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SUBJECT: MATHEMATICS

GRADE: 12

CHAPTER / MODULE: 2

**UNIT / LESSON TOPIC: FACTOR & REMAINDER THEOREM
CALCULUS**

UNIT OUTCOMES

LO 12.2.1.

By the end of this unit, you should be able to :

- ∞ Revise the concepts of gradient and average gradient.
- ∞ Develop an understanding of the concept of instantaneous rate of change.
- ∞ Develop an understanding of the limit concept, one of the most important and powerful tools in Mathematics
- ∞ Define the meaning of the gradient, or slope, of a curve (other than a line) at a point on the curve.
- ∞ Calculate derivatives from the definition of derivative.
- ∞ Recognise the relationship between rate of change, slope of a curve and derivative.

LESSON

AVERAGE GRADIENT & LIMITS

The gradient of a line gives us an indication of how steep a line is relative to the horizontal axis.

A line which joins any two distinct points on any curve is called a secant.

A line which just touches a curve at a single point on the curve is called a tangent.

Limits – when a number “ gets closer or tends towards” another number

EXAMPLES OF LIMITS

If a question looks as follows , it is important to read it correctly :-

$$h \rightarrow 0$$

This reads : h tends to zero and that means we can make h as close to zero as we like.

RATES OF CHANGE

If $s = f(t)$ is the equation of motion, which gives position at time t :

Then $v = \frac{d}{dt}(s)$ gives the velocity at time t ;

And $a = \frac{d}{dt}(v)$ gives the acceleration at any time t .

* If the question says “using first principles”, the formula has to be used.

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

* Before substituting into the formula, find $f(x+h)$ and then put the value into the formula.

* The word $\lim_{h \rightarrow 0}$ must be written up to the last step when h is replaced with 0.

* All x 's must be replaced with $(x+h)$

Eg 1. Find $f'(x)$ by using first principles if $f(x) = 3x^2 - 2x$

If $f(x) = 3x^2 - 2x$ then $f(x+h) = 3(x+h)^2 - 2(x+h)$
 (wherever there is an (x) replace it with a $(x+h)$

Substitute into the formula.....

$$\begin{aligned} & \lim_{h \rightarrow 0} \frac{3(x+h)^2 - 2(x+h) - (3x^2 - 2x)}{h} \\ & \lim_{h \rightarrow 0} \frac{3(x^2 + 2xh + h^2) - 2x - 2h - 3x^2 + 2x}{h} \\ & \lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 - 2h - 3x^2}{h} \\ & \lim_{h \rightarrow 0} \frac{h(6x + 3h - 2)}{h} \\ & \lim_{h \rightarrow 0} 6x + 3h - 2 \\ & = 6x - 2 \end{aligned}$$

Try this example with the students : -

If $f(x) = x^2 + 2x$, determine $f'(x)$ from first principles.

LIMIT EXAMPLE

1. Find the following limit if it exists :-

$$\lim_{h \rightarrow 0} \frac{(2+h)^2 - 4}{h}$$

ANSWER TO EXAMPLE

$$\lim_{h \rightarrow 0} \frac{4 + 4h + h^2 - 4}{h}$$

$$\lim_{h \rightarrow 0} \frac{h(4+h)}{h}$$

$$\lim_{h \rightarrow 0} 4 + h = 4$$

HOMEWORK TASK

A

- ∞ Find $f'(x)$ by using first principles :-

1. $f(x) = -2x^2$

2. $f(x) = 3x^2 - 5x$

B

- ∞ Find the following limits if they exist :-

1. $\lim_{x \rightarrow 2} \frac{x^2 + 3x + 2}{x^2 + 4x + 4}$

2. $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x + 1}$

3. The motion of a particle is given by $s = 5t^2 - 2t$

- (a) Find an expression for velocity
 (b) Find the velocity at $t = 2$
 (c) Find the acceleration of the particle.