eduresourceza.com

Downloaded from eduresourceza.com

LAST PUSH

MATHS IS MATHS

PAPER 1

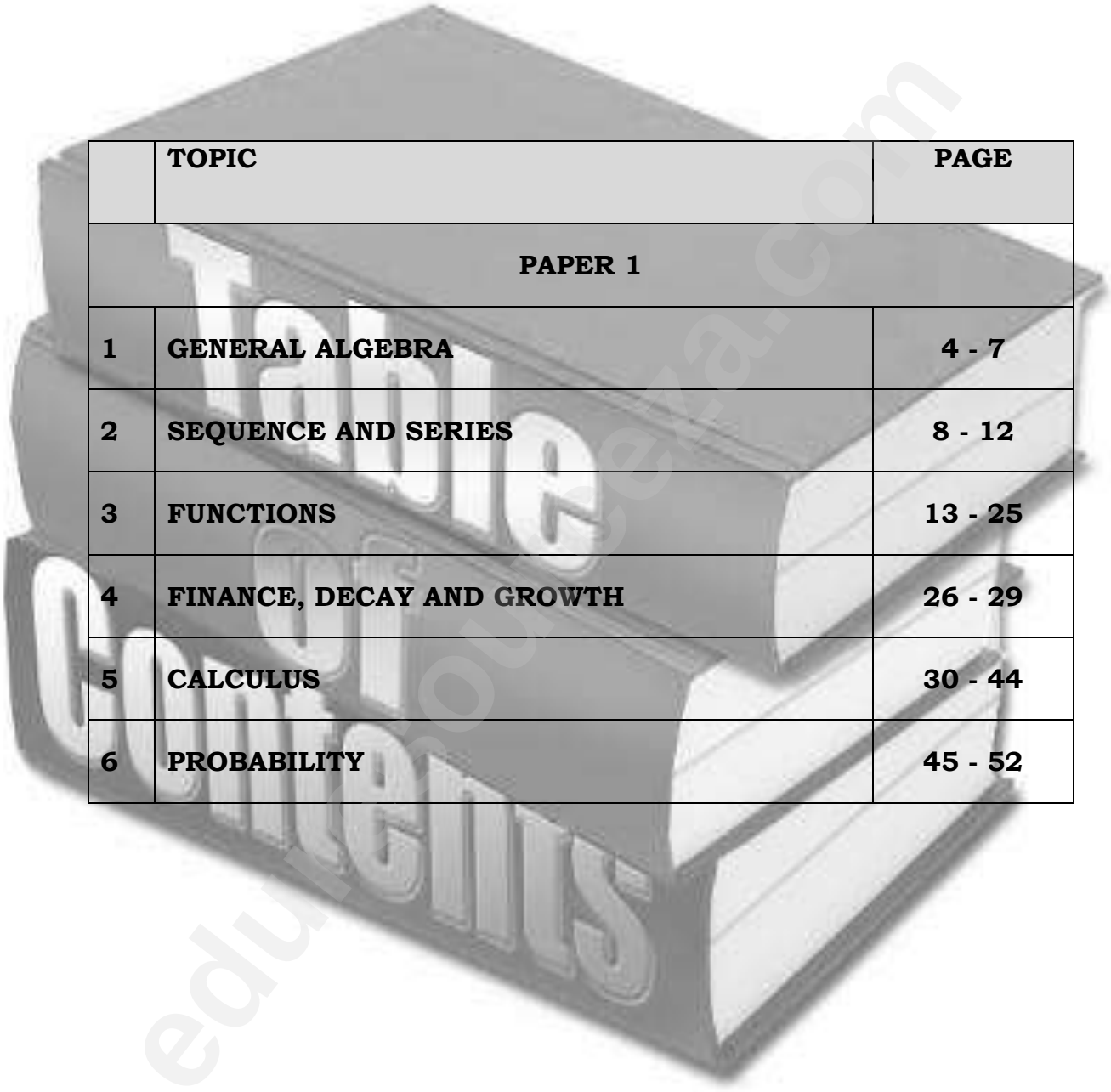
GRADE 12

Downloaded from eduresourceza.com

BY: PAT TSHIKANE

Downloaded from eduresourceza.com

TABLE OF CONTENTS



	TOPIC	PAGE
	PAPER 1	
1	GENERAL ALGEBRA	4 - 7
2	SEQUENCE AND SERIES	8 - 12
3	FUNCTIONS	13 - 25
4	FINANCE, DECAY AND GROWTH	26 - 29
5	CALCULUS	30 - 44
6	PROBABILITY	45 - 52

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

ACKNOWLEDGEMENTS

I would like to express my gratitude to

Lindiwe Nkutha

&

Rachel Taunyane

who saw me through this document, and to all those who provided support, talked things over, read, offered comments and assisted in the editing, proof reading and design.

PAT TSHIKANE

email-ptshikane@yahoo.com

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

GENERAL ALGEBRA

- Write correct quadratic formula and substitute correctly
- Surd form
  1. When squaring both sides include the middle term
  2. Check the validity of both answers. i.e. exclude the invalid one

  Inequalities

  1. Distinguish between critical values and roots
  2. Correct conclusion notation for ‘ or” & ‘and”

- Exponential laws to be revised
- Nature of the roots to be understood in the context of the quadratic formula

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - x - 12 = 0$

1.1.2 $x(x+3) - 1 = 0$ (Leave your answer in simplest surd form)

1.1.3 $x(4-x) < 0$

1.1.4 $x = \dfrac{a^2 + a - 2}{a - 1}$ if a = 888 888 888 888

1.1.5 $\dfrac{x-3}{x+1}$

1.2 Solve the following equations simultaneously:

$y + 7 = 2x$ and $x^2 - xy + 3y^2 = 15$

1.3 Determine the range of the function $y = x + \dfrac{1}{x}, x \neq 0$ and x is real

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 2

2.1 Solve for x:

2.2.1 $x^2 - 9x + 20 = 0$

2.2.2 $3x^2 + 5x = 4$ (correct to TWO decimal places)

2.2.3 $2x^{\frac{-5}{3}} = 64$

2.2.4 $\sqrt{2-x} = x - 2$

2.2.5 $x^2 + 7x < 0$

2.2.6 $\frac{2x-1}{x-3} > 1$

2.2 Given: $(3x - y)^2 + (x-5)^2 = 0$

Solve for x and y

2.3 For which value of k will the equation $x^2 + x = k$ have no real roots?

QUESTION 3

3.1 Solve for x:

3.1.1 $3x^2 - 4x = 0$

3.1.2 $x - 6 + \frac{2}{x} = 0; x \neq 0$ (Leave your answer to TWO decimal places)

3.1.3 $x^{\frac{3}{2}} = 4$

3.1.4 $3^x(x-5) < 0$

3.1.5 $3^{2x} - 8.3^x - 9 = 0$

3.2 Solve for x and y simultaneously

$y = x^2 - x - 6$ and $2x - y = 2$

3.3 Simplify, without the use of a calculator:

$\sqrt{3}.\sqrt{48} - \frac{4^{x+1}}{2^{2x}}$

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

3.4 Given: $f(x)=3(x-1)^2+5$ and $g(x)=3$

3.4.1 Is it possible for $f(x)=g(x)$?

3.4.2 Determine the value(s) of k for which $f(x)=g(x)+k$ has TWO unequal real roots

QUESTION 4

4.1 Solve for *x*:

4.1.1 $(x-2)(4+x)=0$

4.1.2 $3x^2-2x=14$ (Correct to TWO decimal places)

4.1.3 $2^{x+2}+2^x=20$

4.1.4 $4^x+32=12.2^x$

4.2 Solve the following equations simultaneously

$x=2y+3$

$3x^2-5xy=24+16y$

4.3 Solve for *x*: $(x-1)(x-2)<6$

4.4 The roots of a quadratic equation are: $x=\frac{3\pm\sqrt{-k-4}}{2}$, determine the values of *k* if the roots are real.

QUESTION 5

5.1 Solve for *x*:

5.1.1 $x^2-x-20=0$

5.1.2 $2x^2-11x+7=0$ (Correct to TWO decimal places)

5.1.3 $5x^2+4>21x$

5.1.4 $2^{2x}-6.2^x=16$

5.2 Solve for x and y simultaneously

$y+1=2x$

$x^2-xy+y^2=7$

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

5.3 The roots of a quadratic equation are given by $x = \frac{-5 \pm \sqrt{20+8k}}{6}$,

where $k \in \{-3; -2; -1; 0; 1; 2;3\}$

5.3.1 Write down TWO values of k for which the roots will be rational

5.3.2 Write down ONE value of k for which the roots will be non-real

5.4 Calculate a and b if $\sqrt{\frac{7^{2014}-7^{2012}}{12}} = a(7^b)$ and a is not a multiple of 7

eduresourceza.com

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

SEQUENCE & SERIES

- Work with sequences with negative values as well as fractions
- Use the correct notation and language. i.e. n versus T_n
- Simplification of exponents is key (especially for geometric sequence)
- Condition for convergence must be related to linear inequalities
- Meaning of sigma as well as its notation

QUESTION 1

1.1 Given the following quadratic sequence: -2; 0; 3; 7; …n

1.1.1 Write down the value of the next term of this sequence

1.1.2 Determine an expression for the n^{th} term of this sequence

1.1.3 Which term of the sequence will be equal to 322?

1.2 Consider an arithmetic sequence which has the second term equal to 8 and the fifth term equal to 10

1.2.1 Determine the common difference of this sequence

1.2.2 Write down the sum of the first 50 terms of this sequence, using sigma Notation

1.2.3 Determine the sum of the first 50 terms of this sequence

QUESTION 2

The following geometric sequence is given: 10; 5; 2, 5; 1, 25; …

2.1 Calculate the value of the 5^{th} term, T_5, of this sequence

2.2 Determine the nth term, T_n , in terms of n

2.3 Explain why the infinite series 10 + 5 + 2,5; 1, 25 + … converges

2.4 Determine S_∞ - S_n in the form ab^n, where S_n is the sum of the first n terms of the sequence

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 3

3.1 Given the arithmetic series: 18 + 24 + 30 + … + 300

3.1.1 Determine the number of terms in this series

3.1.2 Calculate the sum of this series

3.1.3 Calculate the sum of all the whole numbers up to and including 300 that are NOT divisible by 6

3.2 The first three terms of an arithmetic geometric sequence are 16, 8 and 4 respectively.

3.2.1 Determine the n^{th} term of the sequence

3.2.2 Determine all possible values of n for which the sum of the first n terms of this sequence is greater than 31

3.2.3 Calculate the sum to infinity of this sequence

QUESTION 4

Given the arithmetic series: 2 + 9 + 16 + …(to 251 terms)

4.1 Write down the fourth term of the series

4.2 Calculate the 251^{st} term of the series

4.3 Express the series in sigma notation

4.4 Calculate the sum of series

4.5 How many terms in the series are divisible by 4?

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 5

5.1 Prove that in any arithmetic series in which the first term is a and whose constant difference is d, the sum of the first terms is $S_n = \frac{n}{2}[2a + (n-1)d]$

5.2 Calculate the value of $\sum_{k=1}^{50}(100 - 3k)$

5.3 A quadratic sequence is defined with the following properties:

$T_2 - T_1 = 7$
$T - T_2 = 13$
$T_4 - T_3 = 19$

5.3.1 Write down the value of

(a) $T_5 - T_4$

(b) $T_{70} - T_{69}$

5.3.2 Calculate the value of T_{69} if $T_{89} = 23594$

QUESTION 6

Lesego bought a bonsai (miniature tree) at a nursery. When he bought the tree, its height was 130 mm. Thereafter the height of the tree increased, as shown in below.

INCREASE IN HEIGHT OF THE TREE PER YEAR		
During the first year	**During the second year**	**During the third year**
100mm	70mm	49mm

6.1 Lesego noted the sequence of height increases, namely 100; 70; 49 …, was geometric. During which year will the height of the tree increase by approximately 11, 76mm?

6.2 Lesego plots a graph to represent the height $h(n)$ of the tree (in mm) n years after he bought it. Determine a formula for $h(n)$

6.3 What height will the tree eventually reach?

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 7

Consider the series: S_n = -3 + 5 + 13 + 21 + … to *n* terms

7.1 Determine the general term of the series in the form $T_k = bk + c$

7.2 Write S_n in sigma notation

7.3 Show that $S_n = 4n^2 - 7n$

7.4 Another sequence is defined as:

$Q_1 = -6$

Q2 = -6-3

Q4 = -6-3+5+13

Q5 = -6-3+5+13+21

7.4.1 Write down a numerical expression for Q6

7.4.2 Calculate the value of Q129

QUESTION 8

8.1 A quadratic number pattern $T_n = an^2+bn+c$ has a first term equal to 1. The general term of the first differences is given by $4n+6$

8.1.1 Determine the value a

8.1.2 Determine the formula for T

8.2 Given the series: (1 x 2) + (5 x 6) + (9 x 10) + (13 x 14) + … (81 x 82)

Write down the series in sigma notation. (It is not necessary to calculate the value of the series)

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 9

9.1 Given the quadratic sequence: - 1; -7 ; -11; p; …

9.1.1 Write down the value of p

9.1.2 Determine the nth term of the sequence

9.1.3 The first difference between two consecutive terms of the sequence is 96.

Calculate the values of these two terms.

9.2 The first three terms of the geometric sequence are: 16; 4; 1

9.2.1 Calculate the value of the 12^{th} term. (Leave your answer in simplified exponential form)

9.2.2 Calculate the sum of the first 10 terms of the sequence

9.3 Determine the value of: $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{5}\right)$ …up to 98 factors

QUESTION 10

Consider the infinite geometric series: 45 + 40, 5+ 36, 45 + …

10.1 Calculate the value of the TWELFTH term of the series (correct to TWO decimal places)

10.2 Explain why this series converges

10.3 Calculate the sum to infinity of the series

10.4 What is the smallest value of n for which $S_\infty - S_n < 1$

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

FUNCTIONS

- Thorough understanding between equations of various functions
- Properties of various functions
- Symmetry and reflection should be done on continuous basis
- Integrate topics during, not at the end
- Relate the work covered in Algebra to Graphs
- Emphasize properties of functions and their inverses
- When sketching a graph:
  - Shape is important
  - Intercepts must be labelled
  - Asymptotes must be clearly indicated with a dashed line and the value where it intercepts the axis must be given
  - Include only the required information on the Cartesian Plane
  - Thorough understanding of Exponential and Log functions

QUESTION 1

1.1 Given: $f(x) = \frac{2}{x+1} - 3$

1.1.1 Calculate the coordinates of the y-intercept of f

1.1.2 Calculate the coordinates of the x-intercept of f

1.1.3 Sketch the graph of f in your ANSWER BOOK, showing clearly the asymptotes and the intercepts with the axes

1.1.4 One of the axes of symmetry of f is a decreasing function. Write down the equation of this axis of symmetry

1.2 The graph of an increasing exponential function with equation $f(x) = a.b^x + q$ has the equation of this axis of symmetry.

- Range: $y > -3$
- The points (0; -2) and (1; -1) lie on the graph of f

1.2.1 Determine the equation that defines f

1.2.2 Describe the transformation from $f(x)$ to $h(x) = 2.2^x + 1$

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 2

Given: $f(x) = 2^{x+1} - 8$

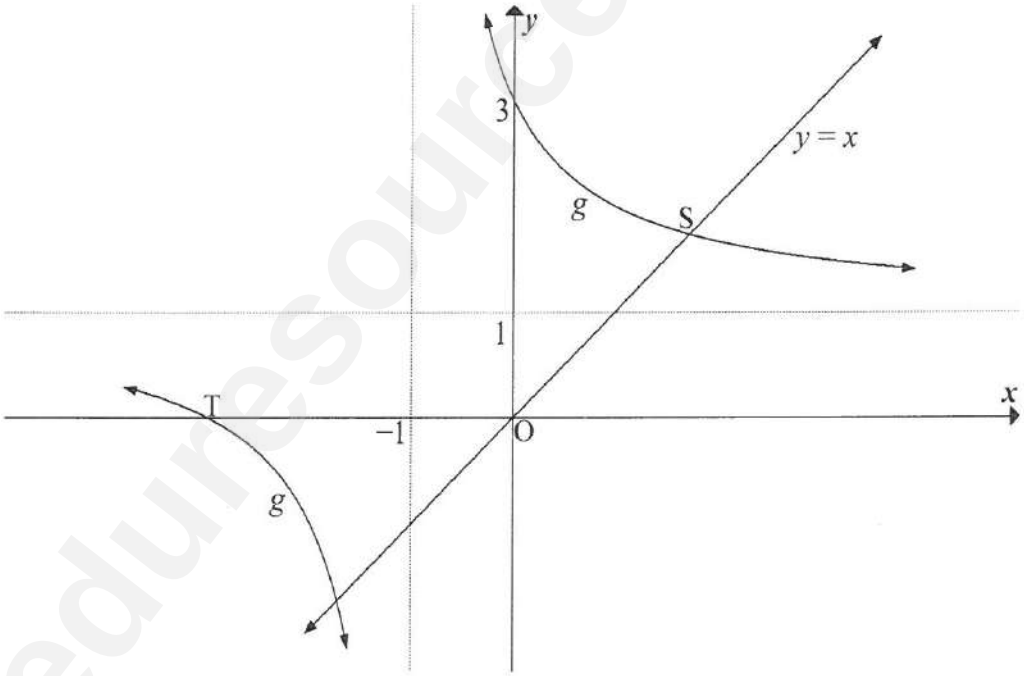
2.1 Write down the equation of the asymptote of f

2.2 Sketch the graph of f . Clearly indicate ALL intercepts with the axes as well as the asymptote

2.3 The graph of g is obtained by reflecting the graph of f in the y-axis. Write down the equation of g

QUESTION 3

The diagram below shows the hyperbola g defined by $g(x) = \frac{2}{x+p} + q$ with asymptotes $y = 1$ and $x = -1$. The graph of g intersects the x-axis at T and the y-axis at (0; 3). The line $y = x$ intersects the hyperbola in the first quadrant at S.



3.1 Write down the values of p and q

3.2 Calculate the x-coordinate of T

3.3 Write down the equation of the vertical asymptote of the graph of h, if $hx) = g(x+5)$

3.4 Calculate the length of OS

3.5 For which values of k will the equation $g(x) = x + k$ have two real roots that are of opposite signs?

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 4

Given: $f(x) = 2^{-x} + 1$

4.1 Determine the coordinates of the y-intercepts

4.2 Sketch the graph of f, clearly indicating ALL intercepts with the axes as well as any asymptotes

4.3 Calculate the average gradient of f between the points on the graph where $x = -2$ and $x = 1$

4.4 If $h(x) = 3f(x)$, write down an equation of the asymptote of h

QUESTION 5

Given: $g(x) = \frac{6}{x+2} - 1$

5.1 Write down the equations of the asymptotes of g

5.2 Calculate:

5.2.1 The y-intercept of g

5.2.2 The x-intercept of g

5.3 Draw the graph of g, showing clearly the asymptotes and the intercepts with the axes

5.4 Determine the equation of the line of symmetry that has a negative gradient, in the form $y = ...$

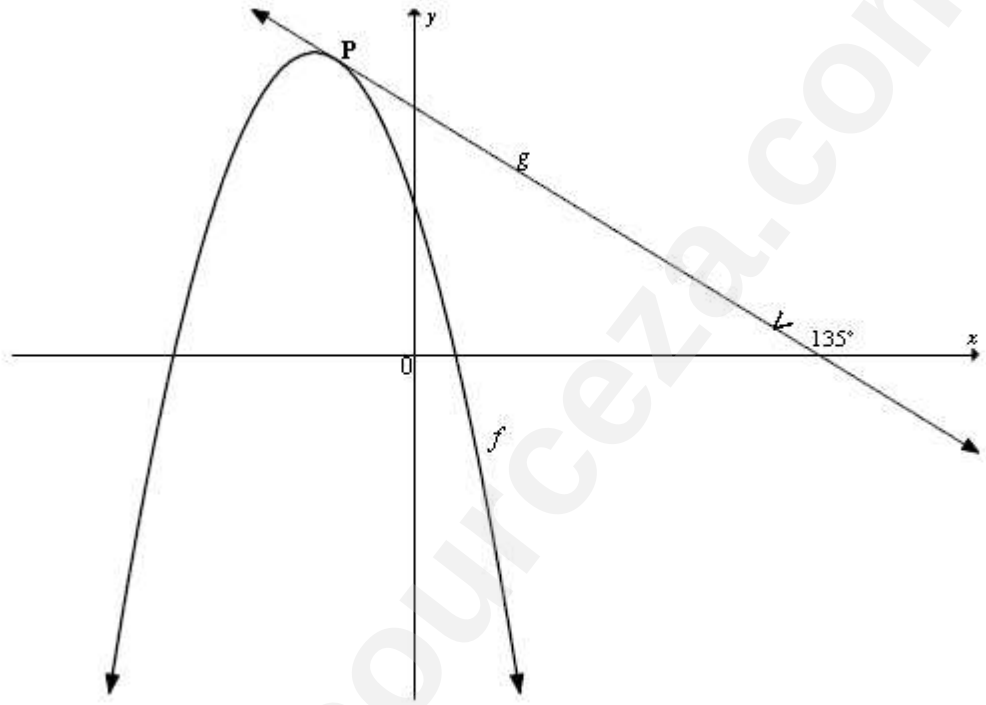
5.5 Determine the value(s) of x for which $\frac{6}{x+2} - 1 \geq -x - 3$

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 6

The sketch below shows the graphs of $f(x) = -2x^2 - 5x + 3$ and $g(x) = ax + q$. The angle of inclination of graph g is 135^0 in the direction of the positive x-axis. P is the point of intersection of f and g is a tangent to the graph of f at P.



6.1 Calculate the coordinates of the turning point of the graph of f

6.2 Calculate the coordinates of P, the point of contact between f and g

6.3 Hence or otherwise, determine the equation of g

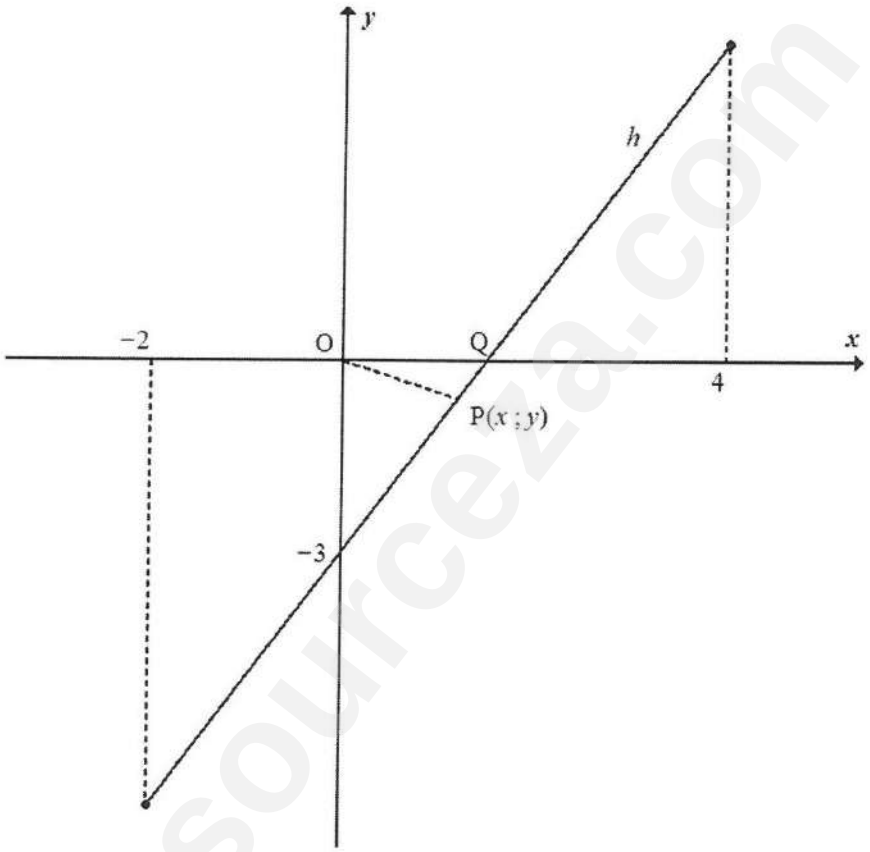
6.4 Determine the values of d for which the line $k(x) = -x + d$ will not intersect the graph of f

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 7

Given: $h(x) = 2x - 3$ for $-2 \leq x \leq 4$. The $x-$intercept of h is Q



7.1 Determine the coordinates of Q

7.2 Write down the domain of h^{-1}

7.3 Sketch the graph of h^{-1} in your ANSWER BOOK, clearly indicating the y-intercept and the end points

7.4 For which value(s) of x will $h(x) = h^{-1}(x)$?

7.5 $P(x; y)$ is the point on the graph of h hat is closest to the origin. Calculate the distance OP

7.6 Given: $h(x) = f'(x)$ where f is a function defined for $-2 \leq x \leq 4$

7.6.1 Explain why f has a local minimum

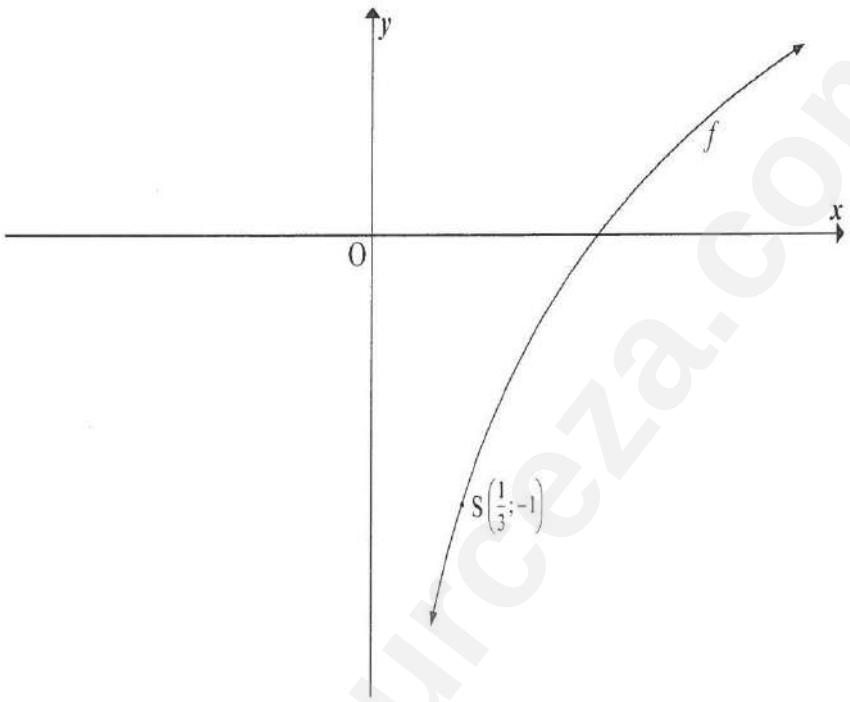
7.6.2 Write down the value of the maximum gradient of the tangent to the graph of f

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 8

Given: $f(x) = \log_a x$ where $a > 0$. $S\left(\frac{1}{3};-1\right)$ is a point on the graph of f



8.1 Prove that $a = 3$

8.2 Write down the equation of h, the inverse of f, in the form $y = ...$

8.3 If $g(x) = -f(x)$, determine the equation of g

8.4 Write down the domain of g

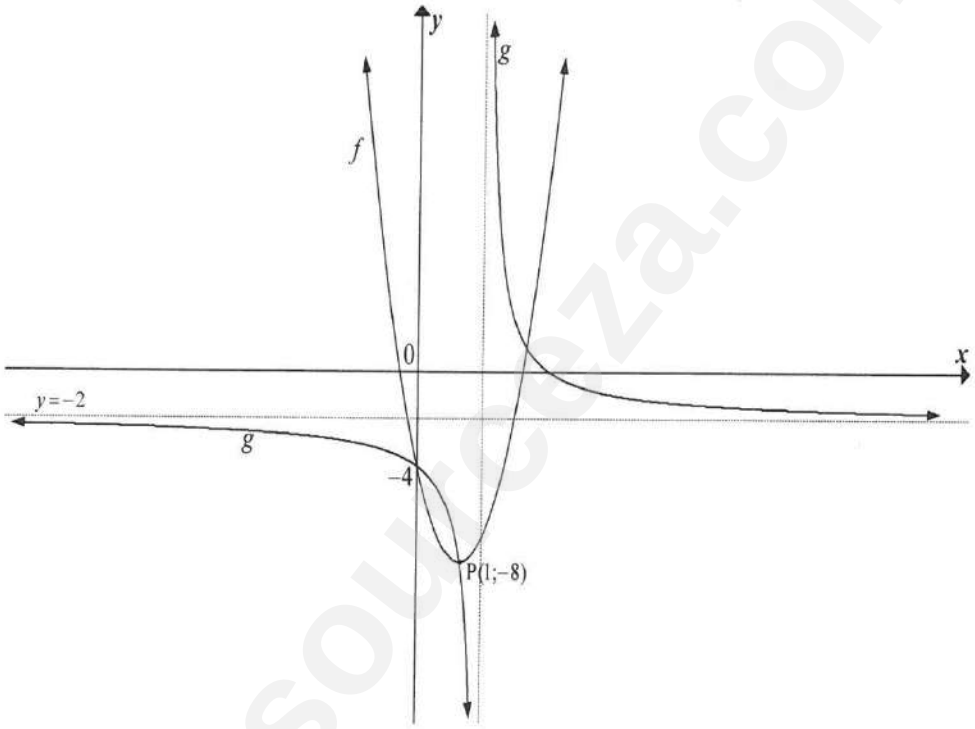
8.5 Determine the values of x for which $f(x) > -3$

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 9

The graphs of the functions $f(x) = a(x+p)^2 + q$ and $g(x) = \frac{k}{x+r} + d$ are sketched below. Both graphs cut the y-axis at -4. One of the points of intersection of the graphs is P (1; -8) Which is also the turning point of f. The horizontal asymptote of g is $y = -2$



9.1 Calculate the values of *a, p* and *q*

9.2 Calculate the values of *k, r* and *d*

9.3 Determine the value(s) of x in the interval $x \leq 1$ for which $g(x) \geq f(x)$

9.4 Determine the value(s) of k for which $f(x) = k$ has two, unequal positive roots

9.5 Write down an equation for the axis of symmetry of g that has a negative gradient

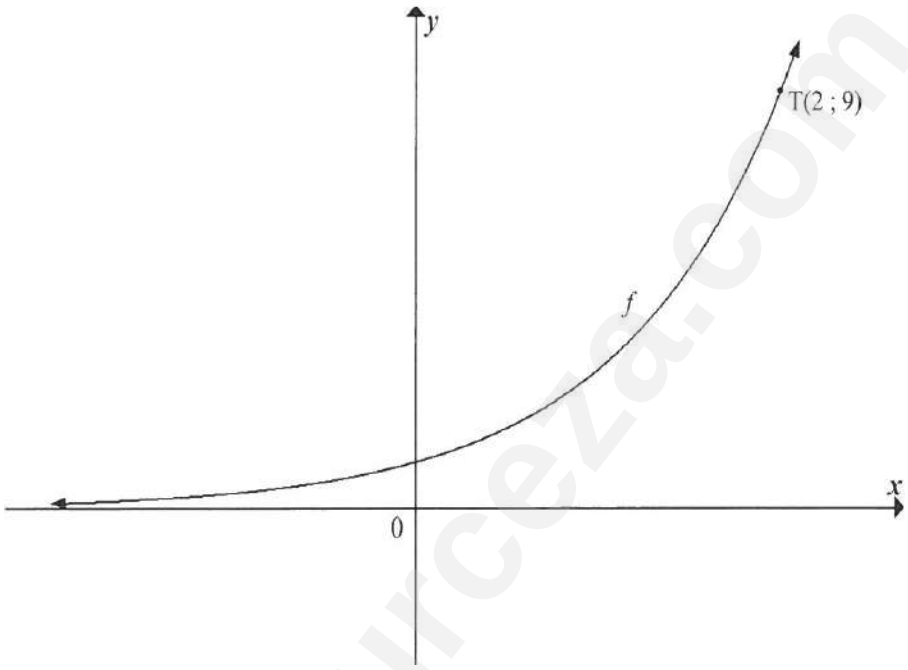
9.6 The point P is reflected in the line determined in QUESTION 9.5 to give the point Q. Write down the coordinates of Q

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 10

The graph of $f(x) = a^x, a > 1$ is shown below. T (2; 9) lies on f



10.1 Calculate the value of a

10.2 Determine the equation of $g(x)$ if $g(x) = f(-x)$

10.3 Determine the value(s) of x for which $f^{-1}(x) \geq 2$

10.4 Is the inverse of f a function? Explain your answer.

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 11

The graph g is defined by the equation $g(x) = \sqrt{ax}$ .The point (8; 4) lies on g

11.1 Calculate the value of a

11.2 If $g(x) > 0$, for what values of x will g be defined?

11.3 Determine the range of g

11.4 Write down the equation of g^{-1}, the inverse of g in the form $y = ...$

11.5 If $h(x) = x - 4$ is drawn, determine ALGEBRAICALLY the point(s) of intersection of h and g

11.6 Hence, or otherwise, determine the value(s) of x for which $g(x) > h(x)$

eduresourceza.com

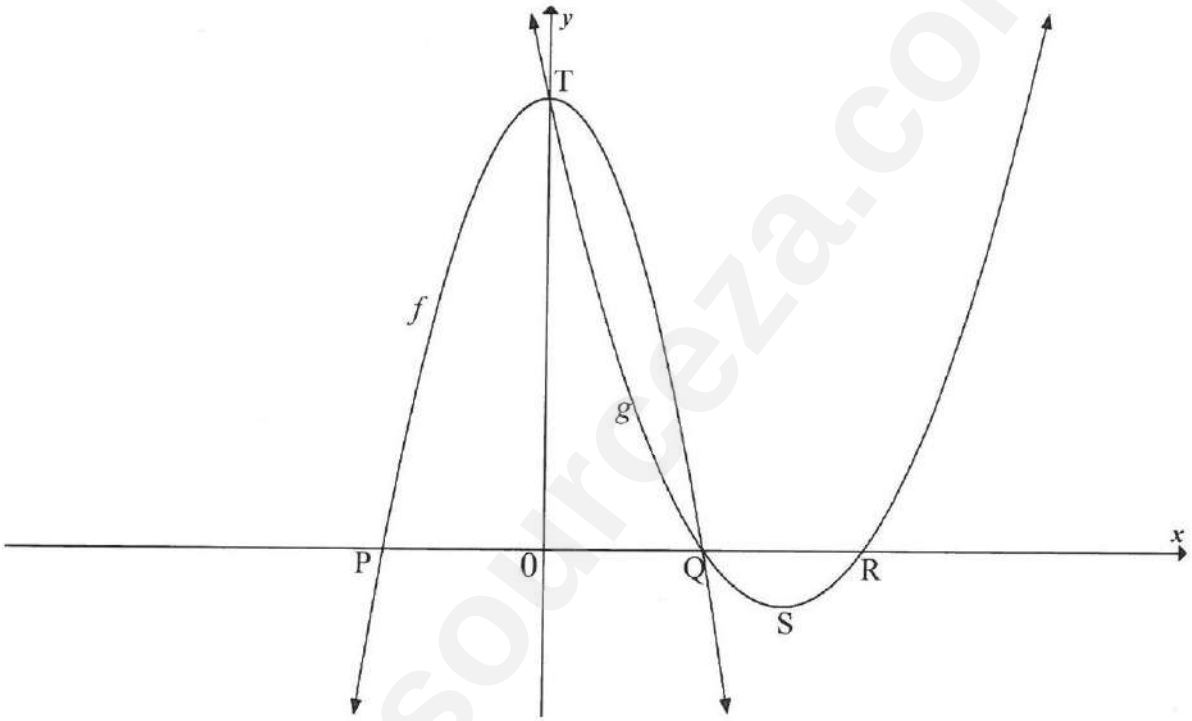
Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 12

12.1 The graphs of $f(x) = 2x^2 + 18$ and $g(X) = ax^2 + bx + c$ are sketched below.

Points P and Q are the x-intercepts of f. Points Q and R are the x-intercepts of g. S is the turning point of g. T is the y-intercept of both f and g.



12.1.1 Write down the coordinates of T

12.1.2 Determine the coordinates of Q

12.1.3 Given that $x = 4{,}5$ at S, determine the coordinates of R

12.1.4 Determine the value(s) of x for which $g''(x) > 0$

12.2 The function defined as $y = \frac{a}{x+p} + q$ has the following properties:

- The domain is $x \in \mathbb{R},\ x \neq -2$
- $y = x + 6$ is in axis of symmetry
- The function is increasing for all $x \in \mathbb{R},\ x \neq -2$

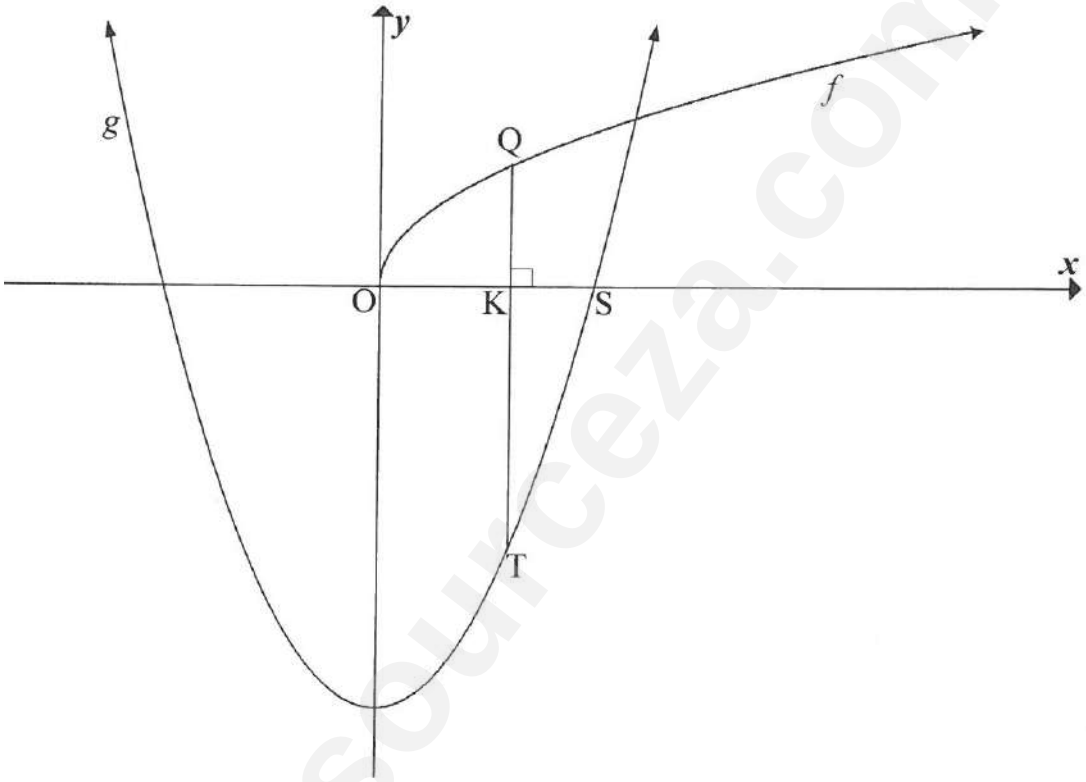
Draw a neat sketch graph of this function. Your sketch must include the asymptotes, if any

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 13

Given: $g(x) = 4x^2 - 6$ and $f(x) = 2\sqrt{x}$. The graphs of g and f are sketched below. S is an x-intercept of g and K is a point between O and S. The straight line QKT with Q on the graph of f and T on the graph of g, is parallel to the y-axis.



13.1 Determine the x-coordinates of S, correct to TWO decimal places

13.2 Write down the coordinates of the turning point of g

13.2.1 Write down the length of QKT in terms of x, where x is the x-coordinate of K

13.2.2 Calculate the maximum length of QT

QUESTION 14

Given: $f(x) = \frac{1}{4}x^2, x \leq 0$

14.1 Determine the equation of f^{-1} in the form $f^{-1}(x) = ...$

14.2 On the same set of axes, sketch the graphs of f and f^{-1}. Indicate clearly the intercepts with the axes, as well as another point on the graph of each of f and f^{-1}

14.3 Is f^{-1} a function? Give a reason for your answer

Downloaded from eduresourceza.com

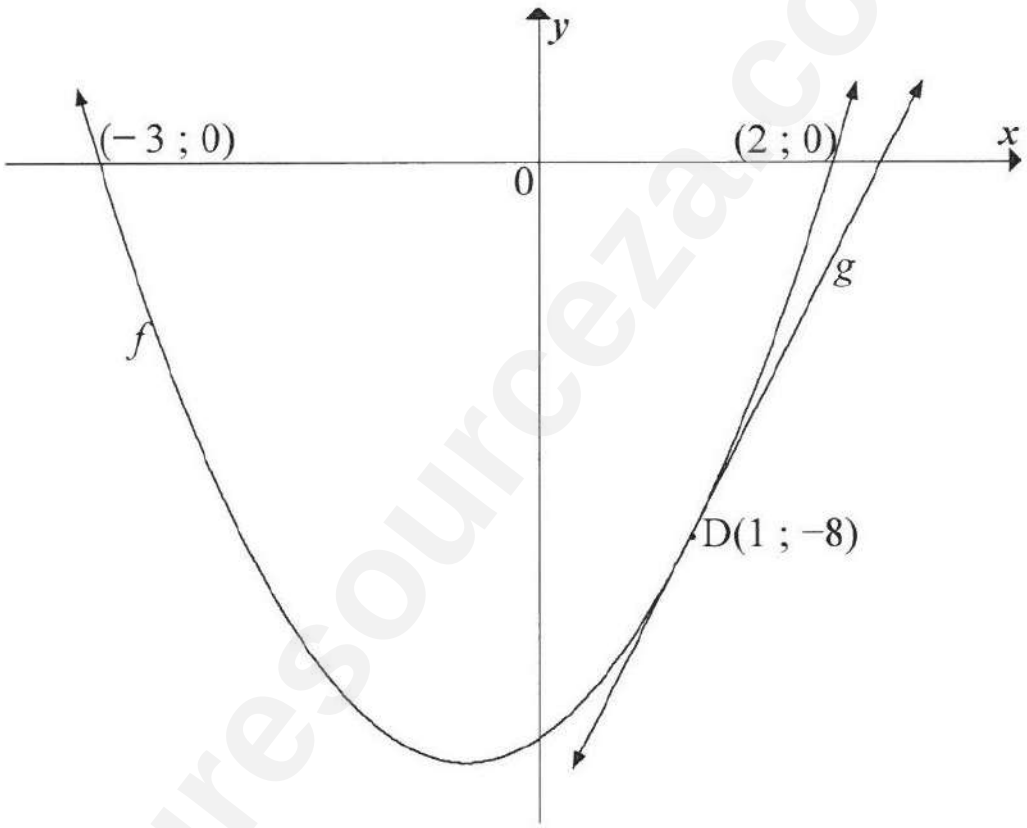
Downloaded from eduresourceza.com

QUESTION 15

The graphs of $f(x) = ax^2 + bx + c$; $a \neq 0$ and $g(x) = mx + k$ are drawn below.

D (1; -8) is a common point on f and g

- f intersects the x-axis at (-3; 0) and (2; 0)
- g is the tangent to f at D



15.1 For what value(s) of x is $f(x) \leq 0$

15.2 Determine the values of a, b and c

15.3 Determine the coordinates of the turning point of f

15.4 Write down the equation of the axis of symmetry of h if $h(x) = f(x-7) + 2$

15.5 Calculate the gradient of g

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 16

Given $h(x) = 16^x$

16.1 Show algebraically that $h(x + \frac{1}{2}) = 4h(x)$

16.2 Calculate, showing ALL calculations.

$$h\left(\frac{1}{4}\right) + 2.h^{-1}(4) - 4^0$$

eduresourceza.com

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

FINANCE, GROWTH & DECAY

- Linear and reducing balance depreciation
- Distinguish between future value and present value formula
- Reinforce the calculation of the number of time periods
- Conversion between nominal and effective rate
- Different compounding
- Comprehension test skills to build context

QUESTION 1

1.1 Nomsa started working on 1 January 1970. At the end of January 1970 and at the end of each month thereafter, she deposited R400 into an annuity fund. She continued doing this until she retired on 31 December 2013.

1.1.1 Determine the total amount of money that she paid into the fund

1.1.2 The interest rate on this fund was 8% p.a. , compounded monthly. Calculate the value of the fund at the time that she retired

1.1.3 On 1 January 2014 Nomsa invested R 2 million in an account paying interest at 10% p.a. compounded monthly. Nomsa withdraws a fixed amount from this account at the end of each month, starting on 31 January 2014. If Nomsa wishes to make monthly withdrawals from this account for 25 years, calculate the maximum amount she could withdraw at the end of each month.

1.2 For each of the three years from 2010 to 2012 the population of town X decreased by 8% per year and the population of town Y increased by 12% per year.

At the end of 2012 the populations of these two towns were equal.

Determine the ratio of the population of town X (call it P_X) to the population of town Y (call it P_Y) at the beginning of 2010.

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 2

2.1 Exactly five years ago Mpume bought a new car for R145 000. The current book value of this car is R72 500. If the car depreciates by a fixed annual rate according to the reducing-balance method, calculate the rate of depreciation

2.2 Samuel took out a home loan for R500 000 at an interest rate of 12% per annum, compounded monthly. He plans to repay this loan over 20 years and his first payment is made one month after the loan is granted.

2.2.1 Calculate the value of Samuel's monthly instalment

2.2.2 Melissa too out for the same amount and at the same interest rate as Samuel. Melissa decided to pay R6 000 at the end of every month.

Calculate how many months it took for Melissa to settle the loan

2.2.3 Who pays more interest, Samuel or Melissa? Justify tour answer.

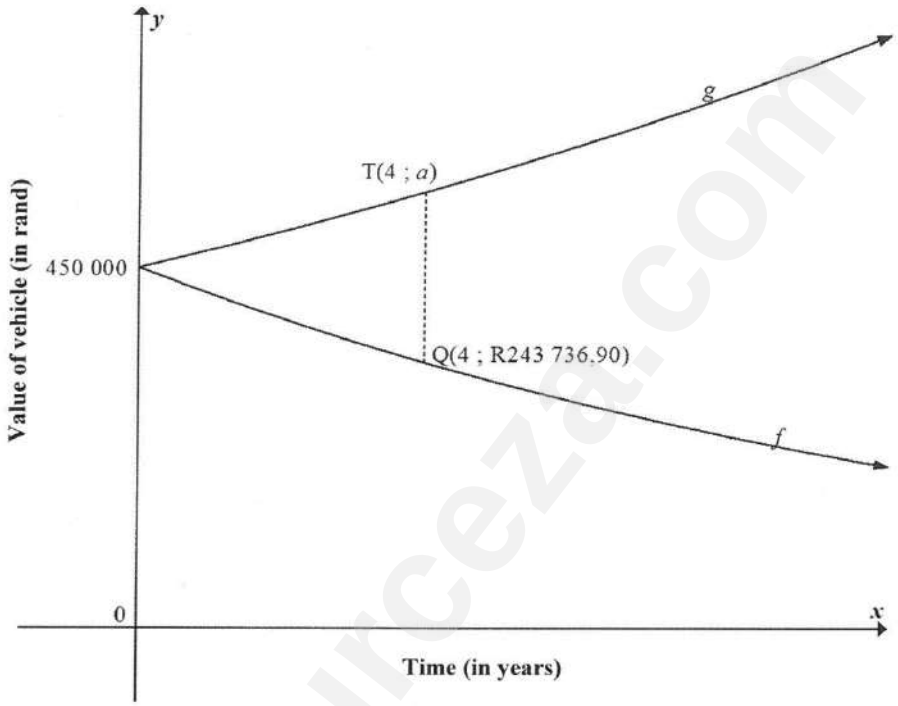
eduresourceza.com

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 3

The graph of f shows the book value of a vehicle x years after the time Joe bought it. The graph of g shows the cost price of a similar new vehicle x years later.



3.1 How much did Joe pay for the vehicle?

3.2 Use the reducing-balance method to calculate the percentage annual rate of depreciation of the vehicle that Joe bought.

3.3 If the average rate of the price increase of the vehicle is 8.1 % p.a., calculate the value of a

3.4 A vehicle that costs R450 000 now, is to be replaced at the end of 4 years. The old vehicle will be used as a trade-in. A sinking fund is created to cover the replacement cost of this vehicle. Payments will be made at the end of each month. The first payment will be made at the end of the 13^{th} month and the last payment will be made at the end of the 48^{th} month. The sinking fund earns interest at a rate of 6, 2% p.a., compounded monthly.

Calculate the monthly payment to the fund.

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 4

Siphokazi bought a house. She paid a deposit of R102 000, which is equivalent to 12% of the selling price of the house. She obtained a loan from from the bank to pay the balance of the selling price. The bank charges her interest of 9% per annum, compounded monthly.

4.1 Determine the selling price of the house

4.2 The period of the loan is 20 years and she starts repaying the loan one month after it was granted. Calculate her monthly instalment

4.3 How much interest will she pay over the period of 20 years? Round your answer correct to the nearest rand

4.4 Calculate the balance of her loan immediately after her 85^{th} instalment

4.5 She experienced financial difficulties after the 85^{th} instalment and did not pay any instalments for 4 months (that is months 86 to 89). Calculate how much Siphokazi owes on her bond at the end of the 89^{th} month.

4.6 She decides to increase her payments to R8 500 per month from the end of the 90^{th} month. How many months will it take to repay her bond after the new payment of R8 500 per month?

QUESTION 5

5.1 Diane invests a lump sum of R5 000 in a savings account for exactly 2 years. The investment earns interest at 10% p.a., compounded quarterly

5.1.1 What is the quarterly interest rate for Diane's investment?

5.1.2 Calculate the amount in Diane's savings account at the end of the 2 years

5.2 Motloi inherits R8100 000. He invests all of his inheritance in a fund which earns interest at a rate of 14% p.a., compounded monthly. At the end of each month he withdraws R10 000 from the fund. His first withdrawal is exactly one month after his initial investment.

5.2.1 How many withdrawals of R10 000 will Motloi be able to make from this fund?

5.2.2 Exactly four years after his initial investment Motloi decides to withdraw all the remaining money in his account and use it as a deposit towards a house.

5.2.2.1 What is the value of Motloi's deposit to the nearest rand?

5.2.2.2 Motloi's deposit is exactly 30% of the purchase price of the house.

What is the purchase price of the house to the nearest rand?

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

CALCULUS

First Principle

- Correct formula and notation
- Correct substitution
- Replacement of x with $f(x + h)$ is critical

Rules for differentiation

- Factorization of sum and difference of two cubes
- Simplification of fractions, exponents and surds
- Differentiate with respect to other variables other than x
- Gradient to the curve

QUESTION 1

1.1 Determine $f'(x)$ from first principles if $f(x) = 3x^2 - 2$

1.2 Determine $\frac{dy}{dx}$ if $y = 2x^{-4} - \frac{x}{5}$

QUESTION 2

2.1 If $f(x) = x^2 - 3x$, determine $f'(x)$ from first principles

2.2 Determine:

2.2.1 $\frac{dy}{dx}$ if $y = \left(x^2 - \frac{1}{x^2}\right)^2$

2.2.3 $D_x\left(\frac{x^3 - 1}{x - 1}\right)$

QUESTION 3

3.1 Determine $f'(x)$ from first principles if $f(x) = x^3$

3.2 Determine the derivative of: $f(x) = 2x^2 + \frac{1}{2}x^4 - 3$

3.3 If $y = (x^6 - 1)^2$, prove that $\frac{dy}{dx} = 12x^5\sqrt{y}$, if $x > 1$

3.4 Given: $f(x) = 2x^3 - 2x^2 + 4x - 1$. Determine the interval on which f is concave up

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 4

4.1 Determine $f'(x)$ from first principles if $f(x)=\frac{2}{x}$

4.2 Determine the derivative of:

4.2.1 $y=3x^2+10x$

4.2.2 $f(x)=\left(x-\frac{3}{x}\right)^2$

QUESTION 5

5.1 Determine the derivative of $f(x)=2x^2+4$ from first principles

5.2 Differentiate:

5.2.1 $f(x)=-3x^2+5\sqrt{x}$

5.2.2 $p(x)=\left(\frac{1}{x^3}+4x\right)^2$

eduresourceza.com

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

Cubic function

- Factorisation of third degree polynomials using remainder theorem or any other method
- Shape is key
- Intercepts with the axis
- Stationery points(use first derivative)
- Concavity and the interval for which it occurs:
  - Concave up $f''(x)$ greater than 0
  - Concave down $f''(x)$ less than 0
- Interpreting the given graph

QUESTION 1

Given: $f(x) = x^3 - 4x^2 - 11x + 30$

1.1 Use the fact that $f(2) = 0$ to write down a factor of $f(x)$

1.2 Calculate the coordinates of the x-intercepts of f

1.3 Calculate the coordinate of the stationary points of f

1.4 Sketch the curve of f in your ANSWER BOOK. Show all intercepts with the axes and turning points clearly.

1.5 For which value(s) of x will $f'(x) < 0$?

QUESTION 2

Given: $h(x) = -x^3 + ax^2 + bx$ and $g(x) = -12x$. P and Q (2; 10) are turning points of h. The graph of h passes through the origin.

2.1 Show that $a = \frac{3}{2}$ and $b = 6$

2.2 Calculate the average gradient of h between P and Q, if it is given that $x = -1$ at P

2.3 Show that the concavity of h changes at $x = \frac{1}{2}$

2.4 Explain the significance of the change in QUESTION 2.3 with respect to h

2.5 Determine the value of x, given $x < 0$, at which the tangent to h is parallel to g

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 3

Given: $f(x) = (x+2)(x^2 - 6x + 9)$

$= x^3 - 4x^2 - 3x + 18$

3.1 Calculate the coordinates of the turning points of the graph of f

3.2 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points

3.3 For which value(s) of x will $x.f'(x) < 0$?

QUESTION 4

Given: $f(x) = 2x^3 - 23x^2 + 80x - 84$

4.1 Prove that $(x-2)$ is a factor of f

4.2 Hence, or otherwise, factorise $f(x)$ fully

4.3 Determine the x-coordinates of the turning points of f

4.4 Sketch the graph of f, clearly labelling ALL turning points and intercepts with the axes

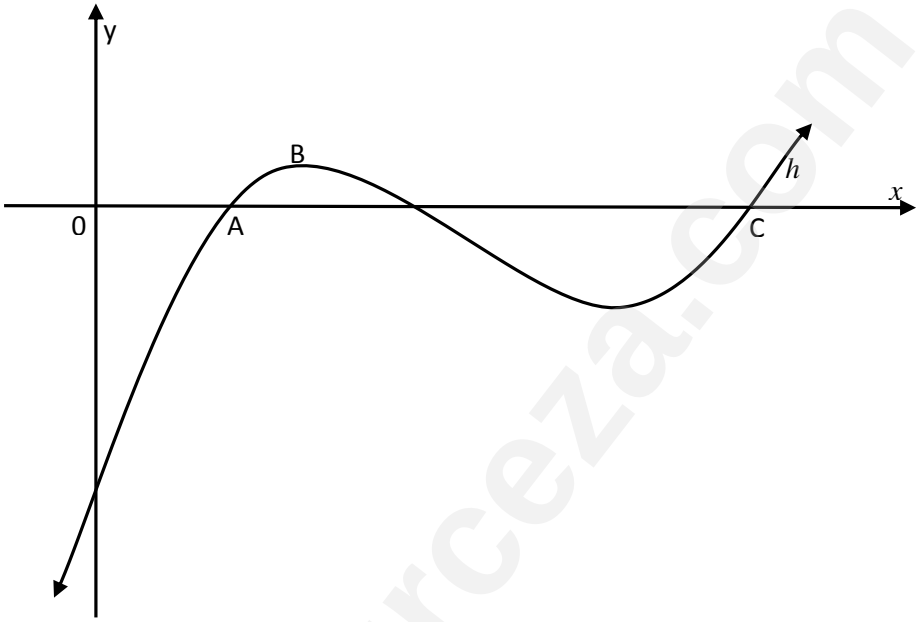
4.5 Determine the coordinates of the y- intercept of the tangent to f that has a slope of 40 and touches f at a point where the x-coordinate is an integer

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 5

The sketch below the graph of $h(x) = x^3 - 7x^2 + 14x - 8$. The x-coordinate of point A is 1. C is another x-intercept of h.



5.1 Determine $h'(x)$

5.2 Determine the x-coordinate of the turning point B

5.3 Calculate the coordinates of C

5.4 The graph of h is concave down for $x < k$. Calculate the value of k

eduresourceza.com

Downloaded from eduresourceza.com

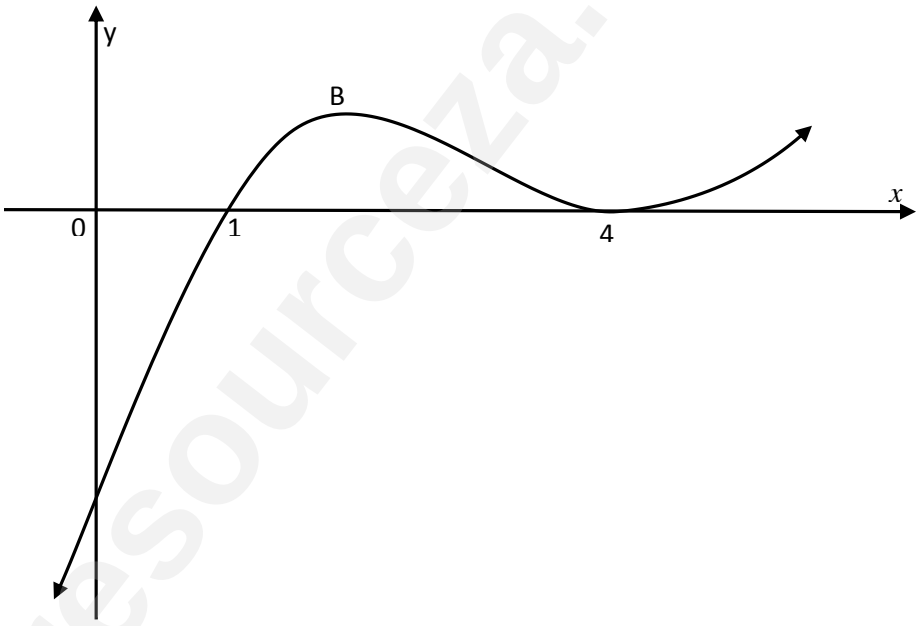
Downloaded from eduresourceza.com

QUESTION 6

6.1 Determine the points on the curve $y = \frac{4}{x}$ where the gradient of the tangent to the curve is -1

6.2 The graph of a cubic function with equation $f(x) = x^3 + ax^2 + bx + c$ is drawn

- $f(1) = f(4) = 0$
- f has a local maximum at B and a local minimum at $x = 4$



6.2.1 Show that $a = -9, b = 24$ and $c = -16$

6.2.2 Calculate the coordinates of B

6.2.3 Determine the value(s) of k for which $f(x) = k$ has negative roots only

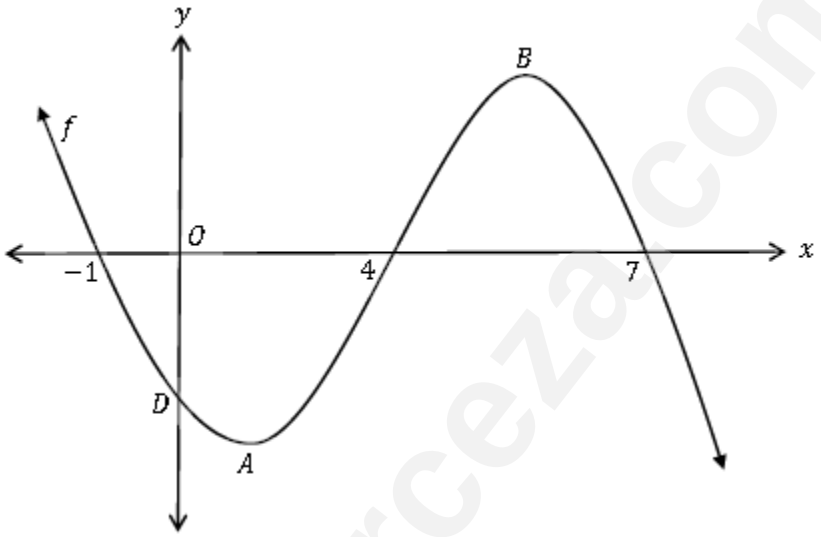
6.2.4 Determine the value(s) of x for which f is concave up

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 7

The sketch below shows the graph of $f(x) = -x^3 + 10x^2 - 17x + d$. The x-intercepts of f are (-1; 0), (4; 0) and (7; 0). A and B are turning points of f and D is the y-intercept of f.
The sketch is not drawn to scale.



7.1 Write down the value of d

7.2 Determine the coordinates of A and B

7.3 Determine the value of x where the concavity of f changes

7.4 Determine the coordinates of the point on f with a maximum gradient

7.5 Determine for which value(s) of x is $f(x).f'(x) > 0$

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 8

The following information about a cubic polynomial, $y = f(x)$ is given:

- $f(-1) = 0$
- $f(2) = 0$
- $f(1) = -4$
- $f(0) = -2$
- $f'(1) = f'(-1) = 0$
- If $x < -1$ then $f'(x) > 0$
- If $x > 1$ then $f'(x) > 0$

8.1 Use this information to draw a neat sketch graph of f using the grid on the DIAGRAM SHEET

8.2 For which value(s) of x is f strictly decreasing?

8.3 Use your graph to determine the x-coordinate of the point of inflection of f

8.4 For which value(s) of x is f concave up?

QUESTION 9

Given $g(x) = ax^3 + bx^2 + cx + d$ as well as the following:

- a < 0
- $g(-3) = g(8) = g(0) = 0$
- $g^1(-1) = g^1(5) = 0$

Draw a neatly labelled sketch of $g(x)$. Show all information. You are not required to label the y-coordinates of the turning point

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

Optimisation

- Calculus of motion
  - ✓ Equation is given
  - ✓ Velocity is the derivative of displacement
  - ✓ Acceleration(2nd derivative) is the derivative of velocity
- Rates of change
  - ✓ Knowledge of formulae for the surface area and volume of right prism is required
  - ✓ A list of relevant formulae will only be provided for the surface area and volume of cones, spheres and pyramids. Learners must select the correct one to use.

QUESTION 1

The displacement, (s) (distance from a fixed point) of a car moving towards a fixed point after t seconds, is given by:

$$s = t^3 - 9t^2 + 24t + 8$$

1.1 Determine an expression for the velocity (v) of the car (rate of change of distance with respect to time)

1.2 Calculate the time t for which the car was reversing

QUESTION 2

A stone thrown vertically into the air reaches a height of h metres after t seconds, and

$h = 120t - 16t^2$.

Find:

2.1 The time taken by the stone to reach its maximum height

2.2 The maximum height reached by the stone

2.3 The speed attained after 2 seconds

2.4 When the speed will be 24 metres per second

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 3

An industrial process requires water to flow through its system as part of the cooling cycle. Water flows continuously through the system for a certain period of time. The relationship between the time (t) from when the water starts flowing and the rate (r) at which the water is flowing through the system is given by the equation:

$$r = -0;2t^2 + 10t$$

3.1 After how long will the water be flowing at the maximum rate?

3.2 After how many seconds does the water stop owing?

QUESTION 4

The profit P (in Rands) from the production of x metres of steel cable per week, is given by:

$$P = 40x - 0,\ 1x^2 - 30$$

4.1 Find an expression for the rate of change of profit per metre of cable produced.

4.2 Find the length of cable to produce per week to ensure maximum profit

4.3 Find at what rate the profit is changing when the length of cable produced per week is 150 m.

QUESTION 5

During an experiment, the temperature T varies with time t according to the equation

$T(t) = 60 + 8t - t^2;\ t \in [0; 10]$

5.1 Determine an expression for the rate of change of temperature with time.

5.2 Determine the rate at which the temperature was changing after 2 seconds

5.3 Determine the maximum temperature

QUESTION 6

The mass, M, in milligrams of a certain bacteria varies according to the function $M(t) = 2t^3 + 10t + 3$, where t is the time in minutes.

Determine:

6.1 The rate of increase in the mass of the bacteria in the 5^{th} minute

6.2 Rate at which the mass grew during the first five minutes

6.3 Mas of the bacteria at the instant when the rate of increase is 226 milligrams per minute

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 7

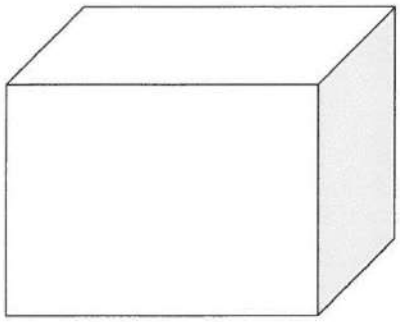
A watertank with an inlet and outlet is used to water a garden. The equation $D = 3 + \frac{1}{2}t^2 - \frac{1}{4}t^3$ gives the depth of water in metres where t is the time in hours that has elapsed since 09:00.

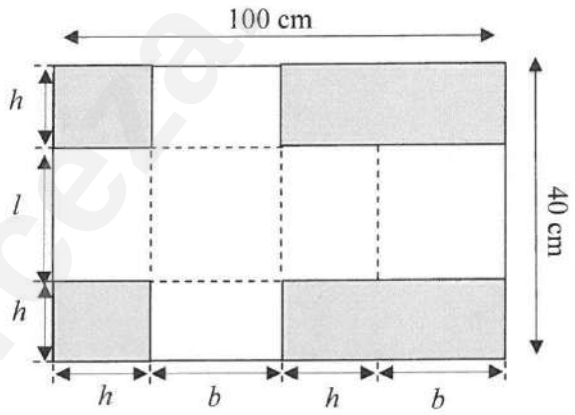
7.1 What is the depth of the water at 11:00

7.2 At what rate does the depth of the water change at 12:00?

7.3 At what time will the inflow of water be the same as the outflow of water?

QUESTION 8





A box is made from a rectangular piece of cardboard, 100 cm by 40cm, by cutting out the shaded areas and folding along the dotted lines as shown in the diagram above.

8.1 Express the length l in terms of the height h

8.2 Hence prove that the volume of the box is given by $V = h(50-h)(40-2h)$

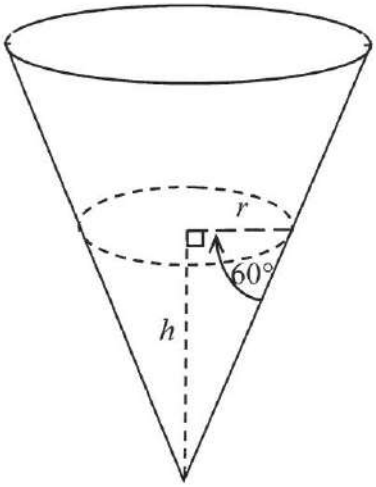
8.3 For which value of h will the volume of the box be a maximum?

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 9

A rain gauge is in the shape of a cone. Water flows into the gauge. The height of the water is h cm when the radius is r cm. The angle between the cone edge and the radius is 60^0, as shown in the diagram below.



Formulae for volume:

$V = \pi r^2 h$ $\quad V = \frac{1}{3}\pi r^2 h$

$V = lbh$ $\quad V = \frac{4}{3}\pi r^3$

9.1 Determine r in terms of h. Leave your answer in surd form

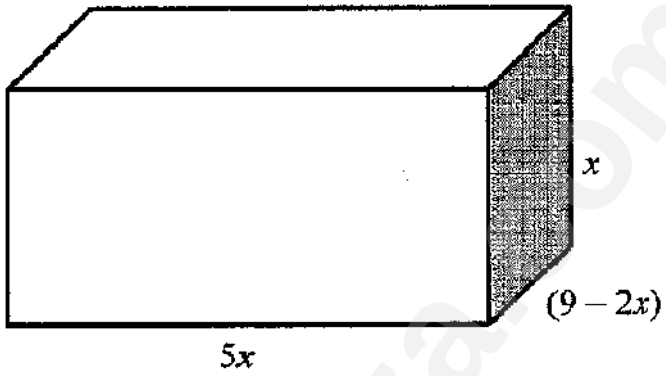
9.2 Determine the derivative of the volume of water with respect to h when h is equal to 9 cm

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 10

A rectangular box has a length of $5x$ units, breadth of $(9-2x)$ units and its height of x units.

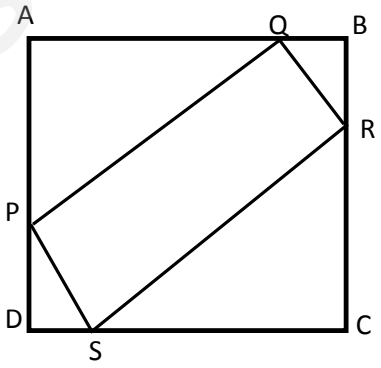


10.1 Show that the volume (V) of the box is given by $V = 45x^2 - 10x^3$

10.2 Determine the value of x for which the box will have maximum volume.

QUESTION 11

ABCD is a square with sides 20 mm each. PQRS is a rectangle that fits inside the square such that QB = BR =DS=DP = k mm



11.1 Prove that the area of PQRS = $-2k(k-20) = 40k - 2k^2$

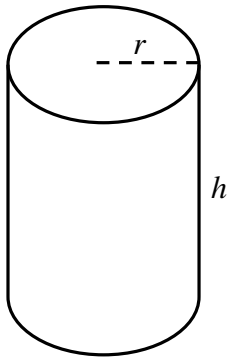
11.2 Determine the value of k for which the area of PQRS is a maximum

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 12

A soft drink can has a volume of 340 cm^3, a height of h cm and a radius of r cm.



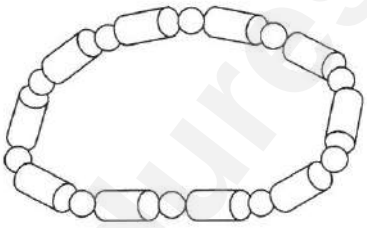
12.1 Express h in terms of r

12.2 Show that the surface area of the can is given by $A(r) = 2\pi r^2 + 680r^{-1}$

12.3 Determine the radius of the can that will ensure that the surface area is a minimum

QUESTION 13

A necklace is made by using 10 wooden spheres and 10 wooden cylinders. The radii, r, of the spheres and the cylinders are exactly the same. The height of each cylinder is h. The wooden spheres and cylinders are to be painted. (Ignore the holes in the spheres and cylinders)



$V = \pi r^2 h$	$S = 2\pi r^2 + 2\pi r h$
$V = \frac{4}{3}\pi r^3$	$S = 4\pi r^2$

13.1 If the volume of a cylinder is 6 cm^3, write h in terms of r

13.2 Show that the total surface area (S) of all the painted surfaces of the necklace is equal to $S = 60\pi r^2 + \frac{120}{r}$

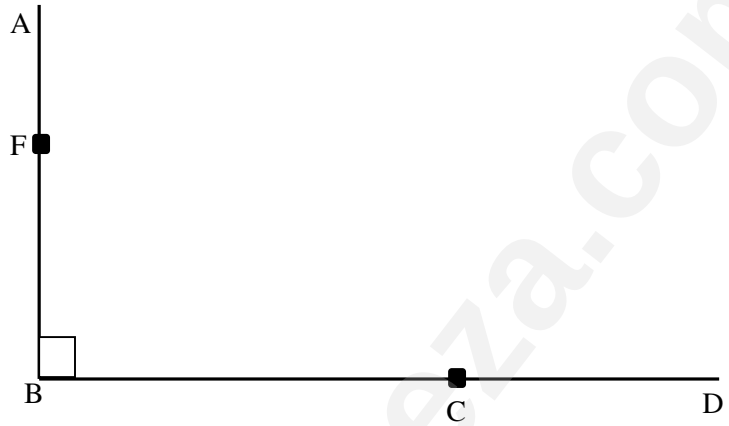
13.3 Determine the value of r so that the least amount of paint will be used

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 14

Two cyclists start to cycle at the same time. One starts at point B and is heading due north to point A, whilst the other starts at point D and is heading due west to point B. The cyclist starting from B cycles at 30km/h while the cyclist starting from D cycles at 40 km/h. The distance between B and D is 100km. After time *t* (measured in hours), they reach points F and C respectively.



14.1 Determine the distance between F and C in terms of *t*

14.2 After how long will the two cyclists be closest to each other?

14.3 What will the distance between the cyclists be at the time determined in QUESTION 10.2

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

PROBABILITY

- Terminology needs to be explained
  - ✓ Complimentary events
  - ✓ Mutually exclusive events
  - ✓ Independent events
- Application of addition and product rules
- Correct notation. n(A) versus P(A)
- Construction and interpretation of probability tools such as Venn diagram, Tree diagram and Contingency table.
- Meaning and Application of Factorial
  - ✓ Codes (repetition versus non-repetition)
  - ✓ Arrangement of words (distinct , letters of the alphabet, e.g. vowels and consonants)
  - ✓ Arrangement of items
  - ✓ Grouping of items
- Calculation of probability based on counting principle

QUESTION 1

Events A and B are such that P (A) = $\frac{1}{4}$ and P (A or B) = $\frac{1}{3}$. Find P (B) (as a simplified fraction) if:

1.1 A and B are mutually exclusive.

1.2 A and B are not mutually exclusive, but A and B are independent events.

QUESTION 2

Events D and K are mutually exclusive. It is given that:

- ❖ P(K) = 3 P(D)
- ❖ P(D or K) = 0,64

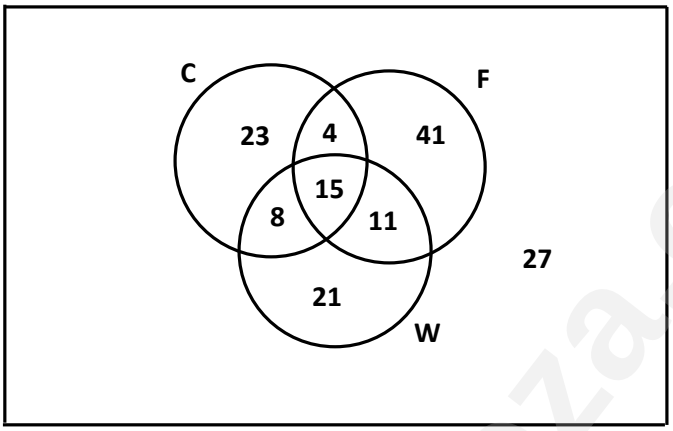
Calculate P(D)

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 3

A school organized a dance for their 150 Grade 12 learners. The learners were asked to indicate their preference for the theme. They had to choose from Casino (C), France (F) and Winter Wonderland (W). The information collected is shown in the Venn diagram below.



3.1 Calculate the probability that a learner, chosen at random:

3.1.1 Does not prefer the Casino or the France theme

3.1.2 Prefers only **TWO** of the given theme choices

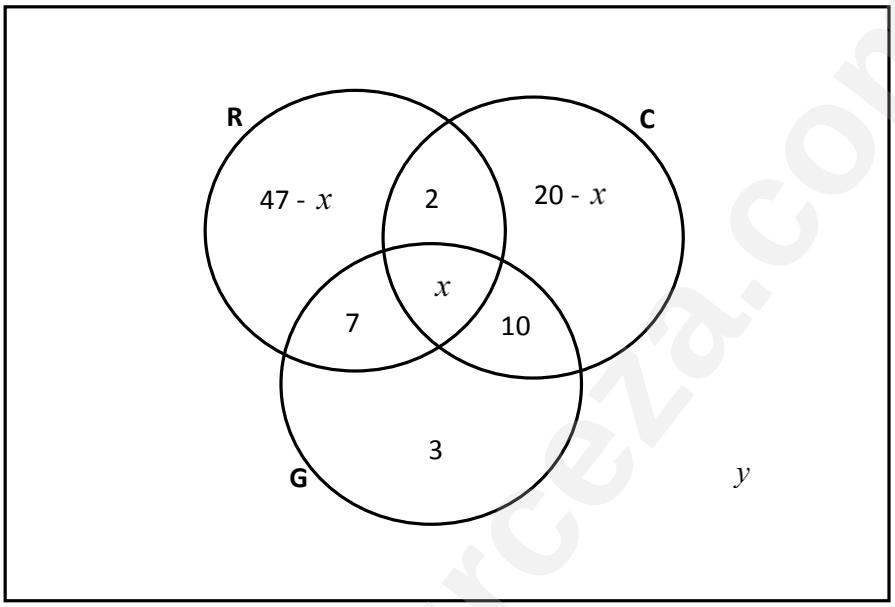
3.2 Show with all working whether the events preferring Casino (C) and preferring France (F) are independent or not.

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 4

There are 115 people in a group. The Venn-diagram below shows the number of people who enjoys listening to radio (R), enjoy gardening (G) and/ or enjoy cooking (C). There are x people who enjoy any of the activities.



4.1 If there are 28 people who enjoy gardening, calculate the value of x.

4.2 Hence determine the value of y.

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 5

A plastic container of tablets contains 3 pink, two green and 5 blue tablets. Two tablets are removed in succession from the container without replacement.

5.1 Draw a tree diagram to represent all outcomes of the above information.

5.2 Round off your answer to 3 decimal digits where necessary, and determine the probability that:

5.2.1 Both tables are blue
5.2.2 At least one pink table is selected

QUESTION 6

6.1 There are 20 boys and 15 girls in a class. The teacher chooses individual learners at random to deliver a speech.

6.1.1 Calculate the probability that the first learner chosen is a boy.

6.1.2 Draw a tree diagram to represent the situation if the teacher chooses three learners, one after the other. Indicate on your diagram ALL possible outcomes.

6.2 Calculate the probability that a boy, then a girl and then another boy is chosen in that order.

6.2.1 Calculate the probability that all three learners chosen are girls.

6.2.2 Calculate the probability that at least one of the learners chosen is a boy.

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 7

In a survey 1 530 skydivers were asked if they had broken a limb. The results of the survey were as follows:

	Broken a limb	**Not broken a limb**	**TOTAL**
MALE	463	b	782
FEMALE	a	c	d
TOTAL	913	617	1 530

7.1 Calculate the values of a, b , c and d

7.2 Calculate the probability of choosing at random in the survey, a female skydiver who has not broken a limb

7.3 Is being a female skydiver and having broken a limb independent?
Use calculations, correct to TWO decimal places, to motivate your answer.

QUESTION 8

The South African Traffic Service is doing a clamp down on speeding. During a recent speeding trap they collected the following data:

	Speeding	**Not Speeding**	**TOTAL**
Male	398	217	615
Female	205	180	385
TOTAL	603	397	1000

8.1 Determine the probability that a driver selected at random is…

8.1.1 Speeding

8.1.2 A female driver and NOT speeding

8.1.3 A male driver and speeding

8.2 Are the events of being a male and speeding independent? Show ALL relevant working to support your answer

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 9

Every client of **NEDBANK** has a personal identity number (PIN) which is made up of 5 digits chosen from the digits 0 to 9

9.1 How many PIN can be made if:

9.1.1 Digits can be repeated

9.1.2 Digits cannot be repeated

9.2 What is the probability that such a PIN will contain at least one nine and the digits can be repeated

QUESTION 10

How many 4 digit numbers **larger than 6 000** can be made using the digits 0, 1, 2, 5, 6, 8, 9 if you may not repeat digits?

QUESTION 11

Three brothers Thapelo, Kgothatso and Phenyo are to run in a race which has eight runners in total. The eight competitors' line up one to a lane, in the lane numbered 1 – 8

11.1 Write down the total number of arrangements at the starting line.

11.2 Find the total number of arrangements in which the three brothers are all next to each other

11.3 Find the probability that Thapelo is in lane 1, Kgothatso in lane 2 and Phenyo is on lane 3

QUESTION 12

Seven cheerleaders each have **one** letter of the word **PIRATES** on their backs. Typically, when the cheerleaders get excited, they might not stay in the correct order

12. 1 How many arrangements could the girls possibly stand in, if order does not matter

12.2 Cheerleaders P and R are good friends, and they are keeping together all the time. How many ways can the cheerleaders arrange themselves if P and R stay together but the others don't?

12.3 What is the probability that cheerleader P will not be the first on the line

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

QUESTION 13

Seven cars of different manufactures, of which 3 are silver, are to be parked in a straight line.

13. 1 In how many different ways can ALL the cars be parked?

13.2 If the three silver cars must be parked next to each other, determine in how many different ways the cars can be parked.

13.3 What is the probability that the silver cars will be parked next to each other?

QUESTION 14

The letters from the word KUTLWANONG are arranged as shown below on separate cards

14.1 How many other "words" can be arranged using all these cards?

14.2 What is the probability that a "word" made, has all the vowels next to each other.

Downloaded from eduresourceza.com

Downloaded from eduresourceza.com

eduresourceza.com

Downloaded from eduresourceza.com