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SUBJECT: PHYSICAL SCIENCES

GRADE: 12

CHAPTER / MODULE: MECHANICS

UNIT / LESSON TOPIC: VERTICAL PROJECTILE MOTION - Equations

By the end of this unit, you should be able to:

- ∞ explain what is a projectile.
- ∞ use equations of motion to determine the height reached by the projectile launched vertically up, the time taken to reach this height and its initial velocity.
- ∞ explain what is terminal velocity.

LESSON

1. Introduction

Equations of motion are equations that are used to describe the motion of a body while experiencing a force as a function of time. These equations apply only to bodies moving linearly (unilaterally) i.e. in one direction with a constant acceleration. The body's motion is considered between two time points: that is, from one initial time point and its final point in time.

2. Revision of Grade 10 and 11 equations of motion

Motion can be described in different ways.

Words: When your friend explains his first experience in driving a car and tells you in detail how he struggles to pull off and stop.

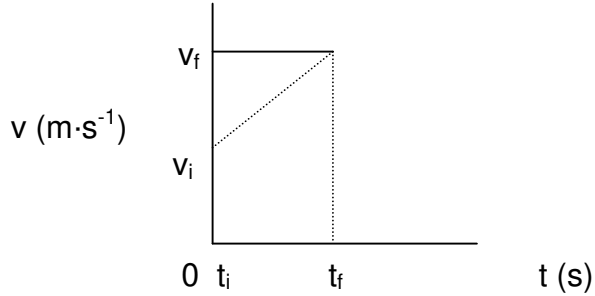
Diagrams: When you draw a sketch to explain a specific movement.

Graphs: We use three different graphs.

1. velocity – time graph
2. acceleration – time graph
3. position – time graph.

Scientists used the results of hundreds of experiments to derive these equations of motion.

Velocity – time graphs



To calculate the acceleration from a velocity time graph: Calculate the gradient of the graph.

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{v_f - v_i}{t_f - t_i} = a$$

$$\text{Therefore } v_f = v_i + a\Delta t \quad \dots\dots\dots (1)$$

To calculate the displacement from a velocity- time graph: calculate the area under the graph.

Square: $l \times b$ (side x side)

Rectangle: $l \times b$

Triangle: $\frac{1}{2} b \times h$

$$\Delta x = \frac{1}{2} b \times l + l \times b = \frac{1}{2}(v_f - v_i) \times (t_f - t_i) + (v_i - 0) \times (t_f - t_i)$$

$$\Delta x = \frac{1}{2}(v_f - v_i)\Delta t + v_i \Delta t \quad \dots\dots\dots (2)$$

From equation (1), $\Delta t = (v_f - v_i) / a$ and substitute into equation 2

$$\text{And obtain } v_f^2 = v_i^2 + 2a\Delta x$$

Then substitute equation 1 into equation 2 i.e. substitute in for v_f

$$\text{And obtain } \Delta x = v_i \Delta t + \frac{1}{2} a \Delta t^2$$

Where v_i = initial velocity in $\text{m}\cdot\text{s}^{-1}$

v_f = final velocity in $\text{m}\cdot\text{s}^{-1}$

a = acceleration in $\text{m}\cdot\text{s}^{-2}$

Δt = change in time in s

Δx = change in position in m

METHOD:

STEP 1: WRITE DOWN ALL THE INFORMATION GIVEN FROM THE QUESTION

STEP 2: IDENTIFY WHICH FORMULA TO USE I.E. IDENTIFY THE KNOWN AND UNKNOWN QUANTITIES

STEP 3: SUBSTITUTE INTO THE EQUATION

STEP 4: INTERPRET THE ANSWER

EXAMPLE 1:

A car is travelling at $5 \text{ m}\cdot\text{s}^{-1}$ and it starts to accelerate at $2 \text{ m}\cdot\text{s}^{-2}$ for 3 s. What distance will the car cover in the 3 s?

ANSWER:

STEP 1: $v_i = 5 \text{ m}\cdot\text{s}^{-1}$
 $a = 2 \text{ m}\cdot\text{s}^{-2}$
 $t = 3 \text{ s}$
 $x = ?$

STEP 2: $\Delta x = v_i \Delta t + \frac{1}{2} a \Delta t^2$

STEP 3: $\Delta x = (5)(3) + \frac{1}{2} (2)(3)^2$
 $= 135 \text{ m}$

STEP 4: The car will travel 135 m in the direction of the motion.

3. Introduction to the use of equations of motion in the vertical direction

A projectile is an object that is given an initial velocity by shooting or throwing etc and once launched, the only force acting on it is the force due to gravity. In the absence of air resistance, the object is free falling.

Terminal velocity is reached when the downward force of gravity and the upward force of air resistance are equal, and now the object falls at a constant velocity as a result of there being no resultant force acting in on the object.

The Equations of Motion for Vertical Projectile Motion

LINEAR MOTION

$$v_f = v_i + a\Delta t$$

$$\Delta x = v_i \Delta t + \frac{1}{2} a \Delta t^2$$

$$v_f^2 = v_i^2 + 2 a \Delta x$$

VERTICAL PROJECTILE MOTION

$$v_f = v_i + g\Delta t$$

$$\Delta y = v_i \Delta t + \frac{1}{2} g \Delta t^2$$

$$v_f^2 = v_i^2 + 2g\Delta y$$

Substitute 'g' for 'a' and 'Δy' for 'Δx' in vertical projectile motion problems

All objects are attracted to the earth with a gravitational force called **WEIGHT**.

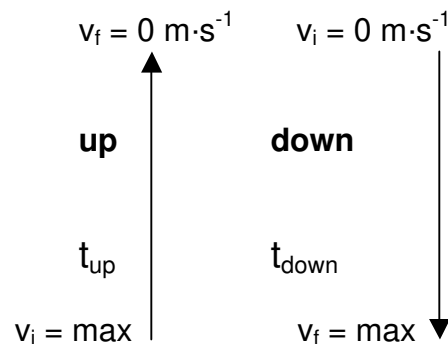
All objects will accelerate towards the earth with a constant acceleration called the gravitational acceleration (g) which has a value of $9,8 \text{ m}\cdot\text{s}^{-2}$.

$g = 9,8 \text{ m}\cdot\text{s}^{-2}$ which is found on the information sheet.

Gravitational acceleration (g) is **ALWAYS** downwards no matter whether the object is being thrown up or falling down.

Important facts concerning Vertical Projectile Motion:

- At the greatest height of the upward motion, $v_f = 0 \text{ m}\cdot\text{s}^{-1}$
- The object will take the same time to reach its greatest height from point of upwards launch as the time taken to fall back to point of launch
($t_{\text{up}} = t_{\text{down}}$)



- Can have motion described by a single set of equations of motion for the upward and downward motion
- If the object is being released from rest or being dropped, its initial velocity is $0 \text{ m}\cdot\text{s}^{-1}$.
- If the object is being thrown upwards, it must start with a maximum velocity and as it moves up, the velocity decreases until it stops.
- These are vectors thus direction is important.

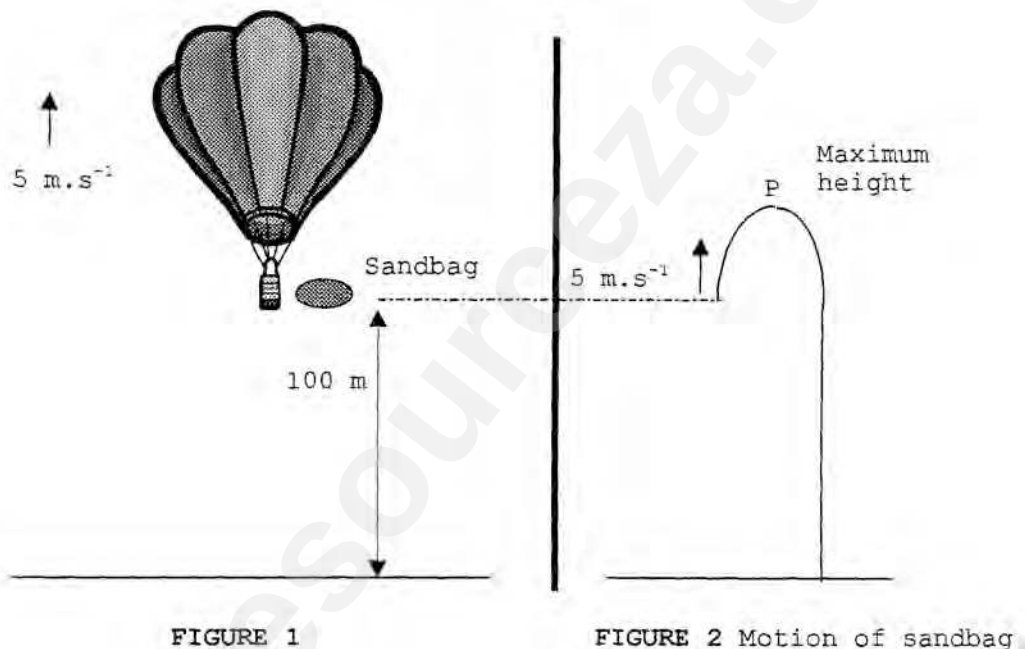
Example 2

A soccer ball is kicked vertically upwards with a velocity of $10 \text{ m}\cdot\text{s}^{-1}$.

- 2.1 Calculate the maximum height reached by the ball.
- 2.2 Calculate the time taken before the soccer ball returns to its original position from where it was kicked.
- 2.3 At what height will the velocity of the soccer ball be $5 \text{ m}\cdot\text{s}^{-1}$?

EXERCISE

A hot-air balloon is rising upwards at a constant velocity of $5 \text{ m}\cdot\text{s}^{-1}$. When the balloon is 100 m above the ground, a sandbag is dropped from it (see FIGURE 1). FIGURE 2 shows the path of the sandbag as it falls to the ground. Ignore air resistance.



- 1.1 What is the acceleration of:
 - 1.1.1 The hot-air balloon while the sandbag is in it? (1)
 - 1.1.2 The sandbag the moment it is dropped from the hot-air balloon? (2)
- 1.2 Determine the maximum height P, **above the ground**, reached by the sandbag after it is released from the hot-air balloon. (3)
- 1.3 Calculate the time taken for the sandbag to reach this maximum height after it has been released. (3)

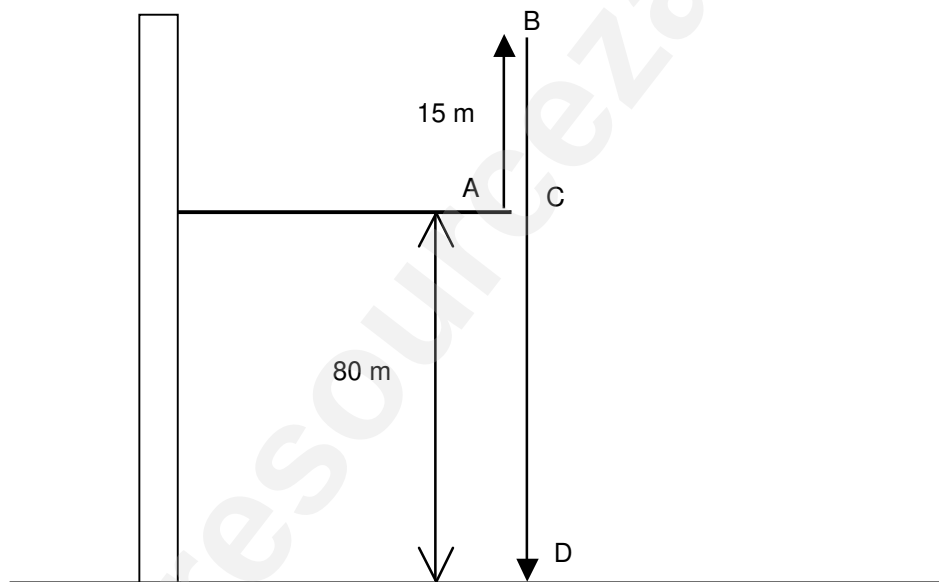
- 1.4 Calculate the total time taken for the sandbag to reach the ground after it has been released. (4)
- 1.5 Will the velocity of the hot-air balloon **INCREASE, DECREASE** or **REMAIN THE SAME** immediately after the sandbag has been released? Explain fully. (4)

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HOMEWORK

QUESTION 1

A cross-bow (bow and arrow) is used to shoot an arrow vertically upwards into the air from the top of a 80 m high platform. The arrow reaches a height of 15 m above the platform and then falls to the ground below. Ignore the effects of air friction.



- 1.1 Calculate the magnitude of the velocity of the arrow at the instant it is shot up into the air from the top of the platform. (4)
- 1.2 Calculate the time it takes for the arrow to reach the ground from the moment it is shot upwards. (4)

[8]

QUESTION 2

The following extract comes from an article in a school newspaper.

THE LAWS OF PHYSICS ARE ACCURATE!

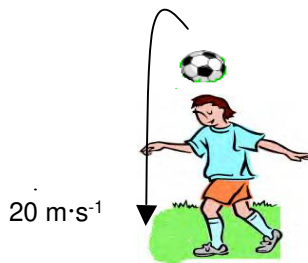
Two construction workers, Alex and Pete, were arguing about whether a smaller brick would hit the ground quicker than a larger brick when both are released from the same height.

Alex said that the larger brick should hit the ground first. Pete argued that the smaller brick would hit the ground first.

- 2.1 Are their statements correct? Give a reason for your answer. (3)
- 2.2 A group of Physical Sciences learners decide to test Alex's and Pete's hypotheses. They drop two bricks, one small and the other much larger, from one of the floors of the school building.
- 2.2.1 Write down TWO precautions they should take to ensure that the result is reliable. (2)
- 2.2.2 Give a reason why, despite all the necessary precautions, they might not get the correct result. (1)
- 2.3 In another experiment, the learners **drop** a brick A from a height of 8 m. After 0,6 s, they **throw** a second brick B downwards from the same height. Both bricks, A and B, hit the ground at the same time.
- Ignore the effects of friction and calculate the speed at which brick B was thrown. (7)
- [13]

QUESTION 3

A soccer player juggles a ball on his head by letting the ball bounce continuously on his head. After the last bounce the ball leaves his head and it takes 2,4 s to reach the ground. The ball lands on the ground at $20 \text{ m}\cdot\text{s}^{-1}$.



- 3.1 Determine the velocity with which the ball leaves the boy's head. (6)
- 3.2 Calculate the maximum height, above the boy's head reached by the ball. (3)
- [9]**

TOTAL: 30