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**LOGARITHMS****Learner note:**

Changing from exponential to logarithmic form in real world problems is the most important concept in this section. This concept is particularly useful in Financial Maths when you are required to solve for  $n$ .

**LESSON OVERVIEW**

1. Introduction session – 5 minutes
2. Typical exam questions:
  - Question 1: 10 minutes
  - Question 2: 5 minutes
  - Question 3: 10 minutes
3. Discussion of solutions: 30 minutes

**SECTION A: TYPICAL EXAM QUESTIONS**

**Question 1** (used with permission from Maths Grade 12 Allcopy Mind Action Series textbook) (10 minutes)  
 A colony of an endangered species originally numbering 1000 was predicted to have a population  $N$  after  $t$  years given by the equation  $N = 1000(0,9)^t$ .

- (a) Estimate the population after 1 year. (2)
- (b) Estimate the population after 2 years. (2)
- (c) After how many years will the population decrease to 200? (5)

**Question 2** (DoE Nov 2008) (5 minutes)  
 R1 570 is invested at 12% per annum. compound interest. After how many years will the investment be worth R23 000? (4)

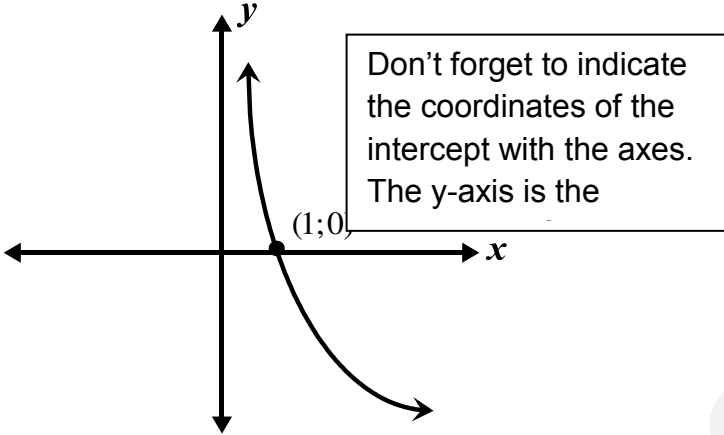
**Question 3** (Link to inverse graphs) (10 minutes)

Given:  $g(x) = \left(\frac{1}{2}\right)^x$

- (a) Write the inverse of  $g$  in the form  $g^{-1}(x) = \dots\dots$  (2)
- (b) Sketch the graph of  $g^{-1}$ . (2)
- (c) Determine graphically the values of  $x$  for which  $\log_{\frac{1}{2}} x < 0$  (1)

**SECTION B: SOLUTIONS AND HINTS**

|      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                     |
|------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------|
| 1(a) | At $t = 1$ ,<br>$N = 1000(0,9)^1$<br>$\therefore N = 900$                                                                                                                                                                                                                                                                                                                                                                                                                            | ✓ $N = 1000(0,9)^1$<br>✓ $N = 900$<br>(2)                                                                                                           |
| 1(b) | At $t = 2$ ,<br>$N = 1000(0,9)^2$<br>$\therefore N = 810$                                                                                                                                                                                                                                                                                                                                                                                                                            | ✓ $N = 1000(0,9)^2$<br>✓ $N = 810$<br>(2)                                                                                                           |
| 1(c) | Here $N = 200$<br>$200 = 1000(0,9)^t$<br>$\therefore 0,2 = (0,9)^t$<br>$\therefore \log_{0,9} 0,2 = t$<br>$\therefore t = 15,27553185$<br>$\therefore t \approx 15 \text{ years } 3 \text{ months}$                                                                                                                                                                                                                                                                                  | ✓ $200 = 1000(0,9)^t$<br>✓ $0,2 = (0,9)^t$<br>✓ $\log_{0,9} 0,2 = t$<br>✓ $t = 15,27553185$<br>✓ $t \approx 15 \text{ yrs } 3 \text{ mnths}$<br>(5) |
| 2    | $A = P(1+i)^n$<br>$\therefore 23000 = 1570(1+0,12)^n$<br>$\therefore \frac{23000}{1570} = (1,12)^n$<br>$\therefore 14,64968153..... = (1,12)^n$<br>$\therefore \log_{1,12} 14,64968153..... = n$<br>$\therefore n = 23,68701789... \text{ years}$<br>$\therefore n = 23 \text{ years } 8 \text{ months}$<br><div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">The 8 months was calculated by multiplying the decimal 0,68701789 by 12</div> | ✓ formula<br>✓ substitution<br>✓ apply log function<br>✓ answer<br>(4)                                                                              |
| 3(a) | $y = \left(\frac{1}{2}\right)^x$<br>$x = \left(\frac{1}{2}\right)^y$<br>$\therefore \log_{\frac{1}{2}} x = y$<br>$\therefore g^{-1}(x) = \log_{\frac{1}{2}} x$<br><div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">Remember that the inverse of a graph is determined by interchanging x and y in the equation of the original graph.</div>                                                                                                | ✓ $x = \left(\frac{1}{2}\right)^y$<br>✓ $g^{-1}(x) = \log_{\frac{1}{2}} x$<br>(2)                                                                   |

|      |                                                                                    |                           |
|------|------------------------------------------------------------------------------------|---------------------------|
| 3(b) |  | ✓ shape<br>✓ (1;0)<br>(2) |
| 3(c) | $\log_{\frac{1}{2}} x < 0$ for $x > 1$                                             | ✓ $x > 1$<br>(1)          |

### SECTION C: ADDITIONAL CONTENT NOTES

If a number is written in exponential form, then the exponent is called the logarithm of the number. For example, the number 64 can be written in exponential form as  $64 = 2^6$ . Clearly, the exponent in this example is 6 and the base is 2. We can then say that the logarithm of 64 to base 2 is 6. This can be written as  $\log_2 64 = 6$ .

The base 2 is written as a sub-script between the “log” and the number 8.

In general, we can rewrite  $\text{number} = \text{base}^{\text{exponent}}$  in logarithmic form as follows:

$$\text{number} = \text{base}^{\text{exponent}}$$

$$\therefore \log_{\text{base}}(\text{number}) = \text{exponent}$$

#### Example

(a) Express the following in logarithmic form:

$$(1) \quad 16 = 4^2$$

$$\therefore \log_4 16 = 2$$

$$(2) \quad x = 2^y$$

$$\therefore \log_2 x = y$$

(b) Express the following in exponential form:

$$(1) \quad \log_3 9 = 2$$

$$\therefore 9 = 3^2$$

$$(2) \quad \log_2 8 = 3$$

$$\therefore 8 = 2^3$$

Other useful laws and definitions (not really assessed but can be useful in certain real world problems)

The following apply for  $x > 0$ ,  $y > 0$ ,  $0 < a < 1$  or  $a > 1$ :

LAW 1

$$\log_a x + \log_a y = \log_a (x \times y)$$

LAW 2

$$\log_a x - \log_a y = \log_a \left( \frac{x}{y} \right)$$

LAW 3

$$\log_a x^m = m \log_a x$$



|             |                        |
|-------------|------------------------|
| DEDUCTION 1 | $\log_a a = 1$         |
| DEDUCTION 2 | $\log_a 1 = 0$         |
| DEDUCTION 3 | $\log_{10} x = \log x$ |

### SECTION D: HOMEWORK

**Question 1** (used with permission from Maths Grade 12 Mind Action Series textbook)

Archaeologists use a specific formula when determining the age of fossils. This formula is the

carbon dating formula and is given by:  $P = \left(\frac{1}{2}\right)^{\frac{n}{5700}}$  where P is the percentage of carbon-14 remaining in the fossils after  $n$  years. Calculate the approximate age of a certain fossil discovered if the percentage of carbon-14 in the fossil is 12,5%.

(4)

**Question 2** (used with permission from Maths Grade 12 Allcopy Mind Action Series textbook)

(a) Determine how many years it would take for the value of a car to depreciate to 50% of its original value, if the rate of depreciation, based on the reducing balance method, is 8% per annum.

(3)

(b) How long will it take for an amount of R50 000 to double if the interest rate is 18% per annum compounded monthly?

(6)

**Question 3**

The graph of  $f : x \rightarrow \log_a x$  passes through the point (16 ; 2).

(a) Calculate the value of  $a$ .

(3)

(b) Write down the equation of the inverse in the form  $f^{-1}(x) = \dots$ .

(2)

(c) Sketch the graphs of  $f$  and  $f^{-1}$  on the same set of axes.

(4)

### SECTION E: SOLUTIONS TO SESSION 15 HOMEWORK

No homework was given in session 15.