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CONSOLIDATIONLearner note:

This consolidation exercise is on Number patterns, Sequences and Series and Calculus graphs.

LESSON OVERVIEW

1. Exam questions: 40 minutes
2. Discussion of test: 20 minutes

SECTION A: TYPICAL EXAM QUESTIONS**QUESTION 1 (DoE Nov 2008)**

1.1 Consider the sequence: $\frac{1}{2}; 4; \frac{1}{4}; 7; \frac{1}{8}; 10; \dots$

1.1.1 If the pattern continues in the same way, write down the next TWO terms in the sequence. (1)

1.1.2 Calculate the sum of the first 50 terms of the sequence. (7)

1.2 Consider the sequence: 8 ; 18 ; 30 ; 44 ; ...

1.2.1 Write down the next TWO terms of the sequence, if the pattern continues in the same way. (2)

1.2.2 Calculate the n^{th} term of the sequence. (6)

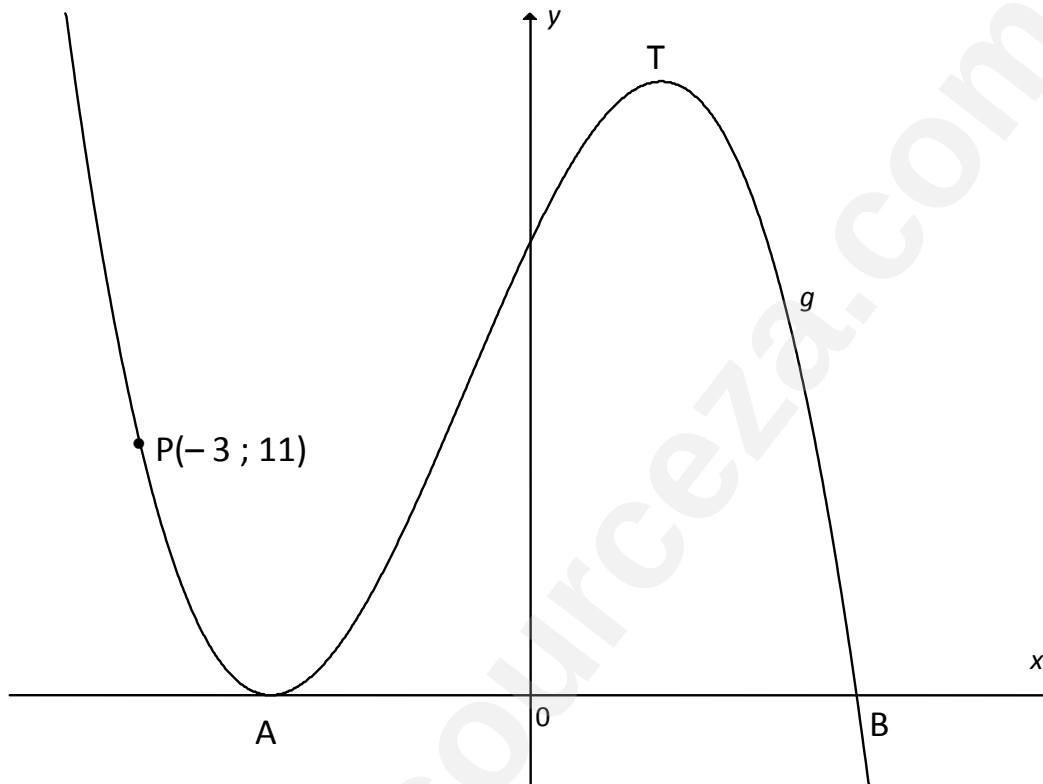
1.2.3 Which term of the sequence is 330? (4)

[20]

QUESTION 2 (DoE Nov 2008)

Sketched below is the graph of $g(x) = -2x^3 - 3x^2 + 12x + 20 = -(2x - 5)(x + 2)^2$

A and T are turning points of g . A and B are the x -intercepts of g . $P(-3 ; 11)$ is a point on the graph.

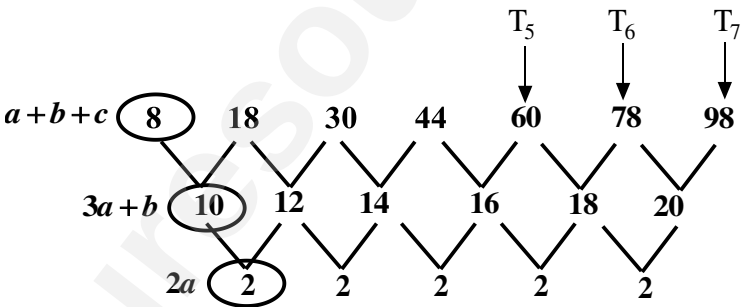


- 2.1 Determine the length of AB. (2)
- 2.2 Determine the x -coordinate of T. (4)
- 2.3 Determine the equation of the tangent to g at $P(-3 ; 11)$, in the form $y = \dots$ (5)
- 2.4 Determine the value(s) of k for which $-2x^3 - 3x^2 + 12x + 20 = k$ has three distinct roots. (3)
- 2.5 Determine the x -coordinate of the point of inflection. (4)

[18]

SECTION B: MARKING MEMORANDUM

QUESTION 1

1.1.1	$\frac{1}{16}$; 13	✓ answers (1)
1.1.2	$S_{50} = 25 \text{ terms of } 1^{\text{st}} \text{ sequence} + 25 \text{ terms of } 2^{\text{nd}} \text{ sequence}$ $S_{50} = \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \text{ to } 25 \text{ terms} \right) + (4 + 7 + 10 + 13 + \dots \text{ to } 25 \text{ terms})$ $S_{50} = \frac{\frac{1}{2} \left(\left(\frac{1}{2} \right)^{25} - 1 \right)}{\frac{1}{2} - 1} + \frac{25}{2} [2(4) + 24(3)]$ $S_{50} = 0,999999\dots + 1000$ $S_{50} = 1001,00$	✓ separating the sequences ✓✓ sum of geometric series ✓✓ sum of arithmetic series ✓✓ final answer (7)
1.2.1	60 ; 78	✓✓ answers (2)
1.2.2	 $\therefore 2a = 2$ $\therefore a = 1$ $\therefore T_n = n^2 + 7n$ $3a + b = 10$ $\therefore 3(1) + b = 10$ $\therefore b = 7$ $a + b + c = 8$ $\therefore 1 + 7 + c = 8$ $\therefore c = 0$	✓ first difference row ✓ second difference ✓ $a = 1$ ✓ $b = 7$ ✓ $c = 0$ ✓ $T_n = n^2 + 7n$ (6)

1.2.3	$330 = n^2 + 7n$ $\therefore -n^2 - 7n + 330 = 0$ $\therefore n^2 + 7n - 330 = 0$ $\therefore n = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(-330)}}{2}$ $\therefore n = \frac{-7 \pm 37}{2}$ $\therefore n = 15 \text{ or } n = -22$ (not valid) Alternatively, factorise: $(n - 15)(n + 22) = 0$	$\checkmark 330 = n^2 + 7n$ $\checkmark n^2 + 7n - 330 = 0$ $\checkmark \checkmark$ Substitution into formula or factorising $\checkmark n = 15 \text{ or } n = -22$ $\checkmark n \neq -22$ (6)
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QUESTION 2

2.1	$-(2x - 5)(x + 2) = 0$ $\therefore x = \frac{5}{2} = 2\frac{1}{2} \text{ or } x = -2$ $\therefore AB = 4\frac{1}{2} \text{ units}$	$\checkmark$ x-intercepts $\checkmark$ length of AB (2)
2.2	$g'(x) = 0$ $-6x^2 - 6x + 12 = 0$ $x^2 + x - 2 = 0$ $(x + 2)(x - 1) = 0$ $x = -2 \text{ or } x = 1$ At T: $x = 1$	$\checkmark -6x^2 - 6x + 12 = 0$ $\checkmark (x + 2)(x - 1) = 0$ $\checkmark x = -2 \text{ or } x = 1$ $\checkmark$ At T: $x = 1$ (4)
2.3	$g'(x) = -6x^2 - 6x + 12$ $\therefore g'(-3) = -6(-3)^2 - 6(-3) + 12$ $\therefore g'(-3) = -54 + 18 + 12$ $\therefore g'(-3) = -24$ $\therefore y - 11 = -24(x - (-3))$ $\therefore y - 11 = -24(x + 3)$ $\therefore y - 11 = -24x - 72$ $\therefore y = -24x - 61$	$\checkmark$ $g'(x) = -6x^2 - 6x + 12$ $\checkmark$ $g'(-3)$ $= -6(-3)^2 - 6(-3) + 12$ $\checkmark g'(-3) = -24$ $\checkmark y - 11 = -24(x + 3)$ $\checkmark y = -24x - 61$ (5)

2.4	y-coordinate of T is: $g(1) = -2(1)^3 - 3(1)^2 + 12(1) + 20$ $= 27$ $\therefore T(1; 27)$ $\therefore 0 < k < 27$ Graphical interpretation of answer: The equation $-2x^3 - 3x^2 + 12x + 20 = k$ is the intersection of the the cubic graph $y = -2x^3 - 3x^2 + 12x + 20$ and the horizontal lines $y = k$. There are three points of intersection for all horizontal lines between $y = 0$ and $y = 27$. The lines $y = 0$ and $y = 27$ intersect the cubic graph in two points only and any line below $y = 0$ or above $y = 27$ will only intersect once with the cubic graph.	$\checkmark g(1) = 27$ $\checkmark\checkmark 0 < k < 27$ (3)
2.5	$g'(x) = -6x^2 - 6x + 12$ $g''(x) = -12x - 6$ $12x + 6 = 0$ $x = -\frac{1}{2}$ <u>Alternative:</u> The x-coordinate of the point of inflection can be determined as follows: $x = \frac{x_A + x_T}{2}$ $\therefore x = \frac{-2 + 1}{2}$ $\therefore x = \frac{-1}{2}$ $\therefore x = -\frac{1}{2}$	$\checkmark$ $g'(x) = -6x^2 - 6x + 12$ $\checkmark g''(x) = -12x - 6$ $\checkmark 12x + 6 = 0$ $\checkmark x = -\frac{1}{2}$ $\checkmark\checkmark x = \frac{-2 + 1}{2}$ $\checkmark\checkmark x = -\frac{1}{2}$ (4)

SECTION E: SOLUTIONS TO SESSION 14 HOMEWORK

No homework was given as session 14 was a consolidation session.

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