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Core Mathematics

Skills Checklist for Paper 1

This skills checklist is to help you identify areas of weakness for extra practice and / or seeking help. It will also assist you in understanding how everything fits together. In each case I have given an example. However, one must be aware that there are infinitely many variations so it is impossible to cover them all!

The high level breakdown for Paper 1 is as follows:

Weighting of Content Areas	
Description	Marks
Algebra and Equations (and inequalities)	25 ± 3
Patterns and Sequences	25 ± 3
Finance, growth and decay	15 ± 3
Functions and Graphs	35 ± 3
Differential Calculus	35 ± 3
Probability	15 ± 3
TOTAL	150

Algebra and Equations (and inequalities)

I know how to:

- ☐ Apply the laws of exponents to

- ☐ simplify expressions

$$\frac{9^{x+1} - 3^{2x}}{3^{-x} \times 3^{3x+1}} \quad \frac{12^x 3^{x+1}}{4^{x+1} 9^x}$$

- ☐ solve equations

$$3^{x+1} - 3^x = 7 \quad 2^{2x} - 6(2^x) + 8 = 0$$

- ☐ Work with fractions

$$\frac{2ab}{\frac{1}{a} + \frac{1}{b}} \quad \frac{x^2 - 1}{x^3 - 1} \div \frac{2x + 2}{2x^2 + 2x + 2}$$

- ☐ Simplify expressions involving surds

$$\frac{\sqrt{20} + \sqrt{12}}{\sqrt{500} + \sqrt{300}}$$

- ☐ Rationalise the denominator of an expression

$$\frac{2}{\sqrt{2}}$$

$$\frac{3}{5 - \sqrt{3}}$$

- ☐ Apply the log laws to

- ☐ simplify expressions without a calculator

$$2\log 2 + 2\log 5 \quad \frac{\log_3 8}{\log_3 2}$$

- ☐ solve equations

$$3^x = 19 \quad \log_x 9 = 2 \quad \log_2 x + \log_2 (x - 2) = 3\log_2 2$$

- ☐ Solve quadratic equations by completing the square

$$x^2 + 4x - 1 = 0 \quad 2x^2 + 3x - 6 = 0 \quad ax^2 + bx + c = 0$$

- ☐ Solve quadratic equations

- ☐ Involving square roots

$$\sqrt{x+2} - x = -2x + 10$$

- ☐ Involving substitutions

$$x^2 - 2x = 18 - \frac{45}{x^2 - 2x}$$

- ☐ Involving fractions

$$\frac{x+2}{x+1} - \frac{3}{x-2} = \frac{1}{x+1}$$

- ☐ Solve word problems involving quadratic equations

R8.00 is divided equally among some people. If there had been 4 more people and R9.00 had been distributed, each would have received 5c less. How many people were there?

- ☐ Determine the nature of the roots of a quadratic equation using $\Delta = b^2 - 4ac$

Determine the nature of the roots of the equation: $10x^2 - x - 2 = 0$ without solving it.

Without solving it, show that the roots of $p^2x^2 + 6px + 9$ are equal for all rational values of p .

- ☐ Use information about the nature of the roots of a quadratic to solve for a variable

For what value(s) of m will the equation $mx^2 + 2 = 4x$ have equal roots?

- ☐ Factorise a cubic function

$$2x^3 + 5x^2 - 4x - 3$$

$$3x^3 + 7x^2 - 4$$

- ☐ Solve a cubic equation

$$x^3 - 4x^2 - 11x + 30 = 0$$

- ☐ Solve simultaneous equations

$$7 = 2x + 3y$$

$$5 = -2x + 3y$$

$$x + 2y = 5$$

$$2y^2 - xy - 4x^2 = 8$$

- ☐ Solve linear inequalities

$$3(2 - x) - 2 \leq 22$$

- ☐ Solve quadratic inequalities

$$x^2 + 2x - 5 > 3$$

- ☐ Use my calculator to solve:

- ☐ Quadratic equations

$$x^2 + 3x - 5 = 0$$

- ☐ Cubic equations

$$x^3 - 7x^2 + 36 = 0$$

- ☐ General equations

$$\text{Solve } 3^x + \log x = 5$$

- ☐ Apply all of the above to solving unseen problems

I know how to:

- ☐ Identify the following types of sequence

- ☐ Arithmetic

Find the next term in: 3 ; 8 ; 13 ; 18 ; ____

Find x if $3x-1$; $2x+4$; $5x+1$ are in arithmetic progression

- ☐ Geometric

Find the next term in: 100 ; -20 ; 5 ; ____

Find x if $3x+1$; $2x+6$; $6x+10$ are in geometric progression

- ☐ Quadratic

Find the next term in: 1 ; 3 ; 6 ; 10 ; ____

- ☐ Find the n^{th} term of

- ☐ An arithmetic sequence

Find T_n for 3 ; 7 ; 11 ;

Which term in the sequence: 25; 14; 3; is equal to -228

- ☐ A geometric sequence

Find T_n for 2 ; 6 ; 18 ;

- ☐ A quadratic sequence

Find T_n for 2 ; 7 ; 14 ; 23 ; 34 ;

- ☐ Insert

- ☐ Arithmetic means

Insert 5 terms between -12 and 30 so that the 7 numbers are in arithmetic sequence

- ☐ Geometric means

Insert 4 geometric means between 4096 and -128

- ☐ Find the sum of

- ☐ An arithmetic series

Determine $2+5+8+11+\dots$ to 23 terms

- ☐ A geometric series

Determine $2+6+18+\dots+118098$

- ☐ Interpret sigma notation

Determine x if $\sum_{k=1}^{23} (2kx+1) = -1633$

Determine n if $\sum_{p=3}^n (2^{-p}) = \frac{127}{512}$

- ☐ Use my calculator for sigma notation

Determine $\sum_{i=1}^6 (i^3 - 5i^2)$

- ☐ Determine whether or not a given geometric series will converge

Justify whether or not 10 ; 11 ; 12.1 ; 13.31 ; $\dots$ will converge.

Determine for which values of x the series: $5-10x+20x^2-40x^3+\dots$ will converge

- ☐ Calculate a sum to infinity

A ball is dropped from a height of 5m. If it bounces indefinitely, each time bouncing back to 80% of the height from which it falls then how far will it travel in total?

- ☐ Apply all of the above to solving unseen problems

The terms 2; x ; y and 9 are such that the first three terms form an arithmetic sequence and the last three terms form a geometric sequence. Calculate the values of x and y .

I know how to:

- ☐ Solve problems involving investments

A man invests R5000 today. After 3 years he withdraws R2000. How much will he have in 5 years from now if interest is paid at 10% per annum compounded monthly for the first two years and then at 16% per annum compounded quarterly for the next three years?

- ☐ Calculate depreciation

- ☐ By straight line

What rate of depreciation will reduce the value of an asset from R500 to R200 in 10 years if the straight line method is used?

- ☐ By reducing balance

What will a R450 000 car be worth in 5 years time if it is depreciated by 23% per annum according to the reducing balance formula?

- ☐ Convert a nominal rate to an effective rate

Convert a rate of 14% per annum compounded quarterly to an effective annual rate.

- ☐ Convert an effective rate to a nominal rate

What annual rate compounded monthly is equivalent to an effective annual rate of 20%?

- ☐ Calculate, for an investment,

- ☐ The amount to invest each period to raise a given sum

How much money will I have immediately after making my 33rd payment of R500 if interest is paid at 12% per annum compounded monthly?

- ☐ The sum raised given a fixed investment

How much must I pay each month to raise R10 000 in 4 years if interest is to be paid at 10% per annum compounded monthly?

- ☐ The number of payments required to raise a sum

I wish to raise an amount of R100 000 by making monthly payments of R1200. How many payments must I make if interest is to be paid at 19% per annum compounded monthly?

- ☐ Calculate, for a loan,

- ☐ The fixed repayment amount

How much must I repay each month if I wish to pay off a loan of R30 000 in two years and interest is charged at 12% per annum compounded monthly?

- ☐ The amount I can borrow given a repayment amount

I wish to borrow some money to buy a car. I will repay it over 5 years with monthly payments of R2340. How much can I borrow if interest is to be charged at 15% per annum compounded monthly?

- ☐ The number of payments required to pay off a loan

I borrow R50 000 and I repay it with monthly payments of R1100. How many payments must I make if interest is charged at 19% per annum compounded monthly?

- ☐ The outstanding balance immediately after making a payment

I repay a loan of R30 000 in two years and interest is charged at 12% per annum compounded monthly? How much will I owe immediately after making the 14th payment?

- ☐ The proportion of a payment going to interest or capital

I repay a loan of R60 000 in seven years and interest is charged at 14% per annum compounded monthly? What percentage of the 60th payment goes to paying off capital?

- ☐ Apply all of the above to solving unseen problems

Functions and Graphs

I know how to:

- ☐ Sketch the following functions

- ☐ Linear

Draw $2x + 4y = 8$ and $y = x + 4$ showing where they intersect the axes and where they intercept one another.

- ☐ Quadratic

Draw $y = x^2 + 5x + 4$

Draw $y = ax^2 + bx + c$ if $a < 0$, $b > 0$ and $c < 0$

- ☐ Hyperbola

Draw $y = \frac{-12}{x+2} + 4$

- ☐ Exponential

Draw $y = 3 \times 2^x + 1$ and $y = \left(\frac{1}{2}\right)^x - 3$

- ☐ Logarithmic

Draw $y = \log_2(x) + 1$ and $y = \log_{\frac{1}{2}}(x - 2)$

- ☐ Find the equations of the following functions given their graphs

- ☐ Linear

Find the equation of a line which passes through $(-2; 7)$ and $(3; -3)$

- ☐ Quadratic

Find the equation of a parabola which has -2 and 3 as x -intercepts and passes through $(2; 8)$

Find the equation of a parabola which has a maximum value of 2 when $x = -1$ and passes through $(1; 14)$

- ☐ Hyperbola

Determine the equation of a hyperbola which passes through $(3; 9)$ and has the following asymptotes: $x = 2$ and $y = 1$

- ☐ Exponential

Determine the equation of an exponential graph which has an asymptote of $y = -2$, and passes through the origin and the point $(2; 16)$

- ☐ Logarithmic

Find the equation of a log graph which has the y -axis as asymptote, an x -intercept of $\frac{1}{8}$ and passes through the point $(1; 3)$

- ☐ Determine the domain of a function

Determine the domain of the following:

$$y = x^2 + 4x + 2$$

$$y = \sqrt{-x^2 - 2x + 8}$$

$$y = \frac{12}{x+2} + 2$$

- ☐ Determine the range of a function

Determine the range of the following:

$$y = x^2 + 4x + 2$$

$$y = \sqrt{-x^2 - 2x + 8}$$

$$y = \frac{12}{x+2} + 2$$

- ☐ Determine the inverse of a function

Determine $f^{-1}(x)$ if

$$f(x) = 3x + 2$$

$$f(x) = 2^x + 1$$

$$f(x) = \log_3(x - 2) + 1$$

$$f(x) = \frac{12}{x+2} - 3$$

- ☐ Transform a function by
- ☐ Shifting it horizontally
 - ☐ Shifting it vertically
 - ☐ Reflecting it in the x -axis
 - ☐ Reflecting it in the y -axis
 - ☐ Reflecting it in the line $y = x$

How could one translate the function $y = x^2 + 4x + 2$ to get the function $y = x^2 - 2x - 1$?

The function $f(x) = x^2 + 2x - 6$ is moved 2 units left and 1 up. Give its new equation in standard form.

The function which results when $y = 2^x - 3$ is reflected in the x -axis

The function which results when $y = \frac{12}{x-2} + 3$ is reflected in the line $y = x$

- ☐ Interpret graphs

For example, given the graphs of two functions f and g , use the graphs to solve:

$$f(x) \leq 0$$

$$f(x) > g(x)$$

Determine the maximum / minimum distance between two graphs using completing the square or calculus

- ☐ Apply all of the above to solving unseen problems

I know how to:

- ☐ Calculate a limit

$$\lim_{x \rightarrow 1} \frac{x^3 + 5x^2 - 4x - 2}{x^2 - 1}$$

- ☐ Calculate a derivative by
 - ☐ First principles

Find $f'(x)$ if $f(x) = -x^2 + 2x - 3$

- ☐ The rules of differentiation

$$y = \frac{x^3 + 5}{2x^2}$$

$$g(x) = \sqrt[4]{x^3} + (x+3)^2 - \frac{x^2 - 4}{x - 2} + \sqrt{3x} + \sqrt{3}x + \sqrt{3}$$

- ☐ Determine the equation of a tangent

Find the equation of the tangent to the curve $y = 3x^2 + 4x - 4$ at the point where $x = -1$

- ☐ Determine the coordinates of a
 - ☐ Turning point

Find the turning points of $f(x) = 2x^3 + 15x^2 - 36x - 81$

- ☐ Point of inflection

Find the point of inflection of $f(x) = x^3 + 6x^2 - 5x + 2$

- ☐ Determine whether a function is increasing, decreasing or stationary at a given point

Given the function $f(x) = x^3 - 6x + 2$ say whether it is increasing, decreasing or stationary at the points:

$$x = -1$$

$$x = 0$$

$$x = \sqrt{2}$$

- ☐ Determine the concavity of a graph

Justifying your answers, describe the concavity of the function:

$f(x) = x^3 - 3x^2 + 5x - 2$ at the points

$$x = -2$$

$$x = 6$$

$$x = 0$$

- ☐ Determine the nature of a stationary point using the second derivative

The function $y = 2x^3 - x^2 - 60x + 108$ at $x = \frac{4}{3}$. Determine whether it is a local maximum or a local minimum.

- ☐ Apply the above to sketch a cubic function

Sketch $y = 2x^3 + 13x^2 - 36$

- ☐ Use a graph to solve an inequality involving function and derivative

For example, given the graph of functions f , use the graph to solve:

$$f'(x) \leq 0$$

$$f(x)f'(x) > 0$$

$$f'(x)f''(x) > 0$$

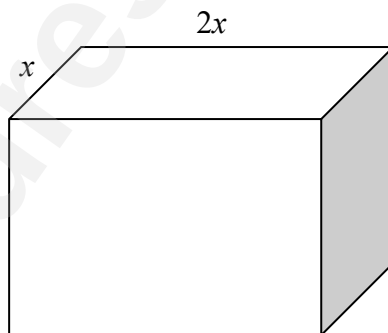
- ☐ Solve optimisation problems

A farmer has 500 boxes of peaches for sale at R6 per box. If he keeps the peaches with the intention of selling them later, the price increases by 10 c per box per week. However, every week 5 boxes become polluted and therefore cannot be sold.

After how many weeks did he sell the peaches if he received R3 182 for them?

For how many weeks should he keep the peaches in order to achieve maximum profit?

The length, breadth and height of a box must add up to 63 cm. John has a box with a length of $2x$ and a breadth of x . Determine the dimensions of the box which will give a maximum volume.



- ☐ Solve problems involving rates of change

The height of a stone (in m), thrown vertically up, after t seconds, is given by $h(t) = 15t - 5t^2$. Find the maximum height the ball reaches and the time it takes to reach that height.

A stone is thrown vertically upwards. Its height (in meters) at time t (in seconds) is given by $s = 30t - 5t^2$. Determine its:

- a) Average velocity in the first second
 - b) Instantaneous velocity after 1.5 seconds
-

During an experiment, the concentration of glucose in the human body, $M(t)$ varied according to the formula: $M(t) = 4.5 - 0.03t^2$ where t is measured in hours and the concentration of glucose is measured in milligrams (mg).

Determine the average rate of change of glucose concentration in the body over the interval between $t=0$ to $t=3$ hours, giving the correct units for your answer.

Determine the instantaneous rate of change of glucose concentration when $t=1$, giving the correct units for your answer.

Did the concentration of glucose increase or decrease during the first three hours? (1)

- ☐ Apply all of the above to solving unseen problems

The graph of $y = ax^2 + bx - 1$ has a local minimum at the point $(2; -13)$. Determine a and b .

Probability

I know how to:

- ☐ Apply the laws of probability to solve probability problems

If $P(\text{taking science}) = 0.3$ and $P(\text{playing rugby}) = 0.5$

Calculate

$$P(\text{science} \cup \text{rugby})$$

$$P(\text{science} \cap \text{rugby})$$

If “taking science” and “playing rugby” are independent of one another

- ☐ Calculate complementary probabilities

If the probability of a hole in one in golf is 0.01% what is the probability of not hitting a hole in one?

- ☐ Test whether or not events are independent

Are the events A and B independent or not? Why?

$$P(A) = 0.3$$

$$P(B) = 0.4$$

$$P(A \cap B) = 0.2$$

- ☐ Test whether or not events are mutually exclusive

Are the events A and B mutually exclusive or not? Why?

$$P(A) = 0.3$$

$$P(B) = 0.4$$

$$P(A \cap B) = 0.2$$

- ☐ Count the number of ways of
- ☐ Arranging unlike objects

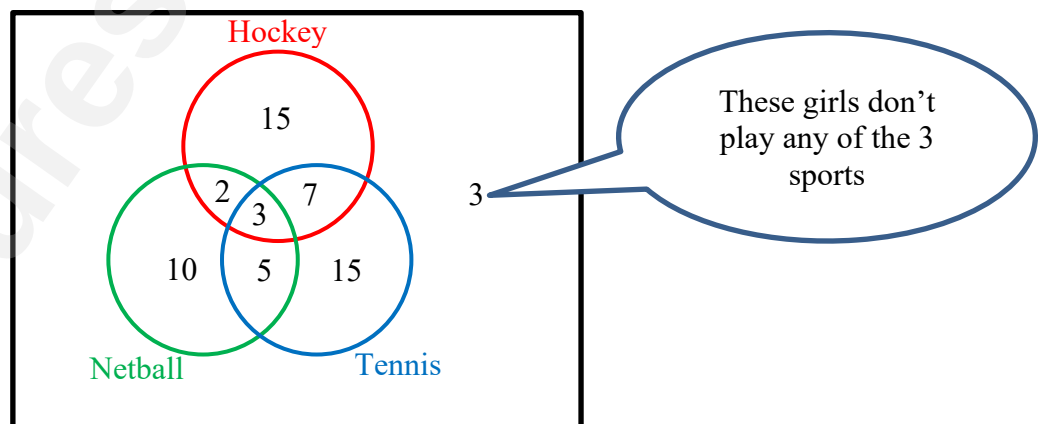
In how many ways can I arrange 7 books on a shelf?

- ☐ Arranging a mixture of unlike and like objects

In how many ways can I arrange 6 books on a shelf if the three hard cover books must be next to one another?

How many different “words” can you make using the letters millennium?

- ☐ Draw and use a Venn Diagram



What is the probability that a girl chosen at random:

1. Plays tennis or hockey?
2. Plays exactly 2 sports?
3. Plays netball but not hockey?
4. Plays at least one of the three sports?

- ☐ Draw and use a Probability tree

A play list on my iPod has 15 songs on. 5 of them are by male artists and the balance by females. If I start playing the play list in random shuffle mode then use a tree diagram to determine the probability that exactly two of the next three songs are by male artists? Note that the iPod will not repeat a song before it has played the entire play list. Express your answer as a percentage to one decimal place.

- ☐ Test for independence from a two-way contingency table

A random sample of 240 people was asked whether or not they watch the sitcom, “Days of our Lives”. The results are recorded in the table below.

	Boys	Girls	Totals
Watch	122	34	160
Don't watch	58	26	80
Totals	180	60	240

By doing an appropriate calculation, determine whether or not viewing preference is dependent on gender.

- ☐ Apply all of the above to solving unseen problems

General

I know how to:

- ☐ Apply all of the above to solving unseen problems

There are many examples available in past papers.

Important disclaimer:

While every effort has been made to ensure that the above list is exhaustive, it may not be!