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SUBJECT: MATHEMATICS

GRADE: 12

CHAPTER / MODULE: 2

UNIT / LESSON TOPIC: CALCULUS

UNIT OUTCOMES

LO 12.2.1.

By the end of this unit, the Learner should be able to :

- ∞ **Revise the concepts of gradient and average gradient.**
- ∞ **Develop an understanding of the concept of instantaneous rate of change.**
- ∞ **Develop an understanding of the limit concept, one of the most important and powerful tools in Mathematics**
- ∞ **Define the meaning of the gradient, or slope, of a curve (other than a line) at a point on the curve.**
- ∞ **Calculate derivatives from the definition of derivative.**
- ∞ **Recognise the relationship between rate of change, slope of a curve and derivative.**

LESSON OVERVIEW (knowledge areas)

INTRODUCTION

Introduction to the rules of differentiation – 20 minutes

Mixed examples of the short method derivatives – 30 minutes

LESSON

*** Before the short method derivative can be done, there are certain things that cannot be in the expression.**

NO BRACKETS

NO SURDS

NO X'S IN THE DENOMINATORS

FACTORISE IF POSSIBLE

* The short method works as follows : The exponent comes down to the front as a whole number and we subtract 1 from the exponent..... ie.

$$f(x) = ax^n \Rightarrow f'(x) = n \cdot ax^{n-1}$$

* These are the different ways we can ask to find the derivative :-

$$f(x) \rightarrow f'(x)$$

$$y \Rightarrow \frac{dy}{dx}$$

$$D_x[\dots] \rightarrow = \dots\dots$$

EXAMPLES OF SHORT METHOD DERIVATIVES

1. Determine the derivative of :-

a) $f(x) = \frac{x^2 + 6x + 8}{x + 2}$

b) $y = 3x^2 + 6x + \frac{2}{x}$

c) $y = \frac{5}{x} + \sqrt{x}$

d) $D_x \left[4x^2 + \frac{2}{x^2} \right]$

e) $f(x) = (2x - 1)^2$

ANSWERS TO THE EXERCISE

1. a)

$$f(x) = \frac{x^2 + 6x + 8}{x + 2}$$

$$f(x) = \frac{(x+4)(x+2)}{x+2}$$

$$f(x) = x + 4$$

$$\therefore f'(x) = 1$$

Factorise if possible, cross out common Bracket.

b)

$$y = 3x^2 + 6x + \frac{2}{x}$$

$$y = 3x^2 + 6x + 2x^{-1}$$

$$\frac{dy}{dx} = 6x + 6 - 2x^{-2}$$

$$= 6x + 6 - \frac{2}{x^2}$$

There must never be a variable in the denominator
If an x goes from the denominator to the numerator
To the exponent becomes negative.

c)

$$y = \frac{5}{x} + \sqrt{x}$$

$$y = 5x^{-1} + x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = -5x^{-2} + \frac{1}{2}x^{-\frac{1}{2}}$$

$$= -\frac{5}{x^2} + \frac{1}{2x^{\frac{1}{2}}}$$

$$= -\frac{5}{x^2} + \frac{1}{2\sqrt{x}}$$

If there is a surd in the question, it must be removed. Remove the x in the denominator by taking it to the top. The answer must always be left with positive exponents

d)

$$D_x \left[4x^2 + \frac{2}{x^2} \right] -$$

$$D_x \left[4x^2 + 2x^{-2} \right] -$$

$$= 8x - 4x^{-3}$$

$$= 8x - \frac{4}{x^3}$$

Get the x to the top of the and make the exponent negative, Leave answers with positive exponents

e)

$$f(x) = (2x-1)^2$$

$$f(x) = (2x-1)(2x-1)$$

$$f(x) = 4x^2 - 2x - 2x + 1$$

$$f(x) = 4x^2 - 4x + 1$$

$$f'(x) = 8x - 4$$

Distribute the two brackets by doing
FOIL

Simplify and differentiate

HOMEWORK TASK

Find the following derivatives :-

1. $f(x) = 2x^3 - x^2 + \frac{1}{x}$

2. $f(x) = (3x + 5)^2$

3. $f(x) = \frac{1}{2x} + \frac{2}{\sqrt{x}}$

4. $f(x) = \frac{x^3 - 8}{x - 2}$

5. $f(x) = \frac{5x^4 - 3x}{x^2}$

ANSWERS TO HOMEWORK TASK

1. $f(x) = 2x^3 - x^2 + x^{-1}$

$$f'(x) = 6x^2 - 2x - x^{-2}$$

$$= 6x^2 - 2x - \frac{1}{x^2}$$

2. $f(x) = 9x^2 + 30x + 25$

$$f'(x) = 18x + 30$$

3. $f(x) = \frac{1}{2}x^{-1} + 2x^{\frac{-1}{2}}$

$$f'(x) = -\frac{1}{2}x^{-2} - x^{\frac{-3}{2}}$$

4. $f(x) = \frac{(x-2)(x^2+2x+4)}{x-2}$

$$f(x) = x^2 + 2x + 4$$

$$f'(x) = 2x + 2$$

5.

$$f(x) = \frac{5x^4 - 3x}{x^2}$$

$$f(x) = \frac{5x^4}{x^2} - \frac{3x}{x^2}$$

$$f(x) = 5x^2 - 3x^{-1}$$

$$f'(x) = 10x + 3x^{-2}$$

$$f'(x) = 10x + \frac{3}{x^2}$$