason/ <i>rede</i> (3)
1.4	The interval to consider: / Die interval: (26,3; 64,7) 10 learners lie outside 1 standard deviation. / 10 leerders lê buite een standaard afwyking.	$\checkmark$ boundaries grense $\checkmark$ 10 (3)
1.5	The mean will increase by 5./ Die gemiddeld sal 5 meer wees, The standard deviation will remain the same./ die standaard afwyking sal dieselfde bly.	$\checkmark$ $\checkmark$ (2)
		[11]



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QUESTION 1

39	42	48	54	62	68	78	78	82	91	93
----	----	----	----	----	----	----	----	----	----	----

1.1	$\bar{x} = 66,82$ or 66,82 thousands	A $\checkmark$ calculation CA $\checkmark$ answer	(2)
1.2	$\sigma = 18,3$	AA $\checkmark$ answer If formula is used CA will apply.	(2)
1.3	$(\bar{x} - \sigma; \bar{x} + \sigma) = (48,52; 85,12)$ $\therefore$ 6 countries	CA $\checkmark$ substitution CA $\checkmark$ answer	(2)
1.4		A $\checkmark$ min & max values A $\checkmark$ quartile 1 value A $\checkmark$ median value & form A $\checkmark$ quartile 3 value	(4)
1.5	The data is skewed to the left/negatively skewed	CA $\checkmark$ answer	(1)
1.6.1	The sum of the data provided for the years must increase.	A $\checkmark$ answer	(1)
1.6.2	IQR will remain the same or change.	A $\checkmark$ answer (both same or change)	(1)
			[13]



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QUESTION 1

1.1	EXAM MARK																													
1.1.1	50	✓ answer	(1) K																											
1.1.2	<table border="1"> <thead> <tr> <th>Interval</th><th>Frequency</th><th>Cumulative frequency</th></tr> </thead> <tbody> <tr><td>$20 \leq x < 30$</td><td>3</td><td>3</td></tr> <tr><td>$30 \leq x < 40$</td><td>3</td><td>6</td></tr> <tr><td>$40 \leq x < 50$</td><td>9</td><td>15</td></tr> <tr><td>$50 \leq x < 60$</td><td>8</td><td>23</td></tr> <tr><td>$60 \leq x < 70$</td><td>12</td><td>35</td></tr> <tr><td>$70 \leq x < 80$</td><td>9</td><td>44</td></tr> <tr><td>$80 \leq x < 90$</td><td>4</td><td>48</td></tr> <tr><td>$90 \leq x < 100$</td><td>2</td><td>50</td></tr> </tbody> </table>	Interval	Frequency	Cumulative frequency	$20 \leq x < 30$	3	3	$30 \leq x < 40$	3	6	$40 \leq x < 50$	9	15	$50 \leq x < 60$	8	23	$60 \leq x < 70$	12	35	$70 \leq x < 80$	9	44	$80 \leq x < 90$	4	48	$90 \leq x < 100$	2	50	✓ first 4 values ✓ last 4 values	(2) K
Interval	Frequency	Cumulative frequency																												
$20 \leq x < 30$	3	3																												
$30 \leq x < 40$	3	6																												
$40 \leq x < 50$	9	15																												
$50 \leq x < 60$	8	23																												
$60 \leq x < 70$	12	35																												
$70 \leq x < 80$	9	44																												
$80 \leq x < 90$	4	48																												
$90 \leq x < 100$	2	50																												
1.1.3	Median = 62 ± 1	✓ 25 on graphs ✓ answer OR ✓ Answer only	(2) R																											
1.1.4	$60 \leq x < 70$	✓ answer	(1) K																											
1.1.5	$50 - 35 = 15$ learners	✓ 35 on graph ✓ answer OR ✓ Answer only	(2) R																											
1.2.1	$IQR = 20 - 11 = 9$	✓ 20-11 ✓ answer OR ✓ Answer only	(2) K																											
1.2.2	2000 - the median score of 20 in 2000 is greater in value than the median score of 17 in 2010. - the upper half (50%) of the students in 2000 scored in the same score range as the upper one-fourth (25%) of the students in 2010. By considering the upper quarter, upper half, and upper three-quarters instead of the lower quarter, lower half, and lower three-quarters, we can conclude that the students as a whole	✓ 2000 ✓ reason	(2) P																											
			[12]																											

(2) P



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Question 1

1.1	<table><thead><tr><th>Interval</th><th>Frequency</th><th>Cummulative frequency</th></tr></thead><tbody><tr><td>$60 < x \leq 70$</td><td>5</td><td>5</td></tr><tr><td>$70 < x \leq 80$</td><td>11</td><td>16</td></tr><tr><td>$80 < x \leq 90$</td><td>22</td><td>38</td></tr><tr><td>$90 < x \leq 100$</td><td>13</td><td>51</td></tr><tr><td>$100 < x \leq 110$</td><td>7</td><td>58</td></tr><tr><td>$110 < x \leq 120$</td><td>3</td><td>61</td></tr></tbody></table>	Interval	Frequency	Cummulative frequency	$60 < x \leq 70$	5	5	$70 < x \leq 80$	11	16	$80 < x \leq 90$	22	38	$90 < x \leq 100$	13	51	$100 < x \leq 110$	7	58	$110 < x \leq 120$	3	61	✓ frequency ✓✓ accumulative frequency (3)
Interval	Frequency	Cummulative frequency																					
$60 < x \leq 70$	5	5																					
$70 < x \leq 80$	11	16																					
$80 < x \leq 90$	22	38																					
$90 < x \leq 100$	13	51																					
$100 < x \leq 110$	7	58																					
$110 < x \leq 120$	3	61																					
1.2	OGIVE Daily sales (in Rand)	✓ angle point ✓ use of cumulative frequency ✓ upper limits ✓ free hand curve (4)																					
1.3	$61 - 51 = 10$	✓✓ answer (2)																					
1.4	Medium = 87 (between 85 and 88)	✓✓ answer (2)																					
		[11]																					



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QUESTION/VRAAG 1

1.1	$k + 70 = 80$ $\therefore k = 10$	✓ $k + 70 = 80$ (1)
1.2	$80 - 7 = p$ OR / OF $63 + 10 = p$ $\therefore p = 73$ $\therefore p = 73$	✓ 73 (1)
1.3	$\bar{x} = \frac{75 \times 2 + 105 \times 16 + 135 \times 28 + 165 \times 17 + 195 \times 10 + 225 \times 7}{80}$ $= \frac{11940}{80}$ $\therefore \bar{x} = 149,25$ Answer only: Full Marks	✓ midpoints/middelwaardes ✓ $\rightarrow 80$ ✓ answer/antwoord (3)
1.4.1	$\bar{x} + \sigma = 186,89$ $\sigma = 37,64$ Answer only: Full Marks	✓ method/metode ✓ answer/antwoord (2)
1.4.2	$q = \bar{x} - \sigma$ $q = 149,25 - 37,64$ $q = 111,61$	✓ answer/antwoord (1)
1.5	value of / waarde van $m = 3$ seconds / sekondes average time elapsed of day 2 will be 3 seconds less / gemiddelde tydverloop van dag 2 is 3 sekondes min der $\bar{x}_{\text{dag 2}} = 146,25$	✓ $m = 3$ ✓ $\bar{x}_{\text{dag 2}} = 146,25$ (2)
		[10]

QUESTION/VRAAG 2

2.1	$y = A + Bx$ $y = 25,42 + 1,32x$	✓ 25,42 ✓ 1,32 ✓ $y = 25,42 + 1,32x$ (3)
2.2	$y = 25,42 + 1,32(36)$ $y = 72,94$	✓ correct substitution/korrekte vervanging ✓ answer/antwoord (2)
2.3	$r = 0,81$ Strong positive/Sterk positiewe	✓ $r = 0,81$ ✓ Strong/Sterk ✓ Positive/Positiewe (3)
		[8]

REGRESSION MODELLING



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QUESTION/VRAAG 2

2.1		✓✓ plotting points/afstreek van punte ✓ regression line/regressielyn (2.3)	(2)
2.2	$a = 29,22$ (29,21542.....) $b = 0,89$ $\therefore y = 29,22 + 0,89x$	✓ calculating value of a and b bereken waardes van a en b ✓ equation for the line/vergelyking vir die lyn	(2)
2.3	On graph/Op grafiek		(1)



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2.4	$r = 0,66$ OR / OF $s_y = \sqrt{\frac{\sum(y - \bar{y})^2}{n}}$ $= \sqrt{\frac{1290,9}{10}} = 11,36$ $s_x = \sqrt{\frac{\sum(x - \bar{x})^2}{n}}$ $= \sqrt{\frac{720,9}{10}} = 8,49$ $b = r \frac{s_y}{s_x}$ then/dan $0,89 = r \frac{11,36}{8,49}$; $r = 0,66$	✓✓ answer/antwoord ✓ S_y, S_x ✓ answer/antwoord	(2)
2.5	Mean/Gemiddelde, $\bar{x} = \frac{559}{10} = 55,90$ (Answer only: Full marks) (Slegs antwoord: Volpunte)	✓✓ answer/antwoord	
	Standard deviation/Standaardafwyking, $\sigma = 11,36$	✓✓ answer/antwoord	(4)
2.6	$55,9 - 11,36$ and $55,9 + 11,36$ [44,54; 67,26] 6 scores/tellings	✓ interval/interval ✓ answer/antwoord	(2)
			[13]



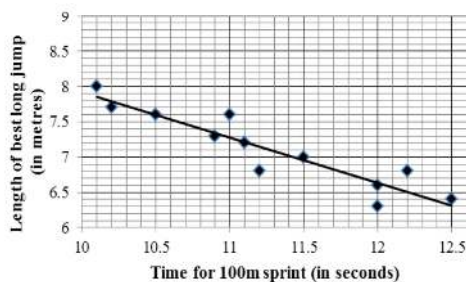
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QUESTION/VRAAG 1

Time for 100 m sprint (in seconds) <i>Tyd vir 100 m-naelloop (in sekondes)</i>	10.1	10.2	10.5	10.9	11	11.1	11.2	11.5	12	12	12.2	12.5
Distance of best long jump (in metres) <i>Afstand van beste sprong in verspring (in meter)</i>	8	7.7	7.6	7.3	7.6	7.2	6.8	7	6.6	6.3	6.8	6.4

SCATTER PLOT / SPREIDIAGRAM



1.1	$a = 14.343... = 14.34$ $b = -0.642... = -0.64$	✓✓ value of a ✓ value of b	(3)
1.2	$y = 14.34 - 0.64(11.7)$ $= 6.85$ OR/OF $y = 6.83$ (calculator / sakrekenaar)	✓ substitution correctly ✓ answer ✓✓ answer	(2) (2)
1.3	The gradient increases / <i>Die gradient neem toe</i> The point (12.3 ; 7.6) lies some distance above the current data. <i>Die punt (12,3 ; 7,6) lê bokant die huidige data.</i>	✓ increases/neem toe ✓ reasoning in words/ <i>redenasië in woorde</i>	(2) (1)



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QUESTION 2

YEAR	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Agriculture (x)	4.2	3.4	3.1	2.7	2.9	3.0	2.9	3.0	2.6	2.5	2.6
Mining (y)	19.2	19.4	19.2	18.5	17.5	17.0	16.8	15.2	14.2	12.8	12.4

2.1	$a = 5.65$ and $b = 3.65$ $y = 5.65 + 3.65x$	A✓ a-value A✓ b-value CA✓ equation	(3)
2.2	$y = 5.65 + 3.65x$ $= 5.65 + 3.65(1.2)$ $= 10.03\%$	A✓ substitution of 1,2 CA✓ answer	(2)
2.3	strong positive correlation of the data $r = 0.7$	CA✓ strong positive A✓ value of correlation coefficient	(2) (1)

QUESTION/VRAAG 2

2.1	A very strong, positive relationship / <i>'n baie sterk positiewe verwantskap</i> The points are close to one another / <i>Die punte lê baie na aan mekaar</i>	✓ statement/bewering ✓ explanation / verduideliking	(2)
2.2	$r = 0.93$	✓ Answer/antwoord	(1)
2.3	$a = 22.83$ $b = 0.66$ $\hat{y} = 22.83 + 0.66x$	✓ a ✓ b ✓ equation/ vergekyking	(3)
2.4	$\hat{y} = 22.830,66(60)$ $= 62.43$ The candidate would have scored 62%	✓ substitution/vervanging ✓ answer/antwoord	(2)
2.5	(82;61)	✓ answer /antwoord	(1)

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QUESTION / VRAAG 2

2.1.1	100	✓ answer / antwoord	(1)
2.1.2	Median / <i>Mediaan</i> = 62	✓ answer / antwoord (Accept / <i>Aanvaar</i> 61 / 62)	(2)
2.1.3		✓ $Q_1 = 37/38$ ✓ $Q_3 = 72/73$ ✓ $Q_2 = 61/62$ min & max / <i>min & maks</i> (10 & 100)	(3)
2.1.4	Skewed to the left / <i>Skeef na links</i>	✓ left / links	(1)
2.2	$b = 20$ $\frac{d-a}{2} = 8$ $2a = d$ sub $\frac{2a-a}{2} = 8$ $a = 16$ $d = 32$ $5 + 16 + 19 + 20 + c + 32 + 35 = 7 \times 22$ $\therefore c = 27$	✓ $b = 20$ ✓ $a = 16$ ✓ $d = 32$ ✓ $c = 27$	(4) (11)



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QUESTION 2

2.1		✓ 4 values ✓ 4 values ✓ 4 values	(3) K
2.2	$r = -0.98$	✓✓ answer -1 rounding	(2) R
2.3	$a = 79.71$ $b = -0.07$ $\hat{y} = 79.71 - 0.07x$	✓ value of a ✓ value of b ✓ equation	(3) R
2.4	$65 = 79.71 - 0.07x$ $-14.71 = -0.07x$ $210.14 \dots \text{min.} = x$ $\therefore 3.5 \text{ hours per week}$	✓ subst ✓ 210.14 min ✓ 3.5 hours	(3) R

[11]



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QUESTION 2

2.1.1	By using a calculator : $a = 76,60$ (76,59564211) $b = 0,1(0,09544928654)$ $\therefore$ equation of line of least squares is $y = 76,60 + 0,10x$	✓ a ✓ b ✓ equation (3)
2.1.2	$y = 76,60 + 0,10(57)$ ≈ 82	✓ substitution ✓ answer (2)
2.1.3	$r = 0,1408374211$ $r \approx 0,14$	✓✓ $r \approx 0,14$ (1)
2.1.4	r value is low which means that the data is spread around the regression line. Therefore prediction using the regression line will be invalid .	✓ invalid (1)
2.2.1	22	✓ 22 (1)
2.2.2	$\frac{1320}{22} = 60$	✓ $\frac{1320}{22}$ ✓ 60 (2)
		[10]



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MODULE 2: ANALYTICAL GEOMETRY

GENERAL ANALYTICAL GEOMETRY

QUESTION/VRAG 3

3.1	$y = 9$	✓ answer/antwoord	(1)
3.2	$AB = 11$ units/eenhede OR/OF $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $AB = \sqrt{(11 - 0)^2 + (9 - 9)^2}$ $AB = 11$ units/eenhede	✓ answer/antwoord	(1)
3.3	D(13; 9)	✓ answer/antwoord	(1)
3.4	Area of $\triangle ABC = \frac{1}{2} AB \cdot CD$ (Oppervlakte van ...) $= \frac{1}{2}(11 \times 8)$ $= 44$ square units/vierkante eenhede	✓ length of CD / lengte van CD ✓ Substitute into correct formula / Vervanging in korrekte formule ✓ answer/antwoord	(3)
3.5	$M = \left[\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right]$ $= \left[\frac{13}{2}, \frac{10}{2} \right]$ $= [6,5 ; 5]$	✓ substitution into midpoint formula/ vervanging in middeelpunt formule ✓ answer/antwoord	(2)



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QUESTION/VRAG 2

2.1	(68 ; 19)	✓ (68 ; 19) (1)
2.2	$\hat{y} = a + bx$ $a = 7,35$ and $en b = 0,98$ $\therefore \hat{y} = 7,35 + 0,98x$	✓ $a = 7,35$ ✓ $b = 0,98$ ✓ equation/ vergelyking (3)
2.3	$\hat{y} = 7,35 + 0,98(83)$ $\hat{y} = 88,69\%$ OR / OF $\hat{y} = 88,35\%$ (calculator / sakrekenaar)	✓ substitute 83 into eq. / vervang 83 in vgl. ✓ 88,69 ✓ 88,35% (2)
2.4	$r = 0,96$ $\therefore$ Very strong positive correlation / Baie Sterk positiewe korrelasie	✓ $r = 0,96$ ✓ Very strong / Baie sterk (2)
2.5	If the data points were to be plotted on a scatter plot diagram, the new plotted points will be shifted downwards with the same constant value. Hence the line of best fit will also shift downwards with the new plotted points, but the slope of the line will remain unchanged. Therefore it has NO influence on the gradient! As die datapunte op 'n spreidiagram geplot moet word, sal die nuwe datapunte afwaarts geskuif word met dieselfde konstante waarde. Vervolgens sal die paslyn ook saam met die nuwe datapunte afwaarts skuif, maar die helling van die lyn sal onveranderd bly. Dus sal dit GEEN invloed op die helling hê nie.	✓ New data points and line of best fit shifted downwards/Nuwe datapunte en paslyn word afwaarts geskuif ✓ No influence on gradient/Geen invloed op helling (2)
		[10]



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3.6	$m_{AC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{9 - 1}{0 - 13}$ $m_{AC} = -\frac{8}{13}$ $y - y_1 = m(x - x_1)$ $y - 5 = \frac{13}{8}(x - 6,5)$ $y = \frac{13}{8}x - 5\frac{9}{16}$	✓ m_{AC} ✓ correct gradient of perpendicular bisector/korrekte gradient van middelloodlyn ✓ Substitution into correct formula/ Vervanging in korrekte formule ✓ answer/antwoord (4)
3.7	$y = \frac{13}{8}(11) - \frac{89}{16}$ $y = \frac{143}{8} - \frac{89}{16}$ $y = \frac{197}{16}$ No, it does not pass through B/Nee, dit gaan nie deur B nie.	✓ substitution into correct equation / vervanging in korrekte vergelyking ✓ answer/antwoord (justification/ regverdiging) (2)
3.8	$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{11 - 13}{8} = -\frac{2}{8} = -\frac{1}{4}$ $\tan \theta = -\frac{1}{4}$ $\theta = 104,04^\circ$ $\angle ABC = 104,04^\circ$	✓ m_{BC} ✓ $\tan \theta$ ✓ answer/antwoord (3)
3.9	$y - y_1 = m(x - x_1)$ $y - 9 = -\frac{8}{13}(x - 13)$ $y = -\frac{8}{13}x + 8 + 9$ $y = -\frac{8}{13}x + 17$	✓ substitution/vervanging ✓ answer/antwoord (2)
		[19]



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QUESTION / VRAAG 3

3.1	$m_{BG} = \frac{-4-4}{-3-1} = 2$	✓ substitution/vervangings ✓ $m_{BG} = 2$
3.2	$y = 2x + c$ $sub(-3; -4)$ $-4 = 2(-3) + c$ OR/OF $sub(1; 4)$ $4 = 2(1) + c$ $\therefore c = 2$ $y = 2x + 2$ $\therefore c = 2$ $y = 2x + 2$ OR/OF $y - y_1 = m(x - x_1)$ $y + 4 = 2(x - (-3))$ OR/OF $y = 2x + 2$ $y - y_1 = m(x - x_1)$ $y - 4 = 2(x - 1)$ $y = 2x + 2$	✓ substitution/vervangings $(-3; -4)/(1; 4)$ ✓ equation/vergelyking OR/OF ✓ substitution/vervangings $(-3; -4)/(1; 4)$ ✓ equation/vergelyking
3.3.1	$m_{BC} = -\frac{1}{2}$ $-\frac{1}{2} = \frac{k - (-4)}{2 - (-3)}$ $-\frac{1}{2} = \frac{k + 4}{5}$ $-5 = 2k + 8$ $2k = -13$ $k = -\frac{13}{2} = -6\frac{1}{2}$	✓ $m_{BC} = -\frac{1}{2}$ ✓ correct substitution/korrekte vervanging ✓ simplification/vereenvoudiging ✓ k value/k-waarde
3.3.2	By translation/Deur translasie $(x; y) \rightarrow (x + 5; y - 2\frac{1}{2})$ $(1; 4) \rightarrow (6; 1\frac{1}{2})$ $D(6; \frac{3}{2})$	✓ method/metode ✓ x value/x-waarde ✓ y value/y-waarde

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QUESTION / VRAAG 3

	OR/OF $\frac{y - (-\frac{13}{2})}{x - 2} = \frac{2}{1}$ $y = 2x - \frac{21}{2} \dots\dots\dots(i)$ $\frac{y - 4}{x - 1} = -\frac{1}{2}$ $2y - 8 = -x + 1$ $2(2x - \frac{21}{2}) - 8 = -x + 1$ $4x - 21 - 8 = -x + 1$ $\therefore x = 6$ $y = 2(6) - \frac{21}{2}$ $\therefore y = \frac{3}{2}$ $D(6; \frac{3}{2})$	OR/OF ✓ method/metode ✓ x value/x-waarde ✓ y value/y-waarde
3.3.3	$m_{CD} = 2$ $\tan \hat{DHX} = 2$ $\hat{DHX} = \tan^{-1}(2)$ $\hat{DHX} = 63,49^\circ$ $\hat{\beta} = 63,49^\circ$ [vertically opposite angles/vertikaal teenoorgeselde hoek]	✓ $m_{CD} = 2$ ✓ $\hat{DHX}$ ✓ $\hat{\beta} = 63,49^\circ$
3.3.4	$y = 2x + c$ $-\frac{13}{2} = 2(2) + c$ $-\frac{21}{2} = c$ $y = 2x - \frac{21}{2}$ $0 = 2x - \frac{21}{2}$ $\therefore x = 5,25$ $GH = 6,25$ OR/OF $Area / Oppvl = \frac{1}{2} GH \cdot y_D$ $= \frac{1}{2} \times 6,25 \times 1,5$ $= 4,69 \text{ square units / vierkanteenhede}$	✓ substitution/vervangings $(2; -\frac{13}{2})$ ✓ $y = 0$ ✓ $GH = 6,25$ ✓ method/metode ✓ y_D ✓ substitution/vervangings ✓ answer/antwoord

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3.1		
3.1.1	$1 = \frac{3+x}{2}$ $2 = 3+x$ $x = -1$	$-2 = \frac{4+y}{2}$ $-4 = 4+y$ $y = -8$ $B(-1; -8)$
3.1.2	$m_{CD} = \frac{0-4}{6-3} = -\frac{4}{3}$	✓ substitution into gradient formula / vervang m in gradient formule ✓ $m_{CD} = -\frac{4}{3}$
3.1.3	$y - 2 = -\frac{4}{3}(x - 11)$ $y = -\frac{4}{3}x + \frac{50}{3}$ OR/OF $y = -\frac{4}{3}x + c$ $2 = -\frac{4}{3}(11) + c$ $c = \frac{50}{3}$ $y = -\frac{4}{3}x + \frac{50}{3}$	✓ substitute m / vervang m ✓ substitute / vervang Q(11; 2)



3.1.4	$CD = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$ OR / OF $= \sqrt{(0 - 4)^2 + (6 - 3)^2}$ R(5;10) midpoint / middelpunt $= \sqrt{25}$ RQ = $\sqrt{(2 - 10)^2 + (11 - 5)^2}$ $CD = 5$ RQ = 10 D is the midpoint of PR / D is die middelpunt van PR C is the midpoint of PQ (line from midpoint of 1 side to 2nd side) / C is die middelpunt van PQ (lyn van middelpunt van 1 sy aan 2de sy) $RQ = 2CD = 10$ (midpoint theorem / middelpuntstelling) $PK = RQ$ $\sqrt{(y+2)^2 + (4-1)^2} = 10$ $\sqrt{(y+2)^2 + (4-1)^2} = 10$ $(y+2)^2 + (4-1)^2 = 10^2$ $y^2 + 4y + 4 + 9 = 100$ $(y+2)^2 = 91$ or / of $y^2 + 4y - 87 = 0$ $y+2 = \pm\sqrt{91}$ $y = \frac{-4 \pm \sqrt{4^2 - 4(1)(-87)}}{2(1)}$ $y = \pm\sqrt{91} - 2$ $y = \frac{-4 \pm \sqrt{364}}{2}$ $y = -11.54$ or / of $y = -11.54$ or / of $y \neq 7.54$ $y \neq 7.54$ $y = -11.54$ or / of $y \neq 7.54$	✓ CD = 5 OR / OF R(5;10) ✓ statement / bewering ✓ RQ = 10 ✓ correct substitution into distance formula / korrekte vervanging in die afstandsformule ✓ simplification / vereenvoudiging ✓ y = -11.54
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3.2		
3.2.1	$m_{PQ} = \tan \theta$ $\tan \theta = 1$ $\theta = 45^\circ$ $\hat{P}_1 = 35^\circ$ vertical opp $\angle$ s / regoorst $\angle$ e $QR \parallel$ to the x-axis / aan die x-as $\hat{T}_1 = 35^\circ + 45^\circ$ ext $\angle$ of Δ / buite $\angle$ v Δ $\hat{T}_1 = 80^\circ$ $\alpha = \hat{T}_1 = 80^\circ$ corr $\angle$ s ST QR / ooreenkomstige $\angle$ e ST QR OR / OF $m_{PQ} = \tan \theta$ $\tan \theta = 1$ $\theta = 45^\circ$ $QR \parallel$ to the x-axis / aan die x-as $\hat{S}_1 = \hat{Q} = 45^\circ$ corr $\angle$ s ST QR / ooreenkomstige $\angle$ a ST QR $\hat{P}_1 = 35^\circ$ vertical opp $\angle$ s / regoorst $\angle$ e $\alpha = 35^\circ + 45^\circ$ ext $\angle$ of Δ / buite $\angle$ v Δ $\alpha = 80^\circ$	✓ $m_{PQ} = \tan \theta$ ✓ $\theta = 45^\circ$ ✓ $\hat{P}_1 = 35^\circ$ ✓ $\hat{T}_1 = 80^\circ$ ✓ $\alpha = \hat{T}_1 = 80^\circ$ OR / OF ✓ $m_{PQ} = \tan \theta$ ✓ $\theta = 45^\circ$ ✓ $\hat{S}_1 = \hat{Q} = 45^\circ$ ✓ $\hat{P}_1 = 35^\circ$ ✓ $\alpha = \hat{T}_1 = 80^\circ$

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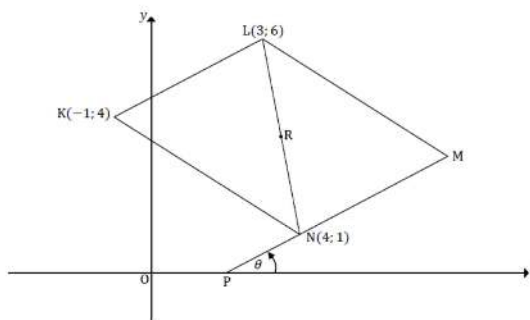


3.2.2	$x \text{ at } / \text{by } U: \frac{U_2 + (-8)}{2} = 1$ $\therefore U_2 = 10 \text{ units} / \text{eenhede}$ $QU = 18 \text{ units} / \text{eenhede}$ $x \text{ at } / \text{by } W = x \text{ at } / \text{by } U = 10$ $y \text{ at } / \text{by } W:$ $y = 10 + \frac{2}{3}$ $= \frac{32}{3}$ $WU = \frac{32}{3} + 5 = \frac{47}{3}$ $\therefore \text{Area } \Delta QWU = \frac{1}{2} (18) \left(\frac{47}{3} \right)$ $= 141 \text{ square units} / \text{eenhede kwadraat}$	✓ $U_2 = 10 \text{ units} / \text{eenhede}$ ✓ $QU = 18 \text{ units} / \text{eenhede}$ ✓ $U = 10$ ✓ $WU = \frac{47}{3}$ ✓ correct substitution in area formula / korrekte vervanging in oppv. formule ✓ 141 square units / eenhede kwadraat
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QUESTION 3



3.1	$m_{KL} = \frac{6 - 4}{3 - (-1)}$ $= \frac{2}{4} \text{ or } \frac{1}{2}$	answer only: full marks A✓ substitution in the correct formula CA✓ answer	(2)
3.1.2	midpoint of R : $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$ $\therefore \left(\frac{3 + 4}{2}, \frac{6 + 1}{2} \right)$ $R \left(\frac{7}{2}, \frac{7}{2} \right)$	answer only: full marks A✓ substitution in the correct formula CA✓ x CA✓ y	(3)
3.1.3	co-ordinates of M: $\frac{x-1}{2} = \frac{7}{2}$ $x-1 = 7 \therefore x = 8$ $\frac{y+4}{2} = \frac{7}{2}$ $y = 7 - 4 \therefore y = 3$ $\therefore M(8; 3)$	answer only: full marks CA✓ $\frac{x-1}{2} = \frac{7}{2}$ CA✓ $\frac{y+4}{2} = \frac{7}{2}$ CA✓ x CA✓ y	(4)
3.2	The equation of NM $m_{NM} = \frac{1}{2}$ [NM KL] $y - y_1 = m(x - x_1)$ $y - 1 = \frac{1}{2}(x - 4)$ $y = \frac{1}{2}x - 1 + 1$ $y = \frac{1}{2}x$	CA✓ gradient CA✓ correct subst. of m and N(4; 1) CA✓ equation	(3)

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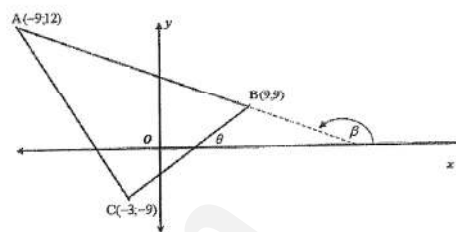




3.3.1	$0 = \frac{1}{2}x - 1$ $x = 2$ $\therefore P(2; 0)$	answer only: full marks	A✓ $y = 0$ CA✓ x	(2)
3.3.2	$\tan \theta = \frac{1}{2}$ $\theta = 26,57^\circ$	answer only: full marks	CA✓ $\tan \theta = m$ CA✓ answer	(2)
3.3.3	$m_{KP} = \frac{0-4}{2-(-1)} = -\frac{4}{3}$ $\tan^{-1}\left(-\frac{4}{3}\right) = 180^\circ - 53,13^\circ = 126,87^\circ$ $KPN = 126,87^\circ - 26,57^\circ = 100,3^\circ$		CA✓ m_{KP} CA✓ inclination of KP A✓ method CA✓ answer	(4)
				[20]



QUESTION 3 / VRAAG 3



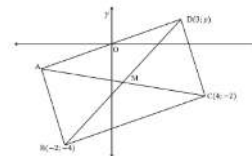
3.1	$m_{AB} = \frac{9-12}{9+9} = -\frac{1}{6}$	✓ subst of A and B / vervang A en B ✓ answer / antwoord (2)
3.2	Extend AB and construct inclination $\angle s\beta$ and θ . <i>Verleng AB en konstrueer $\angle s\beta$ and θ</i> $\tan \beta = -\frac{1}{6}$ $\beta = 170,54^\circ$ $m_{BC} = \frac{9+9}{9+3} = \frac{3}{2}$ $\tan \theta = \frac{3}{2}$ $\theta = 56,31^\circ$ $\hat{ABC} = 56,31^\circ + 9,46^\circ = 65,77^\circ$	✓ $\beta = 170,54^\circ$ ✓ $\tan \theta = \frac{3}{2}$ ✓ $\theta = 56,31^\circ$ ✓ Method/metode ✓ $\hat{ABC} = 65,77^\circ$ (5)
3.3	$M\left(\frac{-3+9}{2}; \frac{-9+9}{2}\right)$ $M(3;0)$	✓ x-value/waarde ✓ y-value/waarde (2)
3.4	$m_{AM} = \frac{12-0}{-9-3} = -1$ $y-0 = -1(x-3)$ $y = -x+3$	✓ $m_{AM} = \frac{12-0}{-9-3}$ ✓ Correct substitution/korrekte vervanging



3.5	$m_{EM} = \frac{n-0}{m-3} = -1$ $\therefore n = -m+3$ $(m-9)^2 + (n-9)^2 = (5\sqrt{5})^2$ Subst: $(m-9)^2 + (-m+3-9)^2 = (5\sqrt{5})^2$ $(m-9)^2 + (-m-6)^2 = (5\sqrt{5})^2$ $m^2 - 18m + 81 + m^2 + 12m + 36 = 125$ $2m^2 - 6m - 8 = 0$ $m^2 - 3m - 4 = 0$ $(m-4)(m+1) = 0$ $m = 4$ or $m = -1$ $n = -(-1) + 3 = 4$ $\therefore K(-1;4)$	✓ equation/vergelyking (3) ✓ $m_{EM} = \frac{n-0}{m-3} = -1$ ✓ $n = -m+3$ ✓ Substitution in dist formula/ vervang in afstandsformule ✓ standard form / standaard vorm ✓ factors/faktore ✓ m-values/waardes ✓ only $m = -1$ ✓ $K(-1;4)$ (8)	[20]
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QUESTION 3

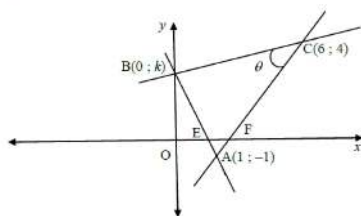


3.1	$AC = BD$ $\sqrt{50} = \sqrt{(3-(-2))^2 + (p-(-4))^2}$ $50 = 25 + p^2 + 8p + 16$ $p^2 + 8p - 9 = 0$ $(p+9)(p-1) = 0$ $p = -9$ or $p = 1$	✓ subst into correct formula ✓ $p^2 + 8p - 9$ ✓ factors ✓ correct p value (4) C
3.2	$M\left(\frac{-2+4}{2}; \frac{-4+1}{2}\right)$ $M\left(\frac{1}{2}; -\frac{3}{2}\right)$	✓ subst into correct formula ✓ $\frac{1}{2}$ ✓ $-\frac{3}{2}$ OR ✓ Answer only (3) R
3.3	$m_{DC} = \frac{1-(-2)}{3-4} = -3$ $m_{DC} = -3$ $m_{EC} = -3$	✓ subst into correct formula ✓ -3 OR ✓ Answer only (2) K
3.4	$y = -3x + c$ $-4 = -3(-2) + c$ $-10 = c$ $y = -3x - 10$	✓ $m = -3$ CA from 3.3 ✓ $c = -10$ (2) R





QUESTION 3



3.1	$AB: y + 3x - 2 = 0$ $\therefore y = -3x + 2$ $\therefore k = 2$	$\checkmark y = -3x + 2$ $\checkmark k = 2$	(2)
3.2	$AC = \sqrt{(1-6)^2 + (-1-4)^2}$ $= \sqrt{50}$ $= 5\sqrt{2}$	$\checkmark$ substitution $\checkmark$ answer	(2)
3.3	$m_{AB} = \frac{-1-2}{1-0} = -3$ $m_{AC} = \frac{4-2}{6-0} = \frac{1}{3}$ $m_{AB} \times m_{AC} = -3 \times \frac{1}{3} = -1$ $\therefore \angle BAC = 90^\circ$	$\checkmark m_{AB} = -3$ $\checkmark m_{AC} = \frac{1}{3}$ $\checkmark m_{AB} \times m_{AC} = -1$	(3)
3.4	$\tan \beta = m_{AC} = 1$ $\beta = 45^\circ$ $\tan \phi = m_{AB} = -3$ $\phi = 18,43^\circ$ $\theta = 45^\circ - 18,43^\circ$ [ext $\angle$ of Δ] $= 26,57^\circ$ OR	$\checkmark \tan \beta = m_{AC} = 1$ $\checkmark \beta$ $\checkmark \phi$ $\checkmark$ method $\checkmark$ answer	(5)



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$AC = \sqrt{(1-6)^2 + (-1-4)^2}$ $= \sqrt{50}$ $= 5\sqrt{2}$	$\checkmark AC = 5\sqrt{2}$
$AB = \sqrt{(0-1)^2 + (2+1)^2}$ $= \sqrt{10}$	$\checkmark AB = \sqrt{10}$
$\sin \theta = \frac{AB}{AC}$ $= \frac{\sqrt{10}}{5\sqrt{2}}$ $= \frac{\sqrt{5}}{5}$ $\therefore \theta = 26,57^\circ$	$\checkmark \sin \theta = \frac{AB}{AC}$ $\checkmark \sin \theta = \frac{\sqrt{10}}{5\sqrt{2}}$ $\checkmark$ answer
OR $\frac{\sin \theta}{AB} = \frac{\sin B}{AC}$ $\sin \theta = \frac{\sqrt{10} \sin 90^\circ}{5\sqrt{2}}$ $= \frac{\sqrt{5}}{5}$ $\therefore \theta = 26,57^\circ$	$\checkmark \frac{\sin \theta}{AB} = \frac{\sin B}{AC}$ $\checkmark AC = 5\sqrt{2}$ $\checkmark AB = \sqrt{10}$ $\checkmark \sin \theta = \frac{\sqrt{10} \sin 90^\circ}{5\sqrt{2}}$ $\checkmark$ answer



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3.5	$\widehat{BDC} = 90^\circ$ altitude $\therefore BC$ is the diameter (line subtends $90^\circ \angle$) midpoint of $BC: \left(\frac{0+6}{2}, \frac{2+4}{2} \right)$ $= (3;3)$ $(x-3)^2 + (y-3)^2 = r^2$ subst $B(0;2)$ $(0-3)^2 + (2-3)^2 = r^2$ $\therefore r^2 = 10$ $(x-3)^2 + (y-3)^2 = 10$ OR $\widehat{BDC} = 90^\circ$ altitude $\therefore BC$ is the diameter (line subtends $90^\circ \angle$) midpoint of $BC: \left(\frac{0+6}{2}, \frac{2+4}{2} \right)$ $= (3;3)$ $(x-3)^2 + (y-3)^2 = r^2$ subst $B(0;2)$ $(0-3)^2 + (2-3)^2 = r^2$ $\therefore r^2 = 10$ $(x-3)^2 + (y-3)^2 = 10$	$\checkmark BC$ is diameter $\checkmark (3;3)$ $\checkmark$ Substitution of $(0;2)$ $\checkmark$ equation $\checkmark BC$ is diameter $\checkmark (3;3)$ $\checkmark$ Substitution of $(0;2)$ $\checkmark$ equation	(4)
3.6	$M_{AC} = \left(\frac{1+6}{2}, \frac{2+1}{2} \right)$ $= \left(\frac{7}{2}, \frac{3}{2} \right)$ $\frac{7}{2} = \frac{x+0}{2}$ $x = 7$ $D(7;1)$	$\checkmark \frac{7}{2}, \checkmark \frac{3}{2}$ $\checkmark x = 7$ $\checkmark y = 1$ (4)	

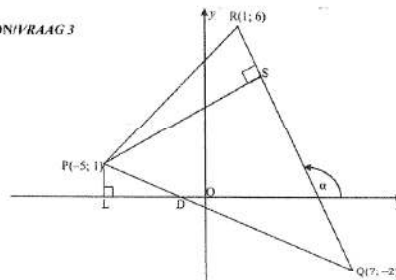
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QUESTION/VRAAG 3

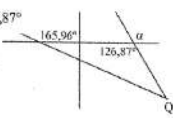


3.1	$x = -5$	$\checkmark x = -5$	(1)
3.2	$m_{PQ} = \frac{-2-6}{7-1}$ $= \frac{-8}{6}$ $= -\frac{4}{3}$	$\checkmark$ Method/Metode $\checkmark$ Answer/Antwoord	(2)
3.3	$PS \perp QR$ $\therefore m_{PS} \times m_{QR} = -1$ $\therefore m_{PS} = \frac{3}{4}$ Through the point / Duur die punt $(-5;1)$ $y-1 = \frac{3}{4}(x+5)$ $y = \frac{3}{4}x + \frac{15}{4} + 1$ $\therefore y = \frac{3}{4}x + \frac{19}{4}$	$\checkmark m_{PS} \times m_{QR} = -1$ [PS $\perp$ QR] $\checkmark m_{PS} = \frac{3}{4}$ $\checkmark$ Subst./Vervang $(-5;1)$ $\checkmark y = \frac{3}{4}x + \frac{19}{4}$	(4)
3.4	$m_{PQ} = \frac{1-(-2)}{-5-7}$ $= \frac{3}{-12}$ $= -\frac{1}{4}$ $\tan \widehat{PQO} = \frac{-1}{4}$	$\checkmark m_{PQ} = -\frac{1}{4}$ $\checkmark \tan \widehat{PQO} = \frac{-1}{4}$ $\checkmark 165,96^\circ$	(3)



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3.5	$\tan \alpha = \frac{-4}{3}$ $\therefore \alpha = -53,13^\circ + 180^\circ = 126,87^\circ$ $165,96^\circ = 126,87^\circ + \angle PQR$ $\therefore \angle PQR = 39,09^\circ$ 	$\checkmark \tan \alpha = \frac{-4}{3}$ $\checkmark \alpha = 126,87^\circ$ $\checkmark 165,96^\circ = 126,87^\circ + \angle PQR$ $\checkmark 39,09^\circ$	(4)
3.6	$\text{area } \triangle PRS = \frac{RS}{QS} = \frac{4}{16} = \frac{1}{4}$ $\text{area } \triangle PQS = \frac{QS}{16} = \frac{1}{4}$ [same height; same base/dieselfde hoogte; dieselfde basis] $\therefore RS = a$ and $QS = 4a$ Length of QR / Lengte van QR: $d_{QR} = \sqrt{(1-7)^2 + (6-(-2))^2}$ $d_{QR} = \sqrt{100} = 10$ units / eenhede $a + 4a = 10$ $\therefore 5a = 10$ $\therefore a = 2$ Length of QS / Lengte van QS = $4(2) = 8$ units / eenhede OR/OF $QR^2 = (1-7)^2 + (6-(-2))^2$ $QR = 10$ $\frac{\text{area } \triangle PQS}{\text{area } \triangle PQR} = \frac{\frac{1}{2} PQ \cdot QS \cdot \sin Q}{\frac{1}{2} PQ \cdot QR \cdot \sin Q}$ $\frac{16x^2}{20x^2} = \frac{QS}{10}$ $\frac{4}{5} = \frac{QS}{10}$ $SQ = \frac{4}{5}(10)$ $= 8$ units / eenhede	$\checkmark \text{area } \triangle PRS = \frac{1}{4}$ $\text{area } \triangle PQS = \frac{1}{4}$ $\checkmark \text{Method/Metode}$ $\checkmark d_{QR} = 10$ $\checkmark QS = 8$ $\checkmark \text{Method/Metode}$ $\checkmark QR = 10$ $\checkmark \text{area } \triangle PQS = \frac{4}{5}$ $\text{area } \triangle PQR = \frac{4}{5}$ $\checkmark \text{Answer/Antwoord}$	(5)



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CIRCLE ANALYTICAL GEOMETRY

QUESTION/VRAG 4			
4.1	$x^2 - 2x + y^2 + 4y = a$ $(x - 1)^2 + (y + 2)^2 = a + 1 + 4$ $(x - 1)^2 + (y + 2)^2 = a + 5$ $\therefore a + 5 = r^2$ $a + 5 = 25$ $a = 20$	$\checkmark \checkmark$ completing the square/voortoel van vierkant $\checkmark$ equating r^2 /gelyk stel aan r^2 $\checkmark r^2 = 25$	(4)
4.2	M(1; -2)	$\checkmark$ answer in coordinate form/antwoord in koördinaat vorm	(1)
4.3	$x^2 - 2x + y^2 + 4y = 20$ A(x; y) $\therefore (4)^2 - 2(4) + y^2 + 4y = 20$ $16 - 8 + y^2 + 4y = 20$ $y^2 + 4y - 12 = 0$ $(y + 6)(y - 2) = 0$ $\therefore y = -6$ or/of $y = 2$ $y = 2$ $A(4; 2) \quad y > 0$	$\checkmark$ substituting into correct formula/vervang in korrekte formule $\checkmark$ standard form/standaardvorm $\checkmark$ factors/faktore $\checkmark$ correct answer/korrekte antwoord	(4)
4.4	$m_{MA} = \frac{2+2}{4-1} = \frac{4}{3}$ $m_{\text{tangent}} = -\frac{3}{4}$ $y - y_1 = m(x - x_1)$ $\therefore y - 2 = -\frac{3}{4}(x - 4)$ $y = -\frac{3}{4}x + 3 + 2$ $\therefore y = -\frac{3}{4}x + 5$	$\checkmark m_{MA} = \frac{4}{3}$ $\checkmark m_{\text{tangent}} = -\frac{3}{4}$ $\checkmark$ substituting into correct formula/vervang in korrekte formule $\checkmark$ answer/antwoord	(4)



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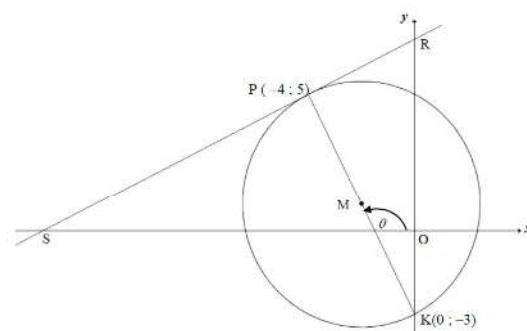
4.5	$M(1; -2), T(-1; -2)$ $MT = \sqrt{(-1-1)^2 + (-2+2)^2}$ $= 2$ $2 < \sqrt{20}$ OR/OF $x^2 - 2x + y^2 + 4y = 20$ T(-1; -2) $\text{LHS} = (-1)^2 - 2(-1) + (-2)^2 + 4(-2)$ $\text{LHS} = 1 + 2 + 4 - 8$ $\text{LHS} = -1$ $\text{LHS} \neq \text{RHS}$ $-1 < 20$ $\therefore T$ lies inside the circle/T lê binne die sirkel	$\checkmark$ substitution/vervang $\checkmark$ simplification/vereenvoudiging $\checkmark$ answer/antwoord	(3)
4.6	$(x-1)^2 + (y+2)^2 = 20$ $(x-1+3)^2 + (y+2-1)^2 = 20$ $(x+2)^2 + (y+1)^2 = 20$	$\checkmark$ substitution into correct formula/vervang in korrekte formule $\checkmark \checkmark$ simplification and answer (x+2) and (y+1) / vereenvoudiging en antwoord (x+2) en (y+1)	(3)



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QUESTION/VRAG 4



4.1.1	$m_{PK} = \frac{5-(-3)}{-4-0}$ $= -2$ $PK \perp SR$ [radius $\perp$ tangent/radial $\perp$ tangent] $\therefore m_{PK} \times m_{RS} = -1$ $\therefore m_{RS} = \frac{1}{2}$	$\checkmark$ substitution P & K into gradient formula $\checkmark$ gradient of PK $\checkmark PK \perp SR$ OR $r \perp$ tangent $\checkmark$ answer	(4)
4.1.2	$y = \frac{1}{2}x + c$ $5 = \frac{1}{2}(-4) + c$ OR/OF $(y-5) = \frac{1}{2}(x-(-4))$ $c = 7$ $y = \frac{1}{2}x + 7$	$\checkmark$ substitution of m and P $\checkmark$ equation	(2)



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4.1.3	$M\left(\frac{-4+0}{2}; \frac{5+(-3)}{2}\right)$ $\therefore M(-2; 1)$ $r^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ $r^2 = (-2+4)^2 + (1-5)^2$ $\therefore r^2 = 20$ $\therefore (x+2)^2 + (y-1)^2 = 20$ or $(\sqrt{20})^2$ OR/OF $M\left(\frac{-4+0}{2}; \frac{5+(-3)}{2}\right) \therefore M(-2; 1)$ $(x+2)^2 + (y-1)^2 = r^2$ $(-4+2)^2 + (5-1)^2 = r^2$ $\therefore r^2 = 20$ $\therefore (x+2)^2 + (y-1)^2 = 20$ or $(\sqrt{20})^2$ OR/OF $M\left(\frac{-4+0}{2}; \frac{5+(-3)}{2}\right) \therefore M(-2; 1)$ $PK = \sqrt{(-4-0)^2 + (5-(-3))^2} = \sqrt{80}$ $r = \frac{\sqrt{80}}{2} = \sqrt{20}$ $\therefore (x+2)^2 + (y-1)^2 = 20$ or $(\sqrt{20})^2$	✓ x value of M ✓ y value of M ✓ $r^2 = 20$ ✓ equation ✓✓ $M(-2; 1)$ $r^2 = 20$ ✓ equation ✓✓ $M(-2; 1)$ $r^2 = 20$ ✓ equation	(4)
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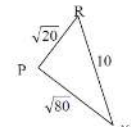


4.1.5	$RS \parallel \text{tangent at } K(0; -3)$ $\therefore m_{RS} = m_{\text{tangent}} = \frac{1}{2}$ $\therefore y = \frac{1}{2}x - 3$ OR/OF $m_{PK} = \frac{1-5}{-2+4} = -2$ $m_{PK} \times m_{\text{tangent}} = -1$ [radius $\perp$ tangent/radialyn] $\therefore m_{\text{tangent}} = \frac{1}{2}$ $\therefore y = \frac{1}{2}x - 3$	✓ gradient ✓ equation ✓ gradient ✓ equation	(2)
4.2	$t \in (-3; 7)$ OR/OF $-3 < t < 7$	✓ -3 (A) ✓ 7 (CA from 4.1.2) ✓ correct inequality ✓ -3 (A) ✓ 7 (CA from 4.1.2) ✓ correct inequality	(3)
4.3	$RS: y = \frac{1}{2}x + 7 \therefore S(-14; 0)$ $SP = \sqrt{(-14-(-4))^2 + (0-5)^2} = \sqrt{100+25} = \sqrt{125}$ $\text{Area } \triangle SMK = \frac{1}{2} \cdot MK \cdot SP$ $= \frac{1}{2}(\sqrt{20})(\sqrt{125})$ $= 25 \text{ square units}$	✓ coordinates of S ✓ length of SP ✓ correct base & height into Area rule ✓ correct substitution ✓ answer	(5)



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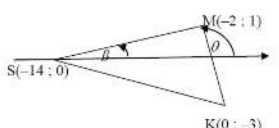


4.1.4	$\tan \theta = m_{PK} = -2$ $\therefore \theta = 180^\circ - 63,43^\circ$ $= 116,57^\circ$ $PKR = 116,57^\circ - 90^\circ$ [ext $\angle$ of $\triangle MOK$] $= 26,57^\circ$ OR/OF  In $\triangle RPK$: $PK = \sqrt{(0-(-4))^2 + (-3-5)^2} = \sqrt{80}$ $PR = \sqrt{(-4-0)^2 + (5-7)^2} = \sqrt{20}$ $RK = 10$ $\cos PKR = \frac{PK^2 + KR^2 - PR^2}{2 \cdot PK \cdot KR} = \frac{(\sqrt{80})^2 + (10)^2 - (\sqrt{20})^2}{2(\sqrt{80})(10)}$ $= \frac{2\sqrt{5}}{5}$ $PKR = 26,57^\circ$	✓ $\tan \theta = -2$ ✓ size of θ ✓ answer ✓ lengths of PK, PR & RK ✓ correct values into cos rule ✓ answer	(3)
	OR/OF $\sin PKR = \frac{\sqrt{20}}{10}$ OR/OF $\cos PKR = \frac{\sqrt{80}}{10}$ $PKR = 26,57^\circ$ $PKR = 26,57^\circ$	✓ lengths of sides ✓ ratio ✓ answer	(3)
	OR/OF $\tan PKR = \frac{\sqrt{20}}{\sqrt{80}}$ $PKR = 26,57^\circ$	✓ lengths of sides ✓ ratio ✓ answer	(3)



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	OR/OF  Let β = inclination of SM/ <i>inclinaste van SM</i> $RS: y = \frac{1}{2}x + 7 \therefore S(-14; 0)$ $SM = \sqrt{(-14-(-2))^2 + (0-1)^2} = \sqrt{145}$ $\tan \beta = \frac{1-0}{-2-(-14)} = \frac{1}{12} \therefore \beta = 4,76^\circ$ $\therefore \angle SMK = 116,57^\circ - 4,76^\circ$ [ext $\angle$ of $\triangle$] $= 111,81^\circ$ $\text{Area } \triangle SMK = \frac{1}{2}(SM)(MK) \sin \angle SMK$ $= \frac{1}{2}(\sqrt{145})(\sqrt{20}) \sin 111,81^\circ$ $= 24,9985 = 25 \text{ square units}$ OR/OF	✓ coordinates of S ✓ length of SM ✓ size of <i>grootte</i> v $\angle SMK$ ✓ correct substitution into area rule ✓ answer	(5)
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 $RS: y = \frac{1}{2}x + 7 \quad \therefore S(-14; 0)$ $SK = \sqrt{(-14-0)^2 + (0+3)^2} = \sqrt{205}$ $\cos \angle SMK = \frac{(\sqrt{145})^2 + (\sqrt{20})^2 - (\sqrt{205})^2}{2(\sqrt{145})(\sqrt{20})} = -\frac{2\sqrt{29}}{29}$ $\angle SMK = 111,80^\circ$ $\text{Area } \triangle SMK = \frac{1}{2}(SM)(KM) \sin \angle SMK$ $= \frac{1}{2}(\sqrt{145})(\sqrt{20}) \sin 111,81^\circ$ $= 24,9985 = 25 \text{ square units}$	✓ coordinates of S ✓ length of SK ✓ size of <i>grootte</i> v $\angle SMK$ ✓ correct substitution into area rule ✓ answer
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(5)

OR/OF Produce KS to T $RS: y = \frac{1}{2}x + 7 \quad \therefore S(-14; 0)$ $SK = \sqrt{(-14-0)^2 + (0+3)^2} = \sqrt{205}$ $SM = \sqrt{(-14-(-2))^2 + (0-1)^2} = \sqrt{145}$ $m_{SK} = -\frac{3}{14} \Rightarrow \angle TSO = 167,91^\circ$ $m_{SM} = \frac{1}{12} \Rightarrow \angle MSO = 4,76^\circ$ $\angle MSK = 180^\circ - 167,91^\circ + 4,76^\circ = 16,85^\circ$ $\text{Area } \triangle SMK = \frac{1}{2}(SM)(SK) \sin \angle MSK$ $= \frac{1}{2}(\sqrt{145})(\sqrt{205}) \sin 16,85^\circ$ $= 24,9985 = 25 \text{ square units}$	✓ coordinates of S ✓ length of SK & SM ✓ size of <i>grootte</i> v $\angle MSK$ ✓ correct substitution into area rule ✓ answer
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(5)



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QUESTION / VRAAG 4

4.1.1 centre <i>middelpunt</i> E(3;1) $(x-3)^2 + (y-1)^2 = r^2$ $(5-3)^2 + (-5-1)^2 = r^2$ $40 = r^2$ $(x-3)^2 + (y-1)^2 = 40$ $x^2 - 6x + 9 + y^2 - 2y + 1 = 40$ $x^2 - 6x + y^2 - 2y = 30$	✓ substitution/<i>vervang</i> E(3; 1) ✓ substitution/<i>vervang</i> P(5; -5) ✓ $40 = r^2$ ✓ standard form/<i>standaardvorm</i>
4.1.2 $m_{rad} = \frac{1-(-5)}{3-5}$ $= \frac{6}{-2} = -3$ $m_{tan} = \frac{1}{3}$ [radius $\perp$ tan] $y = \frac{1}{3}x + c$ $-5 = \frac{1}{3}(5) + c$ $c = -\frac{20}{3}$ $y = \frac{1}{3}x - \frac{20}{3}$	✓ correct substitution/<i>korrekte vervanging</i> ✓ $m_{rad} = -3$ ✓ $m_{tan} = \frac{1}{3}$ ✓ substitution/<i>vervang</i> (5; -5) ✓ equation/<i>vergelyking</i>
4.2.1 centre <i>middelpunt</i> $= \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$ $= \frac{3+5}{2}, \frac{1-5}{2}$ centre <i>middelpunt</i>: (4; -2)	✓ method/<i>metode</i> ✓ x value/<i>x-waarde</i> ✓ y value/<i>y-waarde</i>
4.2.2 $(x-4)^2 + (y+2)^2 = r^2$ $(3-4)^2 + (1+2)^2 = r^2$ $10 = r^2$ $r = \sqrt{10}$	✓ substitution of centre/<i>vervang van middelpunt</i> ✓ substitution/<i>vervang</i> (3;1) ✓ $r = \sqrt{10}$
4.3 $r = \sqrt{40}$ $EC = \sqrt{(9-3)^2 + (3-1)^2}$ $= 2\sqrt{10}$ C is on the circumference / C is op die omtrek	✓ $r = \sqrt{40}$ ✓ distance/<i>afstand</i> EC ✓ motivation/<i>motivering</i>

(18)



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QUESTION / VRAAG 4

 4.1 $\angle TUS = 180^\circ - 101,31^\circ = 78,69^\circ$ adj supp $\angle$s / <i>aangrensende suppl. $\angle$e</i> $m_{TU} = \tan 78,69^\circ = 5$ $c = 6$ $y = 5x + 6$	✓ $\angle TUS = 78,69^\circ$ ✓ $m_{TU} = 5$ ✓ $y = 5x + 6$
4.2 $x - \text{int} / \text{afsnit } y = 0$ $-\frac{1}{5}x + \frac{4}{5} = 0$ $-x + 4 = 0$ $x = 4$ $\therefore S(4; 0)$ $M = \left(\frac{-6+4}{2}, \frac{2+0}{2} \right)$ $\therefore M(-1; 1)$ OR / OF	✓ S(4; 0) ✓ substitute correctly / <i>korrekte vervanging</i> ✓ M(-1; 1)

(3)



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$5x + 6 = -\frac{1}{5}x + \frac{4}{5}$ $\frac{26}{5}x = -\frac{26}{5}$ At M: $x = -1$ $y = -1$ $\therefore M(-1; 1)$ CANDIDATE MUST SHOW CALCULATIONS TO GET MARKS IN THIS QUESTION / KANDIDAAT MOET BEREKENINGS TOON OM PUNTE IN HIERDIE VRAAG TE VERDIEN	✓ $5x + 6 = -\frac{1}{5}x + \frac{4}{5}$ ✓ $x = -1$ ✓ $y = 1$
4.3 $(x+1)^2 + (y-1)^2 = r^2$ $(-6+1)^2 + (2-1)^2 = r^2$ $r^2 = 26$ $(x+1)^2 + (y-1)^2 = 26$ OR / OF $(x+1)^2 + (y-1)^2 = r^2$ $(4+1)^2 + (0-1)^2 = r^2$ $r^2 = 26$ $(x+1)^2 + (y-1)^2 = 26$	✓ substitute / <i>vervang</i> (-6; 2) ✓ $r^2 = 26$ ✓ $(x+1)^2 + (y-1)^2 = 26$ OR / OF ✓ substitute / <i>vervang</i> (4; 0) ✓ $r^2 = 26$ ✓ $(x+1)^2 + (y-1)^2 = 26$
4.4 $m_{SO} = -\frac{1}{5}$ $m_{SO} \times m_{KL} = -1$ $m_{KL} = 5$ radius $\perp$ tan / <i>radius $\perp$ raaklyn</i> $m_{TU} = 5$ proven / <i>reeds bewys</i> $\therefore m_{TU} = m_{KL} = 5$ $KL \parallel TU$	✓ $m_{SO} = -\frac{1}{5}$ ✓ $m_{KL} = 5$ ✓ $m_{TU} = 5$
4.5 $VM = \sqrt{\left(-1 + \frac{1}{2}\right)^2 + (1-7)^2}$ $= 6,02$ radius $= \sqrt{26} = 5,1$ $6,02 > 5,1$ $\therefore V\left(-\frac{1}{2}; 7\right)$ does not lie within the circle / <i>lê nie binne die sirkel nie</i>	✓ $VM = 6,02$ ✓ $6,02 > 5,1$ ✓ conclusion / <i>gevolgtrekking</i>

(3)

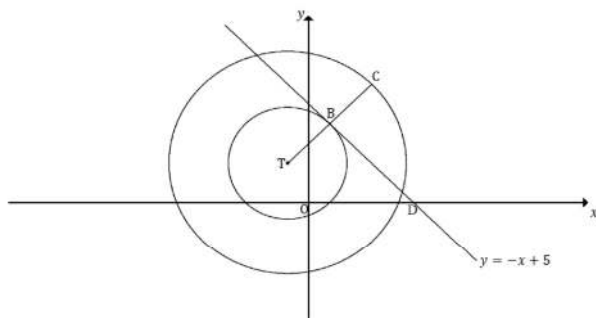
(15)



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QUESTION 4



4.1.	$x^2 + 2x + y^2 - 4y = 27$ $x^2 + 2x + 1 + y^2 - 4y + 4 = 27 + 1 + 4$ $(x + 1)^2 + (y - 2)^2 = 32$ $\therefore T(-1; 2)$ OR $x = \frac{-2}{-2} = -1$ $y = \frac{-4}{-2} = 2$	$\checkmark$ completing square $\checkmark$ LHS $\checkmark$ LHS $\checkmark$ CA $\checkmark$ x CA $\checkmark$ y $\checkmark$ x $\checkmark$ y	(4)
4.2	$m_{TB} \times m_{BD} = -1$ [tangent $\perp$ radius] $m_{TB} = 1$ $y = mx + c$ $2 = 1 \cdot (-1) + c$ $\therefore c = 3$ $\therefore$ Equation of TB is $y = x + 3$	$\checkmark$ $m_{TB} = 1$ $\checkmark$ subst of m and T $\checkmark$ value of c	(3)
4.3	equating TB and BD: $x + 3 = -x + 5$ $2x = 2$ $x = 1$ $y = 1 + 3$ or $y = -1 + 5$ $\therefore B(1; 4)$	$\checkmark$ equating $\checkmark$ x = 1 $\checkmark$ y = 4 CA provided first quadrant.	(3)



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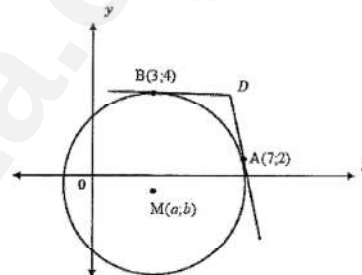
		$\checkmark (x-3)^2 + (y+1)^2 = 25$ $\checkmark$ Midpoint/ <i>middelpunt</i> $\checkmark$ radius (5)
4.1.2	$m_{AM} = \frac{2+1}{7-3} = \frac{3}{4}$ $m_{AD} = -\frac{4}{3}$ $y-2 = -\frac{4}{3}(x-7)$ $y = -\frac{4}{3}x + \frac{34}{3}$	$\checkmark m_{AM} = \frac{2+1}{7-3} = \frac{3}{4}$ $\checkmark$ m of tangent/ <i>m van raaklyn</i> $\checkmark$ Substitution/ <i>vervangings</i> $\checkmark$ equation / <i>vergelyking</i> (4)
4.1.3	Equation of BD: $y = 4$ $-\frac{4}{3}x + \frac{34}{3} = 4$ $-4x + 34 = 12$ $-4x = -22$ $x = \frac{11}{2}$ $D(\frac{11}{2}; 4)$	$\checkmark$ equation of BD/ <i>vergelyking van BD</i> $\checkmark$ equating the 2 tangent/ <i>Stel 2 vergelykings gelyk</i> $\checkmark$ x-value/waarde $\checkmark$ y-value/ waarde (4)
4.2	ADBM is a kite. Two pair of adjacent sides are equal.	$\checkmark$ statement/bewering $\checkmark$ reason /rede (2)
4.3	$MD = \sqrt{(3-\frac{11}{2})^2 + (-1-4)^2} = \frac{5\sqrt{5}}{2}$ $AB = \sqrt{(7-3)^2 + (2-4)^2} = 2\sqrt{5}$ Area ADBM = $\frac{1}{2} \times AB \times MD$ $= \frac{1}{2} \times 2\sqrt{5} \times \frac{5\sqrt{5}}{2}$ $= \frac{25}{2}$ squared units.	$\checkmark$ Length of MD/ <i>lengte van MD</i> $\checkmark$ Length of AB/ <i>lengte van AB</i> $\checkmark$ formula for area of a kite and correct substitution / <i>korrekte formule en vervanging</i> $\checkmark$ answer / <i>antwoord</i> (4)

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4.4	$(x+1)^2 + (y-2)^2 = r^2$ $(1+1)^2 + (4-2)^2 = r^2$ $8 = r^2$ $(x+1)^2 + (y-2)^2 = 8$	CA✓ subst T in circle eq CA✓ subst B CA✓ equation	(3)
4.5	Draw a horizontal line TE with E on BD: Point D(5 : 0) and Point E: 2 = -x + 5 $\therefore E(3 : 2)$ $\text{Area of trap TEDO} = \frac{1}{2}(\text{TE} + \text{OD}) \times \perp h$ $= \frac{1}{2}(4+5) \times 2$ $= 9 \text{ units}$ $\text{Area of } \triangle \text{TBE} = \frac{1}{2}(\text{TB} \times \text{BE})$ $= \frac{1}{2}(\sqrt{8})(\sqrt{8})$ $= 4 \text{ units}$ $\text{Area of } \triangle \text{OTBD} = 9 + 4$ $= 13 \text{ units}^2$	A✓ coordinates of D CA✓ coordinates of E CA✓ length of TE CA✓ subst in area of trap CA✓ subst in correct area of Δ formula A✓ method CA✓ answer	(7)

QUESTION/VRAAG 4



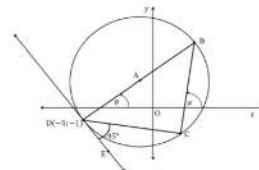
4.1.1	$x^2 - 6x + 9 + y^2 + 2y + 1 = 15 + 9 + 1$ $(x-3)^2 + (y+1)^2 = 25$ $M(3, -1)$ Radius = 5	$\checkmark$ LHS /LK $\checkmark$ RHS/RK $x^2 - 6x + 9 + y^2 + 2y + 1 = 15 + 9 + 1$
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QUESTION 4



4.1.1	$A(-1; 2)$	✓ -1 ✓ 2 A	(2) K
4.1.2	$\frac{x + (-5)}{2} = -1$ $\frac{x - 5}{2} = -1$ $x - 5 = -2$ $x = 3$ $\frac{y + (-1)}{2} = 2$ $\frac{y - 1}{2} = 2$ $y - 1 = 4$ $y = 5$ $B(3; 5)$	✓ both subst into mpt ✓ $x = 3$ ✓ $y = 5$ CA from 4.1.1 OR ✓ ✓ Answer only	(3) C
4.1.3	$m_{AD} = \frac{2 - (-1)}{-1 - (-5)}$ $= \frac{3}{-4}$	✓ subst. CA from 4.1.1 ✓ <i>gradient</i>	(2) K
4.1.4	$\tan \theta = \frac{3}{4}$ $\theta = 36,87^\circ$	✓ $\tan \theta = \frac{3}{4}$ CA from 4.1.3 ✓ answer CA	(2) R
4.1.5	$m_{\text{radius}} = \frac{2}{4} = \frac{1}{2}$ $m_{\text{tangent}} = -\frac{4}{2} = -2$ $y = -\frac{4}{2}x + c$ $-1 = -\frac{4}{2}(-5) + c$ $-1 = \frac{20}{2} + c$ $-1 = \frac{23}{2} + c$ $c = -\frac{23}{2}$ $\therefore y = -\frac{4}{2}x - \frac{23}{2}$ <i>radius $\perp$ tangent</i>	✓ $-\frac{4}{2}$ CA from 4.1.3 penalise if not neg but CA further. ✓ <i>subst</i> $(-5; -1)$ ✓ $c = -\frac{23}{2}$	(3) R

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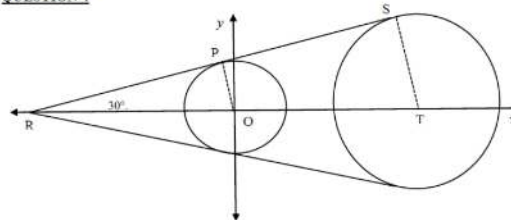
4.2	$\hat{B} = 45^\circ$ [tan, chord theorem / raaklyn, koord stelling] $\alpha = 45^\circ + 36.87^\circ$ [ext $\angle$ of Δ /buite $\angle$ van Δ] $\alpha = 81.87^\circ$ $m_{BC} = \tan 81.87^\circ$ $m_{BC} = 7$	$\checkmark \hat{B} = 45^\circ$ $\checkmark \alpha = 81.87^\circ$ CA from 4.1.4 $\checkmark m_{BC} = \tan 81.87^\circ$ $\checkmark m_{BC} = 7$	(4) P
4.3.1	$x^2 - 6x + 9 + y^2 + 2y + 1 = 8 + 9 + 1$ $(x - 3)^2 + (y + 1)^2 = 18$ $\therefore M(3; -1)$	$\checkmark +9$ both sides $\checkmark +1$ both sides $\checkmark (x - 3)^2$ $\checkmark (y + 1)^2$ <i>answer given</i>	(4) R
4.3.2	$M(3; -1)$ and $A(-1; 2)$ $MA = \sqrt{(3 - (-1))^2 + (-1 - 2)^2}$ $MA = \sqrt{16 + 9}$ $MA = \sqrt{25}$ $MA = 5$ $r_M + r_A = \sqrt{18} + 5$ or $= 9.24$ $MA < r_M + r_A$ $\therefore$ circles intersect / sirkels sny	$\checkmark$ subst $\checkmark 5$ $\checkmark \sqrt{18} + 5$ $\checkmark$ reason for conclusion	(4) P [24]



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QUESTION 4



4.1	$T(6; 0)$	$\checkmark (6; 0)$	(1)
4.2	radius = 3	$\checkmark$ radius = 3	(1)
4.3	$\tan 30^\circ = \frac{\sqrt{3}}{3}$ $0 = \frac{\sqrt{3}}{3}(-3) + c$ $c = \sqrt{3}$ $y = \frac{\sqrt{3}}{3}x + \sqrt{3}$	$\checkmark \tan 30^\circ = \frac{\sqrt{3}}{3}$ $\checkmark$ substitution $\checkmark c = \sqrt{3}$ $\checkmark$ equation	(4)
4.4	$y = -\frac{3}{\sqrt{3}}x + c$ radius $\perp$ tangent $0 = -\frac{3}{\sqrt{3}}(6) + c$ $c = \frac{18}{\sqrt{3}}$ $y = -\frac{3}{\sqrt{3}}x + \frac{18}{\sqrt{3}}$	$\checkmark$ gradient $\checkmark c = \frac{18}{\sqrt{3}}$ $\checkmark$ equation	(3)



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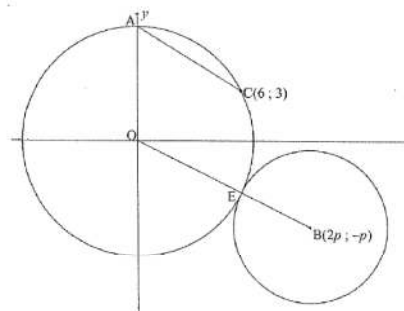
4.5	$\frac{\sqrt{3}}{3}x + \sqrt{3} = -\frac{3}{\sqrt{3}}x + \frac{18}{\sqrt{3}}$ $\frac{9x + 54}{3\sqrt{3}} = \frac{3x + 9}{3\sqrt{3}}$ $12x = 45$ $x = \frac{15}{4} = 3\frac{3}{4}$ $y = \frac{\sqrt{3}}{3}\left(\frac{15}{4}\right) + \sqrt{3}$ $y = \left(\frac{9\sqrt{3}}{4}\right)$ $S\left(\frac{15}{4}; \frac{9\sqrt{3}}{4}\right)$	$\checkmark$ equating $\checkmark$ simplification $\checkmark$ x-value $\checkmark$ substitution $\checkmark$ y-value	(5)
4.6	The distance between centres of circles: $OT = 6$ $r + R = 1 + 3$ $= 4$ The distance between the two circles = $OT - (r + R)$ $= 6 - 4$ $= 2$ units	$\checkmark OT = 6$ $\checkmark r + R = 1 + 3$ $\checkmark 4$ $\checkmark 2$	(4) [18]



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QUESTION/VRAAG 4



4.1	$x^2 + y^2 = r^2$ $(6)^2 + (3)^2 = r^2$ $r^2 = 45$ $\therefore x^2 + y^2 = 45$	$\checkmark r^2 = 45$ $\checkmark$ answer/antwoord	(2)
4.2	$A(0; \sqrt{45})$	$\checkmark 0$ $\checkmark \sqrt{45}$	(2)
4.3	$m_{AC} = \frac{3 - \sqrt{45}}{6 - 0}$ $= \frac{3 - 3\sqrt{5}}{6}$ $= \frac{1 - \sqrt{5}}{2}$ Through the point / Deur die punt $(0; \sqrt{45})$ $y = \frac{1 - \sqrt{5}}{2}x + \sqrt{45}$	$\checkmark$ Method/Metode $\checkmark m_{AC} = \frac{1 - \sqrt{5}}{2}$ $\checkmark$ Equation/Vergelyking	(3)
4.4	$-\sqrt{45} < k < \sqrt{45}$ or / of $-3\sqrt{5} < k < 3\sqrt{5}$ alternative notation / alternatiewe notasie $k \in (-\sqrt{45}; \sqrt{45})$ or / of $k \in (-3\sqrt{5}; 3\sqrt{5})$	$\checkmark -\sqrt{45}$ and / en $\sqrt{45}$ $\checkmark$ Correct notation / Korrekte notasie	(2)
4.5.1	$(x - 2p)^2 + (y + p)^2 = 20$	$\checkmark (x - 2p)^2 + (y + p)^2$ $\checkmark 20$	(2)



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4.5.2	$d_{OH} = \sqrt{45} + \sqrt{20}$ $= 5\sqrt{5}$ or / of $\sqrt{125}$ $OB^2 = (2p)^2 + (p)^2$ [Pythagoras] $(5\sqrt{5})^2 = 4p^2 + p^2$ $125 = 5p^2$ $25 = p^2$ $\therefore p = \pm 5$ But according / Maar volgens diagram $p > 0$ $\therefore p = 5$	$\checkmark d_{OH} = \sqrt{45} + \sqrt{20}$ $\checkmark d_{OH} = 5\sqrt{5}$ $\checkmark OB^2 = (2p)^2 + (p)^2$ $\checkmark p^2 = 25$ $\checkmark p = 5$	(5)
4.6	$x^2 + 4x \cos \theta + y^2 + 8y \sin \theta = -3$ $x^2 + 4x \cos \theta + (2 \cos \theta)^2 + y^2 + 8y \sin \theta + (4 \sin \theta)^2 = -3 + (2 \cos \theta)^2 + (4 \sin \theta)^2$ $\therefore r^2 = -3 + 4 \cos^2 \theta + 16 \sin^2 \theta$ $r^2 = -3 + 4 \cos^2 \theta + 16(1 - \cos^2 \theta)$ $r^2 = -3 + 4 \cos^2 \theta + 16 - 16 \cos^2 \theta$ $r^2 = 13 - 12 \cos^2 \theta$ For / vir $13 - 12 \cos^2 \theta$ to be a maximum / om 'n maksimum te wees: $\cos^2 \theta$ needs to be a minimum / moet 'n minimum wees minimum of / van $\cos^2 \theta = 0$ $\therefore r^2 = 13 - 12(0) = 13$ $\therefore r = \sqrt{13}$ OR / OF $r^2 = -3 + 4(1 - \sin^2 \theta) + 16 \sin^2 \theta$ $r^2 = -3 + 4 - 4 \sin^2 \theta + 16 \sin^2 \theta$ $r^2 = 1 + 12 \sin^2 \theta$ For / vir $1 + 12 \sin^2 \theta$ to be a maximum / om 'n maksimum te wees: maximum of / maksimum van $\sin^2 \theta = 1$ $\therefore r^2 = 1 + 12(1) = 13$ $\therefore r = \sqrt{13}$	$\checkmark$ Complete the square / Voltooi die vierkant $\checkmark r^2 = -3 + 4 \cos^2 \theta + 16 \sin^2 \theta$ $\checkmark 16(1 - \cos^2 \theta)$ $\checkmark r^2 = 13 - 12 \cos^2 \theta$ $\checkmark \cos^2 \theta = 0$ $\checkmark r = \sqrt{13}$ $\checkmark 4(1 - \sin^2 \theta)$ $\checkmark r^2 = -3 + 4 - 4 \sin^2 \theta + 16 \sin^2 \theta$ $\checkmark r^2 = 1 + 12 \sin^2 \theta$ $\checkmark \sin^2 \theta = 1$ $\checkmark r = \sqrt{13}$	(6)

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MODULE 3: TRIGONOMETRY

GENERAL TRIGONOMETRY

QUESTION / VRAAG 5

5.1.1	$\sin 28^\circ = \sin(90^\circ - 62^\circ)$ $= \cos 62^\circ$ $= m$	$\checkmark$ equation / vergelyking $\checkmark$ answer / antwoord	(2)
5.1.2	$\cos 362^\circ = \cos(2^\circ)$ $= \cos(62^\circ - 60^\circ)$ $= \cos 62^\circ \cos 60^\circ + \sin 62^\circ \sin 60^\circ$ $= \frac{1}{2}m + \frac{\sqrt{3}}{2}(\sqrt{1-m^2})$	$\checkmark \cos 2^\circ$ $\checkmark 62^\circ - 60^\circ$ $\checkmark$ expansion / uitbreiding $\checkmark$ substitution / vervanging	(4)
5.2.1	$\frac{\tan(360^\circ - x) \cdot \sin(90^\circ + x)}{\sin(-x)}$ $= \frac{-\tan x \cdot \cos x}{-\sin x}$ $= \frac{\sin x}{\cos x} \times \frac{\cos x}{\sin x}$ $= 1$	$\checkmark -\tan x$ $\checkmark \cos x$ $\checkmark -\sin x$ $\checkmark \frac{\sin x}{\cos x}$ $\checkmark$ answer / antwoord	(5)

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5.3	$4 \sin^2 \theta = 3$ $\sin^2 \theta = \frac{3}{4}$ $\therefore \sin \theta = \pm \frac{\sqrt{3}}{2}$ $\therefore \theta = 120^\circ; 90^\circ < \theta < 180^\circ$ $= \cos \frac{1}{4} \theta \cdot \sin \frac{1}{2} \theta - \tan(3\theta - 45^\circ)$ $= \cos 30^\circ \cdot \sin 60^\circ - \tan(360^\circ - 45^\circ)$ $= \cos 30^\circ \cdot \sin 60^\circ + \tan 45^\circ$ $= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + 1$ $= \frac{3}{2} + 1$ $= \frac{5}{2}$	$\checkmark$ simplification / vereenvoudiging $\checkmark$ value of / waarde van θ $\checkmark$ substitution of / vervanging van θ $\checkmark \cos 30^\circ = \frac{\sqrt{3}}{2}$ $\checkmark \sin 60^\circ = \frac{\sqrt{3}}{2}$ $\checkmark \tan 45^\circ = 1$	(6)
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QUESTION / VRAAG 5

5.1	$\frac{\sin(A - 360^\circ) \cdot \cos(90^\circ + A)}{\cos(90^\circ - A) \cdot \tan(-A)}$ $= \frac{\sin A(-\sin A)}{\sin A(-\tan A)}$ $= \frac{\sin A}{\sin A}$ $= \frac{\sin A}{\cos A}$ $= \tan A$	$\checkmark \sin A$ $\checkmark -\sin A$ $\checkmark \sin A$ $\checkmark -\tan A$ $\checkmark \tan A = \frac{\sin A}{\cos A}$ $\checkmark$ answer	(6)
5.2.1	$r^2 = (\sqrt{34})^2 - (3)^2$ $\therefore r = 5$	$\checkmark$ substitution $\checkmark$ answer	(2)
5.2.2	$\tan \beta = \frac{-5}{3}$	$\checkmark$ correct ratio	(1)

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5.2.3	$\cos 2\beta = 2 \cos^2 \beta - 1$ $= 2\left(\frac{3}{\sqrt{34}}\right)^2 - 1$ $= 2\left(\frac{9}{34}\right) - 1$ $= \frac{16}{34}$ OR $-\frac{8}{17}$ OR / OF $\cos 2\beta = 1 - 2 \sin^2 \beta$ $= 1 - 2\left(-\frac{5}{\sqrt{34}}\right)^2$ $= 1 - 2\left(\frac{25}{34}\right)$ $= \frac{16}{34}$ OR $-\frac{8}{17}$ OR / OF $\cos 2\beta = \cos^2 \beta - \sin^2 \beta$ $= \left(\frac{3}{\sqrt{34}}\right)^2 - \left(-\frac{5}{\sqrt{34}}\right)^2$ $= \frac{9}{34} - \frac{25}{34}$ $= -\frac{16}{34}$ OR $-\frac{8}{17}$	$\checkmark$ compound formula $\checkmark$ substitution $\checkmark$ simplification $\checkmark$ answer $\checkmark$ compound formula $\checkmark$ substitution $\checkmark$ simplification $\checkmark$ answer $\checkmark$ compound formula $\checkmark$ substitution $\checkmark$ simplification $\checkmark$ answer	(4)
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QUESTION / VRAAG 5

5.1.1	$(3)^2 + (-3\sqrt{3})^2 = OB^2$ $OB = 6$	✓ Pythagoras ✓ OB (2)
5.1.2	$\cos(\alpha + 30^\circ)$ $= \cos \alpha \cos 30^\circ - \sin \alpha \sin 30^\circ$ $= \left(\frac{3}{6}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(-\frac{3\sqrt{3}}{6}\right)\left(\frac{1}{2}\right)$ $= \left(\frac{\sqrt{3}}{2}\right)$	✓ expansion/uitbreiding ✓ ratios/verhoudings ✓ answer/antwoord (4)
5.2	$\frac{\sin^2(90^\circ - x) \tan(360^\circ - x)}{\sin(-x)}$ $= \frac{\cos^2 x (-\tan x)}{(-\sin x)}$ $= \frac{\cos^2 x \left(-\frac{\sin x}{\cos x}\right)}{-\sin x}$ $= \cos x$	✓ $\cos^2 x$ ✓ $-\tan x$ ✓ $-\sin x$ ✓ $\frac{\sin x}{\cos x}$ (4)

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QUESTION / VRAAG 5

5.1.1	$x^2 = (\sqrt{1+k^2})^2 - (1)^2$ (Pythagoras) $x^2 = k^2$ $x = k$ $\tan 16^\circ = \frac{1}{k}$	✓ $x = k$ ✓ $\tan 16^\circ = \frac{1}{k}$ (2)
5.1.2	$\cos 32^\circ$ $= \cos 2(16^\circ)$ $= 2\cos^2 16^\circ - 1$ $= 2\left(\frac{k}{\sqrt{1+k^2}}\right)^2 - 1$ OR / OF $\cos 32^\circ$ $= \cos 2(16^\circ)$ $= \cos^2 16^\circ - \sin^2 16^\circ$ $= \left(\frac{k}{\sqrt{1+k^2}}\right)^2 - \left(\frac{1}{\sqrt{1+k^2}}\right)^2$ OR / OF $\cos 32^\circ$ $= \cos 2(16^\circ)$ $= 1 - 2\sin^2 16^\circ$ $= 1 - 2\left(\frac{1}{\sqrt{1+k^2}}\right)^2$	✓ $\cos 2(16^\circ)$ ✓ $2\cos^2 16^\circ - 1$ ✓ correct substitution / korrekte vervanging OR / OF ✓ $\cos 2(16^\circ)$ ✓ $\cos^2 16^\circ - \sin^2 16^\circ$ ✓ correct substitution / korrekte vervanging OR / OF ✓ $\cos 2(16^\circ)$ ✓ $1 - 2\sin^2 16^\circ$ ✓ correct substitution / korrekte vervanging (3)
5.2	$\frac{\cos(90^\circ + x) \sin(x - 180^\circ) - \cos^2(180^\circ - x)}{\cos(-2x)}$ $= \frac{(-\sin x)(-\sin x) - \cos^2 x}{\cos 2x}$ $= \frac{\sin^2 x - \cos^2 x}{\cos 2x}$ OR / OF $= \frac{-(\cos^2 x - \sin^2 x)}{\cos 2x}$ $= -1$	✓ $-\sin x$ ✓ $-\sin x$ ✓ $-\cos^2 x$ ✓ $\cos 2x$ ✓ $-(\cos^2 x - \sin^2 x)$ OR / OF $-\cos 2x$ (6)

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5.3	$\cos(60^\circ + \theta) - \cos(60^\circ - \theta)$ $= [\cos 60^\circ \cos \theta - \sin 60^\circ \sin \theta] - [\cos 60^\circ \cos \theta + \sin 60^\circ \sin \theta]$ $= -2\sin 60^\circ \sin \theta$ $= -2 \cdot \frac{\sqrt{3}}{2} \sin \theta$ $= -\sqrt{3} \sin \theta$ OR / OF $\cos(60^\circ + \theta) - \cos(60^\circ - \theta)$ $= [\cos 60^\circ \cos \theta - \sin 60^\circ \sin \theta] - [\cos 60^\circ \cos \theta + \sin 60^\circ \sin \theta]$ $= \frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta - \frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta$ $= -\sqrt{3} \sin \theta$	✓ method/metode ✓ ratios/verhoudings ✓ answer/antwoord (3)
5.4.1	$LHS = \frac{1 - \sin 2A}{\sin A - \cos A}$ $= \frac{1 - 2\sin A \cos A}{\sin A - \cos A}$ $= \frac{\sin^2 A + \cos^2 A - 2\sin A \cos A}{\sin A - \cos A}$ $= \frac{(\sin A - \cos A)(\sin A + \cos A)}{\sin A - \cos A}$ $= \sin A + \cos A$ OR / OF $RHS = \sin A - \cos A \times \frac{\sin A - \cos A}{\sin A - \cos A}$ $= \frac{\sin^2 A - 2\sin A \cos A + \cos^2 A}{\sin A - \cos A}$ $= \frac{1 - 2\sin A \cos A}{\sin A - \cos A}$	✓ $2\sin A \cos A$ ✓ $\sin^2 A + \cos^2 A$ ✓ factors/faktore ✓ answer/antwoord (4)
5.4.2	$\sin A - \cos A = 0$ $\sin A = \cos A$ $\tan A = 1$ $A = 45^\circ$	✓ $\sin A - \cos A = 0$ ✓ answer/antwoord (2)

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5.3	$\cos 75^\circ \cos 45^\circ - \cos 15^\circ \cos 45^\circ$ $= \cos 75^\circ \cos 45^\circ - \sin 75^\circ \sin 45^\circ$ $= \cos(75^\circ + 45^\circ)$ $= \cos 120^\circ$ $= -\cos 60^\circ$ $= -\frac{1}{2}$ OR / OF $\cos 75^\circ \cos 45^\circ - \cos 15^\circ \cos 45^\circ$ $= \sin 15^\circ \cos 45^\circ - \cos 15^\circ \sin 45^\circ$ $= \sin(15^\circ - 45^\circ)$ $= \sin(-30^\circ)$ $= -\sin 30^\circ$ $= -\frac{1}{2}$	✓ $\cos 75^\circ \cos 45^\circ - \sin 75^\circ \sin 45^\circ$ ✓ $\cos(75^\circ + 45^\circ)$ ✓ $-\cos 60^\circ$ ✓ $-\frac{1}{2}$ OR / OF ✓ $\sin 15^\circ \cos 45^\circ - \cos 15^\circ \sin 45^\circ$ ✓ $\sin(15^\circ - 45^\circ)$ ✓ $-\sin 30^\circ$ ✓ $-\frac{1}{2}$ (4)
5.4.1	$\tan \theta \left(\sin 2\theta + \frac{3\cos^2 \theta}{\sin \theta} \right)$ $= \frac{\sin \theta}{\cos \theta} \left(2\sin \theta \cos \theta + \frac{3\cos^2 \theta}{\sin \theta} \right)$ $= 2\sin^2 \theta + 3\cos \theta$ $= 2(1 - \cos^2 \theta) + 3\cos \theta$ $= -2\cos^2 \theta + 3\cos \theta + 2$	✓ $2\sin \theta \cos \theta$ and / en $\frac{\sin \theta}{\cos \theta}$ ✓ simplification / vereenvoudiging ✓ $1 - \cos^2 \theta$ (3)
5.4.2	$-2\cos^2 \theta + 3\cos \theta + 2 = 0$ $2\cos^2 \theta - 3\cos \theta - 2 = 0$ $(2\cos \theta + 1)(\cos \theta - 2) = 0$ $\cos \theta = -\frac{1}{2} \text{ or / of } \cos \theta = 2$ no solution / geen oplossing ref / verwy $\angle = 60^\circ$ $\theta = \pm 120^\circ + k360^\circ; k \in \mathbb{Z} \quad \text{OR / OF} \quad \theta = 120^\circ + k360^\circ; k \in \mathbb{Z}$ $\theta = 240^\circ + k360^\circ; k \in \mathbb{Z}$	✓ factors / faktore $(2\cos \theta + 1)(\cos \theta - 2)$ ✓ both equations / beide vergelykings $\cos \theta = -\frac{1}{2} \text{ or / of } \cos \theta = 2$ ✓ no solution / geen oplossing ✓ $\theta = \pm 120^\circ + k360^\circ; k \in \mathbb{Z}$ OR / OF $\theta = 120^\circ + k360^\circ; k \in \mathbb{Z}$ $\theta = 240^\circ + k360^\circ; k \in \mathbb{Z}$ (4)

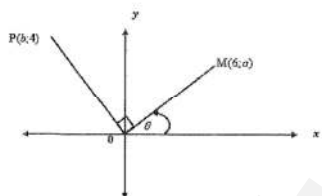
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5.5	$\cos(a+b) = -\frac{\sqrt{2}}{2}$ $a+b=180^\circ-45^\circ$ $a+b=135^\circ$ (1)	$\checkmark a+b=135^\circ$
	$\cos(a-2b) = \frac{1}{2}$ $a-2b=60^\circ$ (2)	$\checkmark a-2b=60^\circ$
	$3b=75^\circ$ $b=25^\circ$ $a=110^\circ$ (1)-(2)	$\checkmark b=25^\circ$ $\checkmark a=110^\circ$
		(4)
		[26]



QUESTION/VRAAG 5



5.1.1	$\cos \theta = \frac{2}{\sqrt{5}}$ $y^2 = (\sqrt{5})^2 - 2^2 = 1$ $y = 1$ $\tan \theta = \frac{a}{6}$ $\frac{a}{6} = \frac{1}{2}$ $a = 3$	$\cos \theta = \frac{2}{\sqrt{5}}$ $\checkmark y = 1$ $\checkmark \frac{a}{6} = \frac{1}{2}$ $\checkmark a = 3$ (3)
5.1.2	$\tan \angle POX = \frac{4}{b}$ $\tan(90^\circ + \theta) = \frac{4}{b}$ $-\frac{1}{\tan \theta} = \frac{4}{b}$ $\tan \theta = -\frac{b}{4} = \frac{1}{2}$ $b = -2$	$\checkmark \tan(90^\circ + \theta) = \frac{4}{b}$ $\checkmark -\frac{1}{\tan \theta} = \frac{4}{b}$ $\checkmark \tan \theta = -\frac{b}{4} = \frac{1}{2}$ $\checkmark \text{answer/antwoord}$ (4)
5.2.1	$\frac{\sin 163^\circ}{\cos 73^\circ} = \frac{\sin(90^\circ + \theta)}{\cos(180^\circ - \theta)}$ $= \frac{\sin(90^\circ + 73^\circ)}{\cos 73^\circ} = \frac{\cos \theta \cdot \tan^2 \theta}{-\cos \theta}$ $= \frac{\cos 73^\circ}{\cos 73^\circ} + \tan^2 \theta$	$\checkmark \cos 73^\circ$ $\checkmark \sin(90^\circ + \theta) = \cos \theta$ $\checkmark \tan^2 \theta$ $\checkmark -\cos \theta$ (4)



QUESTION 5

5.1.1	$\tan \theta = \frac{-\sqrt{3}}{1} = -\sqrt{3}$ $r^2 = (1)^2 + (-\sqrt{3})^2$ Pyth $r^2 = 4$ $r = 2$ $\sin(-\theta) = -\sin \theta$ $= -\left(\frac{-\sqrt{3}}{2}\right) \text{ or } \frac{\sqrt{3}}{2}$	A✓ answer (1)
5.1.2	$\sin(\theta - 60^\circ)$ $= \sin \theta \cos 60^\circ - \cos \theta \sin 60^\circ$ $= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = \frac{-2\sqrt{3}}{4} = \frac{-\sqrt{3}}{2}$	A✓ expansion CA✓ both special ratios CA✓ value of $\cos \theta$ CA✓ answer (3)
5.2	$\frac{\tan(180^\circ - \theta) \sin(90^\circ + \theta)}{\cos 300^\circ \sin(\theta - 360^\circ)}$ $= \frac{-\tan \theta \cdot \cos \theta}{\cos 60^\circ \cdot \sin \theta}$ $= \frac{-\frac{\sin \theta}{\cos \theta} \cdot \cos \theta}{\frac{1}{2} \cdot \sin \theta}$ $= \frac{-1}{\frac{1}{2}} = -2$	A✓ $-\tan \theta$ A✓ $\cos \theta$ A✓ $\cos 60^\circ$ A✓ $\sin \theta$ A✓ identity CA✓ answer (6)
5.3.1	$LHS = \frac{\cos 2x - 1}{\sin 2x}$ $= \frac{1 - 2\sin^2 x - 1}{2\sin x \cos x}$ $= \frac{-2\sin^2 x}{2\sin x \cos x}$ $= \frac{-\sin x}{\cos x}$ $= -\tan x$ $= RHS$	A✓ $1 - 2\sin^2 x$ A✓ $2\sin x \cos x$ A✓ simplification (3)
5.3.2	$x = 90^\circ; 180^\circ; 270^\circ$	A✓ 90° A✓ 180° A✓ 270° (3)
5.3.3	$-\tan 2x = \frac{1}{4}$ $\tan 2x = -\frac{1}{4}$ $2x = 165,96^\circ + k \cdot 180^\circ$ $x = 82,98^\circ + k \cdot 90^\circ; k \in \mathbb{Z}$	A✓ $\tan 2x = -\frac{1}{4}$ A✓ $165,96^\circ$ CA✓ $82,98^\circ$ A✓ $k \in \mathbb{Z}$ (4)
		[24]

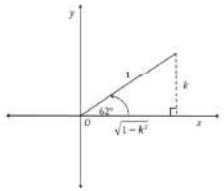


	$= 1 + \frac{\sin^2 \theta}{\cos^2 \theta}$ $= \frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta}$ $= \frac{1}{\cos^2 \theta}$	$\checkmark 1 + \frac{\sin^2 \theta}{\cos^2 \theta}$ $\checkmark \frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta}$ $\checkmark \text{answer/antwoord}$ (7)
5.2.2	$\frac{\sin^2 \theta \cos \theta}{\tan 2\theta(\sin^2 \theta - \cos^2 \theta)}$ $= \frac{\sin^2 \theta \cos \theta}{\frac{\sin 2\theta}{\cos 2\theta}(\sin^2 \theta - \cos^2 \theta)}$ $= \frac{\sin^2 \theta \cos \theta}{\frac{\sin 2\theta}{\cos 2\theta}(-\cos 2\theta)}$ $= \frac{\sin^2 \theta \cos \theta}{-2\sin \theta \cos \theta}$ $= \frac{\sin \theta}{2}$	$\checkmark \frac{\sin 2\theta}{\cos 2\theta}$ $\checkmark 2\sin \theta \cos \theta$ $\checkmark -\cos 2\theta$ $\checkmark \text{simplification/vereenvoudiging}$ $\checkmark \text{answer/antwoord}$ (5)
5.3.	$\cos(x - 60^\circ) = 3 \cos x$ $\cos x \cos 60^\circ + \sin x \sin 60^\circ = 3 \cos x$ $\frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x = 3 \cos x$ $\frac{\sqrt{3}}{2} \sin x = \frac{5}{2} \cos x$ $\sqrt{3} \sin x = 5 \cos x$ $\frac{\sin x}{\cos x} = \frac{5}{\sqrt{3}}$ $\tan x = \frac{5}{\sqrt{3}}$ $x = 70,89^\circ + k \cdot 180^\circ, k \in \mathbb{Z}$	$\checkmark \text{expansion/uitbreiding}$ $\checkmark \text{special angles/bekende hoek}$ $\checkmark \text{simplification/vereenvoudig}$ $\checkmark \frac{\sin x}{\cos x} = \frac{5}{\sqrt{3}}$ $\checkmark \tan x = \frac{5}{\sqrt{3}}$ $\checkmark x = 70,89^\circ + k \cdot 180^\circ, k \in \mathbb{Z}$ (6)
		[25]





QUESTION 5

5.1	If $\sin 40^\circ \cdot \cos 22^\circ + \cos 40^\circ \cdot \sin 22^\circ = k$, then without the use of a calculator, determine the value of the following in terms of k .		
5.1.1	$\sin 40^\circ \cdot \cos 22^\circ + \cos 40^\circ \cdot \sin 22^\circ = k$ $\sin(40^\circ + 22^\circ)$ $\sin 62^\circ = k$	✓ $\sin(40^\circ + 22^\circ)$ ✓ answer A	(2) K
5.1.2	 $\tan(118^\circ)$ $= \tan(180^\circ - 62^\circ)$ $= -\tan 62^\circ$ $= -\frac{k}{\sqrt{1-k^2}}$	✓ $y = k; r = 1$ ✓ $x = \sqrt{1-k^2}$ ✓ $-\tan 62^\circ A$ ✓ answer CA	(4) R
5.1.3	$\frac{\sin 14^\circ \cdot \cos 14^\circ}{2 \sin 14^\circ \cos 14^\circ}$ $= \frac{1}{2} \sin 28^\circ$ $= \frac{1}{2} \cos 62^\circ$ $= \frac{\sqrt{1-k^2}}{2}$	✓ $\frac{1}{2} \sin 28^\circ$ ✓ $\frac{1}{2} \cos 62^\circ$ (do not have to show give 2 marks for answer) ✓ answer CA	(3) C



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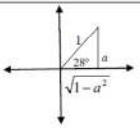
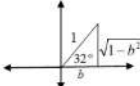
5.2	$LHS = \frac{1 - \cos 2\theta}{\sin 2\theta \times \tan \theta}$ $LHS = \frac{1 - (1 - 2\sin^2 \theta)}{2\sin \theta \cos \theta \times \frac{\sin \theta}{\cos \theta}}$ $LHS = \frac{2\sin^2 \theta}{2\sin \theta \times \frac{\sin \theta}{1}}$ $LHS = 1$ $LHS = RHS$	✓ $1 - 2\sin^2 \theta$ ✓ $2\sin \theta \cos \theta$ ✓ $\frac{\sin \theta}{\cos \theta}$ ✓ $\frac{\sin^2 \theta}{\sin^2 \theta}$	(4) C
5.3	$\sqrt{\sin(180^\circ + A) \cdot \cos(90^\circ + A)} - \tan 45^\circ$ $= \sqrt{-\sin A \cdot -\sin A - 1}$ $= \sqrt{\sin A \cdot \sin A - 1}$ $= \sqrt{-(1 - \sin^2 A)}$ $= \sqrt{-\cos^2 A}$ $\therefore \cos^2 A = 0$ $A = 90^\circ \pm 180^\circ n; n \in \mathbb{Z}$ OR $A = 90^\circ + 360^\circ n \text{ or } A = 270^\circ + 360^\circ n$	✓ $-\sin A$ ✓ $-\sin A$ ✓ -1 ✓ $-\cos^2 A$ ✓ $\cos^2 A = 0$ or $\sin = 1$ or $\cos A = 0$ ✓ answer	(6) C [19]



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QUESTION 5

5.1	$\frac{\sin(A - 180^\circ) \cdot \tan(180^\circ - A) \cdot \cos A}{\cos(90^\circ + A)}$ $= \frac{\sin(-(180^\circ - A)) \cdot (-\tan A) \cdot \cos A}{-\sin A}$ $= \frac{-\sin A \cdot (-\tan A) \cdot \cos A}{-\sin A}$ $= -\frac{\sin A}{\cos A} \cdot \cos A$ $= -\sin A$	✓ $-\tan A$ ✓ $-\sin A$ ✓ $-\sin A$ ✓ identity ✓ answer	(5)
5.2.1	 $\cos 28^\circ = \sqrt{1-a^2}$ OR $\cos^2 28^\circ + \sin^2 28^\circ = 1$ $\cos^2 28^\circ + a^2 = 1$ $\cos^2 28^\circ = 1 - a^2$ $\cos 28^\circ = \sqrt{1-a^2}$	✓ sketch ✓ answer ✓ identity ✓ answer	(2) (2)
5.2.2	 $\cos 64^\circ$ $= \cos(2 \times 32^\circ)$ $= 2\cos^2 32^\circ - 1$ $= 2b^2 - 1$ OR	✓ double angle ✓ answer	(2)



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	$\cos 64^\circ$ $= \cos(2 \times 32^\circ)$ $= 1 - 2\sin^2 32^\circ$ $= 1 - 2(\sqrt{1-b^2})^2$ $= 1 - 2 + 2b^2$ $= 2b^2 - 1$ OR $\cos(2 \times 32^\circ)$ $= \cos^2 32^\circ - \sin^2 32^\circ$ $= b^2 - (\sqrt{1-b^2})^2$ $= b^2 - 1 + b^2$ $= 2b^2 - 1$	✓ double angle ✓ answer (2)	
5.2.3	$\sin 4^\circ$ $= \sin(32^\circ - 28^\circ)$ $= \sin 32^\circ \cos 28^\circ - \cos 32^\circ \sin 28^\circ$ $= (\sqrt{1-b^2})(\sqrt{1-a^2}) - ba$ $= \sqrt{(1-b^2)(1-a^2)} - ab$	✓ compound $\angle$ ✓ expansion ✓ $\sqrt{1-b^2}$ ✓ $\sqrt{1-a^2}$	(4)
5.3	$\frac{1}{1 - \sin x} - \frac{1}{1 + \sin x} = \frac{2 \tan x}{\cos x}$ $LHS: \frac{1}{1 - \sin x} - \frac{1}{1 + \sin x} = \frac{1 + \sin x - (1 - \sin x)}{(1 - \sin x)(1 + \sin x)}$ $= \frac{1 + \sin x - 1 + \sin x}{1 - \sin^2 x}$ $= \frac{2 \sin x}{\cos^2 x}$ $RHS: \frac{2 \tan x}{\cos x} = \frac{2 \frac{\sin x}{\cos x}}{\cos x}$ $= \frac{2 \sin x}{\cos^2 x}$ $\therefore LHS = RHS$	✓ LCD ✓ numerator ✓ $\frac{2 \sin x}{\cos^2 x}$ ✓ $\frac{\sin x}{\cos x}$ ✓ $\frac{2 \sin x}{\cos^2 x}$	(5)



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OR	
$\frac{1}{1-\sin x} - \frac{1}{1+\sin x} = \frac{2 \tan x}{\cos x}$	✓ LCD
LHS: $\frac{1}{1-\sin x} - \frac{1}{1+\sin x} = \frac{1+\sin x - (1-\sin x)}{(1-\sin x)(1+\sin x)}$	✓ numerator
$= \frac{1+\sin x - 1 + \sin x}{1-\sin^2 x}$	✓ simplification
$= \frac{2 \sin x}{\cos^2 x}$	✓ $\frac{2 \sin x}{\cos^2 x}$
$= \frac{2 \sin x}{\cos x \cos x}$	✓ $\frac{\sin x}{\cos x} = \tan x$
$= \frac{2 \tan x}{\cos x}$	(5)
∴ LHS = RHS	
	[18]

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QUESTION/VRAAG 5

5.1.1	$\frac{\sin(180^\circ - 70^\circ) \cdot \tan 60^\circ}{\cos 180^\circ \cdot \tan(180^\circ + 70^\circ) \cdot \sin 20^\circ}$ $\frac{\sin 70^\circ \cdot \tan 60^\circ}{(-1) \tan 70^\circ \cdot \sin 20^\circ}$ $\frac{\sin 70^\circ \cdot \sqrt{3}}{(-1) \cdot \frac{\sin 70^\circ}{\cos 70^\circ} \cdot \cos 70^\circ}$ $= -\sqrt{3}$	✓ $\sin 70^\circ$ ✓ $\cos 180^\circ$ ✓ $\tan 70^\circ$ ✓ $\sin 20^\circ$ ✓ $\sin 70^\circ$ ✓ $\cos 70^\circ$ ✓ $\sin 20^\circ = \cos 70^\circ$ ✓ $-\sqrt{3}$
5.1.2	$\frac{1-2\sin^2 22,5^\circ}{\cos 2(22,5^\circ)}$ $\frac{\cos 45^\circ}{\cos 45^\circ}$ $= \frac{1}{\sqrt{2}}$ $= \frac{\sqrt{2}}{2}$	✓ $1-2\sin^2 22,5^\circ$ ✓ $\cos 2(22,5^\circ)$ ✓ $\cos 45^\circ$ ✓ $\frac{1}{\sqrt{2}}$
5.2.1	$\sin^2 x = 0$ and where / en waar $\tan x$ is undefined / ongedefinieerd ∴ $\sin x = 0$ ∴ $x = 0^\circ$ or / of $x = 180^\circ$ or / of $x = 90^\circ$	✓ $x = 0^\circ$ ✓ $x = 180^\circ$ ✓ $x = 90^\circ$
5.2.2	LHS / LK $\frac{\cos 2x \cdot \tan x}{\sin^2 x}$ $\frac{(\cos^2 x - \sin^2 x) \left(\frac{\sin x}{\cos x} \right)}{\sin^2 x}$ $= \frac{(\cos^2 x - \sin^2 x) \left(\frac{\sin x}{\cos x} \right) \left(\frac{1}{\sin^2 x} \right)}{\sin^2 x}$ $= \frac{\cos^2 x - \sin^2 x}{\sin x \cos x} \cdot \frac{\sin x}{\sin x \cos x}$ $= \frac{\cos^2 x}{\sin x \cos x} - \frac{\sin^2 x}{\sin x \cos x}$ $= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$ $= \frac{\cos x}{\sin x} - \tan x = \text{RHS} / \text{RK}$	✓ $\cos 2x = \cos^2 x - \sin^2 x$ ✓ $\tan x = \frac{\sin x}{\cos x}$ ✓ $\frac{\cos^2 x - \sin^2 x}{\sin x \cos x}$ ✓ $\frac{\cos^2 x}{\sin x \cos x} - \frac{\sin^2 x}{\sin x \cos x}$ ✓ $\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$ (5)

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OR/OF	
LHS / LK $\frac{\cos 2x \cdot \tan x}{\sin^2 x}$ $\frac{(\cos^2 x - \sin^2 x) \left(\frac{\sin x}{\cos x} \right)}{\sin^2 x}$ $= \frac{\cos x \cdot \sin x}{\sin^2 x} - \frac{\cos x}{\sin^2 x}$ $= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$ $= \frac{\cos x}{\sin x} - \tan x = \text{RHS} / \text{RK}$	✓ $\cos 2x = \cos^2 x - \sin^2 x$ ✓ $\tan x = \frac{\sin x}{\cos x}$ ✓ $\frac{\cos x \cdot \sin x}{\sin^2 x}$ ✓ $\frac{\cos x}{\sin^2 x}$ ✓ $\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$ (5) [19]

TRIGONOMETRIC IDENTITIES

QUESTION/VRAAG 6

LHS = $\frac{\cos 2x}{1 + \sin 2x}$ $= \frac{\cos^2 x - \sin^2 x}{1 + 2 \sin x \cos x}$ $= \frac{\sin^2 x + 2 \sin x \cos x + \cos^2 x}{(\cos x + \sin x)(\cos x + \sin x)}$ $= \frac{(\sin x + \cos x)(\sin x + \cos x)}{(\sin x + \cos x)(\sin x + \cos x)}$ $= \frac{\cos x - \sin x}{\sin x + \cos x}$	✓ expansion in numerator / uitbreiding van teller ✓ expansion in denominator / uitbreiding van noemer ✓ $\sin^2 x + \cos^2 x$ ✓ factorisation / faktorisering	[4]
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5.3.1	LHS = $\sin(A+B) - \sin(A-B)$ $= \sin A \cos B + \cos A \sin B - (\sin A \cos B - \cos A \sin B)$ $= \sin A \cos B + \cos A \sin B - \sin A \cos B + \cos A \sin B$ $= 2 \cos A \sin B$ $= \text{RHS}$	✓ compound formula ✓ compound formula (2)
5.3.2	$\sin 77^\circ - \sin 43^\circ = \sin(60^\circ + 17^\circ) - \sin(60^\circ - 17^\circ)$ $= 2 \cos 60^\circ \sin 17^\circ$ $= 2 \times \frac{1}{2} \times \sin 17^\circ$ $= \sin 17^\circ$ OR/OF $\sin 77^\circ - \sin 43^\circ = \sin(60^\circ + 17^\circ) - \sin(60^\circ - 17^\circ)$ $= (\sin 60^\circ \cos 17^\circ + \cos 60^\circ \sin 17^\circ) - (\sin 60^\circ \cos 17^\circ - \cos 60^\circ \sin 17^\circ)$ $= \frac{\sqrt{3}}{2} \cos 17^\circ + \frac{1}{2} \sin 17^\circ - \frac{\sqrt{3}}{2} \cos 17^\circ + \frac{1}{2} \sin 17^\circ$ $= \sin 17^\circ$	✓ $60^\circ + 17^\circ$ ✓ $60^\circ - 17^\circ$ ✓ simplify ✓ $\frac{1}{2}$ ✓ $60^\circ + 17^\circ$ ✓ $60^\circ - 17^\circ$ ✓ expansion ✓ $\frac{1}{2}$ (4) (4) [19]

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QUESTION/FRAG 6

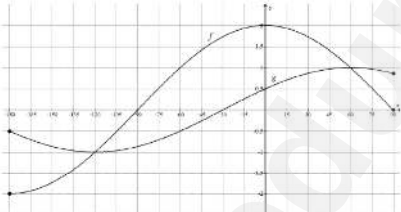
6.1	$\sin 2x = \sin(90^\circ - (x - 30^\circ)) + 360^\circ k$ $2x = 90^\circ - x + 30^\circ + 360^\circ k$ $2x = 120^\circ - x + 360^\circ k$ $x + 2x = 120^\circ + 360^\circ k$ $3x = 120^\circ + 360^\circ k$ $x = 40^\circ + 120^\circ k, k \in \mathbb{Z}$ OR/OF $2x = 180^\circ - (120^\circ - x)$ $2x = 180^\circ - 120^\circ + x$ $2x - x = 60^\circ$ $x = 60^\circ + 360^\circ k$ ALTERNATIVE/ALTERNATIEF: $\cos(90^\circ - 2x) = \cos(x - 30^\circ) + 360^\circ k$ $90^\circ - 2x = x - 30^\circ + 360^\circ k$ $-3x = -120^\circ + 360^\circ k$ $x = 40^\circ - 120^\circ k, k \in \mathbb{Z}$ OR/OF $90^\circ - 2x = 360^\circ - (x - 30^\circ)$ $-x = 300^\circ + 360^\circ k$ $x = -300^\circ + 360^\circ k$ $x = 60^\circ + 360^\circ k$	✓ method/metode ✓ $2x = 90^\circ - x + 30^\circ + 360^\circ k$ ✓ $x = 40^\circ + 120^\circ k, k \in \mathbb{Z}$ ✓ $2x = 180^\circ - (120^\circ - x)$ ✓ $x = 60^\circ + 360^\circ k$	(5)
6.2.1	Period = 360°	✓ answer/antwoord	(1)
6.2.2	$-150^\circ < x < 30^\circ$	✓ values/waardes ✓ inequalities/ongelykhede	(2)



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TRIGONOMETRIC GRAPHS

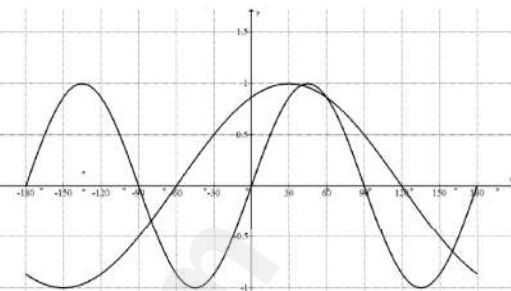
QUESTION/FRAG 8			
8.1	2	✓ answer / antwoord	(1)
8.2		f: ✓ x-int/x-afsnit ✓ shape/vorm g: ✓ x-ints/x-afsnitte ✓ y-int/y-afsnit ✓ shape/vorm	(5)
8.3	$-120^\circ \leq x \leq 60^\circ$	✓ critical values / kritiese waardes ✓ notation/notasie	(2)
8.4	Translate f 30° to the right/Skuif f 30° na regs OR/OF Translate f down by $\frac{1}{2}$ / Skuif f $\frac{1}{2}$ eenheid af	✓ correct translation/korrekte translasië ✓ correct translation/korrekte translasië	(2)
[10]			



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6.2.3



- ✓ x intercepts/x-afsnitte
- ✓ turning points/draaipunte
- ✓ shape/vorm

(3)

6.2.4

 $-180^\circ \leq x \leq -90^\circ$; $-60^\circ \leq x \leq 0^\circ$; $90^\circ \leq x \leq 120^\circ$
 $-180^\circ \leq x \leq -90^\circ$
 $-60^\circ \leq x \leq 0^\circ$
 $90^\circ \leq x \leq 120^\circ$

(3)

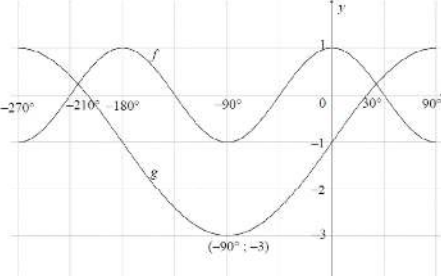
[14]



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QUESTION/FRAG 6

6.1		✓ $(-90^\circ; -3)$ ✓ $(0; -1)$ ✓ x-intercepts: -210° & 30° ✓ shape	(4)
6.2	$\cos 2x = 2 \sin x - 1$ $1 - 2 \sin^2 x = 2 \sin x - 1$ $2 \sin^2 x + 2 \sin x - 2 = 0$ $\sin^2 x + \sin x - 1 = 0$ $\sin x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)}$ $\sin x = \frac{-1 \pm \sqrt{5}}{2}$, since $\sin x = \frac{-1 - \sqrt{5}}{2} < -1$ has no solution	✓ $\cos 2x = 1 - 2 \sin^2 x$ ✓ standard form ✓ using quadratic formula ✓ substitution into quadratic formula	(4)
6.3	$\sin x = \frac{-1 + \sqrt{5}}{2} = 0,618...$ Reference $\angle = 38,17^\circ$ $\therefore x = 38,17^\circ + k \cdot 360^\circ$ or $x = 141,83^\circ + k \cdot 360^\circ$; $k \in \mathbb{Z}$ $\therefore x = 38,17^\circ$ or $-218,17^\circ$ $y = 0,24$ $\therefore$ Points of intersection/snyppunte: $(38,17^\circ; 0,24)$ and $(-218,17^\circ; 0,24)$	✓ $38,17^\circ$ ✓ $141,83^\circ$ ✓ $-218,17^\circ$ ✓ $0,24$	(4)
[12]			



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QUESTION / VRAAG 6

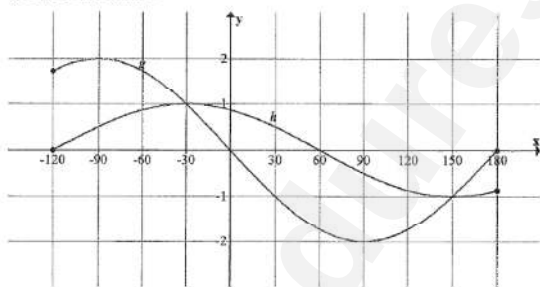
6.1		$\checkmark \left(45^\circ; -\frac{1}{2}\right)$ $\checkmark$ x-intercepts / x-asinite $\checkmark$ shape / vorm $\checkmark$ asymptotes / asymptote (4)
6.2	$y \in [2; 4]$ OR / OF $2 \leq y \leq 4$	$\checkmark y \in [2; 4]$ OR / OF $2 \leq y \leq 4$ (1)
6.3	$x \in [135^\circ; 180^\circ]$ OR / OF $135^\circ \leq x \leq 180^\circ$ $x \in [225^\circ; 270^\circ]$ OR / OF $225^\circ \leq x < 270^\circ$	$\checkmark x \in [135^\circ; 180^\circ]$ OR / OF $135^\circ \leq x \leq 180^\circ$ $\checkmark x \in [225^\circ; 270^\circ]$ OR / OF $225^\circ \leq x < 270^\circ$ (2)
		[7]



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QUESTION/VRAAG 6



6.1	$\cos x \cos 30^\circ - \sin x \sin 30^\circ \geq -2 \sin x$ $\cos(x + 30^\circ) \geq -2 \sin x$ $x \in [-30^\circ; 150^\circ]$	$\checkmark$ manipulation/manipulasie $\checkmark \cos(x + 30^\circ) \geq -2 \sin x$ $\checkmark$ Critical values/kritieke punte $\checkmark$ notation/notasie (4)
6.2	$x \in [-120^\circ; -90^\circ] \cup (150^\circ; 180^\circ]$	$\checkmark$ boundaries and notation/ grense en notasie $\checkmark$ boundaries and notation grense en notasie (4)
		[8]



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QUESTION 6

6.1		A✓ x-intercepts A✓ shape A✓ $(-30^\circ; 1)$ & $(150^\circ; -1)$ A✓ $(-120^\circ; 0)$ & $(180^\circ; -0.87)$ (4)
6.2	Period = 360°	A✓ answer (1)
6.3.1	$-30^\circ < x < 150^\circ$	CA✓ endpoints A✓ correct interval (2)
6.3.2	$x \in (0^\circ; 60^\circ)$	CA✓ endpoints A✓ correct interval (2)
6.4	$y = \cos(x + 30^\circ + 60^\circ)$ $y = \cos(x + 90^\circ)$ $y = -\sin x$	A✓ $\cos(x + 90^\circ)$ CA✓ answer (2)
		[11]



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QUESTION 6

6.1	$f(x) = \sin(x + a)$	
6.2	Sketch	$\checkmark$ x-intercepts $\checkmark$ y-intercept $\checkmark$ Turning points $\checkmark$ end points (4) R
6.3	$f(x) = g(x)$ $\sin(x + 30^\circ) = \cos(3x)$ $\sin(x + 30^\circ) = \sin(90^\circ - 3x)$ $x + 30^\circ = 90^\circ - 3x + 360^\circ n \quad n \in \mathbb{Z}$ $4x = 60^\circ + 360^\circ n$ $x = 15^\circ + 90^\circ n$ or $x + 30^\circ = 180^\circ - (90^\circ - 3x) + 360^\circ n$ $x + 30^\circ = 90^\circ + 3x + 360^\circ n$ $-2x = 60^\circ + 360^\circ n$ $x = -30^\circ - 180^\circ n$	CA from 6.1.1 $\checkmark \sin(90^\circ - 3x)$ $\checkmark$ method in 1st quadrant $\checkmark$ answer $\checkmark$ method in 2nd quadrant $\checkmark$ answer (5) C
6.4	$15^\circ < x < 105^\circ$ or $x \in (15^\circ; 105^\circ)$	$\checkmark$ critical values $\checkmark$ notation CA from 6.3 (2) C
6.1.5	$g(x) = \cos(3x)$ $k(x) = \cos(60^\circ - 3x)$ $k(x) = \cos(3x - 60^\circ)$ $k(x) = \cos(3(x - 20^\circ))$ $\therefore$ translated 20° to the right. / verskuif 20° na regs	$\checkmark$ answer A (2) P
		[14]



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QUESTION 6

[16]

6.1	$\sin(x + 60^\circ) = \cos \frac{1}{2}x$ $\sin(x + 60^\circ) = \sin\left(90^\circ - \frac{1}{2}x\right)$ quadrant 1: $x + 60^\circ = 90^\circ - \frac{1}{2}x + 360k, k \in \mathbb{Z}$ $x + \frac{1}{2}x = 30^\circ + 360k$ $\frac{3}{2}x = 30^\circ + 360k$ $x = 20^\circ + 240k, k \in \mathbb{Z}$ quadrant 2: $x + 60^\circ = 180^\circ - \left(90^\circ - \frac{1}{2}x\right) + 360k, k \in \mathbb{Z}$ $x + 60^\circ = 180^\circ - 90^\circ + \frac{1}{2}x + 360k$ $\frac{1}{2}x = 30^\circ + 360k$ $x = 60^\circ + 720k, k \in \mathbb{Z}$ $x \in \{20^\circ; 60^\circ; 260^\circ\}$ OR	✓ co-function ✓ 1 st quad equation ✓ $x = 20^\circ + 240k$ ✓ 2 nd quad equation ✓ $x = 60^\circ + 720k$ ✓ $x \in \{20^\circ; 60^\circ; 260^\circ\}$
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(7)



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$\sin(x + 60^\circ) = \cos \frac{1}{2}x$ $\cos[90^\circ - (x + 60^\circ)] = \cos \frac{1}{2}x$ quadrant 1 $[90^\circ - (x + 60^\circ)] = \frac{1}{2}x + 360k, k \in \mathbb{Z}$ $90^\circ - x - 60^\circ = \frac{1}{2}x + 360k$ $-\frac{3}{2}x = -30^\circ + 360k$ $x = 20^\circ + 240k, k \in \mathbb{Z}$ quadrant 4 $[90^\circ - (x + 60^\circ)] = 360^\circ - \frac{1}{2}x + 360k, k \in \mathbb{Z}$ $90^\circ - x - 60^\circ = 360^\circ - \frac{1}{2}x + 360k$ $30^\circ - x = 360^\circ - \frac{1}{2}x + 360k$ $-\frac{1}{2}x = 330^\circ + 360k$ $x = -660^\circ + 720k$ $x \in \{20^\circ; 60^\circ; 260^\circ\}$	✓ co-function ✓ 1 st quad equation ✓ $x = 20^\circ + 240k$ ✓ 4 th quad equation ✓ $x = -660^\circ + 720k$ ✓ $x \in \{20^\circ; 60^\circ; 260^\circ\}$ (7)
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6.2.1		g: ✓ x - intercepts ✓ y - intercept ✓ turning points
6.2.2	$20^\circ < x < 60^\circ$ or $260^\circ < x \leq 300^\circ$ OR $x \in (20^\circ; 60^\circ) \cup x \in (260^\circ; 300^\circ]$	✓ $20^\circ < x < 60^\circ$ ✓ $260^\circ < x \leq 300^\circ$ ✓ notation
6.2.3	$120^\circ \leq x \leq 180^\circ$ $x = -60^\circ$ $x = 300^\circ$	✓ $120^\circ \leq x \leq 180^\circ$ ✓ $x = -60^\circ; 300^\circ$

(3)

(3)

(3)

[16]



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QUESTION/VRAAG 6

6.1	$a = 3$ and/en $b = 2$	✓ $a = 3$ ✓ $b = 2$	(2)
6.2	360°	✓ 360°	(1)
6.3	$2 \leq y \leq 4$ or / of $y \in [2;4]$	✓ values/waardes 2 and/en 4 ✓ notation / notasie.	(2)
6.4	$0^\circ < x < 45^\circ$ or / of $90^\circ < x < 135^\circ$ alternative notation / alternatiewe notasie $x \in (0^\circ; 45^\circ) \cup (90^\circ; 135^\circ)$	✓ 0° and/en 45° ✓ 90° and/en 135° ✓ notation / notasie	(3)
6.5	$y = \cos^2 x - \sin^2 x$ $y = \cos 2x$ to coincide with g / om saam te val met g: $y = \sin(90^\circ + 2x)$ $y = \sin 2(x + 45^\circ)$ $\therefore g = 45^\circ$	✓ $y = \cos 2x$ ✓ $\sin(90^\circ + 2x)$ ✓ 45°	(3)

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(3)

[11]

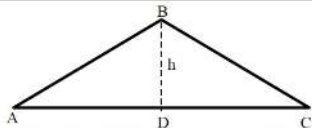


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SOLUTION OF TRIANGLES

QUESTION/VRAG 7

7.1	 Let/Laat $BD = h$ (height of triangle/hoogte van driehoek ABC) Let/Laat $DC = x$ $\therefore AD = (b - x)$ Proof/Bewys: $c^2 = h^2 + (b - x)^2$ $= h^2 + b^2 - 2bx + x^2$ $= b^2 + (h^2 + x^2) - 2bx$ $= b^2 + a^2 - 2bx$ $a^2 = h^2 + x^2$ $\frac{x}{a} = \cos C$ $x = a \cos C$ $\therefore b^2 + a^2 - 2ab \cos C$	✓ AD in terms of x AD in terme van x ✓ c^2 (using/gebruik van Pythagoras) ✓ $(h^2 + x^2) = a^2$ ✓ substitution of x/ vervanging van x	(4)
7.2.1	$\angle PSQ = 90^\circ$ $\angle Q_2 = 90^\circ + \alpha$ In $\triangle QSR$: $SR^2 = SQ^2 + QR^2 - 2SQ \cdot QR \cdot \cos \angle SQR$ $= x^2 + x^2 - 2x \cdot x \cdot \cos(90^\circ + \alpha)$ $= 2x^2 - 2x^2(-\sin \alpha)$ $= 2x^2 + 2x^2(\sin \alpha)$ $= x^2(2 + 2\sin \alpha)$ $\therefore SR = x\sqrt{2(1 + \sin \alpha)}$	✓ angle in semi-circle/hoek in semi-sirkel ✓ use of cos rule/ gebruik van cos-reël ✓ correct substitution/ korrekte vervanging ✓ $x^2(2 + 2\sin \alpha)$	(4)

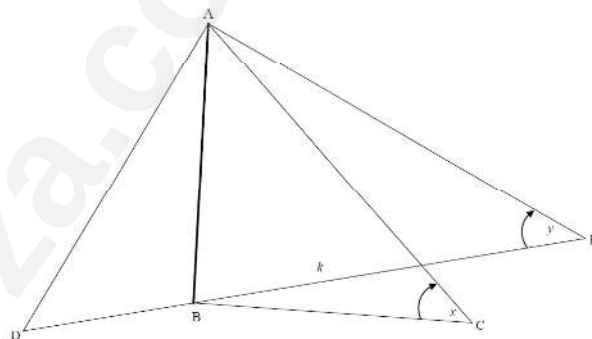


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7.2.2	$5\sqrt{3} = x\sqrt{2(1 + \sin \alpha)}$ $5\sqrt{3} = 5\sqrt{2(1 + \sin \alpha)}$ $\sqrt{3} = \sqrt{2(1 + \sin \alpha)}$ $3 = 2(1 + \sin \alpha)$ $\frac{3}{2} = 1 + \sin \alpha$ $\frac{1}{2} = \sin \alpha$ $\alpha = 30^\circ$ In $\triangle PSQ$ $QS = \sin \alpha$ $PQ = \frac{QS}{\sin \alpha}$ $= \frac{\sin 30^\circ}{5}$ $= 10 \text{ units/eenhede}$	✓ Substitution/ Vervanging ✓ $\sin \alpha = 0,5$ $\alpha = 30^\circ$ ✓ answer/antwoord	(4)
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QUESTION/VRAG 7



7.1	$\angle ABC = 90^\circ$	✓ answer	(1)
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7.2	In $\triangle ABE$: $\frac{AB}{BE} = \tan y$ $AB = k \tan y$ In $\triangle ABC$: $\frac{AB}{AC} = \sin x$ $AC = \frac{AB}{\sin x}$ $= \frac{k \tan y}{\sin x}$	✓ correct ratio ✓ value AB ✓ correct ratio ✓ AC as subject and substitution	(4)
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7.3	$\angle ADC = \angle ACD = \frac{180^\circ - 2x}{2} = 90^\circ - x$ $\frac{DC}{\sin 2x} = \frac{AC}{\sin(90^\circ - x)}$ $\frac{DC}{2 \sin x \cos x} = \frac{AC}{\cos x}$ $DC = \frac{AC(2 \sin x \cos x)}{\cos x}$ $= \frac{k \tan y}{\sin x} \cdot \frac{2 \sin x \cos x}{\cos x}$ $= 2k \tan y$ OR/OF $DC^2 = AD^2 + AC^2 - 2AD \cdot AC \cos 2x$ $= AC^2 + AC^2 - 2AC^2 \cos 2x$ $= 2AC^2(1 - \cos 2x)$ $= 2AC^2(1 - 1 + \sin^2 x)$ $= 4AC^2 \sin^2 x$ $DC = 2AC \sin x$ $= 2 \left(\frac{k \tan y}{\sin x} \right) \sin x$ $= 2k \tan y$	✓ $90^\circ - x$ ✓ subst into sine rule ✓ $2 \sin x \cos x$ $\cos x$ ✓ substitution ✓ substitution into cos rule ✓ factorisation $1 - 2 \sin^2 x$ ✓ DC in terms of AC and $\sin x$ ✓ substitution	(5)
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7.2	OR/OF $DC^2 = AD^2 + AC^2 - 2AD \cdot AC \cos 2x$ $= 2 \left(\frac{k \tan y}{\sin x} \right)^2 - 2 \left(\frac{k \tan y}{\sin x} \right)^2 \cos 2x$ $= \frac{2k^2 \tan^2 y}{\sin^2 x} - \frac{2k^2 \tan^2 y}{\sin^2 x} (1 - 2 \sin^2 x)$ $= \frac{2k^2 \tan^2 y}{\sin^2 x} - \frac{2k^2 \tan^2 y}{\sin^2 x} + 4k^2 \tan^2 y$ $DC = \sqrt{4k^2 \tan^2 y}$ $= 2k \tan y$	✓ correct cos rule ✓ substitution ✓ $1 - 2 \sin^2 x$ ✓ squaring and multiplication ✓ $\sqrt{4k^2 \tan^2 y}$	(5)
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[10]



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QUESTION/VRAAG 7

7.1	$\hat{BDC} = 180^\circ - 2\alpha$ $\frac{y}{\sin \alpha} = \frac{x}{\sin 180^\circ - 2\alpha}$ $y = \frac{x \sin \alpha}{\sin(180^\circ - 2\alpha)}$ $y = \frac{x \sin \alpha}{2 \sin \alpha \cos \alpha}$ $y = \frac{x}{2 \cos \alpha}$ $\cos \theta = \frac{BD}{AB}$ $AB \cos \theta = BD$ $AB = \frac{BD}{\cos \theta}$ $AB = \frac{x}{2 \cos \alpha} \div \cos \theta$ $AB = \frac{x}{2 \cos \alpha \cos \theta}$ $AB = \frac{x}{2 \cos \alpha \cos \theta}$	$\checkmark \hat{BDC} = 180^\circ - 2\alpha$ $\checkmark$ method/metode $\checkmark$ substitution/vervangings $\checkmark 2 \sin \alpha \cos \alpha$ $\checkmark AB = \frac{BD}{\cos \theta}$ $\checkmark$ substitution/vervangings BD $\checkmark$ simplification/vereenvoudiging
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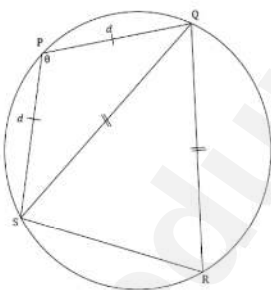
[7]



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QUESTION 7



7.1	$QS^2 = d^2 + d^2 - 2d \cdot d \cdot \cos \theta$ $QS^2 = 2d^2 - 2d^2 \cos \theta$ $QS^2 = 2d^2(1 - \cos \theta)$ $QS = d\sqrt{2(1 - \cos \theta)}$	$\checkmark$ subst into cos-rule $\checkmark$ common factor	(2)
7.2	$\hat{R} = 180^\circ - \theta$ opp. $\angle$'s cyclic quad suppl $\hat{QSR} = 2\theta - 180^\circ$ equal sides, equal angles. $\hat{QSR} = 2\theta - 180^\circ$ sum $\angle$'s Δ $\Delta QRS = \frac{1}{2} \cdot QS \cdot QR \sin \hat{QSR}$ $= \frac{1}{2} \cdot d\sqrt{2(1 - \cos \theta)} \cdot d\sqrt{2(1 - \cos \theta)} \sin(2\theta - 180^\circ)$ $= \frac{1}{2} \cdot d^2 \cdot 2(1 - \cos \theta)(-\sin 2\theta)$ $= -d^2(1 - \cos \theta) \cdot \sin 2\theta$	$\checkmark$ $\hat{QSR}$ $\checkmark$ subst into area rule $\checkmark$ simplify & reduction	(3)

[5]



215



QUESTION / VRAAG 7

7.1	$\hat{D}_1 = 180^\circ - 60^\circ - (120^\circ - \alpha)$ (sum of $\angle$ s of Δ / somvnd $\angle$ ev Δ) $\hat{D}_1 = \alpha$	$\checkmark \hat{D}_1 = 180^\circ - 60^\circ - (120^\circ - \alpha)$ $\checkmark \hat{D}_1 = \alpha$
7.2	$\frac{BD}{\sin 60^\circ} = \frac{4}{\sin \alpha}$ $BD \sin \alpha = 4 \sin 60^\circ$ $BD = \frac{4 \left(\frac{\sqrt{3}}{2} \right)}{\sin \alpha}$ $BD = \frac{2\sqrt{3}}{\sin \alpha}$	$\checkmark$ substitution into correct sin rule / vervang in korrekte sin reël $\checkmark$ simplification / vereenvoudiging $\checkmark$ answer / antwoord
7.3	$\text{In } \Delta ADB: \hat{ADB} = \theta$ ($\angle$'s of a Δ) $\frac{AB}{BD} = \tan \theta$ $AB = BD \cdot \tan \theta$ $= \frac{2\sqrt{3}}{\sin \alpha} \cdot \tan \theta$ $AB = \frac{2\sqrt{3} \tan \theta}{\sin \alpha}$	$\checkmark \hat{ADB} = \theta$ $\checkmark$ trig ratio / trig verhouding $\checkmark$ substitution of BD / vervangings van BD

[3]

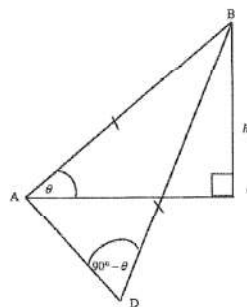
[8]



214



QUESTION/VRAAG 7



7.1	$\text{In } \Delta ARC: \sin \theta = \frac{h}{AB}$ $AB = \frac{h}{\sin \theta}$	$\checkmark AB = \frac{h}{\sin \theta}$
7.2	$\text{In } \Delta ABD: \hat{BAD} = \hat{D} = 90^\circ - \theta$ (sides opposite $\angle$'s) $90^\circ - \theta + 90^\circ - \theta + \hat{ABD} = 180^\circ$ ($\angle$'s of Δ) $\hat{ABD} = 2\theta$	$\checkmark \hat{BAD} = \hat{D} = 90^\circ - \theta$ $\checkmark \hat{ABD} = 2\theta$
7.3	$\frac{AD}{\sin 2\theta} = \frac{AB}{\sin(90^\circ - \theta)}$ $AD = \frac{h}{\sin \theta} \times \sin 2\theta$ $AD = \frac{h}{\sin \theta} \times 2 \sin \theta \cos \theta$ $AD = 2h$	$\checkmark \frac{AD}{\sin 2\theta} = \frac{AB}{\sin(90^\circ - \theta)}$ $\checkmark \cos \theta$ $\checkmark 2 \sin \theta \cos \theta$ $\checkmark$ answer / antwoord

[4]

[7]



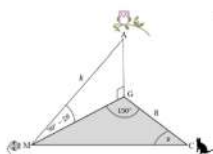
215



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QUESTION 7

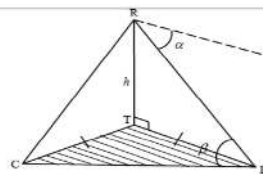


7.1	$\angle AG = 2\theta$	✓ answer	(1) K
7.2	$\frac{MG}{\sin 2\theta} = \frac{k}{\sin 90^\circ}$ $MG = \frac{k \sin 2\theta}{1} \dots \dots (1)$	$\checkmark \frac{MG}{\sin 2\theta} = \frac{k}{\sin 90^\circ} \text{ OR } \frac{MG}{k} = \sin 2\theta$ $\checkmark MG = \frac{k \sin 2\theta}{1}$	(2) C
7.3	$\frac{MC}{\sin 150^\circ} = \frac{MG}{\sin \theta}$ $\frac{MC}{\frac{1}{2}} = \frac{\frac{k \sin 2\theta}{1}}{\sin \theta} \dots \dots (1)$ Subst. (2) in (1) $MC = \frac{k \sin 2\theta}{2 \sin \theta}$ $MC = \frac{k 2 \sin \theta \cos \theta}{2 \sin \theta}$ $MC = k \cos \theta$	$\checkmark \frac{MC}{\sin 150^\circ} = \frac{MG}{\sin \theta}$ $\checkmark \frac{1}{2}$ $\checkmark MC = \frac{\frac{1}{2} MG}{\sin \theta}$ $\checkmark 2 \sin \theta \cos \theta$ answer given	(4) C
7.4	$\Delta MGC = \frac{1}{2} MG \cdot CG \sin 150^\circ$ $\Delta MGC = \frac{1}{2} k \sin 2\theta \cdot 8 \cdot \frac{1}{2}$ $\Delta MGC = 2k \sin 2\theta$ OR $\Delta MGC = \frac{1}{2} MC \cdot CG \sin \theta$ $\Delta MGC = \frac{1}{2} k \cos \theta \cdot 8 \sin \theta$ $\Delta MGC = 4k \cos \theta \sin \theta$ $\Delta MGC = 2k(2 \cos \theta \sin \theta)$ $\Delta MGC = 2k \sin 2\theta$	$\checkmark \text{subst.}$ $\checkmark \text{method}$ answer given OR $\checkmark \text{subst.}$ $\checkmark \text{method}$ answer given	(2) C
[9]			

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QUESTION 7



7.1	$CR = RD$ (congruency OR SAS) $\angle CRD = 180^\circ - 2\beta$ ($\angle$ in Δ)	$\checkmark CR = RD$ $\checkmark$ answer (2)
7.2	$\angle TRD = 90^\circ - \alpha$ $\cos(90^\circ - \alpha) = \frac{h}{RD}$ $RD = \frac{h}{\sin \alpha}$ $CD^2 = \left(\frac{h}{\sin \alpha}\right)^2 + \left(\frac{h}{\sin \alpha}\right)^2 - 2\left(\frac{h}{\sin \alpha}\right)\left(\frac{h}{\sin \alpha}\right)\cos(180^\circ - 2\beta)$ $= \frac{2h^2}{\sin^2 \alpha} - 2\left(\frac{h^2}{\sin^2 \alpha}\right)(-\cos 2\beta)$ $= \frac{2h^2}{\sin^2 \alpha} (1 + \cos 2\beta)$ $= \frac{2h^2}{\sin^2 \alpha} (1 + 2\cos^2 \beta - 1)$ $= \frac{2h^2}{\sin^2 \alpha} (2\cos^2 \beta)$ $= \frac{4h^2}{\sin^2 \alpha} (\cos^2 \beta)$ $\therefore CD = \frac{2h \cos \beta}{\sin \alpha}$	$\checkmark$ ratio $\checkmark RD = \frac{h}{\sin \alpha}$ $\checkmark$ substitution $\checkmark \frac{2h^2}{\sin^2 \alpha} (1 + 2\cos^2 \beta - 1)$ $\checkmark \frac{4h^2}{\sin^2 \alpha} (\cos^2 \beta)$ (5)
7.3	$CD = \frac{2h \cos \beta}{\sin \alpha}$	$\checkmark$ substitution

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$5,4 = \frac{2h \cos 65^\circ}{\sin 51^\circ}$ $h = \frac{5,42 \sin 51^\circ}{2 \cos 65^\circ}$ $h = 4,96$ $h \approx 5$	$\checkmark h$ to the nearest unit (2)
[9]	

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QUESTION/PRAAG 7

7.1	Coordinates of / <i>Koördinate van</i> B(c; 0) $BC = a = \sqrt{(b \cos A - c)^2 + (b \sin A - 0)^2}$ $\therefore a^2 = b^2 \cos^2 A - 2bc \cos A + c^2 + b^2 \sin^2 A$ $= b^2 (\cos^2 A + \sin^2 A) - 2bc \cos A + c^2$ $= b^2 (1) + c^2 - 2bc \cos A$ $= b^2 + c^2 - 2bc \cos A$ ✓ coordinates of / <i>koördinate van</i> B ✓ correct subst. into distance formula / <i>korrekte vervanging in afstandsformule</i> ✓ $b^2 \cos^2 A - 2bc \cos A + c^2 + b^2 \sin^2 A$ ✓ $b^2 (\cos^2 A + \sin^2 A)$	(4)
7.2	S is the nearest to R where RS is the perpendicular height / <i>S is die naaste aan R waar RS die loodregte hoogte is.</i> Let / <i>Laat</i> QS = x $\therefore PS = 55.68 - x$ $(50^2 - x^2) = (60^2 - [55.68 - x]^2) = RS^2$ $2500 - x^2 = 3600 - 3100.26 + 111.36x - x^2$ $2000.26 = 111.36x$ $\therefore x = 17.96 \text{ units / eenhede}$ ✓ Diagram ✓ Pythagoras ✓ Simplify / <i>vereenvoudig</i> ✓ answer / <i>antwoord</i>	(4) [12]



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7.2.1	$\hat{R} = \hat{T} = \hat{P} = 60^\circ$ $QT = 10$ and / <i>en</i> $QR = 50$ $PQ^2 = 60^2 + 50^2 - 2(50)(60) \cos 60^\circ$ $= 3100$ OR / OF $PQ^2 = 10^2 + 60^2 - 2(10)(60) \cos 60^\circ$ $= 3100$ $\therefore PQ = 55.68$	✓ 60° ✓ 10 or/ef 50 ✓ correct subst in cosine-rule / <i>korrekte vervanging in cosreël</i> ✓ 3100	(4)
7.2.2	S is the nearest to R where RS is the perpendicular height / <i>S is die naaste aan R waar RS die loodregte hoogte is.</i> Let / <i>Laat</i> QS = x $\therefore PS = 55.68 - x$ $(50^2 - x^2) = (60^2 - [55.68 - x]^2) = RS^2$ $2500 - x^2 = 3600 - 3100.26 + 111.36x - x^2$ $2000.26 = 111.36x$ $\therefore x = 17.96 \text{ units / eenhede}$ ✓ Diagram ✓ Pythagoras ✓ Simplify / <i>vereenvoudig</i> ✓ answer / <i>antwoord</i>		(4) [12]



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MODULE 4: EUCLIDEAN GEOMETRY

CIRCLE GEOMETRY

QUESTION/PRAAG 9

9.1	Const: Join O to C <i>Konstr: Verbind O met C</i> Construction / <i>Konstruksie</i>: Join CO produced / <i>Verbind CO verleng</i>. Proof / <i>Bewys</i>: Let / <i>Laat</i> $C_1 = x$ and / <i>en</i> $C_2 = y$ $\angle A = x$ [AO = OC = r] $\angle D = x + y$ [OD = OC = r] $\angle AOC = 180^\circ - 2x$ [Sum of angles of / <i>Som v/d hoeke van</i> Δ] $\angle DOC = 180^\circ - (2x + 2y)$ [Sum of angles of / <i>Som v/d hoeke van</i> Δ] But / <i>Maar</i> $\angle AOD = \angle AOC - \angle DOC$ $\angle AOD = 2y$ $\angle AOD = 2 \angle ACD$ ✓ Construction / <i>konstruksie</i> ✓ A and D (in terms of x and y) with reasons <i>A en D in terme van x en y met redes</i> ✓ AOC (in terms of x) with a reason <i>AOC in terme van x met rede</i> ✓ equation for AOD / <i>gelykstel vir AOD</i> ✓ $\angle AOD = 2y$	(5)
9.2	9.2.1 MBC = MCB [MB = MC; radii] $\angle B_3 = C_2 = x$ [BO bisects / <i>halveer</i> MBC and / <i>en</i> OC bisects / <i>halveer</i> MCB] $\angle O_2 = 180^\circ - 2x$ [angles of / <i>hoeke van</i> Δ] ✓ reason / <i>rede</i> ✓ B_3 and / <i>en</i> $C_2 = x$ and reason / <i>en rede</i> ✓ answer / <i>antwoord</i>	(4)
9.2.2	$\angle A = \frac{1}{2} \text{MBC}$ [angle at centre = 2 x angle at circ / <i>middelpuntshoek = 2 x omtrekshoek</i>] $= \frac{1}{2}(180^\circ - 4x)$ [angles of / <i>hoeke van</i> Δ] $\angle A = 90^\circ - 2x$ Therefore / <i>Daarom</i>: $\angle ABO = O_2 - A$ [ext. angle of / <i>buitelshoek van</i> ΔOAB] $= (180^\circ - 2x) - (90^\circ - 2x)$ $= 90^\circ$ AO subtends a right angle / <i>AO onderspan 'n reghoek</i> $\therefore$ AO is a diameter / <i>AO is 'n middellyn</i> ✓ $\angle A = \frac{1}{2} \text{MBC}$ (statement) and reason. <i>(stelling) en rede</i> ✓ substitution and answer / <i>vervanging en antwoord</i> ✓ $\angle ABO = O_2 - A$ (statement) and reason <i>(stelling) en rede</i> ✓ substitution / <i>vervanging</i> ✓ answer / <i>antwoord</i>	(5)

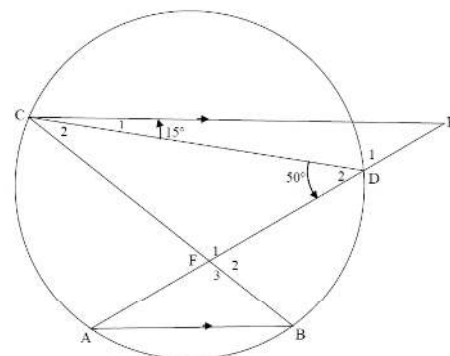


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9.3	In ΔEHO, $OE = x$ and $OH = x - 2$ $EH^2 = x^2 - (x - 2)^2$ [Theorem of Pythagoras] $EH^2 = x^2 - x^2 + 4x - 4$ $= 4(x - 1)$ $\therefore EH = 2\sqrt{x - 1}$ $\therefore EF = 4\sqrt{x - 1}$ line from centre of circle ... / <i>lyn van middelpunt van sirkel ...</i>	✓ $OH = x - 2$ ✓ Theorem of <i>Stelling</i> van Pythagoras ✓ EH ✓ EF [with reason/<i>met rede</i>]	(4)
[19]			

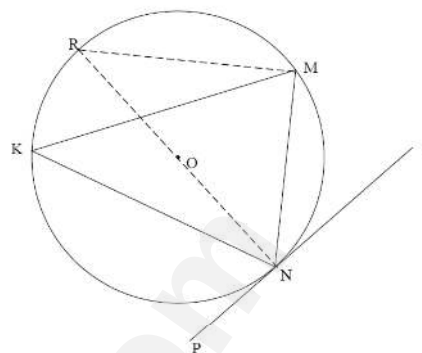
QUESTION/PRAAG 8



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8.1.1	$\hat{E} = 50^\circ - 15^\circ = 35^\circ$ [ext $\angle$ of Δ buite $\angle$ van Δ] $\hat{A} = 35^\circ$ [alt $\angle$ s / verwiss $\angle$ e; CE $\parallel$ AB] OR/OF $\hat{E} = 180^\circ - (130^\circ + 15^\circ) = 35^\circ$ [str line: $\angle$ s of Δ rt lyn; $\angle$ e van Δ] $\hat{A} = 35^\circ$ [alt $\angle$ s / verwiss $\angle$ e; CE $\parallel$ AB]	$\checkmark$ S $\checkmark$ S $\checkmark$ R	(3)
	OR/OF $\hat{B} = 50^\circ$ [$\angle$ s in same segment/ $\angle$ e in dieselfde segment] $\hat{C}_2 + 15^\circ = 50^\circ$ [alt $\angle$ s / verwiss $\angle$ e; CE $\parallel$ AB] $\therefore \hat{C}_2 = 35^\circ$ $\hat{A} = 35^\circ$ [$\angle$ s in same segment/ $\angle$ e in dieselfde segment]	$\checkmark$ S $\checkmark$ S $\checkmark$ R	(3)
8.1.2	$\hat{C}_y = 35^\circ$ [$\angle$ s in same segment/ $\angle$ e in dieselfde segment]	$\checkmark$ S $\checkmark$ R	(2)
8.2	$\hat{C}_2 = \hat{E}$ [from 8.1.1 and 8.1.2] $\therefore$ CF is a tangent to the circle [converse tan chord theorem] $\therefore$ CF is 'n raaklyn aan die sirkel [omgekeerde raakl koordst]	$\checkmark$ S $\checkmark$ R	(2) (7)

QUESTION/VR44G 11



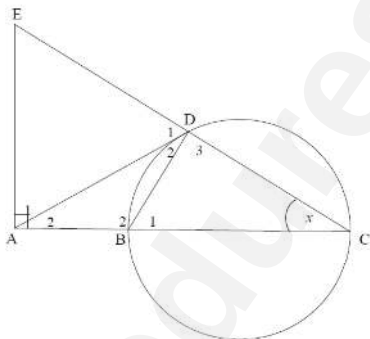
11.1	Construction: Draw diameter NR and draw RM <i>Konstruksie: Trek middellyn NR en verbind RM</i> ONM + MNQ = 90° [radius ⊥ tangent/raaklyn] MNR = 90° [∠ in semi circle/semi-sirkel] ∴ MRN = 180° - (90° + 90° - MNQ) [sum ∠s Δ] = MNQ but MRN = MKN [∠s same segment/∠e dieselfde segment] ∴ MNQ = K	✓ construction ✓ S/R ✓ S/R ✓ S ✓ S/R
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(5)



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11.2



11.2.1(a)	Angle in a semi circle/ <i>Hoek in halfsirkel</i>	✓ R	(1)
11.2.1(b)	Exterior $\angle$ of quad = opp interior $\angle$ / <i>Buite $\angle$ van vierh = teenoorst binne $\angle$</i> OR/OF Opp $\angle$ s of quad supplementary/ <i>Teenoorst $\angle$e van vierh is supplementêr</i>	✓ R	(1)
11.2.1(c)	tangent chord theorem/ <i>raaklyn koord stelling</i>	✓ R	(1)



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11.2.2(a)	In $\triangle AEC$ $\hat{E} = 180^\circ - (90^\circ + x)$ [sum $\angle$s Δ] $= 90^\circ - x$ $\hat{D}_1 = 180^\circ - (90^\circ + x)$ [$\angle$s on a straight line] $= \hat{E} = 90^\circ - x$ $\therefore AD = AE$ [sides opp = $\angle$s / sye teenoor = $\angle$e]	✓ S ✓ S ✓ R	(3)
11.2.2(b)	In $\triangle ADB$ and $\triangle ACD$ $\hat{A}_1 = \hat{A}_2$ [common] $\hat{D}_1 = \hat{C}$ [proven] $\hat{B}_1 = \hat{D}_2 + \hat{D}_3$ [sum $\angle$s Δ] $\therefore \triangle ADB \parallel \triangle ACD$ OR/OF In $\triangle ADB$ and $\triangle ACD$ $\hat{A}_1 = \hat{A}_2$ [common] $\hat{D}_1 = \hat{C}$ [proven] $\therefore \triangle ADB \parallel \triangle ACD$ [$\angle$, $\angle$, $\angle$]	✓ S ✓ S ✓ S ✓ S ✓ R	(3)
11.2.3(a)	$\frac{AD}{AC} = \frac{AB}{AD}$ [Δs] $AD^2 = AC \cdot AB$ $= 3r \times r$ $= 3r^2$	✓ ratio ✓ substitution	(2)
11.2.3(b)	$AD = AE = \sqrt{3}r$ [from 11.2.2(a) & 11.2.3(a)] $AB = r$ and $BC = 2r \therefore AC = 3r$ <u>In $\triangle ACE$:</u> $\tan \hat{E} = \frac{AC}{AE}$ $= \frac{3r}{\sqrt{3}r} = \sqrt{3}$ $\therefore \hat{E} = 60^\circ$ $\therefore \hat{D}_1 = 60^\circ$ [from 11.2.2(a)] $\therefore \hat{A}_1 = 60^\circ$ [$\angle$s of $\Delta = 180^\circ$] $\therefore \triangle ADE$ is equilateral/is gelyksydig	✓ AC ito r ✓ trig ratio ✓ simplification ✓ all 3 $\angle$s = 60°	(4)



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OR/OF $\frac{AD}{AC} = \frac{DB}{CD} \quad [\Delta s]$ $\frac{\sqrt{3}r}{3r} = \frac{DB}{CD}$ $\tan x = \frac{1}{\sqrt{3}}$ $\therefore \text{In } \triangle BDC: x = 30^\circ$ $\therefore \hat{E} = 60^\circ$ $\therefore \hat{D}_1 = 60^\circ \quad [\text{from 11.2.2(a)}]$ $\therefore \hat{A}_1 = 60^\circ \quad [\angle s \text{ of } \Delta = 180^\circ]$ $\therefore \triangle ADE \text{ is equilateral/is gelyksydig}$ OR/OF $\frac{AD}{AC} = \frac{DB}{CD} \quad [\Delta s]$ $\frac{\sqrt{3}r}{3r} = \frac{DB}{CD} \quad \therefore BD = \frac{CD}{\sqrt{3}}$ $DC^2 = BC^2 - DB^2$ $= 4r^2 - \frac{CD^2}{3}$ $3DC^2 = 12r^2 - CD^2$ $4CD^2 = 12r^2$ $DC = \sqrt{3}r$	$\checkmark \frac{\sqrt{3}r}{3r} = \frac{DB}{CD}$ $\checkmark \frac{1}{\sqrt{3}} = \tan x$ $\checkmark x = 30^\circ$ $\checkmark \text{all 3 } \angle s = 60^\circ$ $\checkmark BD = \frac{CD}{\sqrt{3}}$ $\checkmark DC = \sqrt{3}r$	(4)
$EC^2 = EA^2 + AC^2$ $= 3r^2 + 9r^2$ $EC = 2\sqrt{3}r$ $\therefore ED = EC - DC$ $= \sqrt{3}r$ $\therefore ED = EA = AD$ $\therefore \triangle ADE \text{ is equilateral/is gelyksydig}$	$\checkmark EC = 2\sqrt{3}r$ $\checkmark ED = EA = AD$	(4) [20]

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QUESTION / VRAAG 8

	8.1 $\hat{O}_2 = 50^\circ$ $\hat{D}_1 = 25^\circ$ $\angle s$ around a point / $\angle e$ om 'n punt $\angle$ centre = $2 \times \angle$ at circumference midpts $\angle = 2 \times \text{ontreks } \angle$	$\checkmark$ S $\checkmark$ S $\checkmark$ R	(3)
8.2 $\hat{B}_1 = 25^\circ$	tan chord theorem / raaklyn koördstelling	$\checkmark$ S $\checkmark$ R	(2)
8.3 $\hat{BCD} = 120^\circ$ $\hat{B}_2 = 35^\circ$ $\hat{OBC} = \hat{OCB} = 65^\circ$ $\therefore \hat{B}_1 = 65^\circ - 35^\circ$ $\hat{B}_1 = 30^\circ$	opp $\angle s$ of a cyclic quad / teenoorst $\angle e$ v kwh sum of $\angle s$ of a triangle / som $\angle e$ v Δ $\angle s$ opp. equal radii / $\angle e$ teenoor gelyke radiusse	$\checkmark$ S / R $\checkmark$ S $\checkmark$ S	$\checkmark$ answer / antwoord
OR / OF $\hat{BCD} = 120^\circ$ $\hat{B}_2 = 35^\circ$ $\hat{B}_1 + \hat{B}_2 + \hat{B}_3 = 90^\circ$ $\hat{B}_1 = 30^\circ$	OR / OF opp $\angle s$ of a cyclic quad / teenoorst $\angle e$ v kwh sum of $\angle s$ of a triangle / som $\angle e$ v Δ radius $\perp$ tangent / radius $\perp$ raaklyn	$\checkmark$ S / R $\checkmark$ S $\checkmark$ S	$\checkmark$ answer / antwoord

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QUESTION / VRAAG 8

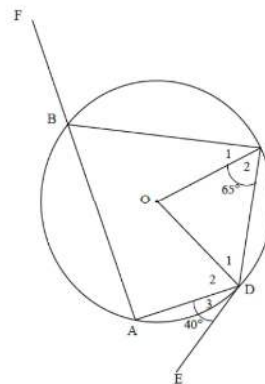
8.1.1 $\hat{H}_2 = 35^\circ$ [tan chord theorem / tan koördteorie] $\hat{E}_4 = \hat{H}_2 = 35^\circ$ [$\angle$ opp = sides / $\angle e$ teenoor = sye]	$\checkmark$ S $\checkmark$ S / R	(3)
8.1.2 $\hat{EKH} = 180^\circ - 2(35^\circ)$ [sum of $\angle$'s in Δ / som van $\angle e$ in Δ] $= 110^\circ$	$\checkmark$ S $\checkmark$ answer / antwoord	(2)
8.1.3 $\hat{G} = 180^\circ - 110^\circ$ [opp $\angle$ of cyclic quad / $\angle e$ teenoor van kwh] $= 70^\circ$	$\checkmark$ S $\checkmark$ R	(2)
8.1.4 $\hat{O}_1 = 220^\circ$ [$\angle$ at centre = $2 \times \angle$ at circumference / midpts $\angle = 2 \times \text{ontreks } \angle$]	$\checkmark$ S $\checkmark$ R	(2)
8.2 $(x-4)^2 + 12^2 = x^2$ [Pythagoras] $x^2 - 8x + 16 + 144 = x^2$ $-8x = -160$ $x = 20$	$\checkmark$ S / R $\checkmark$ expansion / uitbreiding $\checkmark$ simplification / vereenvoudiging $\checkmark$ answer / antwoord	(4) [13]

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QUESTION 8

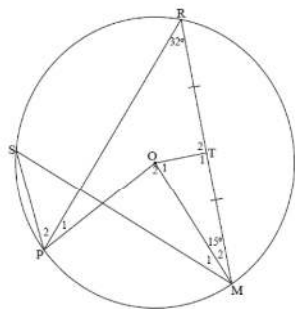
8.1



8.1.1 $\hat{D}_2 + \hat{D}_3 = 90^\circ$ (tan $\perp$ radius) $\therefore \hat{D}_2 = 90^\circ - 40^\circ = 50^\circ$	$\checkmark$ S $\checkmark$ R A $\checkmark$ answer	(3)
8.1.2 $\hat{D}_1 = \hat{C}_2 = 65^\circ$ (angles opposite equal sides) $\hat{D}_1 + \hat{D}_2 = \hat{FBC}$ (ext angle of a cyclic quad) $65^\circ + 50^\circ = \hat{FBC}$ $\therefore \hat{FBC} = 115^\circ$	$\checkmark$ S / R $\checkmark$ S $\checkmark$ R CA $\checkmark$ answer	(4)

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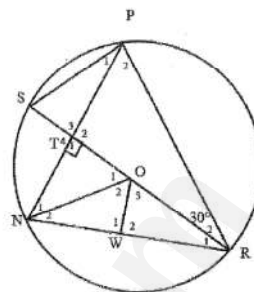




8.2.1	$\hat{S} = 32^\circ$ ($\angle$ s in the same segment)	A✓S A✓R	(2)
8.2.2	$\hat{O}_2 = 64^\circ$ ($\angle$ at centre = $2 \times \angle$ at circumference)	A✓S A✓R	(2)
8.2.3	$\hat{T}_1 = 90^\circ$ (line from centre to midpoint of chord) $\hat{O}_1 + \hat{T}_1 + \hat{M}_2 = 180^\circ$ (sum of angles of Δ) $\hat{O}_1 + 90^\circ + 15^\circ = 180^\circ$ $\hat{O}_1 = 75^\circ$	A✓S/R A✓Method A✓answer	(3) [14]



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8.1.1	$\hat{P}_1 + \hat{P}_2 = 90^\circ$ ($\angle$ in semi-circle/ $\angle$ in semi-sirkel) $\therefore \hat{S} = 60^\circ$ ($\angle$'s of Δ / $\angle$ 'e van Δ)	✓S/R ✓S	(3)
8.1.2	$\hat{N}_1 + \hat{N}_2 = \hat{S} = 60^\circ$ ($\angle$'s in same segment/ $\angle$ 'e in dieselfde segment) $\therefore \hat{R}_1 = 30^\circ$ ($\angle$'s of Δ TNR/ $\angle$ 'e van Δ TNR)	✓S/R ✓S	(3)
8.1.3	$\hat{N}_1 = \hat{R}_1 = 30^\circ$ ($\angle$'s opp equal sides/ $\angle$ 'e teenoor gelyke sye) But $\hat{N}_1 + \hat{N}_2 = 60^\circ$ $\therefore \hat{N}_2 = 30^\circ$	✓S/R ✓S	(2)
8.2	$\hat{W}_2 = 90^\circ$ (line from centre to midpoint/ lyn uit midpt van sirkel na middelpunt van koord) $\hat{T}_1 = \hat{W}_2$ ($= 90^\circ$) $\therefore$ INWO is a cyclic quadrilateral/ Δ 'n koordevierhoek (CONVERSE of exterior $\angle$ of a cyclic quad/ OMGEKEERDE van buitehoek van 'n koordevierhoek)	✓S/R ✓S ✓R	(4) [12]



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QUESTION 8

8.1			
8.1.1	$\hat{A}_2 = 20^\circ$ [equal chords, equal angles/ gelyke koorde; gelyke hoeke]	✓S ✓R	(2) K
8.1.2	$\hat{C}_2 = 90^\circ$ [$\angle$ in semi-circle/ $\angle$ in semi-sirkel]	✓S ✓R	(2) K
8.1.3	$\hat{D} = 70^\circ$ [sum $\angle$ s Δ /som $\angle$ e Δ] $\hat{A}\hat{B}\hat{C} = 110^\circ$ [opp $\angle$ cyclic quad/ teenoorst. $\angle$ v kvh]	✓S & R ✓S ✓R	(3) R



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8.2			
8.2.1	$\hat{A} = x$ [$\angle$ at centre = $2 \angle$ at circuml/ midpts $\angle = 2$ omtreks $\angle$] $\hat{D} = x$ [$\angle$ s in same segment/ $\angle$ e in dieselfde segment] $\hat{C}_2 = x$ [$\angle$ s opp = radii/ $\angle$ e teenoor = radiusse]	✓S ✓R ✓S ✓R ✓S ✓R	(6) R
8.2.2	$\hat{M}_2 = 90^\circ$ [Line from centre to midpoint of chord/ lyn vanaf midpt na midpt van koord] $\hat{O}_2 = 90^\circ - x$ [sum $\angle$ s Δ /som $\angle$ e Δ]	✓S ✓R ✓R	(3) C
8.2.3	$\hat{O}_2 = 90^\circ - x$ [proven/ reeds bewys] $\hat{O}\hat{C}\hat{E} = 90^\circ$ [radius $\perp$ tangent/raaklyn] $\hat{E} = x$ [sum $\angle$ s Δ /som $\angle$ e Δ] $\therefore \hat{E} = \hat{D} = x$ $\therefore$ DOCE is a cyclic quadrilateral. [converse theorem: angles in the same segment] $\therefore$ DOCE is 'n kvh [omgekeerde stelling: hoeke in dies segment.]	✓S ✓R ✓S & R ✓R	(4) C

[20]



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QUESTION 8

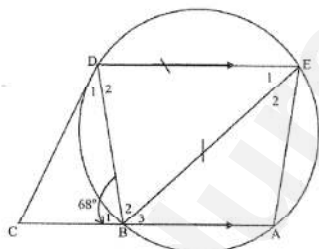
8.1		
8.1	$\hat{D}_1 = 33^\circ$ ($\angle$'s in same circle segment)	$\checkmark$ S $\checkmark$ R (2)
8.2	In ΔDAE : $\hat{E}_1 = 90^\circ$ (line from centre to midpt of chord) $\hat{D}_1 = \hat{C} = 33^\circ$ ($\angle$'s in same circle segment) $\therefore \hat{A}_1 = 57^\circ$ ($\angle$'s of Δ) OR In ΔEBC : $\hat{E}_3 = 90^\circ$ (line from centre to midpt of chord) $\hat{C} = 33^\circ$ (given) $\therefore \hat{B}_1 = 57^\circ$ ($\angle$ s of Δ) $\therefore \hat{B}_1 = \hat{A}_1 = 57^\circ$ ($\angle$'s in same circle segment)	$\checkmark$ S $\checkmark$ R (3) $\checkmark$ S (3)
8.3	$\hat{O}_1 = 2\hat{A}_1 = 114^\circ$ ($\angle$ at center = $2 \times \angle$ at circumference)	$\checkmark$ S $\checkmark$ R (2)



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QUESTION/VRAAG 8



8.1.1	$\hat{D}EA = 68^\circ$ [ext. $\angle$ cyclic quad / buite $\angle$ van kvh]	$\checkmark S \checkmark R$	(2)
8.1.2	$\hat{A} = 112^\circ$ [co-interior $\angle$ s / $ko - binne \angle$ e; $DE \parallel CA$]	$\checkmark S/R$	(1)
8.1.3	$\hat{D}_2 = 68^\circ$ [opp $\angle$ s of cyclic quad / teenoorst. $\angle$ van kvh] OR/OF $\hat{D}_2 = 68^\circ$ [alt $\angle$ s / <i>verwiss</i> $\angle$ e; $DE \parallel CA$]	$\checkmark S \checkmark R$	(2)
8.1.4	$\hat{B}_2 = 68^\circ$ [$\angle$ s opp. = sides / $\angle$ e teenoor = sye]	$\checkmark S/R$	(1)
8.1.5	$\hat{E}_1 = 44^\circ$ [sum of $\angle$ s of Δ /som van binne $\angle$ e van Δ] $\hat{E}_1 = 44^\circ$ [tangent chord theorem / raaklyn – koordstelling]	$\checkmark S/R$	
		$\checkmark S \checkmark R$	(3)
8.2	$\hat{C} = 68^\circ$ [sum of $\angle$ s of Δ /som van binne $\angle$ e van Δ] But / maar $\hat{B}_1 = 68^\circ$ [given / gegee] $DC = DB$ [sides opp. = $\angle$ s / sye teenoor = $\angle$ e] $\therefore \Delta BDC$ is isosceles / <i>gelykbenig</i>	$\checkmark S/R$	(2)
8.3	$\hat{D}_3 = \hat{C} = 68^\circ$ $\therefore DE$ is a tangent / 'n raaklyn [converse tangent chord theorem / omgekeerde raaklyn – koordstelling]	$\checkmark S$	(2)
		$\checkmark R$	(1)

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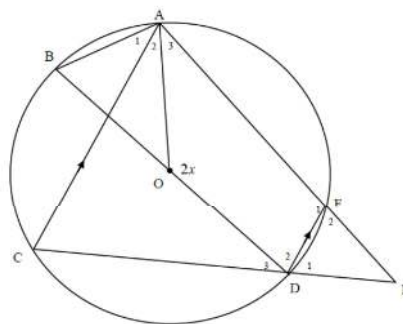
8.4	$\hat{D}_1 + \hat{D}_2 = \hat{A}_1 = 57^\circ$ ($\angle s$ opposite=sides) But $\hat{D}_1 = \hat{C} = 33^\circ$ ($\angle s$ in same circle segment) $\therefore \hat{D}_2 = 24^\circ$ OR $\hat{D}\hat{O}C = 2 \times \hat{B}_1$ ($\angle @centre = 2 \times \angle @circum$) $\hat{D}\hat{O}C = 2 \times 57^\circ$ $= 114^\circ$ $\hat{D}\hat{O}C = \hat{E}_2 + \hat{D}_2$ (ext $\angle$ of Δ) $114^\circ = 90^\circ + \hat{D}_2$ $\therefore \hat{D}_2 = 24^\circ$	$\checkmark S/R$ $\checkmark S$ $\checkmark S/R$ $\checkmark S$
8.5	$\hat{B}_1 + \hat{B}_2 = 90^\circ$ ($\angle$ in semi-circle) $\hat{A}_2 = 57^\circ$ ($\angle s$ of Δ) OR $\triangle ADE \equiv \triangle ABE$ (SAS) $\therefore \hat{A}_1 = \hat{A}_2 = 57^\circ$	$\checkmark S \quad \checkmark R$ $\checkmark S$ $\checkmark S \quad \checkmark R$ $\checkmark S$



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QUESTION/VRAG 9



9.1.1	$\hat{C} = x$ [$\angle$ at centre = $2 \times \angle$ at circumference / midpts $\angle = 2 \times \text{omtreks } \angle$] $\hat{B} = x$ [$\angle$ s in same segment / $\angle$ e in dieselfdesegment] $\hat{D}_1 = x$ [corresp $\angle$ s / ooreenkomstige $\angle$ e; $AC \parallel DF$] $B\hat{A}F = x$ [exterior $\angle$ of cyclic quad / buite $\angle$ kdvh]	$\checkmark$ S R $\checkmark$ S R $\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R
9.1.2	$\hat{D}_1 = \hat{F}_1 = x$ $\therefore DE = FE$ [$\angle$'s opp - sides / $\angle$ e teenoor - sye] $\hat{E} = 180^\circ - 2x$ [sum of $\angle$'s in Δ / som van $\angle$'s in Δ]	$\checkmark \hat{D}_1 = \hat{F}_1$ $\checkmark \hat{E} = 180^\circ - 2x$
9.1.3	$\hat{E} = 180^\circ - 2x$ [sum of $\angle$'s in Δ / som van $\angle$'s in Δ] $A\hat{O}D = 2x$ $\therefore AODE$ is a cyclic quadrilateral / is 'n kdvh [converse of a cyclic quad / omgeek van kdvh]	$\checkmark$ S $\checkmark$ R
9.2	$\frac{AF}{FE} = \frac{CD}{DE}$ [prop theorem / eweredigh] $\frac{12}{8} = \frac{AF}{10}$ $AF = 15$	$\checkmark$ S $\checkmark$ R $\checkmark$ answer / antwoord (3)

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QUESTION / VRAAG 9

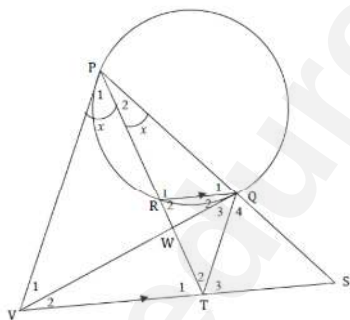
9.1	Equal / gelyk	✓ answer / antwoord	(1)
9.2			
9.2.1	$\hat{D}_1 = \hat{F}_1 = x$ $\hat{F}_1 = \hat{F}_2 = x$	tan chord theorem / raaklyn koordstelling = chords subtend = $\angle s$ = koorde onderstaan = $\angle e$	✓ S / R ✓ S ✓ R (3)
9.2.2	$\hat{F}_2 = \hat{A} = x$ $\hat{D}_1 = \hat{A} = x$ ABDC is a cyclic quad / ABDC is 'n kvh	ext. $\angle$ of cyclic quad / buite $\angle$ v kvh ext $\angle$ = opp int $\angle$ OR converse of ext. $\angle$ of cyclic quad / buite $\angle$ = oorst binne $\angle$ OF omgekeerde buite $\angle$ v kvh	✓ S ✓ R ✓ R (3)
9.2.3	$\hat{B}_1 + \hat{B}_2 = \hat{A}$ $\hat{A} = \hat{D}_1$ $\hat{B}_1 + \hat{B}_2 = \hat{D}_1$ $BE \parallel CD$	tan chord theorem / raaklyn koordstelling proved / reeds bewys correspond $\angle s$ = / ooreenkomst $\angle e$ =	✓ S ✓ R (2)



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QUESTION 9



9.1	$\hat{P}_1 = \hat{Q}_1 = x$ (tan chord theorem) $\hat{Q}_1 = \hat{S} = x$ (corresp $\angle$'s (PQ/VS)) $\hat{S} = x$	A✓ S A✓ R A✓ S A✓ R	(4)
9.2	$\hat{P}_2 = \hat{Q}_2$ (tan chord theorem) $\hat{Q}_2 = \hat{V}_2$ (alt $\angle$'s (PQ/VS)) $\therefore \hat{P}_2 = \hat{V}_2$ $\therefore$ PQTV is a cyclic quadrilateral (converse of angles in the same seg.)	A✓ S A✓ R A✓ S A✓ R A✓ R	(5)
9.3	$\hat{Q}_3 = \hat{P}_1$ ($\angle$'s in the same segment) $\therefore \hat{Q}_3 = \hat{P}_2$ $\therefore$ TQ is a tangent (converse: tan chord theorem)	A✓ S A✓ R A✓ R	(3)
[12]			



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QUESTION/VRAAG 9

9.2.4	$\hat{C}_1 + \hat{C}_2 + \hat{F}_1 + \hat{F}_2 = 180^\circ$ $\hat{C}_1 = \hat{C}_2$ $\hat{F}_1 = \hat{F}_2$ $2\hat{C}_1 + 2\hat{F}_2 = 180^\circ$ $\hat{C}_1 + \hat{F}_2 = 90^\circ$ $\hat{E}_1 = 90^\circ$	opp $\angle$'s of a cyclic quad / teenoorst $\angle e$ v kvh diag rhombus bisect $\angle$ / diag ruit halveer $\angle$ proved / reeds bewys sum of $\angle$'s of Δ / som van $\angle e$ v Δ FC is a diameter of circle FDCE FC is 'n middellyn van sirkel FDCE. OR / OF Let $\hat{F}_1 = \hat{F}_2 = x$ $\hat{C} = 180^\circ - 2x$ $\hat{C}_1 = \hat{C}_2 = 90^\circ - x$ In Δ FDC or / of Δ EFC $\hat{D} = 90^\circ$ or / of $\hat{E} = 90^\circ$ FC is a diameter of circle FDCE. FC is 'n middellyn van sirkel FDCE.	✓ S ✓ R ✓ S ✓ S ✓ R OR / OF ✓ S ✓ R ✓ S ✓ S ✓ R (5)
[14]			

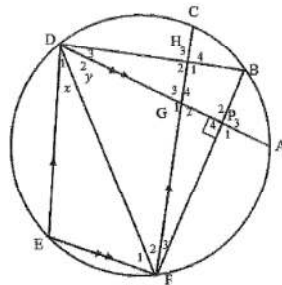


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QUESTION/VRAAG 9

9.1



9.1.1	$\hat{E} = 180^\circ - (x + y)$ (co-interior $\angle$'s, $AD \parallel FE$ / ko-binne $\angle$ 'e; $AD \parallel FE$) $\therefore \hat{B} = x + y$ (opposite $\angle$'s of cyclic quadrilateral / teenoorstaande $\angle$ 'e van koordevierhoek)	✓ S ✓ R ✓ S ✓ R (4)
9.1.2	DEFG a parm (both pair of opposite sides $\parallel$ / albei pare teenoorstaande sye $\parallel$) $\hat{G}_1 = 180^\circ - (x + y)$ (opposite angles of parallelogram / teenoorst $\angle$ 'e van parm) $\hat{G}_2 = \hat{G}_1 = 180^\circ - (x + y)$ (vertically opp $\angle$'s / s'leigoorst $\angle$ 'e) $\hat{G}_4 + \hat{B} = 180^\circ$ HPBG is a cyclic quad / HPBG is 'n koordevierhoek (CONVERSE: opposite $\angle$'s of cyclic quad / OMGEKEERDE: teenoorst $\angle$ 'e van 'n koordevierhoek) $\therefore \hat{H}_1 = \hat{P}_1$ (exterior $\angle$ of cyclic quad / buite $\angle$ van 'n koordevierhoek)	✓ S / R ✓ S ✓ S ✓ S ✓ S ✓ R (6)



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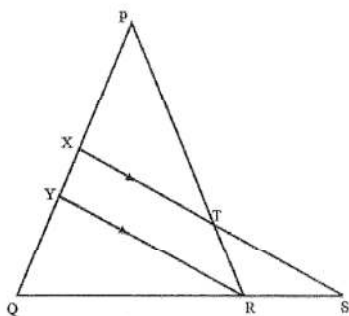
247

**GRADE 12
MATHEMATICS
REVISION**

248



9.2



9.2	$\frac{PT}{TR} = \frac{PX}{XY} = \frac{3}{2}$ (proportionality theorem, $XS \parallel YR$) Ewerdigheidstelling, $XS \parallel YR$	✓S/R
	$\frac{PX}{XQ} = \frac{2}{5}$ $\therefore \frac{PX}{XY} = \frac{3}{2} = \frac{6}{4}$ and/ en $\frac{PX}{XQ} = \frac{6}{15}$ $PX : XY : YQ = 6 : 4 : 11$	✓S ✓manipulation to get equal numerators /manipulasie om tellers dieselfde te kry
	$\therefore \frac{QR}{RS} = \frac{OY}{YX}$ (proportionality theorem, $XS \parallel YR$) Ewerdigheidstelling, $XS \parallel YR$	✓S
	$\therefore \frac{QR}{RS} = \frac{11}{4}$	✓answer (6)

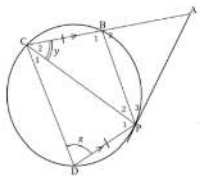
[16]



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9.2



9.2.1	In $\triangle APC$ and / en $\triangle ABP$: 1) $\hat{A} = \hat{A}$ [common $\angle$; gemene $\angle$] 2) $\hat{P}_2 = \hat{C}_2 = y$ [tangent chord theorem/ raaklyn koord stelling] 3) $\hat{APC} = \hat{B}_2$ [sum $\angle$ s Δ /som $\angle$ e Δ] $\therefore \triangle APC \parallel \triangle ABP$ OR In $\triangle APC$ and / en $\triangle ABP$: 1) $\hat{A} = \hat{A}$ [common $\angle$; gemene $\angle$] 2) $\hat{P}_2 = \hat{C}_2 = y$ [tangent chord theorem/ raaklyn koord stelling] $\therefore \triangle APC \parallel \triangle ABP$ ($\angle$; $\angle$; $\angle$)	✓S & R ✓S ✓R ✓S & R ✓S & R ✓S ✓R ✓R ($\angle$; $\angle$; $\angle$)
OR		
	In $\triangle APC$ and / en $\triangle ABP$: 1) $\hat{A} = \hat{A}$ [common $\angle$; gemene $\angle$] 2) $\hat{B}_2 = \hat{D} = x$ [ext $\angle$ of cyc quad / buite $\angle$ koordevh.] $\hat{APC} = \hat{D} = x$ [tangent chord theorem/ raaklyn koord stelling] $\therefore \hat{APC} = \hat{B}_2$ 3) $\hat{P}_2 = \hat{C}_2$ [sum $\angle$ s Δ /som $\angle$ e Δ] $\therefore \triangle APC \parallel \triangle ABP$ OR In $\triangle APC$ and / en $\triangle ABP$: 1) $\hat{A} = \hat{A}$ [common $\angle$; gemene $\angle$] 2) $\hat{B}_2 = \hat{D} = x$ [ext $\angle$ of cyc quad / buite $\angle$ koordevh.] $\hat{APC} = \hat{D} = x$ [tangent chord theorem/ raaklyn koord stelling] $\therefore \hat{APC} = \hat{B}_2$ $\therefore \triangle APC \parallel \triangle ABP$ ($\angle$; $\angle$; $\angle$)	✓S & R ✓S & R ✓S & R ✓S & R ✓S ✓R ✓S & R

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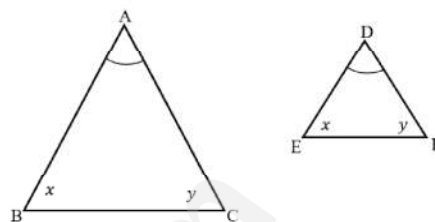


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QUESTION 9

9.1



Constr: On sides AB and AC of $\triangle ABC$, mark points G and H respectively such that $AG = DE$ and $AH = DF$. Draw GH/Merk punt G en H op sy AB en AC van $\triangle ABC$ onderskeidelik af sodanig dat $AG = DE$ en $AH = DF$. Trek GH.	✓ construction/ konstruksie	B/D if no construction
Proof/Bewys: $\triangle AGH = \triangle DEF$ [s, $\angle$, s] $\therefore \hat{AGH} = \hat{E}$ $\hat{AGH} = \hat{B}$ [$\hat{B} = \hat{E}$, given/gegee] $\therefore GH \parallel BC$ [corresp/ooreenk $\angle$ =] $\therefore \frac{AG}{AB} = \frac{AH}{AC}$ [line $\parallel$ one side of Δ] or [prop theorem: $GH \parallel BC$] [lyn $\parallel$ een sy van Δ] of [eweredigheit: $GH \parallel BC$] $\therefore \frac{DE}{AB} = \frac{DF}{AC}$ [constr/konstruksie]	✓ S/R ✓ S ✓ S/R ✓ S ✓ R	(6) K



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9.2.2	$\frac{AP}{AB} = \frac{AC}{AP}$ $\triangle APC \parallel \triangle ABP$ $\therefore AP^2 = AB \cdot AC$	✓ S	(1) K
9.2.3	In $\triangle APC \parallel \triangle CDP$ 1) $\hat{P}_1 = y$ [alt. $\angle$ s/ verwis. $\angle$ e; $CB \parallel DP$] 2) $\hat{D} = \hat{CPA} = x$ [tangent chord theorem/ raaklyn koord stelling] 3) $\hat{A} = \hat{C}_1$ [sum $\angle$ s Δ /som $\angle$ e Δ] $\therefore \triangle APC \parallel \triangle CDP$ OR In $\triangle APC \parallel \triangle CDP$ 1) $\hat{P}_1 = y$ [alt. $\angle$ s/ verwis. $\angle$ e; $CB \parallel DP$] 2) $\hat{D} = \hat{CPA} = x$ [tangent chord theorem/ raaklyn koord stelling] $\therefore \triangle APC \parallel \triangle CDP$ ($\angle$; $\angle$; $\angle$)	✓ S & R ✓ S ✓ R ✓ S & R ✓ S & R ✓ S ✓ R ✓ R ($\angle$; $\angle$; $\angle$)	(4) R
9.2.4	$\frac{AC}{CP} = \frac{PC}{DP}$ [$\triangle APC \parallel \triangle CDP$] $\therefore PC^2 = DP \cdot AC$ $AP^2 + PC^2 = AB \cdot AC + AC \cdot DP$ $= AC(AB + DP)$ $= AC(AB + BC)$ [$DP = BC$] $= AC \cdot AC$ $= AC^2$	✓ S ✓ S ✓ common factor ✓ $= AC(AB + BC)$ answer given	(4) P

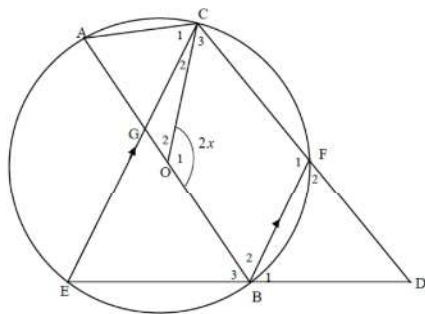
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QUESTION 9



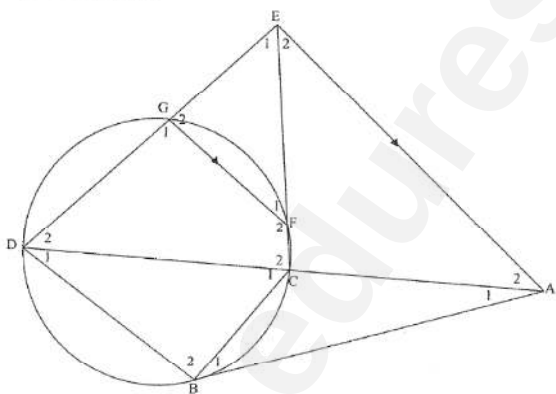
9.1	$\hat{E} = x$ ($\angle$ at centre = $2 \times \angle$ at circum) $\hat{F}_1 = 180^\circ - x$ (opposite $\angle$ s of cyclic quad) $\hat{F}_2 = x$ ($\angle$ s on straight line) OR $\hat{E} = x$ ($\angle$ at centre = $2 \times \angle$ at circum) $\hat{F}_2 = x$ (ext $\angle$ of cyclicquad)	$\checkmark S$ $\checkmark R$ $\checkmark S$ $\checkmark S$ $\checkmark R$ $\checkmark S/R$	 (3) (3)
9.2	$\hat{F}_2 = x$ (ext $\angle$ of cyclic quad OR straight line) $\hat{B}_1 = x = \hat{E}$ (corresponding $\angle$ s, EC BF) $\therefore \hat{F}_2 = \hat{B}_1 = x$ $\therefore DF = DB$ (sides opposite $= \angle$ s)	$\checkmark S$ $\checkmark R$ $\checkmark S$ $\checkmark R$	 (4)



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QUESTION/VRAAG 9



9.1	tangent-chord theorem / raaklyn-hoordsstelling	✓ R	
9.2	In $\triangle ABC$ and/en $\triangle ADB$: $\hat{A}_1 = \hat{A}_1$ [common/gemeenskaplik] $\hat{B}_1 = \hat{D}_1$ [proven/bewys in 9.1] $\therefore \triangle ABC \parallel \triangle ADB$ [$\angle$; $\angle$; $\angle$] OR/OF In $\triangle ABC$ and/en $\triangle ADB$: $\hat{A}_1 = \hat{A}_1$ [common/gemeenskaplik] $\hat{B}_1 = \hat{D}_1$ [proven/bewys in 9.1] $\hat{BCA} = \hat{ABD}$ [$\angle$s of $\hat{\Delta} = 180^\circ / \angle$e van $\hat{\Delta} = 180^\circ$] $\therefore \triangle ABC \parallel \triangle ADB$	✓ S ✓ S ✓ R ✓ S ✓ S ✓ S	(1)

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9.3	$\hat{A} = x \quad (\angle \text{at centre} = 2 \times \angle \text{at circum})$ $\hat{C}_1 + \hat{C}_2 = x \quad (\angle s \text{ opposite } = \text{sides})$ $\hat{C}_2 + \hat{C}_3 = x \quad (\text{ext. } \angle \text{ of cyclic quad})$ $\therefore \hat{C}_1 + \hat{C}_2 = \hat{C}_2 + \hat{C}_3$ $\therefore \hat{C}_1 = \hat{C}_3$ OR $\hat{A} = x \quad (\angle \text{at centre} = 2 \times \angle \text{at circum})$ $\hat{C}_1 + \hat{C}_2 = \hat{A} \quad (\angle s \text{ opposite } = \text{sides})$ $\hat{A} = \hat{F}_2 \quad (\text{exterior } \angle \text{ of cyclic quad})$ $\hat{F}_2 = \hat{C}_2 + \hat{C}_3 \quad (\text{corresponding } \angle s, CE \parallel BF)$ $\therefore \hat{C}_1 + \hat{C}_2 = \hat{C}_2 + \hat{C}_3$ $\therefore \hat{C}_1 = \hat{C}_3$	$\checkmark S$ $\checkmark S$ $\checkmark S \quad \checkmark R$ $\checkmark S$ $\checkmark R$ $\checkmark S \quad \checkmark R$
9.4	$CG = FB, CG \parallel BF \quad (\text{given})$ $\therefore CGBF \text{ is a parm} \quad (\text{one pair opp. sides equal and parallel})$ $BG = FC = 2 \quad (\text{opp. sides of parm.})$ $CF : FD = 2 : 1$ $\frac{BD}{ED} = \frac{FD}{CD} \quad (\text{line } \text{ to one side of } \Delta \text{ or proportionality theorem})$ $\frac{BD}{ED} = \frac{1}{3}$	$\checkmark R$ $\checkmark S/R$ $\checkmark S$ $\checkmark S/R$ $\checkmark \frac{BD}{ED} = \frac{1}{3}$



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9.3	$\hat{E}_2 = \hat{F}_1$ [alternate $\angle$ s / <i>verwys $\angle$e</i> ; EA GF] $\hat{F}_1 = \hat{D}_2$ {ext $\angle$ of cyc quad DGFC/huite $\angle$ v <i>kerkh DGFC</i> } $\therefore \hat{E}_2 = \hat{D}_2$	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R
9.4	In $\triangle AEC$ and/en $\triangle ADE$: $\hat{A}_2 = \hat{A}_2$ [common/ <i>gemeenskaplik</i>] $\hat{E}_2 = \hat{D}_2$ [proven/ <i>bewys</i> in 9.3] $\therefore \triangle AEC \parallel \triangle ADE$ [$\angle$; $\angle$; $\angle$] $\therefore \frac{AE}{AD} = \frac{AC}{AE}$ [from $\parallel$ <i>As / uit</i> $\parallel$ Δe] $\therefore AE^2 = AD \times AC$ $\therefore AE = \sqrt{AD \times AC}$ OR/OF In $\triangle AEC$ and/en $\triangle ADE$: $\hat{A}_2 = \hat{A}_2$ [common/ <i>gemeenskaplik</i>] $\hat{E}_2 = \hat{D}_2$ [proven/ <i>bewys</i> in 9.3] $\angle ACE = \angle AED$ [$\angle$ s of $\Delta = 180^\circ$ / <i>e van $\Delta = 180^\circ$</i>] $\therefore \triangle AEC \parallel \triangle ADE$ $\therefore \frac{AE}{AD} = \frac{AC}{AE}$ [from $\parallel$ <i>As / uit</i> $\parallel$ Δe] $\therefore AE^2 = AD \times AC$ $\therefore AE = \sqrt{AD \times AC}$	$\checkmark$ S $\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ S $\checkmark$ S $\checkmark$ S $\checkmark$ S $\checkmark$ S $\checkmark$ S $\checkmark$ S $\checkmark$ S $\checkmark$ S
9.5	$\frac{AB}{AD} = \frac{AC}{AB}$ [from $\parallel$ <i>As / uit</i> $\parallel$ Δe] from/van 9.2 $AB^2 = AD \times AC$ $= AE^2$ [from/van 9.4] $\therefore AB = AE$	$\checkmark$ S $\checkmark$ S $\checkmark$ S



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QUESTION / VRAAG 10

10.1

$\hat{A} = \hat{D}$ [given/gegee]
 $AQ = DF$ [construction/konstruksie]
 $AP = DE$ [construction/konstruksie]
 $\therefore \triangle APQ \cong \triangle DEF$ [SAS]
 $\hat{P}_1 = \hat{E}$
 $\hat{Q}_1 = \hat{F}$ [$\hat{E} = \hat{F}$]
 $PQ \parallel BC$ [corresp. $\angle$'s / ooreenkomstige $\angle$ 'e]
 $\frac{AP}{AB} = \frac{AQ}{AC}$ [$PQ \parallel BC$]
 But/maar $AP = DE$ and/en $AQ = DF$
 $\therefore \frac{DE}{AB} = \frac{DF}{AC}$ (5)

✓ construction/konstruksie
 ✓ S/R
 ✓ $\hat{P}_1 = \hat{E}$
 ✓ S/R
 ✓ S



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10.2.1

$\angle ADB = 90^\circ$ [$\angle$ in semi-circle / semi-sirkel]
 $\hat{A} = 90^\circ - y$ [$\angle$ in semi-circle / semi-sirkel]
 $\hat{B}_1 = \hat{E}$ [$\angle$ opp = sides / $\angle$ e teenoor = sye]
 $\hat{DCE} = \hat{A} = 90^\circ - y$ [ext. $\angle$ of a cyclic quad = int. opp. $\angle$ / verlengde $\angle$ van 'n Kdvh = binne teenoorg $\angle$]
 $\therefore \hat{D}_1 = 90^\circ$ [sum of $\angle$'s in Δ / som van $\angle$'s in Δ] (5)

10.2.2

$\hat{B}_1 = \hat{D}_2 = y$ [$\angle$ opp = sides / $\angle$ e teenoor = sye]
 $\hat{B}_1 = \hat{B}_2$ [given/gegee]
 $\hat{E} = \hat{D}_2$ [proven/bewys]
 $\therefore \triangle BOD \parallel \triangle BDE$ [$\angle \angle$] (3)

10.2.3

$\frac{DE}{OD} = \frac{BE}{BD}$ [$\parallel \Delta$]
 but/maar $BD = DE$ [given/gegee]
 $\therefore \frac{DE}{OD} = \frac{BE}{DE}$
 $DE^2 = BE \cdot OD$
 ✓ S
 ✓ R
 ✓ S
 ✓ substitution / vervanging (4)

[17]



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QUESTION / VRAAG 10

10.1

NB: NO construction 0 / 5 / GEEN konstruksie 0 / 5

On sides KL and LM of $\triangle KLM$ mark points A and B respectively such that $KA = GH$ and $KB = GI$. Draw AB.
 Op sye KL en LM van $\triangle KLM$ plaas A en B onderskeidelik sodat $KA = GH$ en $KB = GI$. Trek lyn AB.

Proof / Bewys
 In $\triangle KLM$ and $\triangle GHI$
 $KA = GH$ construction / konstruksie
 $\hat{K} = \hat{G}$ given / gegee
 $KB = GI$ construction / konstruksie
 $\therefore \triangle KLM \cong \triangle GHI$ S.S.S
 $\therefore \hat{A}_1 = \hat{H}$
 but $\hat{L} = \hat{L}$ given / gegee
 $\therefore \hat{A}_1 = \hat{H}$
 $\therefore AB \parallel LM$ corr. $\angle$'s = / ooreenkomst. $\angle$ 'e =
 $\frac{KL}{KA} = \frac{LM}{KB}$ line || one side Δ / lyn || een sy v Δ
 $\therefore \frac{KL}{GH} = \frac{LM}{GI}$ (5)

✓ construction / konstruksie
 ✓ S / R
 $\triangle KLM \cong \triangle GHI$ S.S.S
 ✓ S
 ✓ R
 ✓ S / R



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10.2

10.2.1

$\hat{R}_1 = 60^\circ$ equilateral Δ / gelyksydige Δ
 $\hat{W}_1 = \hat{P}_1 + \hat{Q}_1$ ext. $\angle$ of a Δ / buite $\angle$ v Δ
 $= 60^\circ + \hat{Q}_1$
 $= \hat{R}_1 + \hat{Q}_1$
 $\hat{Q}_1 = \hat{R}_2$ $\angle$'s in the same segment / $\angle$ 'e in dieselfde segment
 $\therefore \hat{W}_1 = \hat{T}_1$ (3)

10.2.2

In $\triangle TQR$ and $\triangle QRV$
 1. $\hat{W}_1 = \hat{T}_1$ proved / reeds bewys
 2. $\hat{R}_1 = \hat{T}_2$ equilateral Δ / gelyksydige Δ
 3. $\hat{Q}_1 = \hat{T}_1$ sum $\angle$'s of Δ / som van $\angle$ 'e v Δ
 $\therefore \triangle TQR \parallel \triangle QRV$ (3)

✓ S
 ✓ S
 ✓ S / R
 ✓ S
 ✓ S
 ✓ R

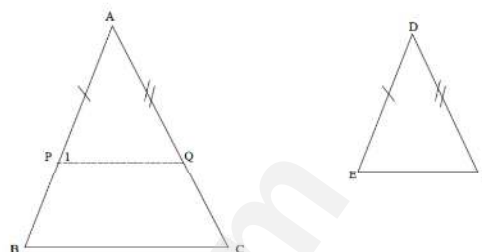


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10.2.3	In ΔTPV and / en ΔWQR 1. $\hat{P}\hat{Q}R = \hat{R}_1$ both 60° / <i>albei</i> 60° ✓ S $\hat{P}\hat{Q}R = \hat{V}_1$ ext. $\angle$ of a cyclic quad. / <i>buite</i> $\angle$ v kvi ✓ S / R $\hat{V}_1 = \hat{R}_1$ ✓ S 2. $\hat{P}_2 = \hat{T}\hat{R}Q$ ext. $\angle$ of a cyclic quad. / <i>buite</i> $\angle$ v kvi ✓ S but / <i>maar</i> $\hat{W}_1 = \hat{T}\hat{R}Q$ proved / <i>reeds bewys</i> $\hat{P}_2 = \hat{W}_1$ 3. $\hat{T} = \hat{Q}_2$ sum of $\angle$s of Δ / <i>somv</i> $\angle$ ev Δ ✓ S $\Delta VPT \parallel \Delta RWQ$ $\angle \angle \angle$ ✓ R $\frac{VP}{RW} = \frac{PT}{WQ} = \frac{VT}{RQ}$ corresponding sides in prop's <i>ooreenkomstige sye in verhouding</i> $\therefore \frac{PT}{WQ} = \frac{PV}{WR}$	
		(6)

QUESTION 10

10.1

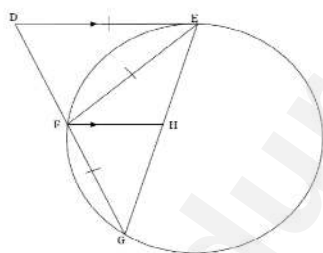


Construct $AP = DE$ and $AQ = DF$ and draw PQ In $\triangle APQ$ and $\triangle DEF$: $AP = DE$ $AQ = DF$ $\hat{A} = \hat{D}$ $\therefore \triangle APQ \cong \triangle DEF$ (SAS) $\vec{P_1} = \vec{E}$, But $\vec{B} = \vec{E} \dots$ (given) $\vec{P_1} = \vec{B}$	$A \checkmark$ construction $A \checkmark \triangle APQ \cong \triangle DEF$ $A \checkmark$ SAS
$PQ \parallel BC$ (corresponding angles $\rightarrow$) $\therefore \frac{AB}{AP} = \frac{AC}{AQ}$ (proportionality theorem; $PQ \parallel BC$) $\therefore \frac{AB}{DE} = \frac{AC}{DF}$ (construction)	$A \checkmark S A \checkmark R$ $A \checkmark S A \checkmark R$



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10.2



10.2.1	$\hat{G}$ (tan chord theorem) $\widehat{EFH}$ (alt $\angle$'s, DE FH) $FEH = \hat{G}$ ($\angle$'s opposite = sides)	$A \vee S \vee R$ $A \vee S \vee R$	
10.2.2(a)	In Δ 's DEF and DGE: 1) $\hat{D}$ is common 2) $\widehat{DEF} = \hat{G}$ (proven) 3) $\widehat{DFE} = \widehat{DEG}$ (sum of angles in triangle) $\Delta DEF \parallel \Delta DGE$ ($\angle \angle \angle$)	$A \vee S$ $A \vee S$ $A \vee R$	(4)
10.2.2(b)	Let $\widehat{DEF} = x^\circ$; $\widehat{EFD} = 2x$ (ext. $\angle$ of Δ) $\hat{D} = 2x$ ($\angle$'s opposite = sides) $\therefore 2x + 2x + x = 180^\circ$ $\therefore x = 30^\circ$ $\therefore \hat{D} = 2x = 72^\circ$	$A \vee S \vee A \vee R$ $A \vee S$ $A \vee$ Equation $A \vee$ value of x	(5)

10.2.3	$\frac{DE}{DG} = \frac{DF}{FG} \quad (\Delta DEF \parallel \Delta DGE)$ $\frac{DE}{DG} = \frac{DF}{FG} \quad (DE = FG)$ $\frac{2}{k+2} = \frac{k}{2}$ $k^2 + 2k = 4$	$A' \sim S$ $A' \sim S$ $A' \sim$ substitution	(3)
10.2.4	$\frac{GH}{GE} = \frac{GF}{GD} = \frac{2}{k+2} \quad (\text{Prop. Intercept theorem DE / FH})$	$A' \sim S$ $A' \sim S$ $A' \sim$ substitution	(2)



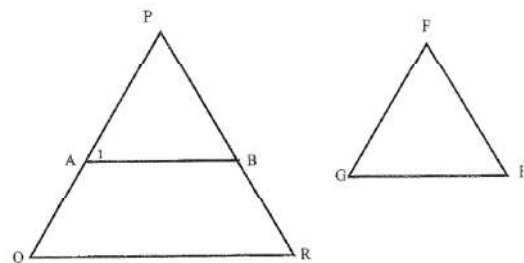
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QUESTION/VRAAG 10

10.1



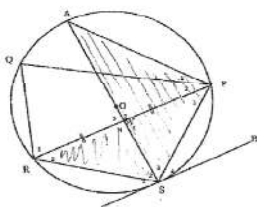
10.1	Construction: Mark off $PA = FG$ and $PB = FH$ and join AB. Konstruksie: $PA = FG$ en $PB = FH$ en verbind AB. $\triangle PAB \cong \triangle FGH$ (SLS) $\hat{A}_1 = \hat{G}$ (congruency) $\hat{C} = \hat{Q}$ (given) $\therefore \hat{A}_1 = \hat{Q}$ $\therefore AB \parallel QR$ (corresponding $\angle$'s =) $\frac{PQ}{PA} = \frac{PR}{PB}$ (line $\parallel$ to one side of Δ) $\therefore \frac{PQ}{PG} = \frac{PR}{FH}$ (construction)	✓ construction konstruksie ✓S/R ✓ ✓S/R ✓S ✓R (6)
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10.2



10.2.1	$\widehat{W}_2 = 90^\circ$ (line from centre to midpoint/ Lyn uit mdpt van sirkel na mdpt van krd) $\hat{S}_4 + \hat{S}_5 = 90^\circ$ (tangent $\perp$ to radius/rkl $\perp$ r) $\widehat{W}_2 = \hat{S}_3 + \hat{S}_4$ $\therefore SB \parallel RP$ (corresponding $\angle$'s = Doreenkomstige $\angle$ 'e =)	$\checkmark S$ $\checkmark R$ $\checkmark S$ $\checkmark R$
10.2.2	In Δ APS and Δ RWS: $\hat{A} = \hat{R}_2$ ($\angle$'s same segment) $\hat{A}PS = \hat{W}_2$ ($\hat{A}PS = 90^\circ$ $\angle$ in semi-circle) $\hat{S}_3 = \hat{S}_2$ ($\angle$'s of Δ) $\therefore \Delta APS \parallel \Delta RWS$ ($\angle \angle \angle$)	$\checkmark$ S/R $\checkmark S$ $\checkmark R$ $\checkmark S$
10.2.3	$\frac{PS}{WS} = \frac{AS}{RS}$ ($\parallel$ Δ 's) $\therefore PS.RS = AS.WS$ But $\Delta RWS = \Delta FWS$ ($S\angle S$) $RS = FS$ $\therefore RS^2 = AS.WS$	$\checkmark S$ $\checkmark S$ $\checkmark$ S/R $\checkmark S$
10.2.4	$RS^2 = RW^2 + WS^2$ (Pythagoras) $AS.WS = RW^2 + WS^2$ ($RS^2 = AS.WS$)	$\checkmark S$ $\checkmark$ substitution/

	$\frac{ASJS}{WS} = \frac{RW^2}{WS} + \frac{WS^2}{WS}$ $\therefore AS = \frac{RW^2}{WS} + WS$	vervanging $\checkmark S$ $\checkmark S$ (4)
		[23]



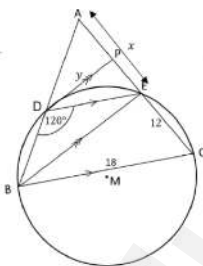
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QUESTION 10



10.1.1	$\frac{AB}{DB} = \frac{AE}{EC}$ [line one side of Δ] or [prop theorem: $DE \parallel BC$] [lyn een sy van Δ] of [eweredigheit: $DE \parallel BC$] $\frac{5}{4} = \frac{x}{12}$ $4x = 60$ $x = 15 \text{ units}$	$\checkmark$ S $\checkmark$ R $\checkmark$ answer	(3) R
10.1.2	$\text{area of } \Delta AEB = \frac{1}{2}AE \cdot h_g$ $\text{area of } \Delta ECB = \frac{1}{2}EC \cdot h_g$ [Same height B / dieselfde hoogtepunt B] $\frac{\text{area of } \Delta AEB}{\text{area of } \Delta ECB} = \frac{AE}{EC}$ $\frac{\text{area of } \Delta AEB}{\text{area of } \Delta ECB} = \frac{15}{12}$ $\frac{\text{area of } \Delta AEB}{\text{area of } \Delta ECB} = \frac{5}{4}$	$\checkmark$ S $\checkmark$ answer CA from 10.1.1	(2) R
	OR		
	$\hat{B} = 60^\circ$ [co-interior $\angle$ s $DE \parallel BC$] / [ko – binne $\angle$ e $DE \parallel BC$] $\hat{C} = 60^\circ$ [opp. $\angle$ s cyclic quad: teenoorste $\angle$ e kdrh] $\hat{A} = 60^\circ$ [$\angle$ s of Δ : $\angle$ e v. Δ] $\therefore AB = BC = AC = 27$		
	$\frac{\text{area of } \Delta AEB}{\text{area of } \Delta ECB} = \frac{\frac{1}{2}AB \cdot AE \sin 60^\circ}{\frac{1}{2}EC \cdot BC \sin 60^\circ}$ $\frac{\text{area of } \Delta AEB}{\text{area of } \Delta ECB} = \frac{27 \times 27}{12 \times 18}$ or $\frac{27 \times 27}{12 \times 27}$ $\frac{\text{area of } \Delta AEB}{\text{area of } \Delta ECB} = \frac{27}{8}$ or $\frac{27}{12}$	$\checkmark$ S $\checkmark$ answer	

10.2	$\hat{C} = 60^\circ$ [opp. $\angle$ s cyclic quad' teenoorst $\angle$ e kdvh] $BE^2 = 12^2 + 18^2 - 2(12)(18)\cos 60^\circ$ $BE^2 = 144 + 324 - 432(\frac{1}{2})$ $BE^2 = 252$ $BE = \sqrt{252}$ $BE = 6\sqrt{7}$ $\frac{AD}{AB} = \frac{DP}{BE}$ [$\triangle ADP \parallel \triangle ABE$] $\frac{5}{9} = \frac{y}{6\sqrt{7}}$ $9y = 5 \times 6\sqrt{7}$ $y = \frac{10\sqrt{7}}{3}$ or 8,82	$\checkmark S$ $\checkmark$ Subst $\checkmark \sqrt{252}$ $\checkmark S \checkmark R$	
	OR	$\checkmark$ answer	(6)
	$\hat{C} = 60^\circ$ [opp. $\angle$ s cyclic quad' teenoorst $\angle$ e kdvh] $BE^2 = 12^2 + 27^2 - 2(12)(27)\cos 60^\circ$ $BE^2 = 144 + 324 - 648(\frac{1}{2})$ $BE^2 = 792$ $BE = \sqrt{792}$ $BE = 6\sqrt{22}$ $\frac{AD}{AB} = \frac{DP}{BE}$ [$\triangle ADP \parallel \triangle ABE$] $\frac{5}{9} = \frac{y}{6\sqrt{22}}$ $9y = 5 \times 6\sqrt{22}$ $y = \frac{10\sqrt{22}}{3}$ or 15,63	$\checkmark S$ $\checkmark$ Subst $\checkmark \sqrt{729}$ $\checkmark S \checkmark R$	
		$\checkmark$ answer	11



QUESTION 10

10.1	Construction: $DG = AB$ and $DH = AC$ ✓ construction In $\triangle ABC$ and $\triangle DEF$ $\hat{A} = \hat{D}$ (given) $AB = DG$ (construction) $AC = DH$ (construction) $\therefore \triangle ABC \cong \triangle DEF$ (SAS) $\therefore \hat{B} = \hat{G}_1$ and $\hat{B} = \hat{E}$ (given) $\therefore \hat{E} = \hat{G}_1$ $\therefore EF \parallel GH$ (corresponding $\angle s =$) $\therefore \frac{DG}{DE} = \frac{DH}{DF}$ (line $\parallel$ to side of $\triangle$) but $AB = DG$ and $AC = DH$ $\therefore \frac{AB}{DE} = \frac{AC}{DF}$ (6)	✓ S/R ✓ S ✓ S/R ✓ S ✓ R

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10.2		
10.2.1	In $\triangle KCB$ and $\triangle AKC$ $\hat{C}_1 = \hat{A}$ (tan-chord) $\hat{K} = \hat{K}$ (common $\angle$) $\hat{KCT} = \hat{B}_2$ (3rd $\angle$ in $\triangle$) $\therefore \triangle KCB \parallel \triangle AKC$ ($\angle \angle \angle$) (3)	✓ S/R ✓ S ✓ R
10.2.2	$\hat{T}_2 = \hat{O}_1 + \hat{A}$ (ext $\angle$ of $\triangle$) $\hat{KCT} = \hat{C}_1 + \hat{C}_2 + \hat{C}_3$ But $\hat{C}_2 + \hat{C}_3 = 90^\circ$ ($\angle s$ in semi-circle) But $\hat{C}_1 = \hat{A}$ (proven) $\hat{KCT} = \hat{A} + 90^\circ$ $\therefore \hat{T}_2 = \hat{KCT} = \hat{A} + 90^\circ$	✓ S/R ✓ S ✓ R ✓ S

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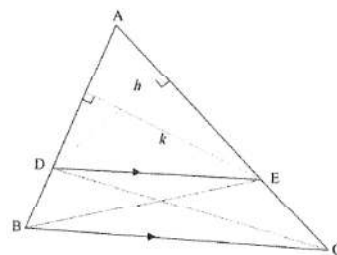
10.2.3	$\triangle COT \parallel \triangle AKC$ $\hat{TCO} = \hat{A}$ ($\angle s$ opp equal sides; $CO = AO$) $\hat{T}_2 = \hat{KCA}$ (proved) $\hat{O}_2 = \hat{K}$ (3rd $\angle$ in $\triangle$) $\triangle COT \parallel \triangle AKC$ ($\angle \angle \angle$)	(4) ✓ S/R ✓ S ✓ R (3)
10.2.4	$\triangle KCB \parallel \triangle AKC$ $\frac{BK}{KC} = \frac{CK}{AK}$ $\therefore BK \cdot AK = KC^2$ $\triangle COT \parallel \triangle AKC$ $\frac{KC}{OT} = \frac{AC}{CT}$ $\therefore KC \cdot CT = AC \cdot OT$ $\therefore KC = \frac{AC \cdot OT}{CT}$ $\therefore KC^2 = \frac{AC^2 \cdot OT^2}{CT^2}$	✓ ratio ✓ S ✓ ratio ✓ S (4) [20]

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QUESTION/VRAAG 10

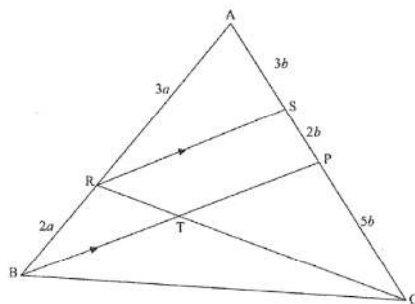
10.1



Construction: Connect DC and BE and draw the altitudes k and h	✓ constr/ konstr
Konstruksie: Verbind DC en BE en trek hoogtelynne k en h.	
$\text{Area } \triangle ADE = \frac{1}{2} \times AD \times h = \frac{AD}{2} \times h$ $\text{Area } \triangle BDE = \frac{1}{2} \times BD \times k = \frac{BD}{2} \times k$	✓ S
$\text{Area } \triangle ADE = \frac{1}{2} \times AE \times h = \frac{AE}{2} \times h$ $\text{Area } \triangle DEC = \frac{1}{2} \times EC \times k = \frac{EC}{2} \times k$	✓ S
but/maar: $\text{Area } \triangle BDE = \text{Area } \triangle DEC$ [DE common base/ gemeenskaplike basis with/ met DE DC] $\therefore \frac{\text{Area } \triangle ADE}{\text{Area } \triangle BDE} = \frac{\text{Area } \triangle ADE}{\text{Area } \triangle DEC}$ $\therefore \frac{AD}{BD} = \frac{AE}{EC}$	✓ S ✓ R ✓ conclusion/ afleiding (6)

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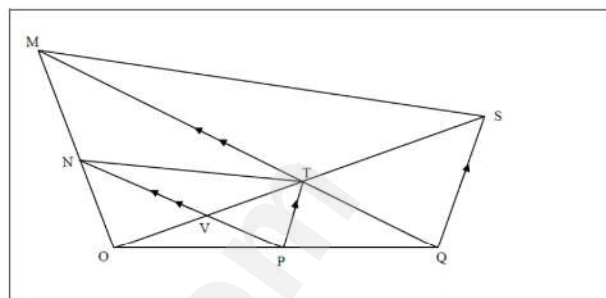




10.2.1	$\frac{AS}{SP} = \frac{AR}{RB}$ $= \frac{3}{2}$ $\frac{AS}{SC} = \frac{3}{7}$	[line side of Δ / <i>lyn sy van Δ</i> OR/OF [RS BP] [AP = PC]	✓S ✓R $\checkmark \frac{3}{2}$ $\checkmark \frac{3}{7}$
10.2.2	$\frac{RT}{TC} = \frac{SP}{PC}$ $= \frac{2}{5}$	[line side of Δ / <i>lyn sy van Δ</i> OR/OF [RS TP]	✓S ✓R ✓answer/antwoord
10.2.3	Area of Δ RAS = $\frac{AS}{SC} = \frac{3}{7}$ Area of Δ RSC = $\frac{SC}{SC} = 1$ [common height / <i>gemeenskaplike hoogte</i>]		$\checkmark \frac{3}{7}$ ✓R
10.2.4	Area Δ TPC = $\frac{1}{2} TC \cdot PC \cdot \sin \angle TPC$ Area Δ RSC = $\frac{1}{2} RC \cdot SC \cdot \sin \angle TPC$ $= \frac{TC}{RC} \cdot \frac{PC}{SC}$ $= \frac{5}{7} \times \frac{5}{7}$ $= \frac{25}{49}$	✓Correct subst into area rule / <i>Korrekte vervang. in area-reël</i> ✓answer/antwoord	(3) [18]

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QUESTION / VRAAG 11

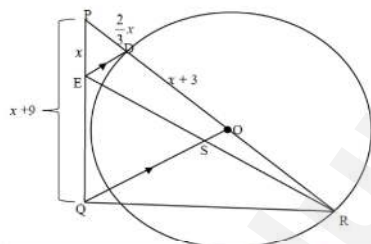


11.1	$\frac{ON}{NM} = \frac{OT}{TS}$ $\frac{ON}{NM} = \frac{OP}{PQ}$ $\therefore \frac{OT}{TS} = \frac{OP}{PQ}$ $\therefore NT \parallel MS$	[prop theorem / <i>eweredigh</i>] [prop theorem / <i>eweredigh</i>]	✓S ✓R ✓S ✓S
11.2	$\frac{ON}{NM} = \frac{OT}{TS} = \frac{3}{5}$ $OT = \frac{3}{8} \times 32$ $= 12$ $VT = \frac{5}{8} \times 12$ $= \frac{15}{2}$ or 7,5	[prop theorem / <i>eweredigh</i>]	✓S R ✓OT ✓method / <i>metode</i> ✓answer / <i>antwoord</i>
			(4) [8]

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QUESTION / VRAAG 11



11.1	$\frac{PE}{EQ} = \frac{PD}{DO}$ $\frac{x}{9} = \frac{3}{x+3}$ $x^2 + 3x = 6x$ $x^2 - 3x = 0$ $x(x-3) = 0$ $x = 0$ or $x = 3$ N.A. / <i>n.v.t</i> DO = 6 DO = OR OR = 6 units / <i>eenhede</i>	line one side Δ POQ OR prop theorem ED OQ / <i>lyn een sy ΔPOQ</i> OF <i>eweredighheidsstelling ED OQ</i> radii / <i>radiusse</i>	✓S ✓R ✓x = 3 ✓OR = 6
11.2	S is the midpoint of RE / <i>S is die middelpunt van RE</i> DE = 2OS DE = 2,8 units / <i>eenhede</i>	midpoint theorem / <i>midpuntstelling</i> same height (DE) / <i>dieselfde hoogte (DE)</i>	✓R ✓answer (2)
11.3	Area Δ PED = $\frac{1}{2} \times PE \times DE$ Area Δ PER = $\frac{1}{2} \times PR \times DE$ $= \frac{1}{2} \times 14 \times \frac{1}{2}$ Area Δ PER = 7 × Area Δ PED = 18,9 units ² / <i>eenhede</i> ²		✓S ✓R ✓18,9 (4) [10]

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