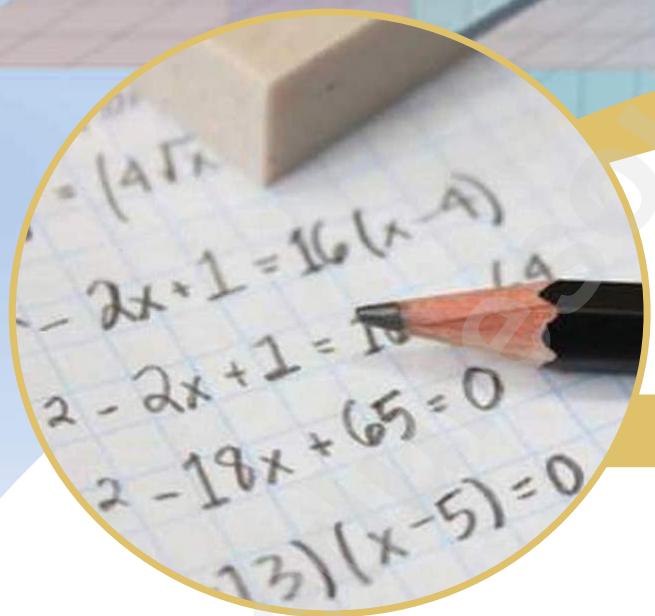
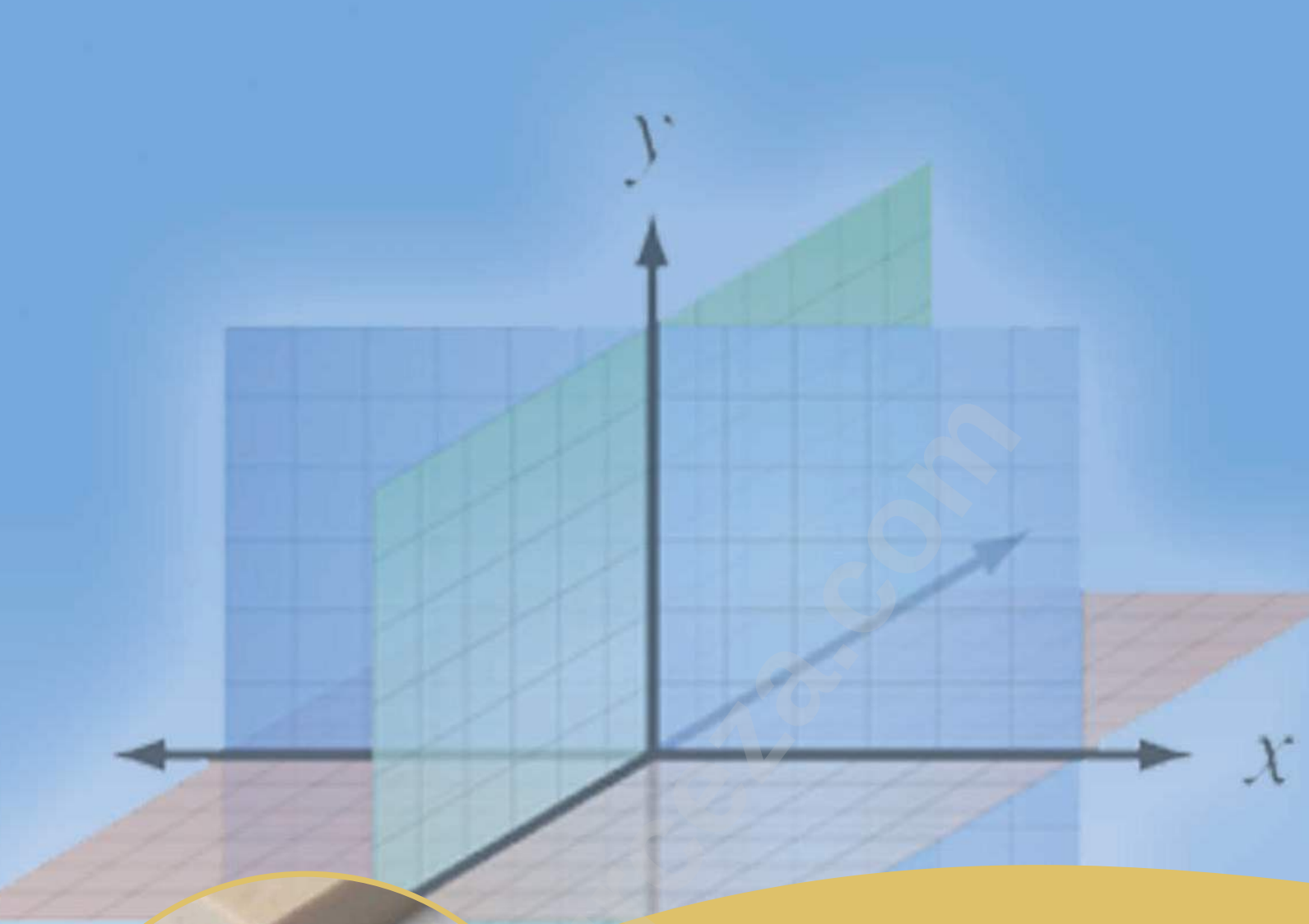




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MATHEMATICS

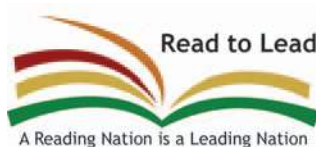
ANALYTICAL GEOMETRY

GRADE 12



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA



Foreword

In order to improve learning outcomes the Department of Basic Education conducted research to determine the specific areas that learners struggle with in Grade 12 examinations. The research included a trend analysis by subject experts of learner performance over a period of five years as well as learner examination scripts in order to diagnose deficiencies or misconceptions in particular content areas. In addition, expert teachers were interviewed to determine the best practice to ensure mastery of the topic by learners and improve outcomes in terms of quality and quantity.

The results of the research formed the foundation and guiding principles for the development of the booklets. In each identified subject, key content areas were identified for the development of material that will significantly improve learner's conceptual understanding whilst leading to improved performance in the subject.

The booklets are developed as part of a series of booklets, with each booklet focussing only on one specific challenging topic. The selected content is explained in detail and include relevant concepts from Grades 10 - 12 to ensure conceptual understanding.

The main purpose of these booklets is to assist learners to master the content starting from a basic conceptual level of understanding to the more advanced level. The content in each booklet is presented in an easy to understand manner including the use of mind maps, summaries and exercises to support understanding and conceptual progression. These booklets should ideally be used as part of a focussed revision or enrichment program by learners after the topics have been taught in class. The booklets encourage learners to take ownership of their own learning and focus on developing and mastery critical content and skills such as reading and higher order thinking skills.

Teachers are also encouraged to infuse the content into existing lesson preparation to ensure in-depth curriculum coverage of a particular topic. Due to the nature of the booklets covering only one topic, teachers are encouraged to ensure learners access to the booklets in either print or digital form if a particular topic is taught.

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2. HOW TO USE THIS BOOKLET

This booklet is designed to clarify the content prescribed for Technical Mathematics. It includes some tips on how to tackle real-life problems on a daily basis. Candidates are expected to have already mastered the content outlined for Grades 8-11.

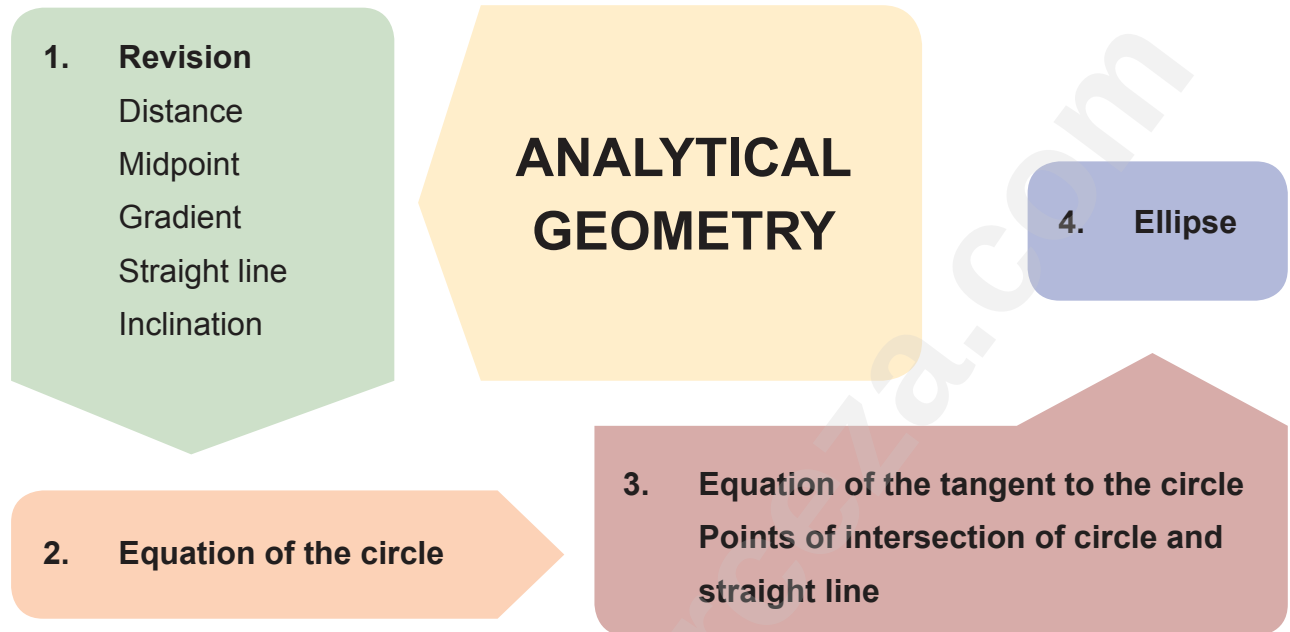
This booklet must be used to master some mathematical rules that you may not have been aware of. The prescribed textbook must also be used.

3. STUDY AND EXAMINATION TIPS

All learners should be able to acquire sufficient understanding and knowledge to:

- develop fluency in computation skills without relying on the use of a calculator;
- generalise, conjecture and try to justify or prove something;
- develop problem-solving and cognitive skills;
- make use of the language of Technical Mathematics;
- identify, investigate and solve problems creatively and critically;
- use the properties of shapes and objects to identify, investigate and solve problems creatively and critically;
- encourage appropriate communication by using descriptions in words, graphs, symbols, tables and diagrams;
- practise Technical Mathematics every day.

4. MIND MAP OF ANALYTICAL GEOMETRY



4.1 Revision of Grades 10 and 11

In **Grade 10** you learnt the following formulae:

1) Distance between A and B

If points A ($x_1; y_1$), B ($x_2; y_2$), C ($x_3; y_3$) and D ($x_4; y_4$), are given:

$$AB = d_{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

2) Gradient of a line AB

$$M_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

3) Mid-point of a line AB

$$M_{AB} = \left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2} \right)$$

4) Equation of a line

$$y = mx + c$$

OR

$$y - y_1 = m(x - x_1)$$

5) **AB // CD (parallel)**

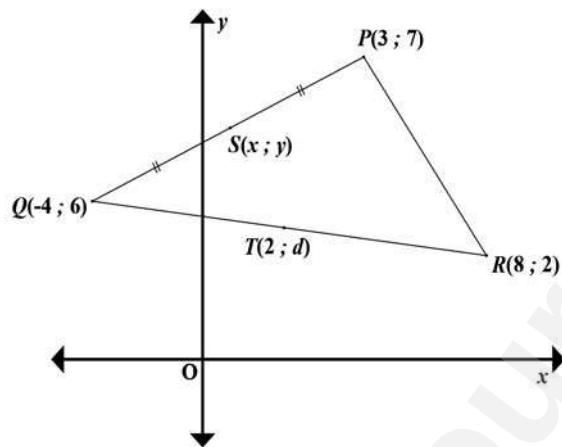
$$m_{AB} = m_{CD}$$

6) **AB ⊥ CD (perpendicular)**

$$m_{AB} \times m_{CD} = -1$$

Examples:

1.1



1.1.1 Determine the gradient of PR.

$$\begin{aligned} M_{PR} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - 6}{2 - (-1)} \\ &= -\frac{2}{3} \end{aligned}$$

Identify the formula.

Substitute values from the required points P and R.

Simplify to get the value of the gradient.

1.1.2 Determine the equation of the straight line passing through P and R in the form $y = \dots$	$M_{PR} = -\frac{2}{3}$ $y = mx + c$ $6 = -\frac{2}{3}(-1) + c$ $6 - \frac{2}{3} = c$ $\frac{16}{3} = c$ $\therefore y = -\frac{2}{3} + \frac{16}{3}$	Find the gradient of PR (as calculated above). Substitute into the equation of the line in standard form. Simplify and make c the subject of the formula. Write the equation of the required line.
1.1.3 Determine the midpoint of SQ.	$M_{SQ} \left(\frac{x_2 + x_1}{2}; \frac{y_2 + y_1}{2} \right)$ $M_{SQ} \left(\frac{4 - 4}{2}; \frac{8 - 3}{2} \right)$ $M_{SQ} \left(0; \frac{11}{3} \right)$	Midpoint formula. Substitute values for points S and Q. Simplify and write the mid-point in coordinate form.

1.1.4 Is PS $\parallel$ QR?

$$m_{PS} = \left(\frac{y_2 - y_1}{x_2 - x_1} \right)$$

$$m_{PS} = \left(\frac{3 - 6}{-4 - (-1)} \right)$$

$$= \frac{3}{3}$$

$$= 1$$

and

$$m_{QR} = \left(\frac{y_2 - y_1}{x_2 - x_1} \right)$$

$$m_{QR} = \left(\frac{4 - 8}{2 - 4} \right)$$

$$= -\frac{4}{2}$$

$$= -2$$

$$m_{PS} \neq m_{QR}$$

PS is not parallel to QR.

1.1.5 Calculate the length SR.

$$SR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(2 + 4)^2 + (4 - 3)^2}$$

$$= \sqrt{(6)^2 + (1)^2}$$

$$= \sqrt{37}$$

Distance formula.

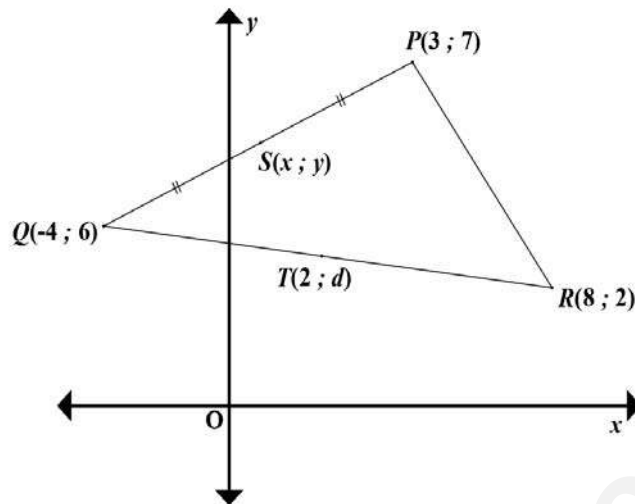
Substitute into formula.

Simplify.

(Leave SR in SURD form if it is not a perfect square.)

For you to do:

1.



1.1	Determine the gradient of QR.	$-\frac{3}{4}$	(2)	✓Substitution ✓Gradient
1.2	Determine the equation of line QR.	$y = -\frac{3}{4}x + 3$	(3)	✓Substitution ✓Simplification ✓Equation of the line
1.3	Calculate the value of d if $T(2;d)$ lies on QR.	$-\frac{3}{4}$	(3)	✓Substitution ✓Simplification ✓Value of d
1.4	Determine the length of QP.	$\sqrt{170}$	(3)	✓Substitution ✓Simplification ✓Length QR
1.5	Determine the coordinates of S, and the midpoint of QP.	$S\left(-\frac{1}{2} ; -\frac{13}{2}\right)$	(4)	✓✓Substitution ✓Simplification ✓Point S

4.2. Coordinate Geometry:

The important formulae for coordinate geometry are:

- Gradient formula: $m_{PS} = \left(\frac{y_2 - y_1}{x_2 - x_1} \right)$
- Distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Mid-point formula: $M_{AB} = \left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2} \right)$
- Straight line equation: $y = mx + c$ or $y - y_1 = m(x - x_1)$
- Inclination: $m = \tan \theta$
- Circle: $x^2 + y^2 = r^2$
- Semi-circle: $y = \pm \sqrt{r^2 - x^2}$

Example:

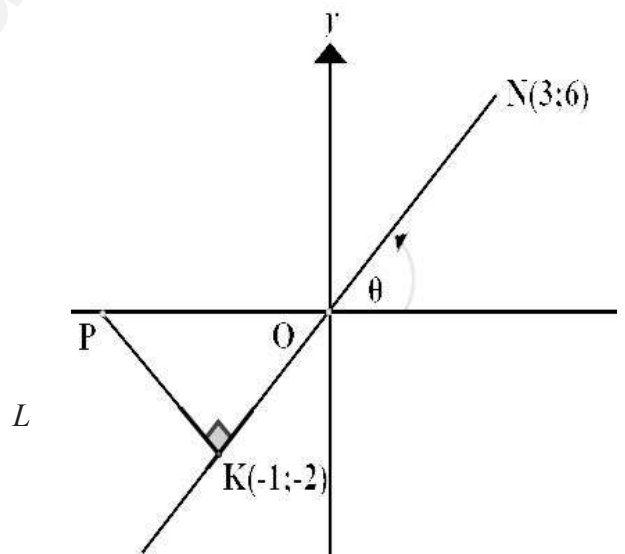
1. In the diagram alongside,
is the midpoint of LN with N(3;6).
PK $\perp$ LN with P on the x-axis.
The angle of inclination of NL is θ .

1.1 Determine:

- 1.1.1 the gradient of NK
- 1.1.2 the size of θ , rounded off to one decimal place.
- 1.1.3 The coordinates of L.
- 1.1.4 The length of NK, rounded off to one decimal place.

1.2 Determine the equation of the:

- straight line parallel to PK
- and which passes through N.



Solutions:

No.	Solution	Marks
1.1.1	$m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \frac{6 + 2}{3 + 1}$ $m = 2$	✓ Correct gradient formula ✓ Correct substitution in correct formula ✓ Answer
1.1.2	$\tan \theta = 2$ $\therefore \theta = 63,4^\circ$	✓ Correct inclination formula ✓ Correct answer
1.1.3	$x = \frac{x_1 + x_2}{2} \quad y = \frac{y_1 + y_2}{2}$ $-1 = \frac{x + 3}{2} \quad -2 = \frac{x + 6}{2}$ $x = -5 \quad y = -10$ $L(-5; -10)$	✓ Correct gradient formula ✓✓ Correct substitution in correct formula ✓ Answer
1.1.4	$NK = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $NK = \sqrt{(3 + 1)^2 + (6 + 2)^2}$ $NK = \sqrt{80}$ $NK = 8,94$	✓ Correct gradient formula ✓ Correct substitution in correct formula ✓ Value rounded off in decimal form
1.2	$m_{nk} = 2 \quad m_{pk} = -\frac{1}{2}$ equation of // line: $y - y_1 = m(x - x_1)$ subst. N(3 ; 6) $y - 6 = -\frac{1}{2} (x - 3)$ $2y - 12 = -(x - 3)$ $2y = -x + 15$ $y = -\frac{1}{2} x + \frac{15}{2}$	✓ for $m_{nk} = 2$ ✓ $m_{pk} = -\frac{1}{2}$ ✓✓ Substitution in correct formula ✓ $y = -\frac{1}{2} x + \frac{15}{2}$

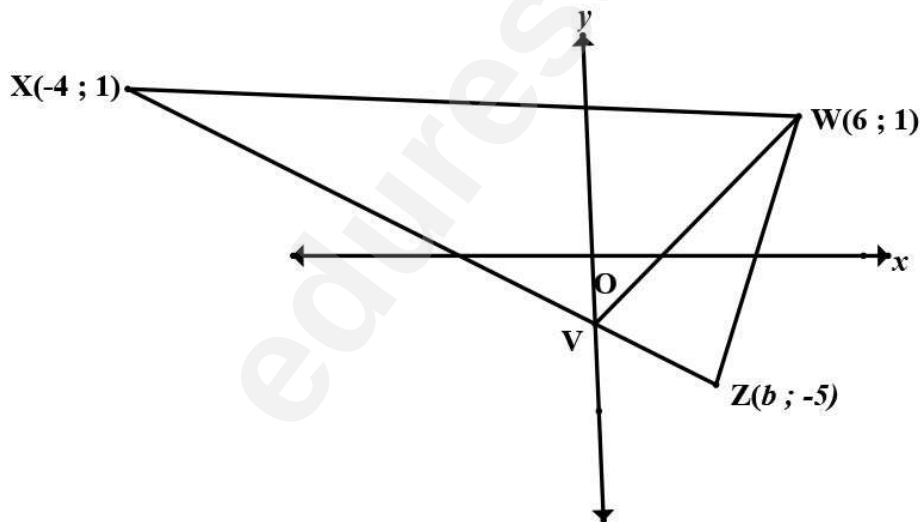
Exercise 1

Determine:

No.	Activity	Marks	Answer
1.1	The length of BC.	3	BC = 10
1.2	The coordinates of E and the midpoint of AB.	2	E(-2;-1)
1.3	The gradient of BC.	2	$m_{BC} = 0,75$
1.4	The equation of the line passing through points D and E.	4	$y = 0,75x + 0,5$
1.5	The size of $\hat{A}BC$.	6	$\hat{A}BC = 8,94^\circ$

Exercise 2

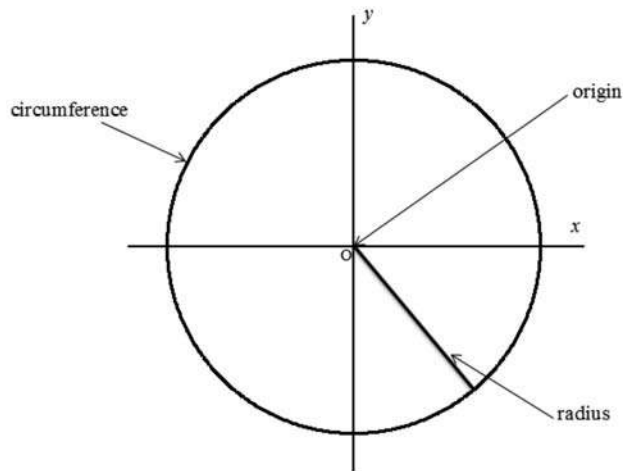
In the diagram, $\triangle XWZ$ is drawn, with: $X(-4;1)$; $W(6;1)$; $Z(b;-5)$. V is a point on the y -axis. The line XVZ has the equation $y = -\frac{3}{4}x + 2$.



No.	Activity	Marks	Answer
1.1	Show that $b = 4$.	1	$b = 4$
1.2	Write down the coordinates of V.	1	$V(0;-2)$
1.3	Determine the equation of the line VW.	3	$y = \frac{1}{2}x - 2$

4.3 Equation of a circle

The equation of a circle is given by $x^2 + y^2 = r^2$, where the centre is at the origin. The radius is defined by the distance from the centre to the circumference of the circle.



Example

Determine the equation of the circle, with the centre at origin and a radius of 3 units.

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 3^2$$

or

$$x^2 + y^2 = 9$$

with the centre at origin and

passing through the point (-3;4)

$$x^2 + y^2 = r^2$$

$$(3)^2 + (4)^2 = r^2$$

$$9 + 16 = r^2$$

$$25 = r^2$$

$$\therefore x^2 + y^2 = 25$$

Explanation

Substitute for radius.

✓ equation

Substitute the coordinates.

Manipulation to find radius

Equation in standard form.

Marks

(1)

(3)

✓ substitution

✓ radius

✓ equation

Equation of the tangent to the circle at a given point

To determine the equation of the tangent, the theorem is used that states: the tangent is perpendicular to the radius of the circle at the point of contact.

Also, use the condition for two lines to be perpendicular, i.e. the product of two lines that are perpendicular is equal to -1 .

Examples

1. Determine the equation of the tangent of with centre $(0 ; 0)$ at the point $(-3 ; 4)$.

Solution

The gradient of the radius to the given point is $-\frac{4}{3}$

The gradient of the tangent is $-\left(-\frac{3}{4}\right) = \frac{3}{4}$

The equation of the tangent is $y = \frac{3}{4}x + c$.

Substitute the values of : $4 = \frac{3}{4}(-3) + c$

$$\therefore c = 4 + \frac{9}{4} = \frac{25}{4}$$

Thus, the equation of the tangent is $y = \frac{3}{4}x + \frac{25}{4}$

2. The gradient of the radius of the circle with centre $(0 ; 0)$ and touching the circle at point R is $\frac{5}{4}$. Determine:
- the co-ordinates of R;
 - the equation of the tangent to the circle that touches the circle at R.

Solution:

- a) The co-ordinates of R are $(-4;5)$ or $(4;-5)$

- b) The gradient of the tangent is $\frac{4}{5}$.

$$\text{The value of } c \text{ is } 5 = \frac{4}{5}(-4) + c \Rightarrow c + 5 = \frac{16}{5} \Rightarrow \frac{25 + 16}{5} = \frac{41}{5}$$

$$\text{The equation is } y = \frac{4}{5}x + \frac{41}{5}.$$

or

$$\text{The value of } c \text{ is } 5 = \frac{4}{5}(4) + c \Rightarrow c - 5 = \frac{16}{5} \Rightarrow \frac{-25 - 16}{5} = -\frac{41}{5}$$

$$\text{The equation is } y = \frac{4}{5}x - \frac{41}{5}$$

Exercises

1. The circle with the centre at the origin passes through point G (2 ; 4).
 - 1.1 Determine the equation of the circle.
 - 1.2 Determine the equation of tangent to the circle at G.
 - 1.3 Determine the points of intersection of the circle and the line with the equation $y = x + 2$.
2. The line intersects the circle at R and T .
 - 2.1 Determine the equation $y = x - 3$ of the circle with centre at the origin passing through R (0;-3) and T (3;0).
 - 2.2 Determine the equation of the tangent of the circle at R.
 - 2.3 Determine the equation of the tangent of the circle at T.

4.4 Ellipse

An ellipse is the set of all points in a plane such that the sum of the distances from T to two fixed points (F_1 and F_2) is a given constant (K), i.e. $TF_1 + TF_2 = K$. F_1 and F_2 are both foci (plural of focus) of the ellipse.

An ellipse has two axes, viz. the major and the minor axis.

The major axis is the segment that contains both foci and has its endpoints on the ellipse. These endpoints are called the vertices. The midpoint of the major axis is the centre of the ellipse.

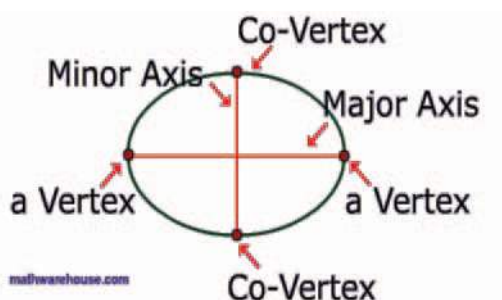
The minor axis is perpendicular to the major axis at the centre, and the endpoints of the minor axis are called co-vertices.

The vertices are at the intersection of the major axis and the ellipse.

The co-vertices are at the intersection of the minor axis and the ellipse.

You can think of an ellipse as an oval.

Picture of an ellipse



4.4.1 STANDARD FORM OF AN ELLIPSE

The general form for the standard equation of an ellipse is:

Horizontal major axis

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 ; \quad a > b$$

Vertical major axis

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 ; \quad a > b$$

Examples

1. Draw a sketch graph of an ellipse defined by $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

Solution:

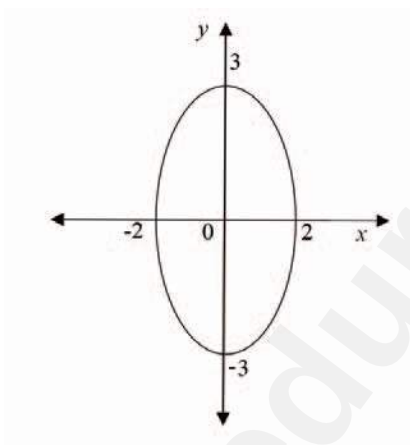
$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$$a^2 = 9 \quad \text{and} \quad b^2 = 4$$

$$\therefore a = 3 \quad \therefore b = 2$$

2. Determine the values of a and b and sketch the graph of the ellipse with the equation:

Solution:

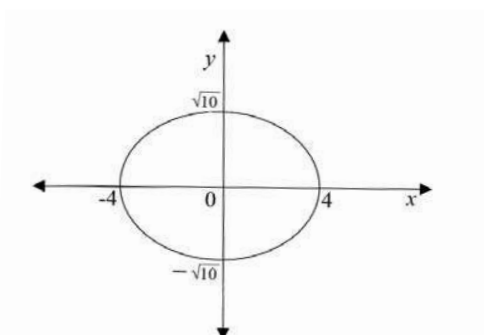


$$\frac{x^2}{16} + \frac{y^2}{10} = 1$$

Solution:

$$a^2 = 16 \quad \text{and} \quad b^2 = 10$$

$$\therefore a = 4 \quad \therefore b = \sqrt{10}$$



Exercises

1. Determine the values of a and b and sketch the graph of an ellipse with the equation:

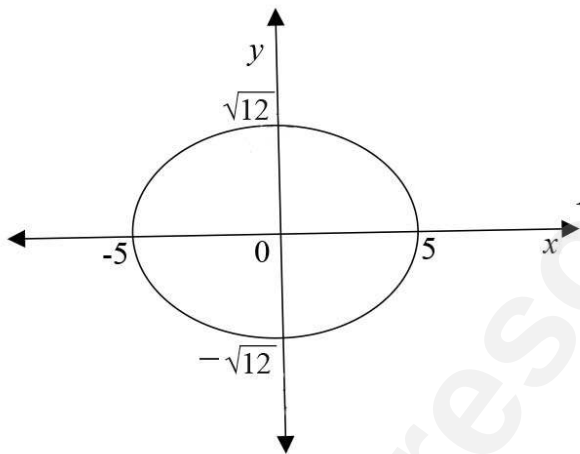
1.1 $\frac{x^2}{9} + \frac{y^2}{12} = 1$

1.2 $\frac{x^2}{36} + \frac{y^2}{25} = 1$

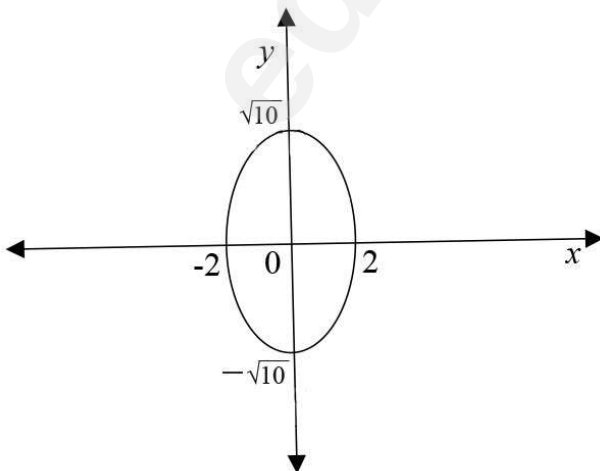
1.3 $\frac{x^2}{49} + \frac{y^2}{24} = 1$

2. Determine the equation of each of the following ellipses:

2.1



2.2



4.5 Coordinate Geometry

The important formulae for coordinate geometry are:

- Gradient formula: $m = \left(\frac{y_2 - y_1}{x_2 - x_1} \right)$
- Distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Mid-point formula: $M = \left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2} \right)$
- Straight line equation: $y = mx + c$ or $y - y_1 = m(x - x_1)$
- Inclination: $m = \tan \theta$
- Circle: $x^2 + y^2 = r^2$
- Semi-circle: $y = \pm \sqrt{r^2 - x^2}$

Example:

1.1 In the diagram alongside, K(-1;-2)

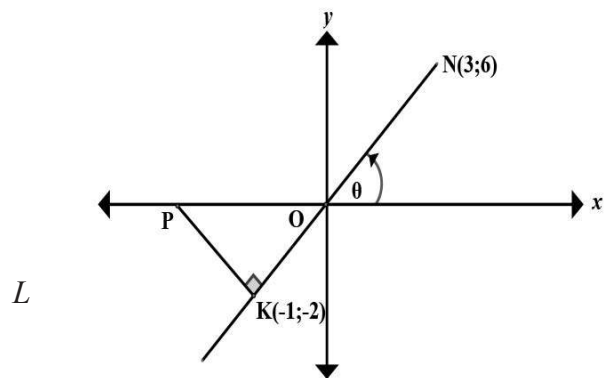
is the midpoint of LN with N(3;6).

PK $\perp$ LN with P on the x -axis.

The angle of inclination of NL is θ .

Determine:

- 1.1.1 the gradient of NK
 - 1.1.2 the size of θ , rounded off to one decimal place.
 - 1.1.3 The coordinates of L.
 - 1.1.4 The length of NK, rounded off to one decimal place.
- 1.2 Determine the equation of the straight line parallel to PK and which passes through N.



Solutions:

No.	Solution	Marks
1.1.1	$m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \frac{6 + 2}{3 + 1}$ $m = 2$	✓ Correct gradient formula ✓ Correct substitution in correct formula ✓ Answer
1.1.2	$\tan \theta = 2$ $\therefore \theta = 63,4^\circ$	✓ Correct inclination formula ✓ Correct answer
1.1.3	$x = \frac{x_1 + x_2}{2} \quad y = \frac{y_1 + y_2}{2}$ $-1 = \frac{x + 3}{2} \quad -2 = \frac{x + 6}{2}$ $x = -5 \quad y = -10$ $L(-5; -10)$	✓ Correct gradient formula ✓✓ Correct substitution in correct formula ✓ Answer
1.1.4	$NK = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $NK = \sqrt{(3 + 1)^2 + (6 + 2)^2}$ $NK = \sqrt{(80)}$ $NK = 8,94$	✓ Correct gradient formula ✓ Correct substitution in correct formula ✓ Value rounded off in decimal form
1.2	$m_{nk} = 2 \quad m_{pk} = -\frac{1}{2}$ equation of // line: $y - y_1 = m(x - x_1)$ subst. N(3 ; 6) $y - 6 = -\frac{1}{2}(x - 3)$ $2y - 12 = -(x - 3)$ $2y = -x + 15$ $y = -\frac{1}{2}x + \frac{15}{2}$	✓ for $m_{nk} = 2$ ✓ $m_{pk} = -\frac{1}{2}$ ✓✓ Substitution in correct formula ✓ $y = -\frac{1}{2}x + \frac{15}{2}$

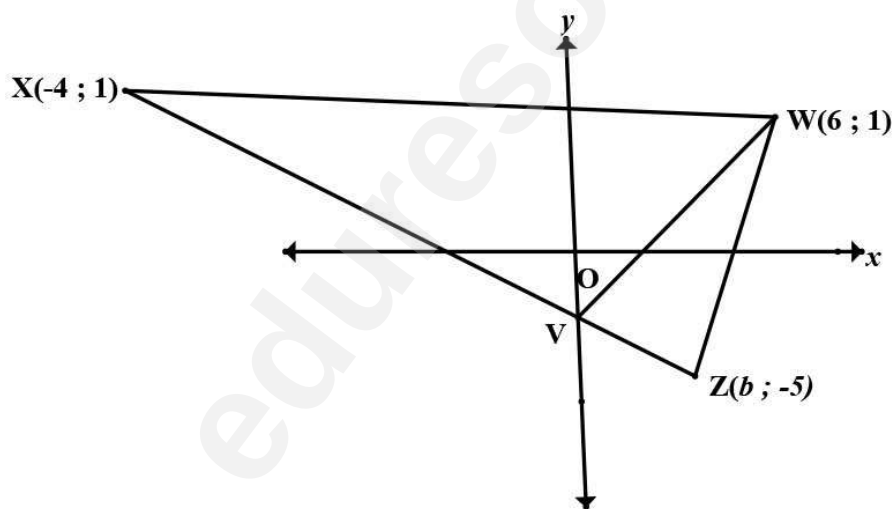
Exercise

Determine:

No.	Activity	Marks	Answer
1.1	The length of BC.	3	BC = 10
1.2	The coordinates of E and the midpoint of AB.	2	E(-2;-1)
1.3	The gradient of BC.	2	$m_{BC} = 0,75$
1.4	The equation of the line passing through points D and E.	4	$y = 0,75x + 0,5$
1.5	The size of $\hat{ABC}$.	6	$\hat{ABC} = 8,94^\circ$

Exercise 2

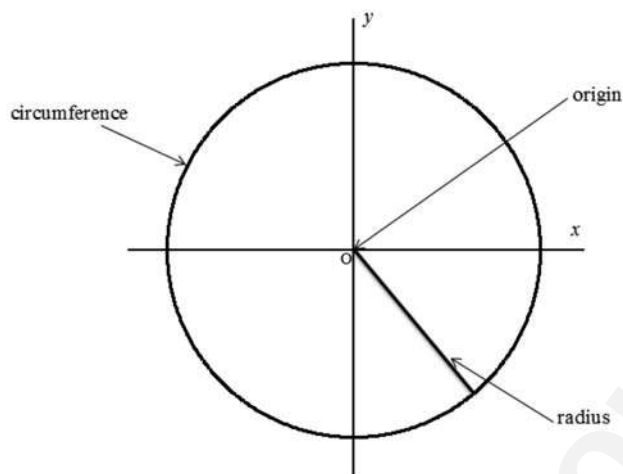
In the diagram, $\triangle XWZ$ is drawn, with: $X(-4;1)$; $W(6;1)$; $Z(b;-5)$. V is a point on the y -axis.
The line XVZ has the equation $y = -\frac{3}{4}x + 2$.



No.	Activity	Marks	Answer
1.1	Show that $b = 4$.	1	$b = 4$
1.2	Write down the coordinates of V .	1	$V(0;-2)$
1.3	Determine the equation of the line VW.	3	$y = \frac{1}{2}x - 2$

4.6 Equation of a circle

The equation of the circle is given by $x^2 + y^2 = r^2$ where the centre is at the origin. The radius is defined by the distance from the centre to the circumference of the circle.



Example

Determine the equation of the circle, with the centre at origin and a radius of 3 units.

Explanation

Marks

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 3^2$$

or

$$x^2 + y^2 = 9$$

with the centre at origin and passing through the point (-3;4)

Substitute for radius.

(1)

✓ equation

(3)

$$x^2 + y^2 = r^2$$

$$(3)^2 + (4)^2 = r^2$$

$$9 + 16 = r^2$$

$$25 = r^2$$

$$\therefore x^2 + y^2 = 25$$

Substitute the coordinates.

✓ substitution

Manipulation of the equation in its form

✓ radius

✓ equation

5 CHECK YOUR ANSWERS

- Show answers/responses to the questions and activities in this section.
- Please ensure accurate correlation to the activity in the previous section.
- Indicate mark allocation (use ticks✓) where required, but more importantly, explain how marks are allocated in the examination.

6 MESSAGE TO GRADE 12 LEARNERS FROM THE WRITERS

Technical Mathematics can be fun, as it requires you to pull together all the pieces of information learnt in the lower grades to answer the Grade 12 examination questions. If one grade was skipped before Grade 12, it will have left a void in the grounding needed to pass this examination.

Please ensure that you know all axioms and corollaries (i.e. all the rules) to answer your questions. Answer Technical Mathematics exemplar papers before you sit the final examinations.

Write the exemplar paper in 3 hours and mark your script on your own, using the memorandum provided to gauge if you are ready for the final paper. The memorandum is also available on the DBE website.

We can assure you that this year's final paper will be similar to those of previous years in both format and style.

7 THANK YOU AND ACKNOWLEDGEMENTS

We hope the explanations are well received in preparation for your final examinations. Mr Leonard Gumani Mudau, Mrs Nonhlanhla Rachel Mthembu, Ms Thandi Mgwanya and Mr Percy Steven Tebeila wish you well.

MATHEMATICS

ANALYTICAL GEOMETRY

GRADE 12

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