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Calculus

Calculus – Establishing what we need Calculus for

In the previous lesson we established that **Calculus** was invented to describe mathematically some situations in the real world – it was invented as a tool for **Mathematical Modelling** (Neill and Shuard). We will continue with the box problem we started in the previous lesson as a way of explaining what differential calculus is used for.

Lesson Outcomes

By the end of this lesson, you should be able to:

- explain the nature of the problem for which differential calculus was invented.

Curriculum Links

LO 2: Functions and Algebra

The learner is able to investigate, analyse, describe and represent a wide range of functions and solve related problems

12.2.7(a) Investigate and use instantaneous rate of change of a variable when interpreting models of situations:

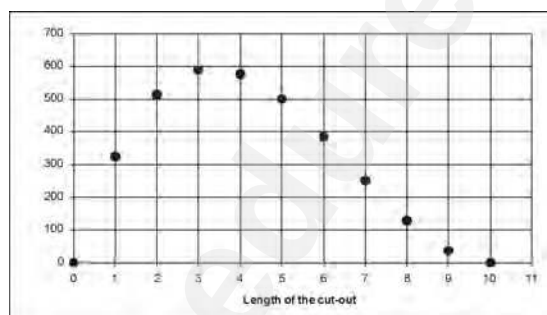
- demonstrating an intuitive understanding of the limit concept in the context of approximating the rate of change or gradient of a function at point; (Linked to task)

(e) Solve practical problems involving optimisation and rates of change.

Lesson Notes

After making a number of boxes and recording the measurements and volumes of the boxes we still had no way of knowing if we had found the box with the largest volume.

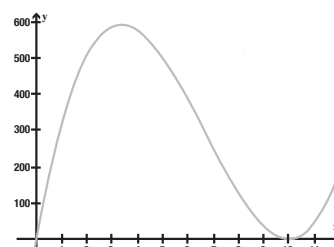
If you plotted the points given in the previous lesson, your graph would look like this:



The volume of the boxes is given by the equation:

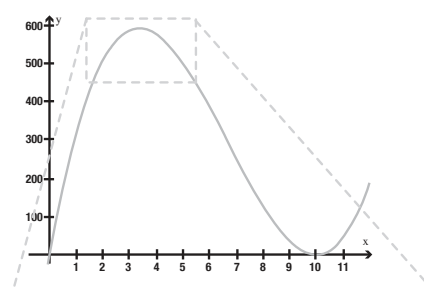
$$\text{volume} = (20 - 2x) \times (20 - 2x) \times x$$

The graph of this function is shown below:

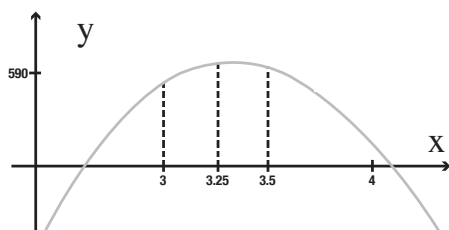


From the graph it can be seen that when $x = 10\text{cm}$, there will be no cardboard left to make up the box, so the volume will be zero.

It seems that the greatest volume occurs for a value of x between 3 and 4. By zooming in you can get a better approximation for this point.

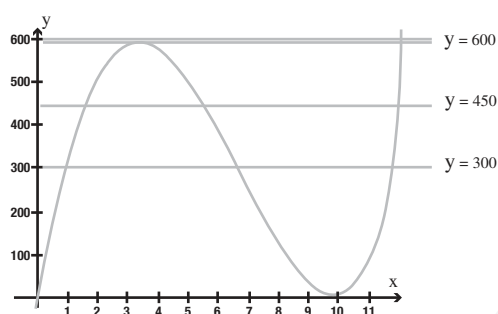


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The greatest volume occurs for a value of x between 3,25 and 3,5. We cannot test every real number between these two points so we are still left with the problem of not knowing at which value of x , the box will have the greatest volume.

We want to find the value of x that will give us the greatest volume – the point at which the graph has a turning point, i.e. we want to find the co-ordinates of the turning point of this graph.



One way of determining the co-ordinates of the turning point would be to draw horizontal lines through the graph representing different possible volumes, until you find the horizontal line (volume) for which the horizontal line and the graph intersect in only two places – that is, you would get two equal roots in the region of interest, and one extra root. For this volume, the horizontal line would be a tangent to the function at the turning point of the function.

Task

Solve the following equations:

$$(20 - 2x) \times (20 - 2x) \times x = 300$$

$$(20 - 2x) \times (20 - 2x) \times x = 450$$

$$(20 - 2x) \times (20 - 2x) \times x = 600$$

