



eduresourceza.com

Sequences and Series

Arithmetic Sequences and Series

In this lesson we will look at a specific kind of sequence or series called an arithmetic sequence or series.

We will determine the general term of an arithmetic sequence and also determine a formula to enable us to find the sum of a number of terms of the arithmetic series.

Lesson Outcomes

By the end of this lesson, you should be able to:

- Identify arithmetic sequences
- Determine the general term of an arithmetic sequence
- Determine the sum of a number of terms in an arithmetic series

Curriculum Links

Learning Outcome 1: Number and number relationships

- Identify and solve problems involving number patterns,
- Correctly interpret sigma notation.
- Prove and correctly select the formula for and calculate the sum of series

Lesson Notes

Some vocabulary:

In arithmetic sequences and series the terms differ by a fixed amount called a common difference. The symbol for a common difference is d .

This means that you can test whether a sequence is arithmetic or not by applying the following rule:

If $T_2 - T_1 = T_3 - T_2 \rightarrow$ the sequence is arithmetic

Examples of arithmetic sequences are as follows:

	Term 1 T_1	Term 2 T_2	Term 3 T_3	Term 4 T_4	Term 5 T_5	Term 6 T_6	Common difference d	Term 8 T_8	Term n T_n
Eg 1	1	2	3	4	5	6	+ 1	8	n
Eg 2	5	8	11	14	17	20	+ 3	$20 + 2 \times 3 = 26$	$5 + (n - 1) \times 3$
Eg 3	15	11	7	3	-1	-5	- 4	$-5 + 2 \times (-4) = -13$	$15 + (n - 1) \times (-4)$
Eg 4	a	$a + d$	$a + 2d$	$a + 3d$	$a + 4d$	$a + 5d$	d	$a + 7d$	$a + (n - 1)d$

Example 4 shows an arithmetic sequence in the general form and gives us an expression for the n th term of the sequence. This enables you to find any term in the sequence as long as you know the value of the first term (a); the common difference (d) and the term number (n). The general rule for the terms of an arithmetic sequence can be said to be:

$$T_n = a + (n - 1)d$$

We can find the sum of a series in the following way. We will use the sequence in example 2 and add the first 6 terms:

Write out the series	5	8	11	14	17	20
Write out in reverse order	20	17	14	11	8	5
Add them	$\overline{25}$	$\overline{25}$	$\overline{25}$	$\overline{25}$	$\overline{25}$	$\overline{25}$

Therefore the sum of the first 6 terms = $(6 \times 25) \div 2 = 75$

We had to divide by 2 because we had written the sequence down twice.

We can repeat the above in the general form

Sequences and Series

Write out the series	a	$a + d$	$a + 2d$	...	$a + (n-2)d$	$a + (n-1)d$
Write out in reverse order	$a + (n-1)d$	$a + (n-2)d$	$a + (n-3)d$	...	$a + d$	a
Add them	$2a + (n-1)d$	$2a + (n-1)d$	$2a + (n-1)d$	$2a + (n-1)d$	$2a + (n-1)d$	$2a + (n-1)d$

Therefore the sum of the first n terms $= S_n = \frac{n \times (2a + (n-1)d)}{2}$

In general:

$$\sum_{i=1}^n a + (i-1)d = \frac{n [2a + (n-1)d]}{2}$$

To summarise:

- We can test for an arithmetic sequence by applying the rule:
 - If $T_2 - T_1 = T_3 - T_2 \rightarrow$ the sequence is arithmetic;
- we have developed a general term for an arithmetic sequence: $T_n = a + (n-1)d$;
- we have developed a formula used to determine the sum of any number of terms of a series: $S_n = \frac{n [2a + (n-1)d]}{2}$
- we have used special shorthand called sigma notation (Σ) to write all of the above neatly: $\sum_{i=1}^n a + (i-1)d = \frac{n [2a + (n-1)d]}{2}$

Task

- Go over the development of the general term and sum formula for an arithmetic series.
- Answer the following questions:
 - Determine the general term formula for the sequence 55; 50; 45; ...
 - Determine the sum formula for the series: 55 + 50 + 45 + ...
Use these formulae to determine:
 - Which individual term has a value of 0?
 - How many terms of the series add up to 0?