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Functions

The Inverse of Exponential Functions

We investigate how to determine the equation of the inverse function of an exponential function.

Lesson Outcomes

By the end of this lesson, you should be able to:

- Determine the graph of an inverse function through point by point plotting
- Determine the equation of the inverse for functions of the form: $f(x) = ax$ and $f(x) = ax^2$

Curriculum Links

LO 2: Functions and Algebra

- 12.2.1 (a) Demonstrate the ability to work with various types of functions and relations including inverses
- 12.2.2 (a) Investigate and generate graphs of the inverse relations of various functions
(b) Determine which inverses are functions and how the domain of the original function needs to be restricted so that the inverse is also a function
- 12.2.3 Identify and use characteristics of functions to sketch graphs of inverses

Lesson Notes

We are going to work with the specific exponential function $f(x) = 2^x$. Below is a table of values for this function.

X	-2	-1	0	1	2	3	-2	-1
$f(x) = 2^x$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	$\frac{1}{4}$	$\frac{1}{2}$

To make up a table of values for the inverse function, we need to take the output values of the function and use them as input values for the inverse function as follows.

Inverse function								
X	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	$\frac{1}{4}$	$\frac{1}{2}$
$f^{-1}(x)$	-2	-1	0	1	2	3	-2	-1

We can use the values from the table to plot the graphs of the function and the inverse function as shown alongside.

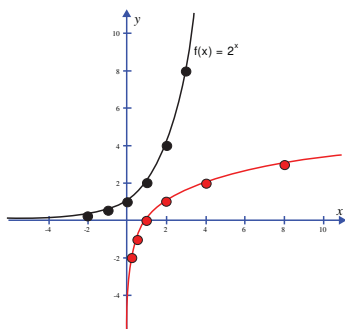
Notice that the x - and y - values have swapped around. We can use this observation to determine the equation of the inverse function.

$$f(x): y = 2^x$$

$$f^{-1}(x): x = 2^y$$

We would rather have the inverse function written in the form $y = \dots$

In order to do so, we use logarithms.



The definition of a logarithm is:

$$\text{If } a^x = b \text{ then } x = \log_a b$$

In simple terms, the logarithm is the index or the exponent of a given number that will give a desired value.

Using that definition we can now tidy up the inverse function.

$$f^{-1}(x): y = \log_2 x$$

We noticed that the function and the inverse are reflections about the line $y = x$. This means that we can actually draw the inverse function of a function using reflection and avoid having to do all that point by point work that we have been using.

Task

Determine by investigation $f^{-1}(x)$ if $f(x) = ax^2$

