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Functions

The Inverse of Linear Functions

We investigate how to determine the equation of the inverse of linear functions.

Lesson Outcomes

By the end of this lesson, you should be able to:

- sketch the graphs of a cubic function
- determine the equation of a tangent to a graph.

Curriculum Links

LO 2: Functions and Algebra

- 12.2.1 (a) Demonstrate the ability to work with various types of functions and relations including inverses
- 12.2.2 (a) Investigate and generate graphs of the inverse relations of various functions
- (b) Determine which inverses are functions and how the domain of the original function needs to be restricted so that the inverse is also a function
- 12.2.3 Identify and use characteristics of functions to sketch graphs of inverses

Lesson Notes

In this lesson we want to determine by investigation the inverse function, $f^{-1}(x)$, of a linear function of the form, $f(x) = ax + q$. To do this we will use the specific function $f(x) = -2x + 6$.

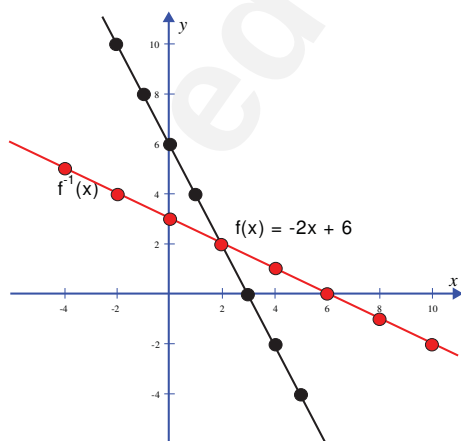
Below is the table of values for this function. The x-values are the input values or the domain of the function and the y-values are the output values or the range of the function.

X	-2	-1	0	1	2	3	4	5
$f(x) = -2x + 6$	10	8	6	4	2	0	-2	-4

To make up a table of values for the inverse function, we need to take the output values of the function and use them as input values for the inverse function as follows.

Inverse function								
X	10	8	6	4	2	0	-2	-4
$f^{-1}(x)$	-2	-1	0	1	2	3	4	5

We can use the table of values to plot the graphs of the function and the inverse function shown below:



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The inverse of the linear function is also a linear function with a constant gradient. Next we use the general point (x;y) and the two points (2;2) and (4;1) to find the equation of the inverse as follows:

$$\frac{y-1}{x-4} = \frac{1-2}{4-2}$$

$$\frac{y-1}{x-4} = \frac{-1}{2}$$

$$y-1 = \frac{-1}{2}(x-4)$$

$$y = \frac{-1}{2}x + 2 + 1$$

$$y = \frac{-1}{2}x + 3$$

$$f^{-1}(x) = \frac{-1}{2}x + 3$$

If we compare the tables of values of the function and its inverse, we will notice that the x- and y-values have just been interchanged. We can use this fact as another way of finding the equation of the inverse. This is shown below:

$$f(x): y = -2x + 6.$$

$$f^{-1}(x): x = -2y + 6. \quad (x\text{- and }y\text{- values have just been interchanged})$$

$$2y = -x + 6$$

$$y = -\frac{1}{2}x + 3$$

We also noticed that the linear function and its inverse are reflections of each other about the line $y = x$ line.

 Task

The general equation of a linear function is $y = ax + q$. Can you find the general inverse equation of a linear function?