



eduresourceza.com

Calculus

Calculus – Rate of change

In this lesson we are going to expand our understanding of the derivative to include the concept – rate of change.

Lesson Outcomes

By the end of this lesson, you should be able to:

- interpret the derivative as a measure of the rate of change of a function

Curriculum Links

LO 2: Functions and Algebra

- 12.2.7 (a) Investigate and use instantaneous rate of change of a variable when interpreting models of situations:
- (e) Solve practical problems involving optimisation and rates of change

Lesson Notes

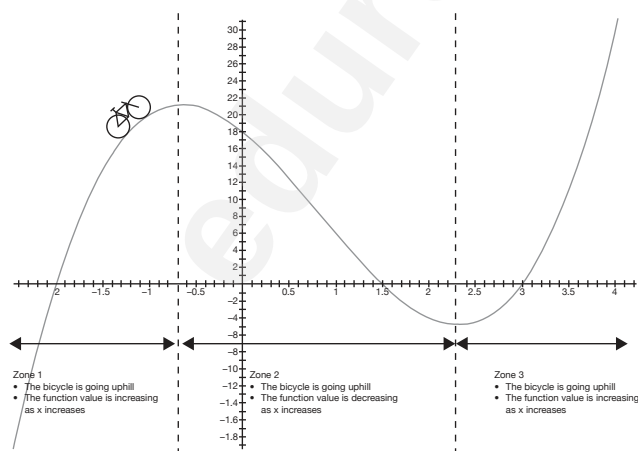
We will consider two main ideas about functions in this lesson:

- What happens to the function value (y -value) as the x -value increases; and
- What happens to the gradient or the rate of change of the function as the x -value increases.

Said differently we will look at the how the function behaves moving from the left hand side of the graph towards the right hand side of the graph.

What happens to the function value (y -value) as the x -value increases:

The graph below shows a bicycle moving from left to right along the graph. Three zones have been identified (see notes below each zone):



We can link the above to our work with derivatives as follows:

- If we drew a tangent to the curve in zone 1 i.e. where the function value is increasing, the gradient of the tangent would be positive. The gradient of the tangent is given by the derivative so the derivative would be positive.
- If we drew a tangent to the curve in zone 2 i.e. where the function value is decreasing, the gradient of the tangent would be negative. The gradient of the tangent is given by the derivative so the derivative would be negative.

What happens to the gradient or the rate of change of the function as the x -value increases:

If you look at the graph in zone 1 again you will notice that up until the point where the function turns, the function is clearly increasing, but in the first half, the function seems to be increasing more quickly than in this second half where the function is not as steep. Having passed the turning point, the function starts decreasing, but it does so slowly at first then does so quite a bit more quickly and then it seems to slow down again.

What you have observed is that not only is the function increasing, but that the rate at which it does so changes – mathematicians speak about the rate of change of the function. The rate of change of a function over an interval is called the average rate of change of a function, while the rate of change at a point is called the instantaneous rate of change. The instantaneous rate of change at a point is given by the value of the derivative at that point.

Calculus

Below is a summary of the ideas that we have discussed above:

	Function value		
	Decreasing	Constant	Increasing
Function value is changing more and more slowly			
Function value is changing at a constant rate			
Function value is changing more and more quickly			

Task

Consider the graph below and:

- Describe what is happening to the function value:
 - In the interval AB
 - In the interval DE
 - At A; B; C; D and E using the words increasing, decreasing and constant;
- Determine at which of the points where the function value is increasing, it is increasing most and at which of the points where the function value is decreasing, it is decreasing most.

